Power Electronics

EE312

ODL WEEK 3 LECTURE

- Contents for this lecture from the course plan for ODL

ODL Week 3
Multilevel Inverters:

Concept of multi-level inverters – Cascaded multi-level inverters

- Topics actually covered in this lecture presentation and division per Question Answer session

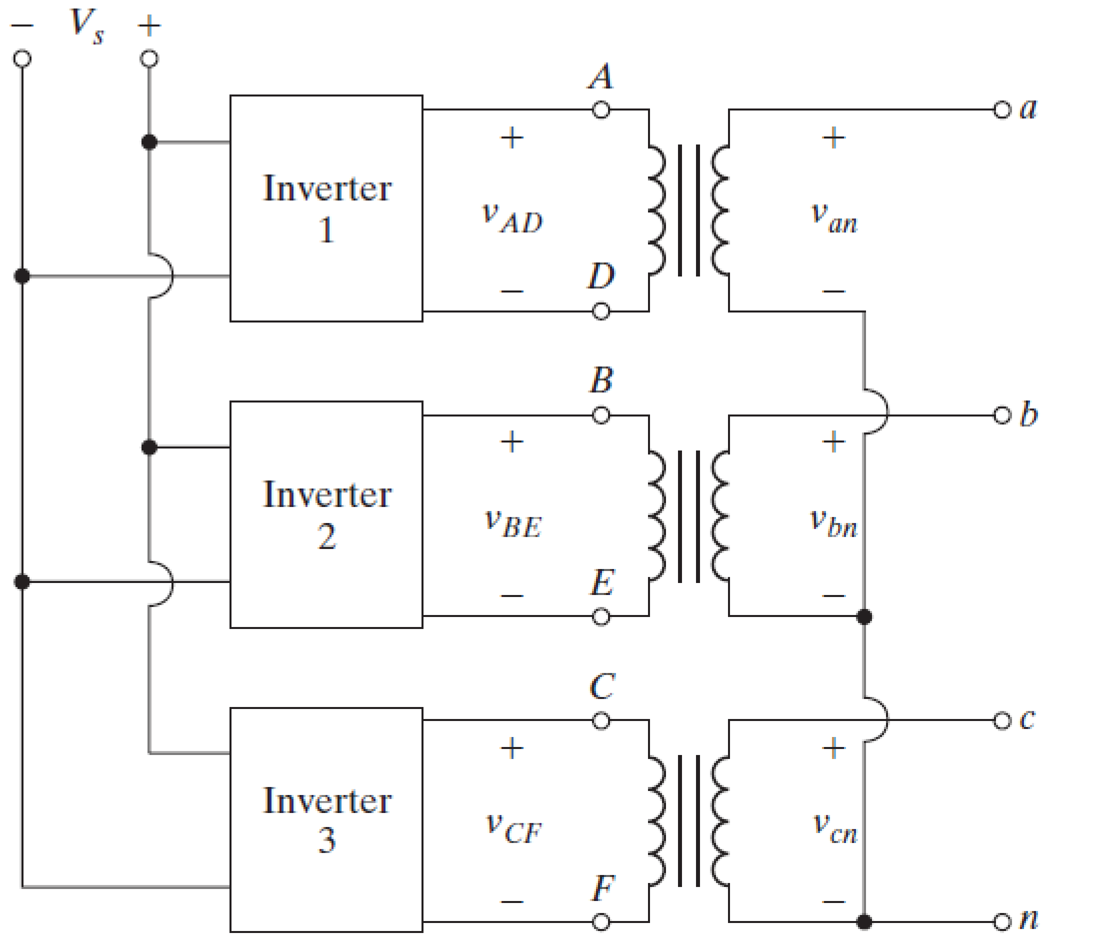
Session No.	**Content of the Lecture presentation** **which students should have covered and can discuss in the Q/A session**
1	Three phase inverters – Bridge circuit and sinusoidal PWM
2	Multi-level inverters – Inverters with independent DC sources
3	Equalizing source power with pattern swapping. Diode Clamped multi-level inverter

- **Note:**

The materials in this presentation are only for the use of students enrolled in this course in the specific campus; these materials are for purposes associated with this course and may not be further disseminated or retained after expiry of the course.

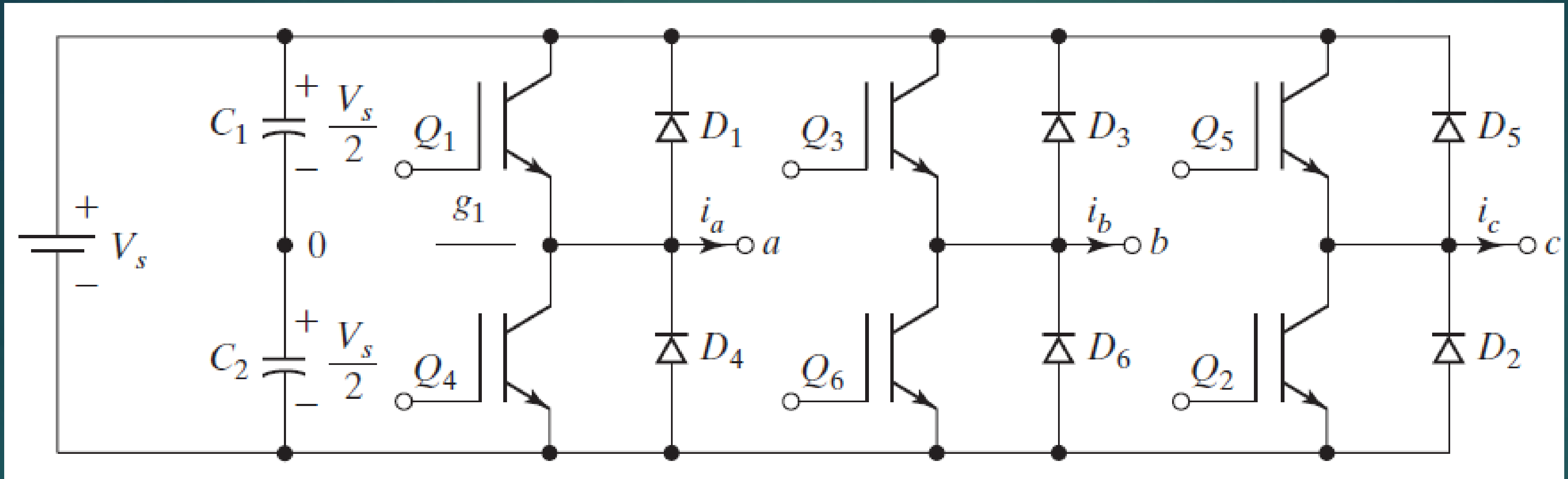

بسم الله الرحمن الرحيم

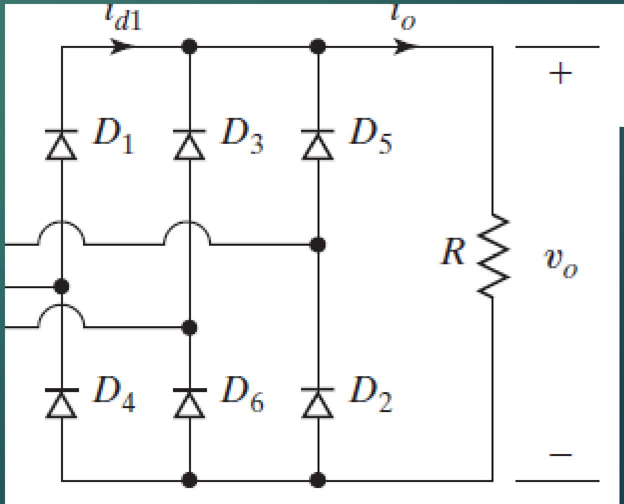
THREE-PHASE INVERTERS



(a) Schematic

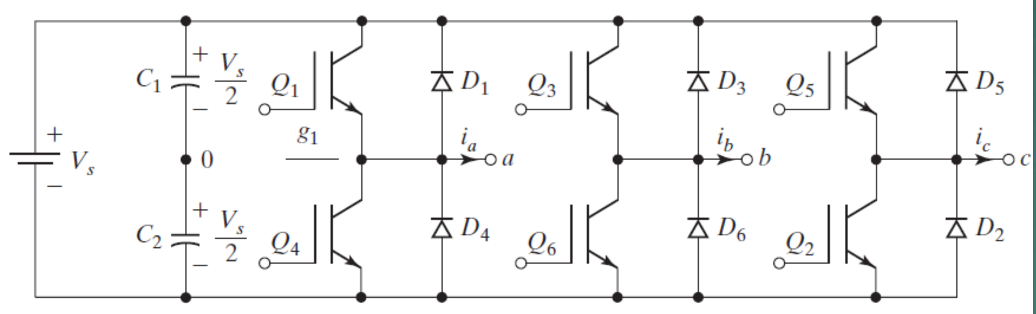
Three-phase bridge inverter

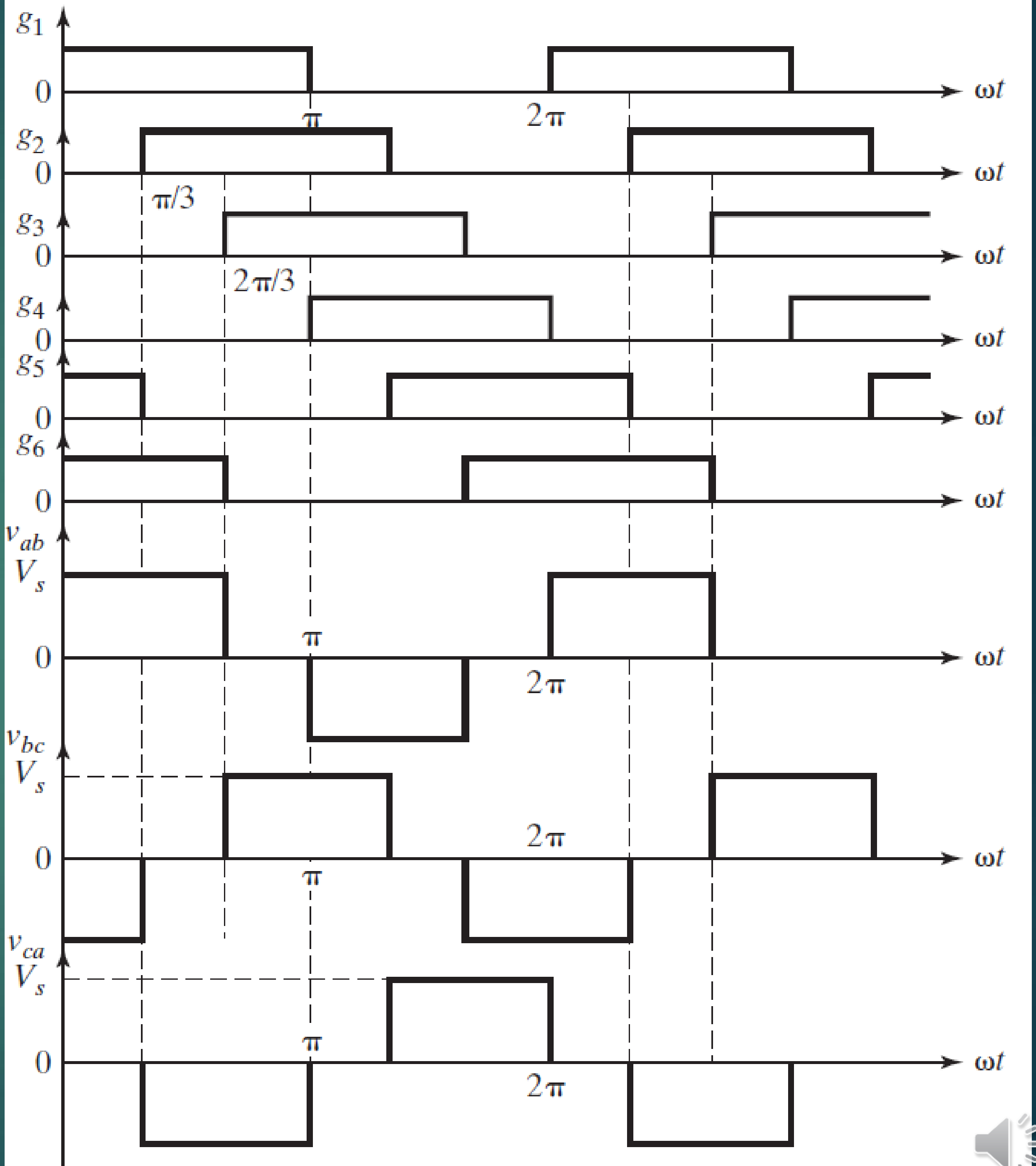




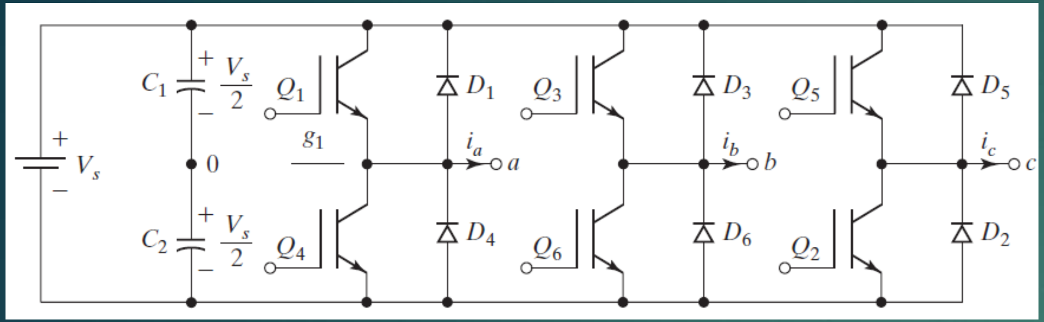
Three phase bridge Rectifier

180-Degree Conduction



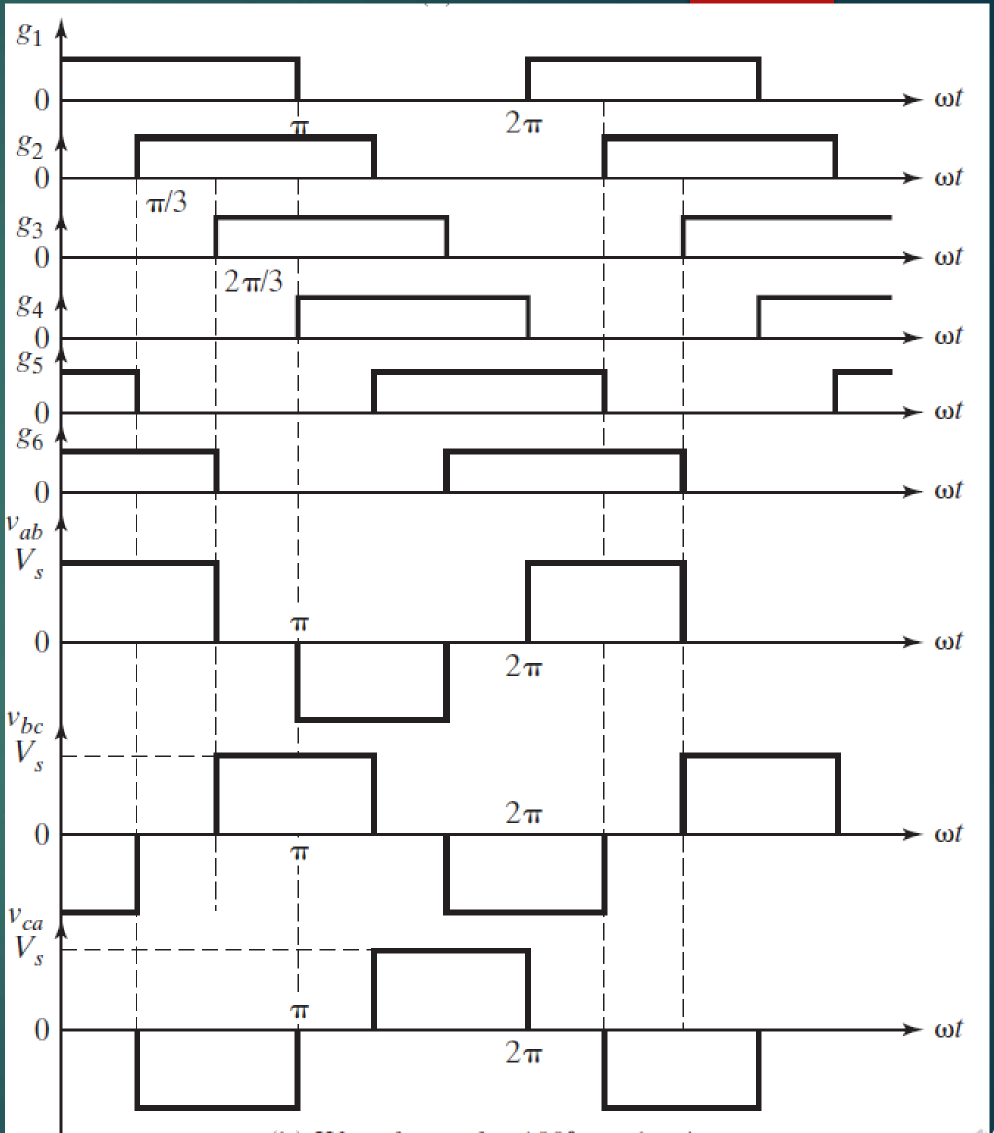


(b) Waveforms for 180° conduction

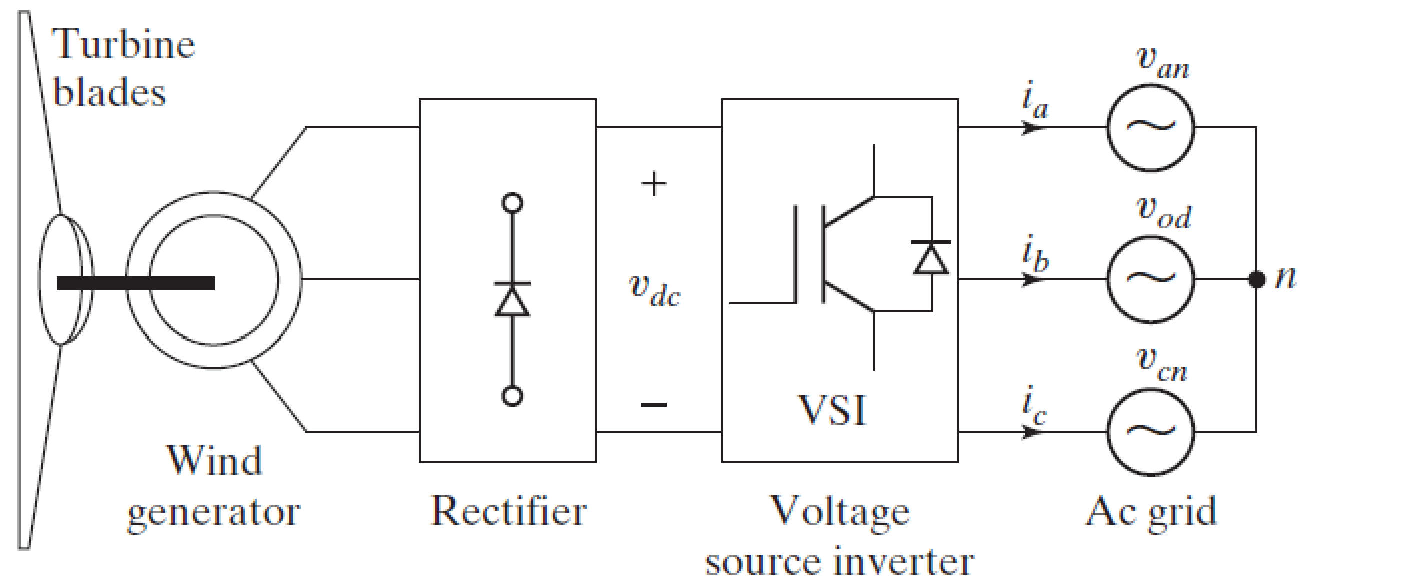


Switch States for Three-Phase Voltage-Source Inverter

State	State No.	Switch States	v_{ab}	v_{bc}	v_{ca}
S_1, S_2, and S_6 are on and S_4, S_5, and S_3 are off	1	100	V_S	0	$-V_S$
S_2, S_3, and S_1 are on and S_5, S_6, and S_4 are off	2	110	0	V_S	$-V_S$
S_3, S_4, and S_2 are on and S_6, S_1, and S_5 are off	3	010	$-V_S$	V_S	0
S_4, S_5, and S_3 are on and S_1, S_2, and S_6 are off	4	011	$-V_S$	0	V_S
S_5, S_6, and S_4 are on and S_2, S_3, and S_1 are off	5	001	0	$-V_S$	V_S
S_6, S_1, and S_5 are on and S_3, S_4, and S_2 are off	6	101	V_S	$-V_S$	0
S_1, S_3, and S_5 are on and S_4, S_6, and S_2 are off	7	111	0	0	0
S_4, S_6, and S_2 are on and S_1, S_3, and S_5 are off	8	000	0	0	0

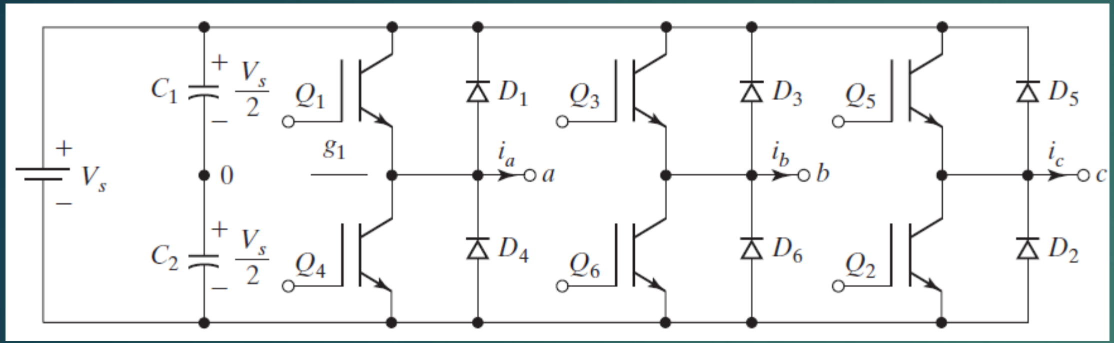


(b) Waveforms for 180° conduction



(c) Wind generator connected to the ac grid through a rectifier and an inverter

Sinusoidal PWM for 3 Phase bridge inverter



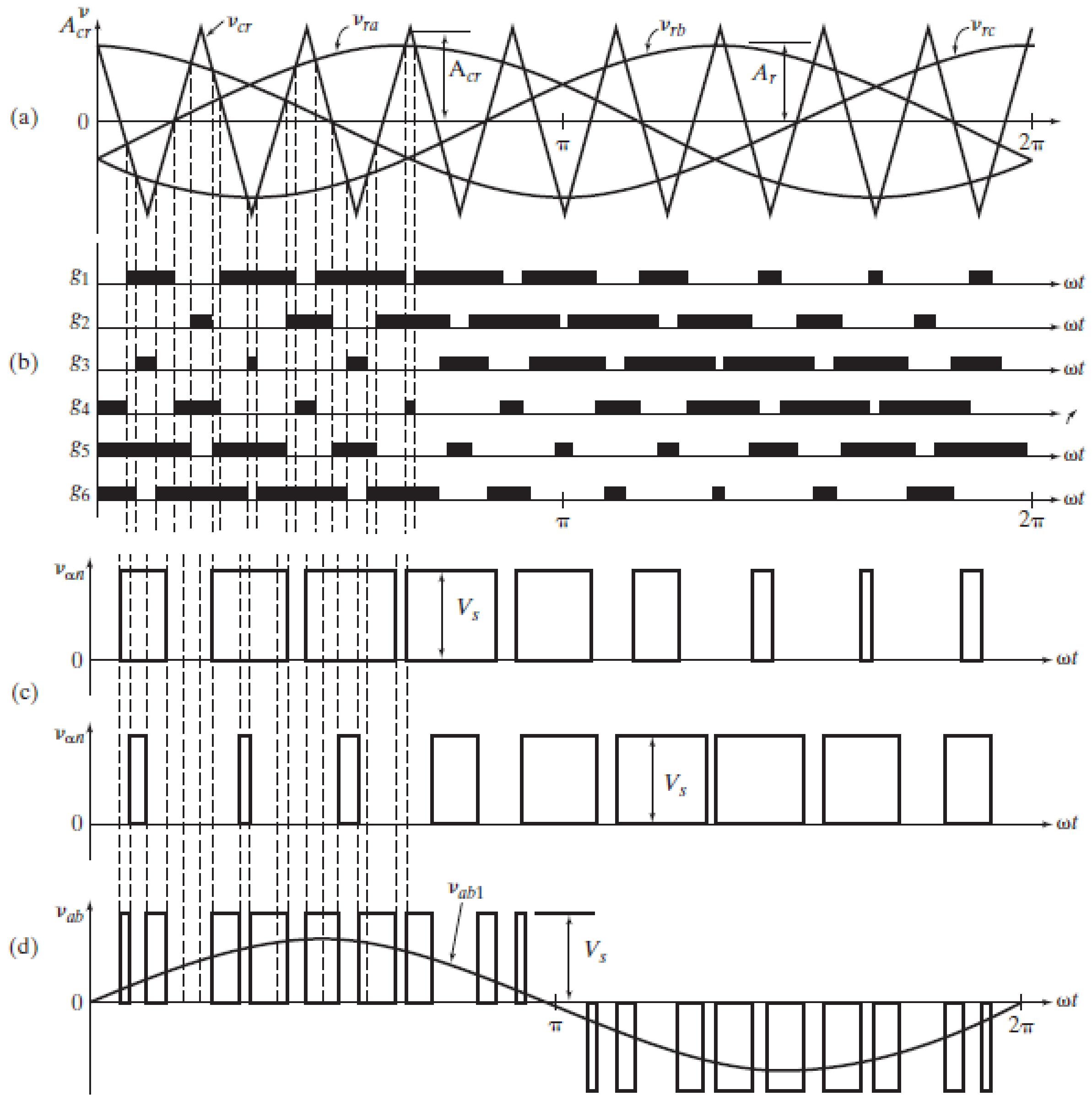


FIGURE 6.20
Sinusoidal pulse-width modulation for three-phase inverter.

For comparison with what you have studied in single phase PWM

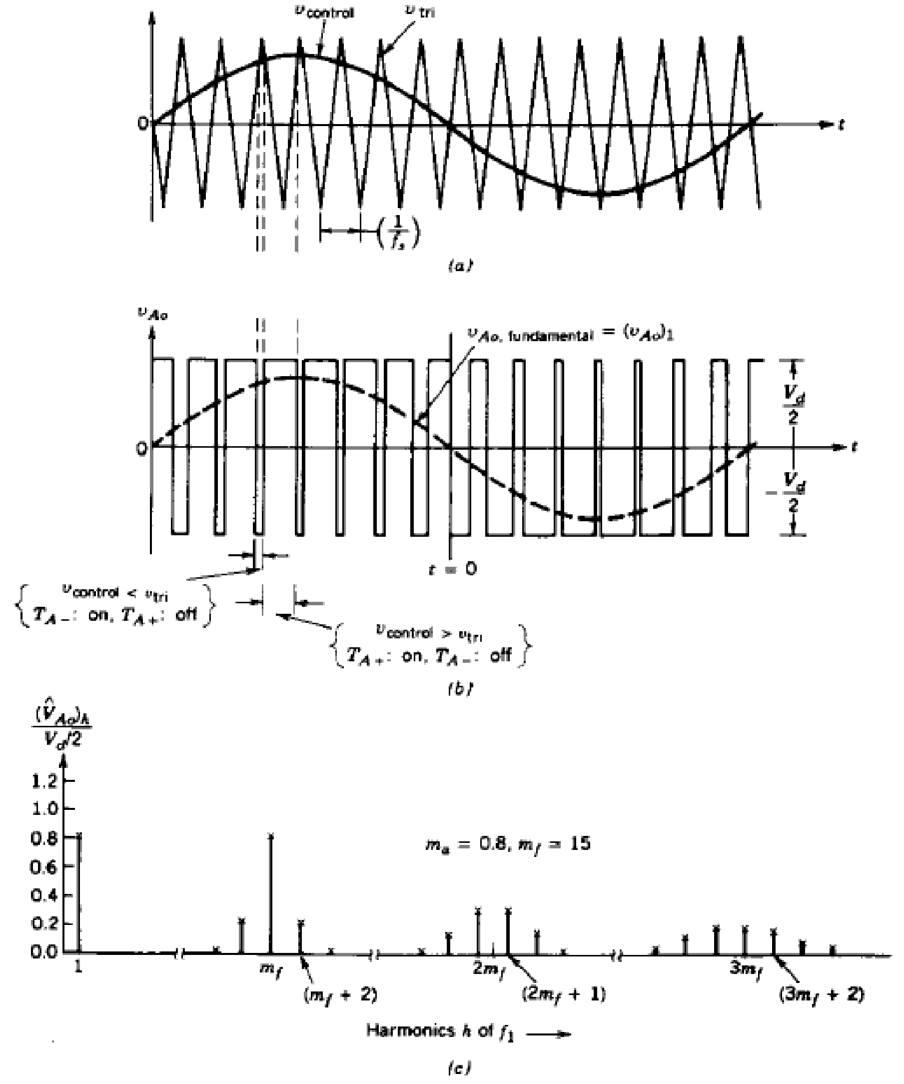
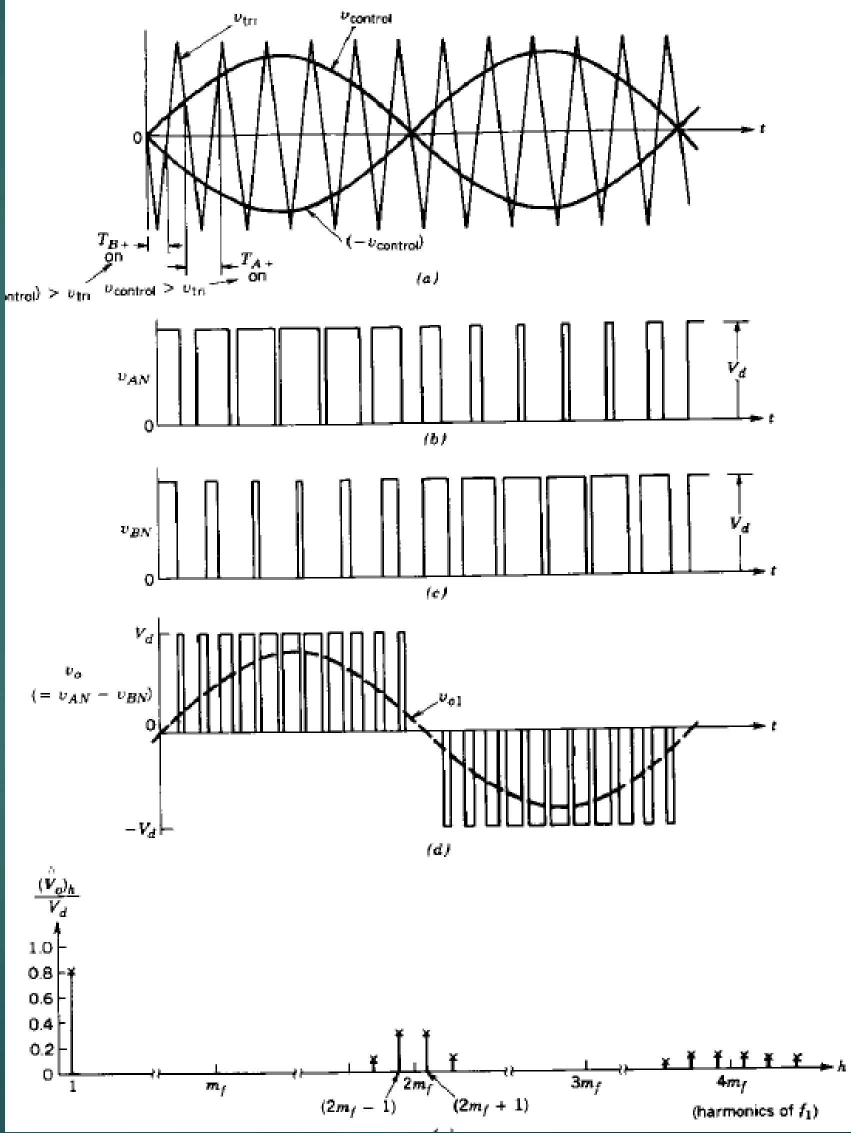


Figure 8-5 Pulse-width modulation.



3 phase SPWM

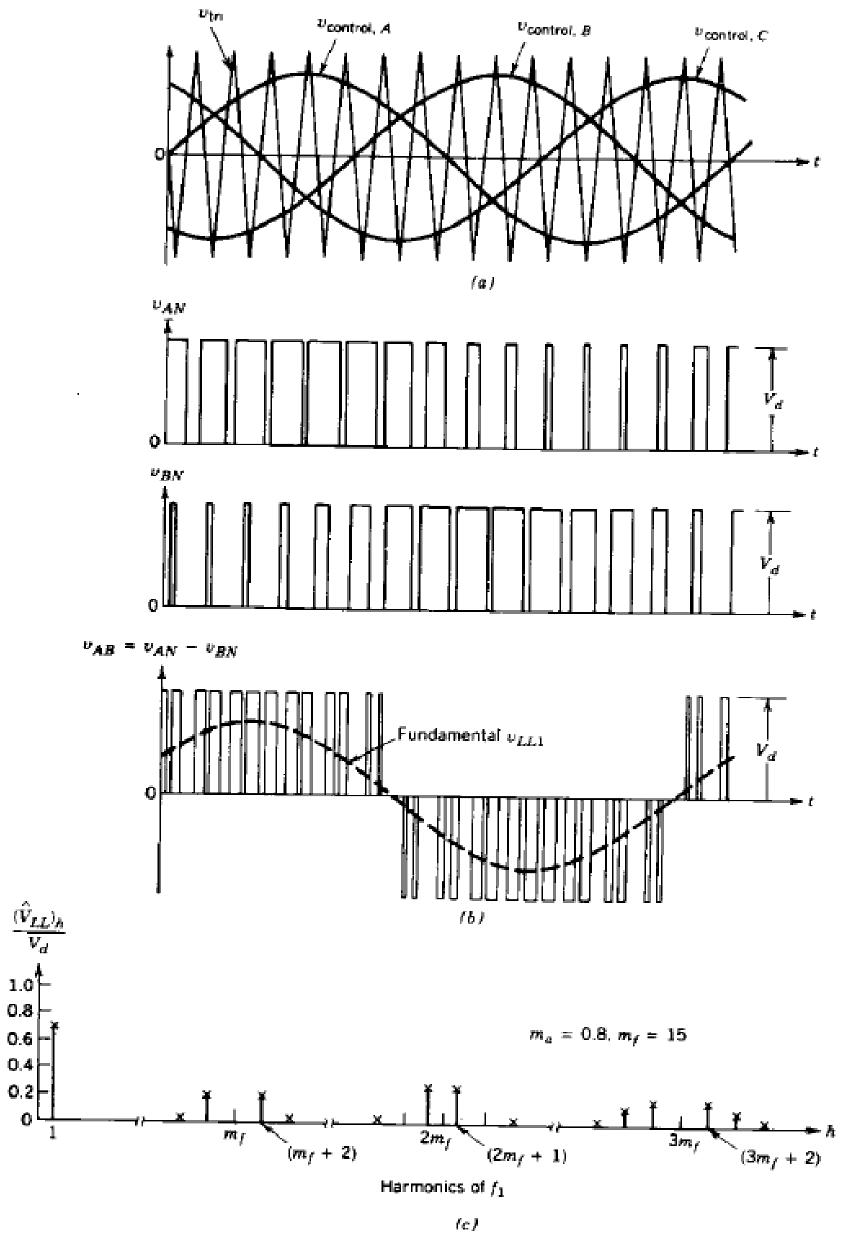
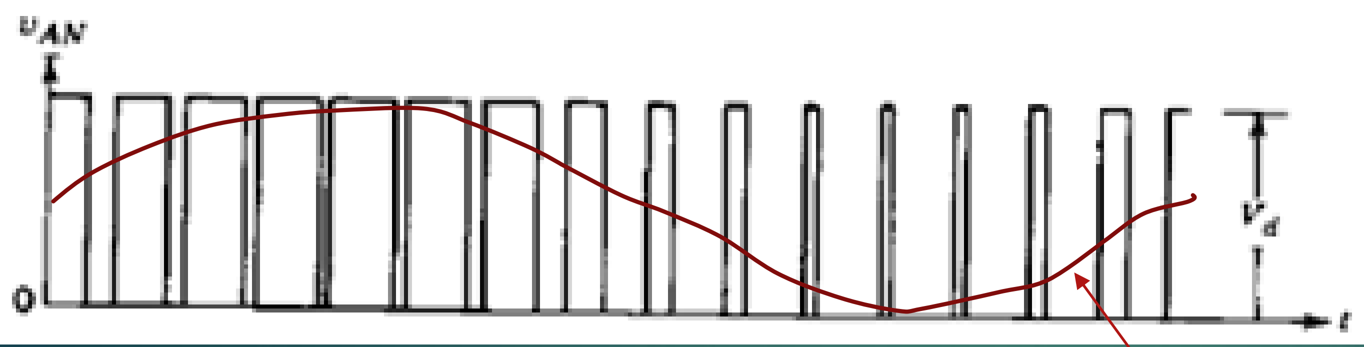
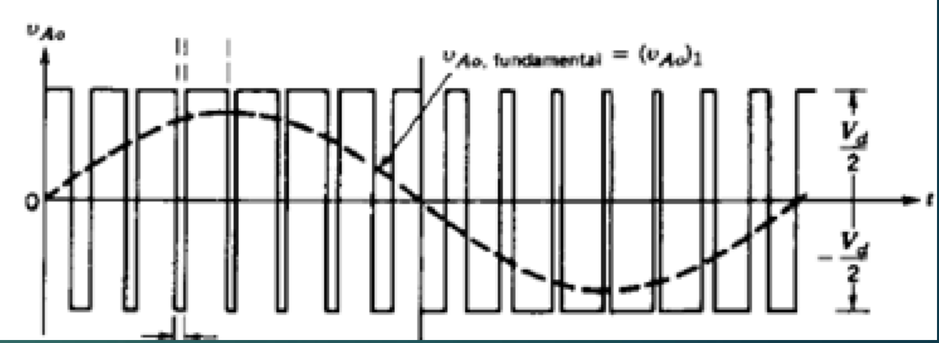
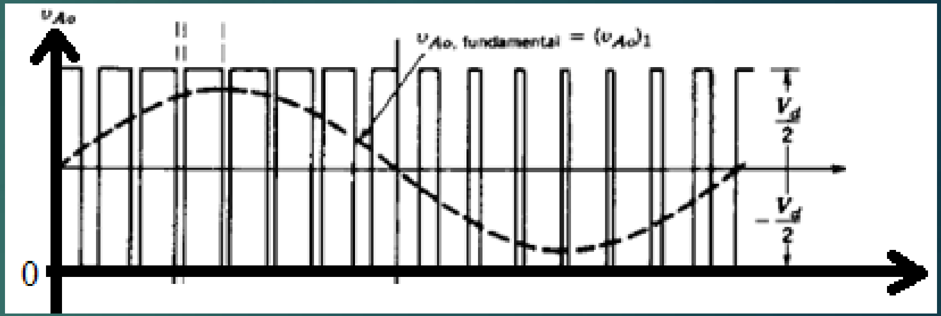


Figure 8-22 Three-phase PWM waveforms and harmonic spectrum.

3 phase SPWM – $V_{LL1\text{-rms}}$ = ….

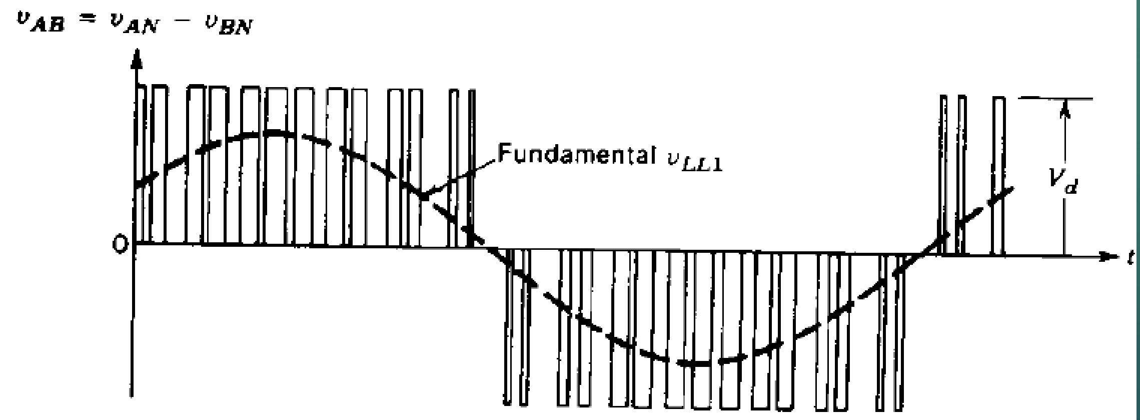






$$(\hat{V}_{AN})_1 = m_a \frac{V_d}{2}$$

$$\underset{\text{(line-line, rms)}}{V_{LL_1}} = \frac{\sqrt{3}}{\sqrt{2}}(\hat{V}_{AN})_1 = \frac{\sqrt{3}}{2\sqrt{2}} m_a V_d \simeq 0.612 m_a V_d$$



Actual amplitude of sine wave will depend upon ma –
For ma = 1,
+ve peak will reach Vd,
In other words,
+ve peak will reach Vd/2 when measured from its own horizontal axis (which is not shown in fig)

8.9 MULTILEVEL INVERTERS

Multilevel Converters with Independent DC Sources

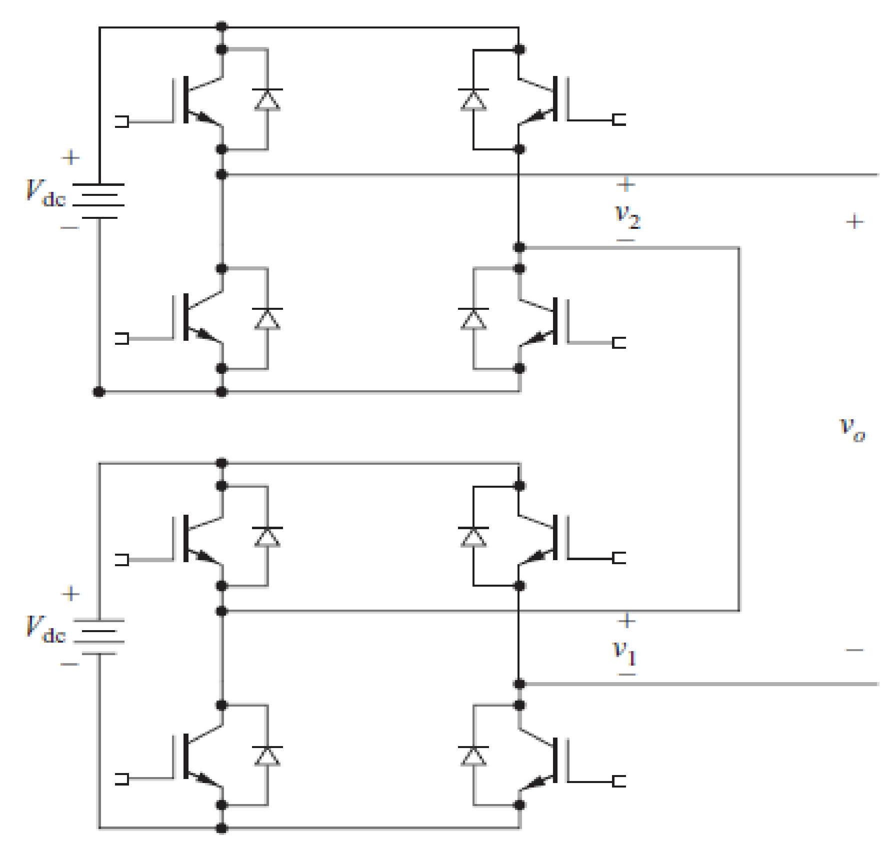
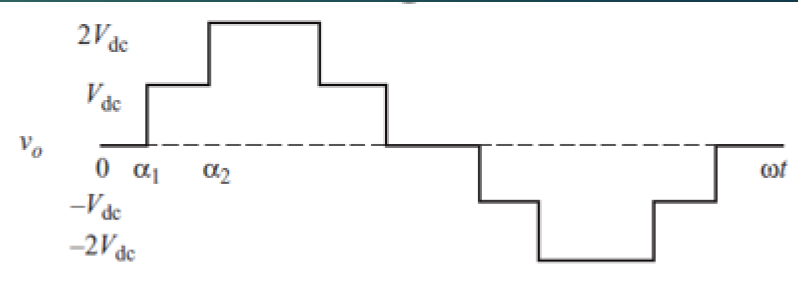


Figure 8-9 An inverter with two dc sources, each with an H bridge implemented with IGBTs.



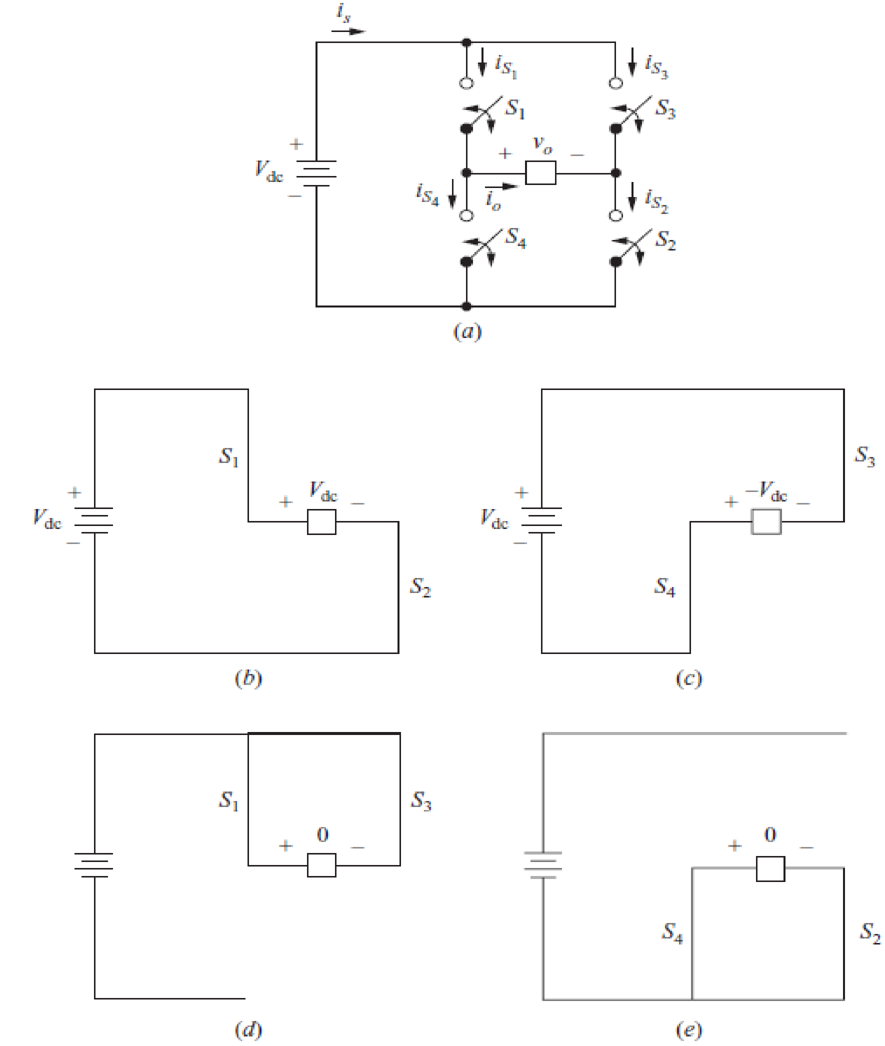


Figure 8-1 (*a*) Full-bridge converter; (*b*) S_1 and S_2 closed; (*c*) S_3 and S_4 closed; (*d*) S_1 and S_3 closed; (*e*) S_2 and S_4 closed.

Fourier series for output voltages of inverter waveforms.

The Fourier series for a periodic function $v_o(\omega t)$ can be expressed as

$$v_o(\omega t) = a_o + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t)$$

For an odd quarter-wave symmetry waveform,

$$a_o = 0 \quad a_n = 0$$

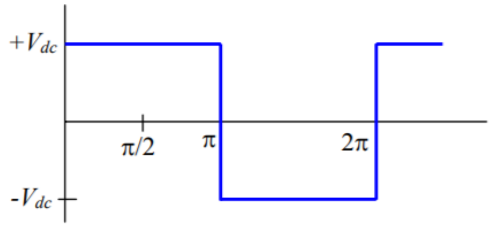
and

$$b_n = \begin{cases} \frac{4}{\pi}\int_0^{\frac{\pi}{2}} v_o \sin(n\omega t)d(\omega t) & \text{for odd } n \\ 0 & \text{for even } n \end{cases}$$

Therefore, $v_o(\omega t)$ can be written as

$$v_o(\omega t) = \sum_{n=odd}^{\infty} b_n \sin(n\omega t)$$

(i) Square-wave



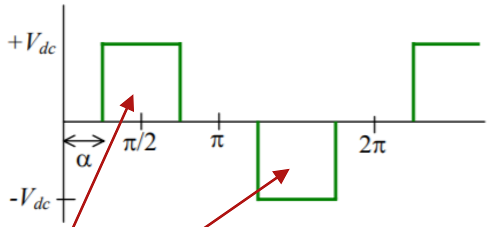
$$b_n = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} v_o \sin(n\omega t)d(\omega t)$$

$$= \frac{4}{\pi}\int_0^{\frac{\pi}{2}} V_{dc} \sin(n\omega t)d(\omega t)$$

$$= \frac{4V_{dc}}{n\pi}[-con(n\omega t]_0^{\frac{\pi}{2}}$$

$$= \frac{4V_{dc}}{n\pi}$$

(ii) Quasi square-wave



$$b_n = \frac{4}{\pi}\int_\alpha^{\frac{\pi}{2}} v_o \sin(n\omega t)d(\omega t)$$

$$= \frac{4}{\pi}\int_\alpha^{\frac{\pi}{2}} V_{dc} \sin(n\omega t)d(\omega t)$$

$$= \frac{4V_{dc}}{n\pi}[-con(n\omega t]_\alpha^{\frac{\pi}{2}}$$

$$= \frac{4V_{dc}}{n\pi}\cos(n\alpha)$$

Audio correction –
These are not on time and off time,
These are +ve and –ve half cycles

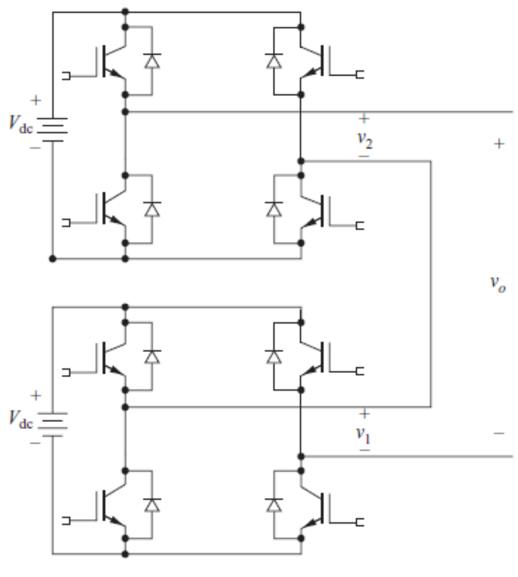


Figure 8-9 An inverter with two dc sources, each with an H bridge implemented with IGBTs.

$$v_o(t) = \frac{a_o}{2} + \sum_{n=1,2,...}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} v_o(\omega t) \cos n\omega t \, d(\omega t)$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} v_o(\omega t) \sin n\omega t \, d(\omega t)$$

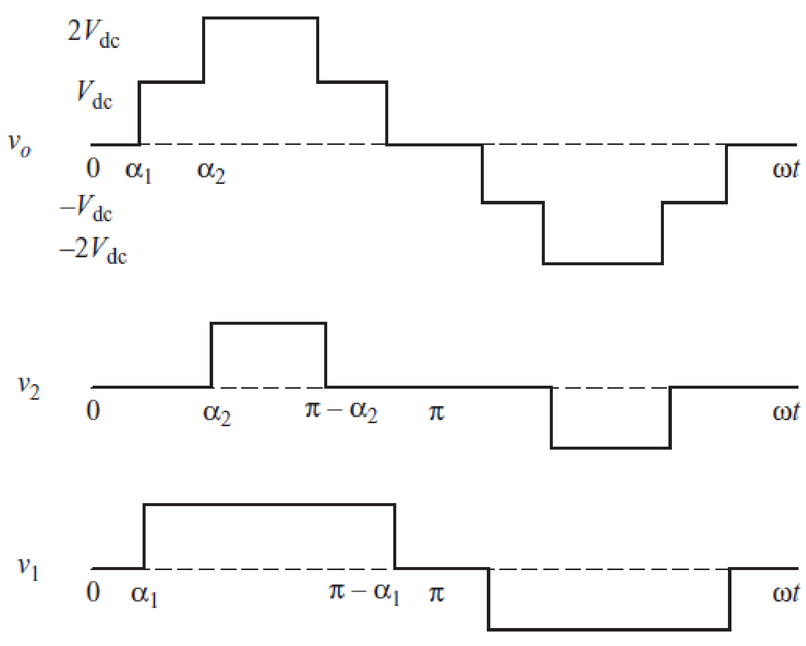


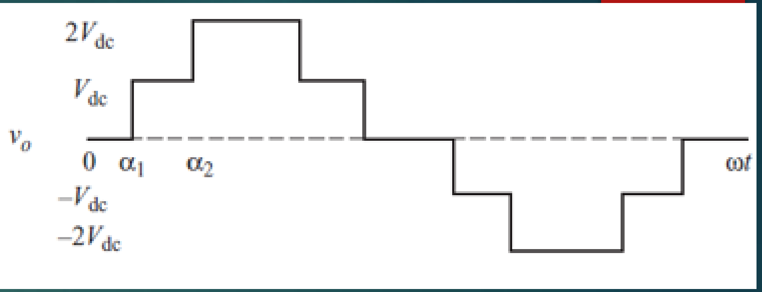
Figure 8-10 Voltage output of each of the H bridges and the total voltage for the two-source multilevel inverter of Fig. 8-9.

$$v_o(t) = \frac{4V_{dc}}{\pi} \sum_{n=1,3,5,7,...}^{\infty} [\cos(n\alpha_1) + \cos(n\alpha_2)]\frac{\sin(n\omega_0 t)}{n} \quad (8\text{-}23)$$

$$V_n = \frac{4V_{dc}}{n\pi}[\cos(n\alpha_1) + \cos(n\alpha_2)] \quad (8\text{-}24)$$

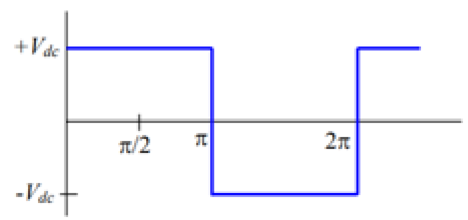
Modulation Index

$$M_i = \frac{V_1}{2(4V_{dc}/\pi)} = \frac{\cos\alpha_1 + \cos\alpha_2}{2} \qquad (8\text{-}25)$$



$$V_n = \frac{4V_{dc}}{n\pi}[\cos(n\alpha_1) + \cos(n\alpha_2)] \qquad (8\text{-}24)$$

(i) Square-wave



$$b_n = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} v_o \sin(n\omega t)d(\omega t)$$

$$= \frac{4}{\pi}\int_0^{\frac{\pi}{2}} V_{dc} \sin(n\omega t)d(\omega t$$

$$= \frac{4V_{dc}}{n\pi}[-con(n\omega t]_0^{\frac{\pi}{2}}$$

$$= \frac{4V_{dc}}{n\pi}$$

Harmonic Elimination

$$\cos(m\alpha_1) + \cos(m\alpha_2) = 0 \qquad (8\text{-}26)$$

$$\cos(\alpha_1) + \cos(\alpha_2) = 2M_i \qquad (8\text{-}27)$$

EXAMPLE 8-6

A Two-Source Multilevel Inverter

For the two-source multilevel inverter of Fig. 8-9 with $V_{dc} = 100$ V: (*a*) Determine the Fourier coefficients through $n = 9$ and the modulation index for $\alpha_1 = 20°$ and $\alpha_2 = 40°$. (*b*) Determine α_1 and α_2 such that the third harmonic ($n = 3$) is eliminated and $M_i = 0.8$.

■ Solution

Using Eq. 8-24 to evaluate the Fourier coefficients,

$$V_n = \frac{4V_{dc}}{n\pi}[\cos(n\alpha_1) + \cos(n\alpha_2)] = \frac{4(100)}{n\pi}[\cos(n20°) + \cos(n40°)]$$

resulting in $V_1 = 217$, $V_3 = 0$, $V_5 = -28.4$, $V_7 = -10.8$, and $V_9 = 0$. Note that the third and ninth harmonics are eliminated. The even harmonics are not present.

The modulation index M_i is evaluated from Eq. (8-25).

$$M_i = \frac{\cos\alpha_1 + \cos\alpha_2}{2} = \frac{\cos 20° + \cos 40°}{2} = 0.853$$

The amplitude of the fundamental frequency voltage is therefore 85.3 percent of that of a square wave of ±200 V.

(*b*) To achieve simultaneous elimination of the third harmonic and a modulation index of $M_i = 0.8$ requires the solution to the equations

$$\cos(3\alpha_1) + \cos(3\alpha_2) = 0$$

and

$$\cos(\alpha_1) + \cos(\alpha_2) = 2M_i = 1.6$$

Using an iterative method, $\alpha_1 = 7.6°$ and $\alpha_2 = 52.4°$.

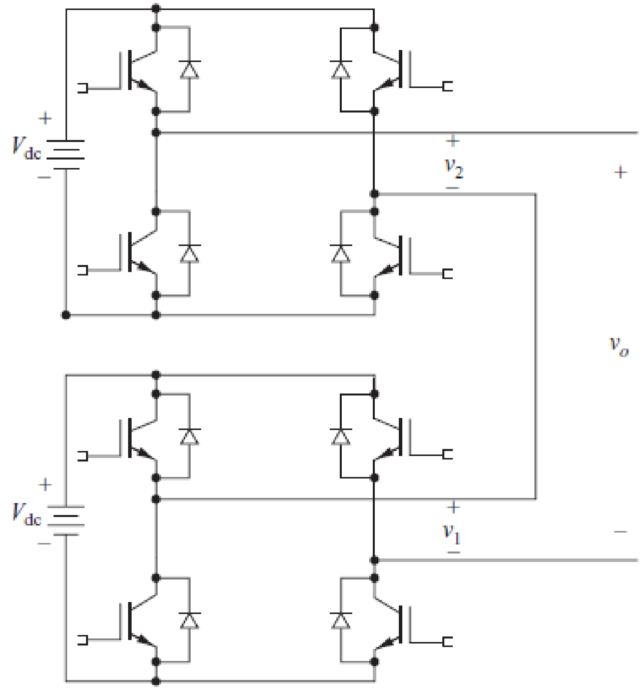


Figure 8-9 An inverter with two dc sources, each with an H bridge implemented with IGBTs.

$$V_n = \frac{4V_{dc}}{n\pi}[\cos(n\alpha_1) + \cos(n\alpha_2)] \qquad (8\text{-}24)$$

$$M_i = \frac{V_1}{2(4V_{dc}/\pi)} = \frac{\cos\alpha_1 + \cos\alpha_2}{2} \qquad (8\text{-}25)$$

$$\cos(m\alpha_1) + \cos(m\alpha_2) = 0 \qquad (8\text{-}26)$$

$$\cos(\alpha_1) + \cos(\alpha_2) = 2M_i \qquad (8\text{-}27)$$

Extending to Multiple DC sources

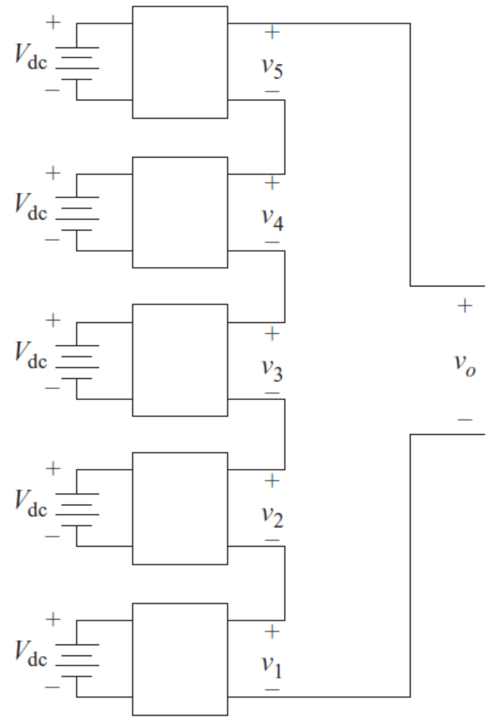


Figure 8-11 A five-source cascade multilevel converter.

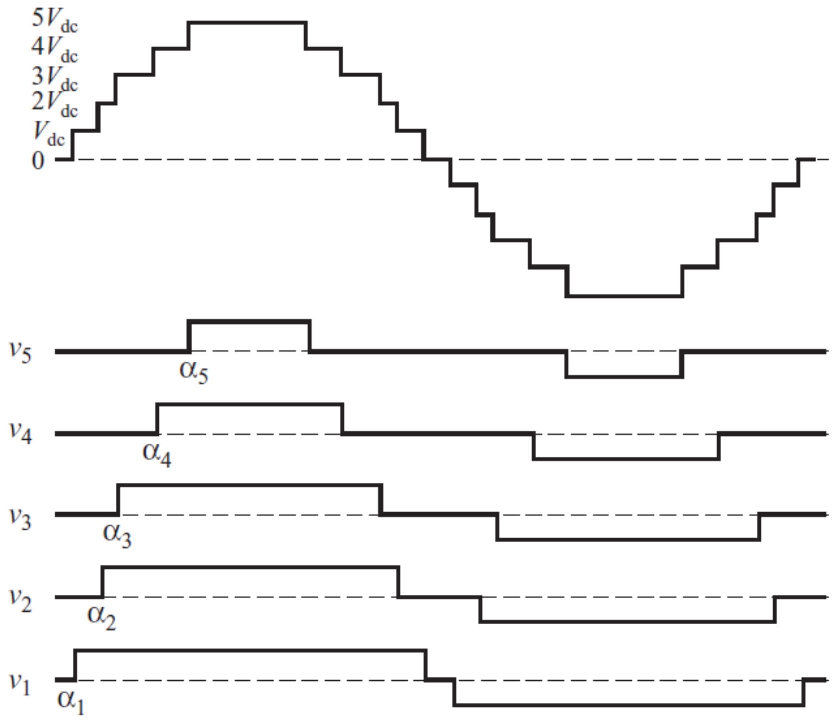


Figure 8-12 Voltages at each H bridge in Fig. 8-11 and the total output voltage.

With k sources – 2k+1 levels may be achieved.

$$v_o(t) = \frac{4V_{dc}}{\pi} \sum_{n=1,3,5,7,\ldots}^{\infty} [\cos(n\alpha_1) + \cos(n\alpha_2) + \cdots + \cos(n\alpha_k)] \frac{\sin(n\omega_0 t)}{n} \quad (8\text{-}28)$$

$$V_n = \frac{4V_{dc}}{n\pi}[\cos(n\alpha_1) + \cos(n\alpha_2) + \cdots + \cos(n\alpha_k)] \quad \text{for } n = 1, 3, 5, 7, \ldots \quad (8\text{-}29)$$

$$M_i = \frac{V_1}{4kV_{dc}/\pi} = \frac{\cos(\alpha_1) + \cos(\alpha_2) + \cdots + \cos(\alpha_k)}{k} \quad (8\text{-}30)$$

$$\cos(m\alpha_1) + \cos(m\alpha_2) + \cdots + \cos(m\alpha_k) = 0 \quad (8\text{-}31)$$

EXAMPLE

A Five-Source Multilevel Inverter

Determine the delay angles required for a five-source cascade multilevel converter that will eliminate harmonics 5, 7, 11, and 13 and will have a modulation index $M_i = 0.8$.

■ **Solution**

The delay angles must satisfy these simultaneous equations:

$$\cos(5\alpha_1) + \cos(5\alpha_2) + \cos(5\alpha_3) + \cos(5\alpha_4) + \cos(5\alpha_5) = 0$$
$$\cos(7\alpha_1) + \cos(7\alpha_2) + \cos(7\alpha_3) + \cos(7\alpha_4) + \cos(7\alpha_5) = 0$$
$$\cos(11\alpha_1) + \cos(11\alpha_2) + \cos(11\alpha_3) + \cos(11\alpha_4) + \cos(11\alpha_5) = 0$$
$$\cos(13\alpha_1) + \cos(13\alpha_2) + \cos(13\alpha_3) + \cos(13\alpha_4) + \cos(13\alpha_5) = 0$$
$$\cos(\alpha_1) + \cos(\alpha_2) + \cos(\alpha_3) + \cos(\alpha_4) + \cos(\alpha_5) = 5M_i = 5(0.8) = 4$$

An iteration method such as the Newton-Raphson method must be used to solve these equations. The result is $\alpha_1 = 6.57°$, $\alpha_2 = 18.94°$, $\alpha_3 = 27.18°$, $\alpha_4 = 45.14°$, and $\alpha_5 = 62.24°$. See the references in the Bibliography for information on the technique.

$$M_i = \frac{V_1}{4kV_{dc}/\pi} = \frac{\cos(\alpha_1) + \cos(\alpha_2) + \cdots + \cos(\alpha_k)}{k} \qquad (8\text{-}30)$$

$$\cos(m\alpha_1) + \cos(m\alpha_2) + \cdots + \cos(m\alpha_k) = 0 \qquad (8\text{-}31)$$

Equalizing Average Source Power with Pattern Swapping

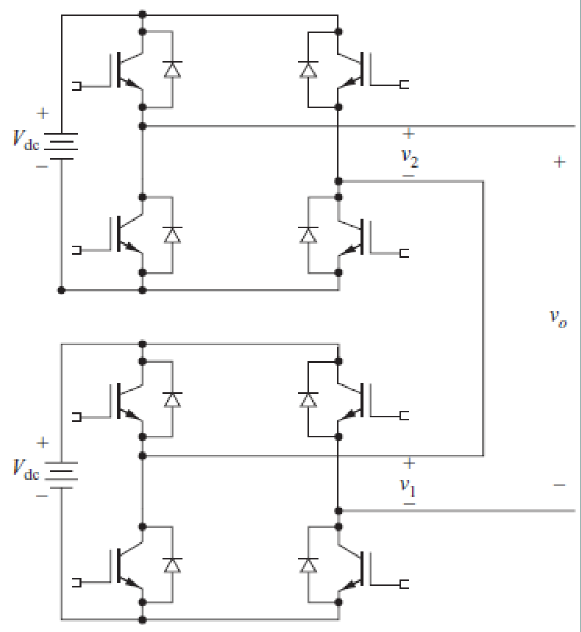


Figure 8-9 An inverter with two dc sources, each with an H bridge implemented with IGBTs.

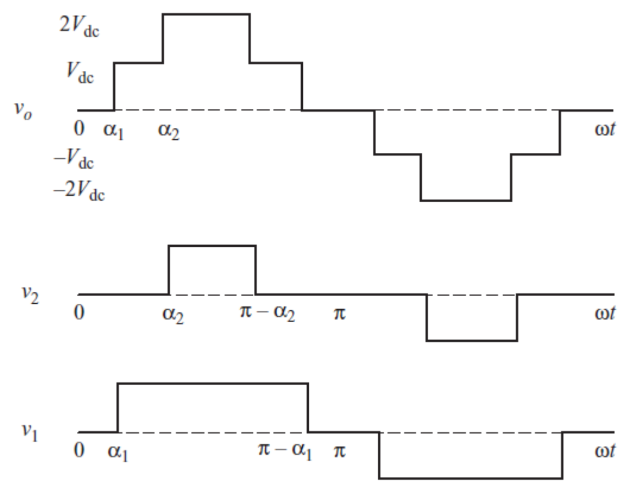


Figure 8-10 Voltage output of each of the H bridges and the total voltage for the two-source multilevel inverter of Fig. 8-9.

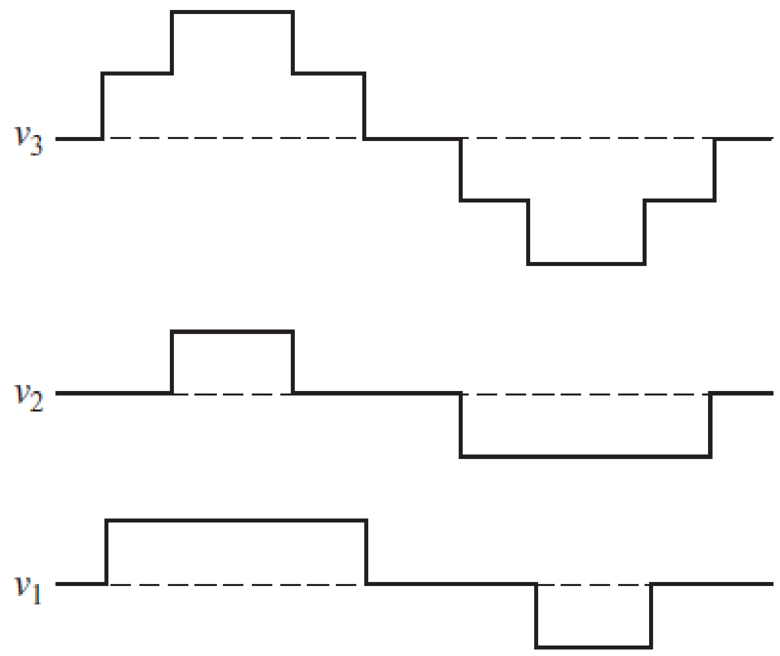


Figure 8-13 Pattern swapping to equalize average power in each source for the two-source inverter of Fig. 8-9.

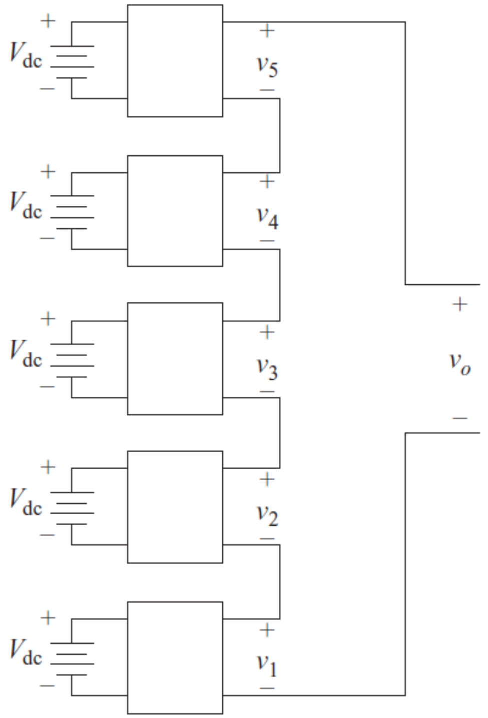


Figure 8-11 A five-source cascade multilevel converter.

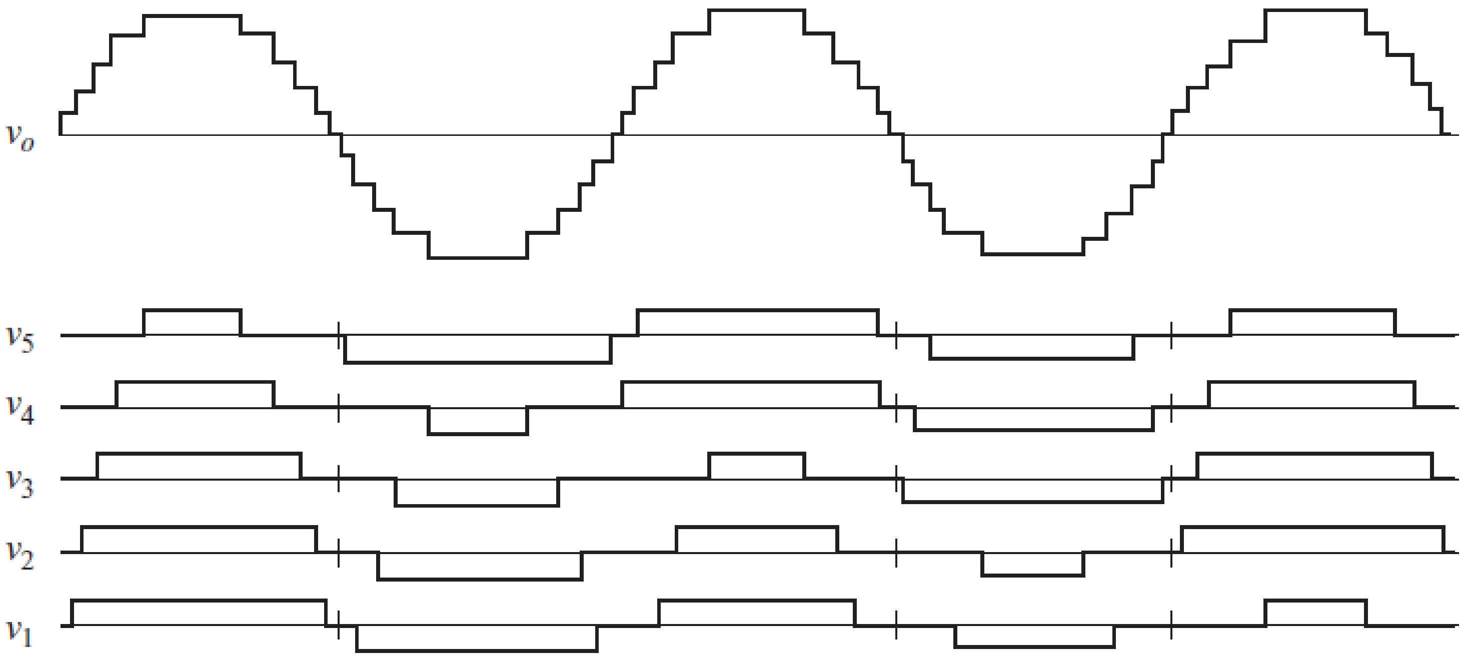
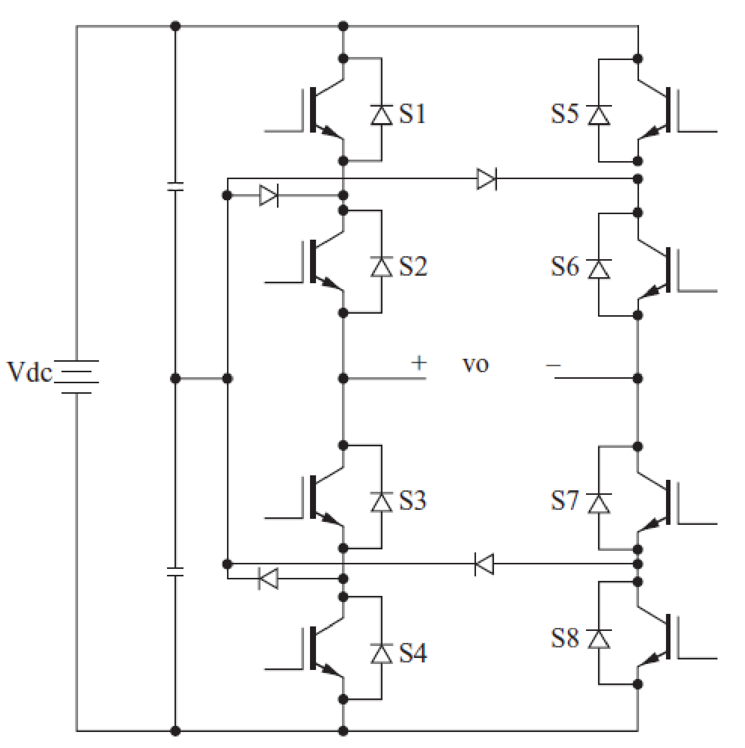


Figure 8-14 Pattern swapping to equalize average source power for the five-source multilevel inverter of Fig. 8-11.

Diode-Clamped Multilevel Inverters



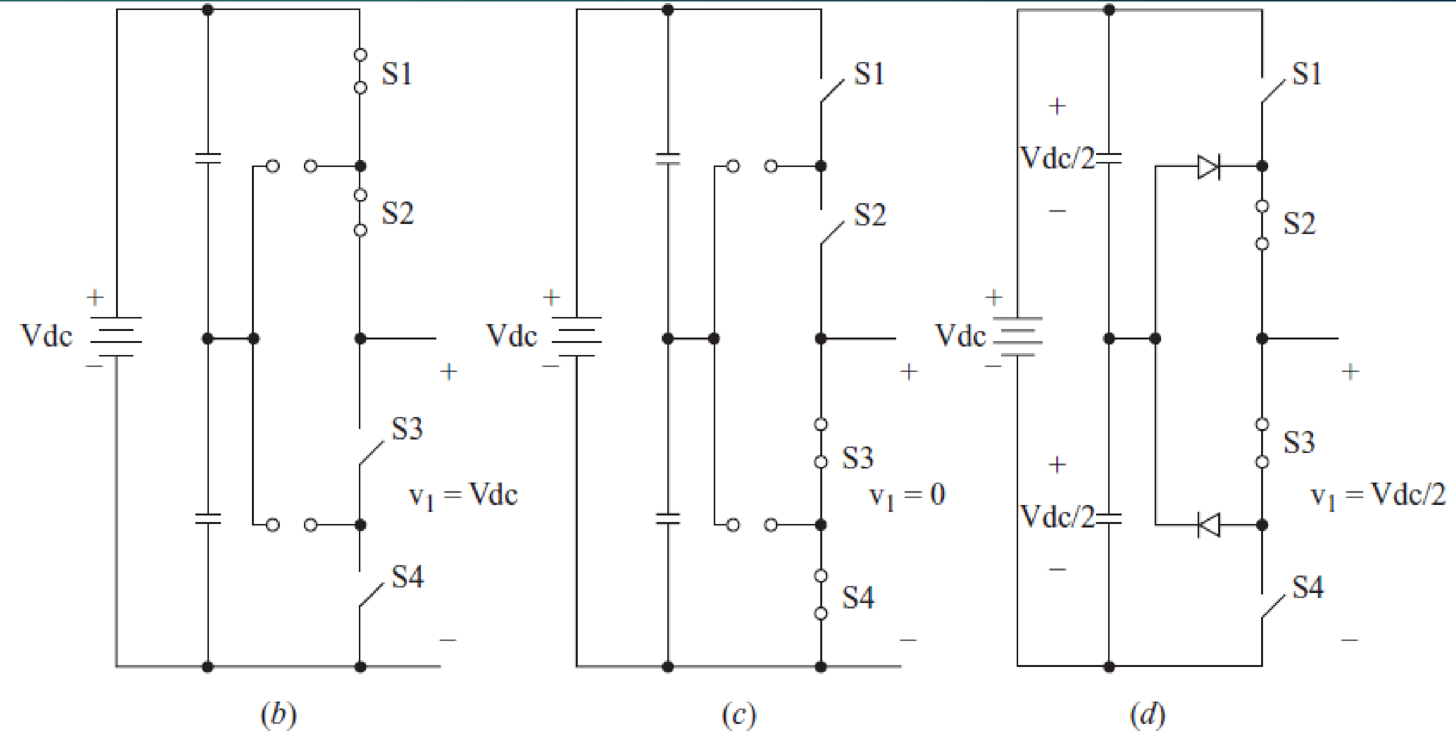
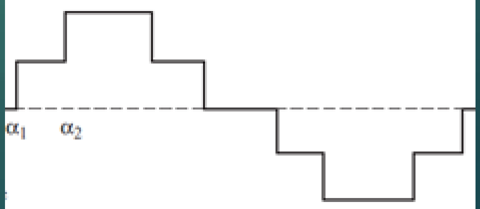


Figure 8-15 (*a*) A diode-clamped multilevel inverter implemented with IGBTs. (*b*) Analysis for one-half of the circuit for $v_1 = V_{dc}$, (*c*) for $v_1 = 0$, and (*d*) for $v_1 = \frac{1}{2}V_{dc}$.

$$v_o \in \left\{ V_{dc}, \frac{1}{2}V_{dc}, 0, -\frac{1}{2}V_{dc}, -V_{dc} \right\} \quad (8\text{-}32)$$



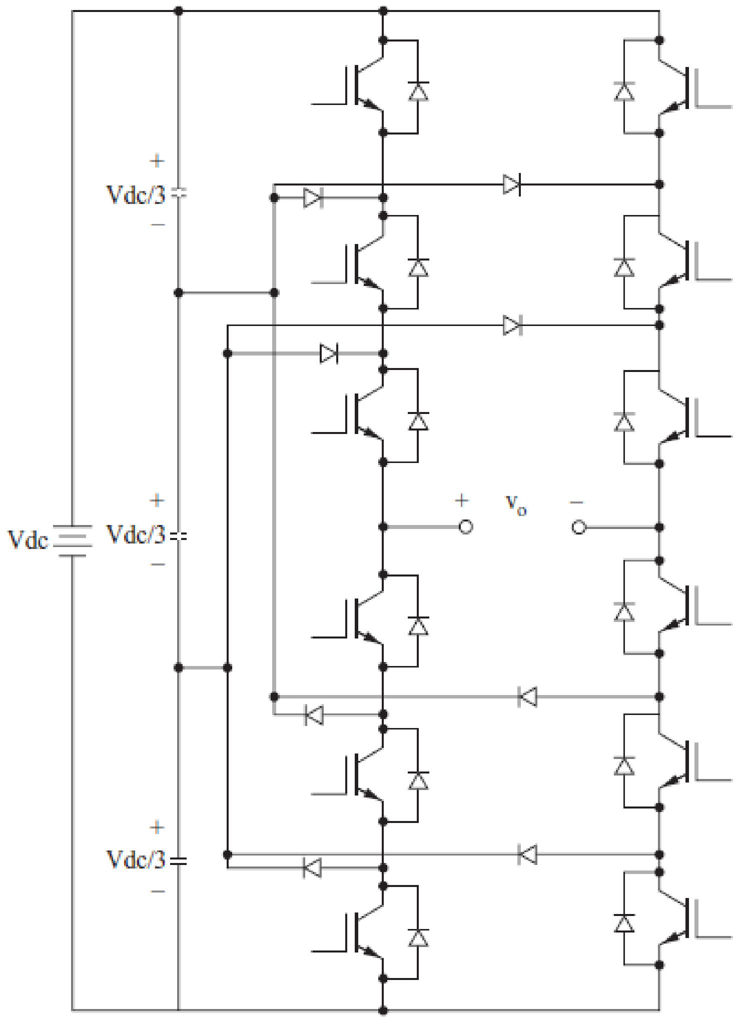
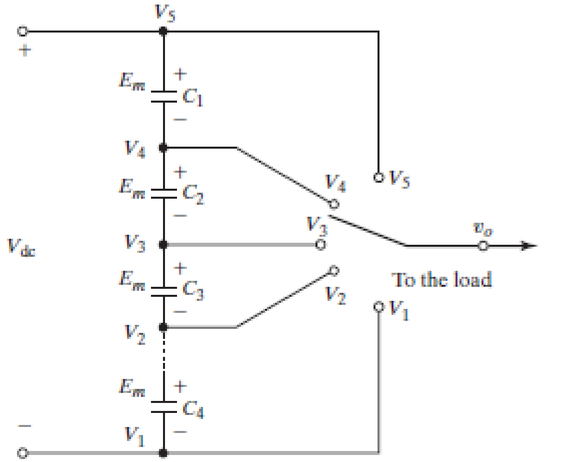


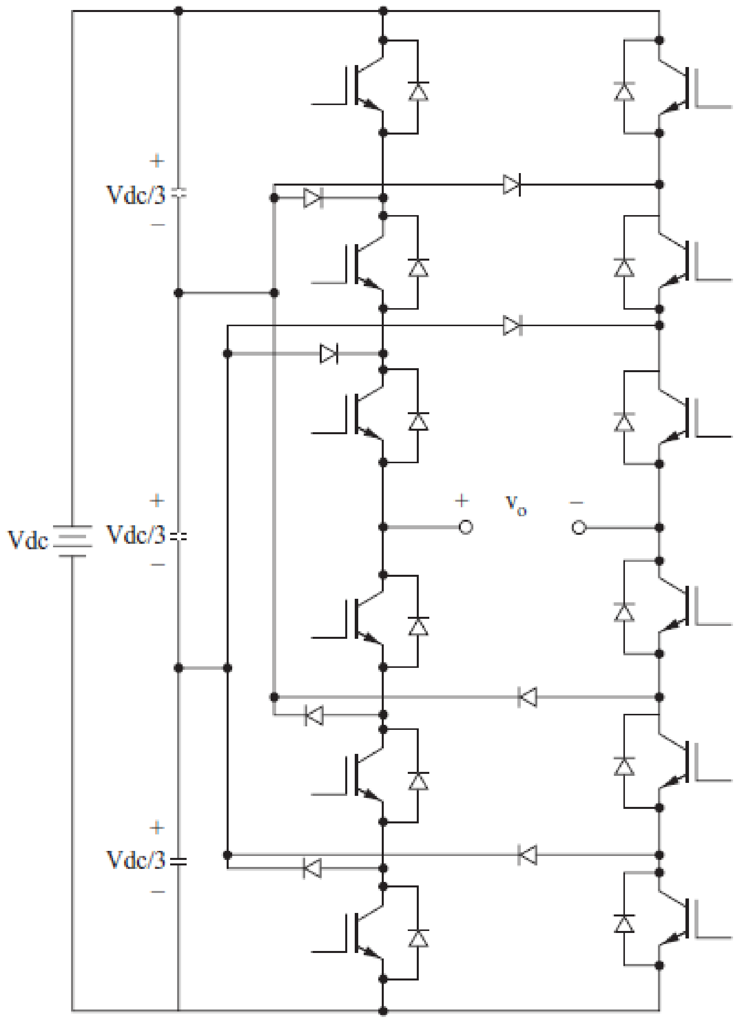
Figure 8-16 A diode-clamped multilevel inverter that produces four voltage levels on each side of the bridge and seven output voltage levels.

$$v_o \in \left\{ V_{dc}, \frac{2}{3}V_{dc}, \frac{1}{3}V_{dc}, 0, -\frac{1}{3}V_{dc}, -\frac{2}{3}V_{dc}, -V_{dc} \right\} \qquad (8\text{-}33)$$

With k capacitors – 2k+1 levels may be achieved.



(b) Schematic of single pole of multilevel inverter by a switch



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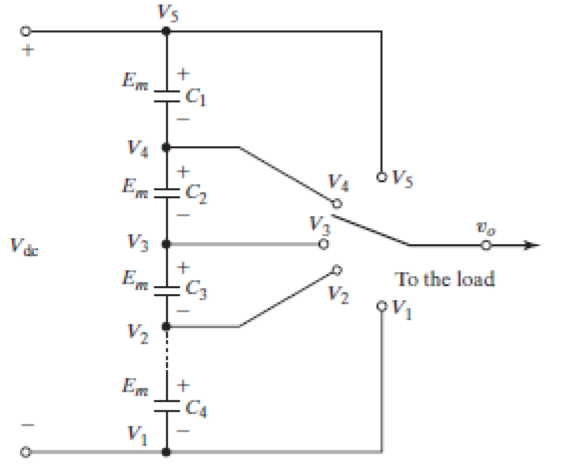

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