Lec_7

Gauss Siedle Method

Solution of Non-Linear Equations

The two *load flow equations* are:

$$P_i = |V_i| \sum_{p=1}^{n} |Y_{ip}||V_p| \cos(\delta_i - \delta_p - \gamma_{ip})$$

$$Q_i = |V_i| \sum_{p=1}^{n} |Y_{ip}||V_p| \sin(\delta_i - \delta_p - \gamma_{ip})$$

These equations provide *the calculated value* of *net* real power and *net* reactive power entering bus 'i'. The equations are non-linear and only a numerical solution is possible. There are different methods could be implemented to solve these equations. Among those are the Gauss-Seidel and Newton-Raphson methods.

Gauss-Seidel Method

Consider a system of non-linear equations having "n" unknowns $x_1, x_2, \ldots\ldots\ldots, x_n$

$$f_1(x_1, x_2, \ldots\ldots\ldots, x_n)$$

$$f_2(x_1, x_2, \ldots\ldots\ldots, x_n)$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots$$

$$f_n(x_1, x_2, \ldots\ldots\ldots, x_n)$$

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Rearranging, then

$$x_i = f_i\,(x_1, x_2, \ldots\ldots\ldots, x_n) \qquad \text{Eq. 1}$$

$$1 \le i \le n$$

Assuming initial values,

$$x_1^o, x_2^o, \ldots\ldots\ldots, x_n^o$$

Substituting the initial values in *Eq. 1*, then

All values are initial values $x_1^o, x_2^o, \ldots\ldots\ldots, x_n^o$

first iteration

first variable

$$x_1^1 = f_1^1(x_1^o, x_2^o, \ldots\ldots\ldots, x_n^o)$$

$x_1 = x_1^1$ from previous step and all other values are initial values $x_2^o, \ldots\ldots\ldots, x_n^o$

$$x_2^1 = f_2^1(x_1^1, x_2^o, \ldots\ldots\ldots, x_n^o)$$

$$x_3^1 = f_3^1(x_1^1, x_2^1, x_3^o \ldots\ldots\ldots, x_n^o)$$

$x_1 = x_1^1$ & $x_2 = x_2^1$ and $x_3^o, \ldots\ldots\ldots, x_n^o$

Or in general

$$x_i^1 = f_i^1(x_1^1, x_2^1, \ldots\ldots, x_i^o, \ldots\ldots, x_n^o)$$

Where 12:08 x_i^1 is the first approximation of x_i using the initial assumed values.

The k^{th} approximation of x_i is:

K^{th} iteration

$$x_i^k = f_i^k(x_1^k, x_2^k, \ldots\ldots x_{i-1}^k, x_i^{k-1}, x_{i+1}^{k-1} \ldots\ldots, x_n^{k-1})$$

i^{th} variable

The changes in the magnitude of each variable x_i^k from its value x_i^{k-1} at the previous iteration is:

$$\Delta x_i = x_i^k - x_i^{k-1}$$

If $\Delta x_i < \varepsilon$ then the solution has converged.

Where, ε is a small value *(for exmple : $\varepsilon = 0.001$)*

EXAMPLE 1:

For the following equation, find an accurate value for x up to 5 decimal places.

$$2x - \log(x) = 7$$

SOLUTION:

Using Gauss-Seidel

$$x = 0.5(7 + \log x)$$

$x^o = 1$ $\quad x^1 = 0.5(7 + \log 1) = 3.5$ $\quad$ 1st iteration

$x^1 = 3.5$ $\quad x^2 = 0.5(7 + \log 3.5) = 3.772034$ $\quad$ 2nd iteration

$x^2 = 3.772034$ $\quad x^3 = 0.5(7 + \log 3.772034) = 3.788287$

$x^3 = 3.788287$ $\quad x^4 = 0.5(7 + \log 3.788287) = 3.789221$

$$x^5 = 3.789274$$

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$x^6 = 3.789278$ $\quad \varepsilon = 0.000004$

EXAMPLE 2:

For the following equations, find an x and y after 4 iterations.

$$x = 0.7 \sin x + 0.2 \cos y \qquad \& \qquad y = 0.7 \cos x - 0.2 \sin y$$

SOLUTION:

Using Gauss-Seidel, assuming initial values

$$x^o = y^o = 0.5 \quad (rad)$$

$$x^1 = 0.7 \sin x^o + 0.2 \cos y^o$$

$$x^1 = 0.7 \sin 0.5 + 0.2 \cos 0.5$$

$$x^1 = 0.51111$$

$$y^1 = 0.7 \cos 0.51111 - 0.2 \sin 0.5$$

$$y^1 = 0.51465$$

$$x^2 = 0.516497 \qquad y^2 = 0.510241$$

$$x^3 = 0.520211 \qquad y^3 = 0.509722$$

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$$x^4 = 0.522520 \qquad y^4 = 0.509007$$

Gauss-Seidel Method for Load Flow Analysis

Advantages

1. ***Simplicity***
2. ***Small computer memory requirement***
3. ***Less computational time per iteration***

Disadvantages

1. ***Slow rate of convergence, and therefore large number of iterations.***
2. ***Increase in the number of iterations as the number of system buses increases.***
3. ***The speed of convergence is affected by the selected slack bus.***

I - G-S Method when PV buses are absent

Assuming a power system in which the voltage *controlled buses are absent*. If the system has *n* buses, then; one bus will be considered as a slack bus and the other *n-1* buses are load buses (***PQ-buses***).

For the Slack or Swing Bus:

$$|V_i| \text{ and } \delta_i = 0 \text{ are known } \& \text{ } P_i \text{ and } Q_i \text{ are unknown}$$

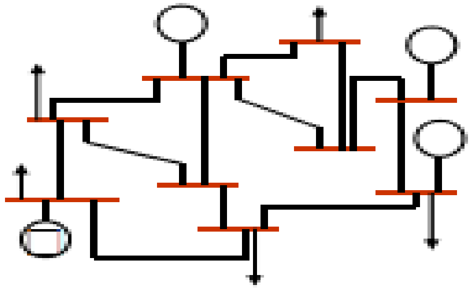
The swing bus voltage is taken as a reference. It is voltage magnitude is known and its phase shift angle is set equal to zero.

For *(n-1)* Load Buses (PQ bus):

$$P_i \text{ and } Q_i \text{ are known } \& \text{ } |V_i| \text{ and } \delta_i \text{ are unknown}$$

Using Gauss-Seidel Method, we assume the ***initial values*** for the magnitude and phase shift angle of *(n-1)* buses. These values are ***updated at each iteration***.

For an 'n' bus system



$$I_{bus} = Y_{bus}\ V_{bus} \qquad \text{..... Eq. 1}$$

For the i^{th} bus of an 'n' bus system, the current entering this bus is:

$$I_i = Y_{i1}\ V_1\ + Y_{i2}\ V_2\ + \ldots\ldots + Y_{ii}\ V_i\ + \ldots Y_{in}\ V_n \qquad \text{..... Eq. 2}$$

$$I_i = Y_{ii} V_i\ + \sum_{\substack{p=1 \\ p \neq i}}^{n} Y_{ip}\, V_p \qquad \text{..... Eq. 3}$$

$$V_i\ = \frac{1}{Y_{ii}} \left(I_i - \sum_{\substack{p=1 \\ p \neq i}}^{n} Y_{ip}\, V_p \right) \qquad \text{..... Eq. 4}$$

In power systems, power is known rather than currents. The complex power injected into the i^{th} bus is:

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$$S_i = P_i + jQ_i = V_i \; I_i^* \qquad \text{..... } Eq.\ 5$$

$$S_i^* = V_i^* I_i \qquad \text{..... } Eq.\ 6$$

OR

$$I_i = \frac{P_i - jQ_i}{V_i^*} \qquad \text{..... } Eq.\ 7$$

Substituting in *Eq. 4*

$$V_i = \frac{1}{Y_{ii}} \left(\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{p=1 \\ p \neq i}}^{n} Y_{ip} V_p \right) \qquad \text{..... } Eq.\ 8$$

Since *bus 1 is the slack bus* "reference", then V_i represents *n-1* set of equations for *i= 2, 3, ..., n*. These equations will be solved using G-S method for the unknowns V_2, V_3, ...V_n.

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NOTES:

1. *Eq. 8* can be written as:

> *NOTE*
> *The values for P and Q are the scheduled values for PQ Bus.*

$$V_i = \frac{1}{V_i^*}\frac{P_i - jQ_i}{Y_{ii}} - \sum_{\substack{p=1 \\ p\neq i}}^{n} \frac{Y_{ip}}{Y_{ii}} V_p \qquad \text{..... Eq. 9}$$

$$V_i = \frac{K_i}{V_i^*} - \sum_{\substack{p=1 \\ p\neq i}}^{n} L_{ip} V_p \qquad \text{..... Eq. 10}$$

$$K_i = \frac{P_i - jQ_i}{Y_{ii}} \qquad \text{and} \qquad L_{ip} = \frac{Y_{ip}}{Y_{ii}}$$

The values for K_i and L_{ip} are computed once in the beginning and used in every iteration.

2. The voltages at all the buses in a power system are close to *1.0* pu. Therefore, we can start the G-S iteration process assuming initial values for the voltages equal to 1.0 and making zero angle.

$$V_2^o = V_3^o = \ldots\ldots V_n^o = 1\angle 0$$

3. 12:09 At each step in the iteration process use the *most updated values* for the voltages to compute the new values for the bus voltages.

$$V_i = \frac{K_i}{V_i^*} - \sum_{\substack{p=1 \\ p \neq i}}^{n} L_{ip} V_p \qquad \text{..... Eq. 11}$$

$$V_i = \frac{K_i}{V_i^*} - \sum_{p=1}^{i-1} L_{ip} V_p - \sum_{p=i+1}^{n} L_{ip} V_p \qquad \text{..... Eq. 12}$$

Therefore, for the ($k^{th}+1$) iteration,

The most updated voltage values are from the previous iteration

$$V_i^{k+1} = \frac{K_i}{(V_i^k)^*} - \sum_{p=1}^{i-1} L_{ip} V_p^{(k+1)} - \sum_{p=i+1}^{n} L_{ip} V_p^k \qquad \text{..... Eq. 13}$$

The most updated voltage values are from the same iteration

$$\text{for } i = 1, 2, \ldots\ldots\ldots n$$

The iteration process is continuous till the convergence occurs, i.e.;

$$|\Delta V_i^{k+1}| = |V_i^{k+1}| - |V_i^k| < \varepsilon \qquad \text{..... Eq. 14}$$

$$\text{for } i = 1, 2, \ldots\ldots\ldots n$$

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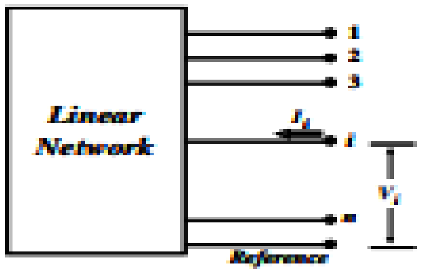
4. The *current and complex power* at i^{th} bus are:



$$I_i = Y_{i1} V_1 + Y_{i2} V_2 + \ldots\ldots + Y_{ii} V_i + \ldots Y_{in} V_n$$

And

$$S_i = P_i + jQ_i = V_i I_i^*$$

$$S_i^* = P_i - Q_i = V_i^* I_i$$

Or

$$P_i - jQ_i = V_i^* (Y_{i1} V_1 + Y_{i2} V_2 + \ldots\ldots + Y_{ii} V_i + \ldots Y_{in} V_n)$$

$$P_i = Re\{ V_i^* (Y_{i1} V_1 + Y_{i2} V_2 + \ldots\ldots + Y_{ii} V_i + \ldots Y_{in} V_n)\}$$

$$Q_i = - Im\{ V_i^* (Y_{i1} V_1 + Y_{i2} V_2 + \ldots\ldots + Y_{ii} V_i + \ldots Y_{in} V_n)\}$$

The two equations are known as the *rectangular form* of the *load flow equations*. They provide *the calculated value* of *net* real power and *net* reactive power *entering* bus '*i*'.

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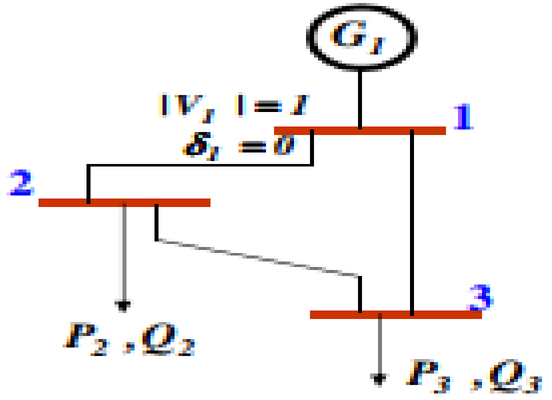
EXAMPLE 3:

For the three bus system. Write the expression for the bus voltages using GS method.



SOLUTION:

The system contains 3 buses, ($n=3$).

i- Select bus 1 as a slack bus "reference".

$$|V_1| = 1 \ and \ \delta_1 = 0$$

ii- Buses 2 and 3 are load buses.

$$P_2, P_3, Q_2 \ and \ Q_3 \ are \ known$$

$$V_2, V_3, \delta_2 \ and \ \delta_3 \ are \ unknown$$

$$V_2 = \frac{K_2}{V_2^*} - \sum_{\substack{p=1 \\ p\neq 2}}^{3} L_{2p} V_p$$

$$V_2 = \frac{1}{V_2^*} \frac{P_2 - jQ_2}{Y_{22}} - \sum_{\substack{p=1 \\ p\neq 2}}^{3} \frac{Y_{2p}}{Y_{22}} V_p$$

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NOTE
The values for P and Q are the scheduled values

$$V_2 = \frac{1}{V_2^*}\frac{P_2 - jQ_2}{Y_{22}} - \left[\frac{Y_{21}}{Y_{22}}V_1 + \frac{Y_{23}}{Y_{22}}V_3\right] \quad \text{..... Eq. 15}$$

$$V_3 = \frac{1}{V_3^*}\frac{P_3 - jQ_3}{Y_{33}} - \left[\frac{Y_{31}}{Y_{33}}V_1 + \frac{Y_{32}}{Y_{33}}V_2\right] \quad \text{..... Eq. 16}$$

Using GS method, *select the initial values* for the unknowns as:

$$V_2^o = V_3^o = 1\angle 0$$

Start the first iteration

The most updated voltage value is the initial value

$$V_2^1 = \frac{1}{(V_2^o)^*}\frac{P_2 - jQ_2}{Y_{22}} - \left[\frac{Y_{21}}{Y_{22}}V_1 + \frac{Y_{23}}{Y_{22}}V_3^o\right] \quad \text{..... Eq. 17}$$

The most updated voltage value is from this iteration

$$V_3^1 = \frac{1}{(V_3^o)^*}\frac{P_3 - jQ_3}{Y_{33}} - \left[\frac{Y_{31}}{Y_{33}}V_1 + \frac{Y_{32}}{Y_{33}}V_2^1\right] \quad \text{..... Eq. 18}$$

Start the second iteration

$$V_2^2 = \frac{1}{(V_2^1)^*}\frac{P_2 - jQ_2}{Y_{22}} - \left[\frac{Y_{21}}{Y_{22}}V_1 + \frac{Y_{23}}{Y_{22}}V_3^1\right] \quad \text{..... Eq. 19}$$

The most updated voltage value is from previous iteration

$$V_3^2 = \frac{1}{(V_3^1)^*}\frac{P_3 - jQ_3}{Y_{33}} - \left[\frac{Y_{31}}{Y_{33}}V_1 + \frac{Y_{32}}{Y_{33}}V_2^2\right] \quad \text{..... Eq. 20}$$

The most updated voltage value is from this iteration

Compare the results for convergence

$$|\Delta V_i^{k+1}| = |V_i^{k+1}| - |V_i^k| \langle \ \varepsilon \qquad for \ \ i = 1,2,\dots\dots\dots n$$

$$|\Delta V_2^2| = |V_2^2| - |V_2^1| \langle \ \varepsilon \quad \text{..... Eq. 21}$$

$$|\Delta V_3^2| = |V_3^2| - |V_3^1| \langle \ \varepsilon \quad \text{..... Eq. 22}$$

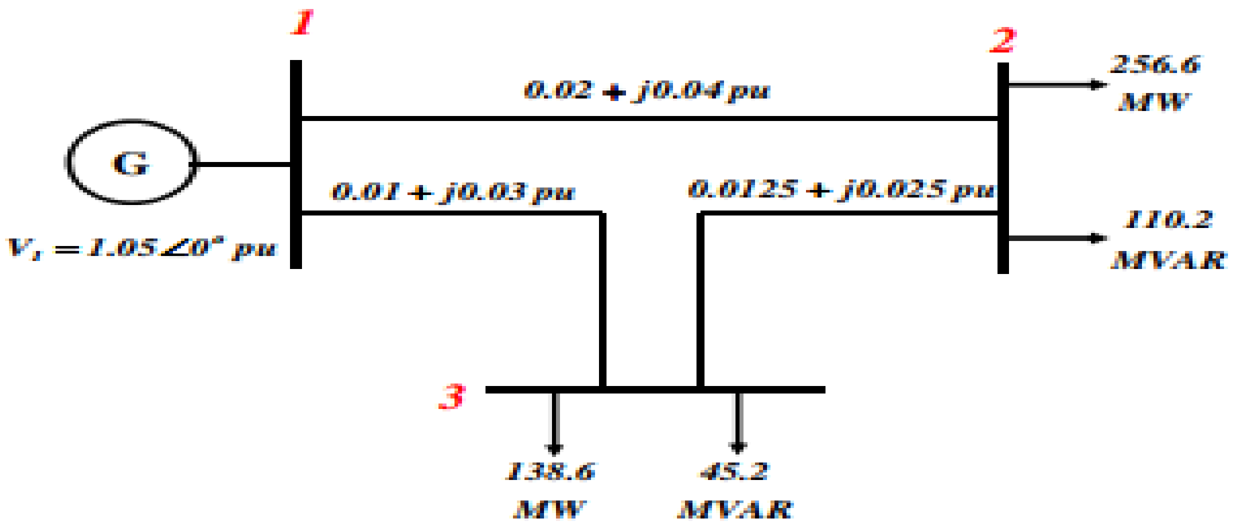
If *Eqs. 21, 22* are not satisfied then start a new iteration.

EXAMPLE 4:

For the system shown in the figure, the line impedances are as indicated in per unit on 100MVA base.

A. ***Using Gauss-Seidel method find the bus voltages after 7 iterations.***

B. ***Using the bus voltages find the Slack bus real and reactive power.***



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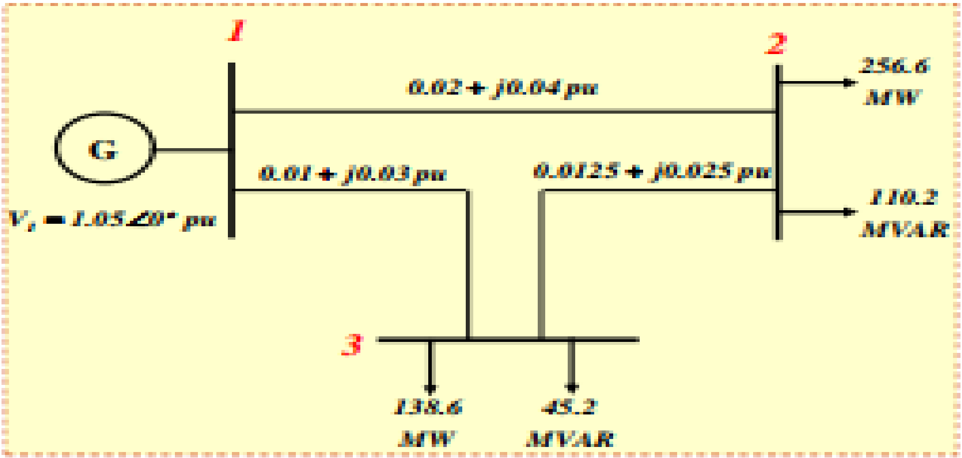
Formulation of the Bus Admittance Matrix

$$y_{12} = \frac{1}{0.02 + j0.04} = 10 - j20 = y_{21}$$

$$y_{13} = \frac{1}{0.01 + j0.03} = 10 - j30 = y_{31}$$

$$y_{23} = \frac{1}{0.0125 + j0.025} = 16 - j32 = y_{32}$$



$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} = \begin{bmatrix} y_{12} + y_{13} & -y_{12} & -y_{13} \\ -y_{21} & y_{21} + y_{23} & -y_{23} \\ -y_{31} & -y_{32} & y_{31} + y_{32} \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} 20 - j50 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j52 & -16 + j32 \\ -10 + j30 & -16 + j32 & 26 - j62 \end{bmatrix}$$

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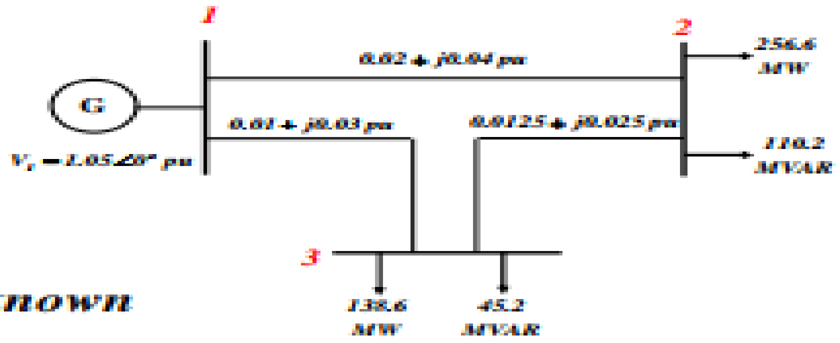
Classification of buses:



Bus 1: Slack Bus

$$V_1 = 1.05\angle 0^\circ \ pu$$

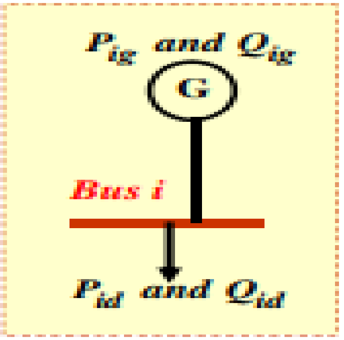
Buses 2 and 3: Load Buses (PQ bus)

P_2, P_3, Q_2 and Q_3 are known

V_2, V_3, δ_2 and δ_3 are unknown

$$P_{2,d} = 256.6MW \qquad Q_{2,d} = 110.2MVAR$$

$$P_{3,d} = 138.6MW \qquad Q_{3,d} = 45.2MVAR$$



$$P_{i,sch} = P_{gi} - P_{di}$$

&

$$Q_{i,sch} = Q_{gi} - Q_{di}$$

$$S_{i,sch} = P_{i,sch} + jQ_{i,sch}$$

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$$S_{2,sch} = (P_{2,g} - P_{2,d}) + j(Q_{2,g} - Q_{2,d})$$

$$S_{2,sch} = \frac{(P_{2,g} - P_{2,d}) + j(Q_{2,g} - Q_{2,d})}{Base\ MVA} \quad pu$$

$$S_{2,sch} = \frac{(0 - 256.6) + j(0 - 110.2)}{100MVA} \quad pu$$

$$S_{2,sch} = -2.566 - j1.102 \quad pu$$

$$S_{3,sch} = -1.386 - j0.452 \quad pu$$

Reminder
The bus admittance matrix
is

$$\begin{bmatrix} 20-j50 & -10+j20 & -10+j30 \\ -10+j20 & 26-j52 & -16+j32 \\ -10+j30 & -16+j32 & 26-j62 \end{bmatrix}$$

Using GS method, select the initial values for the unknowns as:

$$V_2^o = V_3^o = 1\angle 0$$

Start the first iteration

$$V_2^1 = \frac{1}{(V_2^o)^*}\frac{P_{2,sch} - jQ_{2,sch}}{Y_{22}} - \left[\frac{Y_{21}}{Y_{22}}V_1 + \frac{Y_{23}}{Y_{22}}V_3^o\right]$$

$S^*_{i,sch} = P_{i,sch} - jQ_{i,sch}$

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$$V_2^1 = \frac{1}{(1.0)^*}\frac{-2.566 + j1.102}{26 - j52} - \left[\frac{-10 + j20}{26 - j52}1.05 + \frac{-16 + j32}{26 - j52}1.0\right]$$

OR, to simplify the calculations, we have:

$$V_i = \frac{1}{V_i^*}\frac{P_i - jQ_i}{Y_{ii}} - \sum_{\substack{p=1\\p\neq i}}^{n}\frac{Y_{ip}}{Y_{ii}}V_p$$

$$V_2 = \frac{K_2}{V_2^*} - \sum_{\substack{p=1\\p\neq 2}}^{n} L_{2p}V_p$$

$$V_2^1 = \frac{K_2}{(V_2^o)^*} - \left[L_{21}V_1 + L_{23}V_3^o\right]$$

Reminder
The bus admittance matrix
is

$$\begin{bmatrix} 20-j50 & -10+j20 & -10+j30 \\ -10+j20 & 26-j52 & -16+j32 \\ -10+j30 & -16+j32 & 26-j62 \end{bmatrix}$$

The values for K_i and L_{ip} are *computed once* in the beginning and used in every iteration.

$$K_2 = \frac{P_2 - jQ_2}{Y_{22}} \qquad L_{21} = \frac{Y_{21}}{Y_{22}} \quad \textit{and} \quad L_{23} = \frac{Y_{23}}{Y_{22}}$$

K_2 = -0.0367 - j0.031 $\qquad$ L_{21} = -0.3846 $\qquad$ L_{23} = -0.6154

$$V_1 = 1.05\angle 0^o\ pu \quad \textit{and} \quad V_3^o = 1\angle 0$$

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$$V_2^1 = 0.9825 - j0.310$$

$$V_3^1 = \frac{K_3}{(V_3^o)^*} - \left[L_{31} V_1 + L_{32} V_2^1\right]$$

$$K_3 = \frac{P_3 - jQ_3}{Y_{33}} \quad and \quad L_{31} = \frac{Y_{31}}{Y_{33}} \quad and \quad L_{32} = \frac{Y_{32}}{Y_{33}}$$

K_3 = -0.0142 - j0.0164 L_{31} = - 0.4690 + 0.0354i L_{32} = -0.5310 - 0.0354i

$$V_1 = 1.05\angle 0^o \; pu \quad and \quad V_2^1 = 0.9825 - j0.310$$

$$V_3^1 = 1.0011 - j0.0353$$

Start the second iteration $K_2, K_3, L_{21}, L_{23}, L_{31}, L_{32}$ *constants will be the same.*

$$V_2^2 = \frac{K_2}{(V_2^1)^*} - \left[L_{21} V_1 + L_{23} V_3^1\right] \qquad V_2^2 = 0.9816 - j0.0520$$

$$V_3^2 = \frac{K_3}{(V_3^1)^*} - \left[L_{31} V_1 + L_{32} V_2^2\right] \qquad V_3^2 = 1.0008 - j0.0459$$

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Start the third iteration

$K_2, K_3, L_{21}, L_{23}, L_{31}, L_{32}$ *constants will be the same.*

$$V_2^3 = \frac{K_2}{(V_2^2)^*} - \left[L_{21} V_1 + L_{23} V_3^2\right] = 0.9808 - j0.0578$$

$$V_3^3 = \frac{K_3}{(V_3^2)^*} - \left[L_{31} V_1 + L_{32} V_2^3\right] = 1.0004 - j0.0488$$

Start the fourth iteration

$K_2, K_3, L_{21}, L_{23}, L_{31}, L_{32}$ *constants will be the same.*

$$V_2^4 = \frac{K_2}{(V_2^3)^*} - \left[L_{21} V_1 + L_{23} V_3^3\right] = 0.9803 - j0.0594$$

$$V_3^4 = \frac{K_3}{(V_3^3)^*} - \left[L_{31} V_1 + L_{32} V_2^4\right] = 1.0002 - j0.0497$$

After 7 iterations.

$$V_2^7 = 0.9800 - j0.0600 = 0.98183\angle -3.5035^o \ pu$$

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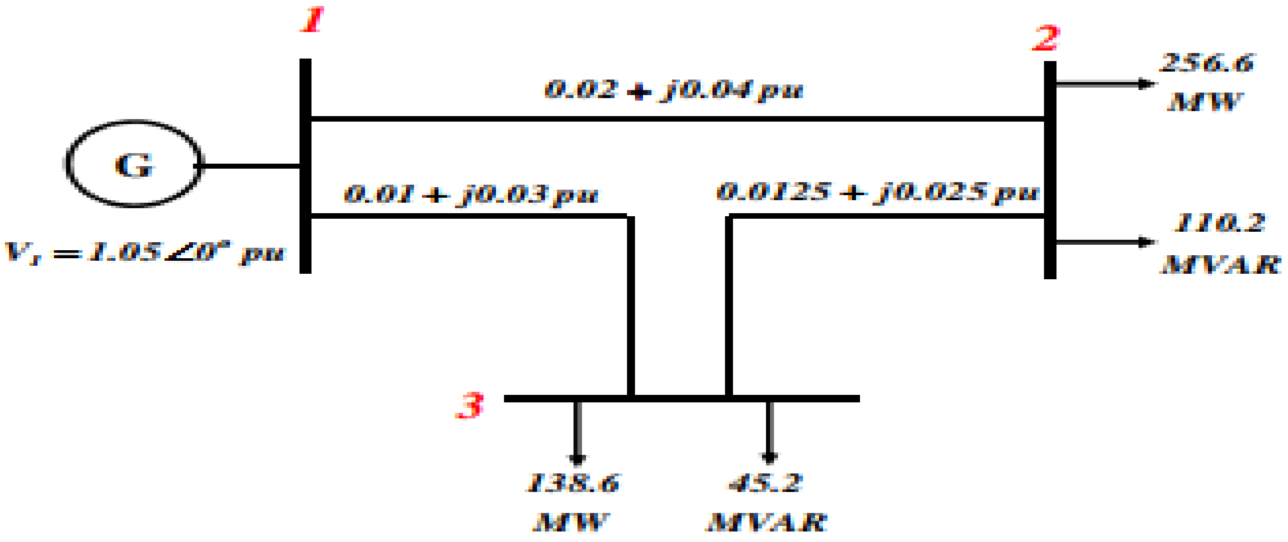
$$V_3^7 = 1.0000 - j0.0500 = 1.00125\angle -2.8624^o \ pu$$

B. Using the bus voltages find the Slack bus real and reactive power.



$$V_1 = 1.05 + j0.0^\circ \text{ pu}$$

$$V_2 = 0.9800 - j0.0600 = 0.98183\angle -3.5035^\circ \quad pu$$

$$V_3 = 1.0000 - j0.0500 = 1.00125\angle -2.8624^\circ \quad pu$$

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Using the *rectangular form* of the load flow equations, then the net active and reactive powers at 1^{th} bus are:

$$P_i = Re\{V_1^*(Y_{11} V_1 + Y_{12} V_2 + Y_{13} V_3)\}$$

$$Q_i = -Im\{V_1^*(Y_{11} V_1 + Y_{12} V_2 + Y_{13} V_3)\}$$

$$P_1 - jQ_1 = V_1^*(Y_{11} V_1 + Y_{12} V_2 + Y_{13} V_3)$$

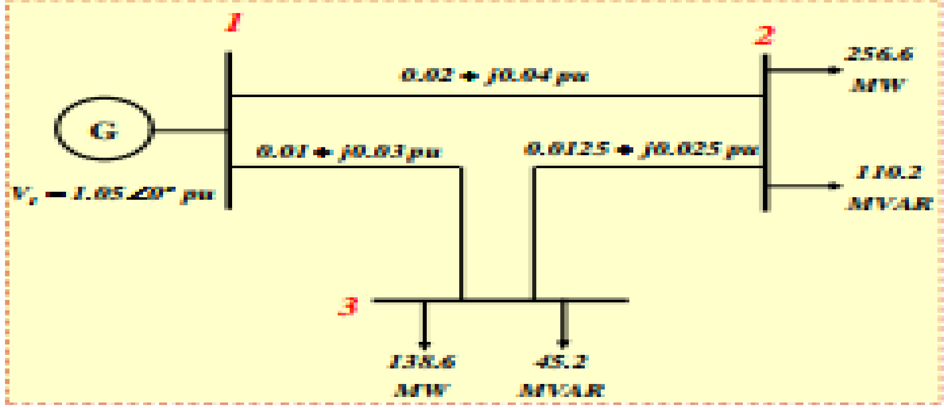
$$P_i - jQ_i = 4.0938 - j1.8894$$

Reminder
The bus admittance matrix is

$$\begin{bmatrix} 20-j50 & -10+j20 & -10+j30 \\ -10+j20 & 26-j52 & -16+j32 \\ -10+j30 & -16+j32 & 26-j62 \end{bmatrix}$$

$$P_1 = 4.0938\,pu$$

$$Q_1 = 1.8894\,pu$$



Base MVA=100

$$P_1 = 409.38\,MVA$$

$$Q_1 = 188.94\,MVA$$

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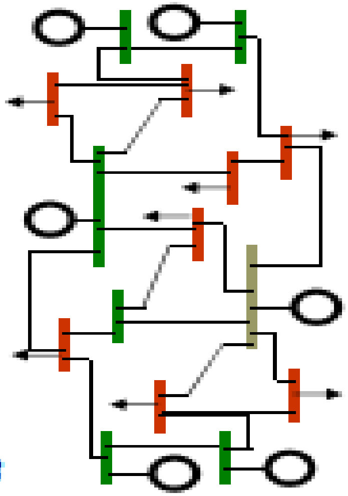
II. Modifying G-S Method when PV buses are present

Assuming a power system has *n* buses, then; one bus will be considered as a slack bus and the other buses are load buses (PQ-buses) and voltage controlled buses (PV-buses). Let the system buses be numbered as:



$$i = 1 \qquad \textit{Salck bus}$$

$$i = 2, 3, \ldots\ldots, m \qquad PV-buses$$

$$i = m+1, m+2, \ldots\ldots, n \qquad PQ-buses$$

For the voltage controlled buses,

$$P_i \text{ and } |V_i| \text{ are known } \quad \& \quad Q_i \text{ and } \delta_i \text{ are unknown}$$

$$|V_i| = |V_i|_{Specified} \qquad \ldots\ldots \textit{Eq. 23}$$

$$Q_{i,min} < Q_i < Q_{i,max} \qquad \ldots\ldots \textit{Eq. 24}$$

The second requirement for the voltage controlled bus may be violated if the bus voltage becomes too high or too small. It is to be noted that we can control the bus voltage by controlling the bus reactive power.

Therefore, during any iteration, if *the PV-bus* *reactive power* violates its limits then set it according to the following rule.

$$Q_i > Q_{i,max} \quad set \quad Q_i = Q_{i,max}$$

$$Q_i < Q_{i,min} \quad set \quad Q_i = Q_{i,min}$$

And *treat this bus as PQ-bus*.

NOTE
For PQ–bus
P_i *and* Q_i *are known*
& $|V_i|$ *and* δ_i *are unknown*

Load flow solution when PV buses are present

a. Calculate Q_i

In the polar form,

$$Q_i = |V_i| \sum_{p=1}^{n} |Y_{ip}||V_p| \sin(\delta_i - \delta_p - \gamma_{ip})$$

For the (k^{th}+1) iteration,

$$Q_i^{(k+1)} = |V_i|_{speci} \sum_{p=1}^{i-1} |Y_{ip}||V_p^{(k+1)}| \sin(\delta_i^{(k)} - \delta_p^{(k+1)} - \gamma_{ip})$$

$$+ |V_i|_{speci} \sum_{p=i}^{n} |Y_{ip}||V_p^{(k)}| \sin(\delta_i^{(k)} - \delta_i^{(k)} - \gamma_{ip})$$

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$$For \quad p = 1 \;\; to \;\; (i-1), \; use \; |V_p| \; \& \; \delta_p \;\; of \;\; (k^{th}+1) iteration$$

$$For \quad p = i \;\; to \;\; n, \; use \; |V_p| \; \& \; \delta_p \;\; of \;\; (k^{th}) iteration$$

$$Set \; |V_i| = |V_i|_{speci}$$

In the rectangular form,

$$P_i - jQ_i = V_i^*(Y_{i1}V_1 + Y_{i2}V_2 + \ldots\ldots + Y_{ii}V_i + \ldots Y_{in}V_n)$$

$$Q_i = -Im\{V_i^*(Y_{i1}V_1 + Y_{i2}V_2 + \ldots\ldots + Y_{ii}V_i + \ldots Y_{in}V_n)\}$$

b. Check Q_i^{k+1} *to see if it is within the limits*

$$Q_{i,min} \langle \; Q_i \; \langle \; Q_{i,max}$$

***Case 1: If the reactive power limits are not violated*,**
calculate V_i^{k+1}

$$V_i^{k+1} = \frac{K_i}{(V_i^k)^*} - \sum_{p=1}^{i-1} L_{ip} V_p^{(k+1)} - \sum_{p=i+1}^{n} L_{ip} V_p^k = |V_i^{k+1}|^{\angle \delta_i^{k+1}}$$

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Use $|V_i|_{speci}$ *and* δ_i^{k+1} For the *PV-bus voltage*.

Reset the magnitude $$|V_i^{k+1}| = |V_i|_{Speci}$$

Voltage magnitude is known for PV bus, therefore the new calculated magnitude will not be used.

$$V_i^{k+1} = |V_i|_{Speci}^{\angle \delta_i^{k+1}}$$

Only the calculated angle will be updated and used.

Case 2: If the reactive power limits are violated,

$$Q_i^{k+1} > Q_{i,max} \quad set \quad Q_i^{k+1} = Q_{i,max}$$

Or

$$Q_i^{k+1} < Q_{i,min} \quad set \quad Q_i^{k+1} = Q_{i,min}$$

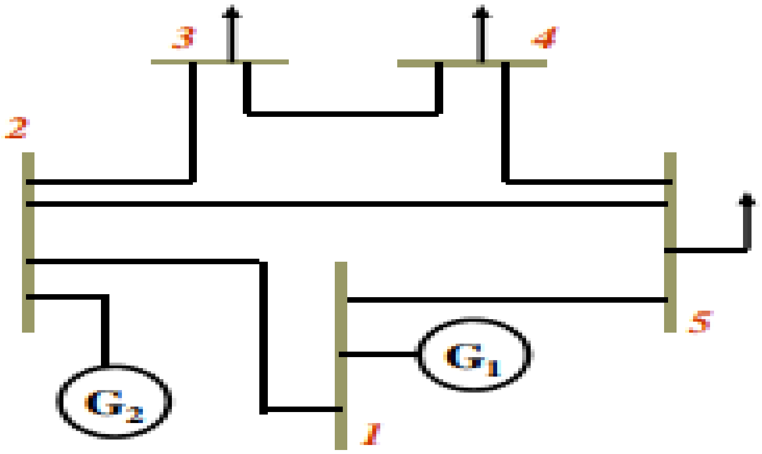
Consider this bus as a *PQ-Bus*, calculate bus voltage V_i^{k+1}

$$V_i^{k+1} = \frac{K_i}{(V_i^k)^*} - \sum_{p=1}^{i-1} L_{ip} V_p^{(k+1)} - \sum_{p=i+1}^{n} L_{ip} V_p^k$$

$$V_i^{k+1} = |V_i^{k+1}|^{\angle \delta_i^{k+1}}$$

The PV-bus becomes PQ-bus and both Voltage magnitude and angle are calculated and used

EXAMPLE 6:



Each line has an impedance of 0.05+j0.15

Line Data for the 5 buses Network

From Bus	To Bus	R	X
1	2	0.0500	0.1500
2	3	0.0500	0.1500
2	4	0.0500	0.1500
3	4	0.0500	0.1500
1	5	0.0500	0.1500
4	5	0.0500	0.1500

The shunt admittance is neglected

Bus Data for the the 5 buses Network Before load flow solution

Bus No.	Bus code	Volt Mag.	Volt Angle	Load MW	Load MVAR	Gen. MW	Gen. MVAR	Q Min.	Q Max.	Inject MVAR
1	Slack	1.0200	0	100	50	?	?	0	0	0
2	PV	1.0200	?	0	0	200	?	20	60	0
3	PQ	?	?	50	20	0	0	0	0	0
4	PQ	?	?	50	20	0	0	0	0	0
5	PQ	?	?	50	20	0	0	0	0	0

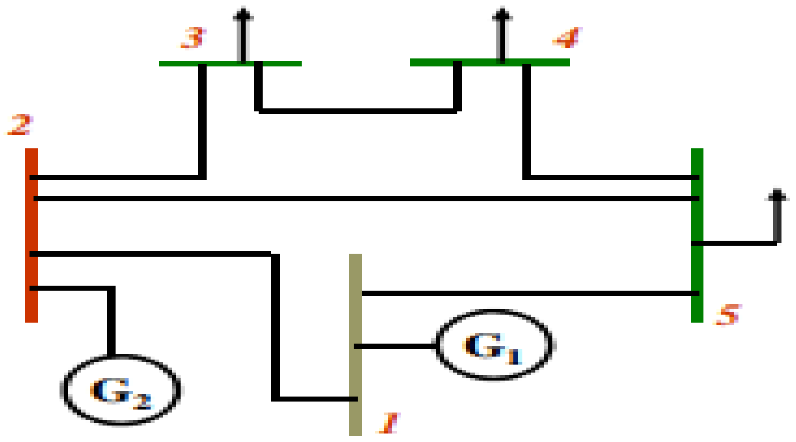
For the '5' bus system

Construct the bus admittance matrix Y_{bus}

Find Q_2, δ_2, V_3, V_4 and V_5

$$Q_{max} = 0.6\ pu$$

$$Q_{min} = 0.2\ pu$$



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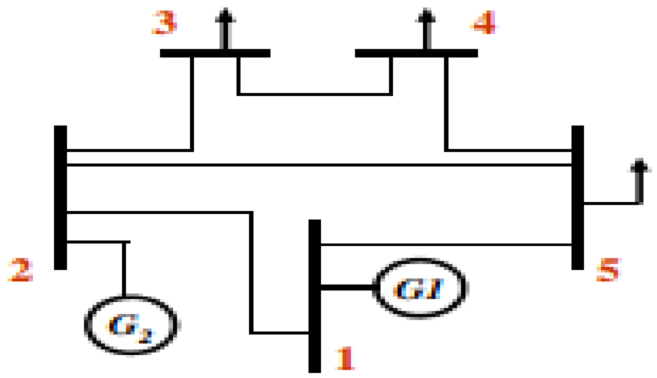
SOLUTION:

Y_{bus} Construction



$$y = \frac{1}{z} = \frac{1}{0.05 + j0.15} = 2 - j6$$

$$Y_{11} = y_{12} + y_{15} = 4 - j12$$

$$Y_{22} = y_{21} + y_{23} + y_{25} = 6 - j18$$

$$Y_{33} = y_{32} + y_{34} = 4 - j12$$

$$Y_{44} = y_{43} + y_{45} = 4 - j12$$

$$Y_{55} = y_{51} + y_{52} + y_{54} = 6 - j18$$

$$Y_{12} = -y_{12} = -2 + j6$$

$$Y_{15} = -y_{15} = -2 + j6$$

$$Y_{13} = Y_{14} = 0$$

$$Y_{bus} = \begin{bmatrix} 4.0 - J12.0 & -2.0 + J6.0 & 0 & 0 & -2.0 + J6.0 \\ -2.0 + J6.0 & 6.0 - J18.0 & -2.0 + J6.0 & 0 & -2.0 + J6.0 \\ 0 & -2.0 + J6.0 & 4.0 - J12.0 & -2.0 + J6.0 & 0 \\ 0 & 0 & -2.0 + J6.0 & 4.0 - J12.0 & -2.0 + J6.0 \\ -2.0 + J6.0 & -2.0 + J6.0 & 0 & -2.0 + J6.0 & 6.0 - J18.0 \end{bmatrix}$$

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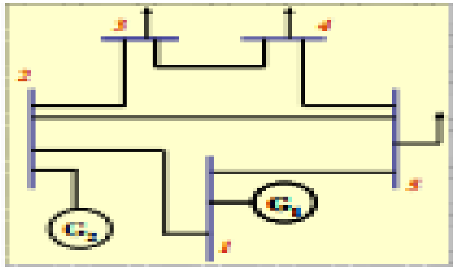
The net scheduled **power** ***injected*** **at each bus is:**



Bus No.	Bus code	Volt Mag.	Volt Angle	Load MW	Load MVAR	Gen. MW	Gen. MVAR	Q Min.	Q Max.	Inject MVAR
1	Slack	1.0200	0	100	50	?	?	0	0	0
2	PV	1.0200	?	0	0	200	?	20	60	0
3	PQ	?	?	50	20	0	0	0	0	0
4	PQ	?	?	50	20	0	0	0	0	0
5	PQ	?	?	50	20	0	0	0	0	0

$$S_{1,sch} = (P_{1,g} - P_{1,d}) + j(Q_{1,g} - Q_{1,d})$$

$$S_{1,sch} = (P_{1,g} - 1.0) + j(Q_{1,g} - 0.5)$$

$$S_{2,sch} = (P_{2,g} - P_{2,d}) + j(Q_{2,g} - Q_{2,d})$$

$$S_{2,sch} = (2.0 - 0) + j(Q_{2,g} - 0)$$

$$S_{3,sch} = (0 - 0.5) + j(0 - 0.2)$$

$$S_{3,sch} = -0.5 - j0.2$$

$$S_{4,sch} = -0.5 - j0.2$$

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$$S_{5,sch} = -0.5 - j0.2$$

The known values are:

$$V_1 = 1.02\angle 0^\circ$$

$$|V_2|_{spec} = 1.02$$

$$Q_{2,min} = 0.2 \quad \text{and} \quad Q_{2,max} = 0.6$$

The bus admittance matrix is

$$\begin{bmatrix} 4.0 - J12.0 & -2.0 + J6.0 & 0 & 0 & -2.0 + J6.0 \\ -2.0 + J6.0 & 6.0 - J18.0 & -2.0 + J6.0 & 0 & -2.0 + J6.0 \\ 0 & -2.0 + J6.0 & 4.0 - J12.0 & -2.0 + J6.0 & 0 \\ 0 & 0 & -2.0 + J6.0 & 4.0 - J12.0 & -2.0 + J6.0 \\ -2.0 + J6.0 & -2.0 + J6.0 & 0 & -2.0 + J6.0 & 6.0 - J18.0 \end{bmatrix}$$

Using GS method, *select the initial values for the unknowns* as:

$$V_3^o = V_4^o = V_5^o = 1\angle 0^o \quad \text{and} \quad \delta_2^o = 0$$

Start the first iteration

Bus 2 is PV Bus

Check Q_2 *is within the limits* $\quad Q_{2,min} < Q_2 < Q_{2,max}$

$$P_i - jQ_i = V_i^*(Y_{i1} V_1 + Y_{i2} V_2 + \dots\dots + Y_{ii} V_i + \dots Y_{in} V_n)$$

$$Q_2^1 = -Im\{V_2^*(Y_{21} V_1 + Y_{22} V_2^o + Y_{23} V_3^o + Y_{24} V_4^o + Y_{25} V_5^o)\}$$

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$$Q_2^1 = 0.2448$$

$$Q_{2,min} < Q_2 < Q_{2,max} \qquad i.e.; \quad 0.20 < 0.2448 < 0.6$$

The reactive power limits are not violated,

Calculate:

$$V_2^1 = \frac{K_2}{(V_2^o)^*} - \left[L_{21} V_1 + L_{23} V_3^o + L_{24} V_4^o + L_{25} V_5^o\right]$$

The values for K_i and L_{ip} are computed once in the beginning and used in every iteration.

$$K_2 = \frac{P_2 - jQ_2}{Y_{22}} \qquad L_{21} = \frac{Y_{21}}{Y_{22}} \qquad L_{23} = \frac{Y_{23}}{Y_{22}} \qquad L_{24} = \frac{Y_{24}}{Y_{22}} \qquad L_{25} = \frac{Y_{25}}{Y_{22}}$$

$S_{2,sch} = 2.0 + j0.2448$

$K_2 = 0.0456 + j0.0959$ $\quad L_{21} = -0.3333$ $\quad L_{23} = -0.3333$ $\quad L_{24} = 0.0$ $\quad L_{25} = -0.3333$

$$V_2^1 = 1.0555\angle 5.1113^o$$

Reset the magnitude

$$|V_2^1| = |V_2|_{Speci} = 1.02$$

Therefore,

$$\delta_2^1 = 5.1113^o$$

$$V_2^1 = 1.02\angle 5.1113^o$$

Voltage magnitude is known and fixed for a PV bus, therefore the new calculated magnitude will not be used.

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Bus 3 is PQ Bus

$$V_3^1 = \frac{K_3}{(V_3^o)^*} - \left[L_{31}V_1 + L_{32}V_2^1 + L_{34}V_4^o + L_{35}V_5^o\right]$$

$$K_3 = \frac{P_3 - jQ_3}{Y_{33}} \qquad L_{31} = \frac{Y_{31}}{Y_{33}} \qquad L_{32} = \frac{Y_{32}}{Y_{33}} \qquad L_{34} = \frac{Y_{34}}{Y_{33}} \qquad L_{35} = \frac{Y_{35}}{Y_{33}}$$

$K_3 =$ -0.0275 - j0.0325 $L_{31} =$ 0.0 $L_{32} =$ -0.5000 $L_{34} =$ -0.5000 $L_{35} =$ 0.0

$$V_3^1 = 0.9806\angle 0.7559^\circ$$

Bus 4 is PQ Bus

$$V_4^1 = \frac{K_4}{(V_4^o)^*} - \left[L_{41}V_1 + L_{42}V_2^1 + L_{43}V_3^1 + L_{45}V_5^o\right]$$

$$K_4 = \frac{P_4 - jQ_4}{Y_{44}} \qquad L_{41} = \frac{Y_{41}}{Y_{44}} \qquad L_{42} = \frac{Y_{42}}{Y_{44}} \qquad L_{43} = \frac{Y_{43}}{Y_{44}} \qquad L_{45} = \frac{Y_{45}}{Y_{44}}$$

$K_4 =$ -0.0275 - j0.0325 $L_{41} =$ 0.0 $L_{42} =$ 0.0 $L_{43} =$ -0.5000 $L_{45} =$ -0.5000

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$$V_4^1 = 0.9631\angle -1.5489^\circ$$

Bus 5 is PQ Bus

$$V_5^1 = \frac{K_5}{(V_5^o)^*} - \left[L_{51} V_1 + L_{52} V_2^1 + L_{53} V_3^3 + L_{54} V_4^1 \right]$$

$K_5 =$ -0.0183 - 0.0217i $L_{51} =$ -0.3333 $L_{52} =$ -0.3333 $L_{53} =$ 0.0 $L_{54} =$ -0.3333

$$V_5^1 = 0.9812\angle -0.0031^o$$

Start the second iteration

Bus 2 is PV Bus

Check Q_2 *is within the limits* $0.2 < Q_2 < 0.6$

$$Q_2^2 = -Im\{V_2^{1*}(Y_{21}\, V_1 + Y_{22}\, V_2^1 + Y_{23}\, V_3^1 + Y_{24}\, V_4^1 + Y_{25}\, V_5^1\,)\}$$

$$Q_2^2 = 0.0290$$

The reactive power limits are violated

$$Q_2 < Q_{i,min} \quad set \quad Q_2 = Q_{i,min} = 0.2$$

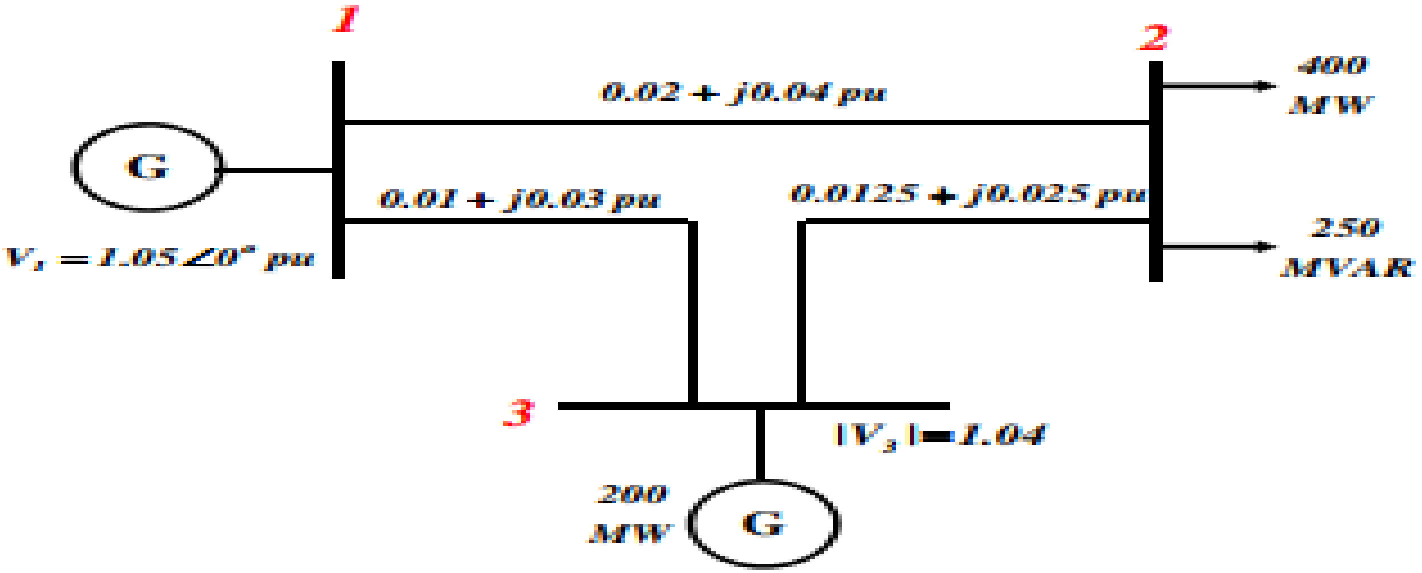
And *treat this bus as PQ-bus* $S_{2,sch} = 2.0 + j0.2$

Use the most updated value of Q_2 to calculate the constant K_2

All Buses 2, 3, 4 and 5 are PQ Buses. Find the bus voltages using GS method

Example 7:

A. The line impedances are as indicated in per unit on 100MVA base. Using Gauss-Seidel method find the power flow solution of the system. Ignoring the limits of Q_3.



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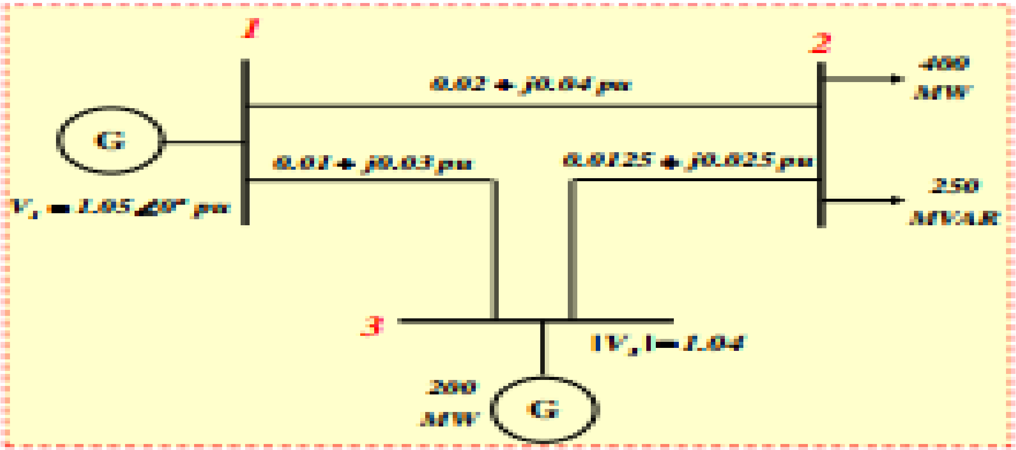
$$y_{12} = \frac{1}{0.02 + j0.04} = 10 - j20 = y_{21}$$

$$y_{13} = \frac{1}{0.01 + j0.03} = 10 - j30 = y_{31}$$

$$y_{23} = \frac{1}{0.0125 + j0.025} = 16 - j32 = y_{32}$$



$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} = \begin{bmatrix} y_{12} + y_{13} & -y_{12} & -y_{13} \\ -y_{21} & y_{21} + y_{23} & -y_{23} \\ -y_{31} & -y_{32} & y_{31} + y_{32} \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} 20 - j50 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j52 & -16 + j32 \\ -10 + j30 & -16 + j32 & 26 - j62 \end{bmatrix}$$

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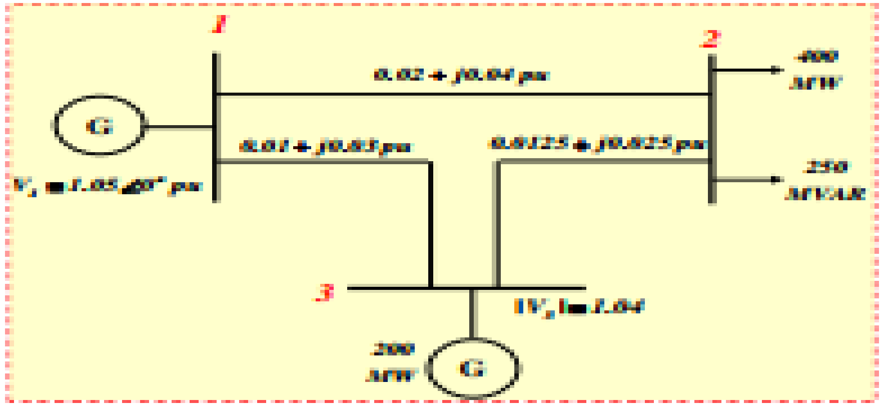
Classification of buses:



Bus 1: Slack Bus

$$V_1 = 1.05\angle 0^\circ \ pu$$

Bus 2: Load Bus (*PQ bus*)

P_2 *and* Q_2 *are known* V_2 *and* δ_2 *are unknown*

$$S_{2,sch} = \frac{(P_{2,g} - P_{2,d}) + j(Q_{2,g} - Q_{2,d})}{Base\ MVA} \ pu$$

$$S_{2,sch} = \frac{(0 - 400) + j(0 - 250)}{100} \ pu$$

$$S_{2,sch} = -4 - j2.5 \ pu$$

Bus 3: Voltage Controlled Bus (*PV bus*)

$|V_3|$ *and* $P_{g,3}$ *are known* $Q_{3,sch}$ *and* δ_3 *are unknown*

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$$P_{3,sch} = 2.0 \ pu$$

Using GS method, select the initial values for the unknowns as:

$V_1 = 1.05\angle 0^o \ pu$ $\quad V_2^o = 1\angle 0$ $\quad |V_3| = 1.04$ $\quad \delta_3^o = 0^o$

Start the first iteration

Bus 2 is PQ Bus

$$V_2 = \frac{K_2}{V_2^*} - \sum_{\substack{p=1 \\ p\neq 2}}^{n} L_{2p} V_p$$

$$V_2^1 = \frac{K_2}{(V_2^o)^*} - \left[L_{21} V_1 + L_{23} V_3^o\right]$$

K2 = - 0.0692 - j0.0423 L21 = - 0.3846 L23 = -0.6154

$$V_2^1 = 0.9746 \text{ - } j0.0423$$

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Bus 3 is PV Bus

$$V_3^1 = \frac{K_3}{(V_3^o)^*} - \left[L_{31} V_1 + L_{32} V_2^1\right]$$

K3 = 0.0274 + j0.0208 L31 = -0.4690 + j0.0354 L32 = -0.5310 - j0.0354

$$V_3^1 = 1.0378 - j0.0052 = 1.0378\angle -0.2854^o$$

Reset the magnitude

$$|V_3^1| = |V_i|_{Speci} = 1.04$$

$$V_3^1 = 1.04\angle -0.2854^o$$

$$V_3^1 = 1.0400 - j0.0052$$

Voltage magnitude is fixed for a PV bus, therefore the new calculated magnitude will not be used.

Start the second iteration

K_2, L_{21}, L_{23} are constants and will be the same.

Bus 2 is PQ Bus

$$V_2^2 = \frac{K_2}{(V_2^1)^*} - [L_{21}V_1 + L_{23}V_3^1]$$

$$V_2^2 = 0.9711 - j0.0434$$

Bus 3 is PV Bus

$$V_3^2 = \frac{K_3}{(V_3^1)^*} - [L_{31}V_1 + L_{32}V_2^2]$$

L_{31} and L_{32} are constants and will be the same.
K_3 is changed as Q_3 change

$$K_3 = \frac{P_3 - jQ_3^2}{Y_{33}}$$

K3 = 0.0305 + j0.0194

$$V_3^2 = 1.0391 - j0.0073 = 1.0391\angle -0.4028^\circ$$

Reset the magnitude

$$V_3^2 = 1.04\angle -0.4028^\circ = 1.0400 - j0.0073$$

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