Lec_5

Transmission Lines

Primary Methods for Power Transfer

The most common methods for transfer of electric power are

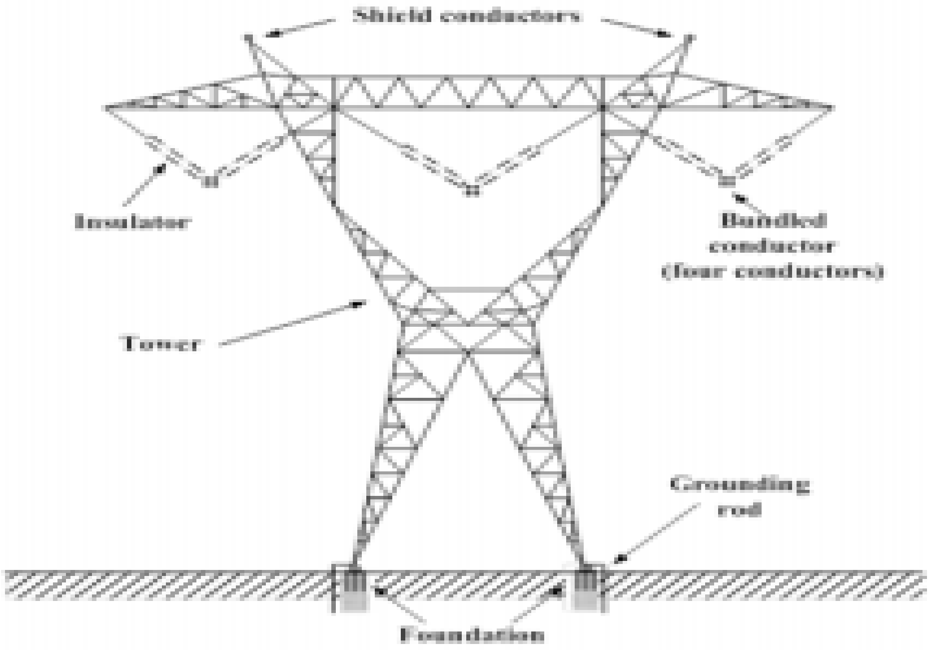
1) Overhead AC
2) Underground AC
3) Overhead DC
4) Underground DC

Transmission lines and cables

- **Extra-high-voltage lines**
  - Voltage: 345 kV, 500 kV, 765 kV
- **High-voltage lines**
  - Voltage: 115 kV, 230 kV
- **Sub-transmission lines**
  - Voltage: 46 kV, 69 kV
- **Distribution lines**
  - Voltage: 2.4 kV to 46 kV, with 15 kV being the most commonly used
- **High-voltage DC lines**
  - Voltage: ±120 kV to ±600 kV

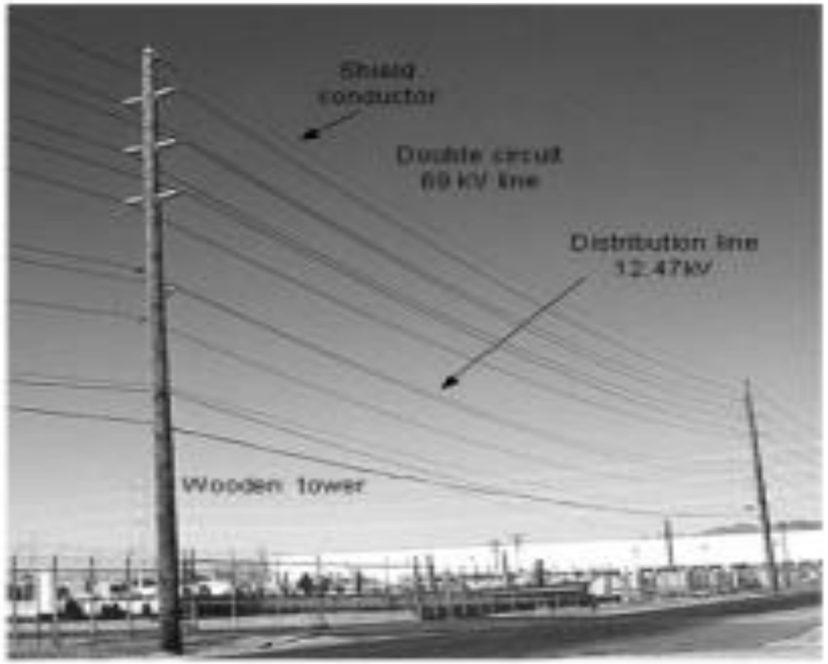
Transmission lines and cables

- Three-phase conductors, which carry the electric current;
- Insulators, which support and electrically isolate the conductors;
- Tower, which holds the insulators and conductors;
- Foundation and grounding; and
- Optional shield conductors, which protect against lightning



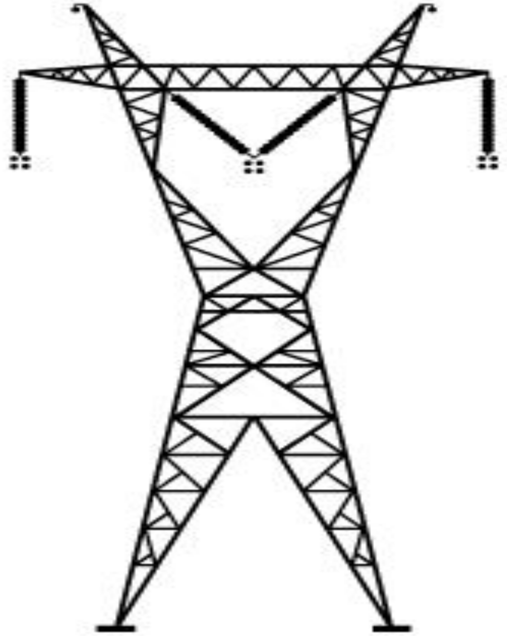
Transmission lines and cables

Distribution Line



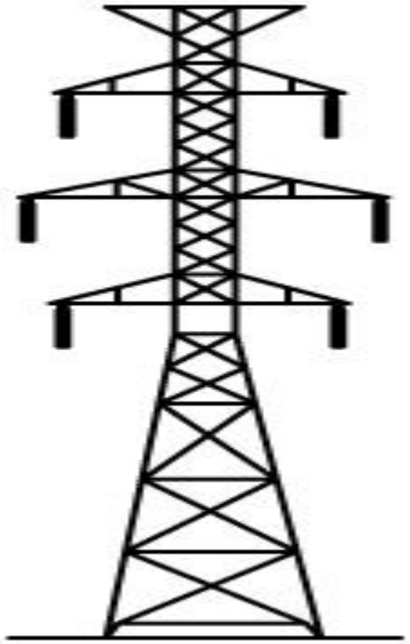
Waist-type tower

- This is the most common type of transmission tower. It's used for voltages ranging from 110 to 735 kV. Because they're easily assembled, these towers are suitable for power lines that cross very uneven terrain.



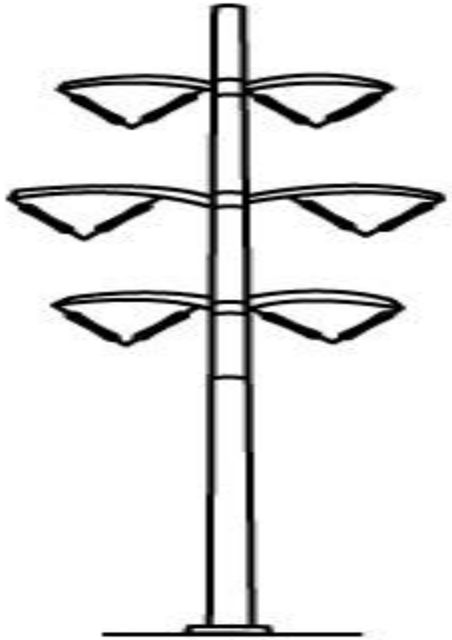
Double-circuit tower

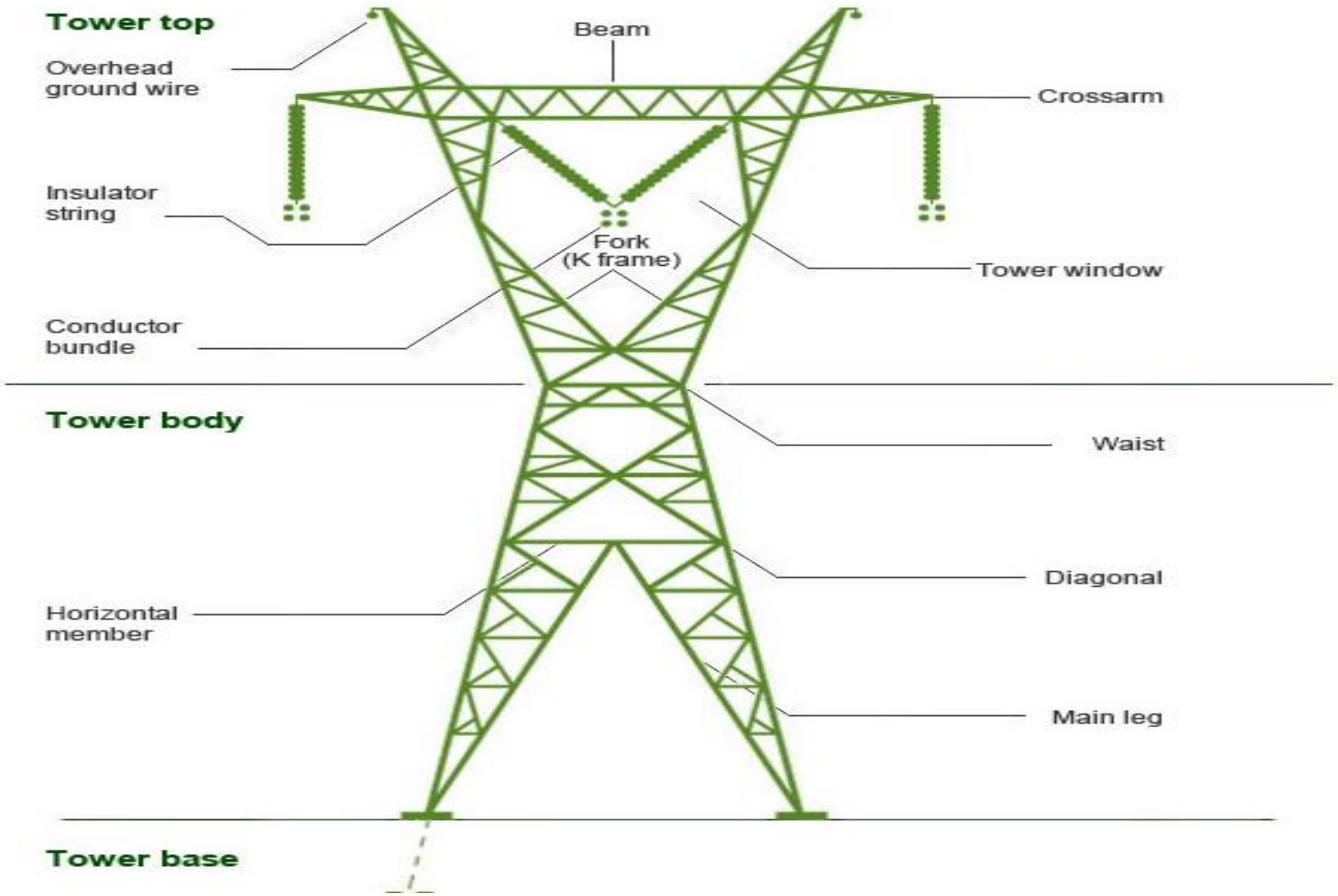
- This small-footprint tower is used for voltages ranging from 110 to 315 kV. Its height ranges from 25 to 60 metres.



Tublar steel pole

- Featuring a streamlined, aesthetic shape, this structure is less massive than other towers, allowing it to blend easily into the environment. For this reason, it's being used more and more in urban centres.




Tower top
Beam
Overhead ground wire
Crossarm
Insulator string
Fork (K frame)
Tower window
Conductor bundle
Tower body
Waist
Diagonal
Horizontal member
Main leg
Tower base

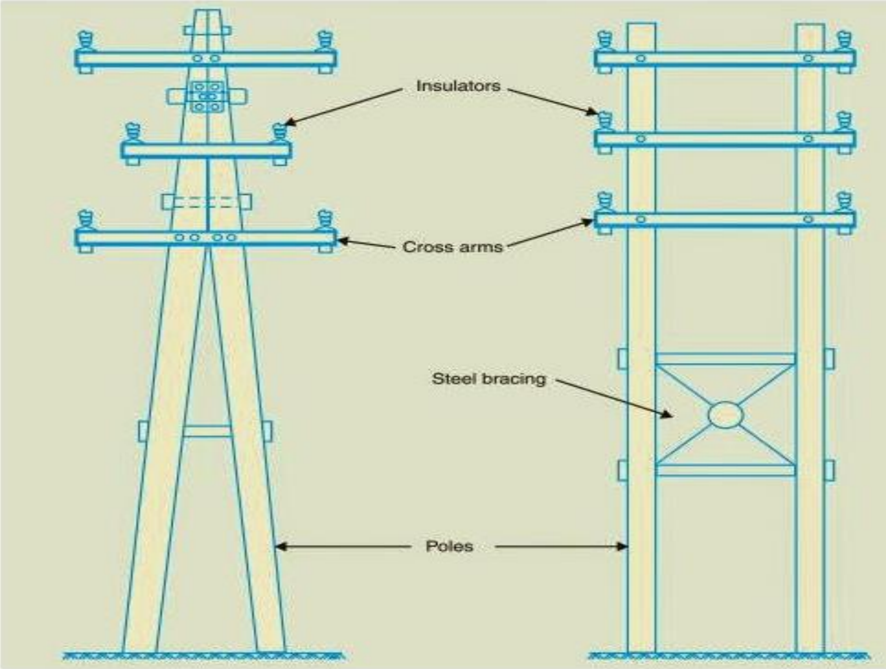
Distribution line poles

- The line supports used for transmission and distribution of electric power are of various types including

- wooden poles
- steel poles

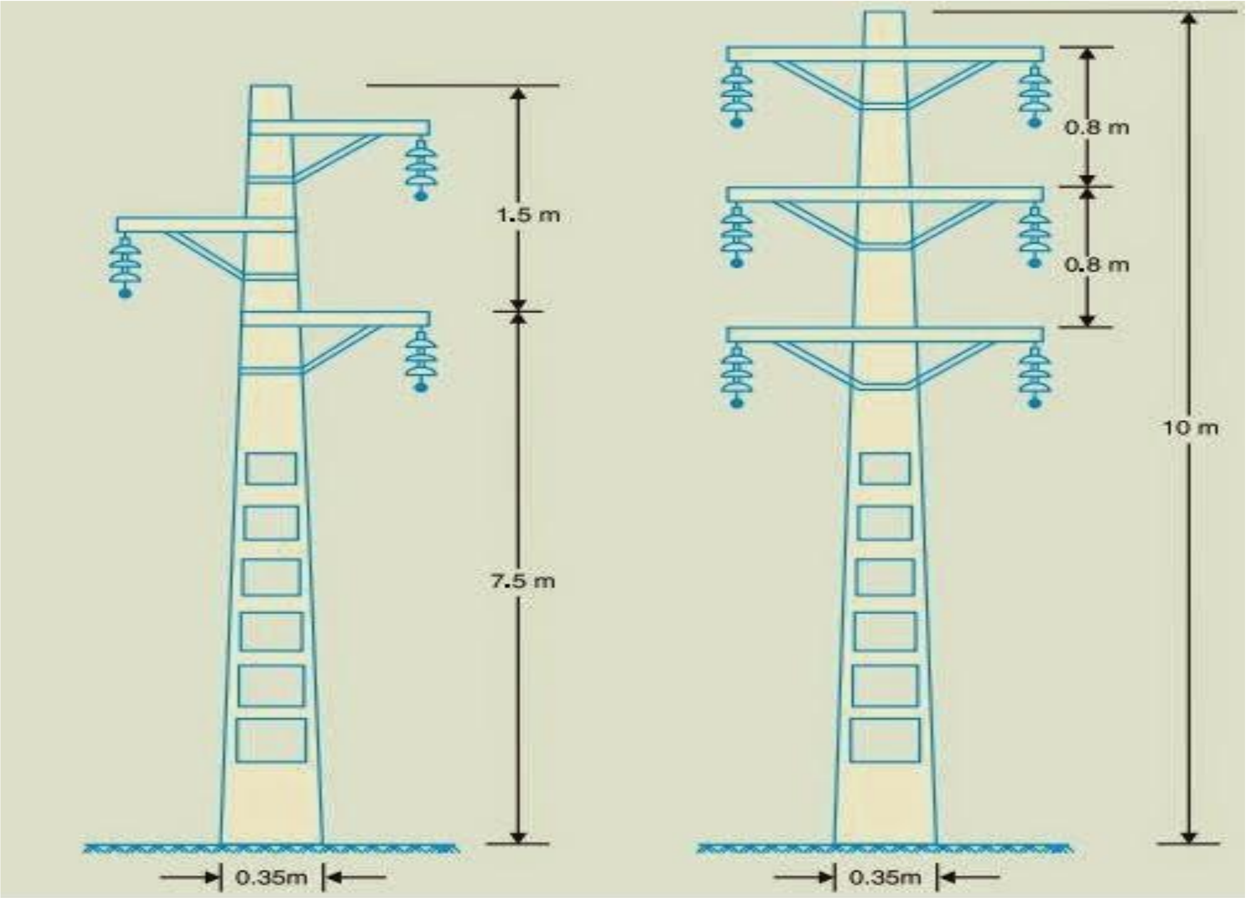
Wooden poles

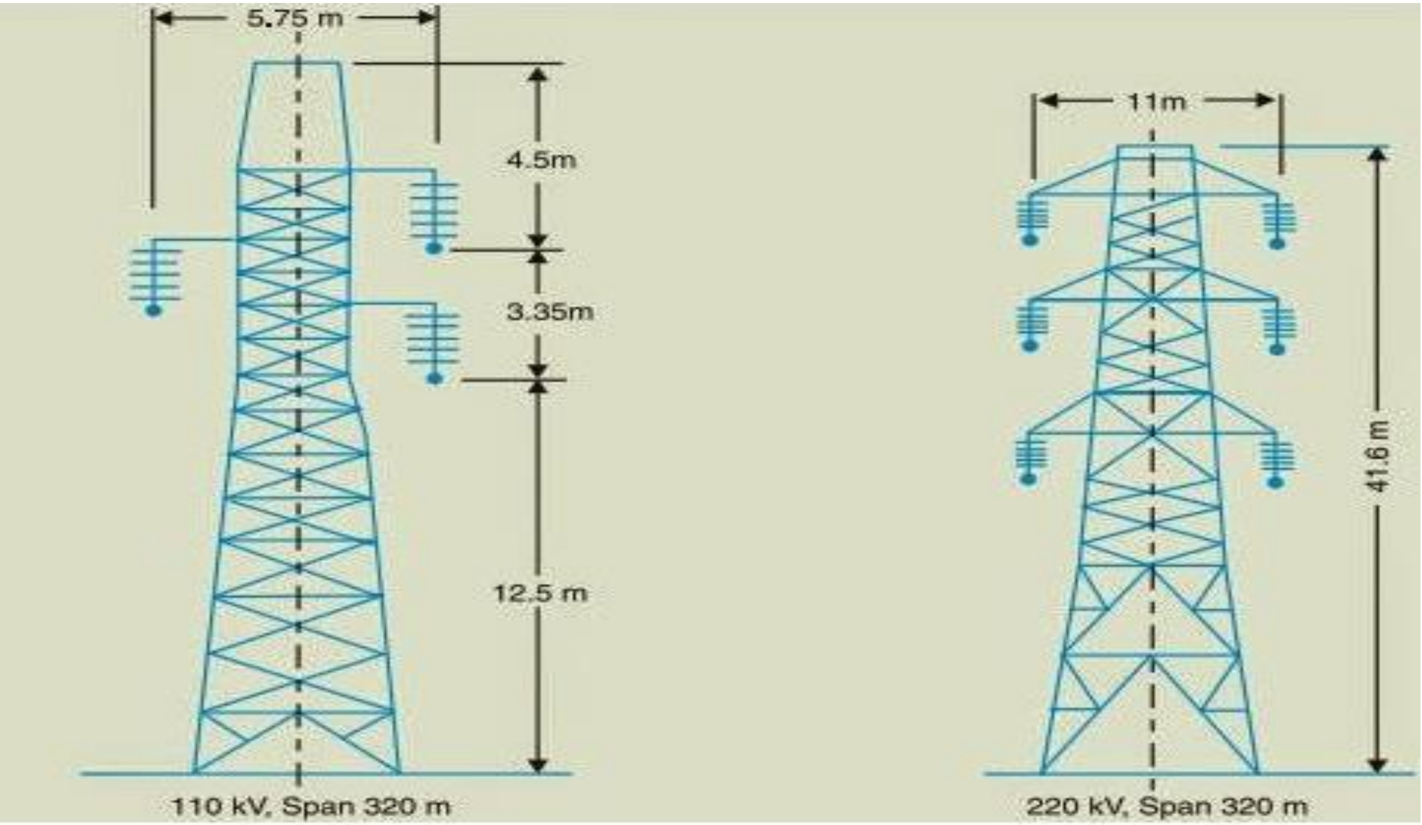
- The main objections to wooden supports are :
- (i) the tendency to rot below the ground level
- (ii) comparatively smaller life (20-25 years)
- (iii) cannot be used for voltages higher than 20 kV
- (iv) less mechanical strength and
- (v) require periodical inspection

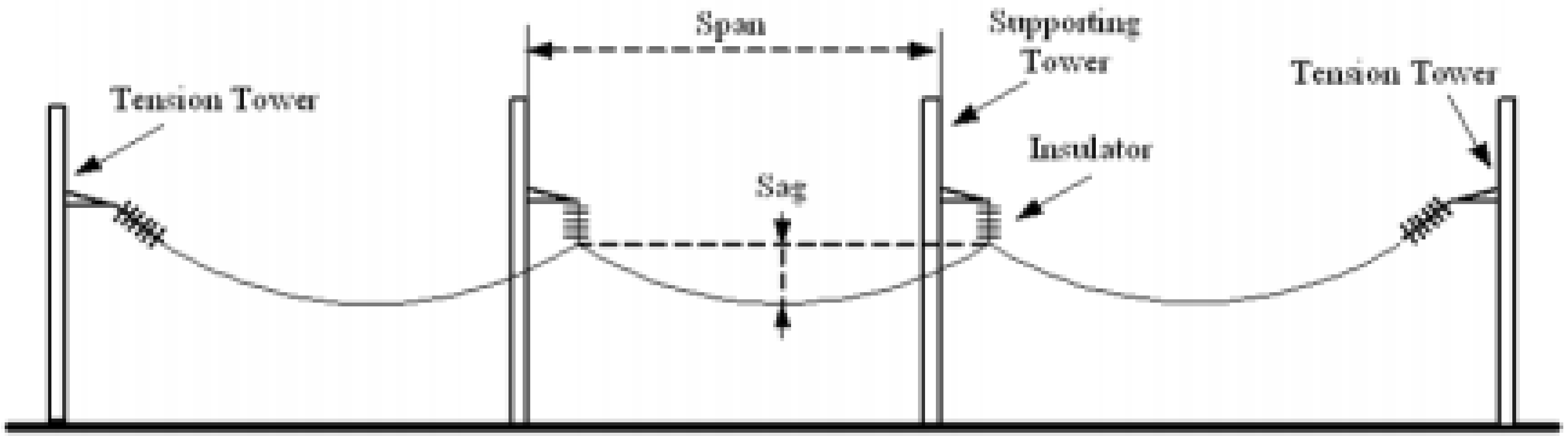

Insulators
Cross arms
Steel bracing
Poles

Steel poles

- The steel poles are often used as a substitute for wooden poles. They possess greater mechanical strength, longer life and permit longer spans to be used. Such poles are generally used for distribution purposes in the cities. This type of support needs to be galvanised or painted in order to prolong its life. The steel poles are of three types (i) rail poles (ii) tubular poles and (iii) rolled steel joints.


1.5 m
7.5 m
0.35m
0.8 m
0.8 m
10 m
0.35m


5.75 m
4.5m
3.35m
12.5 m
110 kV, Span 320 m
11m
41.6 m
220 kV, Span 320 m



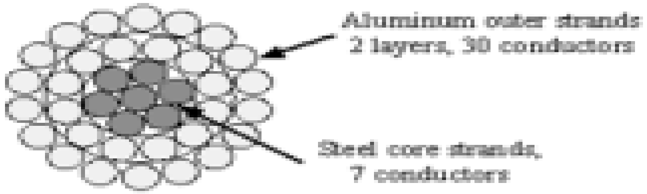
Definition of Parameters

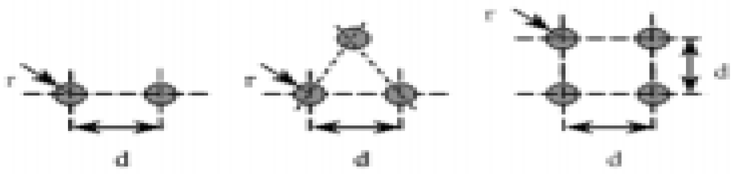
DC Line

Transmission lines and cables

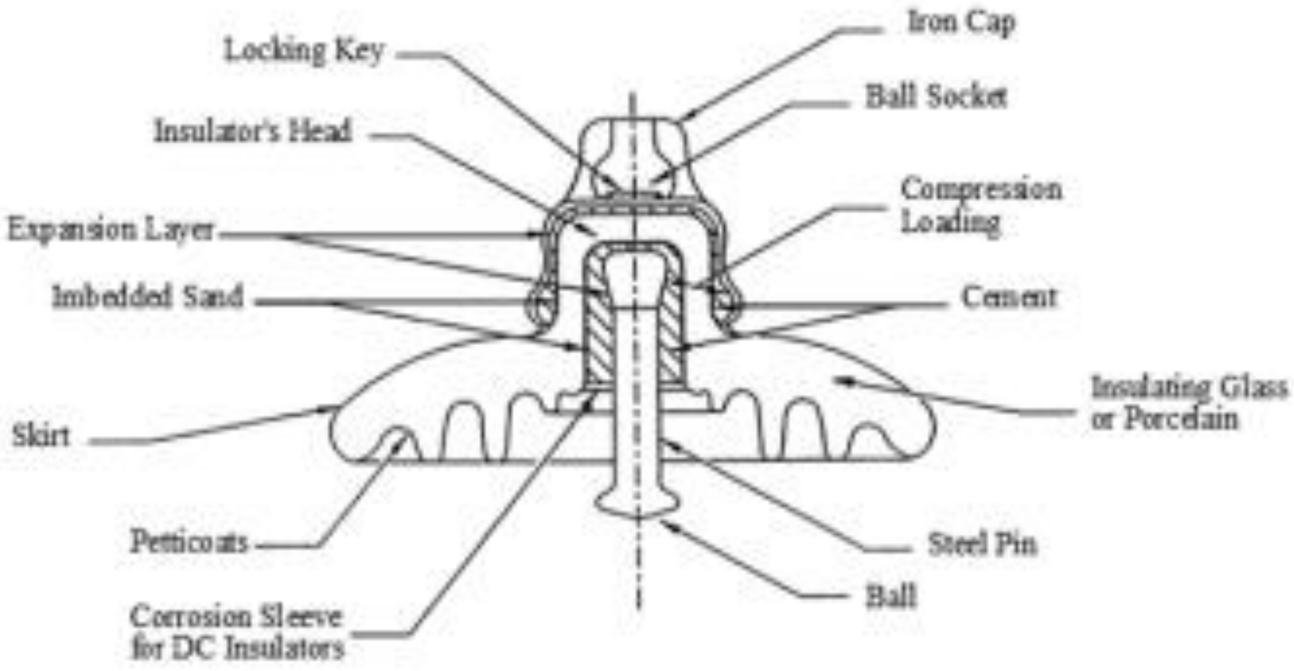
- Aluminum Conductor Steel Reinforced (ACSR);
- All Aluminum Conductor (AAC); and
- All Aluminum Alloy Conductor (AAAC).

- ACSR Coductor





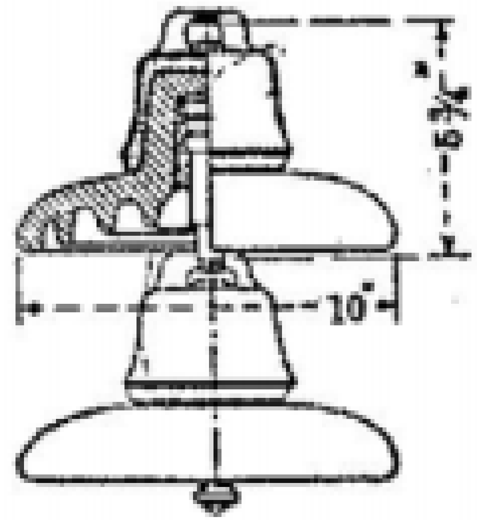
Insulators



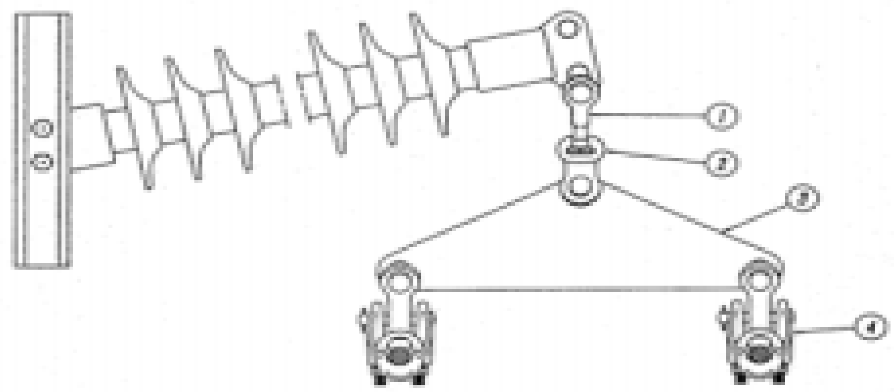
Iron Cap
Locking Key
Ball Socket
Insulator's Head
Compression Loading
Expansion Layer
Imbedded Sand
Cement
Insulating Glass or Porcelain
Skirt
Petticoats
Steel Pin
Ball
Corrosion Sleeve for DC Insulators

Insulator Chain

Line Voltage	Number of Insulators per String
69 kV	4–6
115 kV	7–9
138 kV	8–10
230 kV	12
345 kV	18
500 kV	24
765 kV	30–35

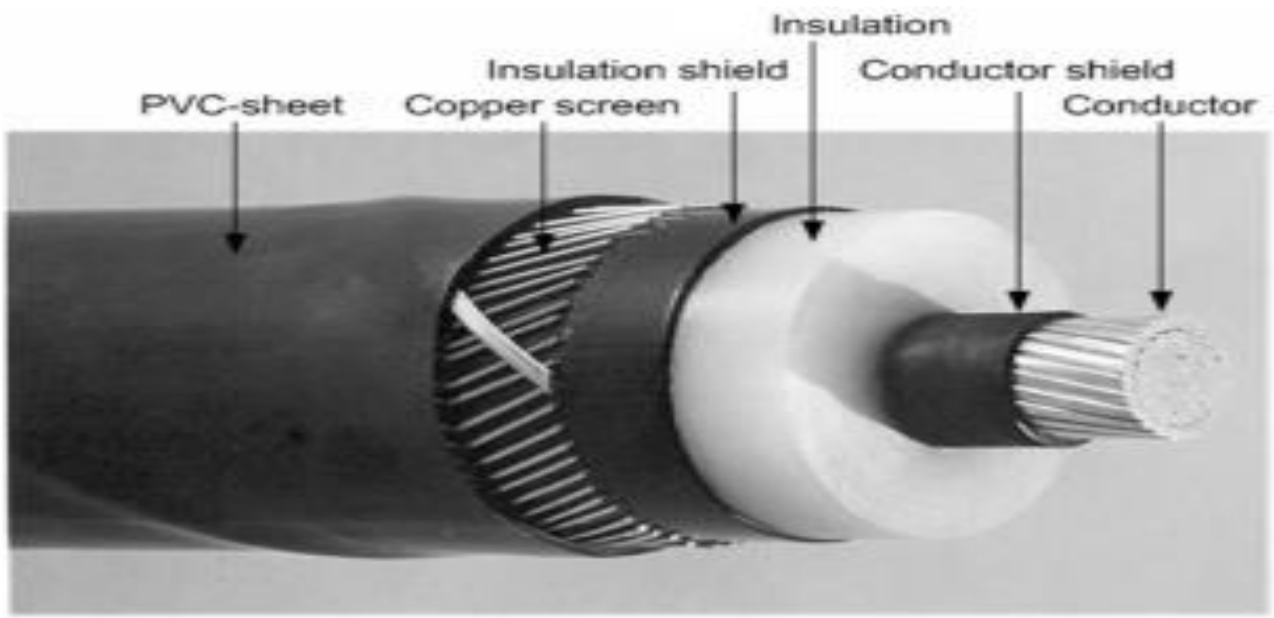


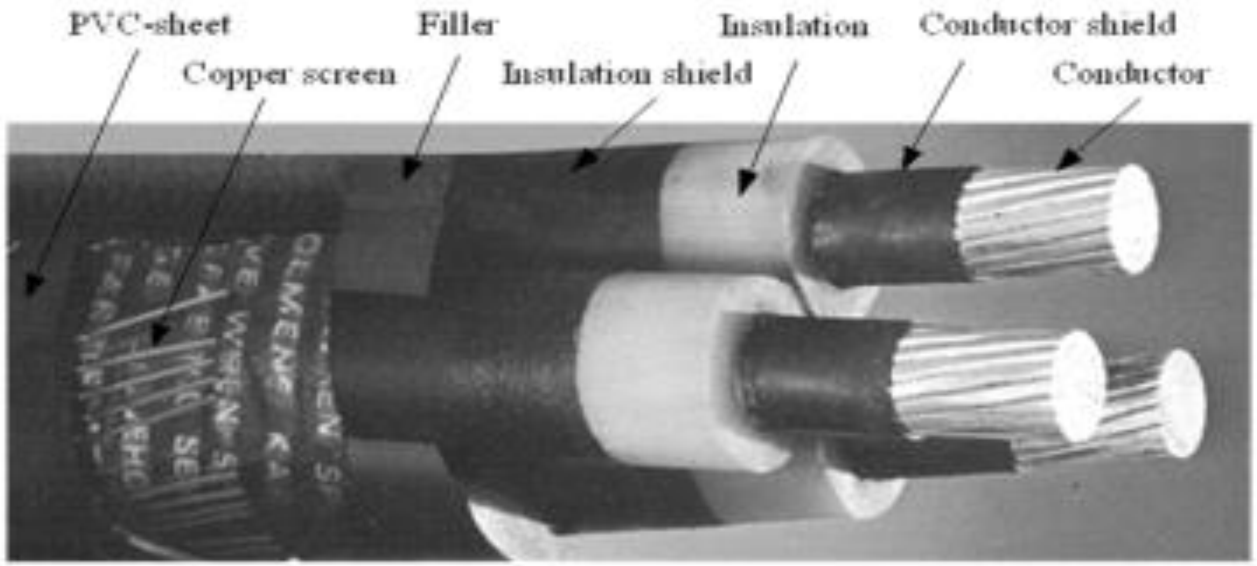
Transmission lines and cables



Line post-composite insulator with yoke holding two conductors.

(1) is the clevis ball,
(2) is the socket for the clevis,
(3) is the yoke plate, and
(4) is the suspension clamp. (*Source*: Sediver)


Insulation
Insulation shield
Conductor shield
PVC-sheet
Copper screen
Conductor


PVC-sheet
Copper screen
Filler
Insulation shield
Insulation
Conductor shield
Conductor

- Transmission lines are classified according to their lengths to:
  - Short: less than 80 km
  - Medium: from 80 km to 240 km
  - Long: longer than 240 km

The DC resistance of a solid round conductor at a specific temperature is given by:

$$R_{DC} = \frac{\rho l}{A}$$

Where:

ρ = Conductor resistivity

L = Conductor length

A = Conductor cross sectional area

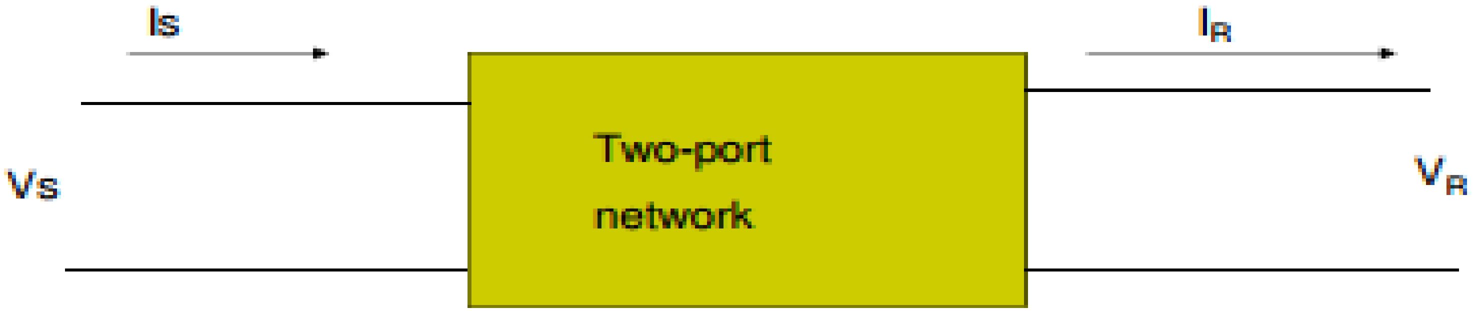
The conductor resistance is affected by three factors:

a) Frequency: skin effect

b) Spiraling

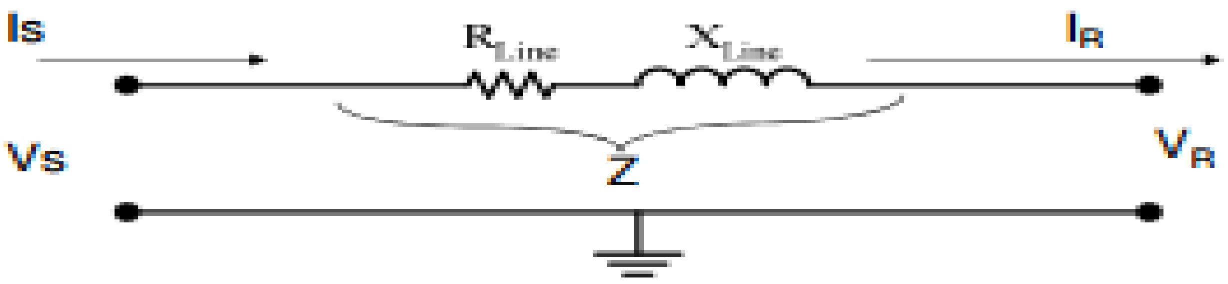
c) Temperature: $R_2 = R_1 \frac{T + t_2}{T + t_1}$

Transmission lines and cables



$$V_s = AV_R + BI_R$$
$$I_s = CV_R + DI_R$$

Short transmission lines



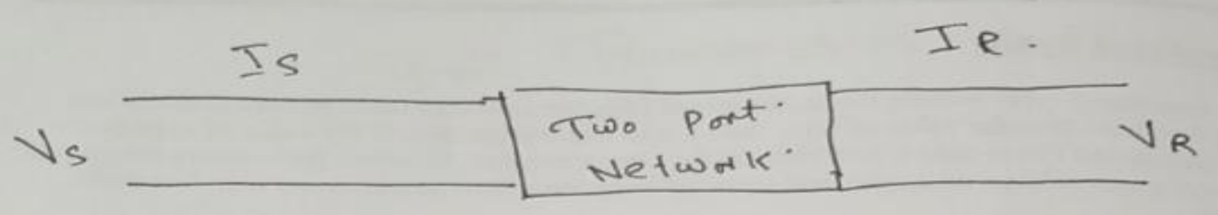
$$V_s = V_R + ZI_R$$

$$I_s = I_R$$

$$A = D = 1$$

$$B = Z$$

$$C = 0$$



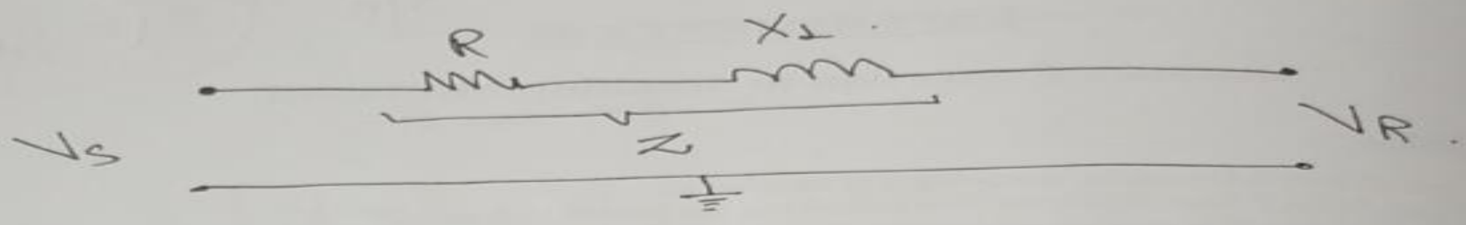
$$V_S = AV_R + BI_R.$$

$$I_S = CV_R + DI_R.$$

$\Rightarrow$ [A, B, C, D

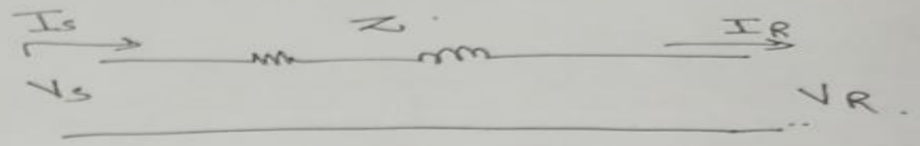
Transmission line Parameter.]

Short Transmission Line :-



In short Transmission Line
Line Impedance Parameters
are lumped.

$Z = R + jx = \underline{zl}$.



equations ?

$$\Rightarrow \left[\begin{array}{l} V_S = V_R + I_R Z \\ I_S = I_R \end{array} \right.$$

Main eq:

$$\left[\begin{array}{l} V_S = A V_R + B I_R \\ I_S = C V_R + D I_R \end{array} \right.$$

Comparing Both eq.

Parameters.

$$\Rightarrow \left[\begin{array}{l} A = 1 \\ B = Z \\ C = 0 \\ D = 1 \end{array} \right.$$

Example 1

- A 220 kV, three phase transmission line is 40 km long. The resistance per phase is 0.15 Ω per km and the inductance per phase is 1.3263 mH per km. Use the short line model to find the voltage and power at the sending end, voltage regulation and efficiency when the line is supplying a three phase load of 381 MVA at 0.8 power factor lagging at 220 kV.

Example 1, Solution

$$Z = (r + j\omega L)l = 6 + j20\,\Omega$$

The receiving voltage per phase is:

$$V_R = \frac{220\angle 0}{\sqrt{3}} = 127\angle 0$$

$$I_R = \frac{S_R^*}{\sqrt{3}V_R} = 1000\angle -36.87$$

$$V_S = V_R + ZI_R = 144.3\angle 4.93kV$$

$$V_S(L-L) = \sqrt{3}V_S = 250kV$$

$$VR = \frac{250-220}{220} = 13.6\%$$

$$P_R = \sqrt{3}220\times 1000\times\cos(36.8) = 304.8MW$$

$$P_s = \sqrt{3}250\times 1000\times\cos(4.93+36.8) = 322.8MW$$

$$\eta = \frac{304.8}{322.8} = 94.4\%$$

Example #1

$l = 40\ Km$

$r = .15\ \Omega/Km$

$L = 1.3263\ mH/Km$

$V_{R1} = 220\ KV.$

$S_1 = 381\ MVA\ at\ .8$

$Z = z\,l.$

$Z = (R + jx)\,l.$

$= (R + j2\pi f L)\,l$

$= (.15 + j\omega.1.3263) \times 40$

$$\boxed{Z = 6 + j20\ \Omega}$$

$$V_R = \frac{220\angle 0}{\sqrt{3}} = 127\angle 0^\circ$$

$$I_R = \frac{S_R}{\sqrt{3}\,V_R} = 1000\ \angle -36.87$$

$$V_S = V_R + Z I_R .$$

$$\boxed{V_S = 144.3 \angle 4.93 \, KV}$$

$$V_{S_{LL}} = \sqrt{3} \, V_S$$

$$\boxed{V_{S_{LL}} = 250 \, KV}$$

Voltage Regulation

$$= \frac{V_S - V_R}{V_R}$$

$$\boxed{V_{Reg.} \% = \frac{250 - 220}{220} = 13.6\%}$$

Medium transmission lines

$$V_s = V_R + Z\left(I_R + \frac{V_R Y}{2}\right) = \left(1 + \frac{YZ}{2}\right)V_R + ZI_R$$

$$I_s = I_R + \frac{V_R Y}{2} + \frac{V_s Y}{2}, \text{ subsitute the value of } V_s$$

$$I_s = Y\left(1 + \frac{YZ}{4}\right)V_R + \left(1 + \frac{YZ}{2}\right)I_R$$

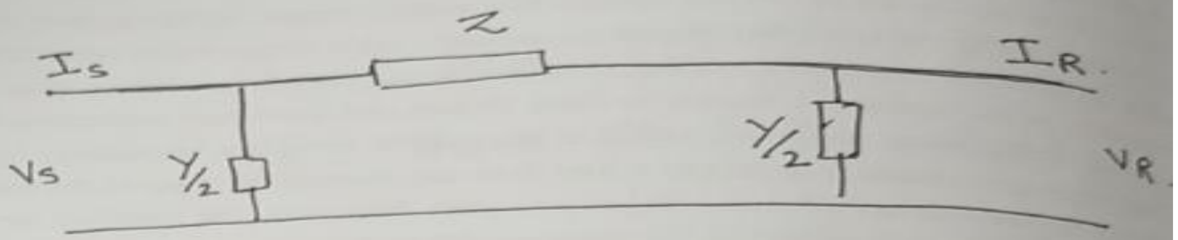
$$A = D = 1 + \frac{YZ}{2}$$

$$B = Z$$

$$C = Y\left(1 + \frac{YZ}{4}\right)$$

Medium Transmission Line:

π-Mode

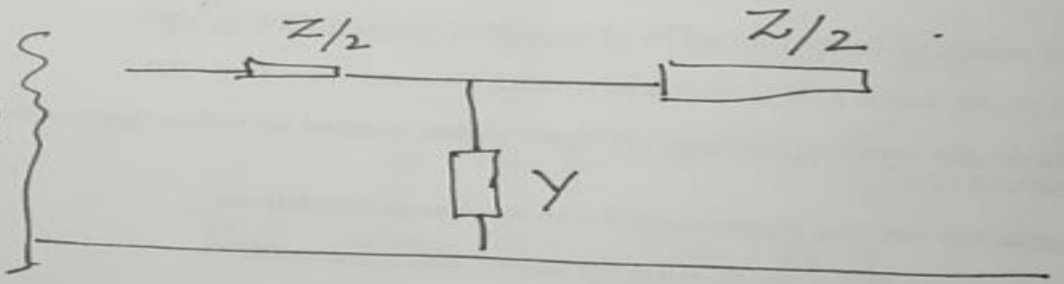


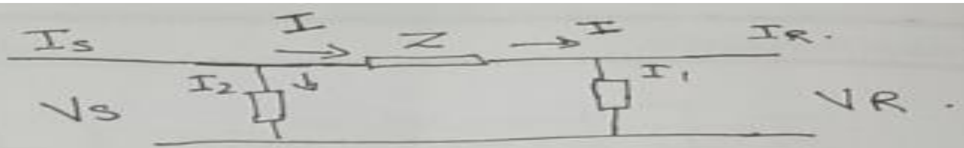
$$Z = zl$$

$$Y = yl$$

$$\Rightarrow \begin{cases} V_s = AV_R + BI_R \\ I_s = CV_R + DI_R \end{cases}$$

T-Model





$$V_S = V_R + IZ$$

$$V_S = V_R + Z(I_1 + I_R).$$

$$= V_R + Z\left(I_R + V_R \cdot \frac{Y}{2}\right).$$

$$= V_R + Z \cdot I_R + \frac{YZ}{2} V_R.$$

$$V_S = \left(1 + \frac{YZ}{2}\right) V_R + I_R Z \longrightarrow (A)$$

$$I_S = I_2 + I_1 + I_R$$

$$= V_S \cdot \frac{Y}{2} + V_R \cdot \frac{Y}{2} + I_R. \quad \text{—} (B)$$

Put eq. A in (B).

$$= \left(1 + \frac{YZ}{2}\right) \cdot \frac{V_R \cdot Y}{2} + I_R Z \cdot \frac{Y}{2}$$

$$+ V_R \frac{Y}{2} + I_R$$

$$= V_R \left[\left(1 + \frac{YZ}{2}\right) + 1 \right] \frac{Y}{2} + \left(1 + \frac{YZ}{2}\right) I_R$$

$$= V_R \left[\frac{2 + YZ + 2}{2} \right] \frac{Y}{2} + \left[\frac{2 + YZ}{2} \right] I_R$$

$$= V_R \cdot Y \left[\frac{4 + YZ}{4} \right] + \left[1 + \frac{YZ}{2} \right] I_R$$

$$I_S = V_R \cdot Y \left[1 + \frac{YZ}{4} \right] + \left(1 + \frac{YZ}{2} \right) I_R \longrightarrow \text{eq. } \boxed{C}$$

$A = 1 + \frac{YZ}{2}$	$C = Y \left(1 + \frac{YZ}{4}\right)$
$B = Z$	$D = 1 + \frac{YZ}{2}$

BY ~~Igori~~ Ignoring the Admittance.

$$A = 1 + \frac{YZ}{2} \qquad C = Y\left(1 + \frac{YZ}{4}\right)$$

$$B = Z \qquad D = 1 + \frac{YZ}{2}$$

$\boxed{Y = 0}$

$$A = 1 \qquad C = 0$$

$$B = Z \qquad D = 1$$

[Medium]. Voltage Regulation :-

$$V_{Reg} = \frac{|V_{R\,NL}| - |V_{R.\,FL}|}{|V_{R\,FL}|}$$

$$\boxed{V_{Reg.} = \frac{\frac{|V_S|}{|A|} - |V_R|}{|V_R|}}$$

$$V_S = AV_R + BI_R \underset{I_R = 0}{\Rightarrow} V_S = A V_R$$

$$V_R = V_S / A$$

Example 2

- A three phase 60 Hz, completely transposed 345kV, 200 km line has two 795,000 cmil 26/2 ACSR conductors per bundle and the following positive sequence line constants:

$z = 0.032 + j0.35\ \Omega/\text{km}$, $y = j4.2*10^{-6}$ S/km. Full load at the receiving end of the line is 700 MW at 0.99 power factor leading and at 95% of rated voltage. Find the following:

  - ABCD parameters of the nominal π circuit
  - Sending end voltage Vs, current Is and power Ps.
  - Percent voltage regulation.
  - Thermal limit.
  - Transmission line efficiency at full load.

a. The total series impedance and shunt admittance values are

$$Z = zl = (0.032 + j0.35)(200) = 6.4 + j70 = 70.29\underline{/84.78^\circ} \quad \Omega$$

$$Y = yl = (j4.2 \times 10^{-6})(200) = 8.4 \times 10^{-4}\underline{/90^\circ} \quad \text{S}$$

From (5.1.15)–(5.1.17),

$$\begin{aligned} A = D &= 1 + (8.4 \times 10^{-4}\underline{/90^\circ})(70.29\underline{/84.78^\circ})\left(\tfrac{1}{2}\right) \\ &= 1 + 0.02952\underline{/174.78^\circ} \\ &= 0.9706 + j0.00269 = 0.9706\underline{/0.159^\circ} \quad \text{per unit} \end{aligned}$$

$$B = Z = 70.29\underline{/84.78^\circ} \quad \Omega$$

$$\begin{aligned} C &= (8.4 \times 10^{-4}\underline{/90^\circ})(1 + 0.01476\underline{/174.78^\circ}) \\ &= (8.4 \times 10^{-4}\underline{/90^\circ})(0.9853 + j0.00134) \\ &= 8.277 \times 10^{-4}\underline{/90.08^\circ} \quad \text{S} \end{aligned}$$

b. The receiving-end voltage and current quantities are

$$\mathrm{V_R} = (0.95)(345) = 327.8 \quad \mathrm{kV_{LL}}$$

$$V_{\mathrm{R}} = \frac{327.8}{\sqrt{3}}\underline{/0^\circ} = 189.2\underline{/0^\circ} \quad \mathrm{kV_{LN}}$$

$$I_{\mathrm{R}} = \frac{700\underline{/\cos^{-1} 0.99}}{(\sqrt{3})(0.95 \times 345)(0.99)} = 1.246\underline{/8.11^\circ} \quad \mathrm{kA}$$

From (5.1.1) and (5.1.2), the sending-end quantities are

$$V_{\mathrm{S}} = (0.9706\underline{/0.159^\circ})(189.2\underline{/0^\circ}) + (70.29\underline{/84.78^\circ})(1.246\underline{/8.11^\circ})$$

$$= 183.6\underline{/0.159^\circ} + 87.55\underline{/92.89^\circ}$$

$$= 179.2 + j87.95 = 199.6\underline{/26.14^\circ} \quad \mathrm{kV_{LN}}$$

$$V_S = 199.6\sqrt{3} = 345.8 \text{ kV}_{LL} \approx 1.00 \quad \text{per unit}$$

$$I_S = (8.277 \times 10^{-4} \underline{/90.08^\circ})(189.2\underline{/0^\circ}) + (0.9706\underline{/0.159^\circ})(1.246\underline{/8.11^\circ})$$

$$= 0.1566\underline{/90.08^\circ} + 1.209\underline{/8.27^\circ}$$

$$= 1.196 + j0.331 = 1.241\underline{/15.5^\circ} \quad \text{kA}$$

and the real power delivered to the sending end is

$$P_S = (\sqrt{3})(345.8)(1.241)\cos(26.14^\circ - 15.5^\circ)$$

$$= 730.5 \quad \text{MW}$$

c. From (5.1.19), the no-load receiving-end voltage is

$$V_{RNL} = \frac{V_S}{A} = \frac{345.8}{0.9706} = 356.3 \quad kV_{LL}$$

and, from (5.1.18),

$$\text{percent VR} = \frac{356.3 - 327.8}{327.8} \times 100 = 8.7\%$$

e. The full-load line losses are $P_S - P_R = 730.5 - 700 = 30.5$ MW and the full-load transmission efficiency is

$$\text{percent EFF} = \frac{P_R}{P_S} \times 100 = \frac{700}{730.5} \times 100 = 95.8\%$$

Example # 02

$z = .032 + j.35\ \Omega/km$

$y = j4.2 \times 10^{-6}\ S/km$

$S_L = 700MW$ at .9 p.f.

at 95% rated voltage.

lenght = 200 km.

$V_L = 345\ KV$

(a).

$Z = zl = (.032 + j.35).200$

$Z = 70.29 \angle 84.78$

$Y = yl = (4.2 \times 10^{-6}).200$

$Y = 8.4 \times 10^{-4} \angle 90^{\circ}$

$A = D = 1 + \frac{YZ}{2} = .97 \angle .159$

$B = Z = 70.29 \angle 84.78$

$C = Y\left(1 + \frac{YZ}{4}\right)$

$= 8.277 \times 10^{-4} \angle 90.08$

(b)

$$V_R = \frac{.95 \times 345}{\sqrt{3}}$$

$$\boxed{V_R = 189.2}$$

$$P_R = \sqrt{3}\, V_{R_{L-L}}\, I_R \cos\theta .$$

$$I_R = \frac{P_R}{\sqrt{3}\, V_{R_{L-L}} \cos\theta} .$$

$$I_R = \frac{700 \angle \cos^{-1} .99}{\sqrt{3}\ (.95 \times 345) \cdot .99}$$

$$\boxed{I_R = 1.246 \angle 8.11 \text{ KA}}$$

$$V_S = AV_R + BI_R$$

$$\boxed{V_S = 199.6 \angle 26.14}$$

$$I_S = CV_R + DI_R$$

$$\boxed{I_S = 1.24 \angle 15.5 \text{ KA}}$$

(c)

$$V_{RNL} = \frac{V_S}{A} = 356.3$$

$$VR = \frac{356.3 - 327.8}{327.8} = 8.7\%$$

$$\boxed{VR = 8.7\%}$$

(e).

$P_R = V_R I_R \cos\theta$.

$P_S = V_S I_S \cos\theta$

$P_R = 700$ MW

$P_S = 730.5$ MW

$$\eta = \frac{P_R}{P_S} = \frac{700}{730.5}$$

$$\boxed{\eta = 95.8\%}$$

Example 3

Consider a 500-kV, 60 Hz three-phase transmission line modeled using the ABCD parameters as follows:

$$V_s = AV_r + BI_r$$
$$I_s = CV_r + AI_r$$
$$A^2 - BC = 1$$

Results of tests conducted at the receiving end the line involving open circuit ($I_r = 0$) and short circuit ($V_r = 0$) are given by:

$$Z_{oc} = \left.\frac{V_s}{I_s}\right|_{I_r=0} = 820\angle\underline{-88.8^\circ}$$

$$Z_{sc} = \left.\frac{V_s}{I_s}\right|_{V_r=0} = 200\angle\underline{78^\circ}$$

Find the line parameters A, B, and C.

Suppose that the load at the receiving end of the line of part b is 750 MVA at nominal voltage, and lagging power factor of 0.83 at rated voltage. Determine the sending end voltage, current, active and reactive power and power factor.

Example 3, Solution

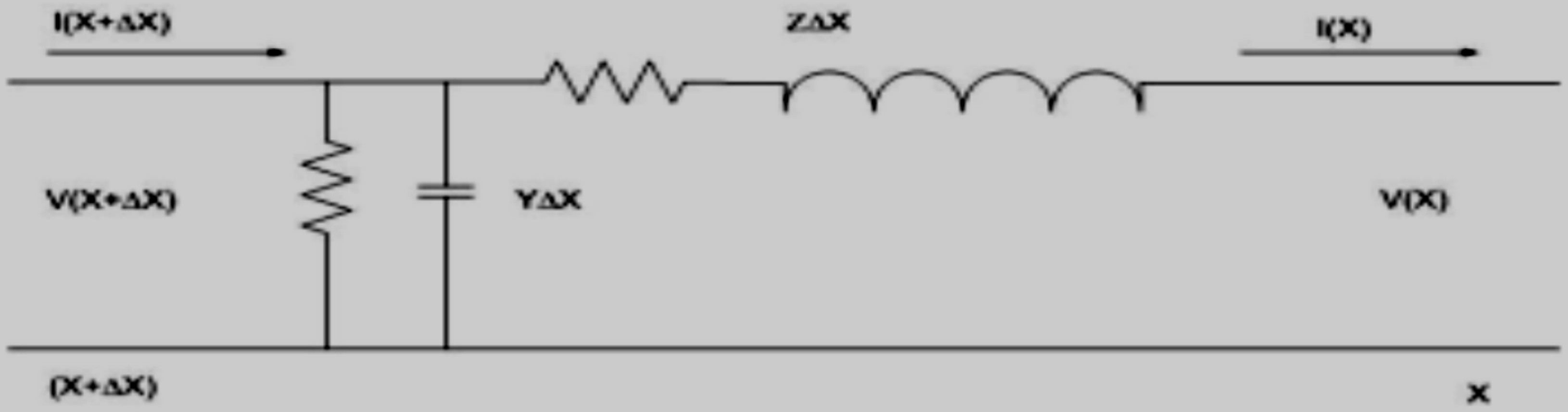
$$\frac{V_s}{I_s} = \frac{AV_r + BI_r}{CV_r + AI_r}$$

$$Z_{oc} = \frac{A}{C} = 820\angle -88.8$$

$$Z_{sc} = \frac{B}{A} = 200\angle 78$$

Then solve for A, B and C and proceed like the previous example.

Long transmission lines



$$z = R + j\omega L \quad \Omega/\text{m}$$

$$y = G + j\omega C \quad \text{S/m}$$

Long transmission lines, cont.

$$V(x+\Delta x)=V(x)+(z\Delta x)I(x)$$

$$\frac{V(x+\Delta x)-V(x)}{\Delta x}=zI(x)$$

Taking the limit as Δx approaches zero:

$$\frac{dV(x)}{dx}=zI(x)$$

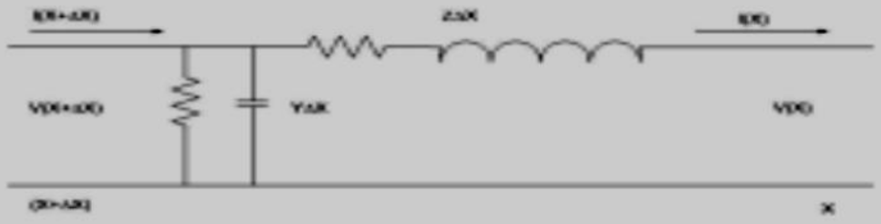
$$I(x+\Delta x)=I(x)+(y\Delta x)V(x+\Delta x)$$

$$\frac{I(x+\Delta x)-I(x)}{\Delta x}=yV(x+\Delta x)$$

Taking the limit as Δx approaches zero:

$$\frac{dI(x)}{dx}=yV(x)$$

$$\frac{d^2V(x)}{dx^2}=z\frac{dI(x)}{dx}=zyV(x) \Longrightarrow \frac{d^2V(x)}{dx^2}-zyV(x)=0$$



$$Let: \gamma^2=zy \Longrightarrow \frac{d^2V(x)}{dx^2}-\gamma^2V(x)=0$$

Long transmission lines, cont.

$$V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x}$$

$\gamma = \sqrt{zy}$ is called the propagation constant

$\gamma = \alpha + j\beta$

$$\frac{dV(x)}{dx} = \gamma A_1 e^{\gamma x} - \gamma A_2 e^{-\gamma x} = zI(x)$$

$$I(x) = \frac{\gamma}{z}\left(A_1 e^{\gamma x} - A_2 e^{-\gamma x}\right) = \sqrt{\frac{y}{z}}\left(A_1 e^{\gamma x} - A_2 e^{-\gamma x}\right) = \frac{A_1 e^{\gamma x} - A_2 e^{-\gamma x}}{Z_c}$$

$Z_c = \sqrt{\dfrac{z}{y}}$ is called the characteristic impedance.

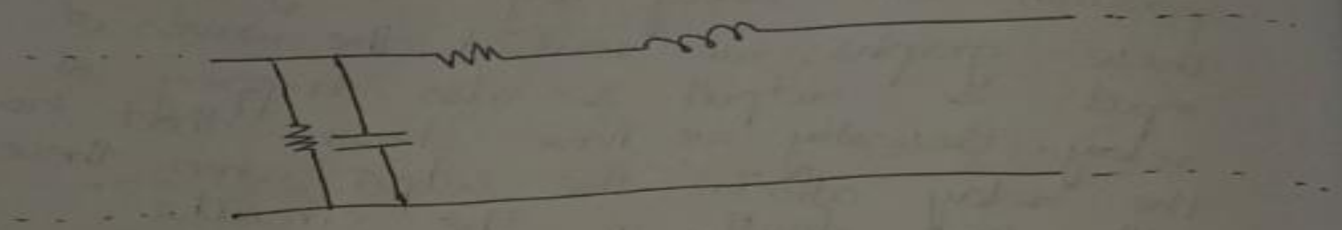
Since $V_R = V(0) = A_1 + A_2$ and $I_R = I(0) = \dfrac{A_1 - A_2}{Z_c}$

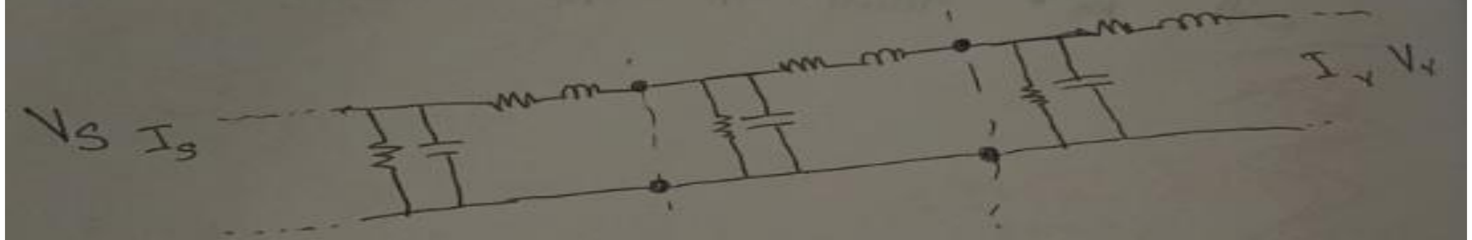
$$A_1 = \frac{V_R + Z_c I_R}{2} \quad \text{and} \quad A_2 = \frac{V_R - Z_c I_R}{2}$$

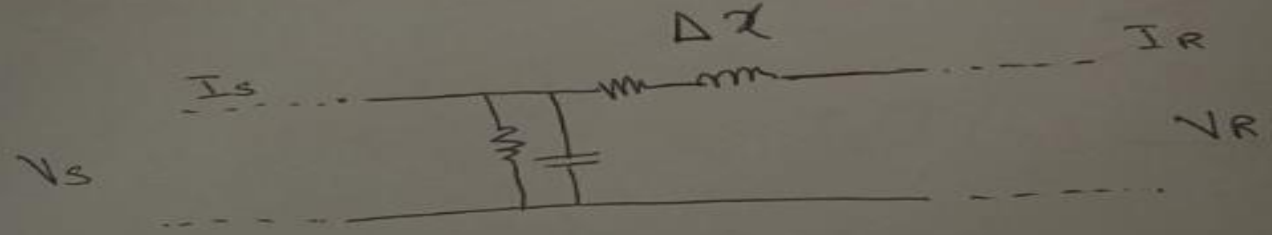
Long Transmission Line

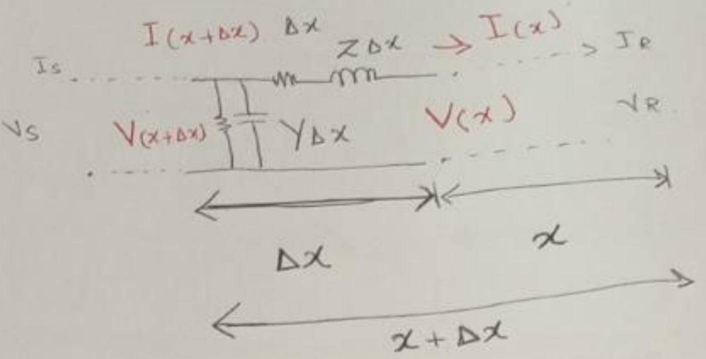
→ For Long Transmission we dont take lumped Parameter.

→ Break Long Transmission is divided into Smaller segments.









$$V(x+\Delta x) = Z\Delta x\, I(x) + V(x).$$

$$\frac{V(x+\Delta x) - V(x)}{\Delta x} = Z\, I(x).$$

$$\underset{\Delta x \to 0}{Lt} \frac{V(x+\Delta x) - V(x)}{\Delta x} = \underset{\Delta x \to 0}{Lt} Z\, I(x).$$

$$\frac{dV(x)}{dx} = Z\, I(x) \longrightarrow (1)$$

$$\frac{d^2V(x)}{dx^2} = Z \cdot \frac{dI(x)}{dx} \rightarrow (2)$$

Current eq.

$$I(x+\Delta x) = V(x+\Delta x)Y \cdot \Delta x + I(x)$$

$$\lim_{\Delta x \to 0} \frac{I(x+\Delta x) - I(x)}{\Delta x} = \lim_{\Delta x \to 0} V(x+\Delta x) \cdot Y.$$

$$\frac{dI(x)}{dx} = Y V(x) \rightarrow (3)$$

$V(x)$ & $I(x)$ in term of V_R & I_R.

$$V(x) = AV_R + BI_R.$$

$$I(x) = CV_R + DI_R.$$

Put eq(3) in eq(2).

$$\frac{d^2 V(x)}{dx^2} = ZY\,V(x)$$

$$\boxed{\frac{d^2 V(x)}{dx^2} - ZY\,V(x) = 0}$$

homogenous eq

$$\boxed{V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x}} \quad (A)$$

⇒ A_1 and A_2 are Integration Constant

$\gamma = \sqrt{ZY}\ m^{-1}$

⇒ γ whose units are m^{-1} is called Propogation Constant.

$$V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x}$$

by taking derivative

$$\frac{dV(x)}{dx} = \gamma A_1 e^{\gamma x} - \gamma A_2 e^{-\gamma x}$$

from eq (1).

$$\frac{dV(x)}{dx} = \gamma A_1 e^{\gamma x} - \gamma A_2 e^{-\gamma x} = Z I(x)$$

Solving for $I(x)$.

B $\leftarrow$ $$I(x) = \frac{A_1 e^{\gamma x} - A_2 e^{-\gamma x}}{Z/\gamma} = \frac{A_1 e^{\gamma x} - A_2 e^{-\gamma x}}{Z_c}$$

$$Z/\gamma \Rightarrow \frac{Z}{\sqrt{YZ}} \Rightarrow \sqrt{Z/Y}$$

$$\boxed{Z_c = \sqrt{Z/Y}\ \Omega} \Rightarrow \text{Charastistic Impedance}$$

$\Rightarrow$

$$\gamma = \sqrt{YZ} = \alpha + j\beta.$$

α = attenuation Constant.

β = Phase Constant.

To solve these eq we use boundary conditions to find constant.

$x = 0$

$V(x) = V_R$ & $I(x) = I_R$.

Apply at eq A & B.

$$V_R = A_1 + A_2 \longrightarrow (x)$$

$$I_R = \frac{A_1}{Z_C} - \frac{A_2}{Z_C}$$

$$Z_C I_R = A_1 - A_2 \longrightarrow (y)$$

Add and subtract x & y.

$$A_1 = \frac{1}{2}(V_R + Z_C I_R) \rightarrow (4)$$

$$A_2 = \frac{1}{2}(V_R - Z_C I_R) \rightarrow (5)$$

Put (4) & 5 in A & B.

$$V(x) = \frac{1}{2}(V_R + Z_C I_R) \cdot e^{\gamma x} + \frac{1}{2}(V_R - Z_C I_R) e^{-\gamma x}$$

$$V(x) = \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right) V_R + Z_C \left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right) I_R$$

$$I(x) = \frac{1}{2 \cdot Z_C}(V_R + Z_C I_R) e^{\gamma x} - \frac{1}{2 Z_C}(V_R - Z_C I_R) e^{-\gamma x}$$

$$I(x) = \frac{1}{Z_C}\left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right) V_R + \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right) I_R.$$

$$V(x) = \cosh(\gamma x) V_R + Z_C \sinh(\gamma x) I_R$$

$$I(x) = \frac{1}{Z_C} \sinh(\gamma x) V_R + \cosh(\gamma x) I_R.$$

$A(x) = D(x) = \cosh(\gamma x) =$

$B(x) = Z_c \sinh(\gamma x) =$

$C(x) = \frac{1}{Z_c} \sinh(\gamma x)$.

Sending end
$x = l$.

$V_S = V(l) = A(l) V_R + B(l) I_R$.
$I_S = I(l) = C(l) V_R + D(l) I_R$

$V_S = \cosh(\gamma l) V_R + Z_c \sinh(\gamma l) I_R$
$I_S = \frac{1}{Z_c} \sinh(\gamma l) V_R + \cosh(\gamma l) I_R$.

$$\cosh \gamma x = \frac{e^{\gamma x} + e^{-\gamma x}}{2}$$

$$\cosh \gamma x = \frac{e^{(\alpha + j\beta)x} + e^{-(\alpha + j\beta)x}}{2}$$

$$\cosh \gamma x = \frac{e^{\alpha x} \cdot e^{j\beta x} + e^{-\alpha x} \cdot e^{-j\beta x}}{2}$$

$$e^{j\theta} = 1\angle\theta = \cos\theta + j\sin\theta.$$

$$= \frac{e^{\alpha x}\angle\beta x + e^{-\alpha x}\angle -\beta x}{2}$$

[β should be in radian.

$$\gamma = \alpha + j\beta \quad \mathrm{m}^{-1} \tag{5.2.37}$$

The quantity γl is dimensionless. Also

$$e^{\gamma l} = e^{(\alpha l + j\beta l)} = e^{\alpha l} e^{j\beta l} = e^{\alpha l} \underline{/\beta l} \tag{5.2.38}$$

Using (5.2.38) the hyperbolic functions cosh and sinh can be evaluated as follows:

$$\cosh(\gamma l) = \frac{e^{\gamma l} + e^{-\gamma l}}{2} = \frac{1}{2}(e^{\alpha l} \underline{/\beta l} + e^{-\alpha l} \underline{/-\beta l}) \tag{5.2.39}$$

and

$$\sinh(\gamma l) = \frac{e^{\gamma l} - e^{-\gamma l}}{2} = \frac{1}{2}(e^{\alpha l} \underline{/\beta l} - e^{-\alpha l} \underline{/-\beta l}) \tag{5.2.40}$$

A three-phase 765-kV, 60-Hz, 300-km, completely transposed line has the following positive-sequence impedance and admittance:

$$z = 0.0165 + j0.3306 = 0.3310\underline{/87.14^\circ} \quad \Omega/\text{km}$$

$$y = j4.674 \times 10^{-6} \quad \text{S/km}$$

Assuming positive-sequence operation, calculate the exact *ABCD* parameters of the line. Compare the exact *B* parameter with that of the nominal π circuit.

$$Z_c = \sqrt{\frac{0.3310\angle 87.14^\circ}{4.674 \times 10^{-6}\angle 90^\circ}} = \sqrt{7.082 \times 10^4\angle -2.86^\circ}$$

$$= 266.1\angle -1.43^\circ \quad \Omega$$

and

$$\gamma l = \sqrt{(0.3310\angle 87.14^\circ)(4.674 \times 10^{-6}\angle 90^\circ)} \times (300)$$

$$= \sqrt{1.547 \times 10^{-6}\angle 177.14^\circ} \times (300)$$

$$= 0.3731\angle 88.57^\circ = 0.00931 + j0.3730 \quad \text{per unit}$$

$$e^{\gamma l} = e^{0.00931} e^{+j0.3730} = 1.0094\underline{/0.3730} \quad \text{radians}$$

$$= 0.9400 + j0.3678$$

and

$$e^{-\gamma l} = e^{-0.00931} e^{-j0.3730} = 0.9907\underline{/-0.3730} \quad \text{radians}$$

$$= 0.9226 - j0.3610$$

Then, from (5.2.39) and (5.2.40),

$$\cosh(\gamma l) = \frac{(0.9400 + j0.3678) + (0.9226 - j0.3610)}{2}$$

$$= 0.9313 + j0.0034 = 0.9313\underline{/0.209^\circ}$$

$$\sinh(\gamma l) = \frac{(0.9400 + j0.3678) - (0.9226 - j0.3610)}{2}$$

$$A = D = \cosh(\gamma l) = 0.9313\underline{/0.209^\circ} \quad \text{per unit}$$

$$B = (266.1\underline{/-1.43^\circ})(0.3645\underline{/88.63^\circ}) = 97.0\underline{/87.2^\circ} \quad \Omega$$

$$C = \frac{0.3645\underline{/88.63^\circ}}{266.1\underline{/-1.43^\circ}} = 1.37 \times 10^{-3}\underline{/90.06^\circ} \quad \text{S}$$

Using (5.1.16), the B parameter for the nominal π circuit is

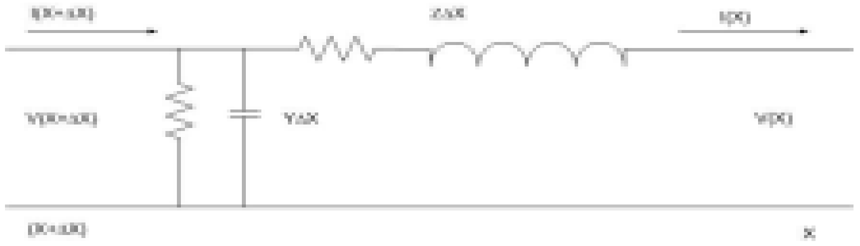
$$B_{\text{nominal } \pi} = Z = (0.3310\underline{/87.14^\circ})(300) = 99.3\underline{/87.14^\circ} \quad \Omega$$

which is 2% larger than the exact value.

Lossless lines

When line losses are neglected, simpler expressions for the line parameters are obtained.

For lossless line, R=G=0 and hence:



$$z = j\omega L \quad \Omega/\text{m}$$
$$y = j\omega C \quad \text{S/m}$$

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}}$$

The characteristics impedance is called the surge impedance and is pure real

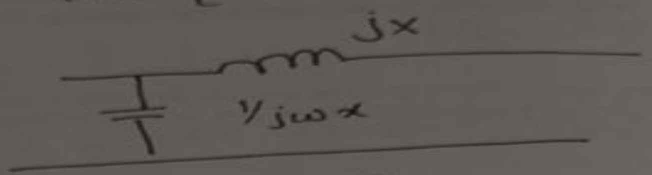
$$\gamma = \sqrt{zy} = \sqrt{(j\omega L)(j\omega C)} = j\omega\sqrt{LC} = j\beta$$

The propagation constant is pure imaginary

Loseless Model:

↳ Assume it is Loseless Model:-

No [R, G].



$R = G = 0.$

$Z = j\omega L.$

$Y = j\omega C.$

$$Z_C = \sqrt{Z/Y} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{L/C}$$

$$\gamma = \sqrt{ZY} = \sqrt{(j\omega L)(j\omega C)}.$$

$$\gamma = j\omega\sqrt{LC} = j\beta$$

Propogation constant.

$$A(x) = \cosh(\gamma x)$$

$$= \frac{e^{\gamma x} + e^{-\gamma x}}{2}$$

For lossless Model.
Replace $\gamma \longrightarrow (j\beta)$.

$$= \frac{e^{j\beta x} + e^{-j\beta x}}{2}$$

$$\boxed{A(x) = \cos(\beta x).}$$

$$\sinh(\gamma x) = \frac{e^{\gamma x} - e^{-\gamma x}}{2}.$$

$$\gamma \longrightarrow j\beta$$

$$\sinh(j\beta x) = \frac{e^{j\beta x} - e^{-j\beta x}}{2}$$

$$= j\sin(\beta x).$$

$B = Z_C \sinh(\gamma x)$.

For lossless lines.

$$\boxed{B = jZ_C \sin(\beta x)}$$

$$C = \frac{1}{Z_C} \sinh(\gamma x)$$

For lossless line.

$$\boxed{C = \frac{j \sin(\beta x)}{Z_C}}$$

Lossless lines

ABCD Parameters

$$A(x) = \cosh(\gamma x) = D(x)$$
$$A(x) = \frac{e^{j\beta x} + e^{-j\beta x}}{2} = \cos(\beta x)$$

$$B = Z_c \sinh(\gamma x)$$
$$\sinh(\gamma x) = \sinh(j\beta x) = \frac{e^{j\beta x} - e^{-j\beta x}}{2} = j\sin(\beta x)$$
$$B = jZ_c \sin(\beta x)$$

$$C = \frac{1}{Z_c}\sinh(\gamma x) = \frac{j\sin(\beta x)}{Z_c}$$

Wavelength

A wavelength is the distance required to change the phase of the voltage or current by 2π.

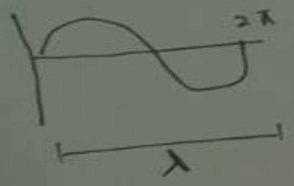
$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}} = \frac{1}{f\sqrt{LC}}$$

$$v = \frac{1}{\sqrt{LC}}$$ Velocity of propagation

Lossless line:

$$V(x) = \cos\beta x \, V_R + j Z_C \sin\beta x \, I_R$$

$$I(x) = \frac{1}{Z_C} j \sin\beta x \, V_R + \cos\beta x \, I_R$$



$$\lambda = \frac{v}{f} \qquad v = \frac{1}{\sqrt{LC}}$$

$$\lambda = \frac{2\pi}{\beta}$$

$$\lambda = \frac{2\pi}{\omega\sqrt{LC}}$$

$$\lambda = \frac{1}{f\sqrt{LC}}$$

Example 5

- A three phase 60 Hz, 500kV, 300 km. The line inductance is 0.97 mH/km and its capacitance is 0.0115 μF/km per phase. Assume a lossless line:

a) Determine the line phase constant β, the surge impedance Zc, velocity of propagation and the line wavelength.

b) The receiving end rated load is 800 MW, 0.8 power factor lagging at 500 kV, determine the sending end quantities.

Example 5, solution

a) For a lossless line:

$$\beta = \omega\sqrt{LC} = 2\pi \times 60\sqrt{0.97 \times 0.0115 \times 10^{-9}} = 0.001259 \text{ rad/km}$$

$$Z_c = \sqrt{\frac{L}{C}} = 290.43\,\Omega \qquad v = \frac{1}{\sqrt{LC}} = 2.994 \times 10^5 \text{ km/s}$$

$$\lambda = \frac{v}{f} = 4990\,km$$

b) The receiving end voltage is: $V_R = \frac{500\angle 0}{\sqrt{3}} = 288.67\angle 0\,kV$

The receiving end current is: $I_R = \frac{800 \times 10^6}{\sqrt{3} \times 500 \times 10^3 \times 0.8}\angle -\cos^{-1}(0.8) = 1154.7\angle -36.87\,A$

Example 5, solution

The sending end voltage is:

$$V_S = \cos(\beta l)V_R + jZ_C \sin(\beta l)I_R = 356.5\angle 16.1\,\text{kV}$$

$$I_S = j\frac{1}{Z_C}\sin(\beta l)V_R + \cos(\beta l)I_R = 902.3\angle -17.9\text{ A}$$

Surge Impedance Loading

Surge impedance loading (SIL) is the power delivered by a lossless line to a load resistance equal to the surge impedance Z_c.

$$I_R = \frac{V_R}{Z_c} \qquad SIL = 3V_R I_R^* = 3\frac{|V_R|^2}{Z_c}$$

$$V(x) = \cos(\beta x)V_R + jZ_c \sin(\beta x) I_R$$

$$V(x) = \cos(\beta x)V_R + jZ_c \sin(\beta x)\left(\frac{V_R}{Z_c}\right)$$

$$V(x) = (\cos(\beta x) + j\sin(\beta x))V_R$$

$$|V(x)| = |V_R|$$

Complex Power Flow Through Transmission Lines

$$V_s = AV_R + BI_R$$

Let

$$A = |A|\angle\theta_A$$
$$B = |B|\angle\theta_B$$

And

$$V_S = |V_S|\angle\delta$$
$$V_R = |V_R|\angle 0$$

$$I_R = \frac{|V_S|\angle\delta - |A|\angle\theta_A |V_R|\angle 0}{|B|\angle\theta_B} \;\longrightarrow\; S_R = 3V_R I^*{}_R$$

The real power at the receiving end of the line is:

$$P_R = \frac{|V_{S(L-L)}||V_{R(L-L)}|\cos(\theta_B - \delta) - |A||V_{R(L-L)}|^2 \cos(\theta_B - \theta_A)}{|B|}$$

For a lossless line, B=jX, θ_A=0, θ_B=90

$$P_R = \frac{|V_{S(L-L)}||V_{R(L-L)}|\sin(\delta)}{X}$$

Complex Power Flow Through Transmission Lines

So the maximum power that can be delivered will be

$$P_{\max} = \frac{|V_{S(L-L)}||V_{R(L-L)}|}{X}$$

This value is called the steady-state stability limit of a lossless line. If an attempt was made to exceed this limit, then synchronous machines at the sending end would lose synchronism with those at the receiving end.

Power Transmission Capabilities

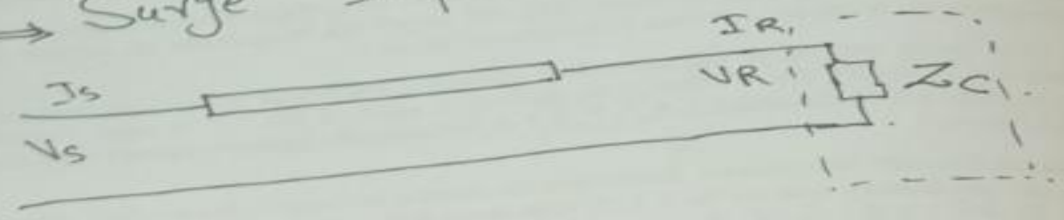
$$P_R = \frac{|V_{S(L-L)}||V_{R(L-L)}|\sin(\delta)}{X}$$

For planning and other purposes, it is very useful to express the power transfer formula in terms of SIL.

For a lossless line: $X = Z_C \sin(\beta l)$ $\Longrightarrow$ $P_R = \dfrac{|V_{S(L-L)}||V_{R(L-L)}|\sin(\delta)}{Z_C \sin(\beta l)}$

$$P_R = \frac{|V_{S(L-L)}|}{V_{rated}} \frac{|V_{R(L-L)}|}{V_{rated}} \frac{(V_{rated})^2}{Z_C} \frac{\sin(\delta)}{\sin(\beta l)} \Longrightarrow \boxed{P_R = \frac{|V_{Spu}||V_{Rpu}|SIL}{\sin(\beta l)}\sin(\delta)}$$

⇒ Surge Impedance Loading :-



$$I_R = \frac{V_R}{Z_C}$$

$$V(x) = \cos\beta x \, V_R + j Z_C \sin\beta x \cdot \frac{V_R}{Z_C}.$$

$$V(x) = (\cos\beta x + j \sin\beta x) V_R$$

$$= 1 \angle \beta x \cdot V_R.$$

$$V(x) = V_R \angle \beta x$$

$$\boxed{|V(x)| = |V_R|.}$$

$$V_S = A V_R + B I_R$$

$$I_R = \frac{V_S - A V_R}{B}$$

$$I_R = \frac{|V_S| \angle \delta - |A| \angle \theta_A \cdot |V_R| \angle 0}{|B| \angle \theta_B}$$

$$S_R = 3 \cdot V_R I_R^*$$

$$S_R = 3 |V_R| \angle 0 \cdot \frac{V_S \angle -\delta - |A| |V_R| \angle -\theta_A}{|B| \angle -\theta_B}$$

$$P_R + jQ_R = \frac{3 |V_R| |V_S| \angle \theta_B - \delta - 3 |A| |V_R^2| \angle \theta_B - \theta_A}{|B|}$$

$$P_R = \frac{3 |V_R| |V_S| \cos(\theta_B - \delta) - 3 |A| |V_R^2| \cos(\theta_B - \theta_A)}{|B|}$$

$$P_R = \frac{|V_{RLL}||V_{SLL}|\cos(\theta_B - \delta) - |A|\,|V_R^2|\cos(\theta_B - \theta_A)}{|B|} \quad (B)$$

For lossless lines.

$$V(x) = \cos\beta x\, V_R + jZ_C \sin\beta x\, I_R.$$

$$B = \underbrace{j}_{\theta_B = 90^\circ}\underbrace{Z_C \sin\beta x}_{\text{Constant}}.$$

$$\boxed{\theta_B = 90^\circ \quad B = jx.}$$

$$A = \cos\beta x$$

$$\boxed{\theta_A = 0^\circ}$$

$\theta_A = 0$, $\theta_B = 90^\circ$, $B = jx$

in eq (B).

$$P_R = \frac{|V_{RLL}||V_{SLL}|\cos(90^\circ - \delta) - |A|\,|V_{RLL}^2|\cos(90 - 0)}{|X|}$$

Complex Power Flow Through Transmission Lines

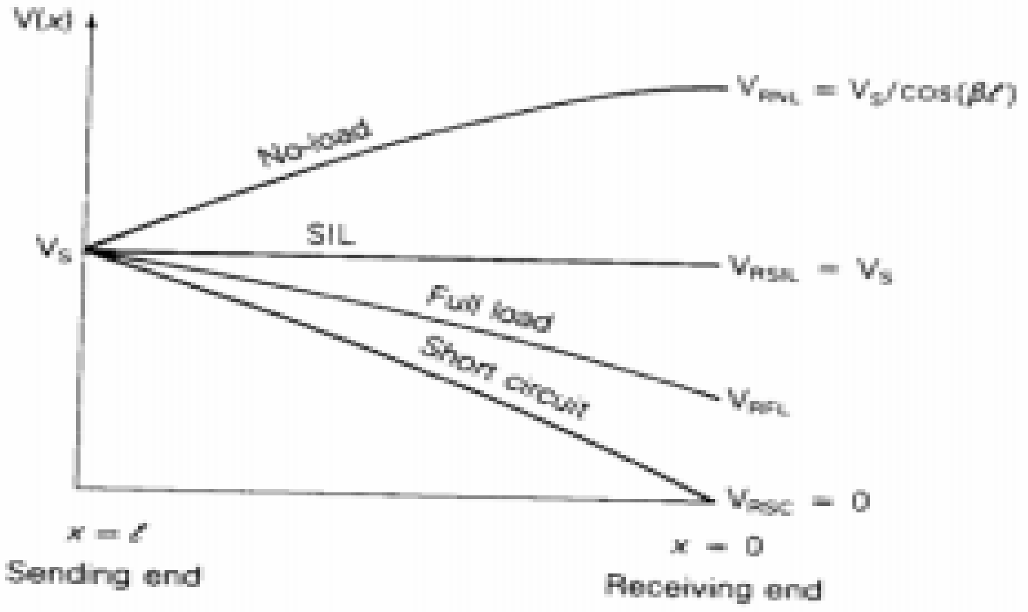
The reactive power at the receiving end of the line is:

$$Q_R = \frac{|V_{S(L-L)}||V_{R(L-L)}|\sin(\theta_B - \delta) - |A||V_{R(L-L)}|^2 \sin(\theta_B - \theta_A)}{|B|}$$

For a lossless line, $B=jX$, $\theta_A=0$, $\theta_B=90$

$$Q_R = \frac{|V_{S(L-L)}||V_{R(L-L)}|}{X}\cos(\delta) - \frac{|V_{R(L-L)}|^2}{X}\cos(\beta l)$$

Voltage Profile under different loading conditions



16:28

-At no-load, $I_{RNL}=0$ and

$V_{NL}(x)=\cos(\beta l) * V_{RNL}$

The no-load voltage increases from $V_S=\cos(\beta l) * V_{RNL}$ at the sending end to V_{RNL} at the receiving end.

-From previous slide, voltage profile is constant at SIL.

-For short circuit, $V_R=0$

-For full load, the receiving voltage will drop depends on the loading conditions.

1. At no-load, $I_{RNL} = 0$ and (5.4.13) yields

$$V_{NL}(x) = (\cos \beta x)V_{RNL} \tag{5.4.22}$$

The no-load voltage increases from $V_S = (\cos \beta l)V_{RNL}$ at the sending end to V_{RNL} at the receiving end (where $x = 0$).

2. From (5.4.18), the voltage profile at SIL is flat.

3. For a short circuit at the load, $V_{RSC} = 0$ and (5.4.13) yields

$$V_{SC}(x) = (Z_c \sin \beta x)I_{RSC} \tag{5.4.23}$$

The voltage decreases from $V_S = (\sin \beta l)(Z_c I_{RSC})$ at the sending end to $V_{RSC} = 0$ at the receiving end.

4. The full-load voltage profile, which depends on the specification of full-load current, lies above the short-circuit voltage profile.

$$P_{R_{max}} = \frac{|V_{RL-L}||V_{SL-L}| \sin\delta}{X}$$

Maximum line loadability

$\Longrightarrow$ Steady state stability limit.

$$B = jX \Rightarrow j Z_C \sin(\beta l).$$

$$P_R = \frac{|V_{S(L-L)}||V_{R(L-L)}| \sin\delta}{Z_C \sin(\beta l)}.$$

$$P_R = \frac{|V_{S(L-L)}|}{V_{Rated}} \frac{|V_{R(L-L)}|}{V_{Rated}} \frac{V_{Rated}^2}{Z_C} \cdot \frac{\sin\delta}{\sin\beta l}$$

$$= \frac{|V_{SPU}||V_{RPU}| \, SIL \, \sin\delta}{\sin(\beta l)}.$$

Example 6

A three phase power of 700 MW is to be transmitted to a substation located 315 km from a source of power. For a preliminary line design assume the following parameters:

V_s = 1 per unit, V_R = 0.9 per unit, λ=5000 km, Z_c=320 Ω and δ=36.87

a) Based on the practical line loadability equation determine a nominal voltage level for the transmission line.

b) For the transmission voltage obtained in (a) calculate the theoretical maximum power that can be transferred by the transmission line.

Example 6, solution

The line phase constant is:

$$\beta l = \frac{2\pi l}{\lambda}\, rad = \frac{360}{5000}(315) = 22.68^{o}$$

The practical line loadability:

$$P_R = \frac{|V_{Spu}||V_{Rpu}|SIL}{\sin(\beta l)}\sin(\delta)$$

$$\Rightarrow \quad 700 = \frac{|1||0.9|SIL}{\sin(22.68)}\sin(36.87) \quad \Rightarrow \quad \text{SIL} = 499 \text{ MW}$$

$$kV_L = \sqrt{(Z_C)(SIL)} = \sqrt{(320)(499.83)} = 400\, kV$$

$$P_{max} = \frac{|V_{Spu}||V_{Rpu}|SIL}{\sin(\beta l)} = 1167 \text{ MW}$$