Bus Admittance Matrix

Lec_3

Formulation of the Bus Admittance Matrix

The first step in developing the mathematical model describing the power flow in the network is the formulation of the bus admittance matrix.

The bus admittance matrix is an $n \times n$ matrix (where n is the number of buses in the system) constructed from the admittances of the equivalent circuit elements of the segments making up the power system.

Most system segments are represented by a combination of shunt elements (connected between a bus and the reference node) and series elements (connected between two system buses).

Node Analysis

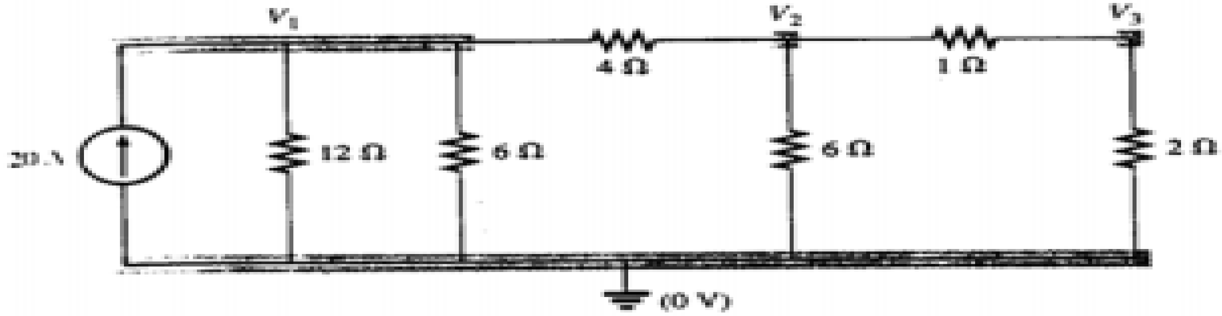


FIG. 8.58

Converting the voltage source to a current source and defining the nodes for the network of Fig. 8.57.

$$
\begin{aligned}
V_1&: \quad \left(\frac{1}{12\,\Omega}+\frac{1}{6\,\Omega}+\frac{1}{4\,\Omega}\right)V_1-\left(\frac{1}{4\,\Omega}\right)V_2+\quad 0 \quad = 20\text{ V}\\
V_2&: \quad \left(\frac{1}{4\,\Omega}+\frac{1}{6\,\Omega}+\frac{1}{1\,\Omega}\right)V_2-\left(\frac{1}{4\,\Omega}\right)V_1-\left(\frac{1}{1\,\Omega}\right)V_3=0\\
V_3&: \quad \left(\frac{1}{1\,\Omega}+\frac{1}{2\,\Omega}\right)V_3-\left(\frac{1}{1\,\Omega}\right)V_2+\quad 0 \quad =0
\end{aligned}
$$

and

$$
\begin{aligned}
0.5V_1 \quad &-0.25V_2 \quad +0 = 20\\
-0.25V_1 \quad &+\frac{17}{\ }V_2 \quad -1V_3 = 0
\end{aligned}
$$

$$\begin{bmatrix} \frac{1}{12}+\frac{1}{6}+\frac{1}{4} & -\frac{1}{4} & 0 \\ \frac{1}{4}+\frac{1}{6}+\frac{1}{1} & -\frac{1}{4} & -\frac{1}{1} \\ 0 & \frac{1}{1} & \frac{1}{1}+\frac{1}{2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$YV = I$$

$\Rightarrow$ Admittance matrix

3- Node Sys

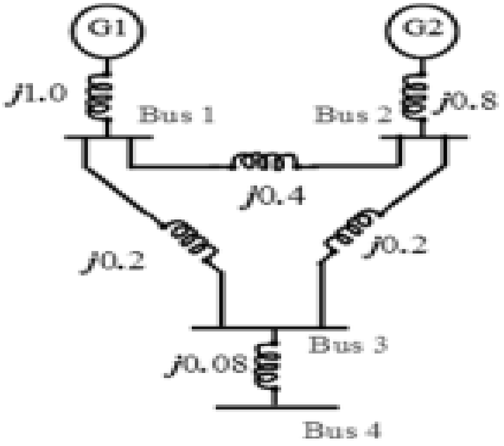
$$Y_{3\times 3} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

Bus
4- Node system.

$$Y_{4\times 4} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$$

Example:

Formulate the *bus admittance matrix* for the network shown in the Figure. The *Impedance diagram* of the system is as indicated. Shunt elements are ignored.



Impedance diagram

Solution:

The *node voltage method* is commonly used for the power system analysis. Where,

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Or

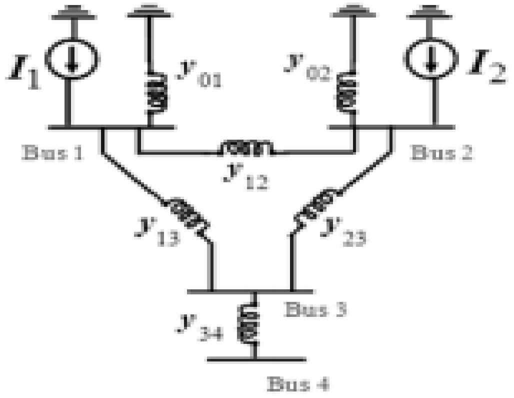
$$I_{bus} = [Y_{bus}]\, V_{bus}$$

The system can be represented in terms of its admittance elements as shown, where:

$$y_{ij} = \frac{1}{Z_{ij}}$$

$$y_{01} = \frac{1}{j1.0} = -j1.0 \qquad y_{12} = -j2.5$$

$$y_{02} = \frac{1}{j0.8} = -j1.25 \qquad y_{13} = y_{23} = -j5.0$$



Admittance diagram

Applying *KCL* at each node (bus), then

$$I_1 = (y_{01} + y_{12} + y_{13})V_1 - y_{12}V_2 - y_{13}V_3$$

$$I_2 = (y_{02} + y_{12} + y_{23})V_2 - y_{12}V_1 - y_{23}V_3$$

$$0 = (y_{31} + y_{32} + y_{34})V_3 - y_{31}V_1 - y_{32}V_2 - y_{34}V_4$$

$$0 = y_{34}V_4 - y_{34}V_3$$

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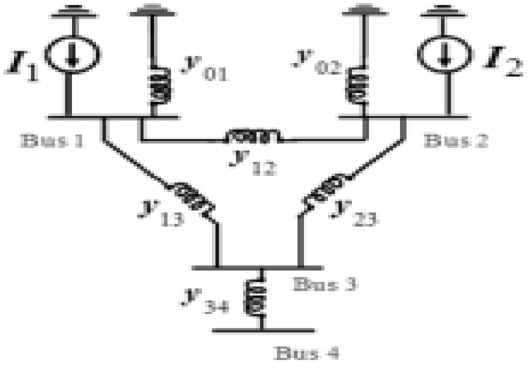
defined: $Y_{11} = y_{01} + y_{12} + y_{13}$ *and* $Y_{12} = Y_{21} = -y_{12}$ $Y_{13} = Y_{31} = -y_{13}$

$Y_{22} = y_{02} + y_{12} + y_{23}$ $Y_{23} = Y_{32} = -y_{23}$ $Y_{34} = Y_{43} = -y_{34}$

$Y_{33} = y_{31} + y_{32} + y_{34}$ $Y_{14} = Y_{41} = 0$ $Y_{42} = Y_{24} = 0$

$Y_{44} = y_{34}$



Then, the Node Voltage Equation is:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$$I_{bus} = [Y_{bus}]\, V_{bus} \qquad Or \qquad V_{bus} = [Y_{bus}^{-1}]\, I_{bus} = [Z_{bus}]\, I_{bus}$$

Substituting the values, then the bus admittance matrix of the network is:

$$Y_{bus} = \begin{bmatrix} -j8.5 & j2.5 & j5.0 & 0 \\ j2.5 & -j8.75 & j5.0 & 0 \\ j5.0 & j5.0 & -j22.5 & j12.5 \\ 0 & 0 & j12.5 & -j12.5 \end{bmatrix}$$

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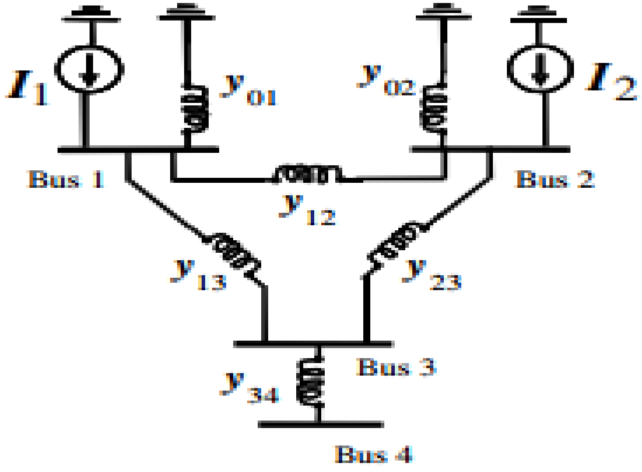
NOTE:

Formulation of the bus admittance matrix follows three simple rules:

1. **The admittance of elements connected between node *k* and *reference* is added to the *(k, k)* entry of the admittance matrix.**

2. **The admittance of elements connected between nodes *j* and *k* is added to the *(j, j)* and *(k, k)* entries of the admittance matrix.**

3. **The negative of the admittance is added to the *(j, k)* and *(k, j)* entries of the admittance matrix.**



$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} (y_{01}+y_{12}+y_{13}) & (-y_{12}) & (-y_{13}) & (0) \\ (-y_{21}) & (y_{02}+y_{21}+y_{23}) & (-y_{23}) & (0) \\ (-y_{31}) & (-y_{32}) & (y_{31}+y_{32}+y_{43}) & (-y_{34}) \\ (0) & (0) & (-y_{43}) & (y_{43}) \end{bmatrix}$$

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Example 7.1. The single-line diagram of a small power system is shown in Fig. 7.3. The corresponding reactance diagram, with reactances specified in per unit, is shown in Fig. 7.4. A generator with emf equal to $1.25\angle 0^\circ$ per unit is connected through a transformer to high-voltage node 3, while a motor with internal voltage equal to $0.85\angle -45^\circ$ is similarly connected to node 4. Develop the nodal

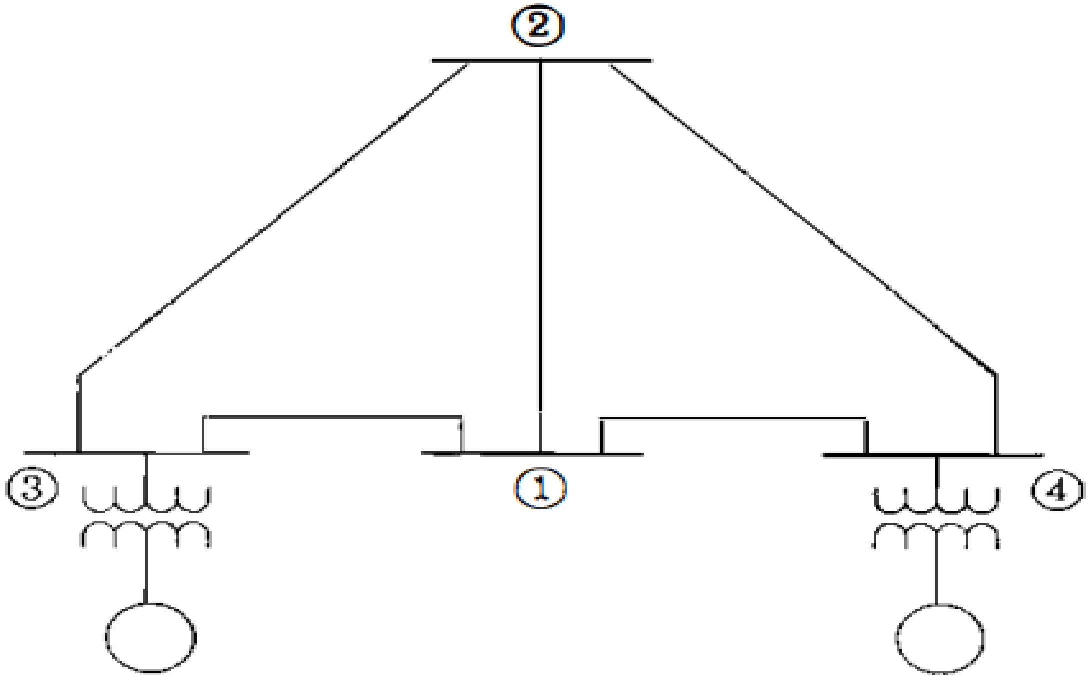

FIGURE 7.3
Single-line diagram of the four-bus system of Example 7.1. Reference node is not shown.

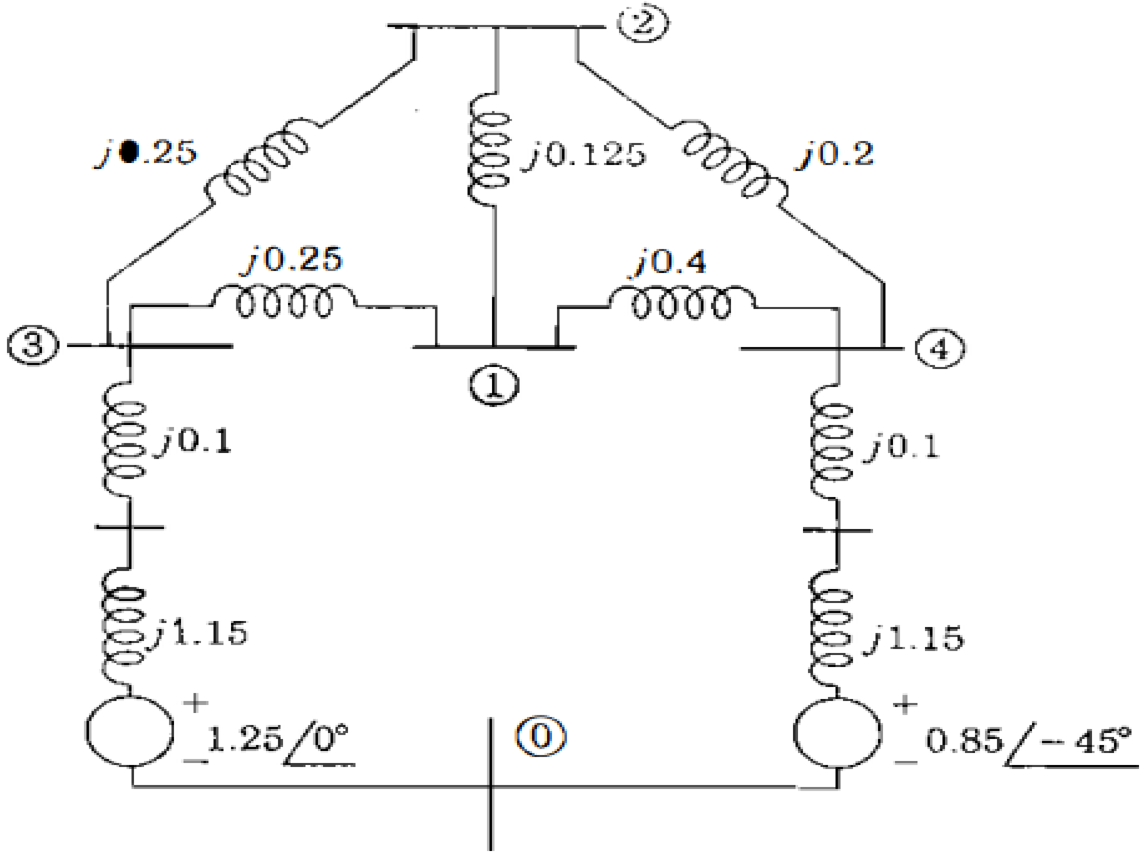

FIGURE 7.4
Reactance diagram for Fig. 7.3. Node 0 is reference, reactances and voltages are in per unit.

$$Y_{21} = Y_{21} = -y_{12} = \frac{-1}{z_{12}} = \frac{-1}{j.125} = j8$$

$$Y_{13} = Y_{31} = -y_{13} = -\frac{1}{z_{13}} = -\frac{1}{j.25} = j4.0$$

$$Y_{14} = Y_{41} = -y_{14} = -\frac{1}{z_{14}} = -\frac{1}{j.4} = j2.5$$

$$Y_{11} = y_{12} + y_{13} + y_{14}$$

$$Y_{22} = y_{21} + y_{23} + y_{24}$$

$$Y_{44} = y_{04} + y_{41} + y_{42}$$

$$\begin{bmatrix} -j14.5 & j8.0 & j4.0 & j2.5 \\ j8.0 & -j17.0 & j4.0 & j5.0 \\ j4.0 & j4.0 & -j8.8 & 0.0 \\ j2.5 & j5.0 & 0.0 & -j8.3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1.00\angle -90^\circ \\ 0.68\angle -135^\circ \end{bmatrix}$$

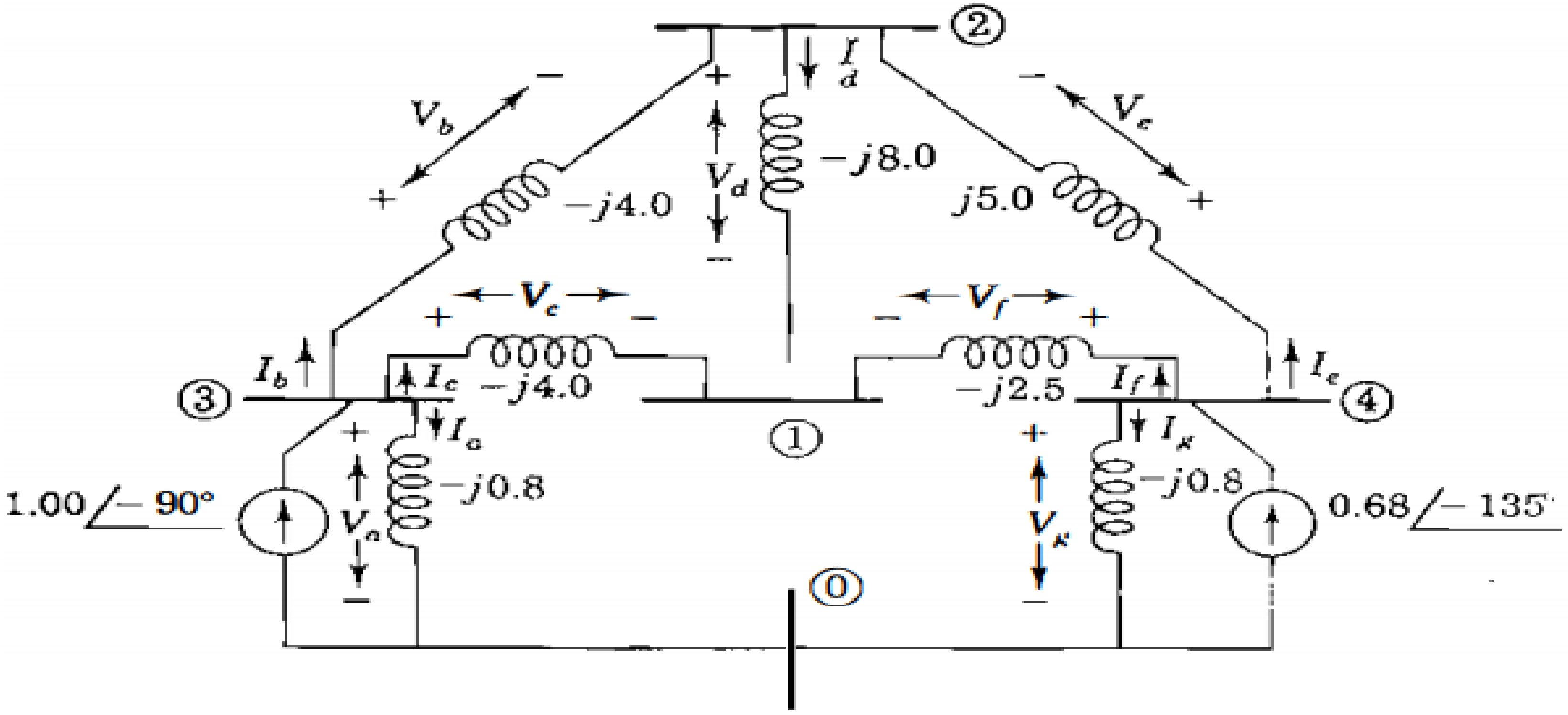

FIGURE 7.5
Per-unit admittance diagram for Fig. 7.4 with current sources replacing voltage sources. Branch names *a* to *g* correspond to the subscripts of branch voltages and currents.

Kron Reduction Method

- Reduced some buses which are not in interest of electric utility company.
- Some reduction methods are used to reduced some buses from system.
- It will help to analyze the large system at lower scale.

$$
\begin{array}{c@{\,}c}
 & \begin{array}{cccc} ① & ② & ③ & ④ \end{array} \\
\begin{array}{c} ① \\ ② \\ ③ \\ ④ \end{array} &
\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}
\end{array}
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} \tag{7.65}
$$

$$
\begin{array}{c@{\,}c}
 & \begin{array}{ccc} ② & ③ & ④ \end{array} \\
\begin{array}{c} ② \\ ③ \\ ④ \end{array} &
\begin{bmatrix} Y_{22}^{(1)} & Y_{23}^{(1)} & Y_{24}^{(1)} \\ Y_{32}^{(1)} & Y_{33}^{(1)} & Y_{34}^{(1)} \\ Y_{42}^{(1)} & Y_{43}^{(1)} & Y_{44}^{(1)} \end{bmatrix}
\end{array}
\begin{bmatrix} V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} I_2 \\ I_3 \\ I_4 \end{bmatrix} \tag{7.66}
$$

$$Y_{jk(\text{new})} = Y_{jk} - \frac{Y_{jp}Y_{pk}}{Y_{pp}} \tag{7.67}$$

where j and k take on all the integer values from 1 to N except p since row p and column p are to be eliminated. The subscript (new) distinguishes the elements in the new $\mathbf{Y}_{\text{bus}}$ of dimension $(N-1) \times (N-1)$ from the elements in the original $\mathbf{Y}_{\text{bus}}$.

Example 7.8. Using Y_{22} as the initial pivot, eliminate node 2 and the corresponding voltage V_2 from the 4×4 system of Example 7.7

$$
\begin{array}{cc}
 & \begin{array}{cccc} \quad 1 \quad & \quad 2 \quad & \quad 3 \quad & \quad 4 \quad \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} &
\begin{bmatrix}
-j16.75 & j11.75 & \boxed{j2.50} & j2.50 \\
\boxed{j11.75} & -j19.25 & j2.50 & j5.00 \\
j2.50 & j2.50 & -j5.80 & 0.00 \\
j2.50 & j5.00 & 0.00 & -j8.30
\end{bmatrix}
\end{array}
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 1.00\angle -90^\circ \\ 0.68\angle -135^\circ \end{bmatrix}
$$

Solution. The pivot Y_{22} equals $-j19.25$. With p set equal to 2 in Eq. (7.67), we may eliminate row 2 and column 2 from $\mathbf{Y}_{bus}$ of Example 7.7 to obtain the new row 1 elements

$$Y_{11(new)} = Y_{11} - \frac{Y_{12}Y_{21}}{Y_{22}} = -j16.75 - \frac{(j11.75)(j11.75)}{-j19.25} = -j9.57792$$

$$Y_{13(new)} = Y_{13} - \frac{Y_{12}Y_{23}}{Y_{22}} = j2.50 - \frac{(j11.75)(j2.50)}{-j19.25} = j4.02597$$

$$Y_{14(new)} = Y_{14} - \frac{Y_{12}Y_{24}}{Y_{22}} = j2.50 - \frac{(j11.75)(j5.00)}{-j19.25} = j5.55195$$

$$Y_{jk(new)} = Y_{jk} - \frac{Y_{jp} \cdot Y_{pk}}{Y_{pp}}$$

(1st) j = 1, 2, 3

(2nd). K = 1, 2, 3.

P = row and column to be eliminated.

2nd Bus is to be eliminated.

$$\underset{jK}{Y_{11\,new}} = \underset{jK}{Y_{11}} - \frac{\underset{jP}{Y_{12}} \cdot \underset{PK}{Y_{21}}}{\underset{PP}{Y_{22}}}$$

$$\underset{jK}{Y_{13\,new}} = \underset{jK}{Y_{13}} - \frac{\underset{jP}{Y_{12}} \; \underset{PK}{Y_{23}}}{\underset{PP}{Y_{22}}}$$

$$Y_{14new} = Y_{14} - \frac{Y_{12} \, Y_{24}}{Y_{22}}$$

$$Y_{33new} = Y_{33} - \frac{Y_{32} \cdot Y_{23}}{Y_{22}}$$

$$Y_{44new} = Y_{44} - \frac{Y_{42} \cdot Y_{24}}{Y_{22}}.$$

$$
\begin{array}{c}
\\
1 \\ 3 \\ 4
\end{array}
\begin{array}{c}
\begin{array}{ccc} \quad 1 \quad & \quad 3 \quad & \quad 4 \quad \end{array} \\
\begin{bmatrix}
-j9.57791 & j4.02597 & j5.55195 \\
j4.02597 & -j5.47532 & j0.64935 \\
j5.55195 & j0.64935 & -j7.00130
\end{bmatrix}
\end{array}
\begin{array}{c}
\\
\begin{bmatrix} V_1 \\ V_3 \\ V_4 \end{bmatrix}
\end{array}
\begin{array}{c}
\\
=
\end{array}
\begin{array}{c}
\\
\begin{bmatrix} 0 \\ 1.00\angle -90^\circ \\ 0.68\angle -135^\circ \end{bmatrix}
\end{array}
$$

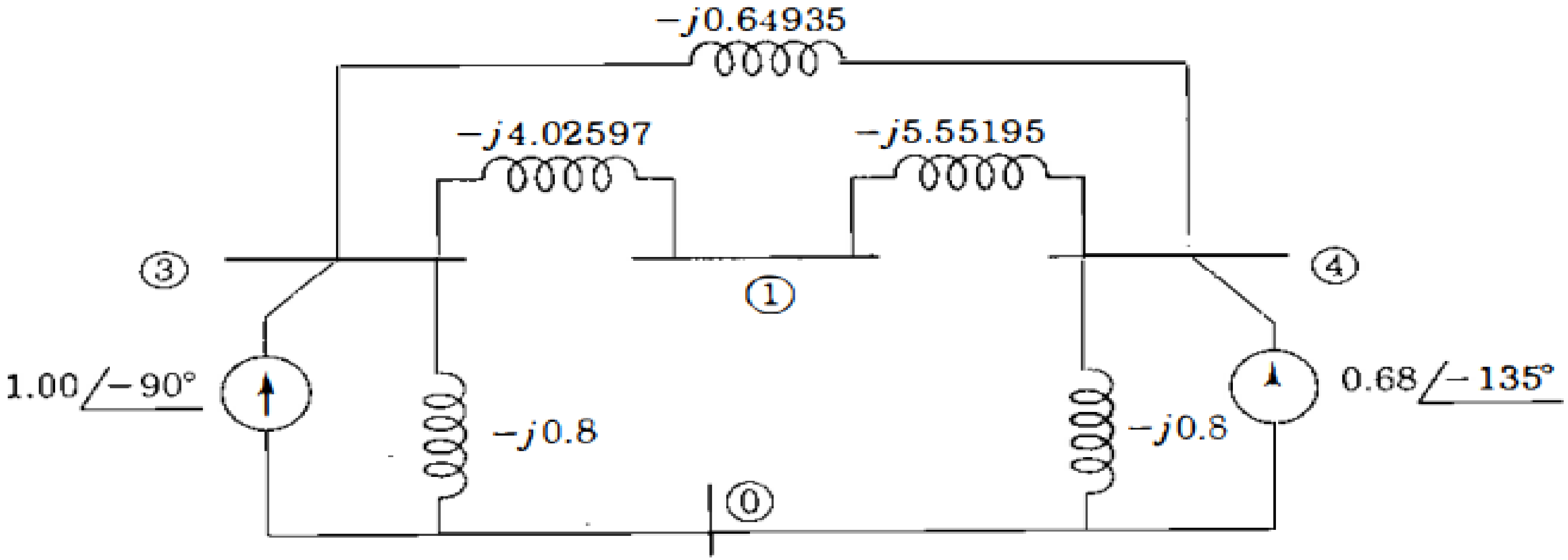

FIGURE 7.17
The Kron-reduced network of Example 7.8.