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Lab # 5

Performance of First order and second order systems

Objectives:

The objective of this lab was to understand and implement the Effect of Disturbance on Control System Performance in MATLAB and use of its command in MATLAB.

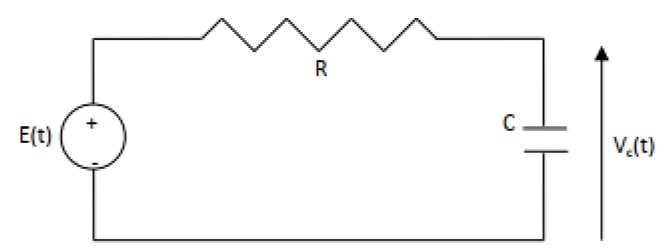
Introduction:

Disturbance signals represent unwanted inputs which affect the control-system's output, and result in an increase of the system error. It is the job of the control-system engineer to properly design the control system to partially eliminate the effects of disturbances on the output and system error. The effect of disturbance is changing the purpose or reduce the efficiency of the output, using closed loop transfer function reduce this effect to minimize the disturbance as possible.

Theory:

It is the distortion or addition of unnecessary data or value in the system through the summer or any other way. It also changes the output of the system hence making the output useless. We will now see the effect of disturbance on the control system of our system and see how it is affecting our output.

An electrical RC-circuit is the simplest example of a first order system. It comprises of a resistor and capacitor connected in series to a voltage supply as shown below on Figure 1.



If the capacitor is initially uncharged at zero voltage when the circuit is switched on, it starts to charge due to the current 'i' through the resistor until the voltage across it reaches the supply voltage. As soon as this happens, the current stops flowing or decays to zero, and the circuit becomes like an open circuit. However, if the supply voltage is removed, and the circuit is closed, the capacitor will discharge the energy it stored again through the resistor. The time it takes the capacitor to charge depends on the time constant of the system, which is defined as the time taken by the voltage across the capacitor to rise to approximately 63% of

the supply voltage. For a given RC-circuit, this time constant is $\tau = RC$ Hence its magnitude depends on the values of the circuit components.
The RC circuit will always behave in this way, no matter what the values of the components. That is, the voltage across the capacitor will never increase indefinitely. In this respect we will say that the system is passive and because of this property it is stable. For the RC-circuit as shown in Fig. 1, the equation governing its behavior is given by

$$\frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}E \quad \text{where } v_c(0) = v_0 \qquad (1)$$

where $V_c(t)$ is the voltage across the capacitor, R is the resistance and C is the capacitance.

The constant $\tau = RC$ is the time constant of the system and is defined as the time required by the system output i.e. $V_c(t)$ to rise to 63% of its final value (which is E). Hence the above equation (1) can be expressed in terms of the time constant as:

$$\tau\frac{dv_c(t)}{dt} + v_c(t) = E \quad \text{where } v_c(0) = v_0 \qquad (1)$$

Obtaining the transfer function of the above differential equation, we get

$$\frac{V_c(s)}{E(s)} = \frac{1}{\tau s+1} \qquad (2)$$

where τ is time constant of the system and the system is known as the first order system. The performance measures of a first order system are its time constant and its steady state.

Exercise 1:

a) Given the values of R and C, obtain the unit step response of the first order system.
   a. R=2KΩ and C=0.01F
   b. R=2.5KΩ and C=0.003F

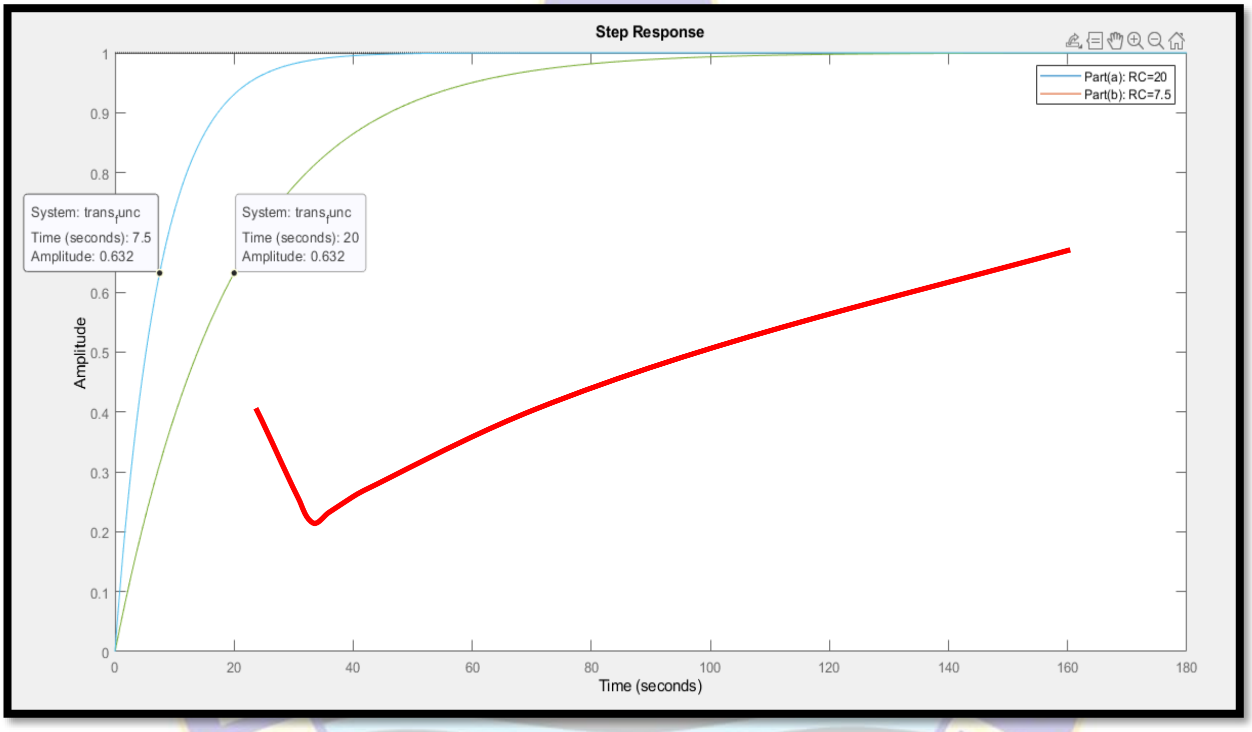
b) Verify in each case that the calculated time constant ($\tau = RC$) and the one measured from the figure as 63% of the final value are same.

c) Obtain the steady state value of the system.

(a): Code:

```
%Calculations
%T = RC     Given R = 2000 C = 0.01
% T = 2000 x 0.01
% T = 20
% TF = 1/(Ts+1)]
% TF = 1/(20s+1)
input = [20 1];
output = [1];
trans_func = tf(output,input);
step(trans_func)
hold on;
%Calculations
%T = RC     Given R = 2500 C = 0.003
```

```
% T = 2500 x 0.003
% T = 7.5
% TF = 1/(Ts+1)]
% TF = 1/(7.5s+1)
input = [7.5 1];
output = [1];
trans_func = tf(output,input);
step(trans_func)
legend('Part(a): RC=20','Part(b): RC=7.5')
```



(b): From the response plotted above the indicated points showing the calculated time constant is equal to the value measured from the figure which is 63% of final value.

(c): From the plot, the steady state value of the system is found to be 1.

Overview Second Order Systems:

Consider the following Mass-Spring system shown in the Figure 2. Where K is the spring constant, B is the friction coefficient, x(t) is the displacement and F(t) is the applied force:

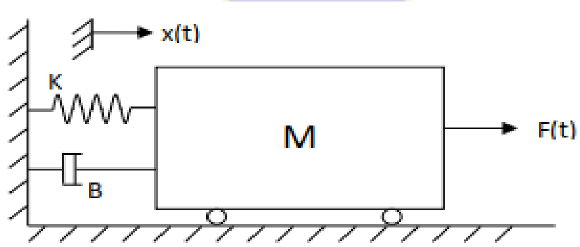


Figure 2. Mass-Spring system

The differential equation for the above Mass-Spring system can be derived as follows

$$M\frac{d^2x(t)}{dt^2} + B\frac{\partial x(t)}{\partial t} + Kx(t) = F(t)$$

Applying the Laplace transformation, we get

$$(Ms^2 + Bs + K) * X(s) = F(s)$$

provided that, all the initial conditions are zeros. Then the transfer function representation of the system is given by

$$TF = \frac{Output}{Input} = \frac{F(s)}{X(s)} = \frac{1}{(Ms^2 + Bs + K)}$$

The above system is known as a second order system.

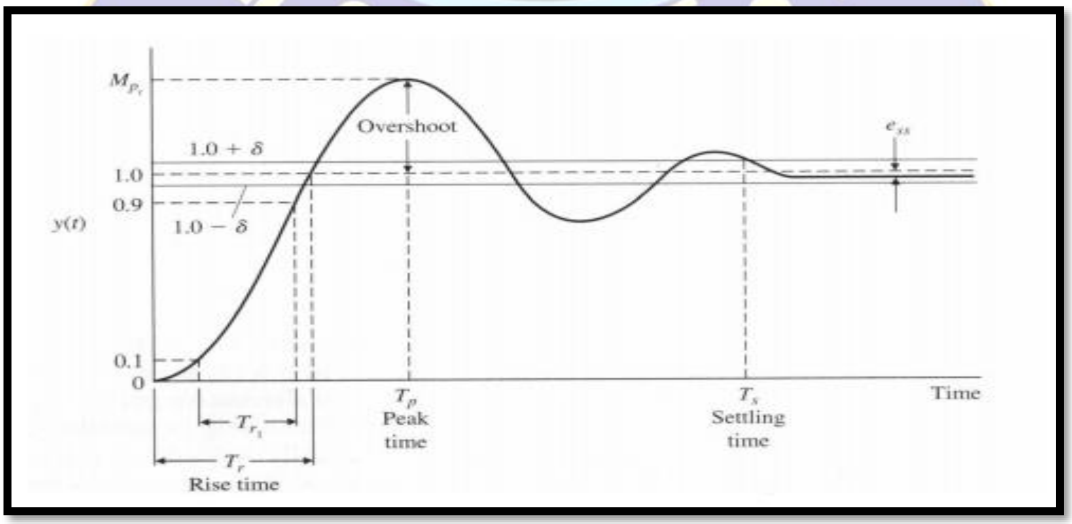
The generalized notation for a second order system described above can be written as

$$Y(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} R(s)$$

With the step input applied to the system, we obtain

$$Y(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

where $0 < \zeta < 1$. The transient response of the system changes for different values of damping ratio, ζ. Standard performance measures for a second order feedback system are defined in terms of step response of a system. Where, the response of the second order system is shown below.



Rise Time: The time for a system to respond to a step input and attains a response equal to a percentage of the magnitude of the input. The 0-100% rise time, T_r, measures the time to 100% of the magnitude of the input. Alternatively, T_{r1}, measures the time from 10% to 90% of the response to the step input.

Peak Time: The time for a system to respond to a step input and rise to peak response.

Overshoot: The amount by which the system output response proceeds beyond the desired response. It is calculated as

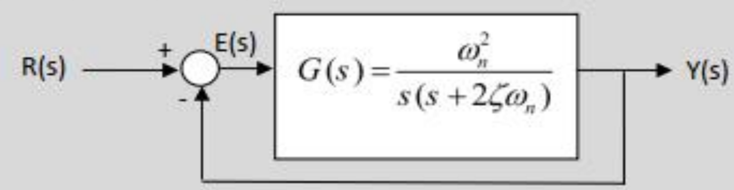
$$P.O. = \frac{M_{P_t} - f\upsilon}{f\upsilon} \times 100\%$$

where M_{Pt} is the peak value of the time response, and f_v is the final value of the response.

Settling Time: The time required for the system's output to settle within a certain percentage of the input amplitude (which is usually taken as 2%). Then, settling time, Ts, is calculated as

$$T_s = \frac{4}{\zeta\omega_n}$$

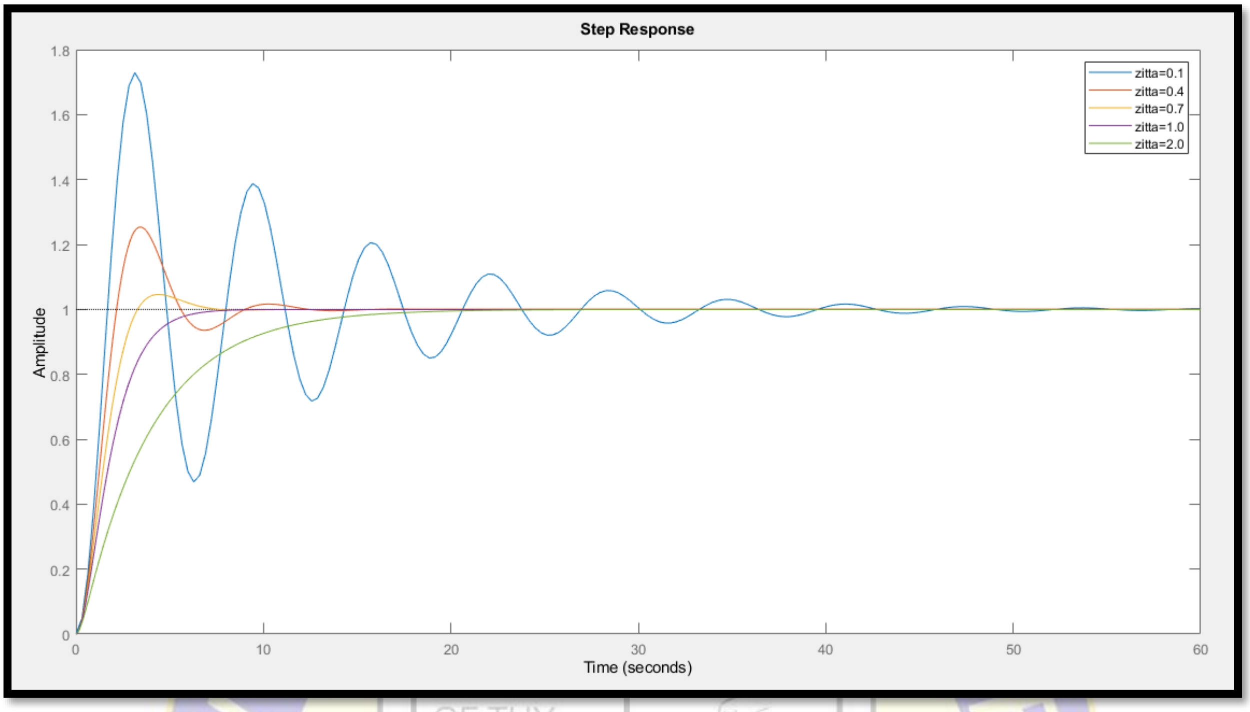
Exercise 2: Effect of damping ratio ζ on performance measures. For a single-loop second order feedback system given below



Find the step response of the system for values of ωn = 1 and ζ = 0.1, 0.4, 0.7, 1.0 and 2.0.

Code:

```
%Calculations
%TF = Y(s)/R(s) = G(s)/(1+G(s)H(s))
%G(s) = w^2/(s^2 + 2*zitta*w*s)
%H(s) = 1
%TF = w^2/(s^2 + 2*zitta*w*s + w^2)
%Given that:
w=1; %Natural frequency
a=[0.1,0.4,0.7,1.0,2.0];   %These are the different values of damping ratio
%TF = 1/(s^2 + 2*zitta*s + 1)
for c=1:5
    zitta = a(c);
    trans_func = tf([1],[1 2*zitta 1])
    step(trans_func)
    stepinfo(trans_func)
    hold on
end
legend('zitta=0.1','zitta=0.4','zitta=0.7','zitta=1.0','zitta=2.0')
```

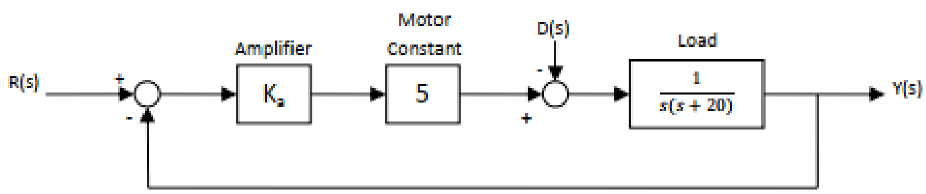


Zeta	Rise Time	% Overshoot	Settling Time	Peak Time	Steady state value
.1	1.1272	72.9156	38.3730	3.1416	1
.4	1.4652	25.3741	8.4094	3.4549	1
.7	2.1268	4.5986	5.9789	4.4078	1
1	3.3579	0	5.8339	9.7900	1
2	8.2308	0	14.8789	27.3269	1

Exercise#3

Exercise 2: Design of a Second order feedback system based on performances.

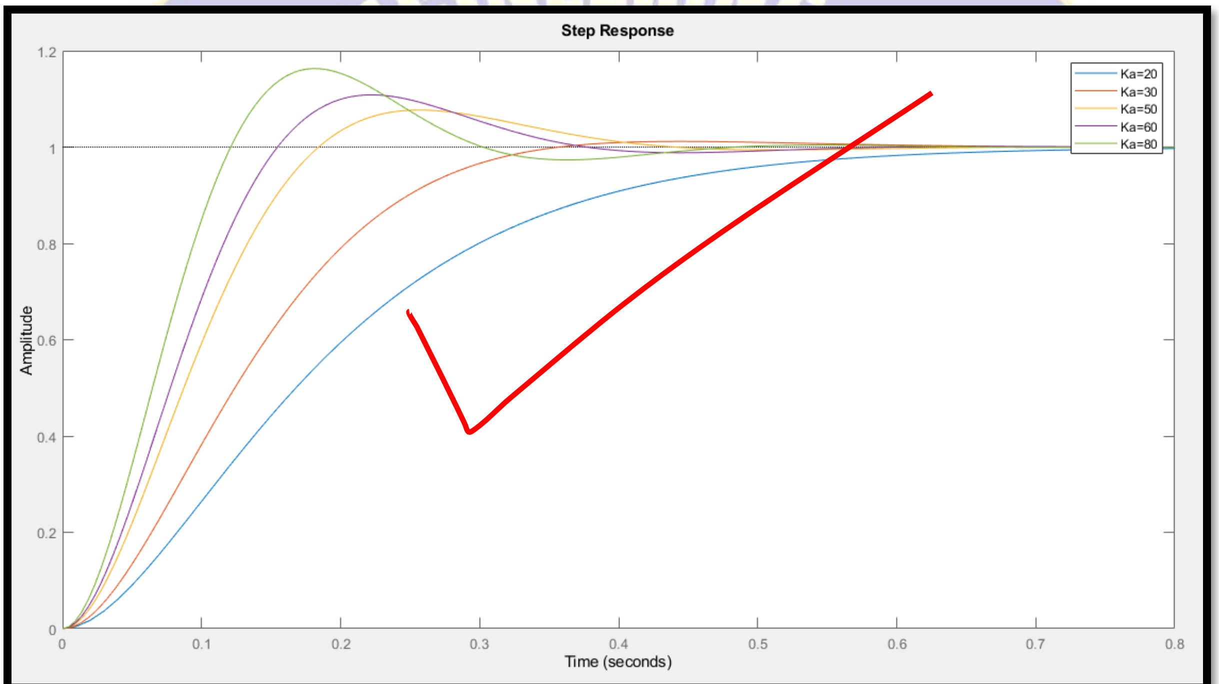
For the motor system given below, we need to design feedback such that the overshoot is limited and there is less oscillatory nature in the response based on the specifications provided in the table. Assume no disturbance (D(s)=0).



Use MATLAB, to find the system performance for different values of K_a and find which value of the gain K_a satisfies the design condition specified. Use the following table.

Code:

```
%Y(s)/R(s) =?
%G(s) = Ka*5*1/(s^2+20*s)      cascaded blocks
%H(s) = 1                      Feedback transfer function
%TF = G(s)/(1+G(s)H(s))
%TF = 5*Ka/(s^2 + 20*s + 5*Ka)
Gain = [20,30,50,60,80];
for c = 1:5
    Ka = Gain(c);
    trans_func = tf([5*Ka],[1 20 5*Ka])
    step(trans_func)
    stepinfo(trans_func)
    hold on
end
legend('Ka=20','Ka=30','Ka=50','Ka=60','Ka=80')
```



K_a	20	30	50	60	80
Percent Overshoot	0	1.175	7	10	16
Settling time	0.58	0.32	0.38	0.34	0.4

From the response plotted above, the **value of gain K_a** which specified the design condition is **<u>30</u>**.

Conclusion:

In this lab, we have learnt about block diagram reduction. We have applied rules of reduction to find out the transfer function and then with the help of MATLAB we have observed the step response of the system. From the response plot, we have measured different parameters like percentage overshoot, setting time, rise time, peak time, etc. We have understood how to find a transfer function of a system.