

# Lec\_5

Transmission Lines

# Primary Methods for Power Transfer

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The most common methods for transfer of electric power are

- 1) Overhead AC
- 2) Underground AC
- 3) Overhead DC
- 4) Underground DC

# Transmission lines and cables

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- **Extra-high-voltage lines**

- Voltage: 345 kV, 500 kV, 765 kV

- **High-voltage lines**

- Voltage: 115 kV, 230 kV

- **Sub-transmission lines**

- Voltage: 46 kV, 69 kV

- **Distribution lines**

- Voltage: 2.4 kV to 46 kV, with 15 kV being the most commonly used

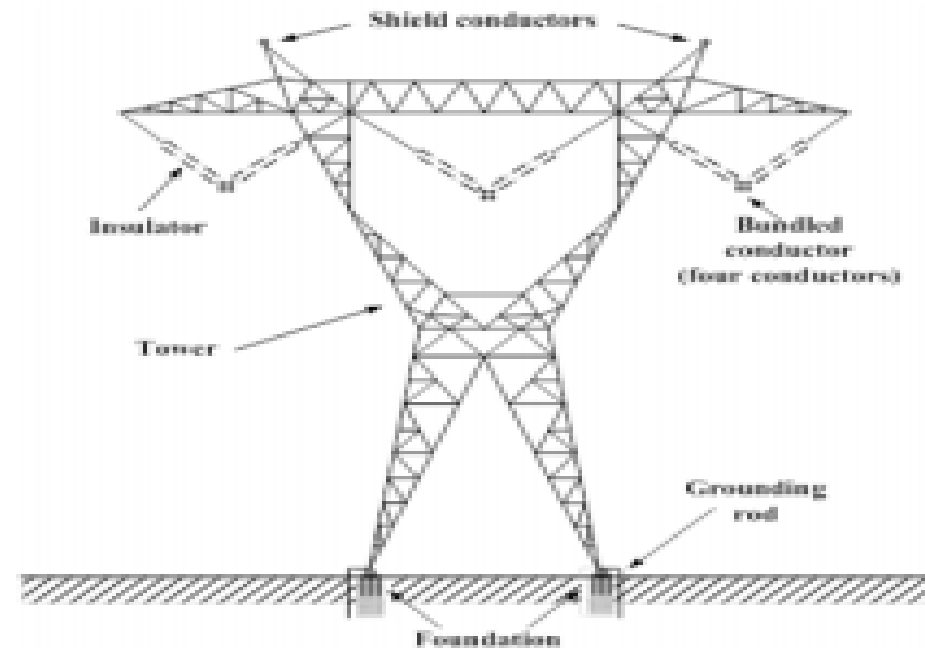
- **High-voltage DC lines**

- Voltage:  $\pm 120$  kV to  $\pm 600$  kV

# Transmission lines and cables

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- ❑ Three-phase conductors, which carry the electric current;
- ❑ Insulators, which support and electrically isolate the conductors;
- ❑ Tower, which holds the insulators and conductors;
- ❑ Foundation and grounding; and
- ❑ Optional shield conductors, which protect against lightning

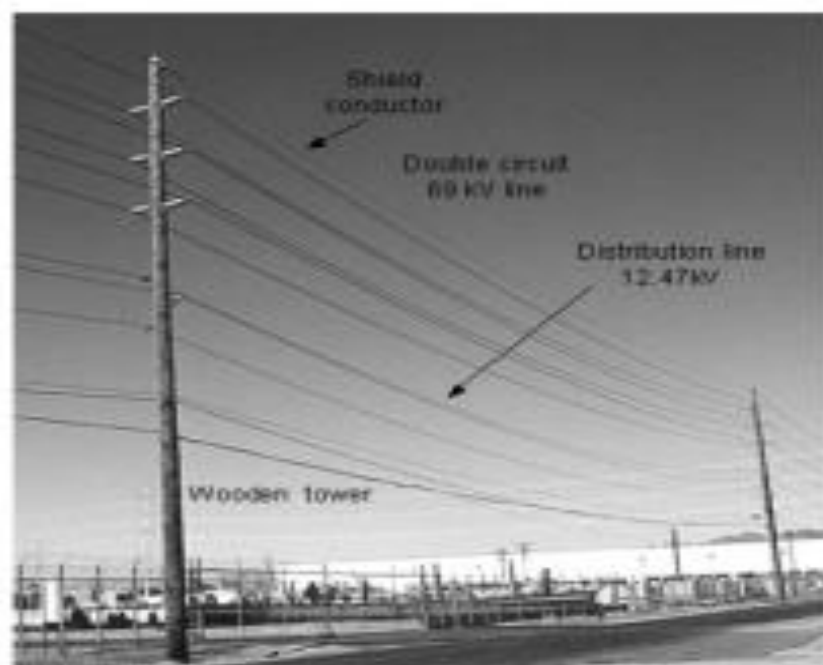


# Transmission lines and cables

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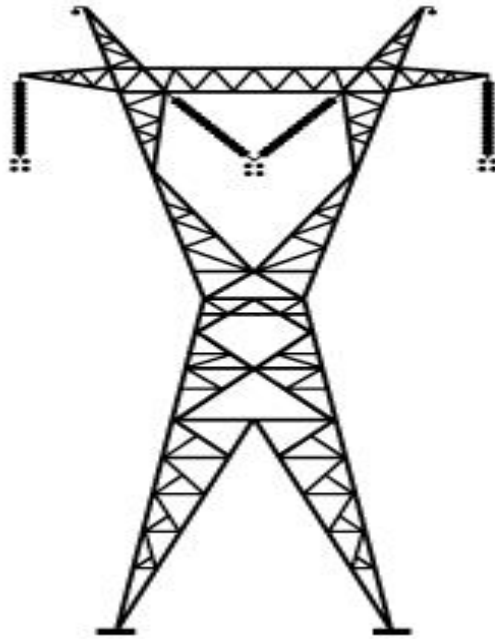


Distribution Line



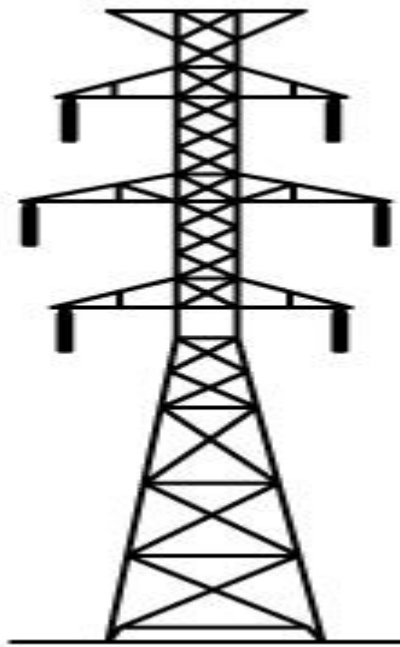
# Waist-type tower

- This is the most common type of transmission tower. It's used for voltages ranging from 110 to 735 kV. Because they're easily assembled, these towers are suitable for power lines that cross very uneven terrain.



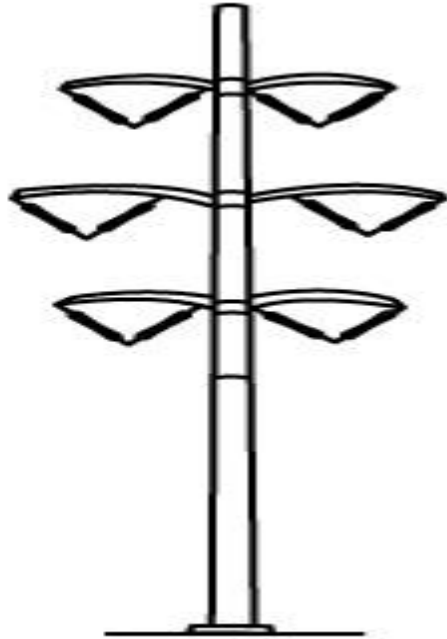
# Double-circuit tower

- This small-footprint tower is used for voltages ranging from 110 to 315 kV. Its height ranges from 25 to 60 metres.

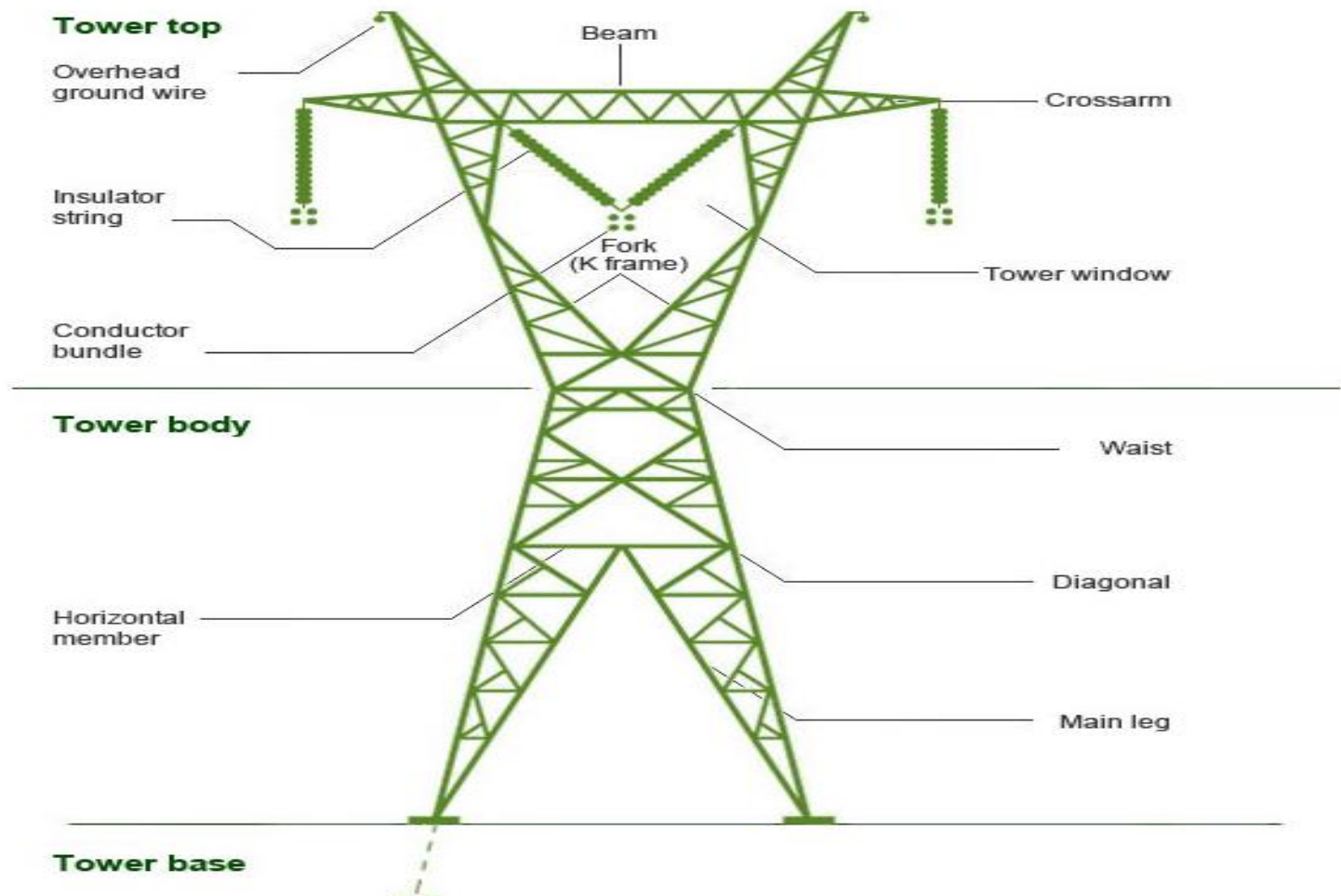


# Tublar steel pole

- Featuring a streamlined, aesthetic shape, this structure is less massive than other towers, allowing it to blend easily into the environment. For this reason, it's being used more and more in urban centres.





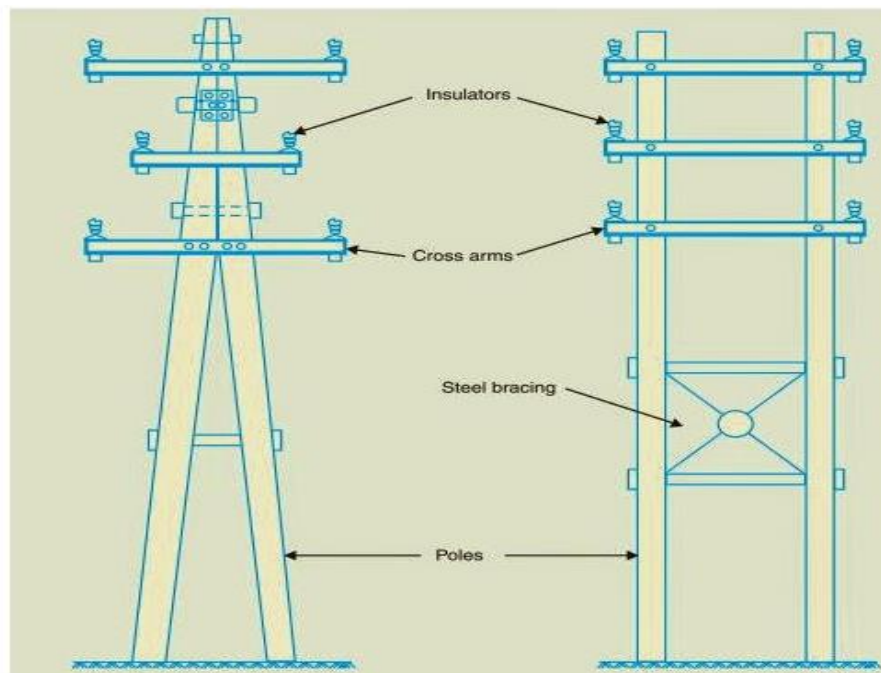


# Distribution line poles

- The line supports used for transmission and distribution of electric power are of various types including
  - wooden poles
  - steel poles

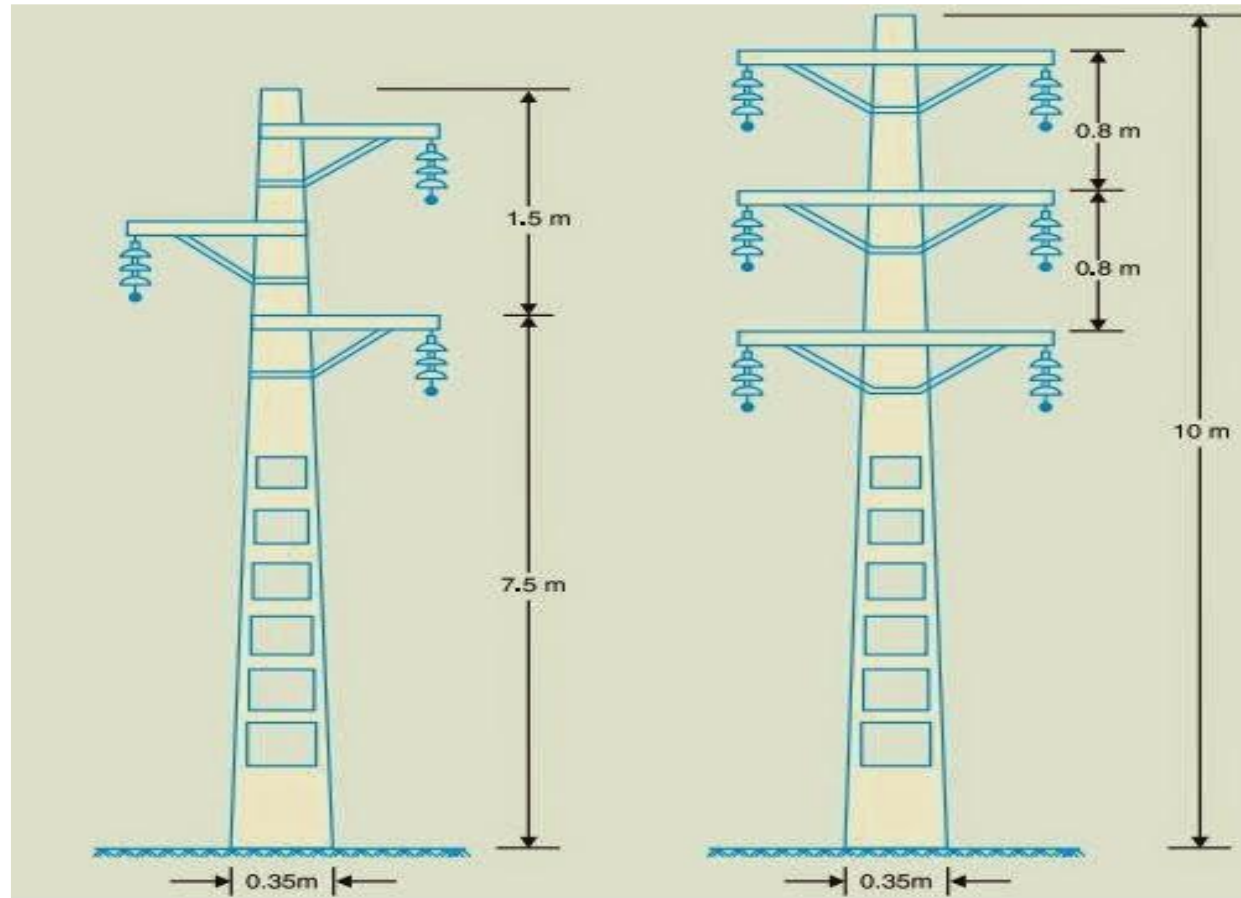
# Wooden poles

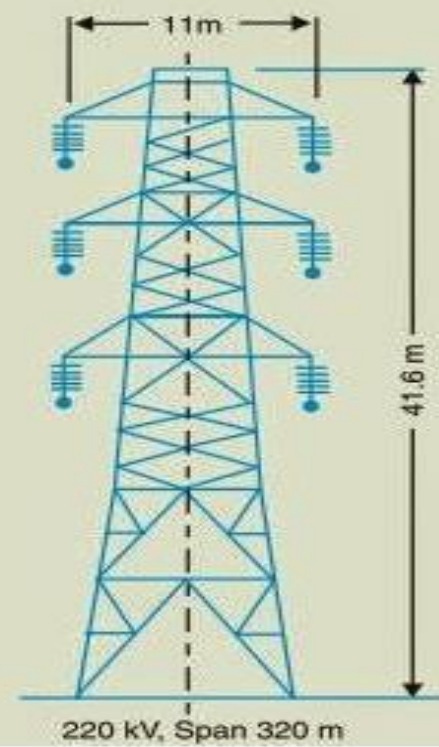
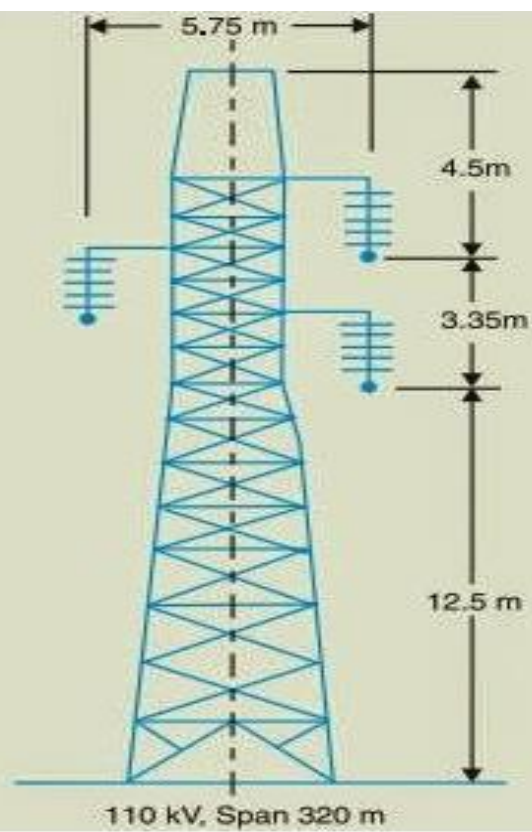
- The main objections to wooden supports are :
- (i) the tendency to rot below the ground level
- (ii) comparatively smaller life (20-25 years)
- (iii) cannot be used for voltages higher than 20 kV
- (iv) less mechanical strength and
- (v) require periodical inspection

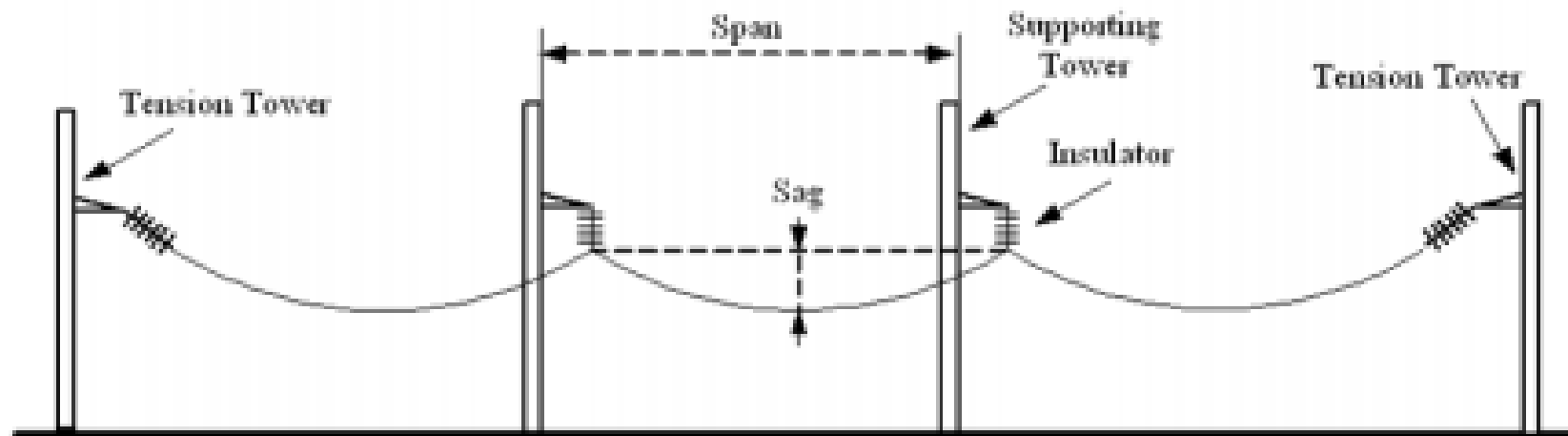


# Steel poles

- The steel poles are often used as a substitute for wooden poles. They possess greater mechanical strength, longer life and permit longer spans to be used. Such poles are generally used for distribution purposes in the cities. This type of support needs to be galvanised or painted in order to prolong its life. The steel poles are of three types (i) rail poles (ii) tubular poles and (iii) rolled steel joints.







Definition of Parameters



## DC Line

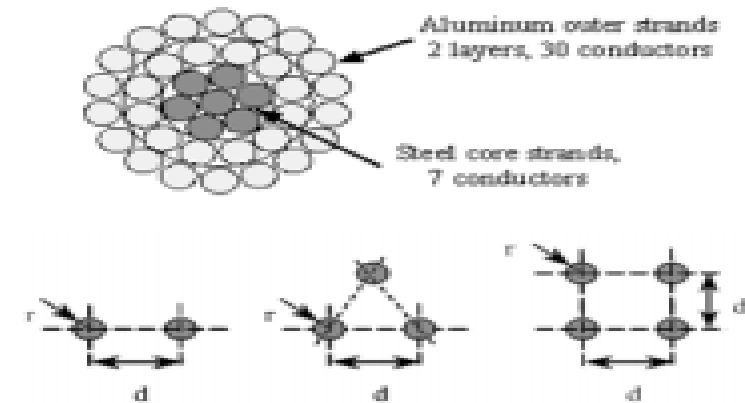
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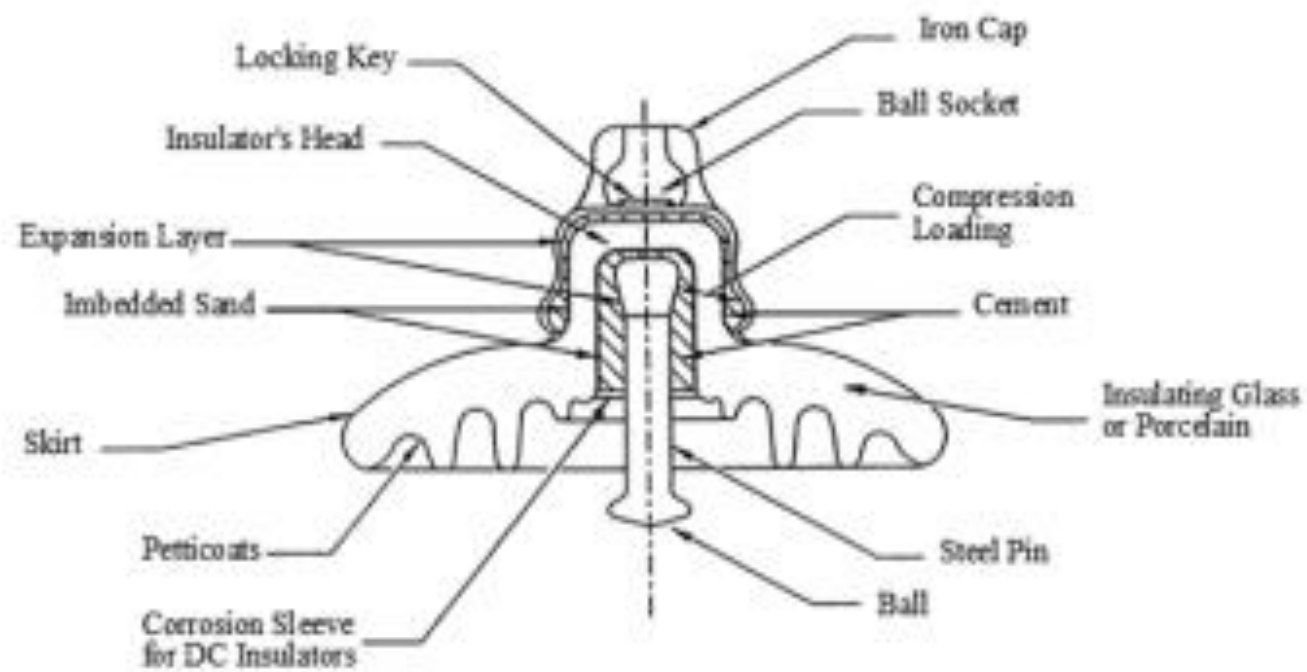
# Transmission lines and cables

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- **Aluminum Conductor Steel Reinforced (ACSR);**
- **All Aluminum Conductor (AAC);**
- and
- **All Aluminum Alloy Conductor (AAAC).**

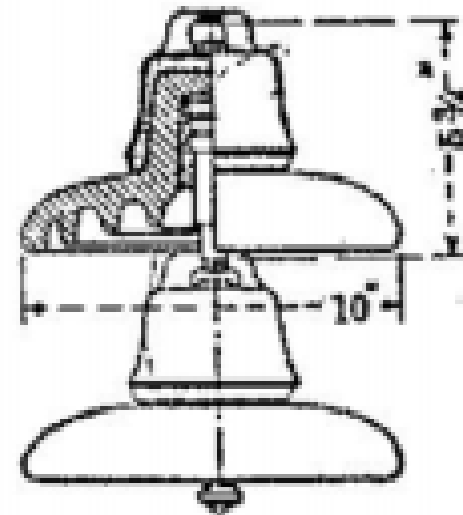


## Insulators



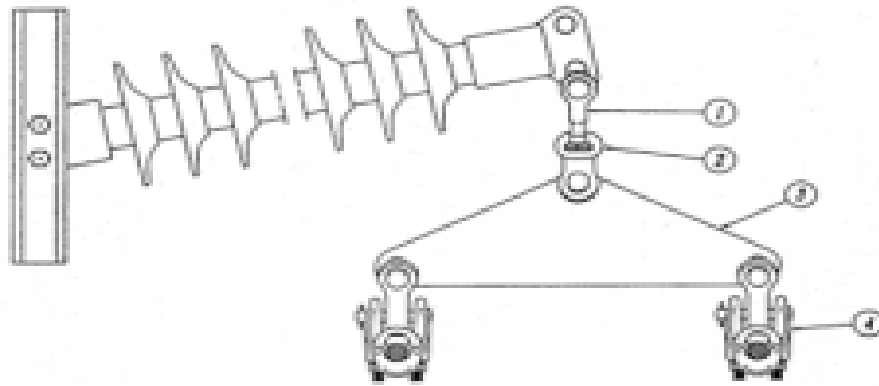
## Insulator Chain

<u>Line Voltage</u>	<u>Number of Insulators per String</u>
69 kV	4-6
115 kV	7-9
138 kV	8-10
230 kV	12
345 kV	18
500 kV	24
765 kV	30-35



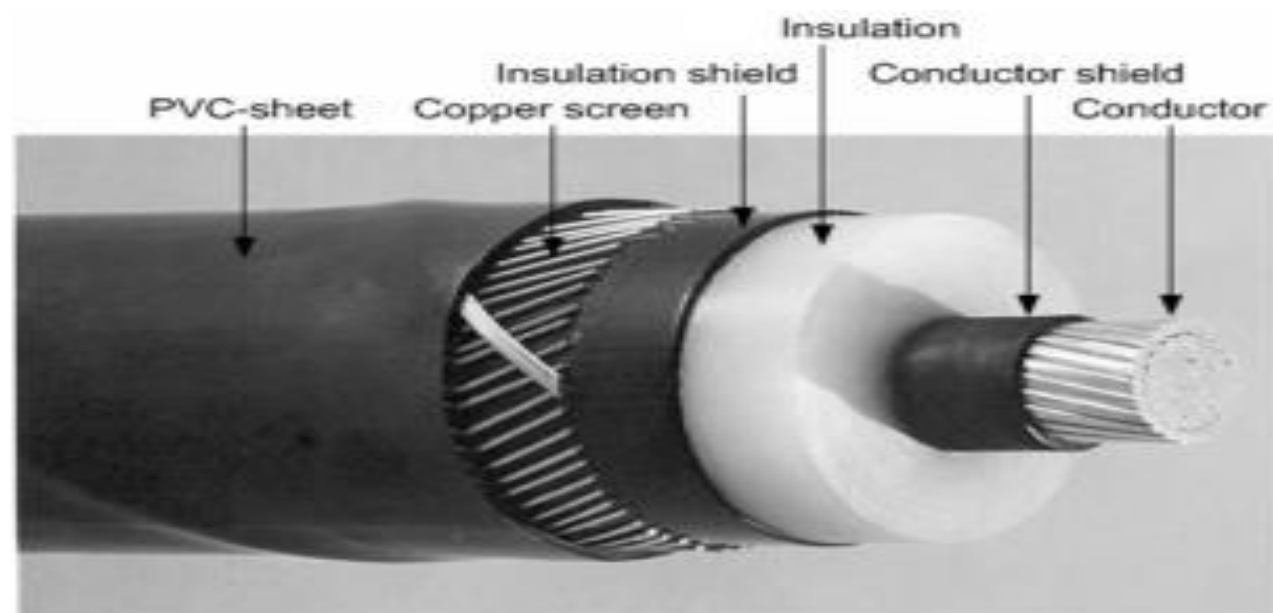
# Transmission lines and cables

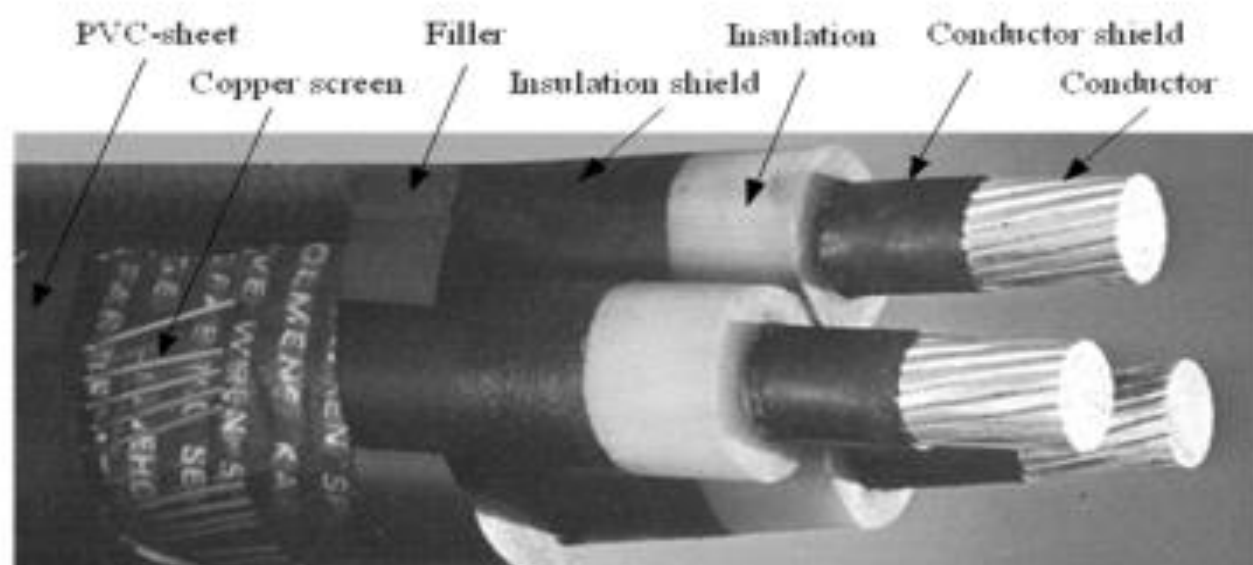
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**Line post-composite insulator with yoke holding two conductors.**

- (1) is the clevis ball,**
- (2) is the socket for the clevis,**
- (3) is the yoke plate, and**
- (4) is the suspension clamp. (*Source: Sediver*)**





- Transmission lines are classified according to their lengths to:
  - Short: less than 80 km
  - Medium: from 80 km to 240 km
  - Long: longer than 240 km



The DC resistance of a solid round conductor at a specific temperature is given by:

$$R_{DC} = \frac{\rho l}{A}$$

Where:

$\rho$  = Conductor resistivity

L = Conductor length

A = Conductor cross sectional area

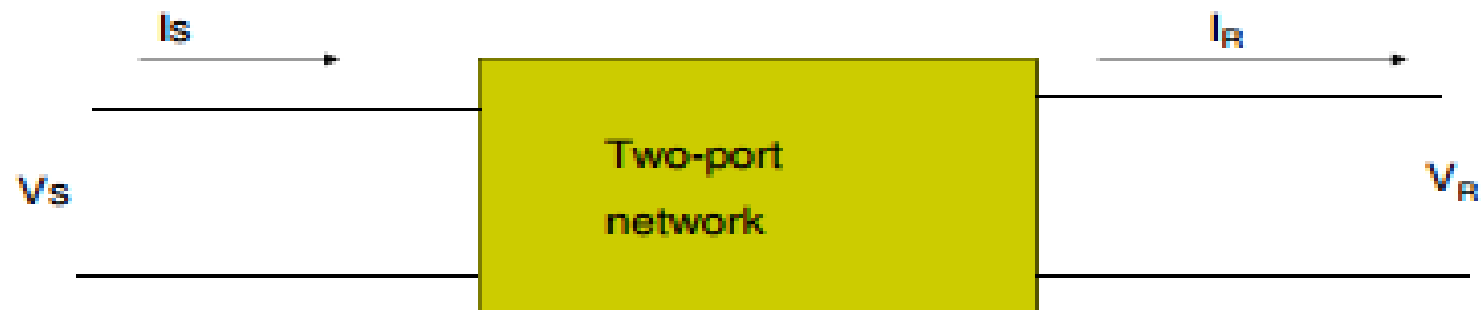
The conductor resistance is affected by three factors:

a) Frequency: skin effect

b) Spiraling

c) Temperature:  $R_2 = R_1 \frac{T + t_2}{T + t_1}$

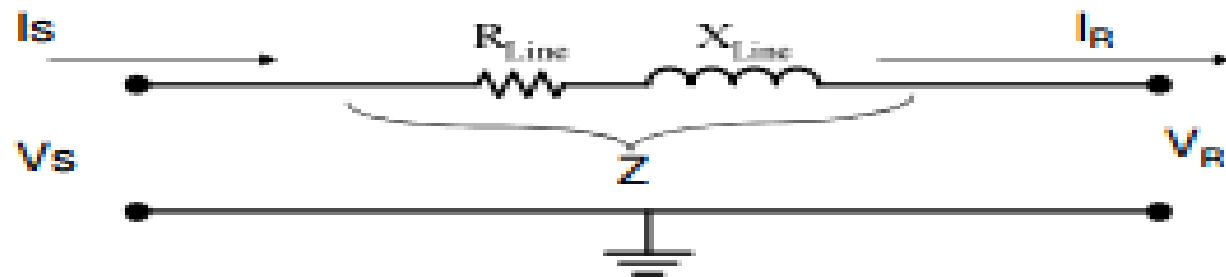
# Transmission lines and cables



$$V_s = AV_R + BI_R$$

$$I_s = CV_R + DI_R$$

# Short transmission lines



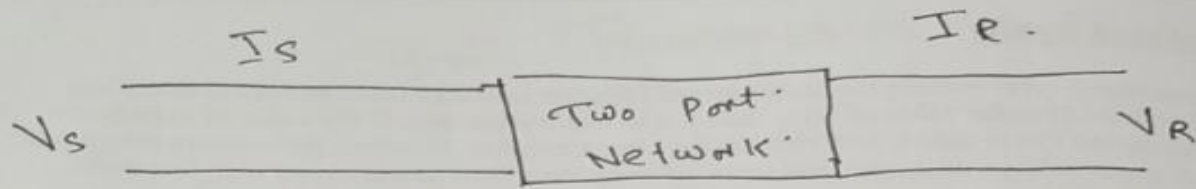
$$V_s = V_R + ZI_R$$

$$I_s = I_R$$

$$A = D = 1$$

$$B = Z$$

$$C = 0$$

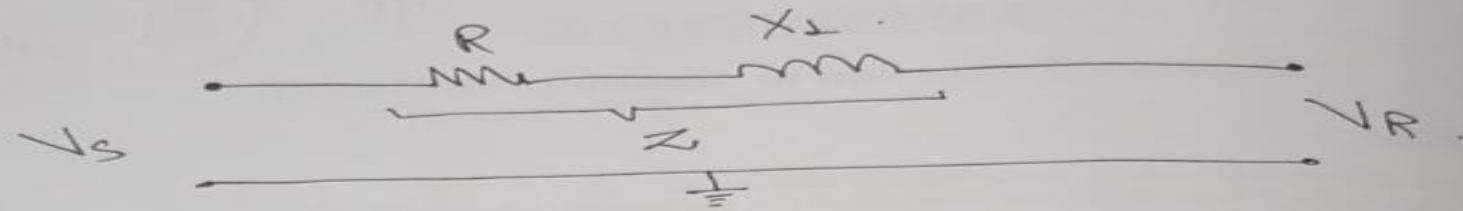


$$V_S = A V_R + B I_R.$$

$$I_S = C V_R + D I_R.$$

$\Rightarrow \left[ \begin{array}{c} A, B, C, D \\ \text{Transmission line} \\ \text{Parameter.} \end{array} \right]$

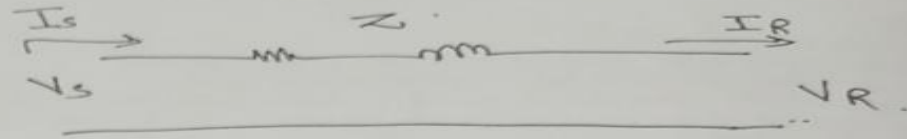
## Short Transmission Line :-



In short Transmission Line  
Line Impedance Parameters  
are lumped.

$$Z = R + jX = \underline{Zl}$$





equations ?

$$\Rightarrow \begin{cases} V_s = V_R + I_R Z \\ I_s = I_R \end{cases}$$

Main  
eq:

$$\begin{cases} V_s = A V_R + B I_R \\ I_s = C V_R + D I_R \end{cases}$$

Comparing Both eq.

$$\Rightarrow \begin{cases} A = 1 \\ B = Z \\ C = 0 \\ D = 1 \end{cases}$$

## Example 1

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- A 220 kV, three phase transmission line is 40 km long. The resistance per phase is  $0.15\ \Omega$  per km and the inductance per phase is 1.3263 mH per km. Use the short line model to find the voltage and power at the sending end, voltage regulation and efficiency when the line is supplying a three phase load of 381 MVA at 0.8 power factor lagging at 220 kV.

## Example 1, Solution

$$Z = (r + j\omega L)l = 6 + j20 \Omega$$

The receiving voltage per phase is:  $\longrightarrow V_R = \frac{220 \angle 0}{\sqrt{3}} = 127 \angle 0$

$$I_R = \frac{S_R^*}{\sqrt{3}V_R} = 1000 \angle -36.87 \quad V_S = V_R + ZI_R = 144.3 \angle 4.93 \text{ kV}$$

$$V_S(L - L) = \sqrt{3}V_S = 250 \text{ kV} \quad \longrightarrow VR = \frac{250 - 220}{220} = 13.6\%$$

$$P_R = \sqrt{3} 220 \times 1000 \times \cos(36.8) = 304.8 \text{ MW} \quad \longrightarrow \quad \eta = \frac{304.8}{322.8} = 94.4\%$$

$$P_s = \sqrt{3} 250 \times 1000 \times \cos(4.93 + 36.8) = 322.8 \text{ MW}$$



Example #1

$$l = 40 \text{ km}$$

$$r = .15 \Omega / \text{km}$$

$$L = 1.3263 \text{ mH} / \text{km}$$

$$V_{R-L} = 220 \text{ KV}$$

$$S_L = 381 \text{ MVA at } .8$$

$$Z = z_l \cdot l$$

$$Z = (R + jX) l$$

$$= (R + j2\pi f L) l$$

$$= (.15 + j\omega \cdot 1.3263) \times 40$$

$$\boxed{Z = 6 + j20 \Omega}$$

$$V_R = \frac{220 \angle 0}{\sqrt{3}} = 127 \angle 0$$

$$I_R = \frac{S_R}{\sqrt{3} V_R} = 1000 \angle -36.87$$

$$V_S = V_R + Z I_R$$

$$V_S = 144.3 \angle 4.93^\circ \text{ kV}$$

$$V_{S_{LL}} = \sqrt{3} V_S$$

$$V_{S_{LL}} = 250 \text{ kV}$$

Voltage Regulation

$$= \frac{V_S - V_R}{V_R}$$

$$V_{Reg.} \% = \frac{250 - 220}{220} = 13.6\%$$

# Medium transmission lines

$$V_s = V_R + Z \left( I_R + \frac{V_R Y}{2} \right) = \left( 1 + \frac{YZ}{2} \right) V_R + Z I_R$$

$$I_s = I_R + \frac{V_R Y}{2} + \frac{V_s Y}{2}, \text{ substitute the value of } V_s$$

$$I_s = Y \left( 1 + \frac{YZ}{4} \right) V_R + \left( 1 + \frac{YZ}{2} \right) I_R$$

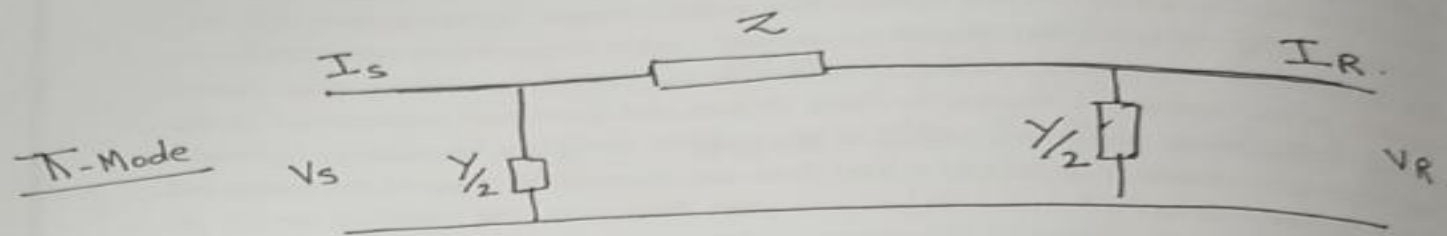


$$A = D = 1 + \frac{YZ}{2}$$

$$B = Z$$

$$C = Y \left( 1 + \frac{YZ}{4} \right)$$

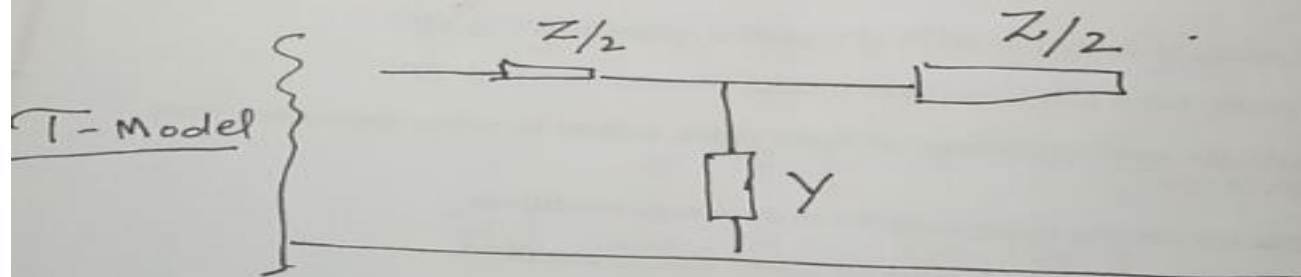
## Medium Transmission Line:

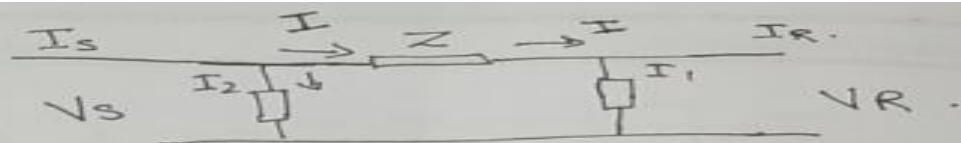


$$Z = z l$$

$$Y = y l$$

$$\Rightarrow \begin{cases} V_s = A V_R + B I_R \\ I_s = C V_R + D I_R \end{cases}$$





$$V_S = V_R + IZ$$

$$V_S = V_R + Z(I_1 + I_R)$$

$$= V_R + Z\left(I_R + \frac{V_R}{Z} \cdot \frac{Y}{2}\right)$$

$$= V_R + Z \cdot I_R + \frac{YZ}{2} V_R$$

$$V_S = \left(1 + \frac{YZ}{2}\right) V_R + I_R Z \rightarrow \textcircled{A}$$

$$I_S = I_2 + I_1 + I_R$$

$$= V_S \cdot \frac{Y}{2} + V_R \cdot \frac{Y}{2} + I_R \quad \text{--- (B)}$$

Put eq. A. in (B) .

$$= \left(1 + \frac{YZ}{2}\right) \cdot \frac{V_R \cdot Y}{2} + I_R Z \cdot \frac{Y}{2} \\ + V_R \cdot \frac{Y}{2} + I_R$$

$$= V_R \left\{ \left( 1 + \frac{YZ}{2} \right) + 1 \right\} \frac{Y}{2} + \left( 1 + \frac{YZ}{2} \right) I_R$$

$$= V_R \left[ \frac{2 + \frac{YZ}{2} + 2}{2} \right] \frac{Y}{2} + \left[ \frac{2 + YZ}{2} \right] I_R$$

$$= V_R \cdot Y \left[ \frac{4 + YZ}{2} \right] + \left[ 1 + \frac{YZ}{2} \right] I_R$$

$$\frac{I_S}{I_R} = V_R \cdot Y \left[ 1 + \frac{YZ}{4} \right] + \left( 1 + \frac{YZ}{2} \right) I_R$$

eq. C

$A = 1 + \frac{YZ}{2}$	$C = Y \left[ 1 + \frac{YZ}{4} \right]$
$B = Z$	$D = 1 + \frac{YZ}{2}$

BY Ignoring ~~the~~ the Admittance

$$\left[ \begin{array}{l} A = 1 + \frac{YZ}{2} \\ B = Z \end{array} \right] \quad \left| \quad \begin{array}{l} C = Y \left( 1 + \frac{YZ}{2} \right) \\ D = 1 + \frac{YZ}{2} \end{array} \right.$$

↓  $\boxed{Y=0}$

$$\begin{array}{l} A = 1 \\ B = Z \end{array}$$

$$\begin{array}{l} C = 0 \\ D = 1 \end{array}$$

[Medium]. Voltage Regulation :-

$$V_{Reg} = \frac{|V_{RNL}| - |V_R \cdot FL|}{|V_R \cdot FL|}$$

$$V_{Reg} = \frac{\frac{|V_S|}{|A|} - |V_R|}{|V_R|}$$

$$\left\{ \begin{array}{l} V_S = A V_R + B I_R \Rightarrow V_S = A V_R \\ I_R = 0 \end{array} \right. \quad \left\{ \begin{array}{l} V_R = V_S / A \end{array} \right.$$

## Example 2

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- A three phase 60 Hz, completely transposed 345kV, 200 km line has two 795,000 cmil 26/2 ACSR conductors per bundle and the following positive sequence line constants:  
 $z = 0.032 + j0.35 \Omega/\text{km}$ ,  $y = j4.2 \times 10^{-6} \text{ S/km}$ . Full load at the receiving end of the line is 700 MW at 0.99 power factor leading and at 95% of rated voltage. Find the following:
  - ABCD parameters of the nominal  $\pi$  circuit
  - Sending end voltage  $V_s$ , current  $I_s$  and power  $P_s$ .
  - Percent voltage regulation.
  - Thermal limit.
  - Transmission line efficiency at full load.



a. The total series impedance and shunt admittance values are

$$Z = z_l = (0.032 + j0.35)(200) = 6.4 + j70 = 70.29/\underline{84.78^\circ} \quad \Omega$$

$$Y = y_l = (j4.2 \times 10^{-6})(200) = 8.4 \times 10^{-4}/\underline{90^\circ} \quad \text{S}$$

From (5.1.15)–(5.1.17),

$$\begin{aligned} A = D &= 1 + (8.4 \times 10^{-4}/\underline{90^\circ})(70.29/\underline{84.78^\circ})\left(\frac{1}{2}\right) \\ &= 1 + 0.02952/\underline{174.78^\circ} \\ &= 0.9706 + j0.00269 = 0.9706/\underline{0.159^\circ} \quad \text{per unit} \end{aligned}$$

$$B = Z = 70.29/\underline{84.78^\circ} \quad \Omega$$

$$\begin{aligned} C &= (8.4 \times 10^{-4}/\underline{90^\circ})(1 + 0.01476/\underline{174.78^\circ}) \\ &= (8.4 \times 10^{-4}/\underline{90^\circ})(0.9853 + j0.00134) \\ &= 8.277 \times 10^{-4}/\underline{90.08^\circ} \quad \text{S} \end{aligned}$$

b. The receiving-end voltage and current quantities are

$$V_R = (0.95)(345) = 327.8 \text{ kV}_{LL}$$

$$V_R = \frac{327.8}{\sqrt{3}} \angle 0^\circ = 189.2 \angle 0^\circ \text{ kV}_{LN}$$

$$I_R = \frac{700 / \cos^{-1} 0.99}{(\sqrt{3})(0.95 \times 345)(0.99)} = 1.246 \angle 8.11^\circ \text{ kA}$$

From (5.1.1) and (5.1.2), the sending-end quantities are

$$\begin{aligned} V_S &= (0.9706 \angle 0.159^\circ)(189.2 \angle 0^\circ) + (70.29 \angle 84.78^\circ)(1.246 \angle 8.11^\circ) \\ &= 183.6 \angle 0.159^\circ + 87.55 \angle 92.89^\circ \\ &= 179.2 + j87.95 = 199.6 \angle 26.14^\circ \text{ kV}_{LN} \end{aligned}$$

$$V_S = 199.6\sqrt{3} = 345.8 \text{ kV}_{LL} \approx 1.00 \text{ per unit}$$

$$\begin{aligned} I_S &= (8.277 \times 10^{-4} \angle 90.08^\circ)(189.2 \angle 0^\circ) + (0.9706 \angle 0.159^\circ)(1.246 \angle 8.11^\circ) \\ &= 0.1566 \angle 90.08^\circ + 1.209 \angle 8.27^\circ \\ &= 1.196 + j0.331 = 1.241 \angle 15.5^\circ \text{ kA} \end{aligned}$$

and the real power delivered to the sending end is

$$\begin{aligned} P_S &= (\sqrt{3})(345.8)(1.241) \cos(26.14^\circ - 15.5^\circ) \\ &= 730.5 \text{ MW} \end{aligned}$$

c. From (5.1.19), the no-load receiving-end voltage is

$$V_{\text{RNL}} = \frac{V_S}{A} = \frac{345.8}{0.9706} = 356.3 \text{ kV}_{\text{LL}}$$

and, from (5.1.18),

$$\text{percent VR} = \frac{356.3 - 327.8}{327.8} \times 100 = 8.7\%$$

e. The full-load line losses are  $P_S - P_R = 730.5 - 700 = 30.5$  MW and the full-load transmission efficiency is

$$\text{percent EFF} = \frac{P_R}{P_S} \times 100 = \frac{700}{730.5} \times 100 = 95.8\%$$

### Example #02

$$Z = .032 + j.35 \Omega/\text{km}$$

$$Y = j4.2 \times 10^{-6} \text{ S/km}$$

$$S_L = 700 \text{ MW at } .9 \text{ p.f.} \\ \text{at } 95\% \text{ rated voltage.}$$

$$\text{length} = 200 \text{ km.}$$

$$V_L = 345 \text{ kV}$$

(a).

$$Z = zL = (.032 + j.35) \cdot 200$$

$$Z = 70.29 \angle 84.78$$

$$Y = yL = (4.2 \times 10^{-6}) \cdot 200$$

$$Y = 8.4 \times 10^{-4} \angle 90^\circ$$

$$A = D = 1 + \frac{YZ}{2} = .97 \angle .159$$

$$B = Z = 70.29 \angle 84.78$$

$$C = Y \left( 1 + \frac{YZ}{4} \right)$$

$$= 8.277 \times 10^{-4} \angle 90.08$$

(b)

$$V_R = \frac{.95 \times 345}{\sqrt{3}}$$

$$\boxed{V_R = 189.2}$$

$$P_R = \sqrt{3} V_{R2} I_R \cos \phi$$

$$I_R = \frac{P_R}{\sqrt{3} V_{R2} \cos \phi}$$

$$I_R = \frac{700 \angle \cos^{-1}.99}{\sqrt{3} (.95 \times 345) \cdot .99}$$

$$\boxed{I_R = 1.246 \angle -8.11 \text{ kA}}$$

$$V_S = A V_R + B I_R$$

$$\boxed{V_S = 199.6 \angle -26.14}$$

$$I_S = C V_R + D I_R$$

$$\boxed{I_S = 1.24 \angle -15.5 \text{ kA}}$$

(c)

$$V_{RL} = \frac{V_S}{A} = 356.3$$

$$V_R = \frac{356.3 - 327.8}{327.8} = 8.7\%$$

$$\boxed{V_R = 8.7\%}$$

(e).

$$P_R = V_R I_R \cos \phi$$

$$P_S = V_S I_S \cos \phi$$

$$P_R = 100 \text{ MW}$$

$$P_S = 1030.5 \text{ MW}$$

$$\eta = \frac{P_R}{P_S} = \frac{100}{1030.5}$$

$$\boxed{\eta = 95.8\%}$$



## Example 3

---

Consider a 500-kV, 60 Hz three-phase transmission line modeled using the ABCD parameters as follows:

$$V_s = AV_r + BI_r$$

$$I_s = CV_r + AI_r$$

$$A^2 - BC = 1$$

Results of tests conducted at the receiving end the line involving open circuit ( $I_r = 0$ ) and short circuit ( $V_r = 0$ ) are given by:

$$Z_{oc} = \left. \frac{V_s}{I_s} \right|_{I_r=0} = 820 \angle -88.8^\circ$$

$$Z_{sc} = \left. \frac{V_s}{I_s} \right|_{V_r=0} = 200 \angle 78^\circ$$

Find the line parameters A, B, and C.

Suppose that the load at the receiving end of the line of part b is 750 MVA at nominal voltage, and lagging power factor of 0.83 at rated voltage. Determine the sending end voltage, current, active and reactive power and power factor.

## Example 3, Solution

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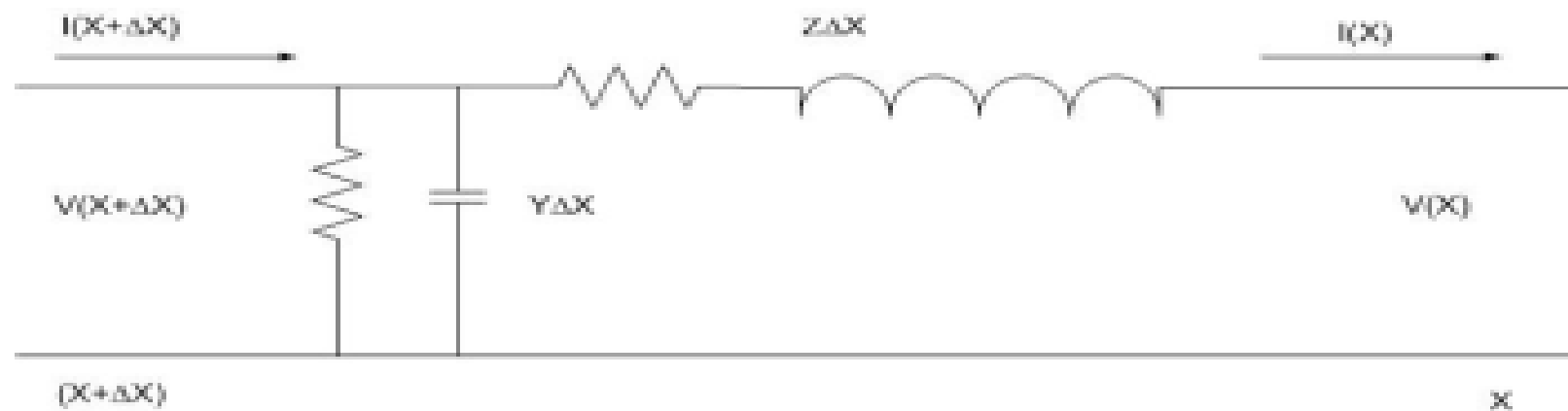
$$\frac{V_s}{I_s} = \frac{AV_r + BI_r}{CV_r + AI_r}$$

$$Z_{oc} = \frac{A}{C} = 820 \angle -88.8$$

$$Z_{sc} = \frac{B}{A} = 200 \angle 78$$

Then solve for A, B and C and proceed like the previous example.

# Long transmission lines



$$z = R + j\omega L \quad \Omega/\text{m}$$

$$y = G + j\omega C \quad \text{S/m}$$

# Long transmission lines, cont.

$$V(x + \Delta x) = V(x) + (z\Delta x)I(x)$$

$$\frac{V(x + \Delta x) - V(x)}{\Delta x} = zI(x)$$

Taking the limit as  $\Delta x$  approaches zero :

$$\frac{dV(x)}{dx} = zI(x)$$

$$\frac{d^2V(x)}{dx^2} = z \frac{dI(x)}{dx} = zyV(x) \quad \longrightarrow$$

$$\text{Let : } \gamma^2 = zy \quad \longrightarrow \quad \frac{d^2V(x)}{dx^2} - \gamma^2V(x) = 0$$

$$I(x + \Delta x) = I(x) + (y\Delta x)V(x + \Delta x)$$

$$\frac{I(x + \Delta x) - I(x)}{\Delta x} = yV(x + \Delta x)$$

Taking the limit as  $\Delta x$  approaches zero :

$$\frac{dI(x)}{dx} = yV(x)$$

$$\frac{d^2V(x)}{dx^2} - zyV(x) = 0$$



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## Long transmission lines, cont.

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$$V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x}$$

$\gamma = \sqrt{zy}$  is called the propagation constant  
 $\gamma = \alpha + j\beta$

$$\frac{dV(x)}{dx} = \gamma A_1 e^{\gamma x} - \gamma A_2 e^{-\gamma x} = zI(x)$$

$$I(x) = \frac{y}{z} (A_1 e^{\gamma x} - A_2 e^{-\gamma x}) = \sqrt{\frac{y}{z}} (A_1 e^{\gamma x} - A_2 e^{-\gamma x}) = \frac{A_1 e^{\gamma x} - A_2 e^{-\gamma x}}{Z_c}$$

$Z_c = \sqrt{\frac{z}{y}}$  is called the characteristic impedance.

Since  $V_R = V(0) = A_1 + A_2$  and  $I_R = I(0) = \frac{A_1 - A_2}{Z_c}$

$$A_1 = \frac{V_R + Z_c I_R}{2} \quad \text{and} \quad A_2 = \frac{V_R - Z_c I_R}{2}$$













