Z-Bus

Lect_4

Zbus

- A power system network can be converted into an equivalent impedance diagram.
- This diagram forms the basis of power flow (or load flow) studies and short circuit analysis.
- **Formation of bus impedance matrix** (also known as Z_{bus} matrix).

$$Z_{bus} = Y_{bus}^{-1}$$

Elements Of The Bus Impedance And Admittance Matrices

- Bus impedance and admittance matrices are inverses of each other.
- Since Y_{bus} is a symmetric matrix, Z_{bus} is also a symmetric matrix.
- Voltage-current relations are given in terms of the Y_{bus} matrix for a 4 bus system

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

- Y_{11} is the admittance measured at bus-1 when buses 2, 3 and 4 are short circuited. The admittance Y_{11} is defined as the **self admittance** at bus-1

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=V_3=V_4=0}$$

- The off diagonal elements are denoted as the **mutual admittances**. The mutual admittance between buses 1 and 2 is defined as Y_{12}

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=V_3=V_4=0}$$

- The mutual admittance Y_{12} is obtained as the ratio of the current injected in bus-1 to the voltage of bus-2 when buses 1, 3 and 4 are short circuited.

$$I = YV$$

$$V = IZ$$

$$Z = Y_{bus}^{-1}$$

$$\boxed{Z_{Matrix} = Y_{\substack{bus \\ Matrix}}^{-1}}$$

$$Y_{11}^{-1} \neq Z_{11}$$

$$Y_{12}^{-1} \neq Z_{12}$$

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=V_3=V_4=0}$$

$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=V_3=V_4=0}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 + Z_{13} I_3 + Z_{14} I_4$$

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=I_3=I_4=0}$$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=I_3=I_4=0}$$

- The voltage-current relation can be written in terms of the Z_{bus} matrix as

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

- The **driving point impedance** at bus-1 is then defined as

$$Z_{11} = \left. \frac{\mathrm{V_1}}{I_1} \right|_{I_2=I_3=I_4=0}$$

- The **transfer impedance** between buses 1 and 2 can be obtained by injecting a current at bus-2 while open-circuiting buses 1, 3 and 4 as

$$Z_{12} = \left. \frac{\mathrm{V_1}}{I_2} \right|_{I_1=I_3=I_4=0}$$

- Z_{11} is not the reciprocal of Y_{11} . Z_{12} is not the reciprocal of Y_{12} .

Modification of Bus Impedance Matrix

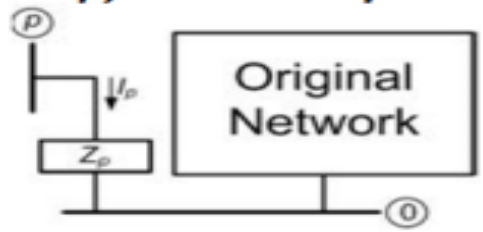
- Four possible cases by which an existing bus impedance matrix can be modified.
- Voltage-current relations in terms of the bus impedance matrix for an n-bus system are given as

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = Z_{orig} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

- The aim is to modify the matrix Z_{orig} when a new bus or line is connected to the power system.

Adding a New Bus to the Reference Bus

- It is assumed that a new bus p (p > n) is added to the reference bus through an impedance Z_p.

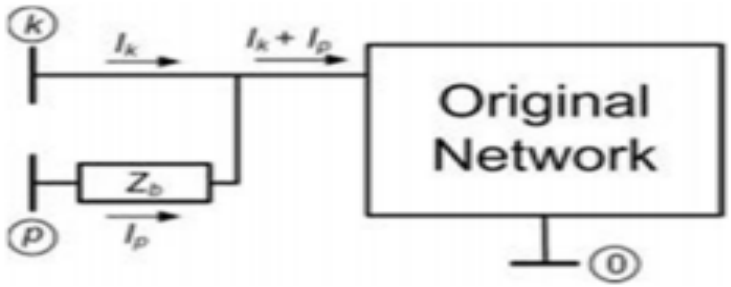


- This bus is only connected to the reference bus, the voltage-current relations the new system are

$$\begin{bmatrix} V_1 \\ \vdots \\ V_n \\ V_p \end{bmatrix} = \begin{bmatrix} & Z_{orig} & & 0 \\ & & & \vdots \\ & & & 0 \\ 0 & \cdots & 0 & Z_p \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_n \\ I_p \end{bmatrix} = Z_{new} \begin{bmatrix} I_1 \\ \vdots \\ I_n \\ I_p \end{bmatrix}$$

Adding a New Bus to an Existing Bus through an Impedance

- A bus, which has not been a part of the original network, is added to an existing bus through a transmission line with an impedance of Z_b .
- Let us assume that p ($p > n$) is the new bus that is connected to bus k ($k < n$) through Z_b.



- Note from this figure that the current I_p flowing from bus p will alter the voltage of the bus k .

Adding a New Bus to an Existing Bus through an Impedance

- that the current I_p flowing from bus p will alter the voltage of the bus k.

$$V_k = Z_{k1}I_1 + Z_{k2}I_2 + \cdots + Z_{kk}(I_k + I_p) + \cdots + Z_{kn}I_n$$

- In a similar way the current I_p will also alter the voltages of all the other buses as

$$V_i = Z_{i1}I_1 + Z_{i2}I_2 + \cdots + Z_{ik}(I_k + I_p) + \cdots + Z_{in}I_n \quad i \neq k$$

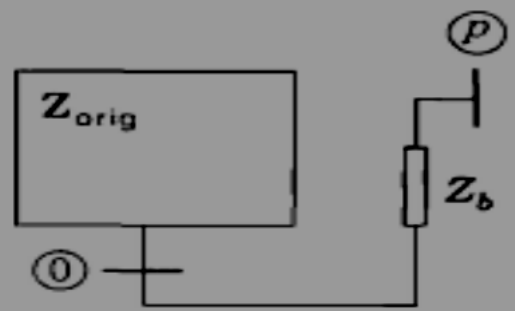
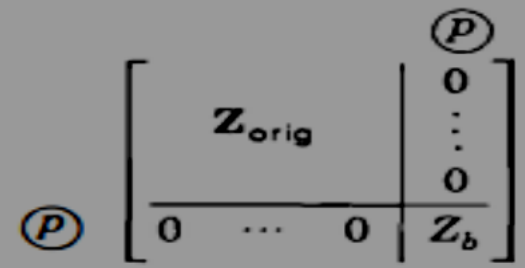
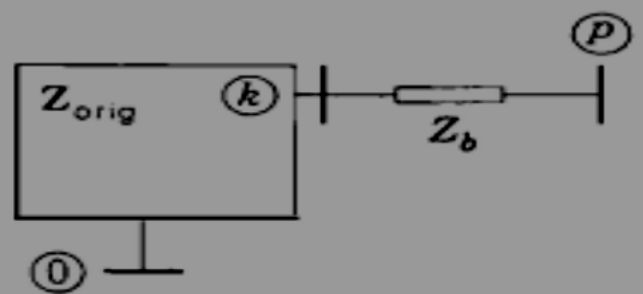
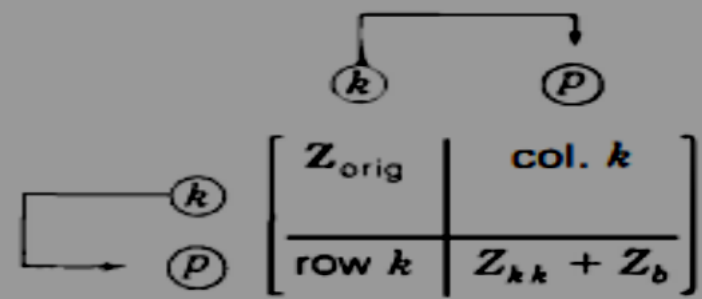
- Furthermore the voltage of the bus p is given by

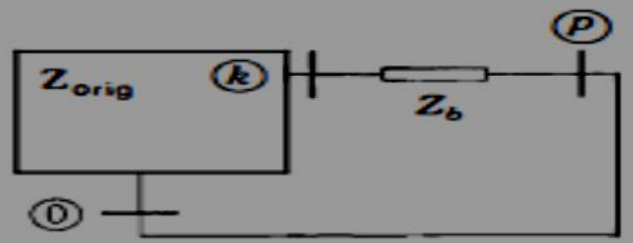
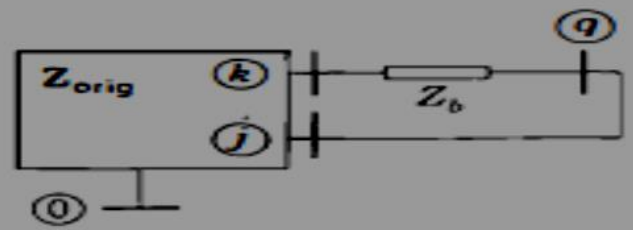
$$V_p = V_k + Z_b I_p$$

$$V_p = Z_{k1}I_1 + Z_{k2}I_2 + \cdots + Z_{kk}I_k + \cdots + Z_{kn}I_n + (Z_{kk}+Z_b)I_p$$

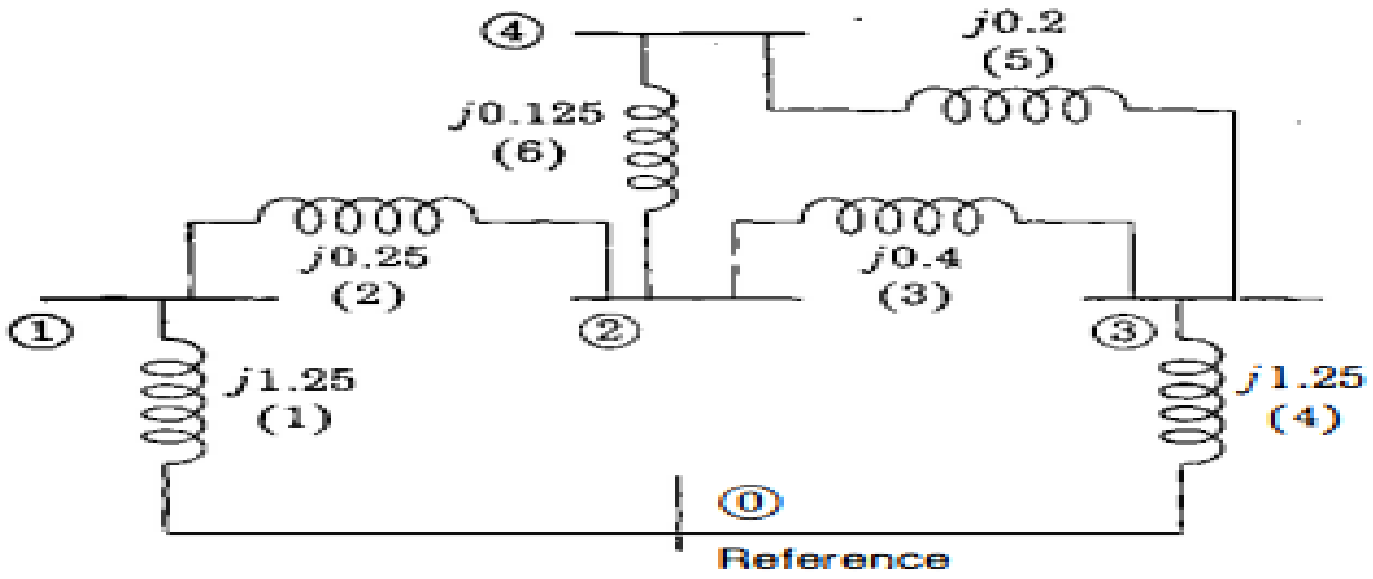
- Therefore the new voltage current relations are

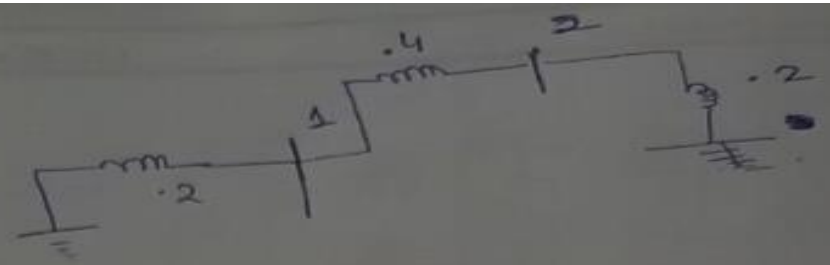
$$\begin{bmatrix} V_1 \\ \vdots \\ V_n \\ V_p \end{bmatrix} = \begin{bmatrix} & Z_{orig} & & Z_{k1} \\ & & & \vdots \\ & & & Z_{kn} \\ Z_{k1} & \dots & Z_{kn} & Z_{kk} + Z_b \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_n \\ I_p \end{bmatrix} = Z_{new} \begin{bmatrix} I_1 \\ \vdots \\ I_n \\ I_p \end{bmatrix}$$

Case	Add branch Z_b from	$Z_{bus\ (new)}$
1	Reference node to new bus (p) 	
2	Existing bus (k) to new bus (p) 	

3	Existing bus (k) to reference node  (Node (p) is temporary.)	• Repeat Case 2 and • Remove row p and column p by Kron reduction
4	Existing bus (j) to existing bus (k)  (Node (q) is temporary.)	• Form the matrix $$\begin{array}{c} \\ q \end{array}\begin{bmatrix} Z_{orig} & \begin{matrix} q \\ \text{col. } j - \text{col. } k \end{matrix} \\ \hline \text{row } j - \text{row } k & Z_{th.,jk} + Z_b \end{bmatrix}$$ where $Z_{th.,jk} = Z_{jj} + Z_{kk} - 2Z_{jk}$ and • Remove row q and column q by Kron reduction

Example 8.4. Determine $\mathbf{Z}_{bus}$ for the network shown in Fig. 8.8, where the impedances labeled 1 through 6 are shown in per unit. Preserve all buses.





$$Z_1 = [.2]$$

new Bus added

$$Z_2 = \left[\begin{array}{c|c} .2 & .2 \\ \hline .2 & .2+.4 \end{array}\right]$$

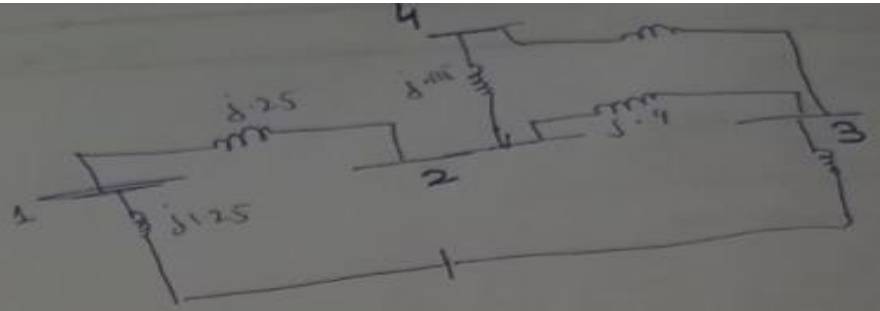
$$Z_2 = \begin{bmatrix} .2 & .2 \\ .2 & .6 \end{bmatrix}$$

Add to reference Bus

$$Z_3 = \left[\begin{array}{cc|c} .2 & .2 & .2 \\ .2 & .6 & .6 \\ \hline .2 & .6 & .8 \end{array}\right]$$

Kron Reduction

↓



1 to 2 [add new bus].

Case 2. 2 to 3 [Add new bus].

Case 3 3 to 0. [Reference bus].

3 to 4 [Add new bus].

Case 4. 4 to 2 [Add bus to existing bus]

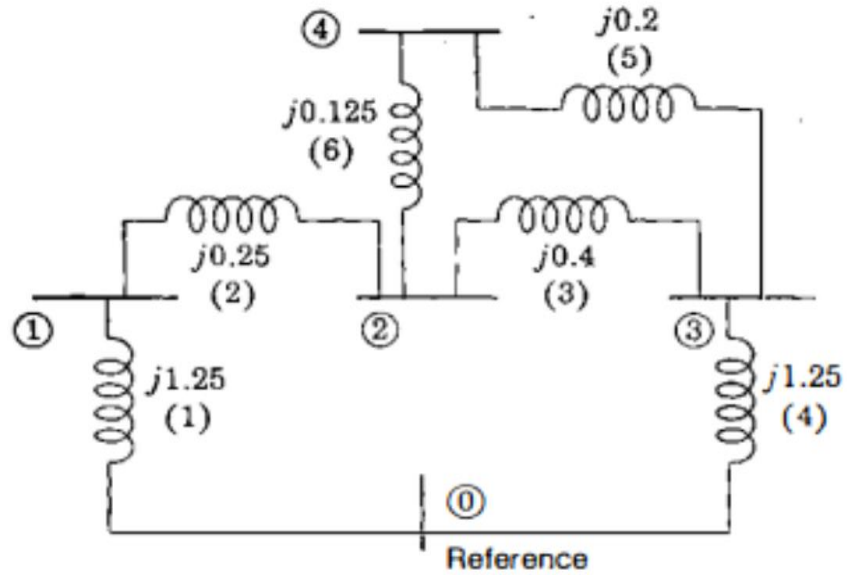
$$[V_1] = \begin{matrix} \\ ① \end{matrix} \begin{matrix} ① \\ [j1.25] \end{matrix} \quad [I_1]$$

We then have the 1×1 bus impedance matrix

$$Z_{bus,1} = \begin{matrix} \\ ① \end{matrix} \begin{matrix} ① \\ [j1.25] \end{matrix}$$

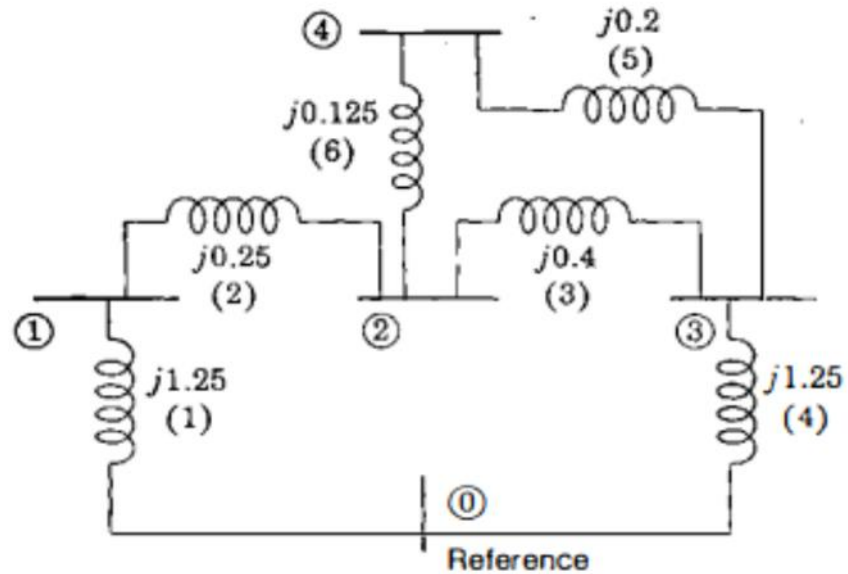
To establish bus ② with its impedance to bus ①, we follow Eq. (8.32) to write

$$Z_{bus,2} = \begin{matrix} \\ ① \\ ② \end{matrix} \begin{matrix} ① & ② \\ \begin{bmatrix} j1.25 & j1.25 \\ j1.25 & j1.50 \end{bmatrix} \end{matrix}$$



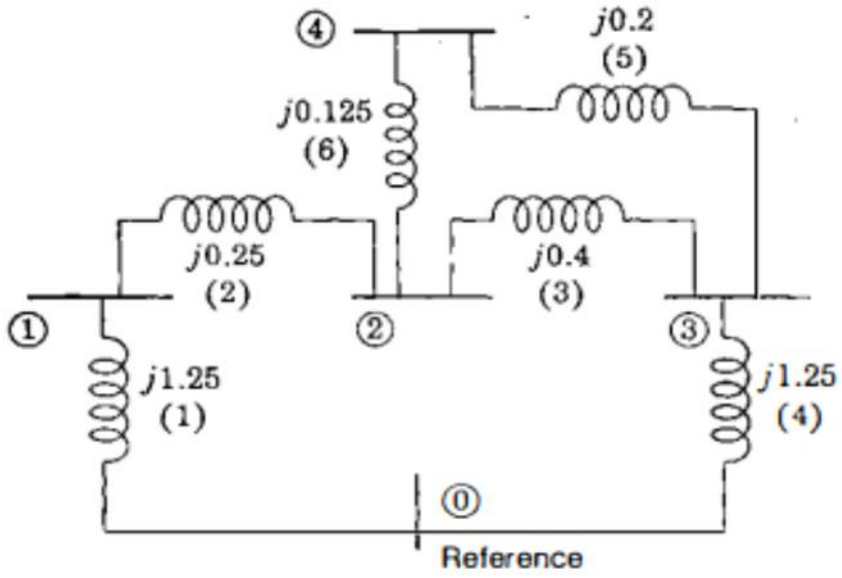
Bus ③ with the impedance connecting it to bus ② is established by writing

$$Z_{bus,3} = \begin{array}{c} \\ ① \\ ② \\ ③ \end{array} \begin{array}{c} \begin{array}{ccc} ① & ② & ③ \end{array} \\ \begin{bmatrix} j1.25 & j1.25 & j.125 \\ j1.25 & j1.50 & j1.50 \\ j1.25 & j1.50 & j1.90 \end{bmatrix} \end{array}$$



Existing bus to reference bus

$$Z_{bus,4} = \begin{array}{c} \\ ① \\ ② \\ ③ \\ ⓟ \end{array} \begin{array}{c} \begin{array}{cccc} ① & ② & ③ & ⓟ \end{array} \\ \left[\begin{array}{cccc} j1.25 & j1.25 & j.125 & j1.25 \\ j1.25 & j1.50 & j1.50 & j.150 \\ j1.25 & j1.50 & j1.90 & j1.90 \\ \hline j1.25 & j1.50 & j1.90 & j3.15 \end{array}\right] \end{array}$$



We now eliminate row p and column p by Kron reduction. Some of the elements of the new matrix from Eq. (8.33) are

$$Z_{11(\text{new})} = j1.25 - \frac{(j1.25)(j1.25)}{j3.15} = j0.75397$$

$$Z_{22(\text{new})} = j1.50 - \frac{(j1.50)(j1.50)}{j3.15} = j0.78571$$

$$Z_{23(\text{new})} = Z_{32(\text{new})} = j1.50 - \frac{(j1.50)(j1.90)}{j3.15} = j0.59524$$

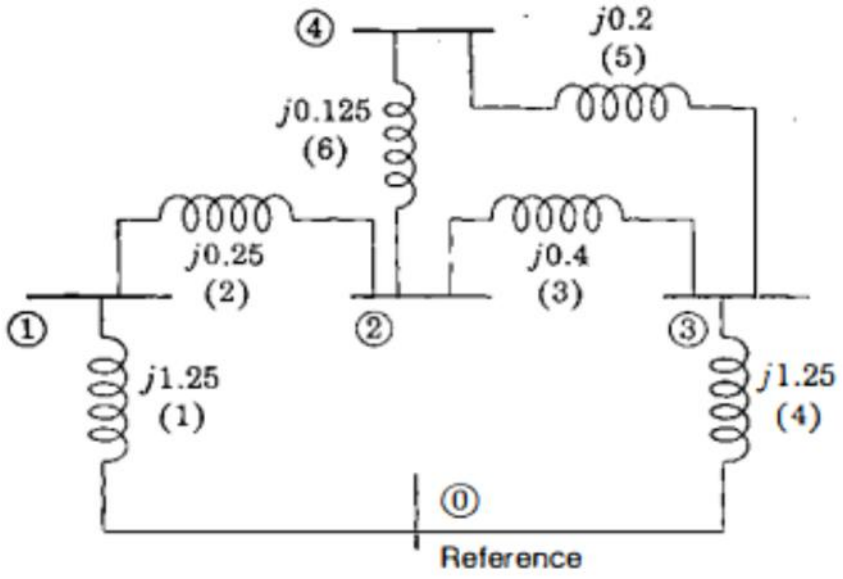
When all the elements are determined, we have

$$Z_{\text{bus.5}} = \begin{array}{c} \\ ① \\ ② \\ ③ \end{array} \begin{array}{c} \begin{array}{ccc} ① & ② & ③ \end{array} \\ \begin{bmatrix} j0.75397 & j0.65476 & j0.49603 \\ j0.65476 & j0.78571 & j0.59524 \\ j0.49603 & j0.59524 & j0.75397 \end{bmatrix} \end{array}$$

To new bus 4

We now decide to add the impedance $Z_b = j0.20$ from bus ③ to establish bus ④ using Eq. (8.32), and we obtain

$$Z_{bus.6} = \begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \end{array} \begin{array}{c} \begin{array}{cccc} ① & ② & ③ & ④ \end{array} \\ \left[\begin{array}{cccc} j0.75397 & j0.65476 & j0.49603 & j0.49603 \\ j0.65476 & j0.78571 & j0.59524 & j0.59524 \\ j0.49603 & j0.59524 & j0.75397 & j0.75397 \\ j0.49603 & j0.59524 & j0.75397 & j0.95397 \end{array}\right] \end{array}$$



Bus 4 to existing bus 2

Finally, we add the impedance $Z_b = j0.125$ between buses ② and ④. If we let j and k in Eq. (8.41) equal 2 and 4, respectively, we obtain the elements for row 5 and column 5

$$Z_{15} = Z_{12} - Z_{14} = j0.65476 - j0.49603 = j0.15873$$

$$Z_{25} = Z_{22} - Z_{24} = j0.78571 - j0.59524 = j0.19047$$

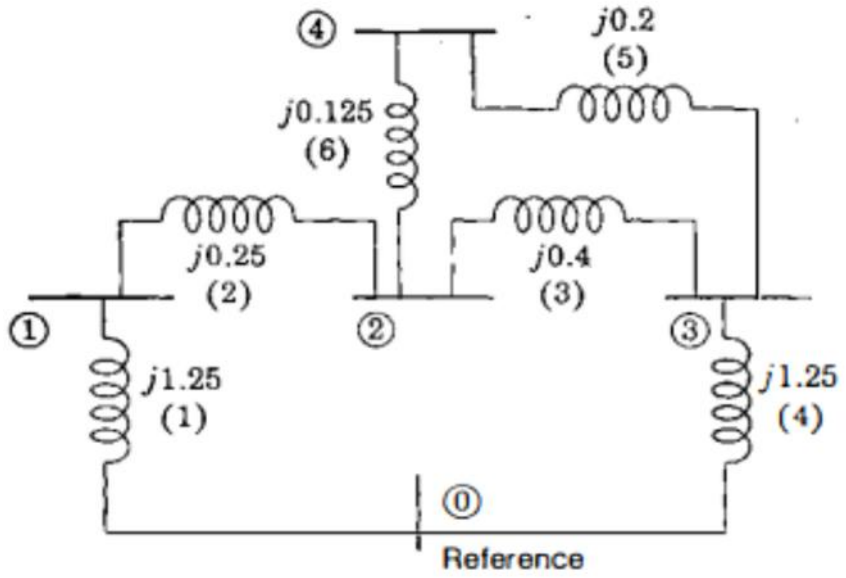
$$Z_{35} = Z_{32} - Z_{34} = j0.59524 - j0.75397 = -j0.15873$$

$$Z_{45} = Z_{42} - Z_{44} = j0.59524 - j0.95397 = -j0.35873$$

and from Eq. (8.42)

$$Z_{55} = Z_{22} + Z_{44} - 2Z_{24} + Z_b$$

$$= j\{0.78571 + 0.95397 - 2(0.59524)\} + j0.125 = j0.67421$$



So, employing $\mathbf{Z}_{bus,6}$ previously found, we write the 5×5 matrix

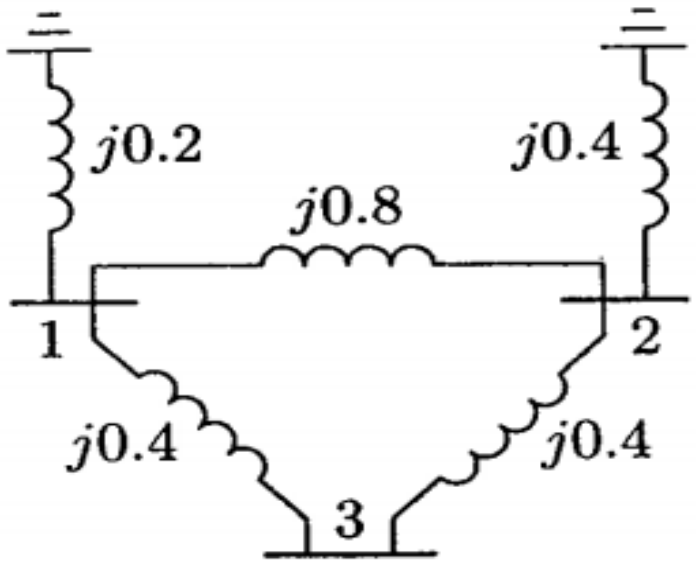
$$
\begin{array}{c}
 \\
⑨
\end{array}
\left[\begin{array}{c|c}
\multicolumn{4}{c|}{\multirow{4}{*}{$\mathbf{Z}_{bus,6}$}} & j0.15873 \\
 & & & & j0.19047 \\
 & & & & -j0.15873 \\
 & & & & -j0.35873 \\
\hline
j0.15873 & j0.19047 & -j0.15873 & -j0.35873 & j0.67421
\end{array}\right]
\begin{array}{c}
 \\
⑨
\end{array}
$$

and from Eq. (8.43) we find by Kron reduction

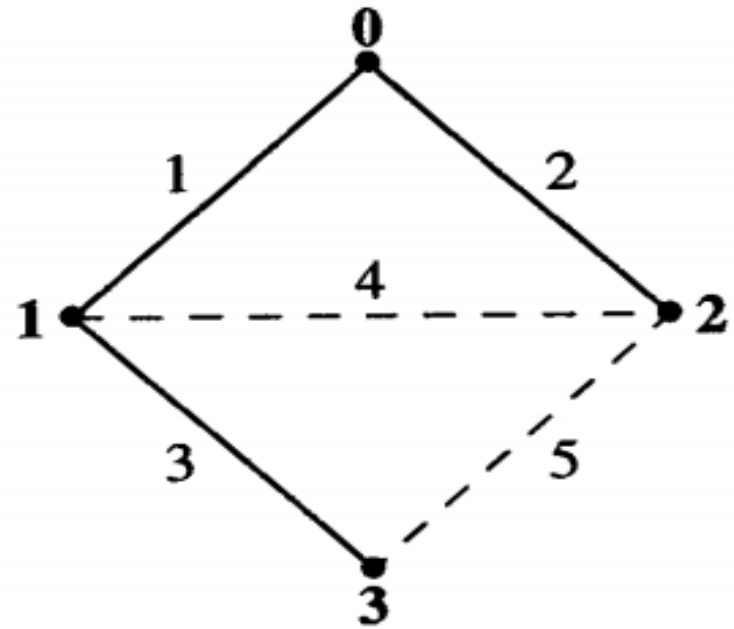
$$
\mathbf{Z}_{bus} = \begin{array}{c}
 \\
① \\
② \\
③ \\
④
\end{array}
\begin{array}{c}
\begin{array}{cccc} ① & ② & ③ & ④ \end{array} \\
\left[\begin{array}{cccc}
j0.71660 & j0.60992 & j0.53340 & j0.58049 \\
j0.60992 & j0.73190 & j0.64008 & j0.69659 \\
j0.53340 & j0.64008 & j0.71660 & j0.66951 \\
j0.58049 & j0.69659 & j0.66951 & j0.76310
\end{array}\right]
\end{array}
$$

Example 9.3

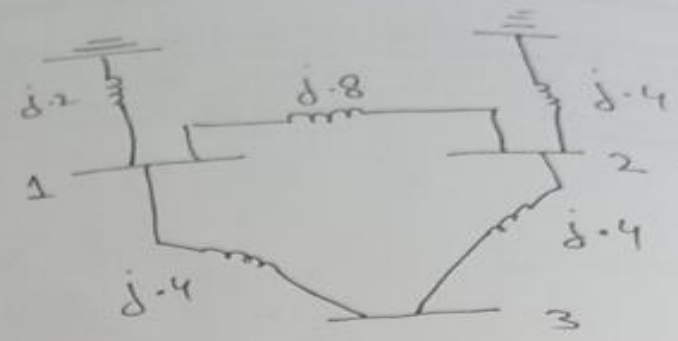
Construct the bus impedance matrix for the network in Example 9.1. The one-line impedance diagram is shown in Figure 9.15(a).



(a)



(b)



Step 1:
Cas 1.

$$Z_1 = [j.2]$$

Step 2. Cas 2.
1 to new bus 2.

$$Z_2 = \begin{bmatrix} j.2 & j.2 \\ j.2 & j1 \end{bmatrix}.$$

Step 3: Cas 3.
Bus 2 to reference bus.

(1)

$$Z_3 = \left[\begin{array}{cc|c} j.2 & j.2 & j.2 \\ j.2 & j1 & j1 \\ \hline j.2 & j1. & j1.4 \end{array}\right]$$

Kron Reduction.

$Z_{2f} = [\quad]_{2\times 2}$.

Step 4:

Case 2: Bus 2 to new Bus 3.

$Z_4 = [\quad]_{3\times 3}$.

Step 5:

Case 4: Existing Bus 3 to Bus 1.

$Z_5 = [\quad]_{4\times 4}$.

Kron Reduction.

$Z_{3f} = [\quad]_{3\times 3}$

8.9. Find $\mathbf{Z}_{bus}$ for the three-bus circuit of Fig. 8.15 by the $\mathbf{Z}_{bus}$ building algorithm of Sec. 8.4.

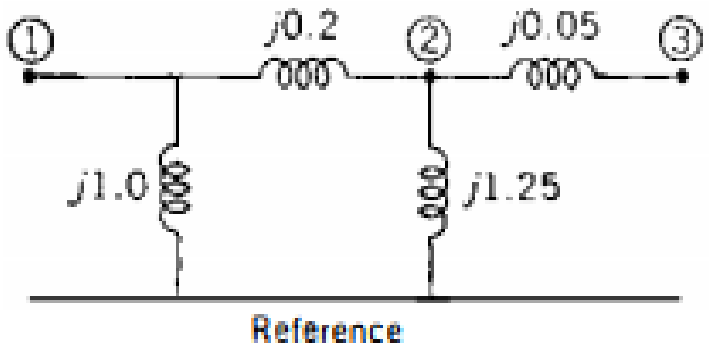


FIGURE 8.15
Circuit for Probs. 8.9, 8.11, and 8.12. Values shown are reactances in per unit.

Z - Bus

Book - Steven, Grainger

Topic 8.3

Example = 8.4

Also can do some examples of Haeli Sadat.
& Related Problems.

Y - Bus

Topic → 7.7
Node Elimination

Y - Bus = Ex # 7.1

Examples & Problem from Haeli Sadat Book.