WELCOME BACK IN NEW SEMESTER!

Instructor: Ali Raza

"Never stop doing your best just because someone doesn't give you credit."

EE-220 Signals and Systems

Session 2019

Week 1

UNIVERSITY OF ENGINEERING AND TECHNOLOGY , LAHORE.

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Teacher's Introduction

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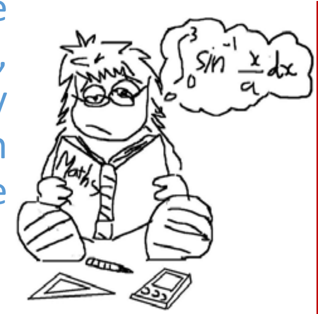
Antenna and Communication Lab

MS in Electrical Engineering – RF and Microwaves, NUST-SEECS

Objective

- To prepare the student for core courses in communication, control and signal processing by giving the student a thorough working knowledge of these techniques.



CLOs

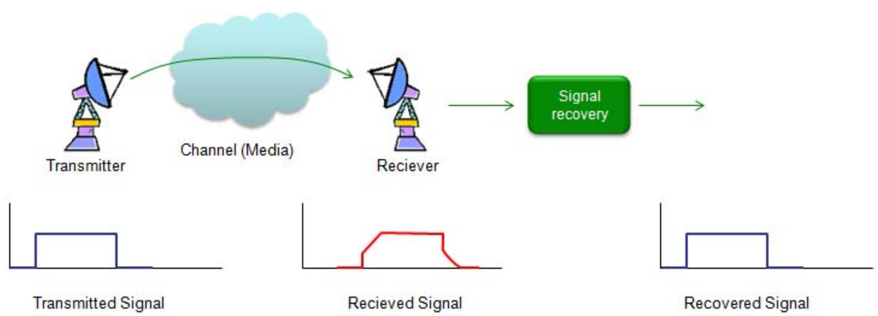
- Define continuous-time signals, discrete-time sequences, corresponding Transforms, their properties, their frequency domain behavior and LTI systems (C1)
- Analyze signals and systems using the Fourier and z transforms (C4)
- Analyze and design LTI systems via software (C3)

Grading Policy

- **Quizzes and Assignments = 30%**
- **Mid Exam = 30%**
- **Final Exam = 40%**

- **Tentatively** there will be 2 quizzes and 2 assignments

Why should we study Signals and Systems ????



Course Books

REQUIRED:

- Signals and Systems by A. V. Oppenheim, A. S. Willsky and S. H. Nawab, 2nd Edition.

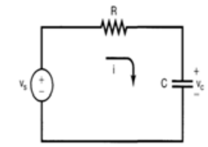
OPTIONAL:

- Digital Signal Processing – Principles, Algorithms and Applications by J. G. Proakis and D. G. Manolakis, 4th Edition, Prentice Hall, 2006.
- Fundamentals of Signals and Systems by M. J. Roberts, McGraw Hill, 2007.
- Linear Systems and Signals by B. P. Lathi, 2nd Edition, Oxford, 2004.

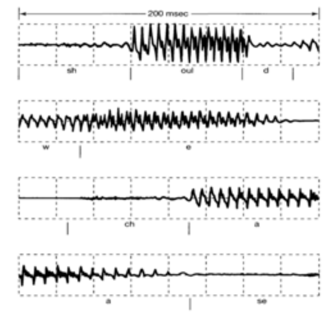
What is a Signal?

- A signal is a pattern of variation of some form
- It can be thought of as variations in a waveform or a sequence that conveys some information
- Electrical signals
  - Voltages and currents in a circuit
- Acoustic signals
  - Sound over time
- Mechanical signals
  - Velocity of a car over time
- Image signals
  - Intensity level of a pixel of image over time/space

Example

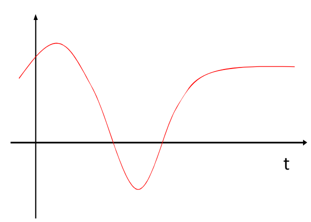






Signal Representation

- Mathematically, signals are represented as a function of one or more independent variables e.g. time, space
- For instance a black and white video signal intensity is dependent on x, y coordinates and time t, f(x,y,t)
- In this course, we shall be exclusively concerned with signals that are a function of a single variable: time

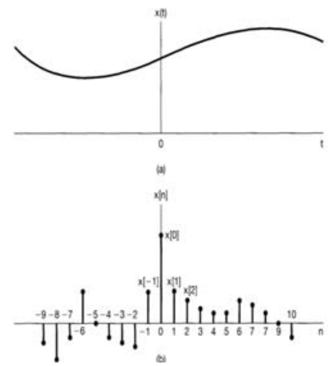


Signal Classifications

- There are two basic types of signals
  - Continuous time signals
  - Discrete-time signals.

Signal Classifications

- **Continuous-Time Signals**
  - Most signals in the real world are continuous time as the scale is infinitesimally fine, for example voltage, velocity...
  - Denoted by $x(t)$, where the time interval may be bounded (finite) or infinite
- **Discrete-Time Signals**
  - Some real world and many digital signals are discrete time as they are sampled, for example pixels, daily stock price (anything that a digital computer processes)
  - Denoted by $x[n]$, where n is an integer value that varies discretely
- **Sampled continuous signal**
  - $x[n] = x(nt)$, where n is sample time

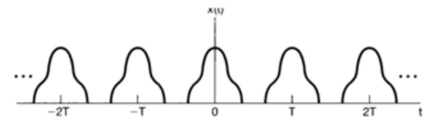


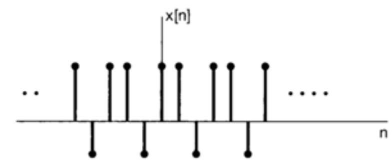
Periodic Signals

- A periodic continuous-time signal $x(t)$ has the property that there is a positive value of T for which

$$x(t) = x(t + T)$$

- In other words, a periodic signal has the property that it is unchanged by a time shift of T

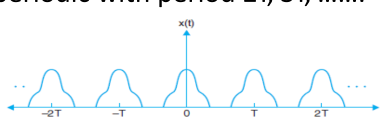




Periodic Signals

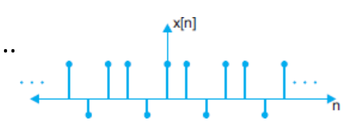
- **Continuous Time Periodic signals**: A periodic signal has the property that it is unchanged by a time shift of T

  $x(t) = x(t + T)$, for all value of t.
- Then x(t) is periodic with time period T.
- The fundamental period T_0 is the smallest positive value of T for which the above equation holds.
- Thus x(t) is also periodic with period 2T, 3T, ...



Periodic Signals

- **Discrete Time Periodic Signals**: A periodic discrete-time signal x[n] has the property that for a positive integer N,
- $x[n] = x[n + N]$, for all values of n.
- The discrete time signal x[n] is periodic with period N if it is unchanged by a time shift of N.
- The fundamental period N_0 is the smallest positive value of N for which the above equation holds.
- Thus x[n] is periodic with period 2N, 3N, ...



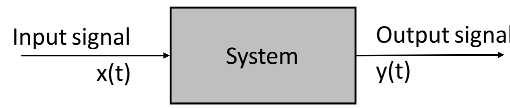
What is a System?

- Systems processes input signals to produce output signals.

- Examples:
  - A circuit involving a capacitor can be viewed as a system that transforms the source voltage (signal) to the voltage (signal) across the capacitor
  - A CD player takes the signal on the CD and transforms it into a signal sent to the loud speaker
  - A communication system is generally composed of three sub-systems, the transmitter, the channel and the receiver. The channel typically attenuates and adds noise to the transmitted signal which must be processed by the receiver

How is a System Represented?

- A system takes a signal as an input and transforms it into another signal



- In a very broad sense, a system can be represented as the ratio of the output signal over the input signal
  - That way, when we "multiply" the system by the input signal, we get the output signal
  - This concept will be firmed up in the coming weeks

Signal Energy and Power

- We are more concerns with the signal strength (Energy/Power) of the signal.
- If v(t) and i(t) are, respectively, the voltage and current across a resistor with resistance R, then the instantaneous power is

$$p(t) = v(t)i(t) = \frac{1}{R}v^2(t).$$

- The total *energy* expended over the time interval $t_1 < t < t_2$ is

$$\int_{t_1}^{t_2} p(t)\,dt = \int_{t_1}^{t_2} \frac{1}{R}v^2(t)\,dt, \qquad \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} p(t)\,dt = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \frac{1}{R}v^2(t)\,dt.$$

Signal Energy and Power

- In this case, the total energy over the time interval t1 < t < t2 in a continuous-time signal x(t) is defined as

$$\int_{t_1}^{t_2} |x(t)|^2 dt, \qquad \sum_{n=n_1}^{n_2} |x[n]|^2,$$

- Power is calculated by dividing total time **t2-t1**, or **n2-n1+1**

Signal Energy and Power

- Furthermore, in many systems we will be interested in examining power and energy in signals over an infinite time interval,

$$E_\infty \triangleq \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2\,dt = \int_{-\infty}^{+\infty} |x(t)|^2\,dt, \qquad E_\infty \triangleq \lim_{N\to\infty} \sum_{n=-N}^{+N} |x[n]|^2 = \sum_{n=-\infty}^{+\infty} |x[n]|^2.$$

$$P_\infty \triangleq \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\,dt$$

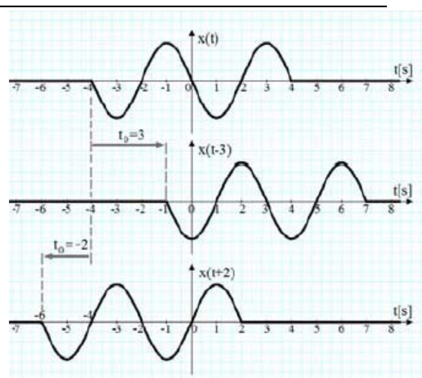
$$P_\infty \triangleq \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2$$

Signal Transformation

- Modification in time axis, independent variable
  - Time shifting
  - Time scaling
  - Time reversal

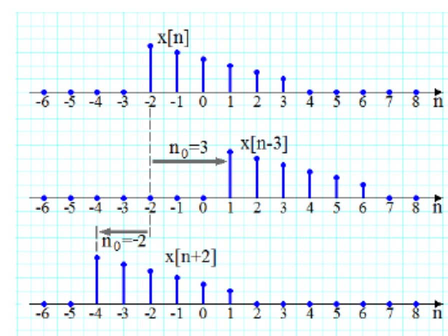
Signal Transformations Continuous Time

- **Time Shifting** (Shifting the Independent variable)
- Delayed/advanced form of original signal
- $x(t-t_o)$: Right Shift if $t_o > 0$, Left Shift if $t_o < 0$.



Signal Transformations Discrete Time

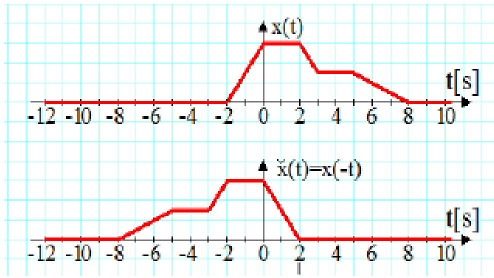
- **Time Shifting** (Shifting the Independent variable)
- $x[n-n_o]$: Right Shift if $n_o > 0$, Left Shift if $n_o < 0$.



Signal Transformations

- **Time Reversal** (flipping the signal around t=0)

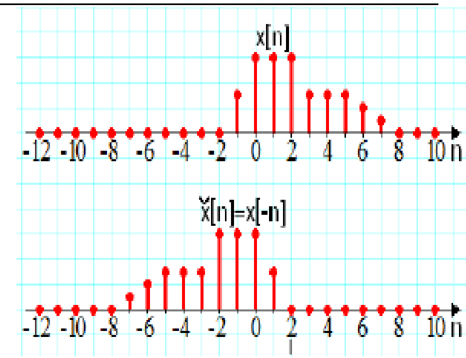
a) x(t)
b) x(-t)



Signal Transformations

- **Time Reversal** (flipping the signal around n=0)

a) x[n]
b) x[-n]

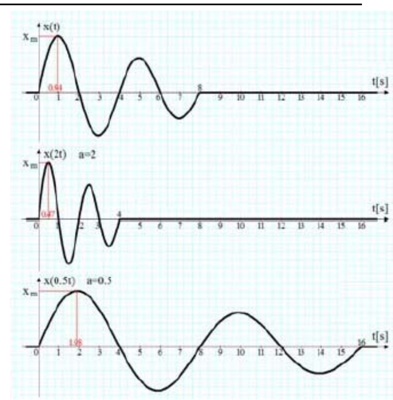


Signal Transformations

- **Time Scaling** (Scaling the Independent variable)
- Compresses or expands the signal by multiplying the time variable by a constant

- y(t) = x(at):

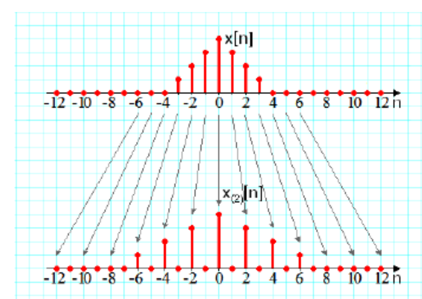
1. a = 1
2. a = 2
3. a = ½ = 0.5



Signal Transformations

- **Time Scaling** (Scaling the Independent variable)

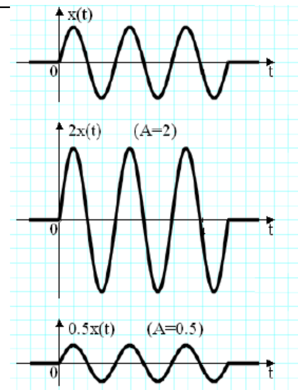
$$x_{(k)}[n] = \begin{cases} x\left[\dfrac{n}{k}\right], & \text{if } k \text{ divides } n \\ 0, & \text{otherwise} \end{cases}$$



Signal Transformations

- **Weighting** (Scaling the dependent variable)

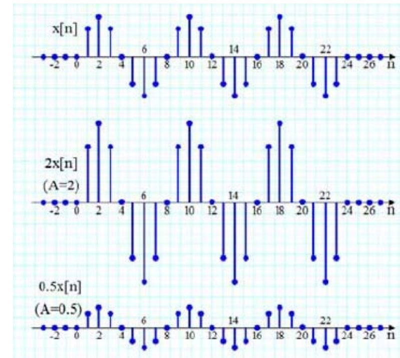
a) Amplification
b) Attenuation



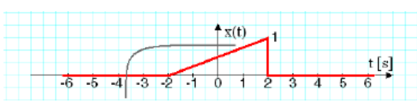
Signal Transformations

- **Weighting** (Scaling the dependent variable)

a) Amplification
b) Attenuation



Example – Signal Transformation



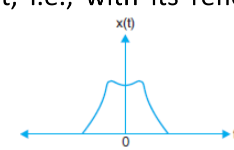
$$x(t) \longrightarrow 2x(-2t-2)$$

- **Remember: Shift before you scale or invert !!**
- **Also solve Examples 1.1, 1.2 and 1.3 for practice**

Even Signals

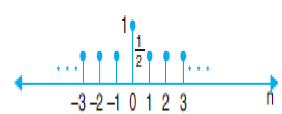
- A continuous signal f(t) is referred to as an even signal if it is identical to its time-reversed counterpart, i.e., with its reflection about the origin.

$$f(t) = f(-t); \quad \text{for all } t$$



- The discrete signal f[n] is said to be even if

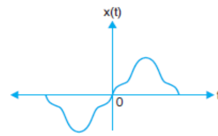
$$f[n] = f[-n]; \quad \text{for all } n$$



Odd Signals

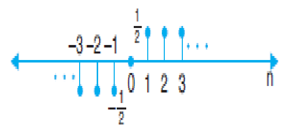
- A continuous signal f(t) is referred to as an odd signal if it is not identical to its time-reversed counterpart, as shown below:

$$f(-t) = -f(t); \quad \text{for all } t$$



- It may be noted that an odd continuous time signal will be zero at origin, i.e., f(0) = 0 at t = 0.
- The signal f[n] is said to be odd if:

$$f[-n] = -f[n]; \quad \text{for all } n$$



Even and Odd Signal Decomposition

- A signal can be decomposed into its even and odd components.
- Decomposition of continuous signal f(t) can be done as:

$$f(t) = f_e(t) + f_o(t)$$

- Here, $f_e(t)$ is the even and $f_o(t)$ is the odd component of continuous signal f(t).
- Obviously, the even function has the property $f_e(-t) = f_e(t)$
- And the odd function has the property $f_o(-t) = -f_o(t)$
- Replacing t by –t in the expression of f(t), we get:

$$f(-t) = f_e(-t) + f_o(-t) = f_e(t) - f_o(t)$$

- Solving from the expression of f(t) and f(–t), we get:

$$f_e(t) = \frac{1}{2}[f(t) + f(-t)] \quad \text{and} \quad f_o(t) = \frac{1}{2}[f(t) - f(-t)]$$

Even and Odd Signal Decomposition

- Decomposition of discrete signal f[n] can be done as in the case of continuous time signal as follows:

$$f[n] = f_e[n] + f_o[n]$$

- Here, $f_e[n]$ is the even and $f_o[n]$ is the odd component of discrete signal f[n].
- Obviously, the even function as usual has the property: $f_e[-n] = f_e[n]$
- And the odd function as usual has the property: $f_o[-n] = -f_o[n]$
- Replacing n by –n in the expression of f[n], we get:

$$f[-n] = f_e[-n] + f_o[-n] = f_e[n] - f_o[n]$$

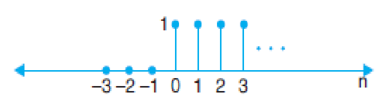
- Solving from the expression of f[n] and f[–n], we get:

$$f_e[n] = \frac{1}{2}[f[n] + f[-n]] \quad \text{and} \quad f_o[n] = \frac{1}{2}[f[n] - f[-n]]$$

Signal Decomposition - Example

- Find the Even and Odd signal components of the signal below:

$$x[n] = \begin{cases} 1, & n > 0 \\ 0, & n < 0 \end{cases}$$



Continuous-Time Exponential and Sinusoidal Signals

$$z = re^{j\theta} \qquad z = x + jy$$

$$r = |z| \qquad |z| = \sqrt{x^2 + y^2}$$

$$x = r\cos\theta \quad y = r\sin\theta \qquad \theta = \tan^{-1}(y/x)$$

$$e^{j\theta} = \cos\theta + j\sin\theta \qquad |e^{j\theta}| = 1$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \qquad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Continuous-Time Exponential and Sinusoidal Signals

- The continuous-time *complex exponential signal* is of the form

$$x(t) = Ce^{at},$$

- where C and *a* are, in general, complex numbers.
- Depending upon the values of these parameters, the complex exponential can exhibit several different characteristics.

Real Exponential Signals

- If C and *a* are real number. (a>0, a<0)