OBJECTIVES

- Establish the ideal characteristics of an ideal operational amplifier.
- Establish the main amplifier configurations.
- Develop an understanding of inverting and noninverting amplification.
- Differentiate between the different summing and subtracting configurations.
- Recognize the various methods for implementing controlled sources.
- Distinguish the use of dual power supply and single power supply.

8–1 THE IDEAL OPERATIONAL AMPLIFIER

An operational amplifier is a direct-coupled amplifier with two (differential) inputs and a single output. It normally requires to be powered by a dual power supply ($+V$ and $-V$ with respect to ground) although later in the chapter we will look at how to connect an operational amplifier to a single power supply. We will define an *ideal* operational amplifier to be one that has the following attributes:

1. It has infinite gain.
2. It has infinite input impedance.
3. It has zero output impedance.
4. It has infinite bandwidth.

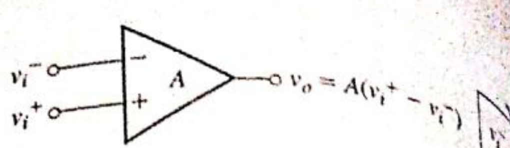
Although no real amplifier* can satisfy any of these requirements, we will see that most modern amplifiers have such large gains and input impedances, and such small output impedances, that a negligibly small error results from assuming ideal characteristics. A detailed study of the ideal amplifier will therefore be beneficial in terms of understanding how practical amplifiers are used as well as in building some important theoretical concepts that have broad implications in many areas of electronics.

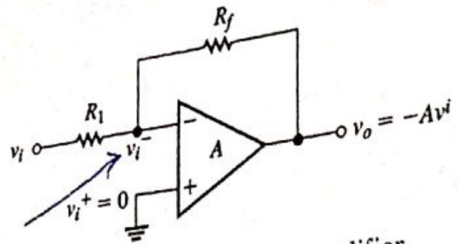
Figure 8–1 shows the standard symbol for an operational amplifier. Note that the two inputs are labeled "+" and "−" and the input signals are correspondingly designated v_i^+ and v_i^-. In relation to our previous discussion of differential amplifiers, these inputs correspond to v_{i1} and v_{i2}, respectively, when the single-ended output is v_{o2}. The + input is called the *noninverting* input and the − input is called the *inverting input.* In many applications, one of the amplifier inputs is grounded, so v_o is in phase with the input if the signal is connected to the noninverting terminal, and v_o is out of phase with the input if the signal is connected to the inverting input. These ideas are summarized in the table accompanying Figure 8–1.

At this point, a legitimate question that may have already occurred to the reader is this: If the gain is infinite, how can the output be anything

*In this chapter, we will h[illegible]ter use the word *amplifier* with the understanding that operational amplifier is meant. We will al[illegible]e term *op-amp,* which is widely used in books, papers, and technical literature.

FIGURE 8–1 Operational amplifier symbol, showing inverting (−) and noninverting (+) inputs





$V_i = 0$
b/c $A = \infty$
$= \frac{V_o}{A}$
$= \frac{V_o}{\infty}$
$= 0$

FIGURE 8–2 An operational-amplifier application in which signal v_i is connected through R_1. Resistor R_f provides feedback. $v_o/v_i^- = -A$.

it is said virtual ground.

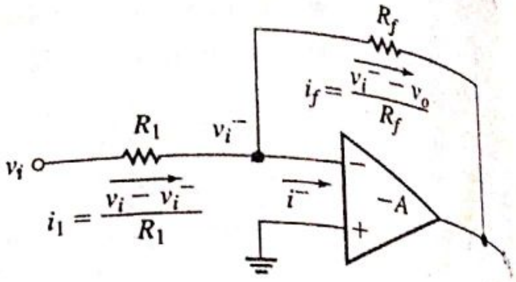


FIGURE 8–3 Voltages and currents resulting application of the signal voltage v_i

other than a severely clipped waveform? Theoretically, if the amp infinite gain, an infinitesimal input voltage must result in an large output voltage. The answer, of course, is that the gain is infinite, just very large. Nevertheless, it *is* true that a very sm voltage will cause the amplifier output to be driven all the w extreme positive or negative voltage limit. The practical answe an operational amplifier is seldom used in such a way that the is applied to an input. Instead, external resistors are connecte around the amplifier in such a way that the signal undergoes vast amplification. The resistors cause gain reduction through signal which we will soon study in considerable detail.

The Inverting Amplifier

Consider the configuration shown in Figure 8–2. In this very useful of an operational amplifier, the noninverting input is grounded, v_i is through R_1 to the inverting input, and feedback resistor R_f is between the output and v_i^-. Let A denote the voltage gain of the $v_o = A(v_i^+ - v_i^-)$. Since $v_i^+ = 0$, we have

$$v_o = -Av_i^-$$

(Note that $v_i \neq v_i^-$.) We wish to investigate the relation between when the magnitude of A is infinite.

Figure 8–3 shows the voltages and currents that result when connected. From Ohm's law, the current i_1 is simply the difference across R_1, divided by R_1:

$$i_1 = (v_i - v_i^-)/R_1$$

Similarly, the current i_f is the difference in voltage across R_f, divi

Letting $A = \infty$, the term v_o/A goes to 0, and we have

$$v_i^- = v_i^+ \tag{8–13}$$

Substituting v_i^+ for v_i^- in (8–10) gives

$$\frac{v_i^+}{R_1} = \frac{v_o - v_i^+}{R_f} \tag{8–14}$$

Solving for v_o/v_i^+ and recognizing that $v_i^+ = v_i$ lead to

$$\frac{v_o}{v_i} = \frac{R_1 + R_f}{R_1} = 1 + \frac{R_f}{R_1} \tag{8–15}$$

We saw (equation 8–8) that when an operational amplifier is connected in an inverting configuration, with $v_i^+ = 0$, the assumption $A = \infty$ gives $v_i^- = 0$ (virtual ground), i.e., $v_i^- = v_i^+$. Also, in the noninverting configuration, the same assumption gives the same result: $v_i^- = v_i^+$ (equation 8–13). Thus, we reach the important general conclusion that *feedback in conjunction with a very large voltage gain forces the voltages at the inverting and noninverting inputs to be approximately equal.*

Equation 8–15 shows that the closed-loop gain of the noninverting amplifier, like that of the inverting amplifier, depends only on the values of external resistors. A further advantage of the noninverting amplifier is that the input impedance seen by v_i is infinite, or at least extremely large in a real amplifier. The inverting and noninverting amplifiers are used in voltage *scaling* applications, where it is desired to multiply a voltage precisely by a fixed constant, or scale factor. The multiplying constant in the inverting amplifier is R_f/R_1 (which may be less than 1), and it is $1 + R_f/R_1$ (which is always greater than 1) in the noninverting amplifier. A wide range of constants can be realized with convenient choices of R_f and R_1 when the gain is R_f/R_1, which is not so much the case when the gain is $1 + R_f/R_1$. For that reason, the inverting amplifier is more often used in precision scaling applications.

Inverting amp

App. of voltage scaling

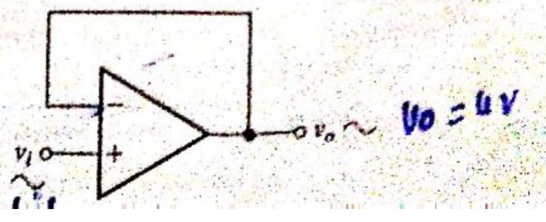
The reader may wonder why it would be desirable or necessary to use an amplifier to multiply a voltage by a number less than 1, since this can also be accomplished using a simple voltage divider. The answer is that the amplifier provides power gain to drive a load. Also, the ideal amplifier has zero output impedance, so the output voltage is not affected by changes in load impedance.

The Voltage Follower

Figure 8–6 shows a special case of the noninverting amplifier used in applications where power gain and impedance isolation are of primary concern. Notice that $R_f = 0$ and $R_1 = \infty$, so, by equation 8–15, the closed-loop gain is $v_o/v_i = 1 + R_f/R_1 = 1$. This configuration is called a *voltage follower* because v_o has the same magnitude and phase as v_i. Like a BJT emitter follower, it has large input impedance and small output impedance and is used as a buffer amplifier between a high-impedance source and a low-impedance load.

Why we always apply feedback at -ve of op-amp: circuit will be stable only when -ve feedback will be given so for +ve it will not be stable.

FIGURE 8–6 The voltage follower



EXAMPLE 8–2

DESIGN

In a certain application, a signal source having 60 kΩ of source impedance produces a 1-V-rms signal. This signal must be amplified to 2.5 V rms and drive a 1-kΩ load. Assuming that the phase of the load voltage is of no concern, design an operational-amplifier circuit for the application.

Solution

Because phase is of no concern and the required voltage gain is greater than 1, we can use either an inverting or noninverting amplifier. Suppose we decide to use the inverting configuration and arbitrarily choose $R_f = 250$ kΩ. Then,

$$\frac{R_f}{R_1} = 2.5 \rightarrow R_1 = \frac{R_f}{2.5} = \frac{250\ \text{k}\Omega}{2.5} = 100\ \text{k}\Omega$$

Note, however, that the signal source sees an impedance equal to $R_1 = 100$ kΩ in the inverting configuration, so the usual voltage division takes place and the input to the amplifier is actually

$$v_i = \left(\frac{R_1}{R_1 + r_S}\right)(1\ \text{V rms}) =$$

$$\left[\frac{100\ \text{k}\Omega}{(100\ \text{k}\Omega) + (60\ \text{k}\Omega)}\right](1\ \text{V rms}) = 0.625\ \text{V rms}$$

Therefore, the magnitude of the amplifier output is

$$v_o = \frac{R_f}{R_1}(0.625\ \text{V rms}) = \frac{250\ \text{k}\Omega}{100\ \text{k}\Omega}(0.625\ \text{V rms}) = 1.5625\ \text{V rms}$$

Clearly, the large source impedance is responsible for a reduction in gain, and it is necessary to redesign the amplifier circuit to compensate for this loss. (Do this, as an exercise.)

In view of the fact that the source impedance may not be known precisely or may change if a replacement source is used, a far better solution is to design a noninverting amplifier. Because the input impedance of this design is extremely large, the choice of values for R_f and R_1 will not depend on the source impedance. Letting $R_f = 150$ kΩ, we have

$$1 + \frac{R_f}{R_1} = 2.5$$

$$\frac{R_f}{R_1} = 1.5$$

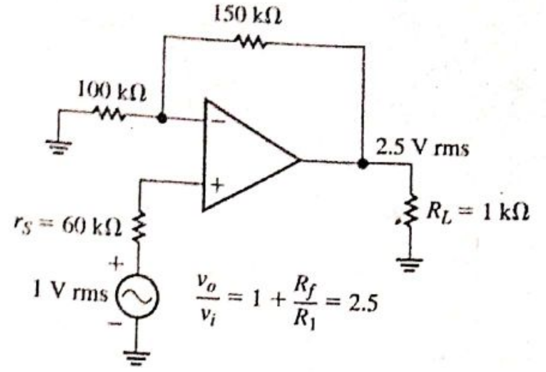
$$R_1 = \frac{R_f}{1.5} = \frac{150\ \text{k}\Omega}{1.5} = 100\ \text{k}\Omega$$

The completed design is shown in Figure 8–7. We can assume that the amplifier has zero output impedance, so we do not need to be concerned with voltage division between the amplifier output and the 1-kΩ load.

The Compensating Resistor R_c

Later in our study of operational amplifiers we will learn about certain nonideal characteristics. One of them is the fact that both inputs in an op-amp take a finite, albeit very small, current called *input bias current*. These two finite input currents can produce a small dc output voltage even when the input voltage is zero. The easiest way to minimize this problem is by including a compensating resistor, R_c, in series with the noninverting input, as shown in Figure 8–8. The value of this resistor as will be ... 10, should

FIGURE 8–7 (Example 8–2)



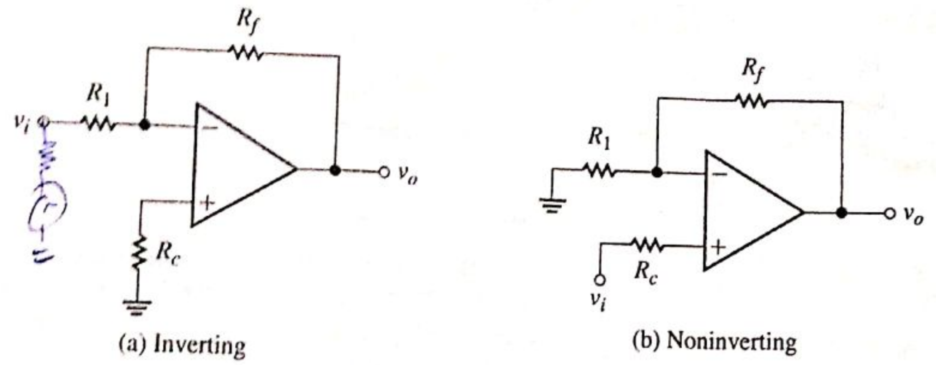


FIGURE 8–8 Using a compensating resistor in inverting and noninverting amplifiers

be approximately equal to the parallel combination of R_f and R_1. Any source resistance present in the circuit should be taken into account as well. For instance, in an inverting amplifier, the compensating resistor should be

$$R_c = R_f \| (r_S + R_1) \quad \text{for inverting} \tag{8–16}$$

since r_S appears in series with R_1.

In the case of a noninverting amplifier, where the source is connected directly to the + input, the *sum* of r_S and R_c should be approximately equal to $R_1 \| R_f$, which means

$$R_c = (R_f \| R_1) - r_S \quad \text{for non inverting} \tag{8–17}$$

Single Power Supply Operation

When an amplifier is to be used with a single power supply, the $-V$ terminal is connected directly to ground and the supply voltage to the $+V$ terminal. The + input must be biased to one-half the supply voltage for proper linear operation. The resulting dc output voltage will also be one-half the supply voltage. Because of this, single power supply operation requires capacitive coupling for both the input signal source and the load resistance or subsequent stage.

Figure 8–9 shows the typical inverting amplifier configuration for single power supply operation. Note the voltage divider that provides the biasing for the + input. The gain formulas remain the same as for normal operation, but the coupling capacitors should be able to pass the lowest frequencies present in the signal; this topic will be covered in detail in a later chapter.

FIGURE 8–9 An inverting amplifier connected to a single power supply

FIGURE 8–10 A noninverting amplifier connected to a single power supply

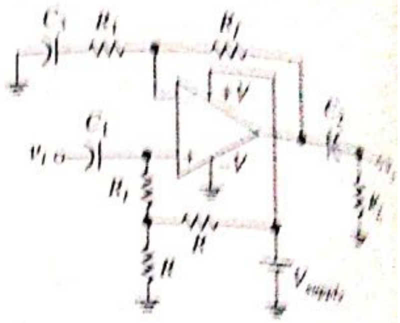


Figure 8–10 shows the noninverting version of the circuit. Note the addition of resistor R_i, which can be used to rise the input resistance seen by the signal source v_i to a particular desired level.

Because these configurations must be used with coupling capacitors, the biasing-compensating resistor is not much of an issue. In other words, any deviation of the output dc voltage from one-half the supply voltage due to bias currents will be irrelevant as far as ac operation is concerned. In any event, deviation can be maintained very small by making the parallel combination of the voltage-divider resistors (plus R_i, if any) match the value of R_f.

8–2 VOLTAGE SUMMATION, SUBTRACTION, AND SCALING

Voltage Summation

We have seen that it is possible to *scale* a signal voltage, that is, to multiply it by a fixed constant, through an appropriate choice of external resistors that determine the closed-loop gain of an amplifier circuit. This operation can be accomplished in either an inverting or noninverting configuration. It is also possible to sum several signal voltages in one operational amplifier circuit and at the same time scale each by a different factor. For example, given inputs v_1, v_2, and v_3, we might wish to generate an output equal to $2v_1 + 0.5v_2 + 4v_3$. The latter sum is called a *linear combination* of v_1, v_2, and v_3, and the circuit that produces it is often called a *linear combination circuit.*

Figure 8–11 shows an inverting amplifier circuit that can be used to sum and scale three input signals. Note that input signals v_1, v_2, and v_3 are applied through separate resistors R_1, R_2, and R_3 to the summing junction of the amplifier and that there is a single feedback resistor R_f. Resistor R_c is the offset compensation resistor discussed previously.

Following the same procedure we used to derive the output of an inverting amplifier having a single input, we obtain for the three-input (ideal) amplifier

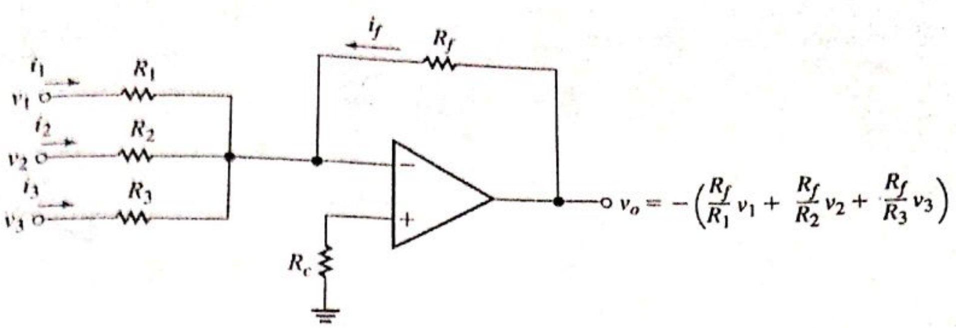


FIGURE 8–11 An operational-amplifier circuit that produces an output equal to the (inverted) sum of three separately scaled input signals

$$i_1 + i_2 + i_3 = -i_f$$

Or, since the voltage at the summing junction is ideally 0,

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} = \frac{-v_o}{R_f}$$

Solving for v_o gives

$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right) \tag{8–18}$$

Equation 8–18 shows that the output is the inverted sum of the separately scaled inputs, i.e., a *weighted* sum, or linear combination, of the inputs. By appropriate choice of values for R_1, R_2, and R_3, we can make the scale factors equal to whatever constants we wish, within practical limits. If we choose $R_1 = R_2 = R_3 = R$, then we obtain

$$v_o = \frac{-R_f}{R}(v_1 + v_2 + v_3) \tag{8–19}$$

and, for $R_f = R$,

$$\boxed{v_o = -(v_1 + v_2 + v_3)} \tag{8–20}$$

The theory can be extended in an obvious way to two, four, or any reasonable number of inputs. In this case, the compensating resistor is obtained from

$$R_c = R_f \| R_1 \| R_2 \| \cdots$$

E 8-3

IGN

1. Design an operational-amplifier circuit that will produce an output equal to $-(4v_1 + 1v_2 + 0.1v_3)$.
2. Write an expression for the output and sketch its waveform when $v_1 = 2 \sin \omega t$ V, $v_2 = +5$ V dc, and $v_3 = -100$ V dc.

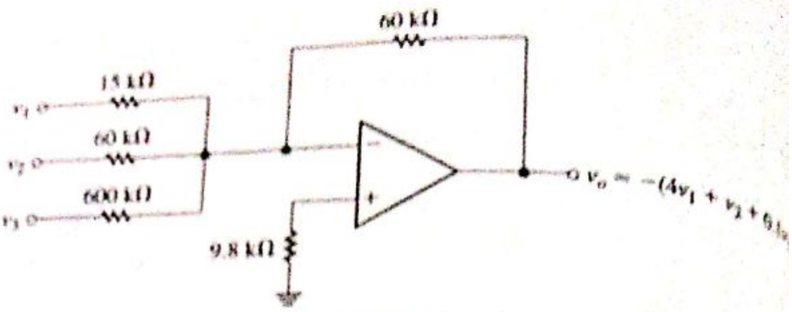
Solution

1. We arbitrarily choose $R_f = 60$ kΩ. Then

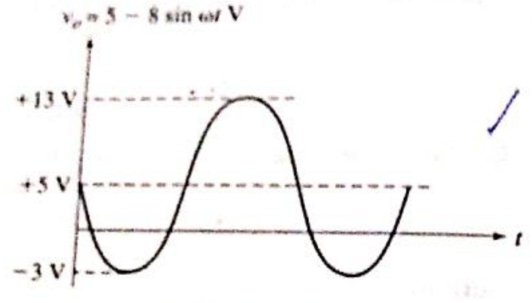
$$\frac{R_f}{R_1} = 4 \Rightarrow R_1 = \frac{60\text{ k}\Omega}{4} = 15\text{ k}\Omega$$

$$\frac{R_f}{R_2} = 1 \Rightarrow R_2 = \frac{60\text{ k}\Omega}{1} = 60\text{ k}\Omega$$

FIGURE 8-12 (Example 8-3)



(a) Circuit



(b) Output waveform

$$\frac{R_f}{R_3} = 0.1 \Rightarrow R_3 = \frac{60\text{ k}\Omega}{0.1} = 600\text{ k}\Omega$$

By equation 8–21, the optimum value for the compensating resistor is $R_c = R_f \| R_1 \| R_2 \| R_3 = (60\text{ k}\Omega) \| (15\text{ k}\Omega) \| (60\text{ k}\Omega) \| (600\text{ k}\Omega) = 9.8\text{ k}\Omega$. The circuit is shown in Figure 8–12(a).

2. $v_o = -[4(2 \sin \omega t) + 1(5) + 0.1(-100)] = -8 \sin \omega t - 5 + 10 = 5 - 8 \sin \omega t$. This output is sinusoidal with a 5-V offset and varies between $5 - 8 = -3$ V and $5 + 8 = 13$ V. It is sketched in Figure 8–12(b).

Figure 8–13 shows a noninverting version of the linear combination circuit. In this case, it can be shown (Exercise 8–13) that

$$v_o = \left(1 + \frac{R_f}{R_g}\right)\left(\frac{R_p}{R_1}v_1 + \frac{R_p}{R_2}v_2 + \frac{R_p}{R_3}v_3\right) \qquad (8\text{-}2$$

where $R_p = R_1 \| R_2 \| R_3$

Although this circuit does not invert the scaled sum, it is somewhat more cumbersome than the inverting circuit in terms of selecting resistor values to provide precise scale factors. Phase inversion is often of ... sequence, but in those applications where a noninverted sum is r... can also be obtained using the inverting circuit of Figure 8–11, ...d a linear ... a unity-gain inverter.

Voltage Subtraction

Suppose we wish to produce an output voltage that equals the difference between two input signals. This operation can be ... using the amplifier in a *differential* mode, where the signal... through appropriate resistor networks to the inverting and ... minals. Figure 8–14 shows the configuration. We can use t...

...be used to su... ...nd v_3 are applied ...tion of the ... is the off... output of ... the three-input

FIGURE 8–13 A noninverting linear combination circuit

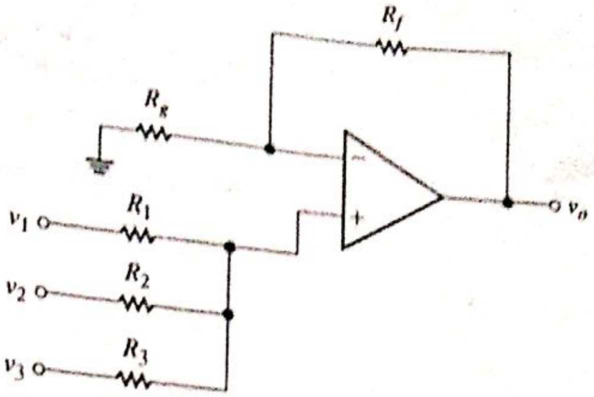
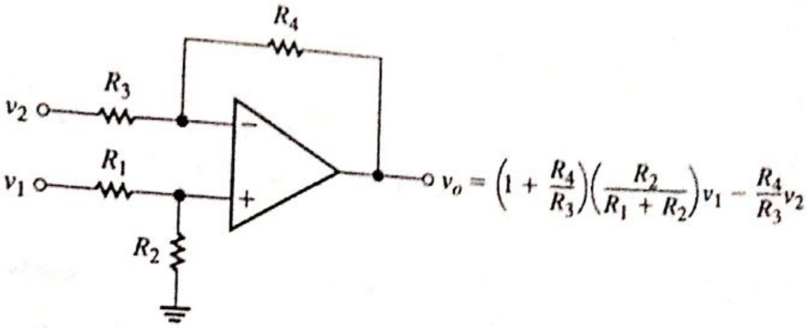


FIGURE 8–14 Using the amplifier in a differential mode to obtain an output proportional to the difference between two scaled inputs



principle to determine the output of this circuit. First, assume that v_2 is shorted to ground. Then

$$v^+ = \frac{R_2}{R_1 + R_2} v_1 \tag{8–22}$$

so

$$v_{o1} = \left(1 + \frac{R_4}{R_3}\right)v^+ = \left(1 + \frac{R_4}{R_3}\right)\left(\frac{R_2}{R_1 + R_2}\right)v_1 \tag{8–23}$$

Assuming now that v_1 is shorted to ground, we have

$$v_{o2} = \frac{-R_4}{R_3} v_2 \tag{8–24}$$

Therefore, with both signal inputs present, the output is

$$v_o = v_{o1} + v_{o2} = \left(1 + \frac{R_4}{R_3}\right)\left(\frac{R_2}{R_1 + R_2}\right) v_1 - \left(\frac{R_4}{R_3}\right)v_2 \tag{8–25}$$

Equation 8–25 shows that the output is proportional to the difference between scaled multiples of the inputs.

To obtain a difference or differential amplifier for which

$$v_o = A(v_1 - v_2) \tag{8–26}$$

where A is the differential gain, select the resistor values in accordance with the following:

$$R_1 = R_3 = R \quad \text{and} \quad R_2 = R_4 = AR \tag{8–27}$$

Substituting these values into (8–25) gives

$$\left(\frac{R + AR}{R}\right)\left(\frac{AR}{R + AR}\right)v_1 - \frac{AR}{R}v_2 = \frac{AR}{R}v_1 - \frac{AR}{R}v_2 = A(v_1 - v_2)$$

as required. When resistor values are chosen in accordance with (8–27), bias compensation resistance $(R_1 \| R_2)$ is automatically the correct $(R_3 \| R_4)$, namely $R \| AR$.

Let the general form of the output of Figure 8–14 be

$$v_o = a_1 v_1 - a_2 v_2$$

where a_1 and a_2 are positive constants. Then, by equation 8–25, we must have

$$a_1 = \left(1 + \frac{R_4}{R_3}\right)\left(\frac{R_2}{R_1 + R_2}\right)$$

and

$$a_2 = \frac{R_4}{R_3}$$

Substituting (8–30) into (8–29) gives

$$a_1 = (1 + a_2)\frac{R_2}{R_1 + R_2}$$

But the quantity $R_2/(R_1 + R_2)$ is always less than 1. Therefore, equation shows that in order to use the circuit of Figure 8–12 to produce $v_o = a_1v_1 -$ a_2v_2, we must have

$$(1 + a_2) > a_1$$

This restriction limits the usefulness of the circuit.

*

EXAMPLE 8–4

DESIGN

Design an op-amp circuit that will produce the output $v_o = 0.5v_1 - 2v_2$

a_1 a_2

Solution

Note that $a_1 = 0.5$ and $a_2 = 2$, so $(1 + a_2) > a_1$. Therefore, it is possible to construct a circuit in the configuration of Figure 8–14.

Comparing v_o with equation 8–25, we see that we must have

$$\left(1 + \frac{R_4}{R_3}\right)\left(\frac{R_2}{R_1 + R_2}\right) = 0.5$$

and

$$\frac{R_4}{R_3} = 2$$

Let us arbitrarily choose $R_4 = 100\ \text{k}\Omega$. Then $R_3 = R_4/2 = 50\ \text{k}\Omega$. Thus

$$\left(1 + \frac{R_4}{R_3}\right)\left(\frac{R_2}{R_1 + R_2}\right) = \frac{3R_2}{R_1 + R_2} = 0.5$$

Arbitrarily choosing $R_2 = 20\ \text{k}\Omega$, we have

$$\frac{3(20\ \text{k}\Omega)}{R_1 + (20\ \text{k}\Omega)} = 0.5$$

$$60\ \text{k}\Omega = 0.5R_1 + (10\ \text{k}\Omega)$$

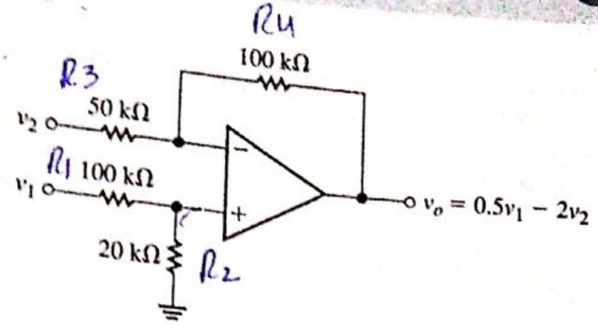
$$R_1 = 100\ \text{k}\Omega$$

The completed design is shown in Figure 8–15.

Basically,

In Example 8–4, we note that the compensation resistance $(R_1 \| R_2 = (100\ \text{k}\Omega) \| (20\ \text{k}\Omega) = 16.67\ \text{k}\Omega)$ is not equal to its optimum $(R_3 \| R_4 = (50\ \text{k}\Omega) \| (100\ \text{k}\Omega) = 33.33\ \text{k}\Omega)$. With some algebraic comp

FIGURE 8–15 (Example 8–4)



we can impose the additional condition $R_1 \| R_2 = R_3 \| R_4$ and thereby force the compensation resistance to have its optimum value. With $v_o = a_1 v_1 - a_2 v_2$, it can be shown (Exercise 8–17) that the compensation resistance $(R_1 \| R_2)$ is optimum when the resistor values are selected in accordance with

$$R_4 = a_1 R_1 = a_2 R_3 = R_2(1 + a_2 - a_1) \tag{8–33}$$

To apply this design criterion, choose R_4 and solve for R_1, R_2, and R_3. In Example 8–4, $a_1 = 0.5$ and $a_2 = 2$. If we choose $R_4 = 100$ kΩ, then $R_1 =$ (100 kΩ)/0.5 = 200 kΩ, $R_2 =$ (100 kΩ)/2.5 = 40 kΩ, and $R_3 =$ (100 kΩ)/2 = 50 kΩ. These choices give $R_1 \| R_2 = 33.3$ kΩ $= R_3 \| R_4$, as required.

Although the circuit of Figure 8–14 is a useful and economical way to obtain a difference voltage of the form $A(v_1 - v_2)$, our analysis has shown that it has limitations and complications when we want to produce an output of the general form $v_o = a_1 v_1 - a_2 v_2$. An alternative way to obtain the difference between two scaled signal inputs is to use *two* inverting amplifiers, as shown in Figure 8–16. The output of the first amplifier is

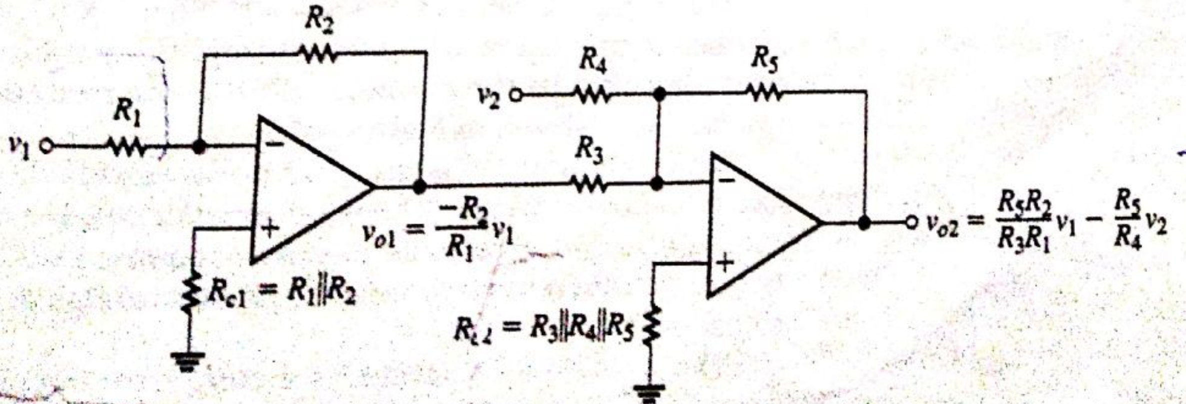
$$v_{o1} = \frac{-R_2}{R_1} v_1 \tag{8–34}$$

and the output of the second amplifier is

$$v_{o2} = -\left(\frac{R_5}{R_3} v_{o1} + \frac{R_5}{R_4} v_2\right) = \frac{R_5 R_2}{R_3 R_1} v_1 - \frac{R_5}{R_4} v_2 \tag{8–35}$$

This equation shows that there is a great deal of flexibility in the choice of resistor values necessary to obtain $v_o = a_1 v_1 - a_2 v_2$, because a large number of combinations will satisfy

$$\frac{R_5 R_2}{R_3 R_1} = a_1 \quad \text{and} \quad \frac{R_5}{R_4} = a_2 \tag{8–36}$$



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