EE-220 Signals and Systems

Session 2019

Week 3

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Last Week

- ❑ Exponential Signals
  - ❑ Continuous and Discrete Time
- ❑ Impulse Function
- ❑ Unit Step Function
- ❑ System
  - ❑ Interconnection of Systems

System Properties

- ❑ A system is said to be ***memoryless*** if its output for each value of the independent variable at a given time is dependent on the input at only that same time.
- ❑ A system is with ***memory*** if its output for each value of the independent variable at a given time is dependent on the input and previous values.

$$y[n] = (2x[n] - x^2[n])^2$$

$$y(t) = x(t)$$

$$y(t) = Rx(t)$$

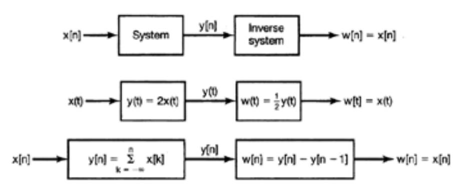
$$y(t) = x(t-1)$$

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)\, d\tau$$

System Properties

- ❑ **Invertibility and Inverse Systems:**
  - ❑ A system is said to be *invertible* if distinct inputs lead to distinct outputs.
  - ❑ If a system is invertible, then an *inverse system* exists



$$y[n] = 0,$$

$$y(t) = x^2(t),$$

System Properties

- ❑ A system is ***causal*** if the output at any time depends on values of the input at only the present and past times.
- ❑ Such a system is often referred to as being ***non anticipative***
- ❑ All memory less systems are causal, since the output responds only to the current value of the input.

$$y(t) = x(t-1) \qquad y[n] = x[n] - x[n+1]$$

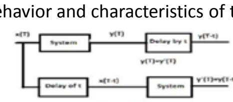
$$y[n] = \sum_{k=-\infty}^{n} x[k] \qquad y(t) = x(t+1)$$

Example 1.12

$$y[n] = \frac{1}{2M+1}\sum_{k=-M}^{+M} x[n-k].$$

System Properties

- ❑ A **stable system** is one in which small inputs lead to responses that do not diverge
  - ❑ A bounded input must produce bounded output signals
- ❑ Example 1.13
- ❑ **Time Invariance:**
- ❑ A system is time invariant if the behavior and characteristics of the system are fixed over time.
- ❑ Example 1.14-1.16



System Properties

- **Linear:** A *linear system,* in continuous time or discrete time, is a system that possesses the important property of superposition
  - *additivity* property
  - *scaling* or *homogeneity* property

1. The response to $x_1(t) + x_2(t)$ is $y_1(t) + y_2(t)$.
2. The response to $ax_1(t)$ is $ay_1(t)$, where a is any complex constant.

- 1.17-1.20

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Chapter 2

LINEAR TIME-INVARIANT SYSTEMS

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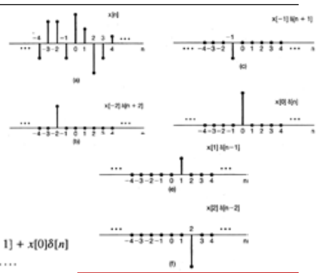
The Representation of Discrete-Time Signals in Terms of Impulses

- Discrete-time unit impulse can be used to construct any discrete-time signal. Discrete-time signal as a sequence of individual impulses.

$$x[-1]\delta[n+1] = \begin{cases} x[-1], & n=-1 \\ 0, & n \neq -1 \end{cases},$$
$$x[0]\delta[n] = \begin{cases} x[0], & n=0 \\ 0, & n \neq 0 \end{cases},$$
$$x[1]\delta[n-1] = \begin{cases} x[1], & n=1 \\ 0, & n \neq 1 \end{cases}.$$



- Therefore, the sum of the five-sequences in the figure equals *x[n]* for - 2 <*n* < 2.

$$x[n] = \ldots + x[-3]\delta[n+3] + x[-2]\delta[n+2] + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + x[2]\delta[n-2] + x[3]\delta[n-3] + \ldots$$

Sifting Property of DT Impulse

$$x[n] = \ldots + x[-3]\delta[n+3] + x[-2]\delta[n+2] + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + x[2]\delta[n-2] + x[3]\delta[n-3] + \ldots$$

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k].$$

- Equation is called the ***sifting property*** of the discrete-time unit impulse

$$u[n] = \sum_{k=0}^{+\infty} \delta[n-k].$$

The Discrete-Time Unit Impulse Response and the Convolution-Sum Representation of LTI Systems

- The importance of the sifting property lies in the fact that it represents *x[n]* as a superposition of scaled versions of shifted unit impulses.
- The response of a linear system to *x[n]* will be the superposition of the scaled responses of the system to each of these shifted impulses.
- Moreover, the property of time in variance tells us that the responses of a time-invariant system to the time-shifted unit impulses are simply time-shifted versions of one another.
- The convolution -sum representation for discrete-time systems that are *both* linear and time invariant results from putting these two basic facts together.

Convolution

- *For x[n]* we can represent the input as a linear combination of shifted unit impulses.

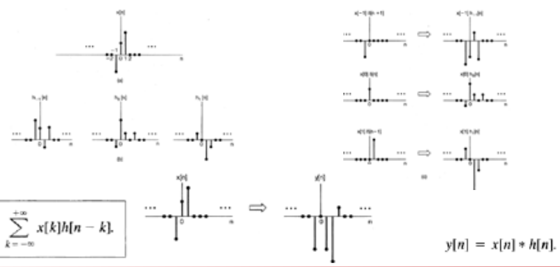
$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k].$$

- Let $h_k[n]$ denotes the response of the linear system to the shifted unit impulse $\delta[n-k]$.
- Specifically, since $\delta[n-k]$ is a time-shifted version of δ[n], the response $h_k[n]$ is a time-shifted version of $h_0[n]$

$$h_k[n] = h_0[n-k]. \quad h[n] = h_0[n].$$

$$\boxed{y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k].}$$

Convolution Sum or Superposition Sum

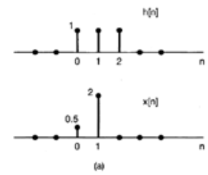


$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k].$$

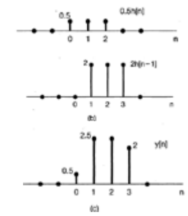
$$y[n] = x[n] * h[n].$$

Convolution

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k].$$

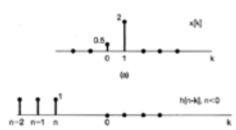


$$y[n] = x[0]h[n-0] + x[1]h[n-1] = 0.5h[n] + 2h[n-1].$$



Convolution

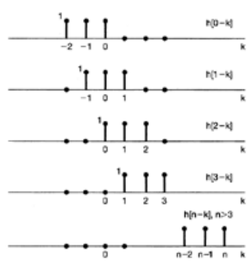
$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k].$$



$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[0-k] = 0.5.$$

$$y[1] = \sum_{k=-\infty}^{\infty} x[k]h[1-k] = 0.5 + 2.0 = 2.5.$$

$$y[2] = \sum_{k=-\infty}^{\infty} x[k]h[2-k] = 0.5 + 2.0 = 2.5.$$



Important Math relations

- **Sum of Geometric Series:**

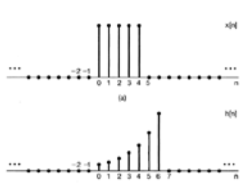
$$y[n] = \sum_{k=0}^{n} \alpha^k = \frac{1-\alpha^{n+1}}{1-\alpha} \qquad \sum_{n=x}^{y} \alpha^n = \frac{\alpha^x - \alpha^{y+1}}{1-\alpha}$$

$$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} \qquad \sum_{n=0}^{\infty} n\,\alpha^n = \frac{\alpha}{1-\alpha^2}$$

Convolution

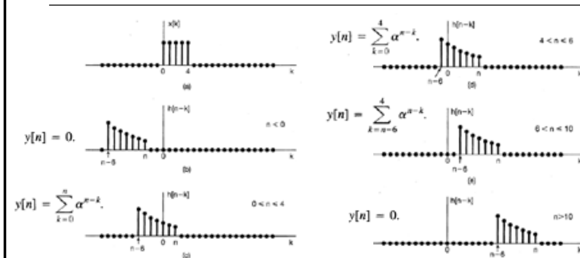
$$x[n] = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$$

$$h[n] = \begin{cases} \alpha^n, & 0 \le n \le 6 \\ 0, & \text{otherwise} \end{cases}$$



Convolution

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k].$$



$$y[n] = 0. \quad n<0$$

$$y[n] = \sum_{k=0}^{n} \alpha^{n-k}. \quad 0<n<4$$

$$y[n] = \sum_{k=0}^{4} \alpha^{n-k}. \quad 4<n<6$$

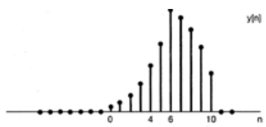
$$y[n] = \sum_{k=n-6}^{4} \alpha^{n-k}. \quad 6<n<10$$

$$y[n] = 0. \quad n>10$$

Convolution

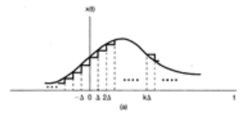
- Using formula of geometric series

$$y[n] = \begin{cases} 0, & n < 0 \\ \dfrac{1-\alpha^{n+1}}{1-\alpha}, & 0 \le n \le 4 \\ \dfrac{\alpha^{n-4}-\alpha^{n+1}}{1-\alpha}, & 4 < n \le 6 \\ \dfrac{\alpha^{n-4}-\alpha^{7}}{1-\alpha}, & 6 < n \le 10 \\ 0, & 10 < n \end{cases},$$



- Practice example 2.5

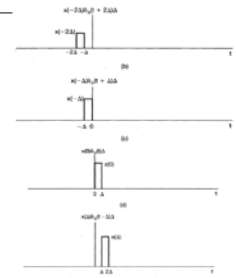
The Representation of Continuous-Time Signals in Terms of Impulses



$$\hat{x}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\delta_\Delta(t-k\Delta)\Delta.$$

$$x(t) = \lim_{\Delta \to 0} \sum_{k=-\infty}^{+\infty} x(k\Delta)\delta_\Delta(t-k\Delta)\Delta.$$

$$x(t) = \int_{-\infty}^{+\infty} x(\tau)\delta(t-\tau)d\tau.$$



Convolution Integral Representation of LTI Systems

$$\hat{y}(t) = \sum_{k=-\infty}^{+\infty} x(k\Delta)\hat{h}_{k\Delta}(t)\Delta.$$

$$y(t) = \lim_{\Delta \to 0} \sum_{k=-\infty}^{+\infty} x(k\Delta)\hat{h}_{k\Delta}(t)\Delta.$$

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h_\tau(t)d\tau. \qquad \boxed{y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau.}$$

$$y(t) = x(t) * h(t).$$