EE-220 Signals and Systems

Session 2019

Week 4

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Last Week

- System Properties
- Shifting Property of Impulse
- Convolution
  - DT Convolution, Examples
  - CT Convolution

Continuous Convolution

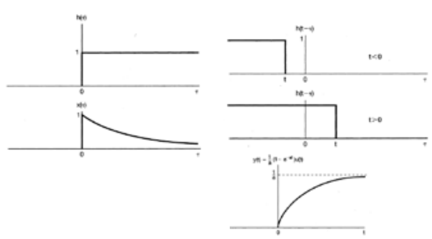
$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = x(t) * h(t)$$

$$x(t) = e^{-at}u(t), \quad a > 0$$

$$h(t) = u(t).$$

$$y(t) = \int_0^t e^{-a\tau}d\tau = -\frac{1}{a}e^{-a\tau}\Big|_0^t = \frac{1}{a}(1 - e^{-at}),$$

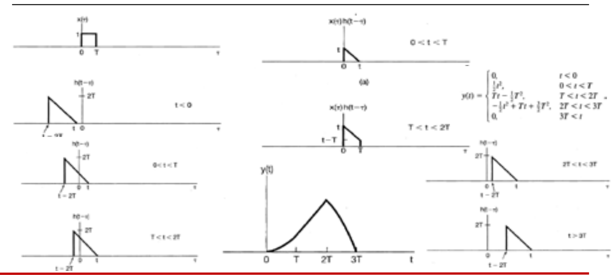
$$y(t) = \frac{1}{a}(1 - e^{-at})u(t),$$



Continuous Convolution

$$x(t) = \begin{cases} 1, & 0 < t < T \\ 0, & \text{otherwise} \end{cases},$$

$$h(t) = \begin{cases} t, & 0 < t < 2T \\ 0, & \text{otherwise} \end{cases}.$$



Review of Convolution

- Starting point will be addition of **lower limits** of independent axis
- End point will be addition of **upper limits** of independent axis
- **DT**
  - Constant Functions – Expand the formula e.g.
    - $x[n] = \delta[n] + 2\delta[n-1],$
    - $h[n] = \delta[n+1] + 3\delta[n] + 2\delta[n-1],$
  - Variables – Use reversal and shifted technique
- **CT**
  - Use reversal and shifted technique

PROPERTIES OF LINEAR TIME-INVARIANT SYSTEMS

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = x[n] * h[n]$$

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = x(t) * h(t)$$

- **The Commutative Property**

$$x[n] * h[n] = h[n] * x[n] = \sum_{k=-\infty}^{+\infty} h[k]x[n-k], \qquad x(t) * h(t) = h(t) * x(t) = \int_{-\infty}^{+\infty} h(\tau)x(t-\tau)d\tau.$$

- **The Distributive Property**

$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n], \quad x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t).$$

The Distributive Property

$$x[n] = \left(\frac{1}{2}\right)^n u[n] + 2^n u[-n],$$

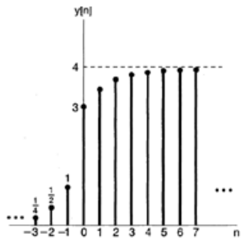
$$h[n] = u[n].$$

$$y[n] = (x_1[n] + x_2[n]) * h[n].$$

$$y[n] = y_1[n] + y_2[n],$$

$$y_1[n] = x_1[n] * h[n]$$

$$y_2[n] = x_2[n] * h[n].$$

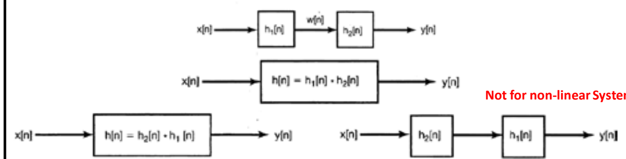


PROPERTIES OF LINEAR TIME-INVARIANT SYSTEMS

- **The Associative Property:**

$$x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n],$$

$$x(t) * [h_1(t) * h_2(t)] = [x(t) * h_1(t)] * h_2(t).$$



Not for non-linear Systems

LTI Systems with and without Memory

$$\boxed{y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = x[n] * h[n]}$$

$$h[n] = 0 \text{ for } n \neq 0.$$

$$h[n] = K\delta[n], \quad y[n] = Kx[n].$$

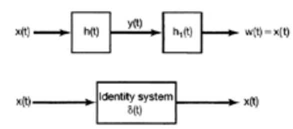
$$h(t) = K\delta(t). \quad y(t) = Kx(t)$$

- For K=1

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k] \qquad x[n] = x[n] * \delta[n]$$

$$x(t) = \int_{-\infty}^{+\infty} x(\tau)\delta(t-\tau)d\tau. \qquad x(t) = x(t) * \delta(t),$$

Invertibility of LTI Systems



$$h(t) * h_1(t) = \delta(t). \qquad h[n] * h_1[n] = \delta[n].$$

Invertibility of LTI Systems

$$y(t) = x(t - t_0).$$

$$h(t) = \delta(t - t_0).$$

$$x(t - t_0) = x(t) * \delta(t - t_0).$$

$$h_1(t) = \delta(t + t_0),$$

$$h(t) * h_1(t) = \delta(t - t_0) * \delta(t + t_0) = \delta(t).$$

Causality for LTI Systems

$$\boxed{y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = x[n] * h[n]}$$

$$h[n] = 0 \quad \text{for } n < 0.$$

$$y[n] = \sum_{k=-\infty}^{n} x[k]h[n-k], \qquad y[n] = \sum_{k=0}^{\infty} h[k]x[n-k].$$

$$h(t) = 0 \quad \text{for } t < 0,$$

$$y(t) = \int_{-\infty}^{t} x(\tau)h(t-\tau)d\tau = \int_{0}^{\infty} h(\tau)x(t-\tau)d\tau.$$

Stability for LTI Systems

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = x[n] * h[n]$$

$$|x[n]| < B \quad \text{for all } n.$$

$$|y[n]| = \left| \sum_{k=-\infty}^{+\infty} h[k]x[n-k] \right|.$$

$$|y[n]| \le \sum_{k=-\infty}^{+\infty} |h[k]||x[n-k]|$$

$$|y[n]| \le B \sum_{k=-\infty}^{+\infty} |h[k]| \quad \text{for all } n.$$

$$\sum_{k=-\infty}^{+\infty} |h[k]| < \infty, \qquad \int_{-\infty}^{+\infty} |h(\tau)| d\tau < \infty.$$

The Unit Step Response of an LTI System

- When input is u[n] or u(t),
- Denoted by s[n], s(t)

$$s[n] = u[n] * h[n].$$

$$s[n] = \sum_{k=-\infty}^{n} h[k]. \qquad h[n] = s[n] - s[n-1].$$

$$s(t) = \int_{-\infty}^{t} h(\tau) d\tau, \qquad h(t) = \frac{ds(t)}{dt} = s'(t).$$