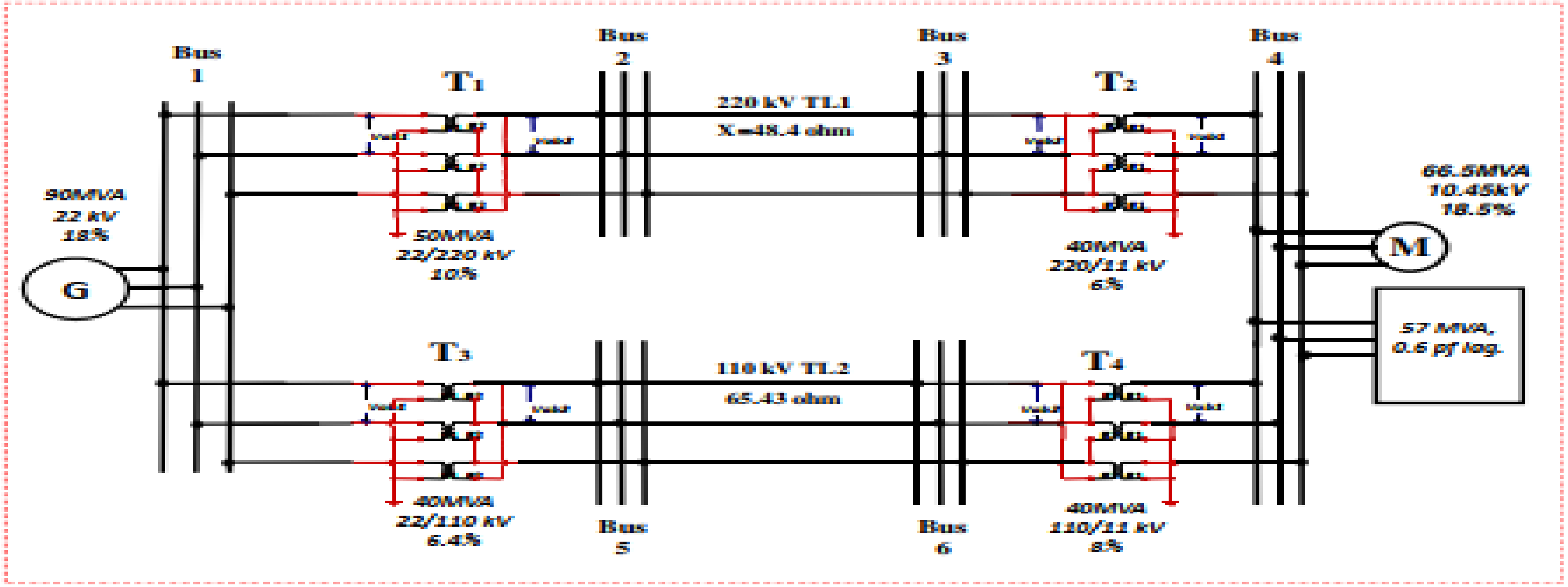
Lec_02

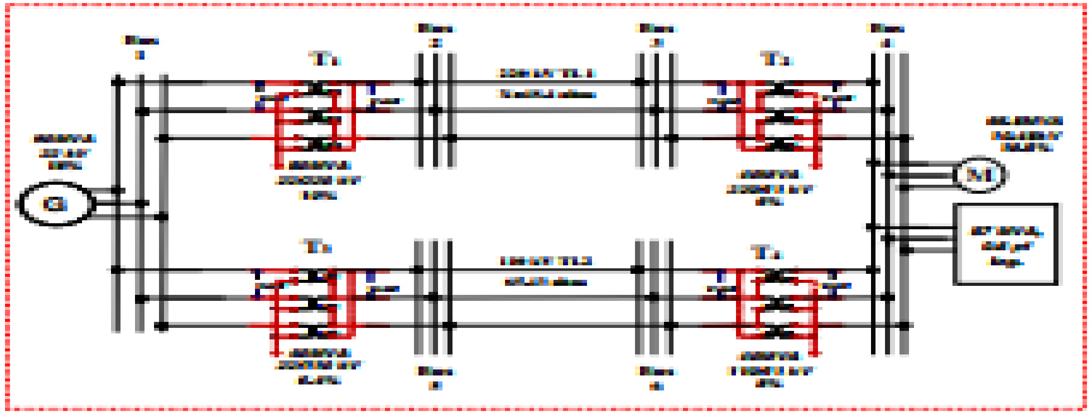
PER UNIT SYSTEM

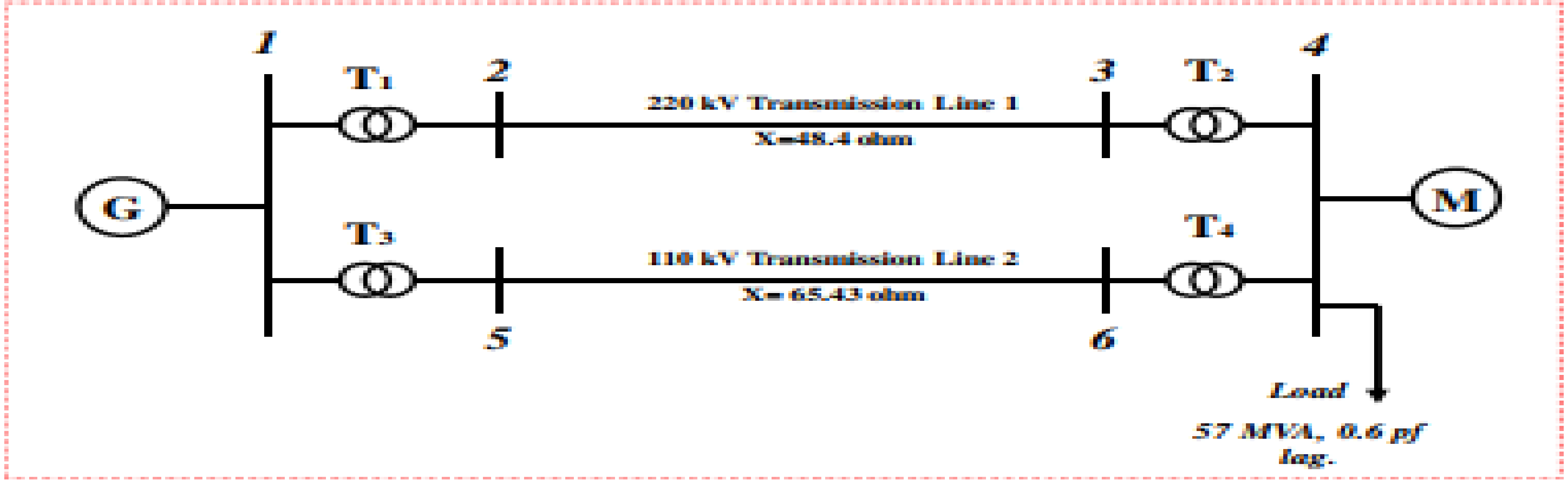
PROBLEM:

Draw the single-line diagram of a three phase power system.



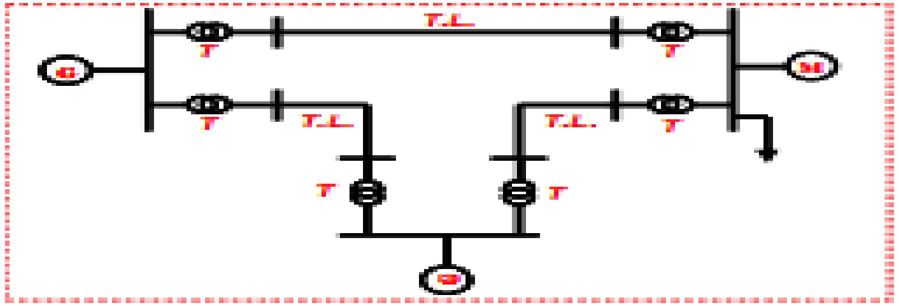
Solution:




1
2
3
4
5
6
T1
T2
T3
T4
G
M
220 kV Transmission Line 1
X=48.4 ohm
110 kV Transmission Line 2
X= 65.43 ohm
Load
57 MVA, 0.6 pf
lag.

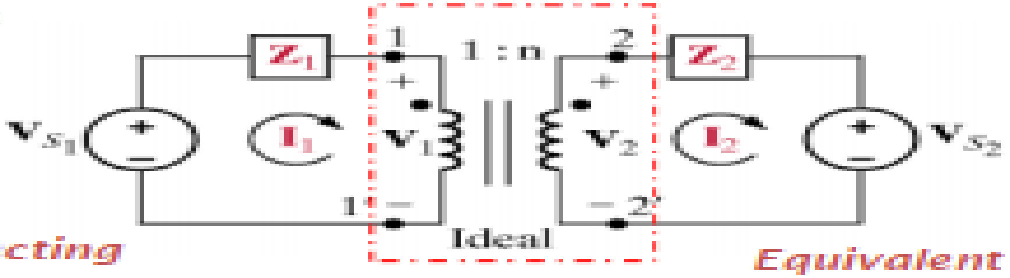
THE PER UNIT SYSTEM

Analyzing interconnected three-phase power systems having *different voltage levels* require huge transformation of all impedances to a single voltage level.

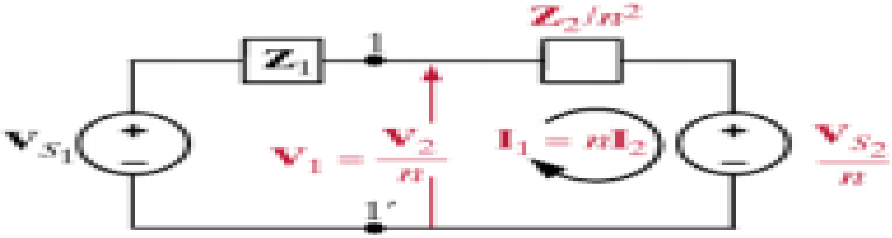


NOTE:

Transformer equivalent circuit as reflected into primary and secondary sides.



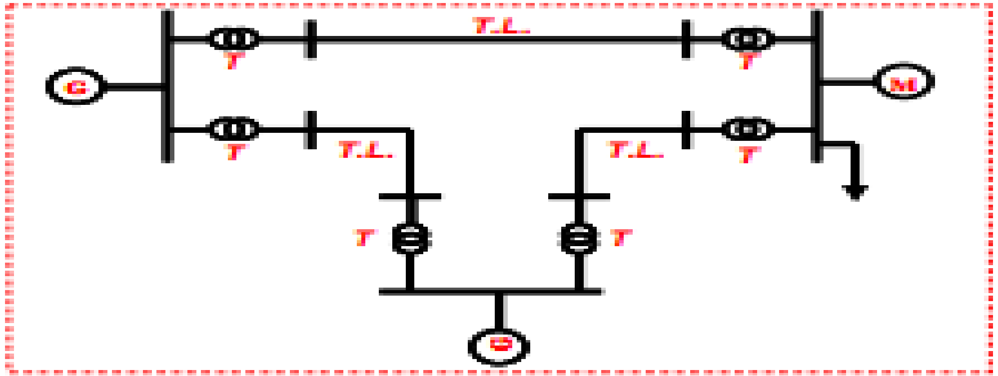
Equivalent circuit reflecting into primary



Equivalent circuit reflecting into secondary

This large mathematical work can be avoided by utilizing the per unit system. Various physical quantities such as power, voltage, current and impedance are expressed as a decimal fraction of base quantities.

The numerical per unit value of any quantity is its ratio to a chosen base quantity of the same dimension. Thus a per unit quantity is a ***normalized*** quantity with respect to the chosen base value. The per unit value of a quantity is thus defined as:



$$p.u.\ value = \frac{Actual\ \ Value}{Base\ \ Value}$$

Note:

The ***nominal voltage*** of lines and equipment is almost always known as well as the ***apparent (complex) power*** in megavolt-amperes, so these two quantities are usually chosen for base value calculation. A minimum of four base quantities are required to complete the per unit system: volt-ampere, voltage, current and impedance. ***Usually three phase MVA and line-to-line voltages are selected for as:***

$(S_{Base})_{3\phi}$ The *base complex power* in mega-volt-amperes (MVA)

$(V_{Base})_{LL}$ the *base voltage* in kilovolts

$$|S_{Base}|_{3\phi} = \sqrt{3}\,|I_{base}|\,|V_{Base}|_{LL} \quad MVA$$

The *base current* in kiloamperes

$$|I_{base}|_{L} = \frac{|S_{Base}|_{3\phi}}{\sqrt{3}\,|V_{Base}|_{LL}}$$

NOTE

$$(S_{Base})_{1\phi} = \frac{(S_{Base})_{3\phi}}{3}$$

$$|S_{Base}|_{1\phi} = |I_{base}|_{\phi}|V_{Base}|_{\phi}$$

$$|V_{Base}|_{1\phi} = \frac{|V_{Base}|_{3\phi}}{\sqrt{3}}$$

The *base impedance* will also be given by

$$|Z_{Base}| = \frac{|V_{Base}|_{\phi}}{|I_{base}|_{\phi}} = \frac{|V_{Base}|_{\phi}^{2}}{|S_{base}|_{\phi}} \; ohms$$

$$|Z_{base}| = \frac{\left[\frac{|V_{Base}|_{LL}}{\sqrt{3}}\right]^2}{\left[\frac{|S_{Base}|_{3\phi}}{3}\right]} = \frac{[\,|V_{Base}|_{LL}\,]^2}{[\,|S_{Base}|_{3\phi}\,]}$$

From the definition of per unit values: $$p.u.\ value = \frac{Actual\ \ Value}{Base\ \ Value}$$

$$S_{p.u.} = \frac{S}{S_{Base}} \qquad V_{p.u.} = \frac{V}{V_{Base}} \qquad I_{p.u.} = \frac{I}{I_{Base}}$$

The per unit impedance:

$$Z_{p.u.} = \frac{Z_{\Omega}}{Z_{Base}} = \frac{Z_{\Omega}\,|S_{Base}|}{|V_{Base}|^2} \; p.u.$$

NOTE

The impedance could be presented in pu or %

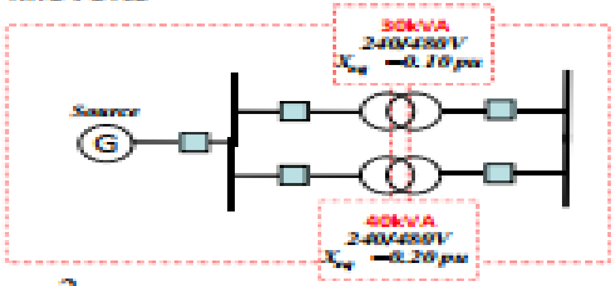
0.2 pu ← → 20%

Base Conversions

If $(Z_{p.u.})_{old}$ an impedance in per unit on a given old base

and $(I_{Base})_{old}$ the base current in kiloamperes

$(V_{Base})_{old}$ the base voltage in kilovolts



It is sometimes required to obtain the per unit value referred to a new base set.

The conversion expression is obtained as:

$$(Z_{p.u.})_{new} = (Z_{p.u.})_{old} \frac{|(S_{Base})_{new}|}{|(S_{Base})_{old}|} \cdot \frac{|(V_{Base})_{old}|^2}{|(V_{Base})_{new}|^2} \; p.u.$$

If the *volt bases are the same*, then

$$(Z_{p.u.})_{new} = (Z_{p.u.})_{old} \frac{|(S_{Base})_{new}|}{|(S_{Base})_{old}|} \; p.u.$$

Per unit system :-

$\underbrace{S \quad V}_{\text{Two value taken.}} \quad \underbrace{I \quad Z}_{\text{Extracted.}}$

$$\left.\begin{matrix} S_{base} \\ V_{base} \end{matrix}\right\} \Rightarrow$$

$$\boxed{I_{base} = \frac{S_{Base}}{\sqrt{3}\, V_{Base\,LL}}}$$

$S = VI$

$I = \frac{S}{V}$

$$\boxed{|Z_{Base}| = \frac{V_{Base\,\phi}}{I_{Base\,\phi}} = \frac{|V_{Base}|^2}{|S_{Base}|_\phi}}$$

$$\boxed{|Z_{Base}| = \frac{\left[V_{Base}/\sqrt{3}\right]^2}{S_{Base}/3} = \frac{\left[V_{Base\,LL}\right]^2}{S_{Base\,3\phi}}}$$

$$\boxed{\text{Per unit Value} = \frac{\text{Actual Value}}{\text{Base Value}}}$$

$V = .9$ P.U.

$I = .8$ P.U.

$$\boxed{S_{P.U} = \frac{S}{S_{Base}}}$$

$$\boxed{V_{P.U} = \frac{V}{V_{Base}}}$$

$$\boxed{I_{P.U} = \frac{I}{I_{Base}}}$$

$$\boxed{Z_{P.U} = \frac{Z}{Z_{Base}}}$$

$$Z_{PU} = \frac{Z_{\Omega}}{Z_{Base}}$$

$$\boxed{Z_{Base} = \frac{V_{Base}^2}{S_{Base}}}$$

$$Z_{PU} = \frac{Z_{\Omega}}{V_{Base}^2 / S_{Base}}$$

$$\boxed{Z_{PU} = \frac{Z_{\Omega} \cdot |S_{Base}|}{|V_{Base}|^2} \text{ P.U}}$$

Base Conversion:-

⇒ Convert The P.U in new Base value.

$$Z_{PU_{new}} = \frac{Z_{\Omega\,old}}{Z_{Base\,new}}$$

$$Z_{PU_{old}} = \frac{Z_{\Omega} \cdot S_{Base}}{V_{Base}^2}$$

$$\frac{Z_{PU\,old} \cdot V_{Base\,old}^2}{S_{Base\,old}} = Z_{\Omega}$$

$$= \frac{(Z_{PU})_{old} |V_{Base\,old}|^2}{|S_{Base\,old}|} \cdot \frac{1}{Z_{Base\,new}}$$

$$= \frac{(Z_{PU\,old}) |V_{Base\,old}|^2}{|S_{Base\,old}|} \cdot \frac{|S_{Base\,new}|}{|V_{Base\,new}|^2}$$

$$Z_{PU_{new}} = Z_{PU\,old} \cdot \frac{|V_{Base\,old}|^2}{|V_{Base\,new}|^2} \cdot \frac{|S_{Base\,new}|}{|S_{Base\,old}|}$$

If voltage are Same.

$$(Z_{pu})_{new} = (Z_{pu})_{old} \cdot \frac{|S_{Base}|_{new}}{|S_{Base}|_{old}} \text{ pu.}$$

Example # 01 :

$$Z = 3.346 + j77.299\ \Omega.$$

Base $\begin{cases} S = 100\ \text{MVA}. \\ V = 735\ \text{KV}. \end{cases}$

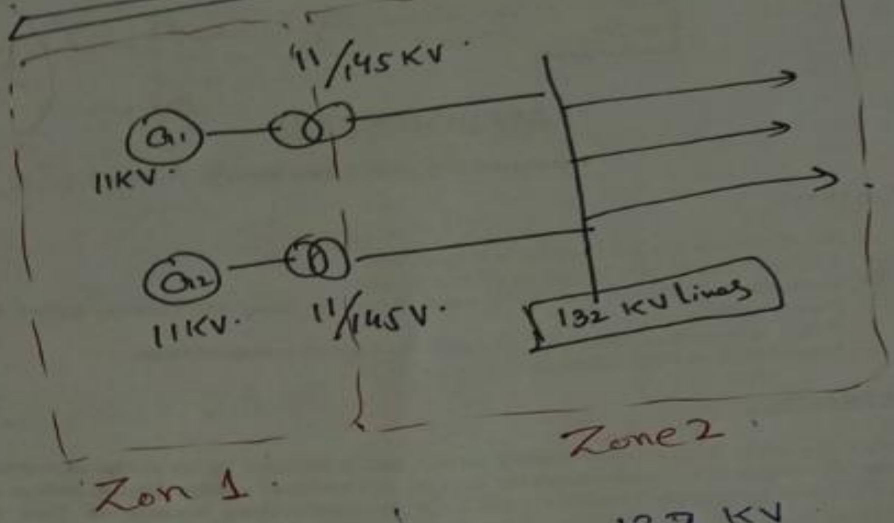
$$Z_{pu} = Z_{\Omega} \cdot \frac{S_{Base}}{|V_{Base}|^2}\ \text{P.U}.$$

$$= Z_{\Omega} \cdot \frac{100 \times 10^6}{(735 \times 10^3)^2}$$

$$= (3.346 + j77.299) \cdot \frac{100 \times 10^6}{(735 \times 10^3)^2}$$

$$= 6.19372 \times 10^{-4} + j1.43 \times 10^{-2}$$

Example#03



$\boxed{10.01\,KV} = \frac{11}{145}(132)$

V_{base}

132 KV

V_{base}

$S_{Base} = 100$ MVA $\xrightarrow{\text{Same}}$ All over Circuit

$V_{Base} = 132$ KV → Different.

$\Rightarrow$ when move different Zones.

(ii) Now convert Reactances into new values.

Generator Reactance:-

$$Z_{pu\,old} = .26 \text{ p.u.}$$

$$Z_{new} = .26 \times \frac{S_{new}}{S_{old}} \times \frac{|V_{old}|^2}{|V_{new}|^2}$$

$$= .26 \times \frac{150}{66.6} \times \frac{(11)^2}{(10.01)^2} = .47 \text{ p.u.}$$

Transformer Reactances:-

$$Z_{new} = .125 \times \frac{150}{75} \times \frac{(145)^2}{(132)^2}$$

$$= .201 \text{ P.U.}$$

Example 1:

Consider a transmission line with an impedance Z= 3.346+j77.299 *ohm*. Assume that Base MAV is 100 and Base kV is 735. What is the *pu* impedance.

Solution:

$$Z_{p.u.} = \frac{Z_{\Omega}}{Z_{Base}} = \frac{Z_{\Omega} \, | S_{Base} |}{| V_{Base} |^2} \, p.u.$$

$$Z_{p.u.} = \frac{Z_{\Omega} \, | MVA, S_{Base} |}{| kV, V_{Base} |^2} \, p.u. = \frac{Z_{\Omega} \, |100|}{|735|^2} = (Z_{\Omega}) * (1.85108 \times 10^{-4})$$

For $R = 3.346$ ohms we obtain

$$R_{\text{p.u.}} = (3.346)\left(1.85108 \times 10^{-4}\right) = 6.19372 \times 10^{-4}$$

For $X = 77.299$ ohms, we obtain

$$X_{\text{p.u.}} = (77.299)\left(1.85108 \times 10^{-4}\right) = 1.430867 \times 10^{-2}$$

Example 2: Convert the impedance of the previous example using a ***new base*** of 200 MVA and 345 kV.

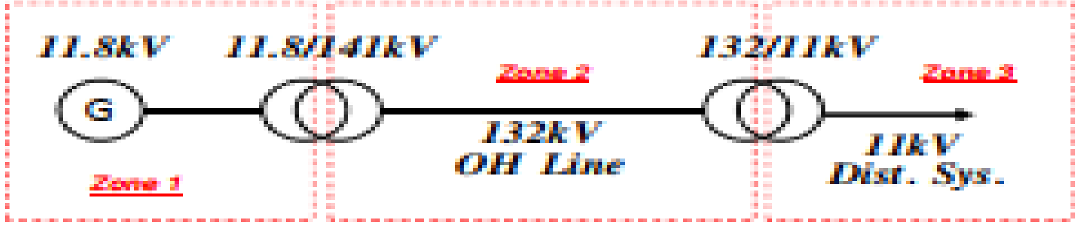
$$(Z_{p.u.})_{old} = 6.19372\times10^{-4} + j1.430867\times10^{-2}$$

The old base is: 100 MVA and 735 kV. & The new base is: 200 MVA and 345 kV.

$$(Z_{p.u.})_{new} = (Z_{p.u.})_{old}\,\frac{|(S_{Base})_{new}|}{|(S_{Base})_{old}|}\cdot\frac{|(V_{Base})_{old}|^2}{|(V_{Base})_{new}|^2}\;p.u.$$

$$(Z_{pu})_{new} = (Z_{p.u.})_{old}\,[\frac{200}{100}]\,[\frac{735}{345}]^2\;p.u.$$

$$(Z_{pu})_{new} = 9.0775(Z_{pu})_{old}$$



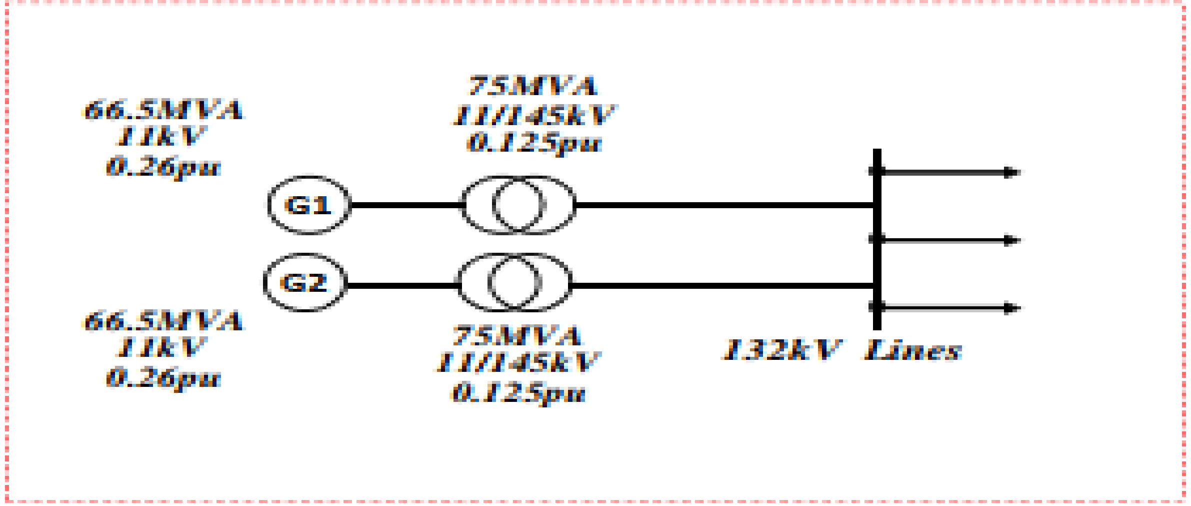
Wrong Selection of Voltage Bases		
11.8kV	132kV	11kV
Right Selection of Voltage Bases		
11.8kV	141kV	$\frac{141}{132}(11)kV$

NOTE: ***kV Base Selection***

The base voltages in the three circuits (*zones*) are related by the turns ratios of the intervening transformers. The transformers quoted may be different from the turns ratios for example a 220/33kV (nominal) transformer may have a turns ratio of 220/34.5kV.

Example 3:

Generators G1 and G2 have a *sub-transient reactance* of 26% on 66.6MVA rating at 11kV, and transformers T1 and T2 a voltage ratio of 11/145kV and an impedance of 12.5% on 75MVA. Choosing 100MVA as base MVA and 132kV as base voltage, find the percentage impedances to new base quantities.



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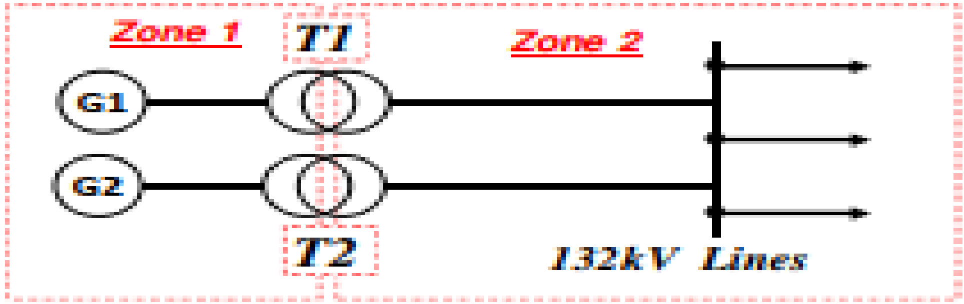
Solution: $S_{Base} = 100 MAV$

$$V_{Base}^{Zone\ 2} = 132kV \quad \text{At overhead Lines}$$



$$\frac{[V_{Base}]_{Gen}^{Zone\ 1}}{[V_{Base}]_{OHL}^{Zone\ 2}} = \frac{[V_{T1}]_{Primary}}{[V_{T1}]_{Secondary}} = \frac{11}{145}$$

$$[V_{Base}]_{Gen}^{Zone\ 1} = \frac{11}{145}(132) = 10.01kV$$

$$(Z_{p.u.})_{new} = (Z_{p.u.})_{old} \frac{|(S_{Base})_{new}|}{|(S_{Base})_{old}|} \cdot \frac{|(V_{Base})_{old}|^2}{|(V_{Base})_{new}|^2} \ p.u.$$

T_1 & T_2 rating
75MVA
11/145kV

a. Generator reactance to new bases is:

$$(Z)_{new} = 26 \frac{100}{66.6} \cdot \frac{(11)^2}{(10.01)^2} = 47.24\% = 0.4724 pu$$

b. Transformer reactance to new bases is:

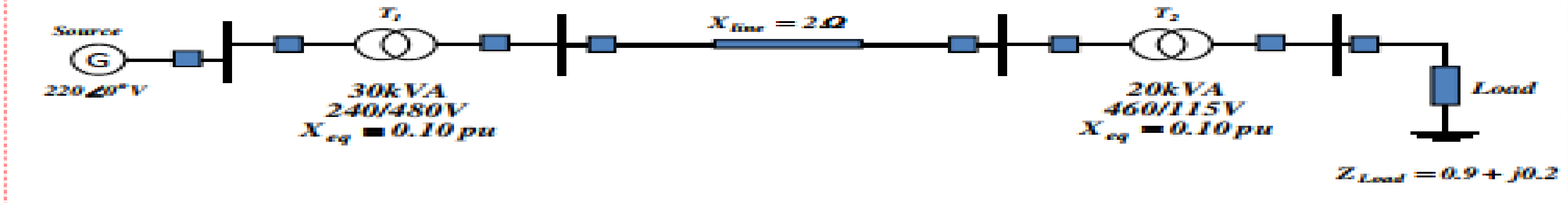
$$(Z)_{new} = 12.5 \frac{100}{75} \cdot \frac{(145)^2}{(132)^2} = 20.1\% = 0.201 pu$$

NOTE

Z = 26 %
Is equivalent to
Z = 0.26pu

Example 4:

For the system shown in the figure. Selecting the base values of 30KVA and 240 V at the generator side.



1. Find all the bases for the system.

$$S_{Base},\ V_{Base},\ I_{Base}\ and\ Z_{Base}$$

2. Draw the per unit reactance circuit.

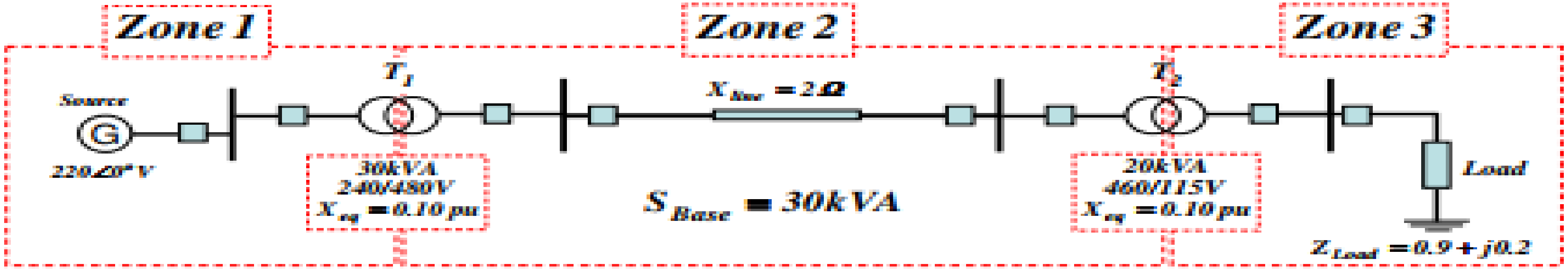
3. Find the actual value of the load current

Reminder

How many zones do we have in this system?

Do you think the selected base voltages in the system zones are related by to the transformers and generator voltages.

Do we have impedances in pu, % or in ohms?



The bases for the system:

$$S_{Base} = 30kVA$$

$$[V_{Base}]_1 = 240V$$

$$\frac{[V_{Base}]_2}{[V_{Base}]_1} = \frac{480V}{240V}$$

$$[V_{Base}]_2 = [V_{Base}]_1 \frac{480V}{240V} = 480V$$

$$\frac{[V_{Base}]_3}{[V_{Base}]_2} = \frac{115V}{460V}$$

$$[V_{Base}]_3 = [V_{Base}]_2 \frac{115V}{460V} = 120V$$

$$[Z_{Base}]_1 = \frac{[V_{Base}]_1^2}{[S_{Base}]_1} = \frac{240^2}{30000} = 1.92\,\Omega$$

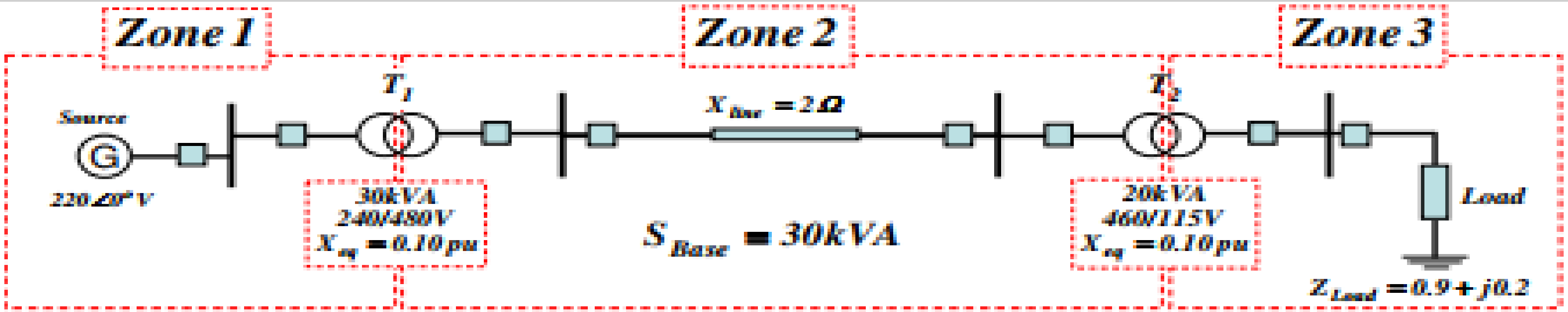
$$[I_{Base}]_1 = \frac{[S_{Base}]_1}{\sqrt{3}[V_{Base}]_1} = \frac{30000}{\sqrt{3}240} = 72.2A$$

$$[Z_{Base}]_2 = \frac{[V_{Base}]_2^2}{[S_{Base}]_2} = \frac{480^2}{30000} = 7.68\,\Omega$$

$$[I_{Base}]_2 = \frac{[S_{Base}]_2}{\sqrt{3}[V_{Base}]_2} = \frac{30000}{\sqrt{3}480} = 36.2A$$

$$[Z_{Base}]_3 = \frac{[V_{Base}]_3^2}{[S_{Base}]_3} = \frac{120^2}{30000} = 0.48\,\Omega$$

$$[I_{Base}]_3 = \frac{[S_{Base}]_3}{\sqrt{3}[V_{Base}]_3} = \frac{30000}{\sqrt{3}120} = 144.3A$$



Draw the per unit circuit

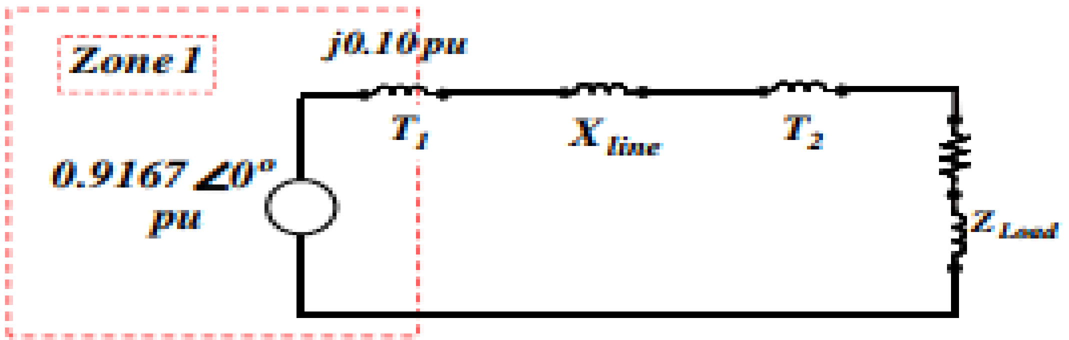
Zone 1

$$S_{Base} = 30kVA$$

$$[V_{Base}]_1 = 240V$$

$$|Z_{Base}| = \frac{|V_{Base}|^2}{|S_{base}|}\ ohms$$

$$[Z_{Base}]_1 = 1.92\,\Omega$$



$$X_{T1} = j0.10\,pu$$

T1 Rating

30kVA

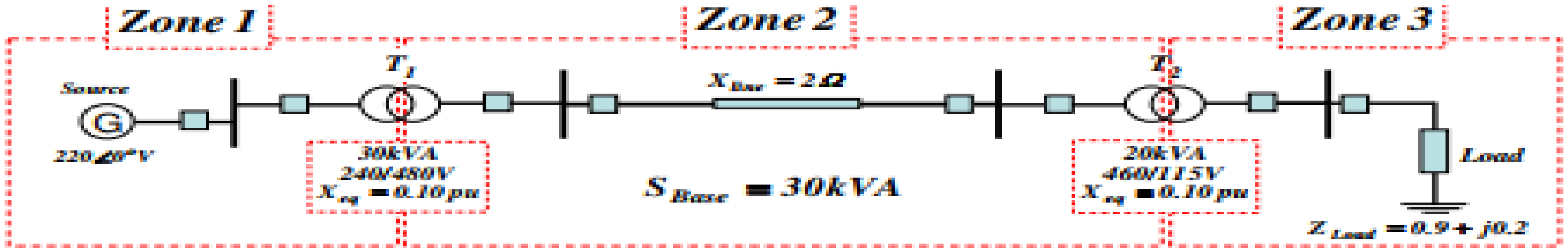
240/480V

Selected Bases

$S_{Base} = 30kVA$

$V_{Base1} = 240V$

$$[V_{Source}]_{pu} = \frac{[V_{source}]_{actual}}{[V_{Base}]_1} = \frac{220\angle 0}{240} = 0.9167\angle 0^\circ\ pu$$



Draw the per unit circuit

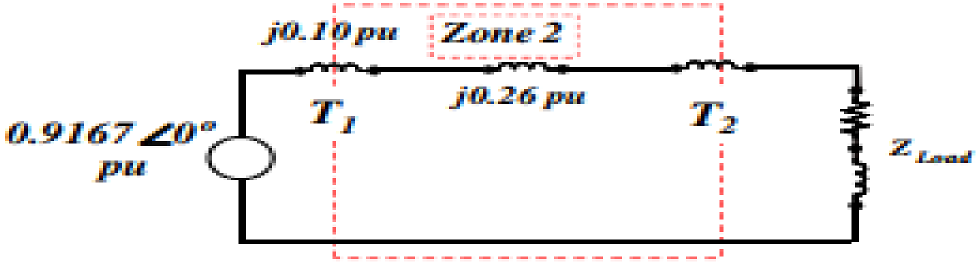
Zone 2

$$S_{Base} = 30kVA$$

$$[V_{Base}]_2 = 480V$$

$$|Z_{Base}| = \frac{|V_{Base}|^2}{|S_{base}|} \text{ ohms}$$

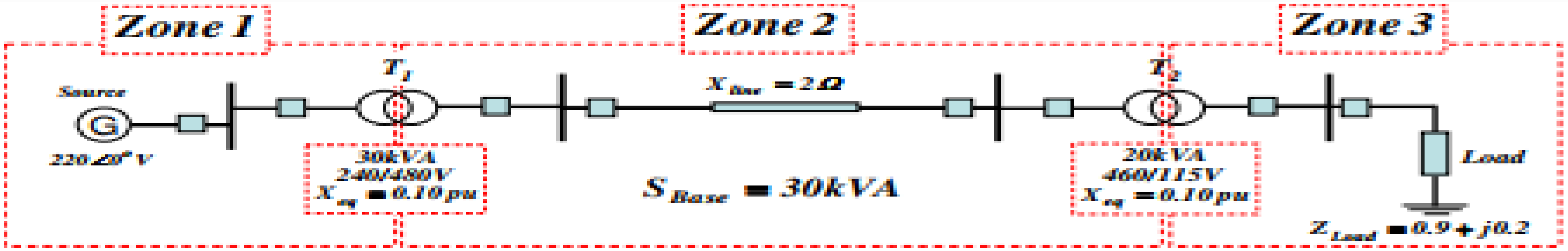
$$[Z_{Base}]_2 = 7.68\,\Omega$$



$$X_{line} = 2\,\Omega$$

$$[X_{line}]_{pu} = \frac{2\,\Omega}{[Z_{Base}]_2} = \frac{2}{7.68}$$

$$[X_{line}]_{pu} = 0.26 \text{ pu}$$



Draw the per unit circuit

Zone 3

$S_{Base} = 30kVA$

$$\frac{[V_{Base}]_2}{[V_{Base}]_3} = \frac{460}{115}$$

$$[V_{Base}]_2 = 480V$$

$$\frac{480}{[V_{Base}]_3} = \frac{460}{115}$$

$$[V_{Base}]_3 = 120V$$

$$[X_{New}^{T_2}]_{pu} = [X_{Old}]_{pu}\,\frac{[S_{Base}]_{New}}{[S_{Base}]_{Old}}\,\frac{[V_{Base}]_{Old}^2}{[V_{Base}]_{New}^2}$$

T2 Rating
20kVA
460/115V

Selected Bases
S_{Base} = 30 kVA
$V_{Base\,3}$ = 120V

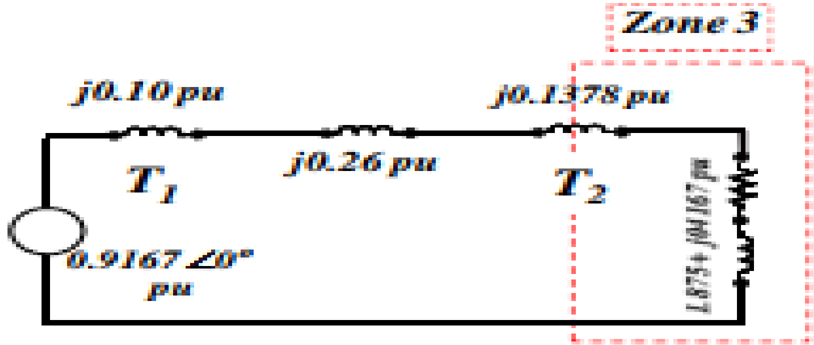
$$[X_{New}^{T_2}]_{pu} = 0.10\,\frac{(30000)}{(20000)}\,\frac{115^2}{120^2} = 0.1378\,pu$$

$$|Z_{Base}| = \frac{|V_{Base}|_{\phi}^2}{|S_{base}|_{\phi}}\ ohms$$

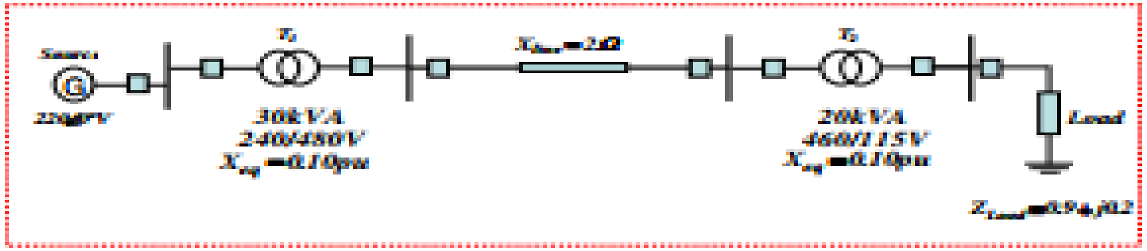
$$[Z_{Base}]_3 = 0.48\,\Omega$$

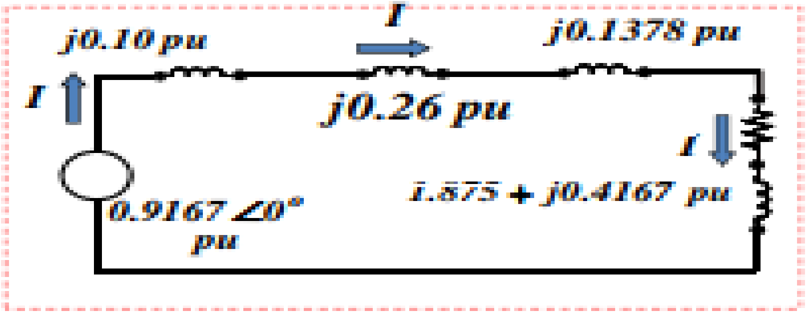
$$[Z_{load}]_{pu} = \frac{0.9 + j0.2}{0.48}$$

$$[Z_{load}]_{pu} = 1.875 + j0.4167\ pu$$



2. The per unit reactance circuit.





3. The actual load current

$$I_{pu} = \frac{V_{pu}}{(Z_{Total})} = \frac{0.9167\angle 0^{o}}{j0.1 + j0.26 + j0.1378 + 1.875 + j0.4167}$$

$$I_{pu} = 0.4395\angle -26.01^{o} \quad pu$$

$$[I_{actual}]_{Load} = I_{pu} * I_{Base} = (0.4395\angle -26.01^{o} \ pu)(144.3A$$

$$[I_{actual}]_{Load} = I_{pu} * I_{Base} = 63.3\angle -26.01^{o} A$$

Reminder

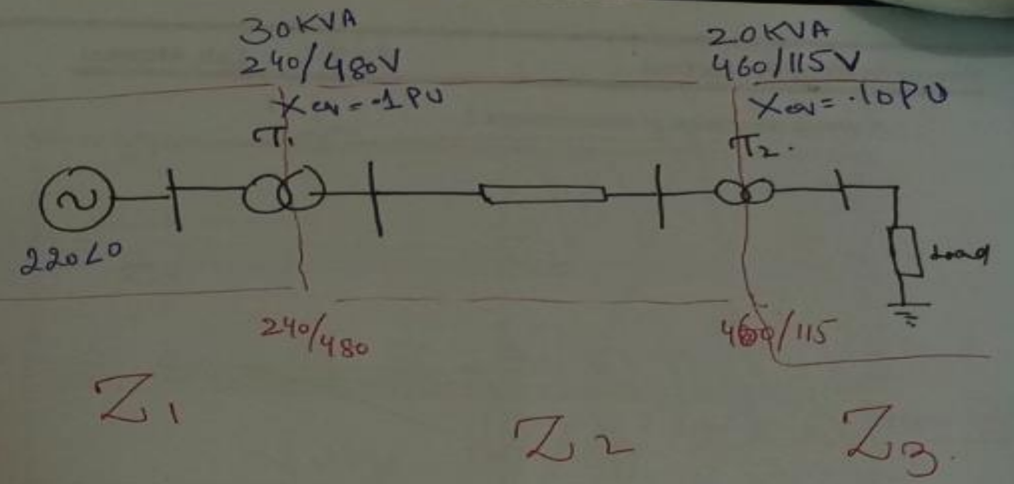
The same *pu* current is running in the circuit. To get the *actual value* for the current we have to *select the right base* current

$$[I_{Base}]_1 = 72.2A$$

$$[I_{Base}]_2 = 36.2A$$

$$[I_{Base}]_3 = 144.3A$$

Example=4

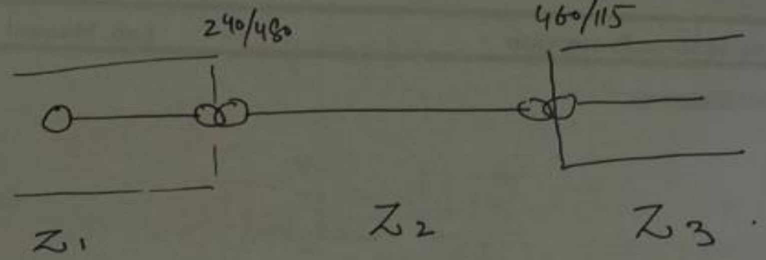


$\Longrightarrow$ $S_{Base} = 30$ KVA.
$V_{Base} = 240$V

2
$\Rightarrow$ Make Zones.
$\Rightarrow$ S value Remain Same

$\Rightarrow$ V will change according to Zone.

$\Rightarrow$ V will change w.r.t Turn Ratio of Transformer.



fix ⇒ [$S = 30$ KVA

Voltge in 3 Zones $V_{Base} = 240$

Zone 1 :

$$\boxed{V_{Base\,1} = 240}$$

Zone 2 :

$$V_{Base\,2} = \underset{(V_{Base1})}{240} \cdot \left[\frac{480}{240}\right]$$

$$\boxed{V_{Base\,2} = 480}$$

Zone 3 :- $V_{Base\,2} \left[\frac{115}{460}\right]$

$$= 480 \left[\frac{115}{460}\right]$$

$$\boxed{V_{Base_3} = 120\,V}$$

Impedance Base in Zones

Zone 1 :-

$$[Z_{Base}]_1 = \frac{[V_{Base1}]^2}{S_{Base1}}$$

$$= \frac{240^2}{30K} = 1.92\,\Omega$$

Zone 2 :-

$$[Z_{Base}]_2 = \frac{[V_{Base2}]^2}{S_{Base2}}$$

$$= \frac{480^2}{30K} = 7.68\,\Omega$$

Zone 3 :-

$$[Z_{Base}]_3 = \frac{[V_{Base3}]^2}{S_{Base3}}$$

$$= \frac{120^2}{30K} = .48\,\Omega$$

Per unit value :-

Zone 1 :-

$$V_{source\,pu} = \frac{V_{source}}{V_{pu}} = \frac{220\angle 0}{240}$$

$$\boxed{V_{sor\,pu} = .9167\angle 0\ pu}$$

Zone 2 :-

$$X_{line} = 2\Omega$$

$$X_{line\,pu} = \frac{X_{line}}{[Z_{Base2}]} = \frac{2}{7.68}$$

$$\boxed{X_{line\,pu} = .26\ pu}$$

$$X_{T_1} = X_{pu\,old} \cdot \frac{S_{new}}{S_{old}} \cdot \frac{V_{old}^2}{V_{new}^2}$$

Same   Same

$$\frac{\cancel{30k}}{\cancel{30k}} \cdot \frac{\cancel{480^2}}{\cancel{480^2}}$$

$$= .1$$

$$\boxed{X_{T_1} = .1\ pu}$$

Zone 3 :-

$$X_{T2} = X_{T old} \cdot \frac{S_{new}}{S_{old}} \cdot \frac{\{V_{old\ base}\}^2}{\{V_{new\ old}\}^2}$$

$$= .1 \cdot \frac{30K}{20K} \cdot \frac{115^2}{120^2}$$

$$\boxed{X_{T_2\ pu} = .1378\ PU}$$

$$Z_{load} = .9 + j.2$$

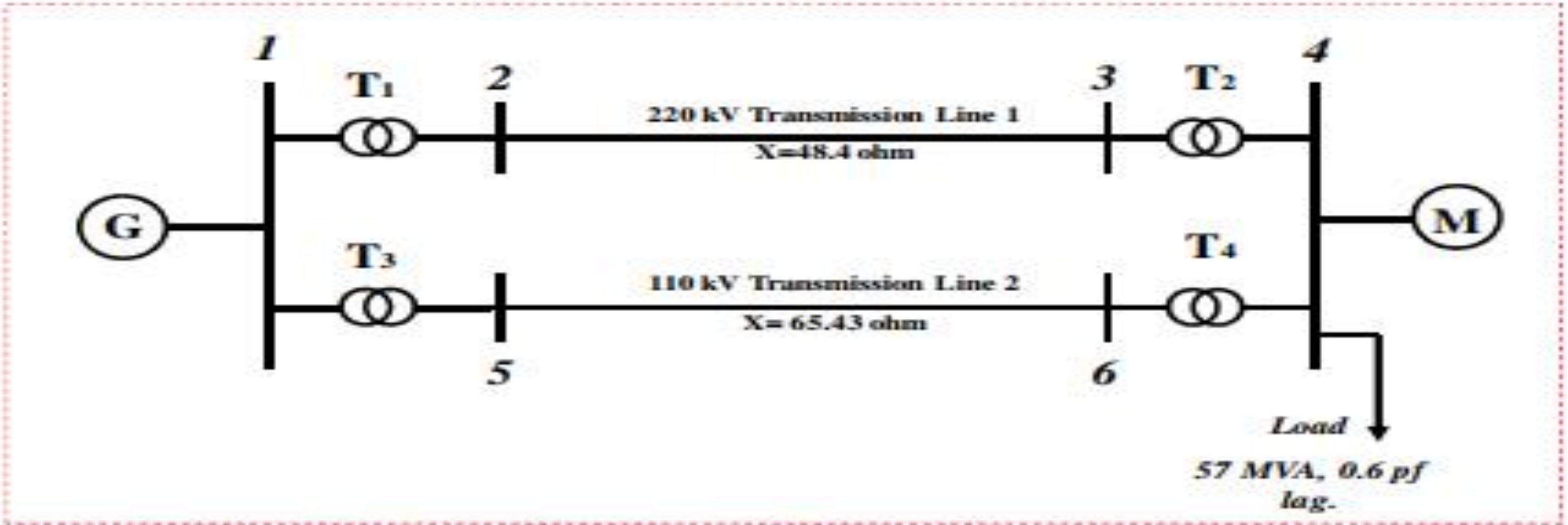
$$\{Z_{base}\}_3 = .48\Omega$$

$$Z_{load\ pu} = \frac{.9 + j.2}{.48}$$

$$\{Z_{load}\}_{pu} = 1.87 + j.4167\ PU$$

Example 5:

The single-line diagram of a three phase power system is shown in the Figure. Select a common base of 100 MVA and 22 kV on the generator side.



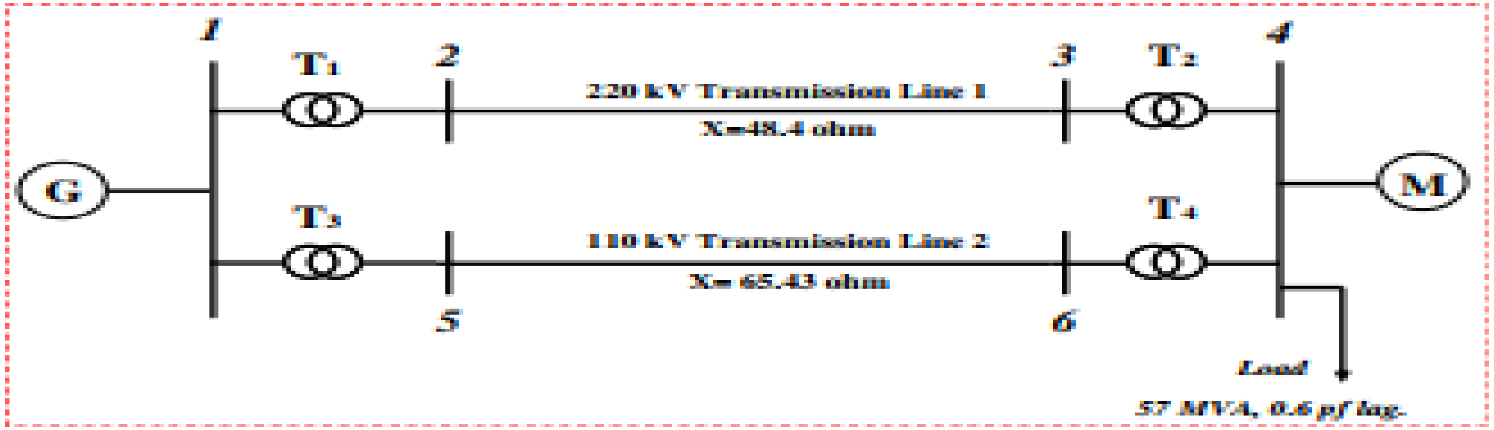
1. Draw an impedance diagram of the system.

Think about this:

What are the Voltage and Power bases?

Make sure that the base voltages in the system zones are related by the turns ratios of the intervening transformers.

How many zones do we have in this system?



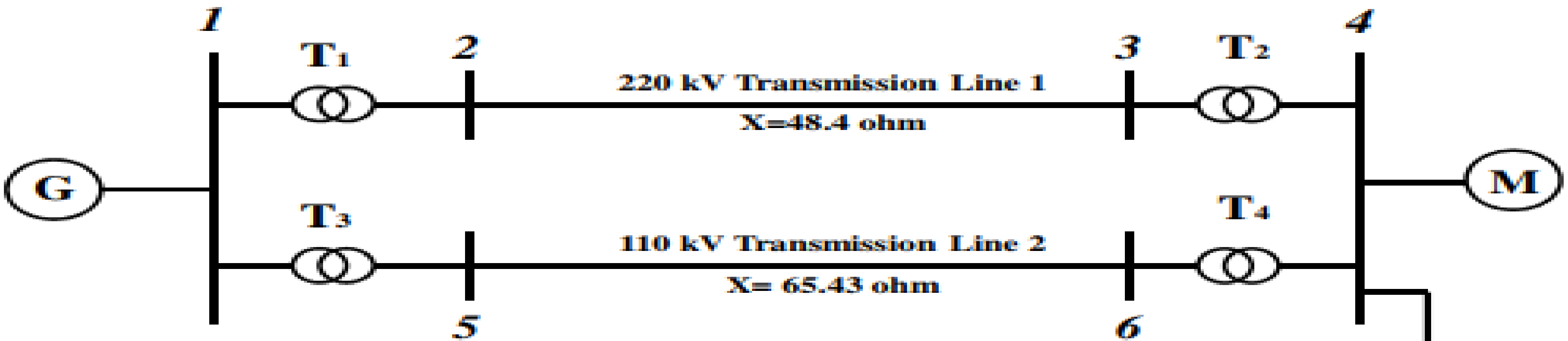
	MVA	Voltage	X
G	90	22 kV	18 %
T1	50	22/220 kV	10 %
T2	40	220/11 kV	06 %
T3	40	22/110 kV	6.4 %
T4	40	110/11 kV	08 %
M	66.5	10.45kV kV	18.5 %

Do we have impedances in **pu**, **%** or in **ohms**?

Is there any difference between system voltages and selected bases?

Is there any difference between system Rated MVA and Base MVA?

Is there any difference between system Rated kV and Operating kV?



Load

57 MVA, 0.6 pf lag.

	MVA	Voltage	X
G	90	22 kV	18 %
T1	50	22/220 kV	10 %
T2	40	220/11 kV	06 %
T3	40	22/110 kV	6.4 %
T4	40	110/11 kV	08 %
M	66.5	10.45kV	18.5 %

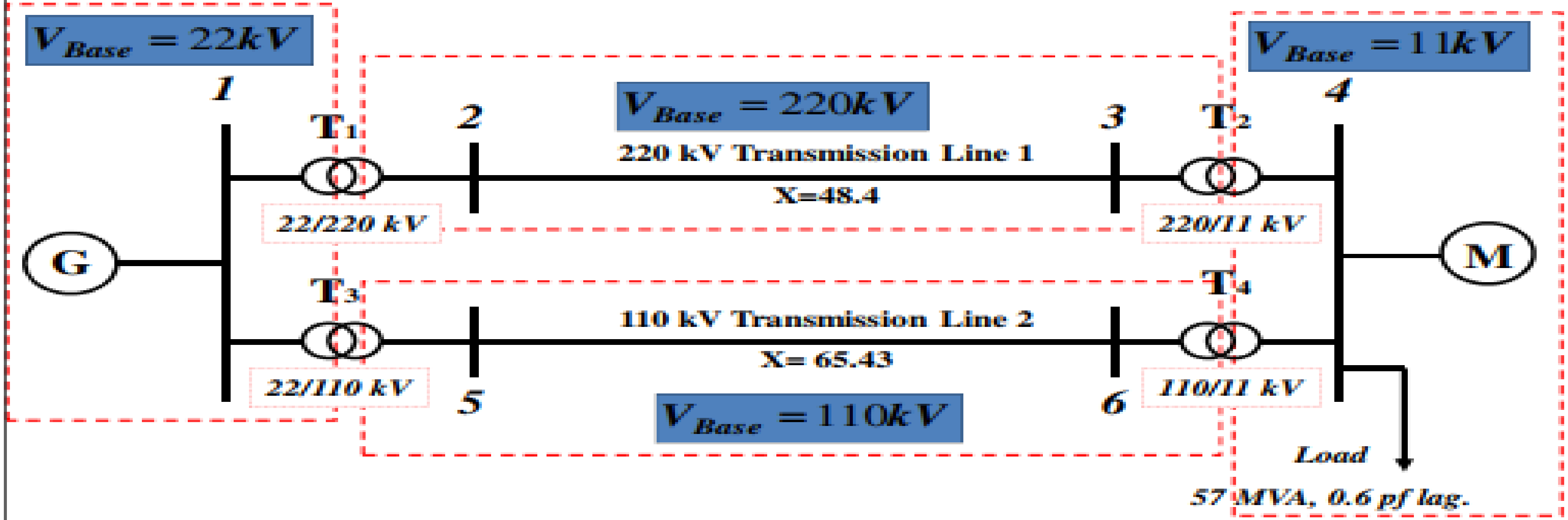
Note:
X% is calculated according to MVA and kV of each part of the network.

Solution:

The base voltages in the system zones are related by the turns ratios of the transformers.

Voltage bases must be determined for all sections of the network.
A common base of ***100 MVA*** and ***22 kV*** on the generator side is selected.

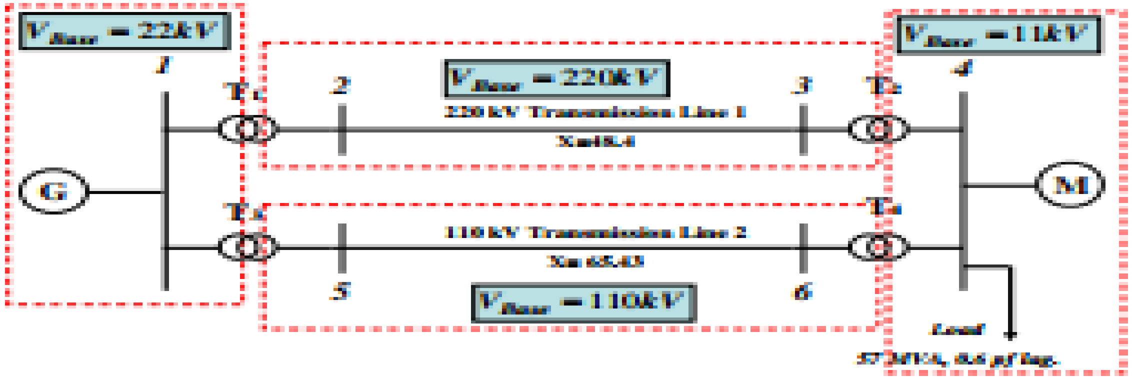
The voltage levels that exist in the system are 11kV, 22kV, 110kV and 220kV. Select the voltage bases as:



For transformers and generator

$$(Z_{p.u.})_{new} = (Z_{p.u.})_{old} \; \frac{|(S_{Base})_{new}|}{|(S_{Base})_{old}|} \cdot \frac{|(V_{Base})_{old}|^2}{|(V_{Base})_{new}|^2} \; p.u.$$

Since the *volt bases* are the same for *transformers and generator*, then



$$(Z_{p.u.})_{new} = (Z_{p.u.})_{old} \; \frac{|(S_{Base})_{new}|}{|(S_{Base})_{old}|}$$

	MVA	Voltage	X
G	90	22 kV	18 %
T1	50	22/220 kV	10 %
T2	40	220/11 kV	06 %
T3	40	22/110 kV	6.4 %
T4	40	110/11 kV	08 %

$$X_G = 0.18 \, \frac{100}{90} = 0.20 \;\; pu$$

$$X_{T1} = 0.10 \, \frac{100}{50} = 0.20 \;\; pu$$

$$X_{T2} = 0.06 \, \frac{100}{40} = 0.15 \;\; pu$$

$$X_{T3} = 0.064 \, \frac{100}{40} = 0.16 \;\; pu$$

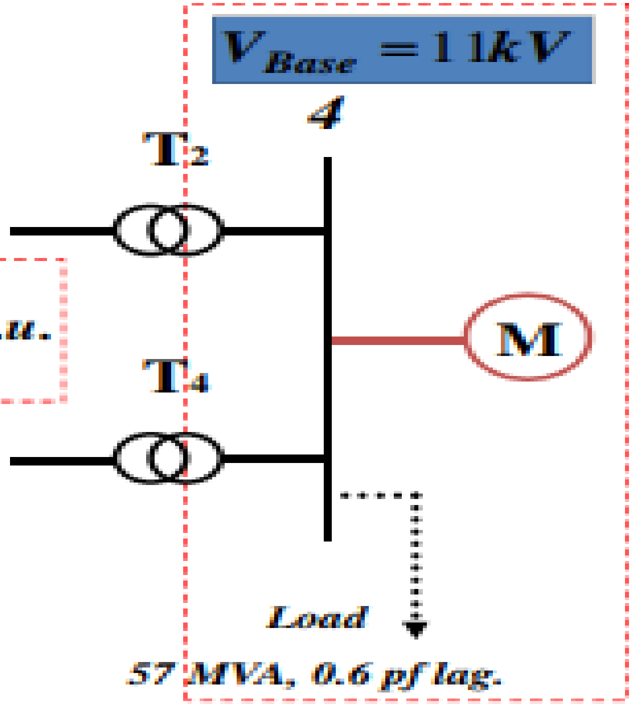
$$X_{T4} = 0.08 \, \frac{100}{40} = 0.20 \;\; pu$$

For the motor

	MVA	Voltage	X
M	66.5	10.45kV	18.5 %

$$(Z_{p.u.})_{new} = (Z_{p.u.})_{old} \frac{|(S_{Base})_{new}|}{|(S_{Base})_{old}|} \cdot \frac{|(V_{Base})_{old}|^2}{|(V_{Base})_{new}|^2} \; p.u.$$

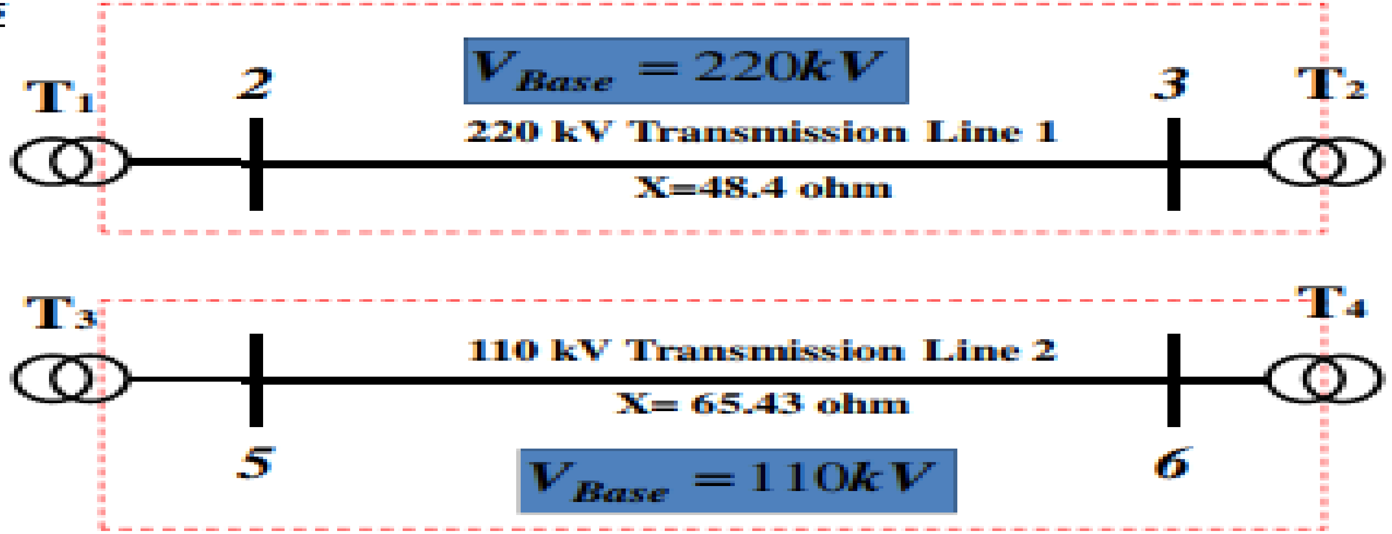
$$X_M = 0.185 \, \frac{100}{66.5}\left(\frac{10.45}{11}\right)^2 = 0.25 \; pu$$



For the Transmission Lines

$X_{\text{Line 1}} = 48.4$ ohm

$X_{\text{Line 2}} = 65.43$ ohm



$$Z_{p.u.} = \frac{Z_{\Omega}}{Z_{Base}}$$

$$|Z_{Base}| = \frac{|V_{Base}|^2}{|S_{base}|}$$

$$\boxed{Z_{p.u.} = \frac{Z_{\Omega}\,|MVA,\ S_{Base}|}{|kV,\ V_{Base}|^2}\ p.u.}$$

$$X_{line\,1} = \frac{48.4(100)}{(220)^2} = 0.10\ p.u. \qquad X_{line\,2} = \frac{65.43(100)}{(110)^2} = 0.54\ p.u.$$

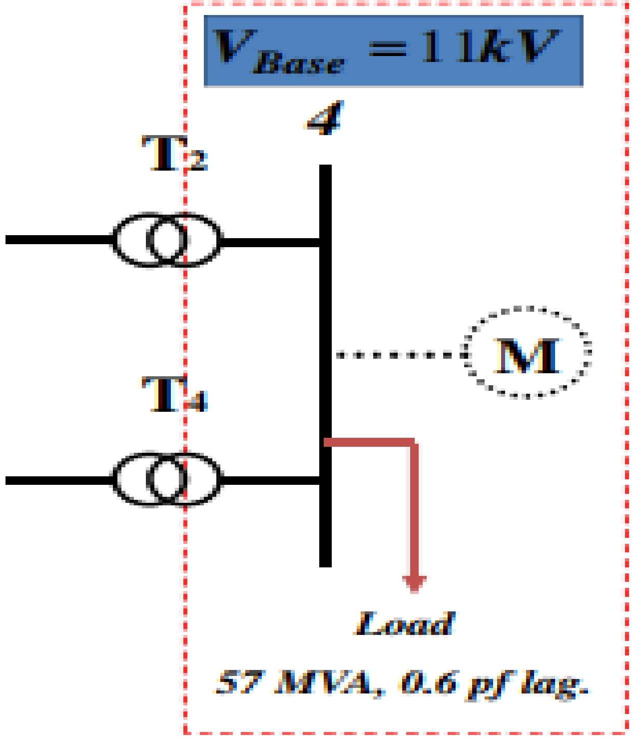
For the Load

Load = 57 MVA, 0.6 pf lag.

$$S_{3\phi} = \sqrt{3}(V_{LL})(I_L)^* = \sqrt{3}(V_{LL})\left(\frac{V_\phi}{(Z_L)}\right)^*$$

$$= \sqrt{3}(V_{LL})\left(\frac{V_{LL}}{(\sqrt{3}Z_L)}\right)^* = \frac{|V_{LL}|^2}{Z_L{}^*}$$

$$Z_{Load} = \frac{(V_{L-L})^2}{S^*_{L(3\phi)}} \, ohm$$



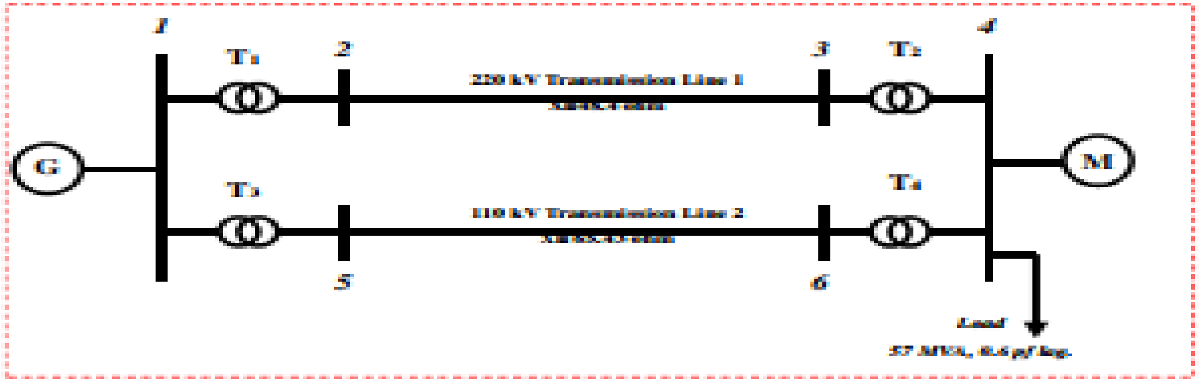
$$S_{L(3\phi)} = 57\angle 53.13 \ MVA \qquad V_{L-L} = 10.45 \ kV$$

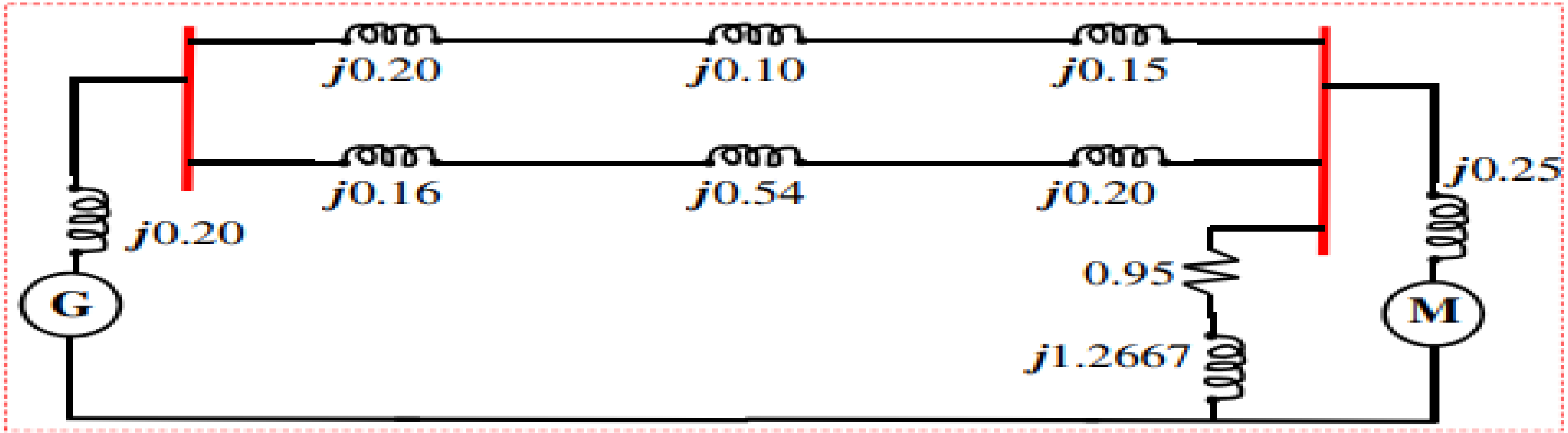
$$Z_{p.u.} = \frac{Z_\Omega \,|\, MVA, S_{Base} \,|}{|\, kV, V_{Base} \,|^2} \, p.u.$$

$$Z_{Load} = \frac{(10.45)^2}{57\angle -53.13} = 1.1495 + j1.53267 \, ohm$$

$$(Z_{Load})_{pu} = \frac{Z_{Load}\,(MVA,\,100)}{(kV,\,11)^2} = 0.95 + j1.2667 \, p.u.$$

1. The per unit impedance diagram of the system is:





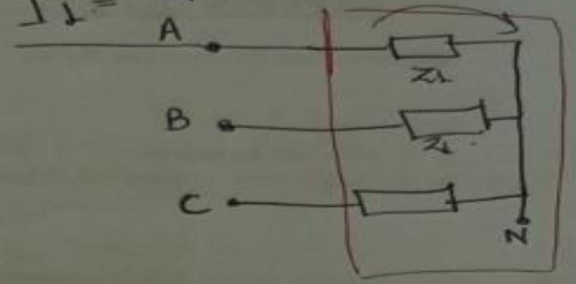
S for VL

Example 5 portion for static load

→ Static load is mostly Y-connected.

$I_L = I_\phi$

VAN



57 MVA .6 pf lag.

$$S_{3\phi} = 3 V_\phi I_\phi^*$$

$$I_\phi = \frac{V_\phi}{Z_L}$$

$$V_L = \sqrt{3} V_\phi$$

$$= 3 \cdot \frac{V_L}{\sqrt{3}} I_\phi^*$$

$$= \sqrt{3} \frac{V_L}{\cancel{}} \cdot \left(\frac{V_\phi}{Z_L}\right)^*$$

$$= \sqrt{3} V_L \cdot \frac{V_L}{\sqrt{3} Z_L^*}$$

$$S_{3\phi} = \frac{(V_L)^2}{Z_L^*}$$

$$\boxed{Z_{Load} = \frac{(V_{L-L})^2}{S^*_{L(3\phi)}}}$$

$$S_{L(3\phi)} = 57\angle 53.13 \text{ MVA}$$

$$V_{LL} = 10.45 \text{ KV}$$

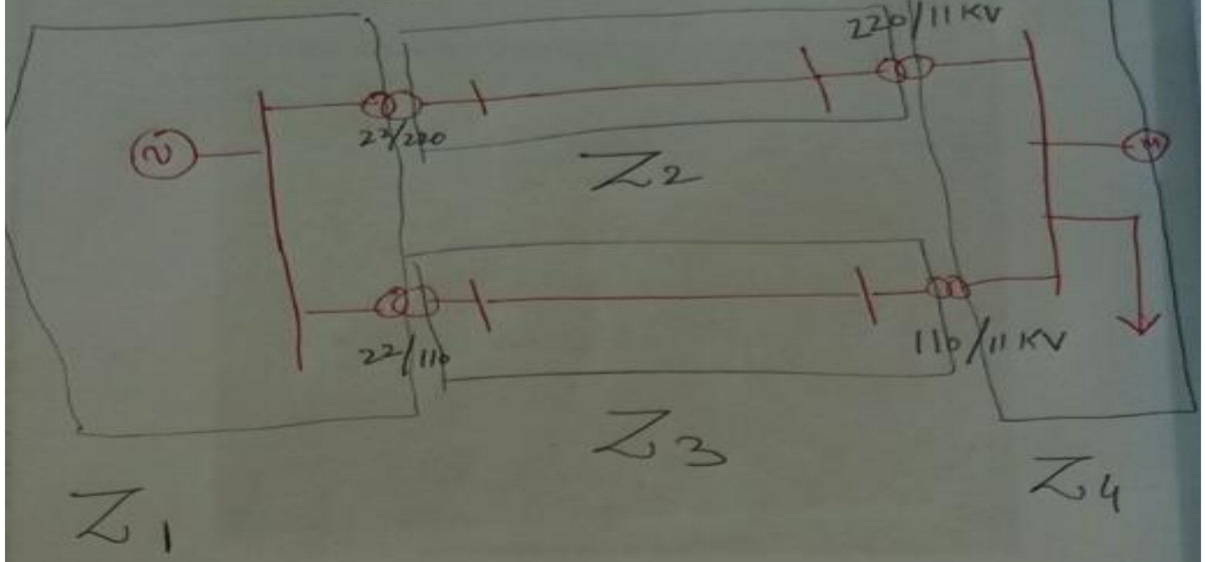
$$Z_{Load} = \frac{(10.45)^2}{57\angle -53.13} = 1.149 + j1.5\ \Omega$$

$$Z_{Load\ PU} = \frac{Z_{Load} \cdot S}{V^2}$$

$$\boxed{Z_{Load\ PU} = \frac{Z_{Load} \cdot 100 \text{ MVA}}{11^2} = .95 + j1.2 \text{ PU}}$$

Zones in Example 5

$$V_{Base} = 22\ KV$$



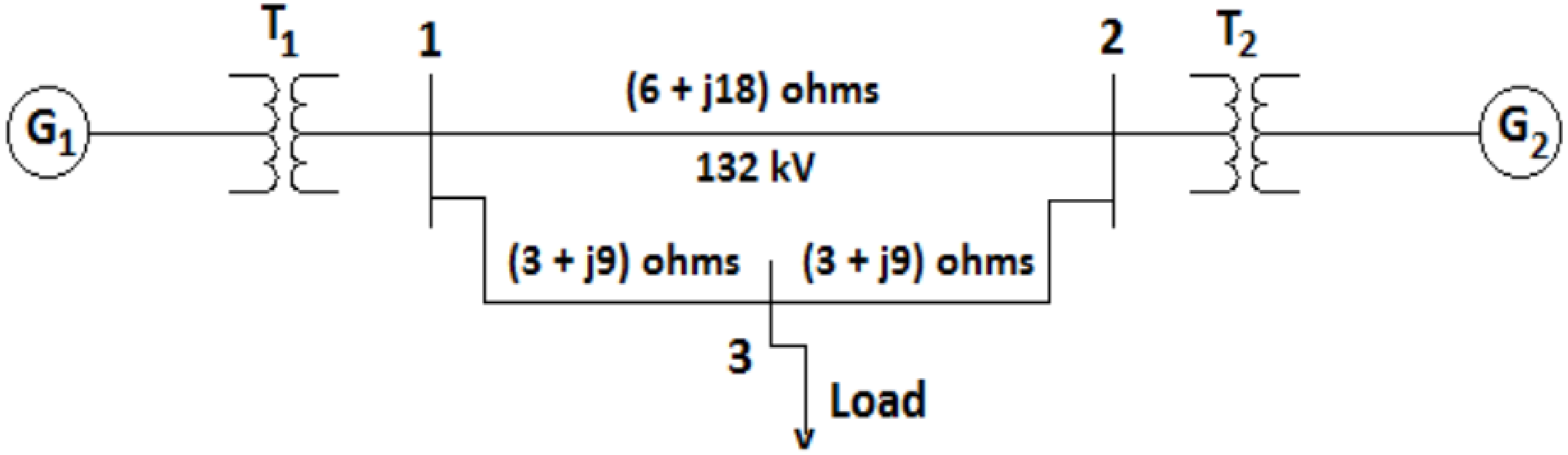
$$Z_1 = 22\ KV$$

$$\text{Zone } 2 = 22 \cdot \frac{220}{22} = 220\ KV$$

$$\text{Zone } 3 = 22 \cdot \frac{110}{22} = 110\ KV$$

$$\text{Zone } 4 = \left\{ \begin{array}{l} 110 \cdot \dfrac{11}{110} = 11\ KV \\ 220 \cdot \dfrac{11}{220} = 11\ KV \end{array} \right.$$

A single line diagram of a single phase circuit is shown in Fig. 1. Draw the per unit impedance diagram assuming the base values of 100 MVA and 100 kV in the transmission line sections.



The Generators and Transformers are rated as follows:

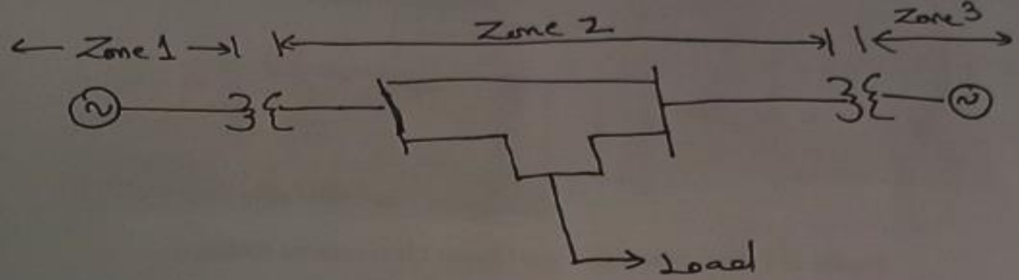
Generator 1:	50 MVA,	12.2 kV,	$X'' = 0.20$ pu
Generator 2:	25 MVA,	13.8 kV,	$X'' = 0.15$ pu
Transformer 1:	80 MVA,	12.2/132 kV,	$X = 10$ %
Transformer 2:	100 MVA,	13.8/132 kV,	$X = 15$%
Load:	60 MVA,	0.8 pf lagging at 124 kV	

Solution:-

Base value in Transmission Line Zone

$S_b = 100\,MVA.$

$V_b = 100\,KV.$



Zone $\cancel{2}$ 1 :-

$S_b = 100\ MVA.$

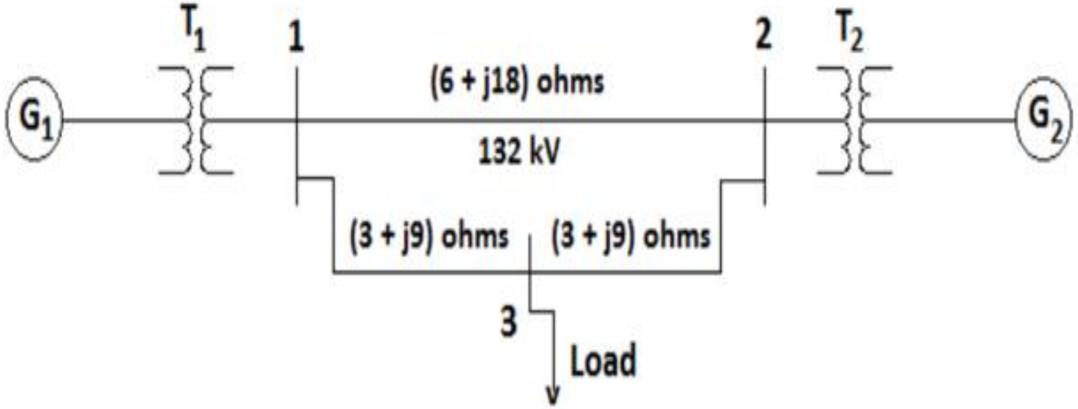
$V_b = 100 \times \frac{12.2}{13.2} = 9.24\,KV$

$X_{G_1} = .2 \times \frac{12.2^2}{9.24^2} \times \frac{100}{50} = j.697\,PU$

$X_{T_1} = .10 \times \frac{12.2^2}{9.24^2} \times \frac{100}{80} = j.2179\,PU$

The Generators and Transformers are rated as follows:

Generator 1:	50 MVA,	12.2 kV,	X" = 0.20 pu
Generator 2:	25 MVA,	13.8 kV,	X" = 0.15 pu
Transformer 1:	80 MVA,	12.2/132 kV,	X = 10 %
Transformer 2:	100 MVA,	13.8/132 kV,	X = 15%
Load:	60 MVA,	0.8 pf lagging at 124 kV	



Zone 2:

$S_b = 100$ MVA.

$V_b = 100$ KV.

$$Z_{line_1} = \frac{6 + j18}{100^2} \times 100 = .06 + j.18 \text{ PU}$$

$$Z_{line_2} = .03 + j.09 \text{ PU} + \quad .03 + j.09 \text{ PU}$$

$$S_{load} = 60 \, (\cos\theta + j\sin\theta).$$

$$= 60 \, (.8 + j.6).$$

$$\boxed{S_{load} = 48 + j36 \text{ MVA}}$$

$$Z_{load} = \frac{124^2}{48 - j36}$$

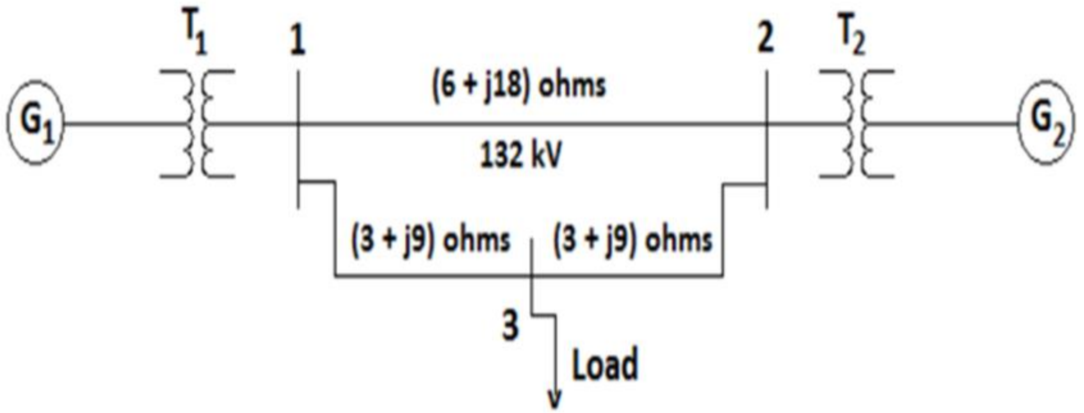
$$= 205 + j153.7 \, \Omega$$

$$Z_{PU} = \frac{205 + j153.7}{100^2} \cdot 100 = 2.05 + j1.53$$

$$Z_{PU} = 2.05 + j1.53 \text{ PU}$$

The Generators and Transformers are rated as follows:

Generator 1:	50 MVA,	12.2 kV,	X" = 0.20 pu
Generator 2:	25 MVA,	13.8 kV,	X" = 0.15 pu
Transformer 1:	80 MVA,	12.2/132 kV,	X = 10 %
Transformer 2:	100 MVA,	13.8/132 kV,	X = 15%
Load:	60 MVA,	0.8 pf lagging at 124 kV	



Zone 3:-

$S_b = 100\,MVA$.

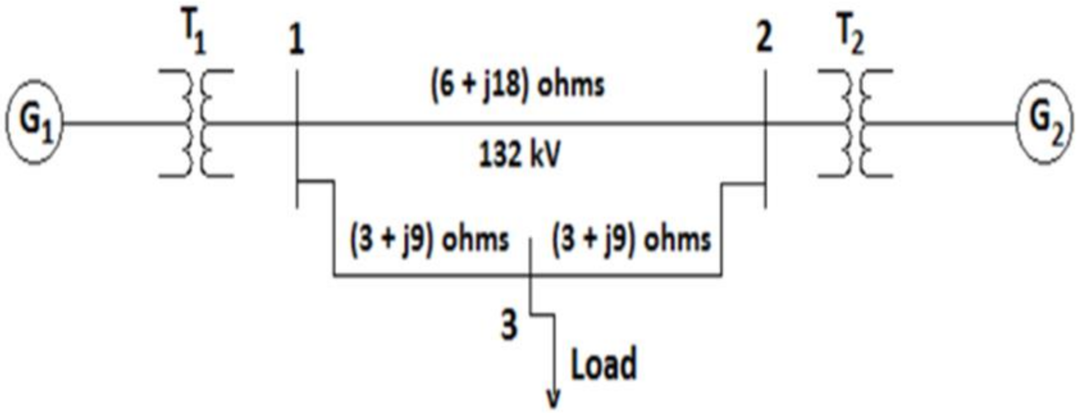
$$V_b = 100 \times \frac{13.8}{132} = 10.45\,KV.$$

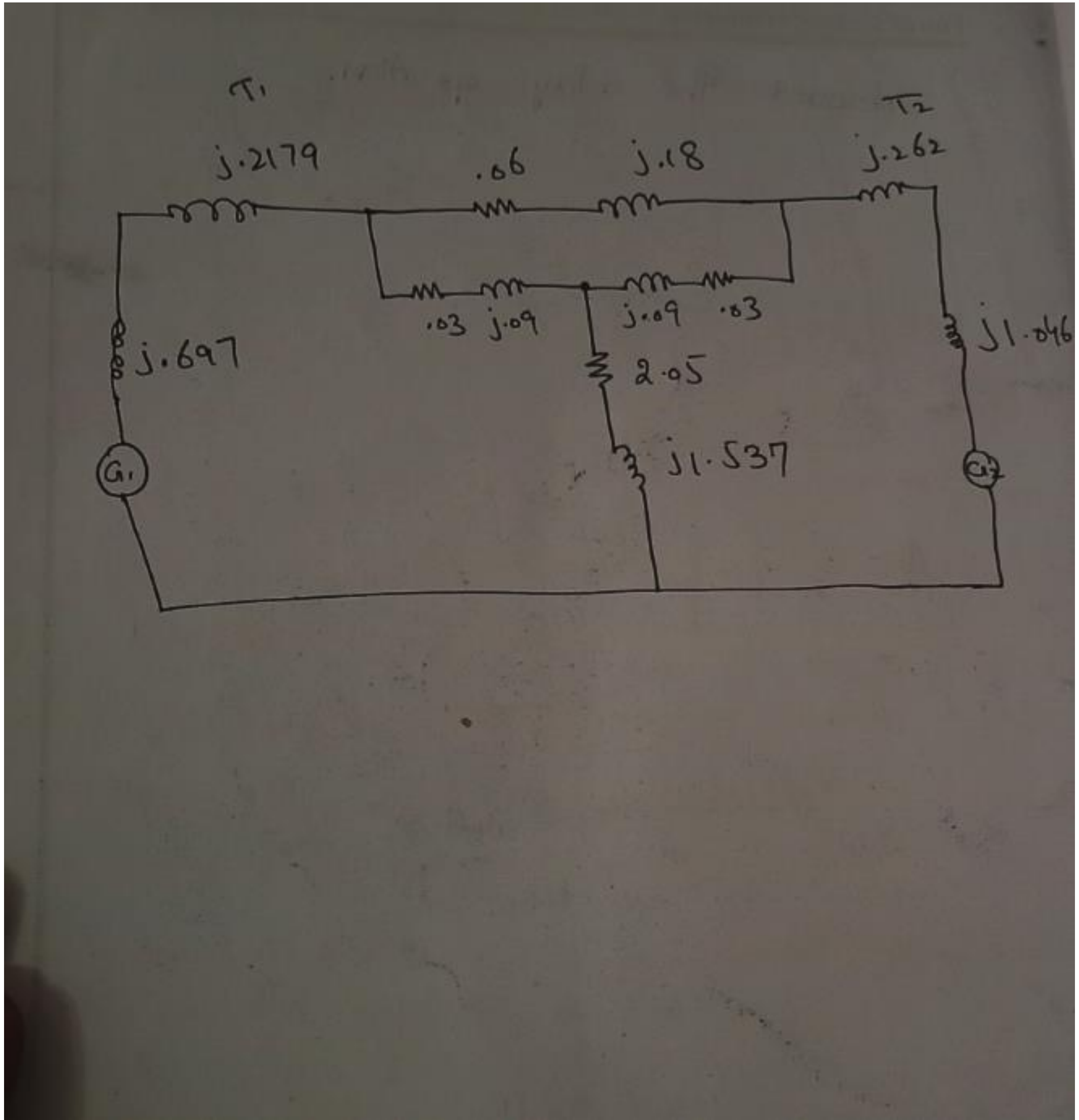
$$X_{G_2} = .15 \times \frac{13.8^2}{10.45^2} \times \frac{100}{25} = j1.046\,PU.$$

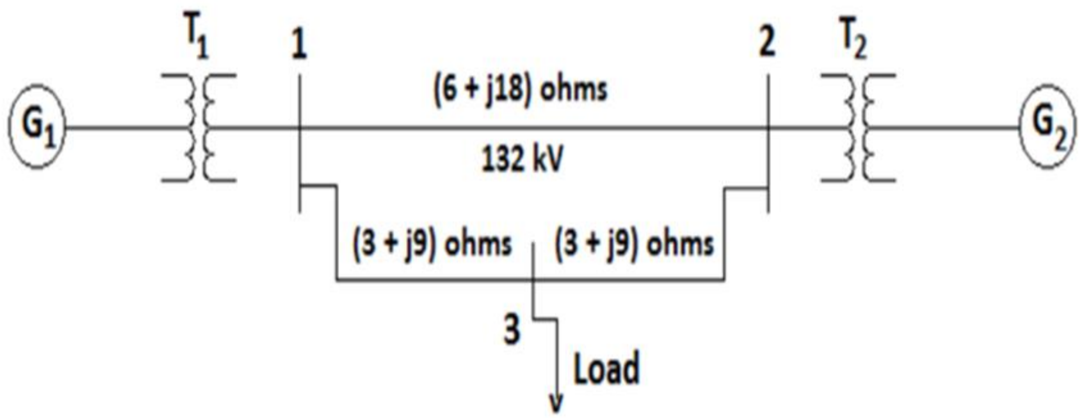
$$X_{T_2} = .15 \times \frac{13.8^2}{10.45^2} \times \frac{100}{100} = j.262\,PU$$

The Generators and Transformers are rated as follows:

Generator 1:	50 MVA,	12.2 kV,	X" = 0.20 pu
Generator 2:	25 MVA,	13.8 kV,	X" = 0.15 pu
Transformer 1:	80 MVA,	12.2/132 kV,	X = 10 %
Transformer 2:	100 MVA,	13.8/132 kV,	X = 15%
Load:	60 MVA,	0.8 pf lagging at 124 kV	




T1
j.2179
.06
j.18
T2
j.262
.03
j.09
j.09
.03
j.697
2.05
j1.046
j1.537
G1
G2


T1
1
2
T2
G1
G2
(6 + j18) ohms
132 kV
(3 + j9) ohms
(3 + j9) ohms
3
Load