EE-220 Signals and Systems

Session 2019

Week 5

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Last Week

- Convolution
  - CT Convolution, Examples
  - Review
- System Properties
- Unit Step Response

Linear Constant-Coefficient Differential Equations

- Let us consider a first-order differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t),$$

- We must solve the differential equation.
- To solve a differential equation, we must specify one or more auxiliary conditions

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Linear Constant-Coefficient Differential Equations

$$x(t) = Ke^{3t}u(t),$$

$$\frac{dy(t)}{dt} + 2y(t) = x(t),$$

$$y(t) = y_p(t) + y_h(t),$$

$$\frac{dy(t)}{dt} + 2y(t) = x(t), \qquad \frac{dy(t)}{dt} + 2y(t) = 0.$$

Linear Constant-Coefficient Differential Equations

- A general Nth-order linear constant-coefficient differential equation is given by

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}.$$

- Homogeneous Solution $\displaystyle\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = 0.$

- For N=0 $\displaystyle y(t) = \frac{1}{a_0}\sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}.$

Finite Impulse Response (FIR) System

- Nth-order linear constant-coefficient difference equation

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]. \qquad y[n] = \frac{1}{a_0}\left\{\sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k y[n-k]\right\}.$$

- Homogeneous Solution $\displaystyle\sum_{k=0}^{N} a_k y[n-k] = 0.$

- N=0

$$y[n] = \sum_{k=0}^{M}\left(\frac{b_k}{a_0}\right)x[n-k]. \qquad h[n] = \begin{cases} \frac{b_n}{a_0}, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}.$$

Infinite Impulse Response (IIR) Systems

$$y[n] - \frac{1}{2}y[n-1] = x[n]. \quad x[n] = K\delta[n].$$

$$y[n] = x[n] + \frac{1}{2}y[n-1].$$

- In this case, since $x[n] = 0$ for $n \le -1$, the condition of initial rest implies that $y[n] = 0$ for $n \le -1$, so that we have as an initial condition $y[-1] = 0$. Starting from this initial condition, we can solve for successive values of $y[n]$

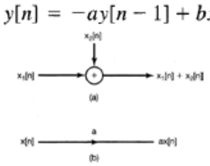
$$y[0] = x[0] + \frac{1}{2}y[-1] = K,$$
$$y[1] = x[1] + \frac{1}{2}y[0] = \frac{1}{2}K,$$
$$y[2] = x[2] + \frac{1}{2}y[1] = \left(\frac{1}{2}\right)^2 K,$$
$$\vdots$$
$$y[n] = x[n] + \frac{1}{2}y[n-1] = \left(\frac{1}{2}\right)^n K.$$

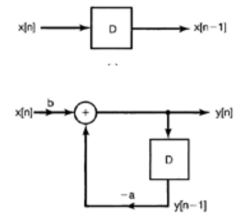
$$h[n] = \left(\frac{1}{2}\right)^n u[n].$$

Block Diagram Representations of First-Order Systems

$$y[n] + ay[n-1] = bx[n].$$

$$y[n] = -ay[n-1] + bx[n].$$

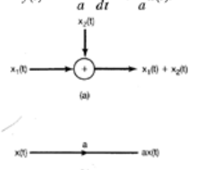


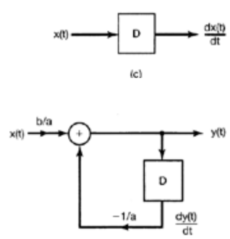


Block Diagram Representations of First-Order Systems

$$\frac{dy(t)}{dt} + ay(t) = bx(t).$$

$$y(t) = -\frac{1}{a}\frac{dy(t)}{dt} + \frac{b}{a}x(t).$$

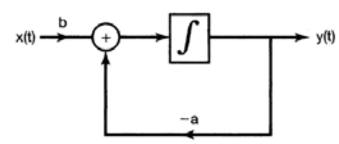




Block Diagram Representations of First-Order Systems

$$\frac{dy(t)}{dt} + ay(t) = bx(t).$$

$$y(t) = \int_{-\infty}^{t} \left[bx(\tau) - ay(\tau)\right] d\tau.$$



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Chapter 3

FOURIER SERIES REPRESENTATION OF PERIODIC SIGNALS

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THE RESPONSE OF LTI SYSTEMS TO COMPLEX EXPONENTIALS

- It is advantageous in the study of LTI systems to represent signals as linear combinations of basic signals that possess the following two properties:
  - The set of basic signals can be used to construct a broad and useful class of signals.
  - The response of an LTI system to each signal should be simple enough in structure to provide us with a convenient representation for the response of the system to any signal constructed as a linear combination of the basic signals.
- Complex exponentials have above properties e^{st} and z^n
  - c and n are complex.

THE RESPONSE OF LTI SYSTEMS TO COMPLEX EXPONENTIALS

- The importance of complex exponentials in the study of LTI systems stems from the fact that the response of an LTI system to a complex exponential input is the same complex exponential with only a change in amplitude;

$$\text{continuous time: } e^{st} \longrightarrow H(s)e^{st},$$
$$\text{discrete time: } z^{n} \longrightarrow H(z)z^{n},$$

- A signal for which the system output is a (possibly complex) constant times the input is referred to as an ***eigenfunction*** of the system, and the amplitude factor is referred to as the system's ***eigenvalue***.

CT Eigenvalue

$$y(t) = \int_{-\infty}^{+\infty} h(\tau)x(t-\tau)\,d\tau$$
$$= \int_{-\infty}^{+\infty} h(\tau)e^{s(t-\tau)}\,d\tau.$$

$$y(t) = e^{st}\int_{-\infty}^{+\infty} h(\tau)e^{-s\tau}\,d\tau.$$

$$H(s) = \int_{-\infty}^{+\infty} h(\tau)e^{-s\tau}\,d\tau.$$

$$y(t) = H(s)e^{st},$$

DT Eigenvalue

$$x[n] = z^{n},$$

$$y[n] = \sum_{k=-\infty}^{+\infty} h[k]x[n-k]$$
$$= \sum_{k=-\infty}^{+\infty} h[k]z^{n-k} = z^{n}\sum_{k=-\infty}^{+\infty} h[k]z^{-k}.$$

$$y[n] = H(z)z^{n},$$

$$H(z) = \sum_{k=-\infty}^{+\infty} h[k]z^{-k}.$$

Eigenfunctions with Linearity

- Let x(t) correspond to a linear combination of three complex exponentials; that is,

$$x(t) = a_1e^{s_1t} + a_2e^{s_2t} + a_3e^{s_3t}.$$

$$a_1e^{s_1t} \longrightarrow a_1H(s_1)e^{s_1t},$$
$$a_2e^{s_2t} \longrightarrow a_2H(s_2)e^{s_2t},$$
$$a_3e^{s_3t} \longrightarrow a_3H(s_3)e^{s_3t},$$

$$y(t) = a_1H(s_1)e^{s_1t} + a_2H(s_2)e^{s_2t} + a_3H(s_3)e^{s_3t}.$$

$$x(t) = \sum_k a_ke^{s_kt},$$
$$y(t) = \sum_k a_kH(s_k)e^{s_kt}.$$

$$x[n] = \sum_k a_kz_k^n,$$
$$y[n] = \sum_k a_kH(z_k)z_k^n.$$

Example 3.1

$$y(t) = x(t-3).$$

$$x(t) = e^{j2t}$$

$$y(t) = e^{j2(t-3)} = e^{-j6}e^{j2t}.$$

$$H(s) = \int_{-\infty}^{+\infty} \delta(\tau-3)e^{-s\tau}\,d\tau = e^{-3s},$$

$$H(j2) = e^{-j6}.$$

$$y(t) = \cos(4(t-3)) + \cos(7(t-3)).$$

$$x(t) = \tfrac{1}{2}e^{j4t} + \tfrac{1}{2}e^{-j4t} + \tfrac{1}{2}e^{j7t} + \tfrac{1}{2}\,e^{-j7t}.$$

$$y(t) = \tfrac{1}{2}e^{-j12}e^{j4t} + \tfrac{1}{2}e^{j12}e^{-j4t} + \tfrac{1}{2}e^{-j21}e^{j7t} + \tfrac{1}{2}e^{j21}e^{-j7t},$$

$$y(t) = \tfrac{1}{2}e^{j4(t-3)} + \tfrac{1}{2}e^{-j4(t-3)} + \tfrac{1}{2}e^{j7(t-3)} + \tfrac{1}{2}e^{-j7(t-3)}$$
$$= \cos(4(t-3)) + \cos(7(t-3)).$$

CONTINUOUS-TIME PERIODIC SIGNALS

- A signal is periodic if, for some positive value of T,

$$x(t) = x(t+T) \quad \text{for all } t.$$

$$x(t) = \cos\omega_0t \qquad x(t) = e^{j\omega_0t}.$$

$$\phi_k(t) = e^{jk\omega_0t} = e^{jk(2\pi/T)t}, \quad k = 0, \pm1, \pm2, \ldots.$$

- Each of these signals has a fundamental frequency that is a multiple of w_0, and therefore, each is periodic with period T

$$x(t) = \sum_{k=-\infty}^{+\infty} a_ke^{jk\omega_0t} = \sum_{k=-\infty}^{+\infty} a_ke^{jk(2\pi/T)t}$$

Example 3.2

$$x(t) = \sum_{k=-3}^{+3} a_k e^{jk2\pi t},$$

$$a_0 = 1,$$

$$a_1 = a_{-1} = \frac{1}{4},$$

$$a_2 = a_{-2} = \frac{1}{2},$$

$$a_3 = a_{-3} = \frac{1}{3}.$$

$$x(t) = 1 + \frac{1}{4}(e^{j2\pi t} + e^{-j2\pi t}) + \frac{1}{2}(e^{j4\pi t} + e^{-j4\pi t}) + \frac{1}{3}(e^{j6\pi t} + e^{-j6\pi t}).$$

$$x(t) = 1 + \frac{1}{2}\cos 2\pi t + \cos 4\pi t + \frac{2}{3}\cos 6\pi t.$$