Name	Urwa Maryam, Umar Hayyat, Mateen Abbas
Roll no.	2019-EE-352, 2019-EE-360, 2019-EE-384
Marks	4.5 10-12-21

Lab Manual # 10

QUBE−Stability and Second Order System Analysis

Objectives

In this lab, we will learn about

- Stable, marginally stable, and unstable system
- Underdamped second-order system
- Calculation of Damping ratio and natural frequency
- Finding out the peak time and percent overshoot time-domain specifications

Prerequisites

- QUBE-Servo Integration laboratory experiment.
- Filtering laboratory experiment.

Introduction

Servo Model:

The QUBE-Servo voltage-to-speed transfer function is

$$P_{v-s}(s) = \frac{\Omega_m(s)}{V_m(s)} = \frac{K}{\tau s + 1}, \longrightarrow 1$$

where $K \cong 23.2\, rad/(V-s)$ is the model steady-state gain, $\tau = 0.13s$ is the model time constant, $\Omega_m(s) = L\,[\omega_m(t)]$ is the motor speed (i.e. speed of load disk), and $V_m(s) = L\,[v_m(t)]$ is the applied motor voltage. If desired, you can conduct an experiment to find more precise model parameters, K and τ, for your particular servo (e.g. performing the Bump Test Modeling lab).

The voltage-to-position process transfer function can be found by integrating the above equation

$$P_{v-p} = \frac{\Theta_m(s)}{V_m(s)} = \frac{K}{s(\tau s + 1)} \longrightarrow 2$$

where $\Theta_m(s) = L\,[\theta_m(t)]$ is the load gear position.

Stability:

Definition for Bounded-Input Bounded-Output (BIBO) stability is:

1. A system is stable if every bounded input yields a bounded output.
2. A system is unstable if any bounded input yields an unbounded output. The stability of a system can be determined from its poles:

- Stable systems have poles only in the left-hand plane.
- Unstable systems have at least one pole in the right-hand plane and/or poles of multiplicity greater than 1 on the imaginary axis.
- Marginally stable systems have one pole on the imaginary axis and the other poles in the left-hand plane.

Tasks:

- Determine the stability of the voltage-to-speed servo system.
- Determine the stability of the voltage-to-position servo system.

Calculations and comments on the stability

Using Equation 1

1) Characteristic Equation: $\tau s + 1 = 0$

$$\boldsymbol{s = -\frac{1}{\tau} = -7.69}$$

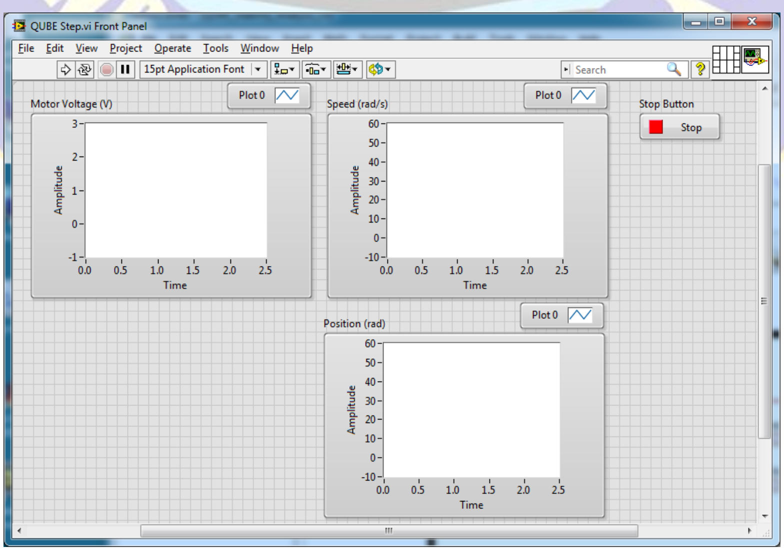
Using Equation 2

2) Characteristic Equation: $s(\tau s + 1) = 0$

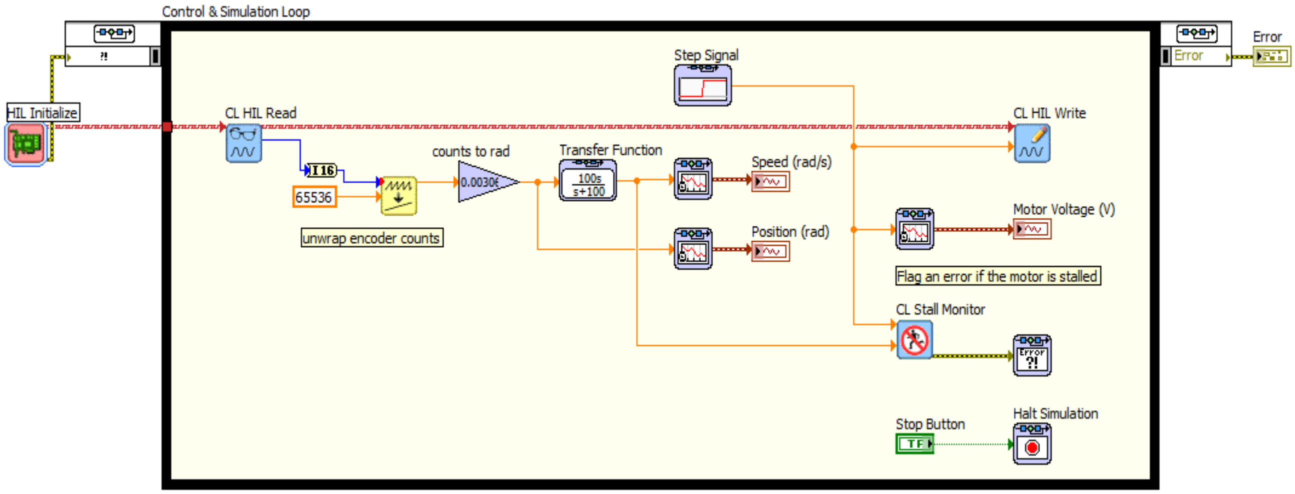
$$\boldsymbol{s = 0} \quad ; \quad \boldsymbol{s = -\frac{1}{\tau} = -7.69}$$

Roots of characteristic equation 1, show that system pole is in left half plane. So, ***speed response*** is stable.
Roots of characteristic equation 2, show that system poles are in left half plane and at origin. So, ***position response*** is marginally-stable because of pole at origin.

Based on the VIs already designed in QUBE-Servo Integration and Filtering labs, design a VI that applies a step of 1V to the motor and reads the servo velocity and the position as shown in Figure 1. Configure the Simulation Loop to run for 2.5 seconds.



(a) Front Panel



(b) Block Diagram

Figure 1: Measuring speed and position when applying a step

Front Panel:

- **Based on the measured/observed *speed* response and the BIBO stability principle, comments on the stability of the system. How does this compare with your results from the pole analysis?**

By giving the step input, step response is observed which shows that the ***speed*** response of system is stable w.r.t. BIBO stability principle. The response become constant after covering the transient period. The pole analysis also shows that speed response of servo motor is stable because pole is in left half plane and on real axis.

- **Based on the measured/observed *position* response and the BIBO stability principle, comments on the stability of the system. How does this compare with your results from the pole analysis?**

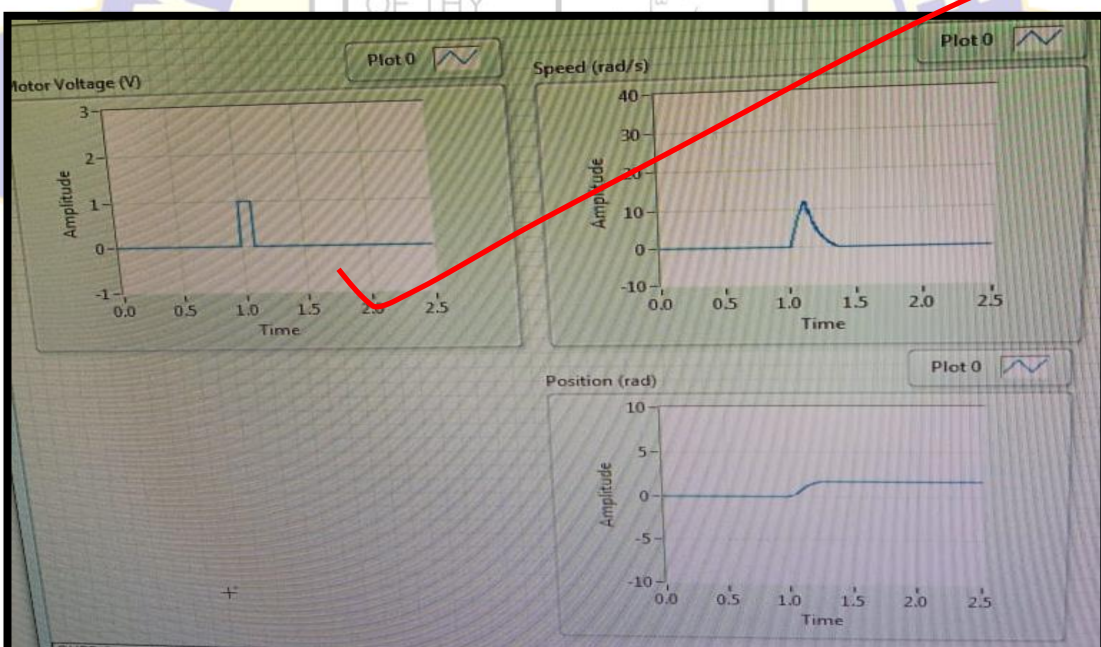
By giving the step input, step response is observed which shows that the ***position*** response of system is marginally stable w.r.t. BIBO stability principle. ~~The pole analysis also shows that position response of servo motor is marginally stable because of one pole at origin~~.

- **Is there an input where the open-loop servo position response is BIBO stable? If so, modify the VI to include your input, test it on the servo, and show the position response. Based on this result, how could you define marginal stability?**

Hint: Try an impulse (short step) and compare the position for impulse response with the step response observed earlier.

Applying the impulse response (waveform shown below), it is concluded that system is stable for impulse input. The system response become stable/constant after covering a transient period. For impulse input the position response is BIBO stable. Based on the result, we can say "*A system is marginally stable if the system is stable for some bounded inputs and unstable for others*".

<u>Front Panel:</u>



<u>Conclusion</u>

In this exercise we conclude that stability of system is defined by the nature of input. For some input the system is stable while it is unstable for some other inputs. If bounded input is applied and response is bounded than system is stable otherwise system is unstable. Marginal stability of the system in term of bounded input bounded output stability criteria is defined as: *the system is stable for some bounded inputs and unstable for other.*

Second Order Step Response:

The *standard second-order* transfer function has the form

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \quad \longrightarrow 3$$

where ω_n is the natural frequency and ζ is the damping ratio. The properties of its response depend on the values of the parameters ω_n and ζ .

Consider a second-order system as shown in the above equation when subjected to a step input given by

$$R(s) = \frac{R_0}{s},$$

with a step amplitude of $R_0 = 1.5$. The system response to this input is shown in Figure 2, where the red trace is the output response $y(t)$ and the blue trace is the step input $r(t)$.

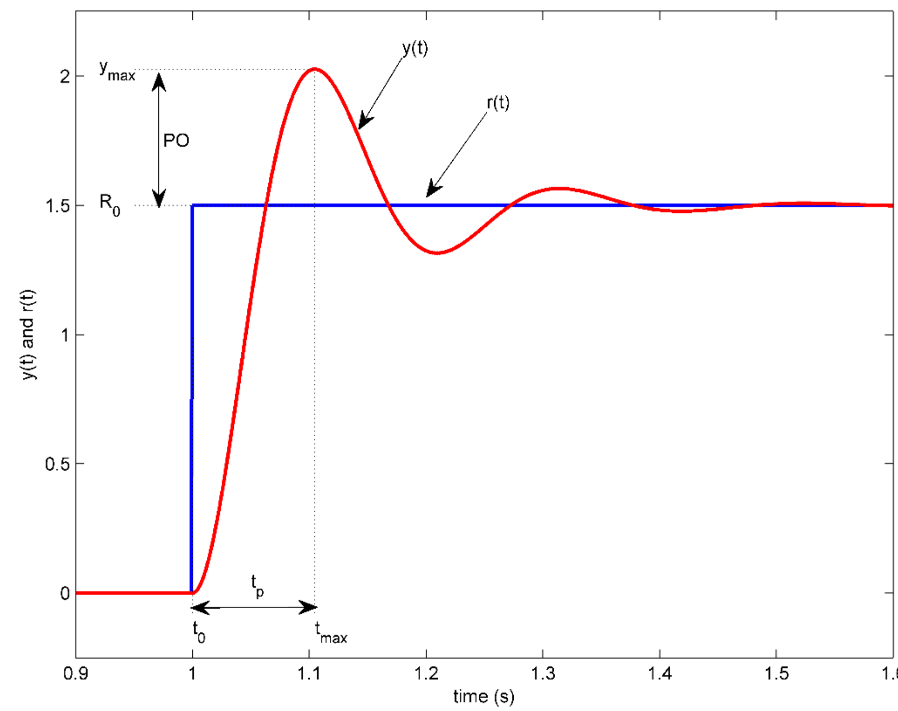


Figure 2 Standard second order step response

Peak Time and Percent Overshoot:

` The maximum value of the response is denoted by the variable y_{max} and it occurs at a time t_{max}. For a response similar to Figure 2, the percent overshoot is found using

$$PO = \frac{100\,(y_{max} - R_0)}{R_0}.$$

From the initial step time t_0 the time it takes for the response to reach its maximum value is

$$t_p = t_{max} - t_0$$

This is called the *peak time* of the system.

In a second-order system, the amount of overshoot depends solely on the damping ratio parameter, and it can be calculated using the equation

$$PO = 100e^{\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)}.$$

The peak time depends on both the damping ratio and natural frequency of the system, and it can be derived as

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}.$$

Generally speaking, the damping ratio affects the shape of the response while the natural frequency affects the speed of the response.

Unity Feedback:

The unity-feedback control loop shown in Figure 3 will be used to control the position of the QUBE-Servo. The input to the plant is the motor voltage $V_m(s)$. The reference is the desired motor position $R(s) = \Theta_d(s)$ and the output $Y(s)$ is the actual position $Y(s) = \Theta_m(s)$.

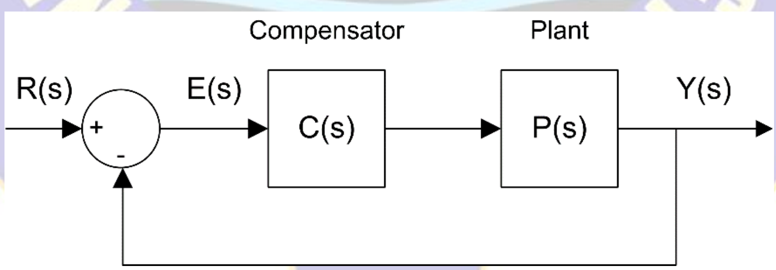


Figure 3 Unity Feedback Loop

The voltage-to-position transfer function is given by

$$P_{v-p} = \frac{\Theta_m(s)}{V_m(s)} = \frac{K}{s(\tau s + 1)}$$

where $\Theta_m(s) = L\,[\theta_m(t)]$ is the load gear position.

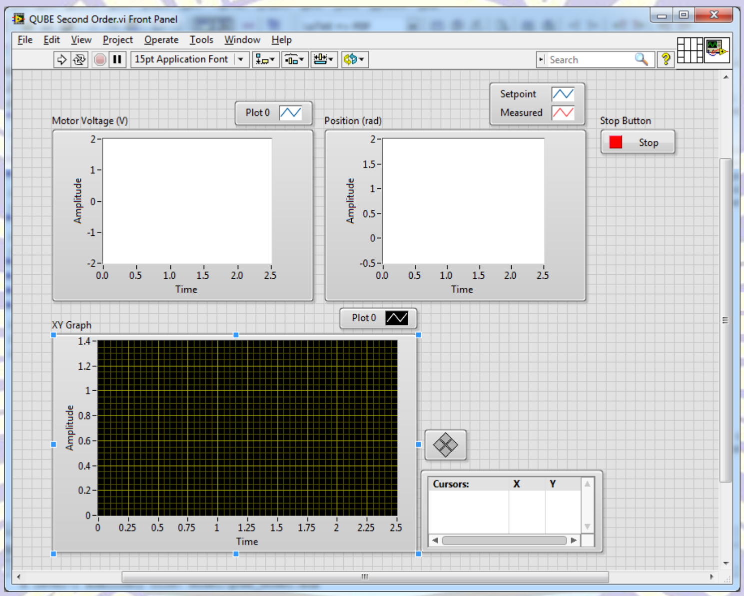
The controller is denoted by $C\,(s)$ whose value is unity. $(C(s) = 1)$

The closed-loop transfer function of the QUBE-Servo position control from the reference input $R(s) = \Theta_d(s)$ to the output $Y(s) = \Theta_m(s)$ using unity feedback as shown in Figure 3 is

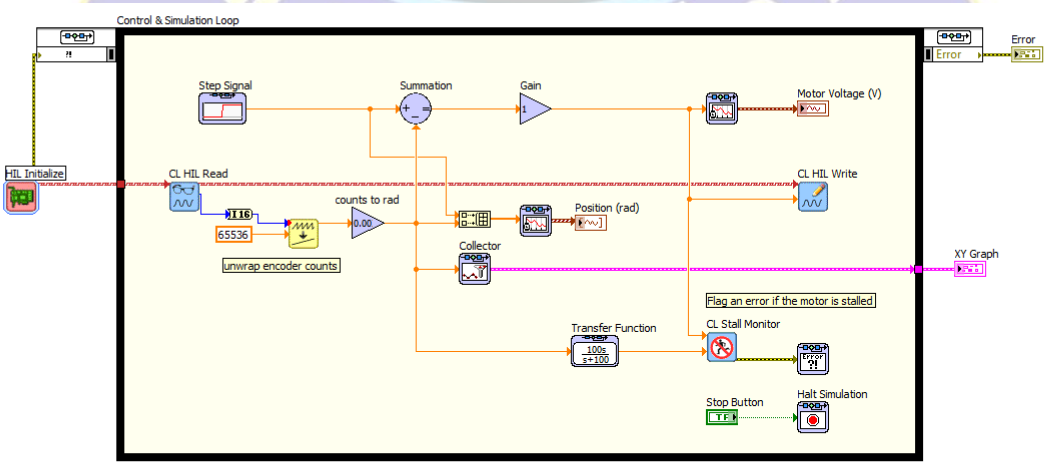
$$\frac{\Theta_m(s)}{\Theta_d(s)} = \frac{\frac{K}{\tau}}{s^2 + \frac{1}{\tau}s + \frac{K}{\tau}} \longrightarrow 4$$

Tasks

Design the VI as shown in Figure 4(b) implementing the unity feedback control. A step reference of 1 rad is applied at 1 second and the controller runs for 2.5 seconds. This is the desired position. To apply your step for a 2.5 seconds, set the *Final Time* of the Simulation Loop to 2.5 (instead of *inf*). As shown in Figure 4, the unity feedback step response is "saved" using the Collector block from the *Control Design & Simulation* | *Simulation* | *Utilities* palette and displayed in an XY Graph. Use the cursors to take the measurements.



(a) Front Panel



(b) Block Diagram

Figure 4: Unity feedback position control of QUBE-Servo

- Given the QUBE-Servo closed-loop equation under unity feedback system and the model parameters K and τ which were already obtained from the bump modeling test, find the natural frequency and damping ratio of the system.

Comparing Equation 3 and 4:

$$\omega_n = \sqrt{\frac{K}{\tau}} = \sqrt{\frac{23.2}{0.13}} = 13.3\ rad/s$$

$$2\zeta\omega_n = \frac{1}{\tau}$$

$$\zeta = \frac{1}{2\omega_n\tau} = \frac{1}{2(13.3)(0.13)} = 0.287$$

- Based on your obtained ω_n and ζ, also find the peak time and percent overshoot using the formulas.

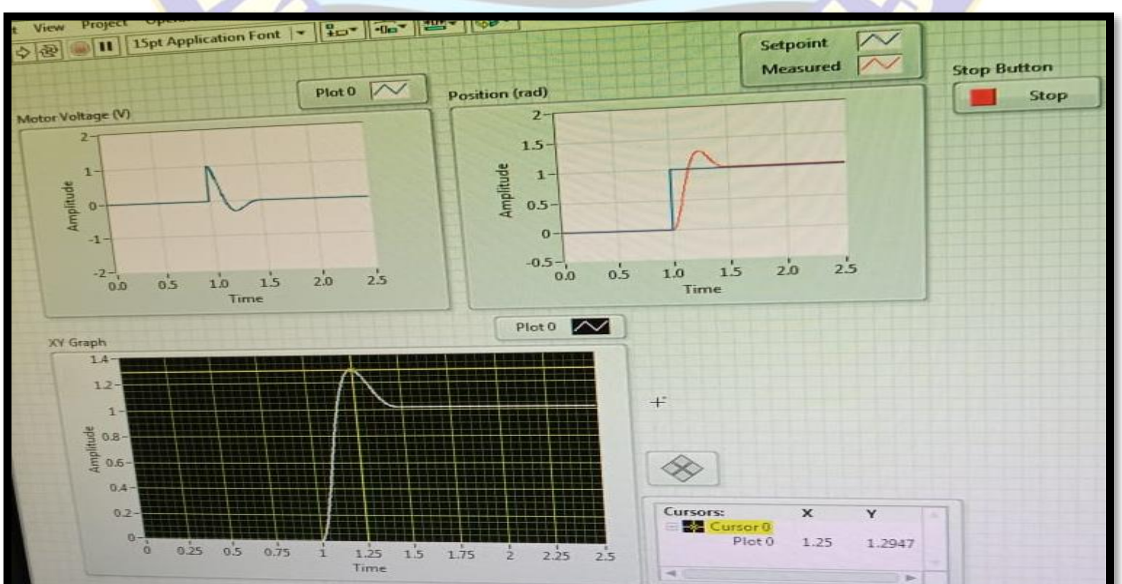
$$PO = 100e^{\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)}.$$

PO = 100 x $e^{\left(\frac{-\pi(0.287)}{\sqrt{1-0.287^2}}\right)}$
PO = 100 x 0.39
PO = 39%

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}.$$

$$t_p = \frac{\pi}{13.3\sqrt{1-0.287^2}} = \frac{\pi}{12.74} = 0.247\ s$$

<u>Unity feedback QUBE-Servo step response (Front Panel)</u>



- Now evaluate the peak time and percent overshoot from the response and compare that with your theoretical results.

  Hint: Use the cursor palette in the XY Graph to measure points off the plot.

$$PO = \frac{100\,(y_{max} - R_0)}{R_0}.$$

PO = 100 x (1.2947 – 1)/1
PO = 29.4%

$t_p = t_{max} - t_0$ = 1.25 - 1
$t_p = 0.25\ s$

Why there is difference b/w $PO_{Theoretical}$ & $PO_{Graphical}$?

The difference b/w these two values is because of internal friction and losses within the motor which we didn't account in our calculations.

Conclusion:

In this lab, we performed stability test with Qube Servo motor. We performed calculation to find out pole location and then predicted about the stability of system. We did the task on hardware and found out whether the predicted results are same or not. We understood the concept of stable, unstable and marginally stable system. We analyzed the second order underdamped system. We calculated the natural frequency and damping ratio. From these values we calculated the percentage overshoot and peak time and then compared the theoretical values with the graphical values.