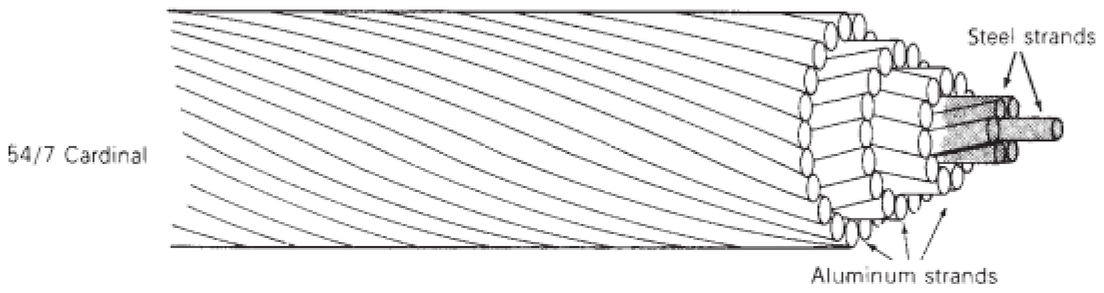
TRANSMISSION LINE PARAMETERS

Lec_10

• One of the most common conductor types is aluminum conductor, steel-reinforced (ACSR), which consists of layers of aluminum strands surrounding a central core of steel strands.

FIGURE 4.1

Typical ACSR conductor



- Other conductor types include the all-aluminum conductor (AAC), all aluminum-alloy conductor (AAAC), aluminum conductor alloy-reinforced (ACAR), and aluminum-clad steel conductor (Alumoweld)

LINE RESISTANCE

The resistance of the conductor is very important in transmission efficiency evaluation and economic studies. The dc resistance of a solid round conductor at a specified temperature is given by

$$R_{dc} = \frac{\rho l}{A} \tag{4.1}$$

where ρ = conductor resistivity
l = conductor length
A = conductor cross-sectional area

The conductor resistance is affected by three factors: frequency, spiraling, and temperature.

The conductor resistance increases as temperature increases. This change can be considered linear over the range of temperature normally encountered and may be calculated from

$$R_2 = R_1 \frac{T + t_2}{T + t_1} \tag{4.2}$$

INDUCTANCE OF A SINGLE CONDUCTOR

A current-carrying conductor produces a magnetic field around the conductor. The magnetic flux lines are concentric closed circles with direction given by the right-hand rule. With the thumb pointing in the direction of the current, the fingers of the right hand encircled the wire point in the direction of the magnetic field. When the current changes, the flux changes and a voltage is induced in the circuit. By definition, for nonmagnetic material, the inductance L is the ratio of its total magnetic flux linkage to the current I, given by

$$L = \frac{\lambda}{I} \tag{4.3}$$

where λ = flux linkages, in Weber turns.

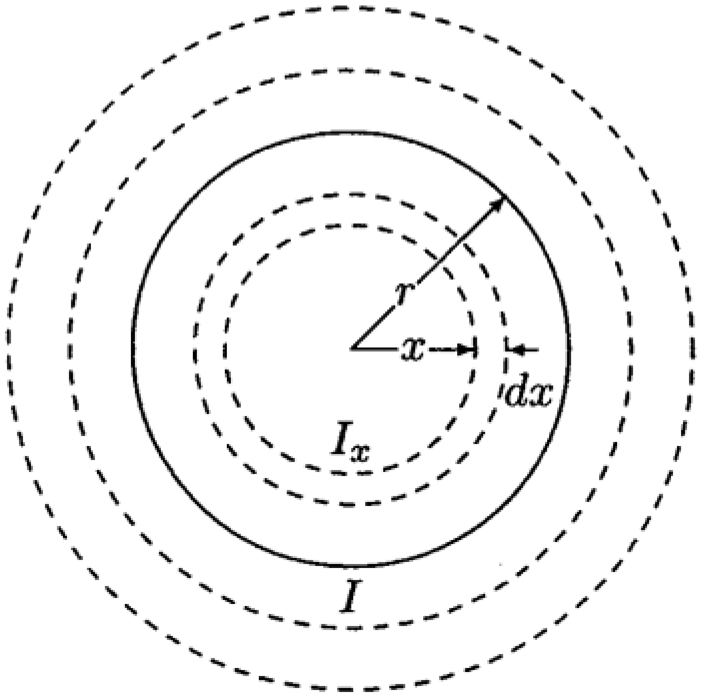


FIGURE 4.3
Flux linkage of a long round conductor.

$$\int_0^{2\pi x} H_x \cdot dl = I_x \tag{4.4}$$

or

$$H_x = \frac{I_x}{2\pi x} \tag{4.5}$$

4.4.1 INTERNAL INDUCTANCE

A simple expression can be obtained for the internal flux linkage by neglecting the skin effect and assuming uniform current density throughout the conductor cross section, i.e.,

$$\frac{I}{\pi r^2} = \frac{I_x}{\pi x^2} \tag{4.6}$$

Substituting for I_x in (4.5) yields

$$H_x = \frac{I}{2\pi r^2}x \tag{4.7}$$

For a nonmagnetic conductor with constant permeability μ_0, the magnetic flux density is given by $B_x = \mu_0 H_x$, or

$$B_x = \frac{\mu_0 I}{2\pi r^2}x \tag{4.8}$$

where μ_0 is the permeability of free space (or air) and is equal to $4\pi \times 10^{-7}$H/m. The differential flux $d\phi$ for a small region of thickness dx and one meter length of the conductor is

$$d\phi_x = B_x dx \cdot 1 = \frac{\mu_0 I}{2\pi r^2} x dx \tag{4.9}$$

The flux $d\phi_x$ links only the fraction of the conductor from the center to radius x. Thus, on the assumption of uniform current density, only the fraction $\pi x^2/\pi r^2$ of the total current is linked by the flux, i.e.,

$$d\lambda_x = (\frac{x^2}{r^2}) d\phi_x = \frac{\mu_0 I}{2\pi r^4} x^3 dx \tag{4.10}$$

The total flux linkage is found by integrating $d\lambda_x$ from 0 to r.

$$
\begin{aligned}
\lambda_{int} &= \frac{\mu_0 I}{2\pi r^4}\int_0^r x^3\,dx \\
&= \frac{\mu_0 I}{8\pi}\ \text{Wb/m} \qquad (4.11)
\end{aligned}
$$

From (4.3), the inductance due to the internal flux linkage is

$$
L_{int} = \frac{\mu_0}{8\pi} = \frac{1}{2}\times 10^{-7}\ \text{H/m} \qquad (4.12)
$$

Note that L_{int} is independent of the conductor radius r.

4.4.2 INDUCTANCE DUE TO EXTERNAL FLUX LINKAGE

Consider H_x external to the conductor at radius $x > r$ as shown in Figure 4.4. Since the circle at radius x encloses the entire current, $I_x = I$ and in (4.5) I_x is replaced by I and the flux density at radius x becomes

$$B_x = \mu_0 H_x = \frac{\mu_0 I}{2\pi x} \tag{4.13}$$

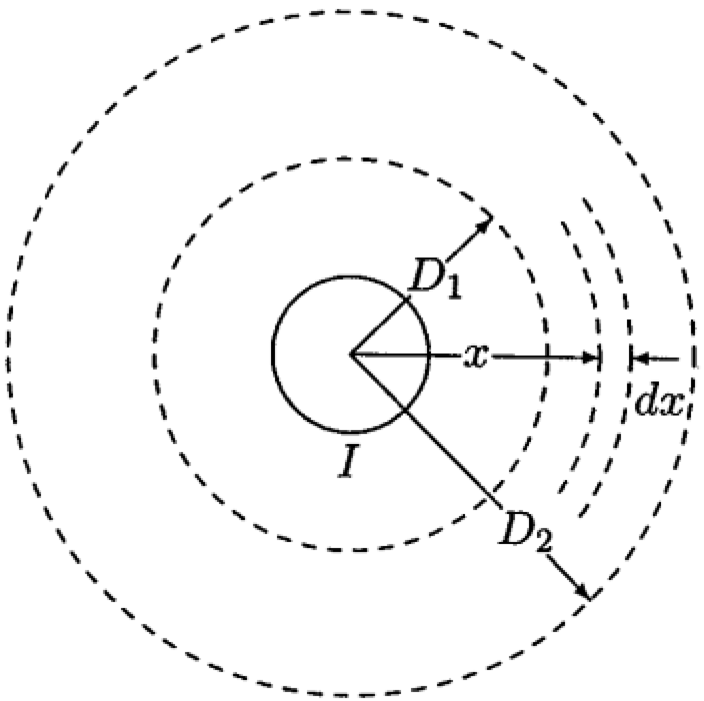


FIGURE 4.4
Flux linkage between D_1 and D_2.

Since the entire current I is linked by the flux outside the conductor, the flux linkage $d\lambda_x$ is numerically equal to the flux $d\phi_x$. The differential flux $d\phi_x$ for a small region of thickness dx and one meter length of the conductor is then given by

$$d\lambda_x = d\phi_x = B_x dx \cdot 1 = \frac{\mu_0 I}{2\pi x} dx \tag{4.14}$$

The external flux linkage between two points D_1 and D_2 is found by integrating $d\lambda_x$ from D_1 to D_2.

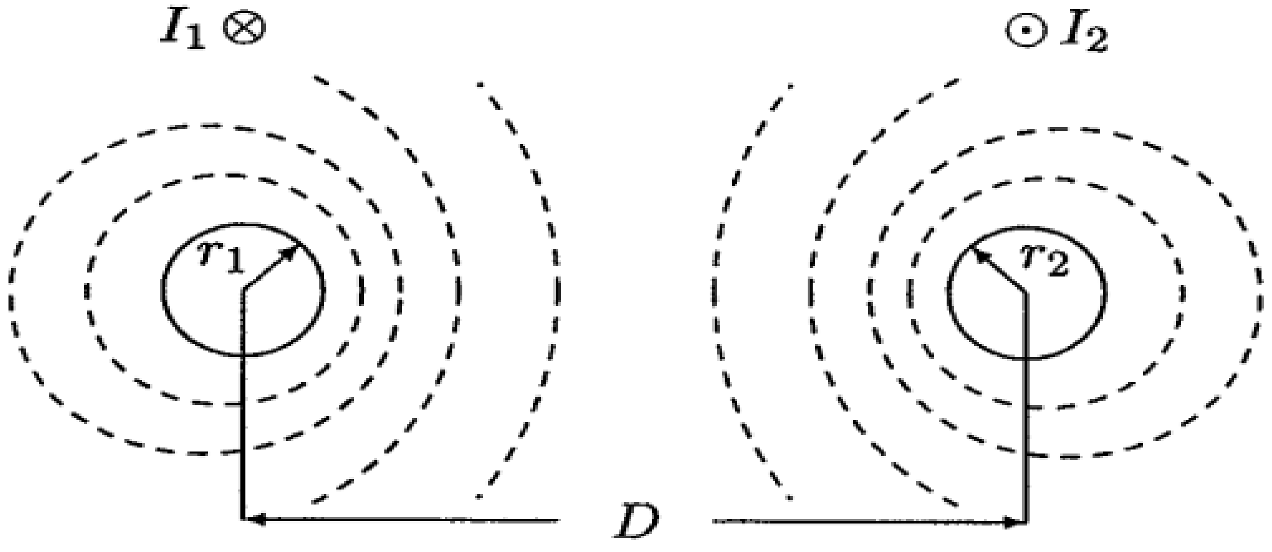
$$\begin{aligned}
\lambda_{ext} &= \frac{\mu_0 I}{2\pi} \int_{D_1}^{D_2} \frac{1}{x} dx \\
&= 2 \times 10^{-7} I \ln \frac{D_2}{D_1} \quad \text{Wb/m}
\end{aligned} \tag{4.15}$$

The inductance between two points external to a conductor is then

$$L_{ext} = 2 \times 10^{-7} \ln \frac{D_2}{D_1} \quad \text{H/m} \tag{4.16}$$

4.5 INDUCTANCE OF SINGLE-PHASE LINES

Consider one meter length of a single-phase line consisting of two solid round conductors of radius r_1 and r_2 as shown in Figure 4.5. The two conductors are separated by a distance D. Conductor 1 carries the phasor current I_1 referenced into the page and conductor 2 carries return current $I_2 = -I_1$. These currents set up magnetic field lines that links between the conductors as shown.



Inductance of conductor 1 due to internal flux is given by (4.12). The flux beyond D links a net current of zero and does not contribute to the net magnetic flux linkages in the circuit. Thus, to obtain the inductance of conductor 1 due to the net external flux linkage, it is necessary to evaluate (4.16) from $D_1 = r_1$ to $D_2 = D$.

$$L_{1(ext)} = 2 \times 10^{-7} \ln \frac{D}{r_1} \quad \text{H/m} \tag{4.17}$$

The total inductance of conductor 1 is then

$$L_1 = \frac{1}{2} \times 10^{-7} + 2 \times 10^{-7} \ln \frac{D}{r_1} \quad \text{H/m} \tag{4.18}$$

Equation (4.18) is often rearranged as follows:

$$\begin{aligned} L_1 &= 2 \times 10^{-7} \left(\frac{1}{4} + \ln \frac{D}{r_1} \right) \\ &= 2 \times 10^{-7} \left(\ln e^{\frac{1}{4}} + \ln \frac{1}{r_1} + \ln \frac{D}{1} \right) \\ &= 2 \times 10^{-7} \left(\ln \frac{1}{r_1 e^{-1/4}} + \ln \frac{D}{1} \right) \end{aligned} \tag{4.19}$$

Let $r_1' = r_1 e^{-\frac{1}{4}}$, the inductance of conductor 1 becomes

$$L_1 = 2 \times 10^{-7} \ln \frac{1}{r_1'} + 2 \times 10^{-7} \ln \frac{D}{1} \quad \text{H/m} \tag{4.20}$$

Similarly, the inductance of conductor 2 is

$$L_2 = 2 \times 10^{-7} \ln \frac{1}{r_2'} + 2 \times 10^{-7} \ln \frac{D}{1} \quad \text{H/m} \tag{4.21}$$

If the two conductors are identical, $r_1 = r_2 = r$ and $L_1 = L_2 = L$, and the inductance per phase per meter length of the line is given by

$$L = 2 \times 10^{-7} \ln \frac{1}{r'} + 2 \times 10^{-7} \ln \frac{D}{1} \quad \text{H/m} \tag{4.22}$$

The term $r' = re^{-\frac{1}{4}}$ is known mathematically as the *self-geometric mean distance* of a circle with radius r and is abbreviated by GMR. r' can be considered as the radius of a fictitious conductor assumed to have no internal flux but with the same inductance as the actual conductor with radius r. GMR is commonly referred to as *geometric mean radius* and will be designated by D_s. Thus, the inductance per phase in millihenries per kilometer becomes

$$L = 0.2 \ln \frac{D}{D_s} \quad \text{mH/km} \tag{4.23}$$

4.6 FLUX LINKAGE IN TERMS OF SELF- AND MUTUAL INDUCTANCES

The series inductance per phase for the above single-phase two-wire line can be expressed in terms of self-inductance of each conductor and their mutual inductance. Consider one meter length of the single-phase circuit represented by two coils characterized by the self-inductances L_{11} and L_{22} and the mutual inductance L_{12}. The magnetic polarity is indicated by dot symbols as shown in Figure 4.6.

The flux linkages λ_1 and λ_2 are given by

$$\begin{aligned} \lambda_1 &= L_{11}I_1 + L_{12}I_2 \\ \lambda_2 &= L_{21}I_1 + L_{22}I_2 \end{aligned} \tag{4.24}$$

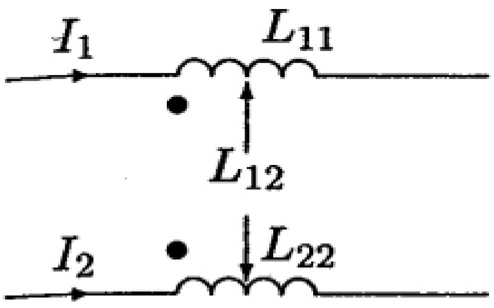


FIGURE 4.6
The single-phase line viewed as two magnetically coupled coils.

Since $I_2 = -I_1$, we have

$$\begin{aligned}\lambda_1 &= (L_{11} - L_{12})I_1 \\ \lambda_2 &= (-L_{21} + L_{22})I_2\end{aligned} \tag{4.25}$$

Comparing (4.25) with (4.20) and (4.21), we conclude the following equivalent expressions for the self- and mutual inductances:

$$L_{11} = 2 \times 10^{-7} \ln \frac{1}{r'_1}$$

$$L_{22} = 2 \times 10^{-7} \ln \frac{1}{r'_2}$$

$$L_{12} = L_{21} = 2 \times 10^{-7} \ln \frac{1}{D} \tag{4.26}$$

The concept of self- and mutual inductance can be extended to a group of n conductors. Consider n conductors carrying phasor currents I_1, I_2, ..., I_n, such that

$$I_1 + I_2 + \cdots + I_i + \cdots + I_n = 0 \tag{4.27}$$

Generalizing (4.24), the flux linkages of conductor i are

$$\lambda_i = L_{ii} I_i + \sum_{j=1}^{n} L_{ij} I_j \qquad j \neq i \tag{4.28}$$

$$\lambda_i = 2 \times 10^{-7} \left(I_i \ln \frac{1}{r_i'} + \sum_{j=1}^{n} I_j \ln \frac{1}{D_{ij}} \right) \quad j \neq i \tag{4.29}$$

4.7 INDUCTANCE OF THREE-PHASE TRANSMISSION LINES

4.7.1 SYMMETRICAL SPACING

Consider one meter length of a three-phase line with three conductors, each with radius r, symmetrically spaced in a triangular configuration as shown in Figure 4.7.

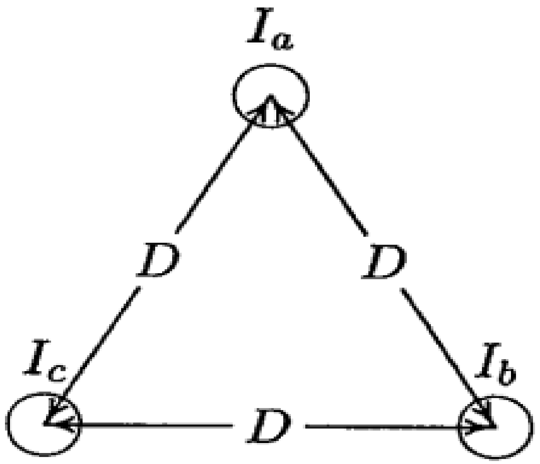


FIGURE 4.7
Three-phase line with symmetrical spacing.

Assuming balanced three-phase currents, we have

$$I_a + I_b + I_c = 0 \tag{4.30}$$

From (4.29) the total flux linkage of phase a conductor is

$$\lambda_a = 2 \times 10^{-7} \left(I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D} + I_c \ln \frac{1}{D} \right) \tag{4.31}$$

Substituting for $I_b + I_c = -I_a$

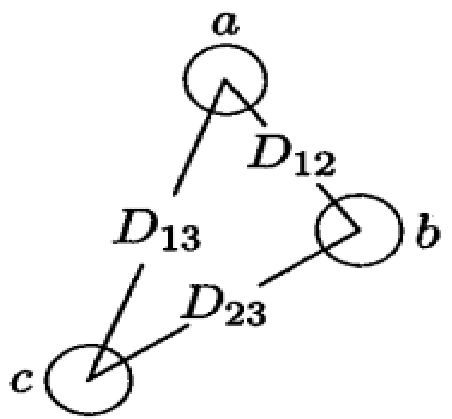
$$\begin{aligned} \lambda_a &= 2 \times 10^{-7} \left(I_a \ln \frac{1}{r'} - I_a \ln \frac{1}{D} \right) \\ &= 2 \times 10^{-7} I_a \ln \frac{D}{r'} \end{aligned} \tag{4.32}$$

Because of symmetry, $\lambda_b = \lambda_c = \lambda_a$, and the three inductances are identical. Therefore, the inductance per phase per kilometer length is

$$L = 0.2 \ln \frac{D}{D_s} \quad \text{mH/km} \tag{4.33}$$

4.7.2 ASYMMETRICAL SPACING

Practical transmission lines cannot maintain symmetrical spacing of conductors because of construction considerations. With asymmetrical spacing, even with balanced currents, the voltage drop due to line inductance will be unbalanced. Consider one meter length of a three-phase line with three conductors, each with radius r. The Conductors are asymmetrically spaced with distances shown in Figure 4.8.



The application of (4.29) will result in the following flux linkages.

$$
\begin{aligned}
\lambda_a &= 2 \times 10^{-7} \left(I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D_{12}} + I_c \ln \frac{1}{D_{13}} \right) \\
\lambda_b &= 2 \times 10^{-7} \left(I_a \ln \frac{1}{D_{12}} + I_b \ln \frac{1}{r'} + I_c \ln \frac{1}{D_{23}} \right) \\
\lambda_c &= 2 \times 10^{-7} \left(I_a \ln \frac{1}{D_{13}} + I_b \ln \frac{1}{D_{23}} + I_c \ln \frac{1}{r'} \right)
\end{aligned}
\tag{4.34}
$$

or in matrix form

$$
\lambda = LI \tag{4.35}
$$

$$L = 2 \times 10^{-7} \begin{bmatrix} \ln \frac{1}{r'} & \ln \frac{1}{D_{12}} & \ln \frac{1}{D_{13}} \\ \ln \frac{1}{D_{12}} & \ln \frac{1}{r'} & \ln \frac{1}{D_{23}} \\ \ln \frac{1}{D_{13}} & \ln \frac{1}{D_{23}} & \ln \frac{1}{r'} \end{bmatrix} \tag{4.36}$$

For balanced three-phase currents with I_a as reference, we have

$$\begin{aligned} I_b &= I_a \angle 240^\circ = a^2 I_a \\ I_c &= I_a \angle 120^\circ = a I_a \end{aligned} \tag{4.37}$$

where the operator $a = 1\angle 120°$ and $a^2 = 1\angle 240°$. Substituting in (4.34) results in

$$\begin{aligned} L_a &= \frac{\lambda_a}{I_a} = 2 \times 10^{-7}\left(\ln\frac{1}{r'} + a^2 \ln\frac{1}{D_{12}} + a \ln\frac{1}{D_{13}}\right) \\ L_b &= \frac{\lambda_b}{I_b} = 2 \times 10^{-7}\left(a \ln\frac{1}{D_{12}} + \ln\frac{1}{r'} + a^2 \ln\frac{1}{D_{23}}\right) \\ L_c &= \frac{\lambda_c}{I_c} = 2 \times 10^{-7}\left(a^2 \ln\frac{1}{D_{13}} + a \ln\frac{1}{D_{23}} + \ln\frac{1}{r'}\right) \end{aligned} \tag{4.38}$$

4.7.3 TRANSPOSE LINE

A per-phase model of the transmission line is required in most power system analysis. One way to regain symmetry in good measure and obtain a per-phase model is to consider transposition. This consists of interchanging the phase configuration every one-third the length so that each conductor is moved to occupy the next physical position in a regular sequence. Such a transposition arrangement is shown in Figure 4.9.

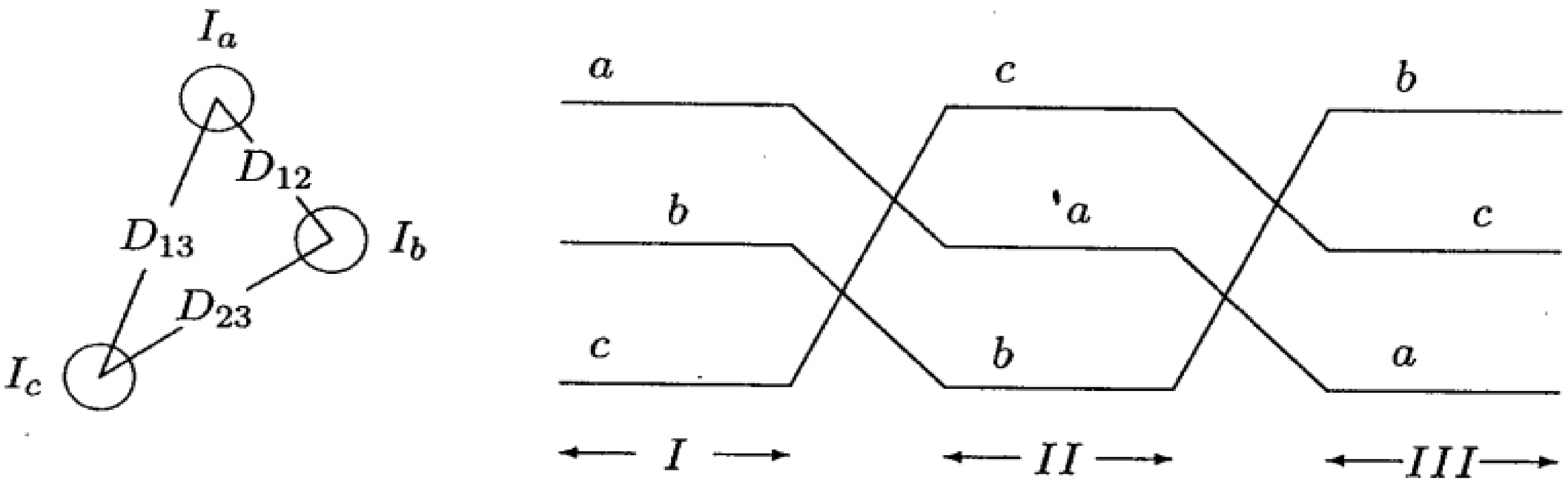


FIGURE 4.9
A transposed three-phase line.

Since in a transposed line each phase takes all three positions, the inductance per phase can be obtained by finding the average value of (4.38).

$$L = \frac{L_a + L_b + L_c}{3} \tag{4.39}$$

Noting $a + a^2 = 1\angle 120° + 1\angle 240° = -1$, the average of (4.38) becomes

$$L = \frac{2 \times 10^{-7}}{3}\left(3 \ln \frac{1}{r'} - \ln \frac{1}{D_{12}} - \ln \frac{1}{D_{23}} - \ln \frac{1}{D_{13}}\right)$$

or

$$L = 2 \times 10^{-7} \left(\ln \frac{1}{r'} - \ln \frac{1}{(D_{12} D_{23} D_{13})^{\frac{1}{3}}} \right)$$

$$= 2 \times 10^{-7} \ln \frac{(D_{12} D_{23} D_{13})^{\frac{1}{3}}}{r'} \tag{4.40}$$

or the inductance per phase per kilometer length is

$$L = 0.2 \ln \frac{GMD}{D_S} \quad \text{mH/km} \tag{4.41}$$

where

$$GMD = \sqrt[3]{D_{12} D_{23} D_{13}} \tag{4.42}$$

This again is of the same form as the expression for the inductance of one phase of a single-phase line. GMD (geometric mean distance) is the equivalent conductor spacing. For the above three-phase line this is the cube root of the product of the three-phase spacings. D_s is the geometric mean radius, GMR. For stranded conductor D_s is obtained from the manufacturer's data. For solid conductor, $D_s = r' = re^{-\frac{1}{4}}$.

4.8 INDUCTANCE OF COMPOSITE CONDUCTORS

In the evaluation of inductance, solid round conductors were considered. However, in practical transmission lines, stranded conductors are used. Also, for reasons of economy, most EHV lines are constructed with bundled conductors. In this section an expression is found for the inductance of composite conductors. The result can be used for evaluating the GMR of stranded or bundled conductors. It is also useful in finding the equivalent GMR and GMD of parallel circuits. Consider a single-phase line consisting of two composite conductors x and y as shown in Figure 4.10. The current in x is I referenced into the page, and the return current in y is $-I$. Conductor x consists of n identical strands or subconductors, each with radius r_x. Conductor y consists of m identical strands or subconductors, each with radius r_y. The current is assumed to be equally divided among the subconductors. The current per strand is I/n in x and I/m in y. The application of (4.29) will result in the following expression for the total flux linkage of conductor a

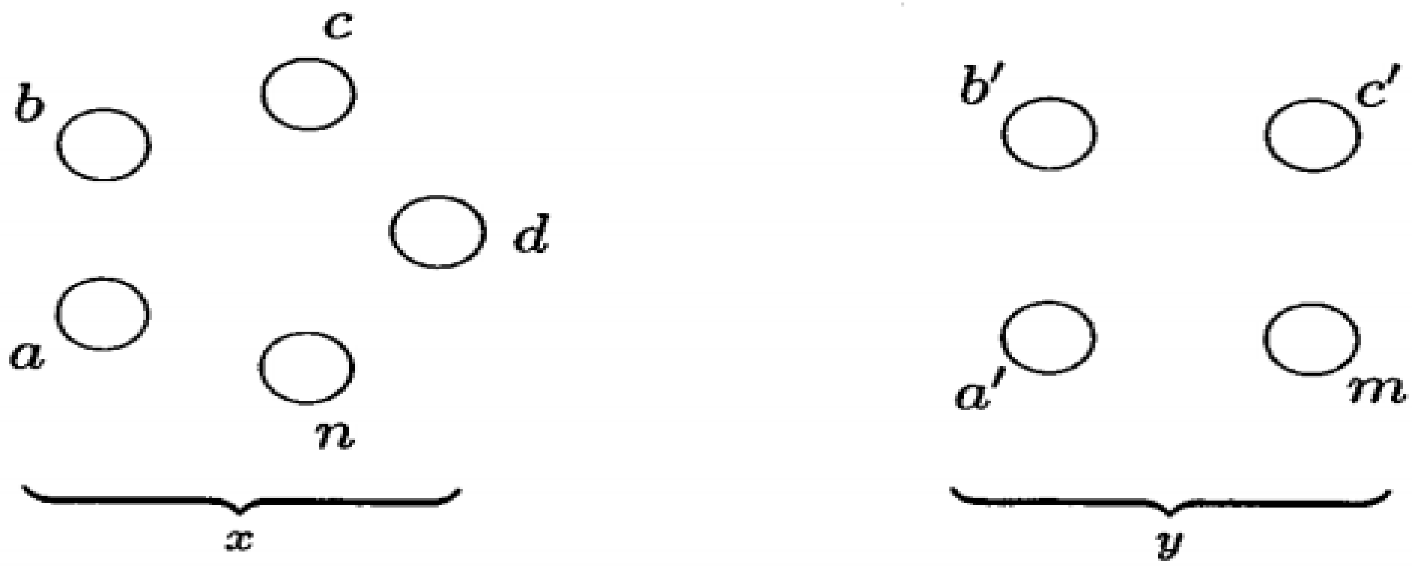


FIGURE 4.10
Single-phase line with two composite conductors.

$$\lambda_a = 2 \times 10^{-7} \frac{I}{n} \left(\ln \frac{1}{r'_x} + \ln \frac{1}{D_{ab}} + \ln \frac{1}{D_{ac}} + \cdots + \ln \frac{1}{D_{an}} \right)$$
$$-2 \times 10^{-7} \frac{I}{m} \left(\ln \frac{1}{D_{aa'}} + \ln \frac{1}{D_{ab'}} + \ln \frac{1}{D_{ac'}} + \cdots + \ln \frac{1}{D_{am}} \right)$$

or

$$\lambda_a = 2 \times 10^{-7} I \ln \frac{\sqrt[m]{D_{aa'} D_{ab'} D_{ac'} \cdots D_{am}}}{\sqrt[n]{r'_x D_{ab} D_{ac} \cdots D_{an}}} \tag{4.43}$$

The inductance of subconductor a is

$$L_a = \frac{\lambda_a}{I/n} = 2n \times 10^{-7} \ln \frac{\sqrt[m]{D_{aa'} D_{ab'} D_{ac'} \cdots D_{am}}}{\sqrt[n]{r'_x D_{ab} D_{ac} \cdots D_{an}}} \tag{4.44}$$

Using (4.29), the inductance of other subconductors in x are similarly obtained. For example, the inductance of the subconductor n is

$$L_n = \frac{\lambda_n}{I/n} = 2n \times 10^{-7} \ln \frac{\sqrt[m]{D_{na'} D_{nb'} D_{nc'} \cdots D_{nm}}}{\sqrt[n]{r'_x D_{na} D_{nb} \cdots D_{nc}}} \tag{4.45}$$

The average inductance of any one subconductor in group x is

$$L_{av} = \frac{L_a + L_b + L_c + \cdots + L_n}{n} \tag{4.46}$$

Since all the subconductors of conductor x are electrically parallel, the inductance of x will be

$$L_x = \frac{L_{av}}{n} = \frac{L_a + L_b + L_c + \cdots + L_n}{n^2} \tag{4.47}$$

substituting the values of L_a, L_b, $L_c, \cdots, L_n$ in (4.47) results in

$$L_x = 2 \times 10^{-7} \ln \frac{GMD}{GMR_x} \text{ H/meter} \tag{4.48}$$

where

$$GMD = \sqrt[mn]{(D_{aa'} D_{ab'} \cdots D_{am}) \cdots (D_{na'} D_{nb'} \cdots D_{nm})} \tag{4.49}$$

and

$$GMR_x = \sqrt[n^2]{(D_{aa} D_{ab} \cdots D_{an}) \cdots (D_{na} D_{nb} \cdots D_{nn})} \tag{4.50}$$

where $D_{aa} = D_{bb} \cdots = D_{nn} = r'_x$

GMD is the mnth root of the product of the mnth distances between n strands of conductor x and m strands of conductor y. GMR_x is the n^2 root of the product of n^2 terms consisting of r' of every strand times the distance from each strand to all other strands within group x.

Example 4.1

A stranded conductor consists of seven identical strands each having a radius r as shown in Figure 4.11. Determine the GMR of the conductor in terms of r.

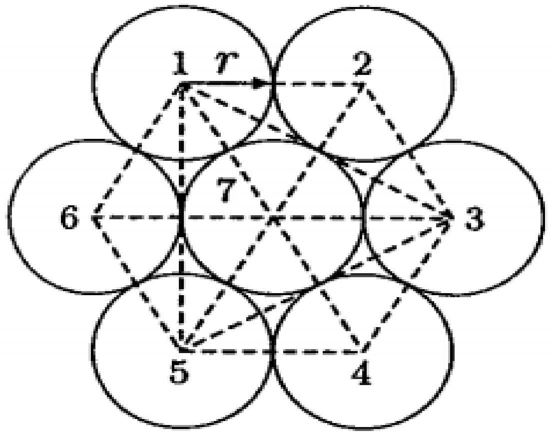


FIGURE 4.11
Cross section of a stranded conductor.

From Figure 4.11, the distance from strand 1 to all other strands is:

$$D_{12} = D_{16} = D_{17} = 2r$$
$$D_{14} = 4r$$
$$D_{13} = D_{15} = \sqrt{D_{14}^2 - D_{45}^2} = 2\sqrt{3}\,r$$

From (4.50) the GMR of the above conductor is

$$GMR = \sqrt[49]{(r' \cdot 2r \cdot 2\sqrt{3}\,r \cdot 4r \cdot 2\sqrt{3}\,r \cdot 2r \cdot 2r)^6 \cdot r'(2r)^6}$$

$$= r\sqrt[7]{(e)^{-\frac{1}{4}}\,(2)^{6}\,(3)^{\frac{6}{7}}\,(2)^{\frac{6}{7}}}$$
$$= 2.1767r$$

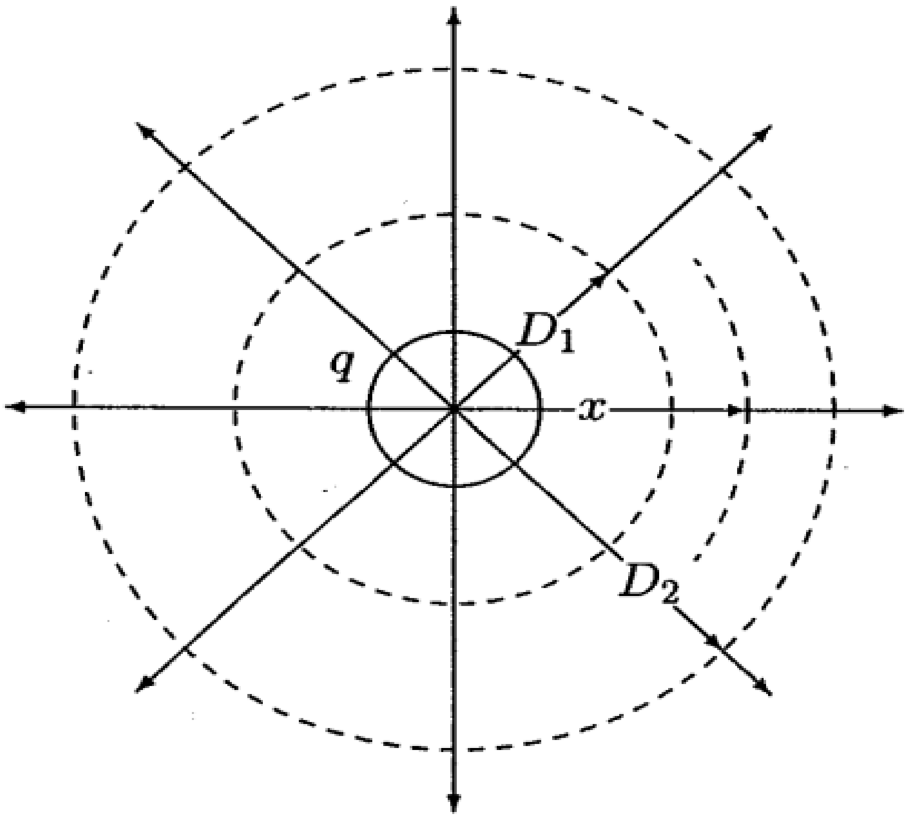
With a large number of strands the calculation of GMR can become very tedious. Usually these are available in the manufacturer's data.

4.10 LINE CAPACITANCE

Transmission line conductors exhibit capacitance with respect to each other due to the potential difference between them. The amount of capacitance between conductors is a function of conductor size, spacing, and height above ground. By definition, the capacitance C is the ratio of charge q to the voltage V, given by

$$C = \frac{q}{V} \tag{4.59}$$

Consider a long round conductor with radius r, carrying a charge of q coulombs per meter length as shown in Figure 4.14.



conductor. The intensity of the field at any point is defined as the force per unit charge and is termed *electric field intensity* designated as E. Concentric cylinders surrounding the conductor are equipotential surfaces and have the same electric flux density. From Gauss's law, for one meter length of the conductor, the electric flux density at a cylinder of radius x is given by

$$D = \frac{q}{A} = \frac{q}{2\pi x(1)} \tag{4.60}$$

The electric field intensity E may be found from the relation

$$E = \frac{D}{\varepsilon_0} \tag{4.61}$$

where ε_0 is the permittivity of free space and is equal to 8.85×10^{-12} F/m. Substituting (4.60) in (4.61) results in

$$E = \frac{q}{2\pi\varepsilon_0 x} \tag{4.62}$$

The potential difference between cylinders from position D_1 to D_2 is defined as the work done in moving a unit charge of one coulomb from D_2 to D_1 through the electric field produced by the charge on the conductor. This is given by

$$V_{12} = \int_{D_1}^{D_2} E dx = \int_{D_1}^{D_2} \frac{q}{2\pi\varepsilon_0 x} dx = \frac{q}{2\pi\varepsilon_0} \ln \frac{D_2}{D_1} \tag{4.63}$$

The notation V_{12} implies the voltage drop from 1 relative to 2, that is, 1 is understood to be positive relative to 2. The charge q carries its own sign.

4.11 CAPACITANCE OF SINGLE-PHASE LINES

Consider one meter length of a single-phase line consisting of two long solid round conductors each having a radius r as shown in Figure 4.15. The two conductors are separated by a distance D. Conductor 1 carries a charge of q_1 coulombs/meter and conductor 2 carries a charge of q_2 coulombs/meter. The presence of the second conductor and ground disturbs the field of the first conductor. The distance of separation of the wires D is great with respect to r and the height of conductors is much larger compared with D. Therefore, the distortion effect is small and the charge is assumed to be uniformly distributed on the surface of the conductors.

Assuming conductor 1 alone to have a charge of q_1, the voltage between conductor 1 and 2 is

$$V_{12(q_1)} = \frac{q_1}{2\pi\varepsilon_0} \ln \frac{D}{r} \tag{4.64}$$

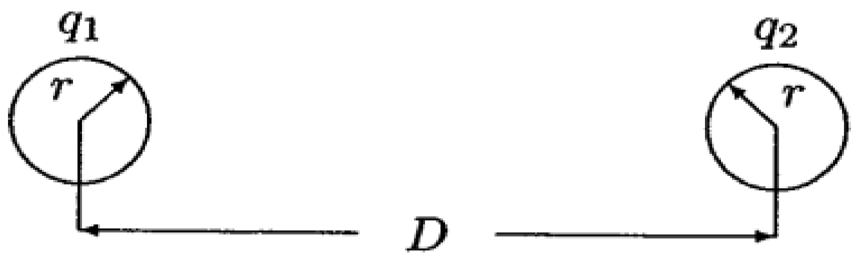


FIGURE 4.15
Single-phase two-wire line.

Now assuming only conductor 2, having a charge of q_2, the voltage between conductors 2 and 1 is

$$V_{21(q_2)} = \frac{q_2}{2\pi\varepsilon_0} \ln \frac{D}{r}$$

Since $V_{12(q_2)} = -V_{21(q_2)}$, we have

$$V_{12(q_2)} = \frac{q_2}{2\pi\varepsilon_0} \ln \frac{r}{D} \tag{4.65}$$

From the principle of superposition, the potential difference due to presence of both charges is

$$V_{12} = V_{12(q_1)} + V_{12(q_2)} = \frac{q_1}{2\pi\varepsilon_0} \ln \frac{D}{r} + \frac{q_2}{2\pi\varepsilon_0} \ln \frac{r}{D} \tag{4.66}$$

For a single-phase line $q_2 = -q_1 = -q$, and (4.66) reduces to

$$V_{12} = \frac{q}{\pi\varepsilon_0} \ln \frac{D}{r} \quad \text{F/m} \tag{4.67}$$

From (4.59), the capacitance between conductors is

$$C_{12} = \frac{\pi \varepsilon_0}{\ln \frac{D}{r}} \quad \text{F/m} \tag{4.68}$$

Equation (4.68) gives the line-to-line capacitance between the conductors. For the purpose of transmission line modeling, we find it convenient to define a capacitance C between each conductor and a neutral as illustrated in Figure 4.16. Since the

voltage to neutral is half of V_{12}, the capacitance to neutral $C = 2C_{12}$, or

$$C = \frac{2\pi\varepsilon_0}{\ln\frac{D}{r}} \quad \text{F/m} \tag{4.69}$$

Recalling $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m and converting to μF per kilometer, we have

$$C = \frac{0.0556}{\ln\frac{D}{r}} \quad \mu\text{F/km} \tag{4.70}$$

4.13 CAPACITANCE OF THREE-PHASE LINES

Consider one meter length of a three-phase line with three long conductors, each with radius r, with conductor spacing as shown Figure 4.18.

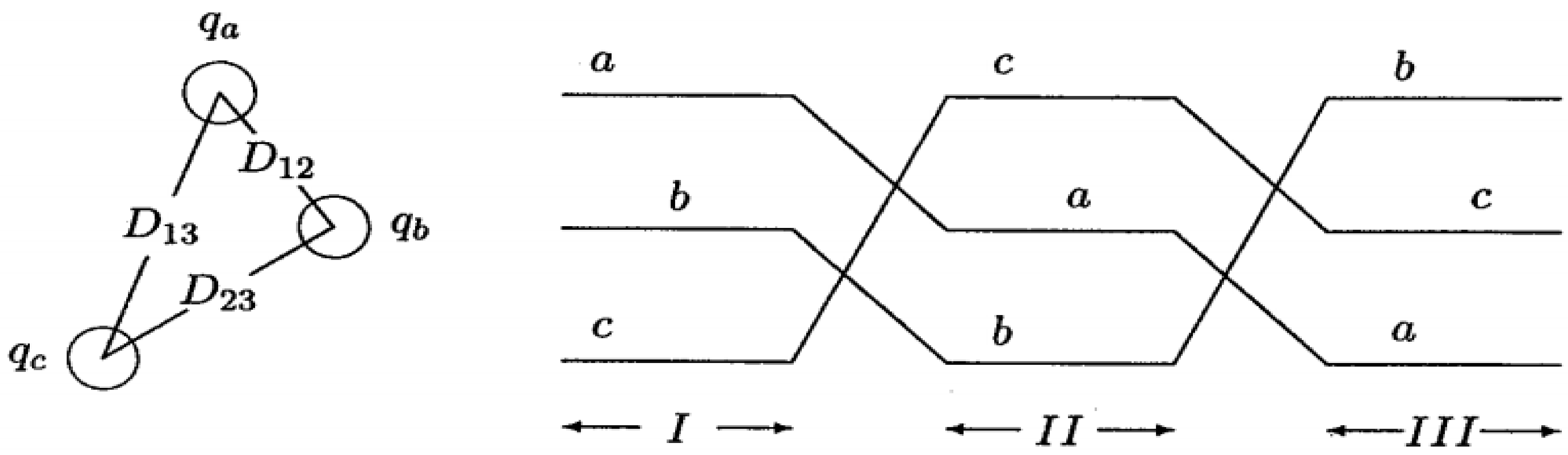


FIGURE 4.18
Three-phase transmission line.

Since we have a balanced three-phase system

$$q_a + q_b + q_c = 0 \tag{4.73}$$

We shall neglect the effect of ground and the shield wires. Assume that the line is transposed. We proceed with the calculation of the potential difference between a and b for each section of transposition. Applying (4.72) to the first section of the transposition, V_{ab} is

$$V_{ab(I)} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\frac{D_{12}}{r} + q_b \ln\frac{r}{D_{12}} + q_c \ln\frac{D_{23}}{D_{13}}\right) \tag{4.74}$$

Similarly, for the second section of the transposition, we have

$$V_{ab(II)} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\frac{D_{23}}{r} + q_b \ln\frac{r}{D_{23}} + q_c \ln\frac{D_{13}}{D_{12}}\right) \tag{4.75}$$

and for the last section

$$V_{ab(III)} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\frac{D_{13}}{r} + q_b \ln\frac{r}{D_{13}} + q_c \ln\frac{D_{12}}{D_{23}}\right) \tag{4.76}$$

The average value of V_{ab} is

$$V_{ab} = \frac{1}{(3)2\pi\varepsilon_0}\left(q_a \ln \frac{D_{12}D_{23}D_{13}}{r^3} + q_b \ln \frac{r^3}{D_{12}D_{23}D_{13}} \right.$$
$$\left. + q_c \ln \frac{D_{12}D_{23}D_{13}}{D_{12}D_{23}D_{13}}\right) \tag{4.77}$$

or

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln \frac{(D_{12}D_{23}D_{13})^{\frac{1}{3}}}{r} + q_b \ln \frac{r}{(D_{12}D_{23}D_{13})^{\frac{1}{3}}}\right) \tag{4.78}$$

Note that the GMD of the conductor appears in the logarithm arguments and is given by

$$GMD = \sqrt[3]{D_{12}D_{23}D_{13}} \tag{4.79}$$

Therefore, V_{ab} is

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln \frac{GMD}{r} + q_b \ln \frac{r}{GMD}\right) \tag{4.80}$$

Similarly, we find the average voltage V_{ac} as

$$V_{ac} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln \frac{GMD}{r} + q_c \ln \frac{r}{GMD}\right) \tag{4.81}$$

Adding (4.80) and (4.81) and substituting for $q_b + q_c = -q_a$, we have

$$V_{ab} + V_{ac} = \frac{1}{2\pi\varepsilon}\left(2q_a \ln \frac{GMD}{r} - q_a \ln \frac{r}{GMD}\right) = \frac{3q_a}{2\pi\varepsilon_0} \ln \frac{GMD}{r} \tag{4.82}$$

For balanced three-phase voltages,

$$\begin{aligned} V_{ab} &= V_{an}\angle 0^\circ - V_{an}\angle -120^\circ \\ V_{ac} &= V_{an}\angle 0^\circ - V_{an}\angle -240^\circ \end{aligned} \tag{4.83}$$

Therefore,

$$V_{ab} + V_{ac} = 3V_{an} \tag{4.84}$$

Substituting in (4.82) the capacitance per phase to neutral is

$$C = \frac{q_a}{V_{an}} = \frac{2\pi\varepsilon_0}{\ln \frac{GMD}{r}} \quad \text{F/m} \tag{4.85}$$

or capacitance to neutral in μF per kilometer is

$$C = \frac{0.0556}{\ln \frac{GMD}{r}} \quad \mu\text{F/km} \tag{4.86}$$