LOAD FLOW (*NEWTON-RAPHSON METHOD*)

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Newton - Raphson Method

• Newton Raphson is the most widely used method for solving simultaneous non-linear algebraic equations.

• It is a successive approximation procedure based on an initial estimate of the unknown and the use of the Taylor's expansion.

• Consider the solution of the one-dimensional equation given by: $f(x) = c$

• If $x^{(0)}$ is an initial estimate of the solution and $\Delta x^{(0)}$ is a small deviation from the correct solution, we must have:

$$f(x^{(0)} + \Delta x^{(0)}) = c$$

• Taylor series expansion yields:

$$f(x)^{(0)} + \left(\frac{df}{dx}\right)^{(0)} \Delta x^{(0)} + \frac{1}{2!}\left(\frac{d^2 f}{dx^2}\right)^{(0)} \left(\Delta x^{(0)}\right)^2 + \ldots = c$$

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Newton - Raphson Method

• **Assume that $\Delta x(0)$ is a small, then the 2nd and higher order terms can be neglected.**

$$\Delta c^{(0)} = c - f(x^{(0)}) = \left(\frac{df}{dx}\right)^{(0)} \Delta x^{(0)}$$

• **The second approximation will be:**

$$x^{(1)} = x^{(0)} + \Delta x^{(0)} = x^{(0)} + \frac{\Delta c^{(0)}}{\left(\frac{df}{dx}\right)^{(0)}}$$

• **Successive use of the above procedure yields the Newton – Raphson algorithm.**

$$\Delta c^{(k)} = c - f(x^{(k)}) = \left(\frac{df}{dx}\right)^{(k)} \Delta x^{(k)}$$

$$x^{(k+1)} = x^{(k)} + \Delta x^{(k)}$$

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Example 1

Use the Newton Raphson technique to find a root of the following equation, assume initial estimate = 6: $f(x) = x^3 - 6x^2 + 9x - 4 = 0$

Solution: $\frac{df(x)}{dx} = 3x^2 - 12x + 9$

$$\Delta c^{(0)} = c - f(x^{(0)}) = 0 - \left[(6)^3 - 6(6)^2 + 9(6) - 4\right] = -50$$

$$\left(\frac{df)}{dx}\right)^{(0)} = 3(6)^2 - 12(6) + 9 = 45$$

$$\Delta x^{(0)} = \frac{\Delta c^{(0)}}{\left(\frac{df}{dx}\right)^{(0)}} = \frac{-50}{45} = -1.11$$

$$x^{(1)} = x^{(0)} + \Delta x^{(0)} = 6 - 1.111 = 4.8889$$

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Example 1, continue

The subsequent iterations result in:

$$x^{(2)} = x^{(1)} + \Delta x^{(1)} = 4.8889 - \frac{13.4431}{22.037} = 4.2789$$

$$x^{(3)} = x^{(2)} + \Delta x^{(2)} = 4.2789 - \frac{2.9981}{12.5797} = 4.0405$$

$$x^{(4)} = x^{(3)} + \Delta x^{(3)} = 4.0405 - \frac{0.3748}{9.4914} = 4.0011$$

Example 2

Use the Newton-Raphson technique to find the solution of the following equation, Assume initial solution = 1

$$f(x) = x^2 - 2 = 0$$

Solution: $$\frac{df(x)}{dx} = 2x$$

$$\Delta c^{(0)} = c - f(x^{(0)}) = 0 - \left[(1)^2 - 2\right] = +1$$

$$\left(\frac{df}{dx}\right)^{(0)} = 2(1) = 2$$

$$\Delta x^{(0)} = \frac{\Delta c^{(0)}}{\left(\frac{df}{dx}\right)^{(0)}} = \frac{1}{2} = 0.5$$

$$x^{(1)} = x^{(0)} + \Delta x^{(0)} = 1 + 0.5 = 1.5$$

Example 2

$$\Delta c^{(1)} = c - f(x^{(1)}) = 0 - \left[(1.5)^2 - 2\right] = -.25$$

$$\left(\frac{df}{dx}\right)^{(1)} = 2(1.5) = 3$$

$$\Delta x^{(1)} = \frac{\Delta c^{(1)}}{\left(\frac{df}{dx}\right)^{(1)}} = \frac{-0.25}{3} = -0.0833$$

$$x^{(2)} = x^{(1)} + \Delta x^{(1)} = 1.5 - 0.0833 = 1.416$$

An then go to the next iteration and so on.

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Newton - Raphson Method

- **For n-dimensional equations:**

$$f_1(x_1, x_2, \ldots x_n) = c_1$$
$$f_2(x_1, x_2, \ldots x_n) = c_2$$
$$.$$
$$.$$
$$f_n(x_1, x_2, \ldots x_n) = c_n$$

Writing the Taylor's series expansion of the left hand side and neglecting the higher order terms:

$$(f_1)^{(0)} + \left(\frac{\partial f_1}{\partial x_1}\right)^{(0)} \Delta x_1{}^{(0)} + \left(\frac{\partial f_1}{\partial x_2}\right)^{(0)} \Delta x_2{}^{(0)} + \ldots \left(\frac{\partial f_1}{\partial x_n}\right)^{(0)} \Delta x_n{}^{(0)}) = c_1$$

$$(f_2)^{(0)} + \left(\frac{\partial f_2}{\partial x_1}\right)^{(0)} \Delta x_1{}^{(0)} + \left(\frac{\partial f_2}{\partial x_2}\right)^{(0)} \Delta x_2{}^{(0)} + \ldots \left(\frac{\partial f_2}{\partial x_n}\right)^{(0)} \Delta x_n{}^{(0)}) = c_2$$

$$.$$

$$(f_n)^{(0)} + \left(\frac{\partial f_n}{\partial x_1}\right)^{(0)} \Delta x_1{}^{(0)} + \left(\frac{\partial f_n}{\partial x_2}\right)^{(0)} \Delta x_2{}^{(0)} + \ldots \left(\frac{\partial f_n}{\partial x_n}\right)^{(0)} \Delta x_n{}^{(0)}) = c_n$$

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Newton - Raphson Method

- **Or in matrix form:**

$$\begin{bmatrix} c_1-(f_1)^{(0)} \\ c_2-(f_2)^{(0)} \\ . \\ . \\ c_n-(f_n)^{(0)} \end{bmatrix} = \underbrace{\begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^{(0)} & \left(\frac{\partial f_1}{\partial x_2}\right)^{(0)} & \ldots\ldots & \left(\frac{\partial f_1}{\partial x_n}\right)^{(0)} \\ \left(\frac{\partial f_2}{\partial x_1}\right)^{(0)} & \left(\frac{\partial f_2}{\partial x_2}\right)^{(0)} & \ldots\ldots & \left(\frac{\partial f_2}{\partial x_n}\right)^{(0)} \\ . & & & \\ \left(\frac{\partial f_n}{\partial x_1}\right)^{(0)} & \left(\frac{\partial f_n}{\partial x_2}\right)^{(0)} & \ldots\ldots & \left(\frac{\partial f_n}{\partial x_n}\right)^{(0)} \end{bmatrix}}_{\text{Jacobian Matrix}} \begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \\ . \\ \Delta x_n^{(0)} \end{bmatrix}$$

- **In short form:** $\Delta C^{(k)} = J^{(k)} \Delta X^{(k)}$

So: $\Delta X^{(k)} = \left[J^{(k)}\right]^{-1} \Delta C^{(k)}$

And the Newton-Raphson algorithm becomes $X^{(k+1)} = X^{(k)} + \Delta X^{(k)}$

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Example 3

Use the Newton Raphson method to solve the following system of equations, assume initial estimate = 1:

$$x_1^2 - x_2^2 + x_3^2 = 11$$

$$x_1 x_2 + x_2^2 - 3x_3 = 3$$

$$x_1 - x_1 x_3 + x_2 x_3 = 6$$

Solution:

$$J = \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right) & \left(\frac{\partial f_1}{\partial x_2}\right) & \left(\frac{\partial f_1}{\partial x_3}\right) \\ \left(\frac{\partial f_2}{\partial x_1}\right) & \left(\frac{\partial f_2}{\partial x_2}\right) & \left(\frac{\partial f_2}{\partial x_3}\right) \\ \left(\frac{\partial f_3}{\partial x_1}\right) & \left(\frac{\partial f_3}{\partial x_2}\right) & \left(\frac{\partial f_3}{\partial x_3}\right) \end{bmatrix} = \begin{bmatrix} 2x_1 & -2x_2 & 2x_3 \\ x_2 & x_1+2x_2 & -3 \\ 1-x_3 & x_3 & -x_1+x_2 \end{bmatrix}$$

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Example 3 ,continue

$$J^{(1)} = \begin{bmatrix} 2(1) & -2(1) & 2(1) \\ (1) & (1)+2(1) & -3 \\ 1-(1) & (1) & -(1)+(1) \end{bmatrix} = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 3 & -3 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\Delta C^{(1)} = \begin{bmatrix} c_1 - (f_1)^{(0)} \\ c_2 - (f_2)^{(0)} \\ c_3 - (f_3)^{(0)} \end{bmatrix} = \begin{bmatrix} 11-1 \\ 3-(-1) \\ 6-(1) \end{bmatrix} = \begin{bmatrix} 10 \\ 4 \\ 5 \end{bmatrix}$$

$$\Delta X^{(1)} = \left[J^1\right]^{-1} \Delta C^{(1)} = \begin{bmatrix} 4.75 \\ 5.00 \\ 5.25 \end{bmatrix} \quad \text{And} \quad X^{(1)} = X^{(0)} + \Delta X^{(1)} = \begin{bmatrix} 5.75 \\ 6.00 \\ 6.25 \end{bmatrix}$$

And the process continues till the error is within certain limits.

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Example 4

Use the Newton Raphson method to solve the following system of equations, assume initial estimate = 1:

$$f_1(x) = 2x_1^2 + x_2^2 - 8 = 0$$
$$f_2(x) = x_1^2 - x_2^2 + x_1 x_2 - 4 = 0$$

Solution:

$$J = \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right) & \left(\frac{\partial f_1}{\partial x_2}\right) \\ \left(\frac{\partial f_2}{\partial x_1}\right) & \left(\frac{\partial f_2}{\partial x_2}\right) \end{bmatrix} = \begin{bmatrix} 4x_1 & +2x_2 \\ 2x_1 + x_2 & x_1 - 2x_2 \end{bmatrix}$$

$$J^{(1)} = \begin{bmatrix} 4(1) & -2(1) \\ 2(1)+1 & 1-2(1) \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$$

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Example 4-continue

$$J^{(1)-1} = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}^{-1} = \frac{1}{-4-6}\begin{bmatrix} -1 & -2 \\ -3 & +4 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & -0.4 \end{bmatrix}$$

$$\Delta C^{(1)} = \begin{bmatrix} c_1 - (f_1)^{(0)} \\ c_2 - (f_2)^{(0)} \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\Delta X^{(1)} = \left[J^1\right]^{-1} \Delta C^{(1)} = \begin{bmatrix} 1.1 \\ 0.3 \end{bmatrix} \quad \text{And} \quad X^{(1)} = X^{(0)} + \Delta X^{(1)} = \begin{bmatrix} 2.1 \\ 1.3 \end{bmatrix}$$

And the process continues till the error is within certain limits.

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Newton - Raphson Method for power system

- **As previously derived:**

$$P_i = |V_i| \sum_{p=1}^{n} |Y_{ip}||V_p| \cos(\delta_i - \delta_p - \gamma_{ip}) \qquad (1)$$

$$Q_i = |V_i| \sum_{p=1}^{n} |Y_{ip}||V_p| \sin(\delta_i - \delta_p - \gamma_{ip}) \qquad (2)$$

These two equations constitute a set of nonlinear algebraic equations in term of independent variables:

a) Voltage magnitude in per unit.

b) Phase angle in radian.

We have two equations for each load bus and one equation for each voltage controlled bus.

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Newton - Raphson Method for power system

• Expanding the previous equations using Taylor's series and neglecting all higher order terms results in the following:

$$\begin{bmatrix} \Delta P_2^{(k)} \\ \vdots \\ \Delta P_n^{(k)} \\ \hline \Delta Q_2^{(k)} \\ \vdots \\ \Delta Q_n^{(k)} \end{bmatrix} = \left[\begin{array}{ccc|ccc} \left(\frac{\partial P_2}{\partial \delta_2}\right)^{(k)} & \cdots & \left(\frac{\partial P_2}{\partial \delta_n}\right)^{(k)} & \left(\frac{\partial P_2}{\partial |V_2|}\right)^{(k)} & \cdots & \left(\frac{\partial P_2}{\partial |V_n|}\right)^{(k)} \\ \vdots & & \vdots & \vdots & & \vdots \\ \left(\frac{\partial P_n}{\partial \delta_2}\right)^{(k)} & \cdots & \left(\frac{\partial P_n}{\partial \delta_n}\right)^{(k)} & \left(\frac{\partial P_n}{\partial |V_2|}\right)^{(k)} & \cdots & \left(\frac{\partial P_n}{\partial |V_n|}\right)^{(k)} \\ \hline \left(\frac{\partial Q_2}{\partial \delta_2}\right)^{(k)} & \cdots & \left(\frac{\partial Q_2}{\partial \delta_n}\right)^{(k)} & \left(\frac{\partial Q_2}{\partial |V_2|}\right)^{(k)} & \cdots & \left(\frac{\partial Q_2}{\partial |V_n|}\right)^{(k)} \\ \vdots & & \vdots & \vdots & & \vdots \\ \left(\frac{\partial Q_n}{\partial \delta_2}\right)^{(k)} & \cdots & \left(\frac{\partial Q_n}{\partial \delta_n}\right)^{(k)} & \left(\frac{\partial Q_n}{\partial |V_2|}\right)^{(k)} & \cdots & \left(\frac{\partial Q_n}{\partial |V_n|}\right)^{(k)} \end{array}\right] \begin{bmatrix} \Delta \delta_2^{(k)} \\ \vdots \\ \Delta \delta_n^{(k)} \\ \hline \Delta |V_2^{(k)}| \\ \vdots \\ \Delta |V_n^{(k)}| \end{bmatrix} \quad (3)$$

Or in short

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

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Newton - Raphson Method for power system

• The terms ΔP_i and ΔQ_i are the difference between the scheduled and calculated values. Therefore:

$$\Delta P_i^{(k)} = P_i^{sch} - P_i^{(k)} \quad (4)$$

$$\Delta Q_i^{(k)} = Q_i^{sch} - Q_i^{(k)} \quad (5)$$

• The new estimates for bus voltages are:

$$\delta_i^{(k+1)} = \delta_i^{(k)} + \Delta \delta_i^{(k)} \quad (6)$$

$$|V_i^{(k+1)}| = |V_i^{(k)}| + \Delta |V_i^{(k)}| \quad (7)$$

Note:

For voltage controlled buses, the voltage magnitudes are known. Therefore, if m buses of the system are voltage controlled, m equations involving ΔQ and ΔV and the corresponding columns of the Jacobian matrix are eliminated.

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Newton - Raphson Method for power system

The procedure for power flow solution by the Newton-Raphson method is as follows:

1. **For load buses, voltage magnitudes and angles are set. For voltage regulated buses, only the voltage angles are set.**
2. **For load buses, ΔP and ΔQ and for voltage controlled ΔP are calculated from the equations 4 and 5.**
3. **The elements of the Jacobian matrix are calculated.**
4. **The linear equations are solved by optimally ordered triangular factorization and Gaussian elimination.**
5. **The new voltage magnitudes and phases are computed using equations 6 and 7.**
6. **The process is continued until:**

$$\left|\Delta P_i^{(k)}\right| \leq \varepsilon$$

$$\left|\Delta Q_i^{(k)}\right| \leq \varepsilon$$

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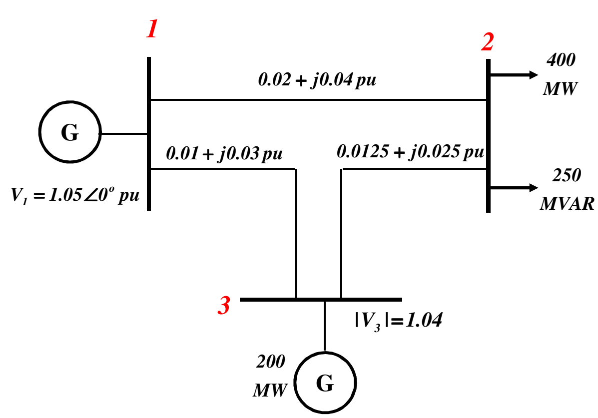
***Example 5*:**

The line impedances are as indicated in per unit on 100MVA base. Using Newton Raphson method find the power flow solution of the system.



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The expressions for the real and reactive power are as follows:

$$P_2 = |V_2| \sum_{p=1}^{3} |Y_{2p}||V_p| \cos(\delta_2 - \delta_p - \gamma_{2p})$$

$$P_2 = |V_2||V_1||Y_{21}|\cos(\delta_2 - \delta_1 - \gamma_{21}) + |V_2|^2|Y_{22}|\cos(-\gamma_{22}) + |V_2||V_3||Y_{23}|\cos(\delta_2 - \delta_3 - \gamma_{23})$$

$$P_3 = |V_3| \sum_{p=1}^{3} |Y_{3p}||V_p| \cos(\delta_3 - \delta_p - \gamma_{3p})$$

$$P_3 = |V_3||V_1||Y_{31}|\cos(\delta_3 - \delta_1 - \gamma_{31}) + |V_3||V_2||Y_{32}|\cos(\delta_3 - \delta_2 - \gamma_{32}) + |V_3|^2|Y_{31}|\cos(-\gamma_{33})$$

$$Q_2 = |V_2| \sum_{p=1}^{3} |Y_{2p}||V_p| \sin(\delta_2 - \delta_p - \gamma_{2p})$$

$$Q_2 = |V_2||V_1||Y_{21}|\sin(\delta_2 - \delta_1 - \gamma_{21}) + |V_2|^2|Y_{22}|\sin(-\gamma_{22}) + |V_2||V_3||Y_{21}|\sin(\delta_2 - \delta_3 - \gamma_{23})$$

$$Q_2 = |V_2||V_1||Y_{21}|\sin(\delta_2 - \delta_1 - \gamma_{21}) - |V_2|^2|Y_{22}|\sin(\gamma_{22}) + |V_2||V_3||Y_{21}|\sin(\delta_2 - \delta_3 - \gamma_{23})$$

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The Jacobian Matrix

$$J = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} \end{bmatrix}$$

Elements of the Jacobian matrix can be calculated as follows:

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1||Y_{21}|\sin(\gamma_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}|\sin(\gamma_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\sin(\gamma_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial |V_2|} = |V_1||Y_{21}|\cos(\gamma_{21} - \delta_2 + \delta_1) + 2|V_2||Y_{22}|\cos(\gamma_{22}) + |V_3||Y_{23}|\cos(\gamma_{23} - \delta_2 + \delta_3)$$

The other Jacobian elements will be calculated to finalize the Jacobian matrix

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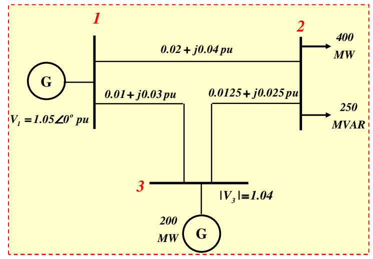
Classification of buses:



Bus 1: Slack Bus

$$V_1 = 1.05\angle 0^o \ pu$$

Bus 2: Load Bus (PQ bus)

P_2 *and* Q_2 *are known* $\qquad$ V_2 *and* δ_2 *are unknown*

$$S_{2,sch} = \frac{(P_{2,g} - P_{2,d}) + j(Q_{2,g} - Q_{2,d})}{Base\ MVA} \ pu$$

$$S_{2,sch} = \frac{(0 - 400) + j(0 - 250)}{100} \ pu$$

$$S_{2,sch} = -4 - j2.5 \ pu$$

Bus 3: Voltage Controlled Bus (PV bus)

$|V_3|$ *and* $P_{g,3}$ *are known* $\qquad$ $Q_{3,sch}$ *and* δ_3 *are unknown*

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$$P_{3,sch} = 2.0 \ pu$$

Using N-R method, select the initial values for the unknowns as:

$$V_1 = 1.05\angle 0^o \ pu \qquad V_2^o = 1\angle 0 \qquad |V_3| = 1.04$$

$$\delta_2^o = 0^o \qquad \delta_3^o = 0^o$$

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = -4.0 - (-1.14) = -2.86$$

$$\Delta P_3^{(0)} = P_2^{sch} - P_3^{(0)} = 2.0 - (0.5616) = 1.4384$$

$$\Delta Q_2^{(0)} = Q_2^{sch} - Q_2^{(0)} = -2.5 - (-2.28) = -0.220$$

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We are ready now to use the N-R technique:

$$\begin{bmatrix}\Delta P_2\\ \Delta P_3\\ \Delta Q_3\end{bmatrix}=\begin{bmatrix}\frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|}\\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|}\\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|}\end{bmatrix}\begin{bmatrix}\Delta \delta_2\\ \Delta \delta_3\\ \Delta |V_2|\end{bmatrix}$$

$$\begin{bmatrix}-2.86\\ 1.4384\\ -0.22\end{bmatrix}=\begin{bmatrix}54.28 & -33.28 & 24.86\\ -33.28 & 66.04 & -16.64\\ -27.14 & 16.64 & 49.72\end{bmatrix}\begin{bmatrix}\Delta \delta_2^{(0)}\\ \Delta \delta_3^{(0)}\\ \Delta |V_2|^{(0)}\end{bmatrix}$$

$\Delta\delta_2^{(0)} = -0.045263$ ⇨ $\delta_2^{(1)} = 0 + (-0.045263) = -0.045263$

$\Delta\delta_3^{(0)} = -0.007718$ ⇨ $\delta_3^{(1)} = 0 + (-0.007718) = -0.007718$

$\Delta|V_2|^{(0)} = -0.026548$ ⇨ $|V_2|^{(1)} = 1 + (-0.026548) = 0.97345$

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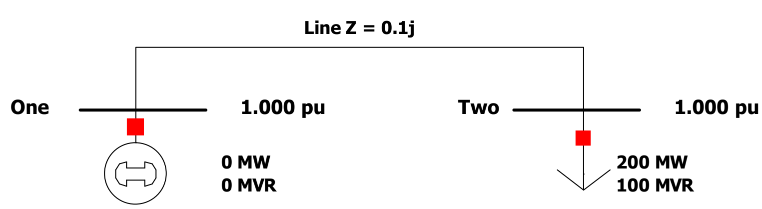
This is the end of the fist iteration, the process will be repeated for the second iteration and so on.

***Example 6*:**

For the two bus power system shown below, use the Newton-Raphson power flow to determine the voltage magnitude and angle at bus two. Assume that bus one is the slack and SBase = 100 MVA.



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Example 6-solution:

$$\mathbf{x} = \begin{bmatrix} \theta_2 \\ |V_2| \end{bmatrix} \qquad \mathbf{Y}_{bus} = \begin{bmatrix} -j10 & j10 \\ j10 & -j10 \end{bmatrix}$$

$$P_i = |V_i| \sum_{p=1}^{n} |Y_{ip}||V_p| \cos(\delta_i - \delta_p - \gamma_{ip})$$

$$Q_i = |V_i| \sum_{p=1}^{n} |Y_{ip}||V_p| \sin(\delta_i - \delta_p - \gamma_{ip})$$

$$P_2 = |V_2| \sum_{p=1}^{2} |Y_{2p}||V_p| \cos(\delta_2 - \delta_p - \gamma_{2p})$$

$$P_2 = |V_2||Y_{21}||V_1| \cos(\delta_2 - \delta_1 - \gamma_{21}) + |V_2||Y_{22}||V_2| \cos(\delta_2 - \delta_2 - \gamma_{22})$$

$$P_2 = |V_2||10| \sin(\delta_2) = 0$$

$$Q_2 = |V_2| \sum_{p=1}^{2} |Y_{2p}||V_p| \sin(\delta_2 - \delta_p - \gamma_{2p})$$

$$Q_2 = |V_2||Y_{21}||V_1| \sin(\delta_2 - \delta_1 - \gamma_{21}) + |V_2|^2 Y_{22}| \sin(\delta_2 - \delta_2 - \gamma_{22})$$

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$$Q_2 = |V_2|(-10) \cos(\delta_2) + |V_2|^2 10 = 0$$

Example 6-solution:

The Jacobian Matrix

$$J = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial |V_2|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial |V_2|} \end{bmatrix}$$

$$J = \begin{bmatrix} 10|\mathrm{V}_2|\cos\delta_2 & 10\sin\delta_2 \\ 10|\mathrm{V}_2|sin\delta_2 & -10\cos\delta_2 + 20|\mathrm{V}_2| \end{bmatrix}$$

$$X^{(0)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad J^{(0)} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \qquad \begin{bmatrix} \Delta P_2^{(0)} \\ \Delta Q_2^{(0)} \end{bmatrix} = \begin{bmatrix} (0-2)-0 \\ (0-1)-0 \end{bmatrix}$$

$$X^{(1)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}^{-1} \begin{bmatrix} -2 \\ -1 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix}$$

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This is the end of the fist iteration, the process will be repeated for the second iteration and so on.