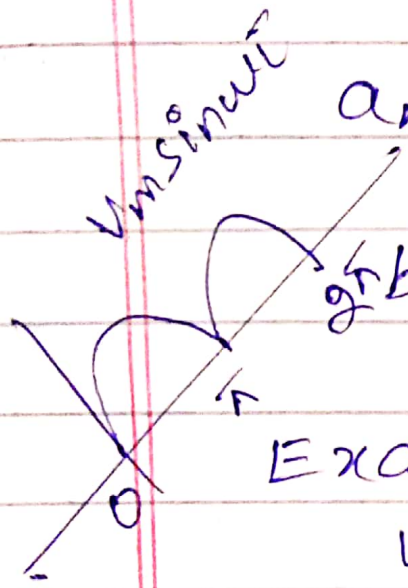
Power Electronics

Fourier series

$I_{dc} = \frac{a_0}{2}$ $\quad a_0 = \frac{2}{T}\int_0^T v_o(t)\,dt$

$a_n = \frac{2}{T}\int_0^T v_o(t)\cos n\omega t\,dt$

$b_n = \frac{2}{T}\int_0^T v_o(t)\sin n\omega t\,dt$



Example 3.2

$$v_o(t) = V_{dc} + \sum_{n=1,2,3}^{\infty} (a_n\cos n\omega t + b_n\sin n\omega t)$$

$$V_{dc} = \frac{1}{T}\int_0^T V_m\sin\omega t\,d(\omega t)$$

$$= \frac{2}{T}\int_0^{\pi} V_m\sin\omega t\,d(\omega t)$$

$$= \frac{1}{\pi}\int_0^{\pi} V_m\sin\omega t\,d(\omega t)$$

$$= \frac{V_m}{\pi}\Big[-\cos\omega t\Big]_0^{\pi}$$

$$= \frac{V_m}{\pi}\left[-\cos\pi + \cos 0\right]$$

$$\boxed{V_{dc} = \frac{2V_m}{\pi}}$$

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$$a_n = \frac{2}{T}\int_0^T V_o(t)\cos n\omega t \, d\omega t,$$

$$a_n = \frac{2\times 2}{2\pi}\int_0^{T/2} V_m \sin\omega t \cos n(\omega t) \, d\omega t,$$

$$a_n = \frac{V_m}{\pi}\int_0^{\pi} 2\sin\omega t \cos n\omega t \, d\omega t,$$

$$= \frac{V_m}{\pi}\int_0^{\pi} (\sin(1+n)\omega t + \sin(1-n)\omega t) \, d\omega t$$

$$= \frac{V_m}{\pi}\left[\left|\frac{-\cos(1+n)\omega t}{1+n}\right|_0^{\pi} + \left|\frac{-\cos(1-n)\omega t}{1-n}\right|_0^{\pi}\right]$$

$$= \frac{V_m}{\pi}\left[\frac{1}{1+n}\left[-\cos(1+n)\pi + 1\right] + \frac{1}{1-n}\left[-\cos(1-n)\pi + \cos 0\right]\right]$$

$$= \frac{V_m}{\pi}\left[\frac{1}{n+1}\left[1 - \cos(1+n)\pi\right] + \frac{1}{1-n}\left[1 - \cos(1-n)\pi\right]\right)$$

$$= \frac{V_m}{\pi}\left[\frac{(1-n)\left[1-\cos(1+n)\pi\right] + (n+1)\left[1-\cos(1-n)\pi\right]}{(n+1)(1-n)}\right]$$

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$$= \frac{V_m}{\pi}\left[\frac{1 - \cos(n+1)\pi - \cancel{n} + n\cos(n+1)\pi + \cancel{n} - n\cos(1-n)\pi + 1 - \cos(1-n)\pi}{(1+n)(1-n)}\right]$$

$$= \frac{V_m}{\pi}\left[\frac{2 - \cos(n+1)\pi + n\cos(n+1)\pi - n\cos(1-n)\pi - \cos(1-n)\pi}{(1+n)(1-n)}\right]$$

Put $n = 1$

$$= \frac{V_m}{\pi}\left[\frac{2 - \cos(2\pi) + \cos(2\pi) - \cos(0) - \cos(0)}{(1+n)(1-n)}\right]$$

$$= \frac{V_m}{\pi}\left[0\right] = 0$$

Put $n = 3$

$1+3$
$4-3$
$= 0$

$$= \frac{V_m}{\pi}\left[\frac{2 - \cos(4\pi) + 3\cos(4\pi) - 3\cos(2\pi) - \cos(2\pi)}{(1+n)(1-n)}\right]$$

$$= 0$$

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$2-(-1)+2(-1)$
$-2(-1)-(-1)$
$=3-2+2+1$
$=4$

For $n = 1, 3, 5$

$a_n = 0$

For ~~$a_n = ?$~~ $n = 2$

$$= \frac{V_m}{\pi(1+n)(1-n)} \left[\frac{2 - \cos(3\pi) + 2\cos(3\pi) - 2\cos(\pi) - \cos(\pi)}{} \right]$$

$$= \frac{4V_m}{\pi(1+n)(1-n)}$$

$$= \frac{-4V_m}{\pi(n+)(n-1)}$$

For $n = 2, 4, 6$

$$a_n = \frac{-4V_m}{\pi(n+1)(n-1)}$$

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when n = Even

$$a_n = -\frac{4V_m}{\pi(n-1)(n+1)}$$

when n = odd

a_n = zero

$$b_n = \frac{2}{T}\int_0^T V_m \sin\omega t \sin n\omega t \, d\omega t$$

$$= \frac{-V_m}{\pi}\int_0^{\pi} -2\sin\omega t \sin n\omega t \, d\omega t$$

$$= \frac{-V_m}{\pi}\left[\int_0^{\pi} \cos(n+1)\omega t \, d\omega t - \int_0^{\pi} \cos(1-n)\omega t \, d\omega t\right]$$

$$= \frac{-V_m}{\pi}\left[\left[\frac{\sin(n+1)\omega t}{n+1}\right]_0^{\pi} - \left[\frac{\sin(1-n)\omega t}{1-n}\right]_0^{\pi}\right]$$

$= 0$ For Any n

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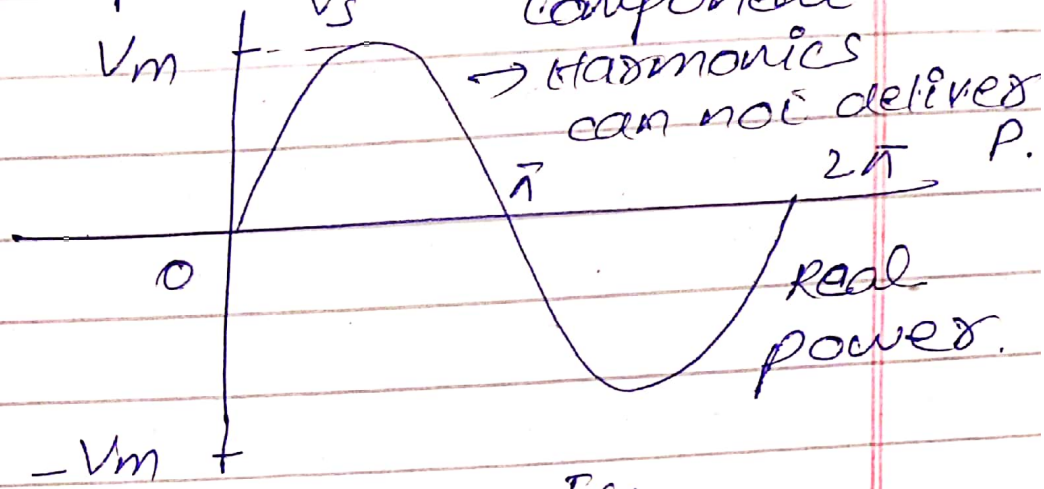
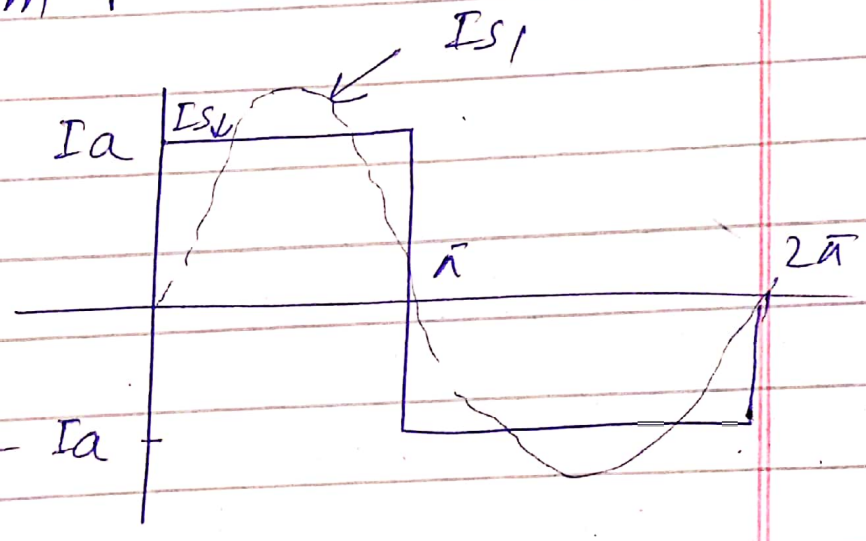
Real Power is only delivered by Fundamental component.

$$PF = \frac{V_s I_{s_1} \cos\phi}{V_s I_s}$$

Example 3.4 :





Let us apply Fourier Series

$$i_s(t) = I_{dc} + \sum_{n=1,2,3}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$I_{dc} = \frac{1}{T}\int_0^T i_s(t)\, d(\omega t)$$

$$= \frac{1}{2\pi}\int_0^{2\pi} I_a\, d(\omega t)$$

$$= 0$$

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$$a_n = \frac{2}{T}\int_0^T i_s(t)\cos n\omega t \, d\omega t$$

$$= \frac{1}{\pi}\int_0^{2\pi} I_a \cos n\omega t \, d\omega t$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} I_a \cos n\omega t \, d\omega t - \int_{\pi}^{2\pi} I_a \cos n\omega t \, d\omega t\right]$$

$$= \frac{1}{\pi}\left[I_a \left|\frac{\sin n\omega t}{n}\right|_0^{\pi} - I_a \left|\frac{\sin n\omega t}{n}\right|_{\pi}^{2\pi}\right]$$

$$= 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} I_a \sin n\omega t \, d\omega t$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} I_a \sin n\omega t \, d\omega t - \int_{\pi}^{2\pi} I_a \sin n\omega t \, d\omega t\right]$$

$$= \frac{1}{\pi}\left[I_a \left|\frac{-\cos n\omega t}{n}\right|_0^{\pi} + I_a \left|\frac{\cos n\omega t}{n}\right|_{\pi}^{2\pi}\right]$$

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$$= \frac{1}{\pi}\left[\frac{I_a}{n}\left(-\cos\pi n + \cos 0\right) + \frac{I_a}{n}\left(\cos 2\pi n - \cos \pi n\right)\right]$$

For n = even

$$b_n = 0$$

For n = odd

$$b_n = \frac{1}{\pi}\left[\frac{4I_a}{n}\right]$$

$$= \frac{4I_a}{n\pi}$$

$$i_s(t) = \frac{4I_a}{\pi}\left(\sin\omega t + \frac{\sin 3\omega t}{3} + \frac{\sin 5\omega t}{5} + \cdots\right)$$

$$I_{s_1(peak)} = \frac{4I_a}{\pi} \rightarrow \text{peak value}$$

RMS value is

$$I_{s_1} = \frac{4I_a}{\pi\sqrt{2}} = 0.90 I_a$$

As Input current is square wave

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RMS value of Input current.

$$I_s = I_a$$

$$HF = THD = \left(\left(\frac{I_s}{I_{s1}}\right)^2 - 1\right)^{1/2}$$

$$= \left(\left(\frac{1}{0.90}\right)^2 - 1\right)^{1/2}$$

$$= 0.4843$$

$$= 48.43\%$$

(b) PF = ?

$$= \frac{I_{s1}}{I_s} \cos\phi \quad (\because \phi = 0)$$

$$= \frac{0.90}{1} = 0.9 \text{ (lagging)}$$

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Example NO 10.1

$$i_s(t) = a_0 + \sum_{1,2}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$a_0 = \frac{1}{2\pi} \int_{\alpha}^{2\pi+\alpha} I_a \, d\omega$$

$$= 0$$

$$a_n = \frac{1}{\pi} \int_{\alpha}^{2\pi+\alpha} I_a \cos n\omega t \, d\omega t$$

$$= \frac{1}{\pi} \left[\int_{\alpha}^{\pi+\alpha} I_a \cos n\omega t \, d\omega t + \int_{\pi+\alpha}^{2\pi+\alpha} -I_a \cos n\omega t \, d\omega t \right]$$

$$= \frac{1}{\pi} \left[\frac{I_a}{n} \left| \sin n\omega t \right|_{\alpha}^{\pi+\alpha} - \frac{I_a}{n} \left| \sin n\omega t \right|_{\pi+\alpha}^{2\pi+\alpha} \right]$$

$$= \frac{1}{\pi} \left[\frac{I_a}{n} \left(\sin(n\alpha + n\pi) - \sin n\alpha \right) - \frac{I_a}{n} \left(\sin(2\pi n + \alpha n) - \sin(\pi n + \alpha n) \right) \right]$$

For n = even $a_n = 0$

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For odd

$$= \frac{1}{\pi}\left[\frac{Ia}{n}\left(\sin(\alpha+\pi) - \sin\alpha\right)\right.$$

$$\left. - \frac{Ia}{n}\left(\sin(2\pi+\alpha) - \sin(\alpha+\pi)\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{Ia}{n}\left(-\sin\alpha - \sin\alpha\right) - \frac{Ia}{n}\left(\sin\alpha + \sin\alpha\right)\right]$$

$$= \frac{1}{\pi}\left[-\frac{4Ia\sin\alpha}{n}\right] = -\frac{4Ia\sin\alpha}{n\pi}$$

For n = odd

Fourier series can be written as

$$i_s(t) = a_0 + \sum_{n=1,3,5}^{\infty} c_n \sin(n\omega t + \phi_n)$$

$$c_n = (a_n^2 + b_n^2)^{1/2}$$

$$\phi_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$$

$$c_n = \frac{4Ia}{n\pi}$$

Rms value of nth harmonic

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$$C_n = \frac{4 I a}{\sqrt{2}\, n \pi}$$

$$C_n = \frac{2\sqrt{2}\, I a}{n \pi}$$

$$\phi_n = \tan^{-1}\left(-\frac{\frac{4Ia}{n\pi}\sin n\alpha}{\frac{4Ia}{n\pi}\cos n\alpha} \right)$$

$$= \tan^{-1}\left(-\tan n\alpha \right)$$

$$\phi_n = -n\alpha$$

$$I_{s1} = \frac{2\sqrt{2}\, I a}{\pi}$$

$$I_s = I a$$

$$HF = \left(\left(\frac{I_s}{I_{s1}}\right)^2 - 1 \right]^{1/2}$$

$$= \left[\left(\frac{I a}{\frac{8 I a}{\pi}} \right)^2 - 1 \right)^{1/2}$$

$$= \left[\left(\frac{\pi^2}{8^2} \right) - 1 \right)^{1/2}$$

$$= 0.483 \text{ OR } 48.3\%$$

$$DF = \cos\phi_1 = \cos(-\alpha)$$

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Example NO 10.4

$$V_{dc} = \frac{3\sqrt{3}\,V_m}{\pi}\cos\alpha$$

$$0.5\,V_m = \frac{3\sqrt{3}\,V_m}{\pi}\cos\alpha$$

$$\frac{0.5\pi}{3\sqrt{3}} = \cos\alpha$$

$$\alpha = \cos^{-1}\left(\frac{0.5\pi}{3\sqrt{3}}\right)$$

$$V_s = 208/\sqrt{3} = 120.1\,V$$

$V_m \rightarrow$ peak value of phase voltage

$$V_m = \sqrt{2} \times V_s$$

$$= \sqrt{2} \times 120.1$$

$$= 169.83\,V$$

$$0.5 = \frac{V_{dc}}{V_{dm}} = V_m$$

$$V_{dc} = 0.5 \times V_{dm}$$

$$V_{dm} = \frac{3\sqrt{3}\,V_m}{\pi}$$

$$= 280.89\,V$$

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$$V_{dc} = 0.5 \times 280.89$$
$$= 140.45V$$

(a)

$$\cos\alpha = 0.5$$
$$\alpha = (\pi/3) = 60^\circ$$

(b)

$$I_{dc} = \frac{V_{dc}}{R}$$
$$= \frac{140.45}{10} = \boxed{14.05A}$$

$$I_{rms} = \frac{V_{rms}}{R}$$

$$V_{rms} = \sqrt{3}\, V_m \left(\frac{1}{2} + \frac{3\sqrt{3}}{4\pi} \cos(2\alpha)\right)^{1/2}$$

$$= \sqrt{3} \times 169.83 \left(\frac{1}{2} + \frac{3\sqrt{3}}{4\pi} \cos\left(2 \times \frac{\pi}{3}\right)\right)^{1/3}$$

$$V_{rms} = 159.29V$$

$$I_{rms} = \frac{V_{rms}}{R} = \boxed{15.92A}$$

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(c) Out of 6 pulses, Thyristor conduct 2 pulses.

$$I_A = \frac{I_{dc}}{3} = \frac{14.05}{3}$$

$$= 4.68A$$

$$I_R = I_{rms}\sqrt{2/6}$$

$$= 9.2A$$

(d) $$\eta = \frac{I_{dc}V_{dc}}{I_{rms}V_{rms}}$$

(e) $$TUF = \frac{P_{dc}}{VI}$$

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