Fast Decoupled Method

Lec_09

Power system transmission lines have a very high X/R ratio. For such a system, real power changes ΔP are less sensitive to changes in the voltage magnitude and are most sensitive to changes in phase angle $\Delta\delta$. Similarly, reactive power is less sensitive to changes in angle and are mainly dependent on changes in voltage magnitude. Therefore, it is reasonable to set elements $\boldsymbol{J_2}$ and $\boldsymbol{J_3}$ of the Jacobian matrix to zero. Thus, (6.54) becomes

$$\begin{bmatrix} \Delta \boldsymbol{P} \\ \Delta \boldsymbol{Q} \end{bmatrix} = \begin{bmatrix} J_1 & \mathbf{0} \\ \mathbf{0} & J_4 \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\delta} \\ \Delta |\boldsymbol{V}| \end{bmatrix} \tag{6.68}$$

$$\Delta P = J_1 \Delta\delta = [\frac{\partial P}{\partial \delta}]\ \Delta\delta \tag{6.69}$$

$$\Delta Q = J_4 \Delta|V| = [\frac{\partial Q}{\partial |V|}]\ \Delta|V| \tag{6.70}$$

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) - |V_i|^2 |Y_{ii}| \sin \theta_{ii}$$

Replacing the first term of the above equation with $-Q_i$, as given by (6.53), results in

$$\begin{aligned} \frac{\partial P_i}{\partial \delta_i} &= -Q_i - |V_i|^2 |Y_{ii}| \sin \theta_{ii} \\ &= -Q_i - |V_i|^2 B_{ii} \end{aligned}$$

Where $B_{ii} = |Y_{ii}| \sin \theta_{ii}$ is the imaginary part of the diagonal elements of the bus admittance matrix. B_{ii} is the sum of susceptances of all the elements incident to bus

Where $B_{ii} = |Y_{ii}| \sin \theta_{ii}$ is the imaginary part of the diagonal elements of the bus admittance matrix. B_{ii} is the sum of susceptances of all the elements incident to bus i. In a typical power system, the self-susceptance $B_{ii} \gg Q_i$, and we may neglect Q_i. Further simplification is obtained by assuming $|V_i|^2 \approx |V_i|$, which yields

$$\frac{\partial P_i}{\partial \delta_i} = -|V_i| B_{ii} \tag{6.71}$$

Under normal operating conditions, $\delta_j - \delta_i$ is quite small. Thus, in (6.56) assuming $\theta_{ii} - \delta_i + \delta_j \approx \theta_{ii}$, the off-diagonal elements of $\boldsymbol{J_1}$ becomes

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j|B_{ij}$$

Further simplification is obtained by assuming $|V_j| \approx 1$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i| B_{ij} \tag{6.72}$$

Similarly, the diagonal elements of J_4 described by (6.61) may be written as

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i||Y_{ii}|\sin\theta_{ii} - \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j)$$

replacing the second term of the above equation with $-Q_i$, as given by (6.53), results in

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i||Y_{ii}|\sin\theta_{ii} + Q_i$$

Again, since $B_{ii} = Y_{ii}\sin\theta_{ii} \gg Q_i$, Q_i may be neglected and (6.61) reduces to

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i|B_{ii} \tag{6.73}$$

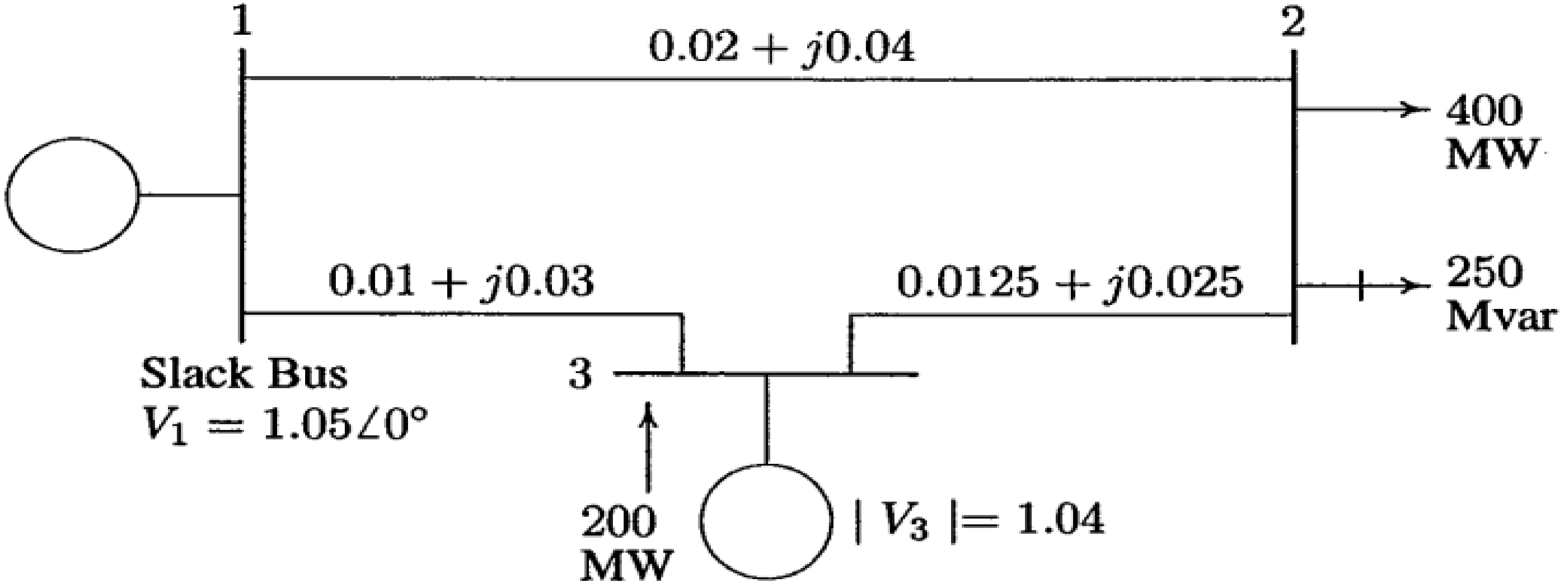
$$\frac{\Delta P}{|V_i|} = -B' \, \Delta\delta \tag{6.75}$$

$$\frac{\Delta Q}{|V_i|} = -B'' \, \Delta|V| \tag{6.76}$$

Here, $\boldsymbol{B'}$ and $\boldsymbol{B''}$ are the imaginary part of the bus admittance matrix $\boldsymbol{Y_{bus}}$. Since the elements of this matrix are constant, they need to be triangularized and inverted only once at the beginning of the iteration. $\boldsymbol{B'}$ is of order of $(n-1)$. For voltage-controlled buses where $|V_i|$ and P_i are specified and Q_i is not specified, the corresponding row and column of $\boldsymbol{Y_{bus}}$ are eliminated. Thus, $\boldsymbol{B''}$ is of order of $(n-1-m)$, where m is the number of voltage-regulated buses. Therefore, in the fast decoupled power flow algorithm, the successive voltage magnitude and phase angle changes are

$$\Delta\delta = -[B']^{-1}\frac{\Delta P}{|V|} \tag{6.77}$$

$$\Delta|V| = -[B'']^{-1}\frac{\Delta Q}{|V|} \tag{6.78}$$



1
2
0.02 + j0.04
400
MW
0.01 + j0.03
0.0125 + j0.025
250
Mvar
Slack Bus
$V_1 = 1.05\angle 0°$
3
200
MW
$|V_3| = 1.04$

The bus admittance matrix of the system as obtained in Example 6.10 is

$$Y_{bus} = \begin{bmatrix} 20 - j50 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j52 & -16 + j32 \\ -10 + j30 & -16 + j32 & 26 - j62 \end{bmatrix}$$

In this system, bus 1 is the slack bus and the corresponding bus susceptance matrix for evaluation of phase angles $\Delta\delta_2$ and $\Delta\delta_3$ is

$$B' = \begin{bmatrix} -52 & 32 \\ 32 & -62 \end{bmatrix}$$

The inverse of the above matrix is

$$[B']^{-1} = \begin{bmatrix} -0.028182 & -0.014545 \\ -0.014545 & -0.023636 \end{bmatrix}$$

$$
\begin{aligned}
P_2 &= |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2^2||Y_{22}|\cos\theta_{22} \\
&\quad +|V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3) \\
P_3 &= |V_3||V_1||Y_{31}|\cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\cos(\theta_{32} \\
&\quad -\delta_3 + \delta_2) + |V_3^2||Y_{33}|\cos\theta_{33} \\
Q_2 &= -|V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - |V_2^2||Y_{22}|\sin\theta_{22} \\
&\quad -|V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)
\end{aligned}
$$

$$S_2^{sch} = -\frac{(400 + j250)}{100} = -4.0 - j2.5 \quad \text{pu}$$

$$P_3^{sch} = \frac{200}{100} = 2.0 \quad \text{pu}$$

The slack bus voltage is $V_1 = 1.05\angle 0$ pu, and the bus 3 voltage magnitude is $|V_3| = 1.04$ pu. Starting with an initial estimate of $|V_2^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, and $\delta_3^{(0)} = 0.0$, the power residuals are computed from (6.63) and (6.64)

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = -4.0 - (-1.14) = -2.86$$
$$\Delta P_3^{(0)} = P_3^{sch} - P_3^{(0)} = 2.0 - (0.5616) = 1.4384$$
$$\Delta Q_2^{(0)} = Q_2^{sch} - Q_2^{(0)} = -2.5 - (-2.28) = -0.22$$

The fast decoupled power flow algorithm given by (6.77) becomes

$$\begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \end{bmatrix} = -\begin{bmatrix} -0.028182 & -0.014545 \\ -0.014545 & -0.023636 \end{bmatrix} \begin{bmatrix} \frac{-2.8600}{1.0} \\ \frac{1.4384}{1.04} \end{bmatrix} = \begin{bmatrix} -0.060483 \\ -0.008909 \end{bmatrix}$$

Since bus 3 is a regulated bus, the corresponding row and column of $\boldsymbol{B'}$ are eliminated and we get

$$B'' = [-52]$$

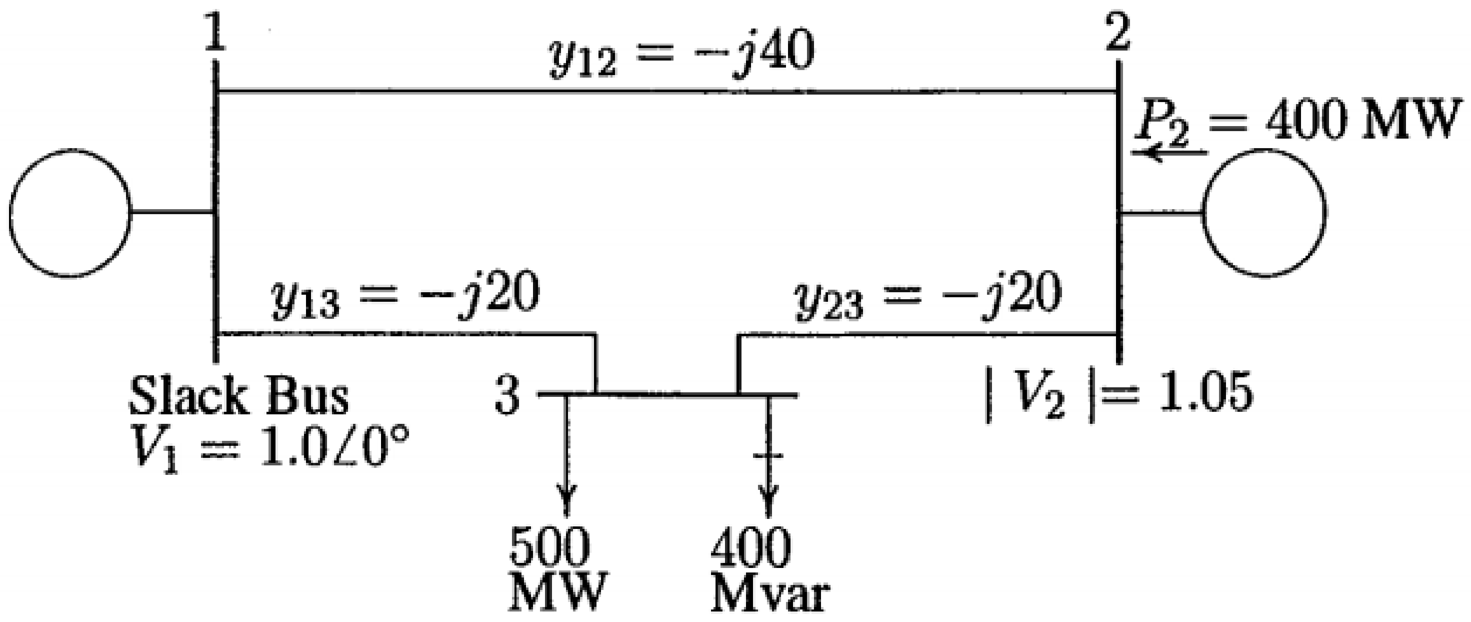
From (6.78), we have

$$\Delta|V_2| = -\left[\frac{-1}{52}\right]\left[\frac{-.22}{1.0}\right] = -0.0042308$$

The new bus voltages in the first iteration are

$$
\begin{aligned}
\Delta\delta_2^{(0)} &= -0.060483 & \delta_2^{(1)} &= 0 + (-0.060483) = -0.060483 \\
\Delta\delta_3^{(0)} &= -0.008989 & \delta_3^{(1)} &= 0 + (-0.008989) = -0.008989 \\
\Delta|V_2^{(0)}| &= -0.0042308 & |V_2^{(1)}| &= 1 + (-0.0042308) = 0.995769
\end{aligned}
$$

6.12. Figure 6.25 shows the one-line diagram of a simple three-bus power system with generation at buses 1 and 2. The voltage at bus 1 is $V = 1.0\angle 0^\circ$ per unit. Voltage magnitude at bus 2 is fixed at 1.05 pu with a real power generation of 400 MW. A load consisting of 500 MW and 400 Mvar is taken from bus 3. Line admittances are marked in per unit on a 100 MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.


1
$y_{12} = -j40$
2
$P_2 = 400$ MW
$y_{13} = -j20$
$y_{23} = -j20$
Slack Bus
$V_1 = 1.0\angle 0^\circ$
3
$|V_2| = 1.05$
500
MW
400
Mvar

(a) Show that the expression for the real power at bus 2 and real and reactive power at bus 3 are

$$P_2 = 40|V_2||V_1|\cos(90^\circ - \delta_2 + \delta_1) + 20|V_2||V_3|\cos(90^\circ - \delta_2 + \delta_3)$$
$$P_3 = 20|V_3||V_1|\cos(90^\circ - \delta_3 + \delta_1) + 20|V_3||V_2|\cos(90^\circ - \delta_3 + \delta_2)$$
$$Q_3 = -20|V_3||V_1|\sin(90^\circ - \delta_3 + \delta_1) - 20|V_3||V_2|\sin(90^\circ - \delta_3 + \delta_2) + 40|V_3|^2$$

(b) Using Newton-Raphson method, start with the initial estimates of $V_2^{(0)} = 1.0 + j0$ and $V_3^{(0)} = 1.0 + j0$, and keeping $|V_2| = 1.05$ pu, determine the phasor values of V_2 and V_3. Perform two iterations.
(c) Check the power flow solution for Problem 6.12 using **lfnewton** and other required programs. Assume the regulated bus (bus # 2) reactive power limits are between 0 and 600 Mvar.

(a) Show that the expression for the real power at bus 2 and real and reactive power at bus 3 are

$$P_2 = 40|V_2||V_1|\cos(90^\circ - \delta_2 + \delta_1) + 20|V_2||V_3|\cos(90^\circ - \delta_2 + \delta_3)$$
$$P_3 = 20|V_3||V_1|\cos(90^\circ - \delta_3 + \delta_1) + 20|V_3||V_2|\cos(90^\circ - \delta_3 + \delta_2)$$
$$Q_3 = -20|V_3||V_1|\sin(90^\circ - \delta_3 + \delta_1) - 20|V_3||V_2|\sin(90^\circ - \delta_3 + \delta_2) + 40|V_3|^2$$

(b) Using Newton-Raphson method, start with the initial estimates of ${V_2}^{(0)} = 1.05 + j0$ and ${V_3}^{(0)} = 1.0 + j0$, and keeping $|V_2| = 1.05$ pu, determine the phasor values of V_2 and V_3. Perform two iterations.

(c) Check the power flow solution for Problem 6.12 using the **lfnewton** and other required programs. Assume the regulated bus (bus # 2) reactive power limits are between 0 and 600 Mvar.

By inspection, the bus admittance matrix in polar form is

$$Y_{bus} = \begin{bmatrix} 60\angle -\frac{\pi}{2} & 40\angle \frac{\pi}{2} & 20\angle \frac{\pi}{2} \\ 40\angle \frac{\pi}{2} & 60\angle -\frac{\pi}{2} & 20\angle \frac{\pi}{2} \\ 20\angle \frac{\pi}{2} & 20\angle \frac{\pi}{2} & 40\angle -\frac{\pi}{2} \end{bmatrix}$$

(a) The power flow equation with voltages and admittances expressed in polar form is

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

Substituting the elements of the bus admittance matrix in the above equations for P_2, P_3, and Q_3 will result in the given equations.

(b) Elements of the Jacobian matrix are obtained by taking partial derivatives of the given equations with respect to δ_2, δ_3 and $|V_3|$.

$$\frac{\partial P_2}{\partial \delta_2} = 40|V_2||V_1|\sin(\frac{\pi}{2} - \delta_2 + \delta_1) + 20|V_2||V_3|\sin(\frac{\pi}{2} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial \delta_3} = -20|V_2||V_3|\sin(\frac{\pi}{2} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial |V_3|} = 20|V_2|\cos(\frac{\pi}{2} - \delta_2 + \delta_3)$$

$$\frac{\partial P_3}{\partial \delta_2} = -20|V_3||V_2|\sin(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial \delta_3} = 20|V_3||V_1|\sin(\frac{\pi}{2} - \delta_3 + \delta_1) + 20|V_3||V_2|\sin(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial |V_3|} = 20|V_1|\cos(\frac{\pi}{2} - \delta_3 + \delta_1) + 20|V_2|\cos(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_3}{\partial \delta_2} = -20|V_3||V_2|\cos(\frac{\pi}{2} - \delta_3 + \delta_2)$$
$$\frac{\partial Q_3}{\partial \delta_3} = 20|V_3||V_1|\cos(\frac{\pi}{2} - \delta_3 + \delta_1) + 20|V_3||V_2|\cos(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_3}{\partial |V_3|} = -20|V_1|\sin(\frac{\pi}{2} - \delta_3 + \delta_1) - 20|V_2|\sin(\frac{\pi}{2} - \delta_3 + \delta_2) + 80|V_3|$$

The load and generation expressed in per units are

$$P_2^{sch} = \frac{400}{100} = 4.0 \;\; \text{pu}$$
$$S_3^{sch} = -\frac{(500 + j400)}{100} = -5.0 - j4.0 \;\; \text{pu}$$

The slack bus voltage is $V_1 = 1.0\angle 0$ pu, and the bus 2 voltage magnitude is $|V_2| = 1.05$ pu. Starting with an initial estimate of $|V_3^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, and $\delta_3^{(0)} = 0.0$, the power residuals are

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = 4.0 - (0) = 4.0$$
$$\Delta P_3^{(0)} = P_3^{sch} - P_3^{(0)} = -5.0 - (0) = -5.0$$
$$\Delta Q_3^{(0)} = Q_3^{sch} - Q_3^{(0)} = -4.0 - (-1.0) = -3.0$$

Evaluating the elements of the Jacobian matrix with the initial estimate, the set of linear equations in the first iteration becomes

$$\begin{bmatrix} -2.8600 \\ 1.4384 \\ -0.2200 \end{bmatrix} = \begin{bmatrix} 63 & -21 & 0 \\ -21 & 41 & 0 \\ 0 & 0 & 39 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta|V_3^{(0)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, the new bus voltages in the first iteration are

$$\begin{aligned} \Delta\delta_2^{(0)} &= 0.0275 & \delta_2^{(1)} &= 0 + 0.0275 = 0.0275 \ \text{ radian} = 1.5782^\circ \\ \Delta\delta_3^{(0)} &= -0.1078 & \delta_3^{(1)} &= 0 + (-0.1078) = -0.1078 \ \text{ radian} = -6.1790^\circ \\ \Delta|V_3^{(0)}| &= -0.0769 & |V_3^{(1)}| &= 1 + (-0.0769) = 0.9231 \ \text{ pu} \end{aligned}$$

6.13. For Problem 6.12:

(a) Obtain the power flow solution using the fast decoupled algorithm. Perform two iterations.

(b)Check the power flow solution for Problem 6.12 using the **decouple** and other required programs. Assume the regulated bus (bus # 2) reactive power limits are between 0 and 600 Mvar.

(a) In this system, bus 1 is the slack bus and the corresponding bus susceptance matrix for evaluation of phase angles $\Delta\delta_2$ and $\Delta\delta_3$ form the bus admittance matrix in Problem 6.12 is

(a) In this system, bus 1 is the slack bus and the corresponding bus susceptance matrix for evaluation of phase angles $\Delta\delta_2$ and $\Delta\delta_3$ form the bus admittance matrix in Problem 6.12 is

$$B' = \begin{bmatrix} -60 & 20 \\ 20 & -40 \end{bmatrix}$$

The inverse of the above matrix is

$$[B']^{-1} = \begin{bmatrix} -0.02 & -0.01 \\ -0.01 & -0.03 \end{bmatrix}$$

The expressions for real power at bus 2 and 3 and the reactive power at bus 3 are given in Problem 6.12. The slack bus voltage is $V_1 = 1.0\angle 0$ pu, and the bus 2 voltage magnitude is $|V_3| = 1.05$ pu. Starting with an initial estimate of $|V_3^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, and $\delta_3^{(0)} = 0.0$, the power residuals are computed from (6.63) and (6.64)

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = 4 - (0) = 4$$
$$\Delta P_3^{(0)} = P_3^{sch} - P_3^{(0)} = -5 - (0) = -5$$
$$\Delta Q_3^{(0)} = Q_3^{sch} - Q_3^{(0)} = -4 - (-1) = -3$$

The fast decoupled power flow algorithm given by (6.77) becomes

$$\begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \end{bmatrix} = -\begin{bmatrix} -0.02 & -0.0 \\ -0.01 & -0.03 \end{bmatrix} \begin{bmatrix} \frac{4}{1.05} \\ \frac{-5}{1.0} \end{bmatrix} = \begin{bmatrix} .0262 \\ -0.1119 \end{bmatrix}$$

Since bus 2 is a regulated bus, the corresponding row and column of $\boldsymbol{B'}$ are eliminated and we get

$$B'' = [-40]$$

From (6.78), we have

$$\Delta|V_3| = -\left[\frac{-1}{40}\right]\left[\frac{-3}{1.0}\right] = -0.075$$

The new bus voltages in the first iteration are

$$\begin{aligned}
\Delta\delta_2^{(0)} &= 0.0262 \qquad \delta_2^{(1)} = 0 + (0.0262) = 0.0262 \;\text{ radian} = 1.5006^\circ \\
\Delta\delta_3^{(0)} &= -0.1119 \quad \delta_3^{(1)} = 0 + (-0.1119) = -0.1119 \;\text{ radian} = -6.4117^\circ \\
\Delta|V_3^{(0)}| &= -0.075 \quad |V_2^{(1)}| = 1 + (-0.075) = 0.925 \;\text{ pu}
\end{aligned}$$