and

$$g(x_2, y_2) = -0.01055.$$

Further continuation gives

$$f_x\big|_{(x_2,y_2)} = 1.2617, \quad f_y\big|_{(x_2,y_2)} = -3.1829$$

$$g_x\big|_{(x_2,y_2)} = 3.1829, \quad g_y\big|_{(x_2,y_2)} = 1.2617$$

and

$$J = \begin{vmatrix} f_x & f_y \\ g_x & g_y \end{vmatrix} = 11.7227 \neq 0$$

which yield

$$h = 0.006379, \qquad k = -0.0773$$

Thus, the third approximation is

$$x_3 = x_2 + h = 0.8901, \qquad y_3 = y_2 + k = 0.5926$$

and

$$f(x_3, y_3) = -0.0127, \quad g(x_3, y_3) = 0.0152$$

The three iterations are tabulated as

i	x_i	y_i	$f(x_i, y_i)$	$g(x_i, y_i)$
1	0.9	0.7	–0.394	–0.042
2	0.8837	0.6003	–0.0327	–0.0106
3	0.8901	0.5926	–0.0127	0.0152

EXERCISES

2.1 Find the real root of the equation, $x^3 - 3x - 5 = 0$ by the bisection method.

2.2 Find the real root of the equation, $x^3 + x - 3 = 0$, using Regula-Falsi method, correct to four places of decimal.

2.3 Find the real root of the equation $x^6 - x^4 - x^3 - 1 = 0$, which lies between 1.4 and 1.5, correct to four places of decimal, by the method of false position, obtained after three successive approximations.

2.4 Use Regula-Falsi method to find the real roots of the equation $x^3 - \sin x + 1 = 0$ correct to four decimal places after three successive approximations between $(-2, -1)$.

2.5 Explain the method of false position for finding a real root of the equation $f(x) = 0$, and hence derive the general formula.

2.6 Use Regula-Falsi method to compute the root of the equation $\cos x = xe^x = 0$.

2.7 Find the root of the equation $2x = \cos x + 3$, correct to three decimal places using iteration method.

2.8 Find a root of the equation $x \log_{10} x = 4.77$ by Newton-Raphson method, correct to two decimal places.

2.9 Explain the Newton-Raphson method to find a root of the equation $f(x) = 0$, and hence derive its iteration formula.

2.10 Geometrically explain Newton-Raphson method to find a root of the equation $f(x) = 0$ and hence derive the general formula.

2.11 Obtain the real root of the equation $x^3 - 3x - 5 = 0$ using Newton-Raphson method, after third iteration.

2.12 Find a real root of the equation, $x^4 - x - 10 = 0$ using Newton-Raphson method correct to four decimal places.

2.13 Apply Newton-Raphson method to determine a root of the equation $\cos x = xe^x$ correct to three decimal places, using the initial approximation $x_0 = 1$.

2.14 Set up the Newton's scheme of iteration for finding the p-th root of positive number N.

2.15 Obtain the cube root of 12 using Newton-Raphson iteration.

2.16 Find the first approximation of the root of the equation $x^3 - 3x - 5 = 0$ using Muller's method, which lies between 2 and 3.

2.17 Find the first approximation to the root of the equation

$$f(x) = \sin x - \frac{x}{2} = 0$$

near $x = 2.0$, using Muller's method.

2.18 Using Graeffe's root squaring method, find the roots of the equation $x^3 - 4x^2 + 3x + 1 = 0$ with the help of a calculator.

2.19 Using the method of false position, find the root of $x \sin x - 1 = 0$ which lies in the interval (0, 2).

2.20 Find the quadratic factors of

$$x^4 - 5.7x^3 + 26.7x^2 - 42.21x + 69$$

Using Bairstows method with $(x^2 - 1.5x + 4.3)$ as a starting factor.

2.21 Using Bairstows method, find the quadratic factors of the polynomial

$$2x^4 + 7x^3 - 4x^2 + 29x + 14$$

with $(x^2 + 5x + 2)$ as a starting factor.

2.22 Find the solution of

$$f(x, y) = x^3 - 3xy^2 + 1 = 0$$

$$g(x, y) = 3x^2y - y^3 = 0$$

taking (1, 1) as the initial approximation using Newton's method.

2.23 Using Newton's method, find the solution of

$$f(x, y) = 4x^2 + y^2 + 2xy - y - 2 = 0$$

$$g(x, y) = 2x^2 + 3xy + y^2 - 3 = 0$$

taking (0.4, 0.9) as the initial approximation.

Answers to Exercises

CHAPTER 2

2.1 Given $f(x) = x^3 - 3x - 5$, we note $f(2) = -3, f(3) = 13$. Hence, a real root lies between $x_1 = 2$ and $x_2 = 3$. Various approximations are given below:

n	x_{n+1}	$f(x_{n+1})$
2	2.5	3.125
3	2.25	−0.3594
4	2.375	1.2715
5	2.3125	0.4290
6	2.28125	0.0281

2.2 Given $f(x) = x^3 + x - 3$, we note $f(1) = -1, f(2) = 7$. Hence, a real root lies between $x_1 = 1$ and $x_2 = 2$. Various approximations are given below:

n	x_{n+1}	$f(x_{n+1})$
2	1.125	−0.45117
3	1.17798	−0.1874
4	1.1994	−0.07519
5	1.2079	−0.02972
6	1.21125	−0.011693
7	1.21257	-4.5705×10^{-3}
8	1.21308	-1.78507×10^{-3}
9	1.21328	-7.13168×10^{-4}
10	1.21336	-2.794298×10^{-4}

Therefore, the required root is 1.2134

2.3 Given $f(x) = x^6 - x^4 - x^3 - 1$, we note that $f(1.4) = -0.056064$, $f(1.5) = 1.953125$. Hence, the real root lies between $x_1 = 1.4$ and $x_2 = 1.5$. Various approximations to the root are given below:

n	x_{n+1}	$f(x_{n+1})$
2	1.40279	−0.012735
3	1.40342	-2.86077×10^{-3}
4	1.40356	-6.609198×10^{-4}

Hence, the required, real root is 1.4036.

2.4 Given $f(x) = x^3 - \sin x + 1$. We note that $f(-1) = 0.84147$, $f(-2) = -6.0907$. Since $f(-1)$ and $f(-2)$ are of opposite signs, the real root lies between -2 and -1. Let $x_1 = -1$, $x_2 = -2$, using Regula–Falsi method, various approximations to the root are given below:

n	x_{n+1}	$f(x_{n+1})$
2	−1.12139	0.49055
3	−1.18688	0.25526
4	−1.21959	0.12495

Thus, the required root after three successive approximations is −1.2196.

2.6 Given $f(x) = \cos x - xe^x$, we observe $f(0) = 1$, $f(1) = -2.177979$. Hence, the root lies between $x_1 = 0$ and $x_2 = 1$. Using Regula–Falsi method, various approximations to the root are given below:

n	x_{n+1}	$f(x_{n+1})$
2	0.31467	0.51987
3	0.44673	0.20355
4	0.49402	0.70802×10^{-1}
5	0.50995	0.23608×10^{-1}
6	0.51520	0.77601×10^{-2}
7	0.51692	0.25389×10^{-2}
8	0.51749	0.82936×10^{-3}

2.7 Rewrite the equation in the form $x = (\cos x + 3)/2 = \phi(x)$. We observe $f(0) = -4$, $f(\pi/2) = \pi - 3 > 0$, showing that a root lies in the interval $(0, \pi/2)$. Also, $|\phi'(x)| = \sin(x/2) < 1$ for all x in $(0, \pi/2)$. We start with $x_0 = \pi/2$. Then the successive iterations are

$$x_1 = \phi(x_0) = 1.5, \qquad f(x_1) = 0.07074$$

$$x_2 = \frac{1}{2}\left(\cos\frac{1.5 \times 180}{\pi} + 3\right) = 1.5353686, \qquad f(x_2) = -0.0353169$$

$$x_3 = 1.5177, \qquad f(x_3) = 0.01766$$

$$x_4 = 1.526, \qquad f(x_4) = -7.21865 \times 10^{-3}$$

$$x_5 = 1.522, \qquad f(x_5) = 4.77696 \times 10^{-3}$$

$$x_6 = 1.524, \qquad f(x_6) = -1.22075 \times 10^{-3}$$

Hence, the root is 1.524.

2.8 $f(x) = x \log_{10} x - 4.77$

$f'(x) = \log_{10} x + \log_{10} e = \log_{10} x + 0.4343$

Since, $e = 2.71828$, $\log_{10} e = 0.4343$.

Newton's–Raphson formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n \log_{10} x_n - 4.77}{\log_{10} x_n + 0.4343}$$

Note that $f(6) = -0.1011, f(7) = 1.1457$. Therefore, the required root lies in the interval (6, 7). Taking $x_0 = 7$ as the initial approximation, the successive approximations are

$$x_1 = 7 - \frac{1.145686}{1.2794} = 6.1045, \qquad f(x_1) = 0.026$$

$$x_2 = 6.1045 - \frac{0.026}{1.21995} = 6.0832, \qquad f(x_2) = 3.23613 \times 10^{-5}$$

$$x_3 = 6.0832 - \frac{3.23613 \times 10^{-5}}{1.21843} = 6.0832, \qquad f(x_3) = 3.23613 \times 10^{-5}$$

Hence, the required root is 6.0832.

2.11 $f(x) = x^3 - 3x - 5, f'(x) = 3x^2 - 5, f''(x) = 6x, f(2) = -3, f(3) = 13$. Taking $x_0 = 3$, the successive iterations by Newton's method are

n	x_n	$f(x_n)$
1	2.45833	2.48164
2	2.29431	0.19399
3	2.27915	1.59433×10^{-3}

2.12 $f(x) = x^4 - x - 10, f'(x) = 4x^3 - 1, f''(x) = 12x^2, f(1) = -10, f(2) = 4$. Taking $x_0 = 2$, the successive iterations by Newton's method are

n	x_n	$f(x_n)$
1	1.87096	0.38268
2	1.85577	4.6201×10^{-3}
3	1.85558	-6.2104×10^{-5}

2.13 0.5175

2.14 $x_{n+1} = \dfrac{1}{p}\left[\dfrac{(p-1)x_n^p + N}{x_n^{p-1}}\right]$

2.15 Taking $x_0 = 2.5$, the first approximation is 2.30666

2.16 Let $f(x) = x^3 - 3x - 5$, then $f(1) = -7, f(2) = -3, f(3) = 13$. Muller's method can be conveniently started by taking

$$x_{i-2} = 1, \quad x_{i-1} = 2 \quad x_i = 3$$
$$f_{i-2} = -7, \quad f_{i-1} = -3 \quad f_i = 13$$

The first approximation is

$$x_{i+1} = x_i + \lambda h_i = 3 - 0.74043 = 2.25957$$

2.17 Muller's method can be conveniently started by taking

$$x_{i-2} = 1.8, \qquad x_{i-1} = 2.0 \qquad x_i = 2.2$$
$$f_{i-2} = 0.07385, \qquad f_{i-1} = -0.09070 \qquad f_i = -0.29150$$

The first approximation is given by

$$x_{i+1} = x_i + \lambda h_i = 2.2 - (1.52373)\,(0.2) = 1.89525$$

2.18 $i = 1$, the new polynomial is $x^3 - 10x^2 + 17x - 1 = 0$

$i = 2$, the new polynomial is $x^3 - 66x^2 + 269x - 1 = 0$

$i = 3$, the new polynomial is $x^3 - 3818x^2 + 72229x - 1 = 0$

The corresponding approximations of the roots are

$$\sqrt{\frac{1}{17}} = 0.2425, \quad \sqrt{\frac{17}{10}} = 1.3038, \quad \sqrt{10} = 3.1623$$

$$\sqrt[4]{\frac{1}{269}} = 0.2469, \quad \sqrt[4]{\frac{269}{66}} = 1.4208, \quad \sqrt[4]{66} = 2.8503$$

$$\sqrt[8]{\frac{1}{72229}} = 0.2469, \quad \sqrt[8]{\frac{72229}{3818}} = 1.4441, \quad \sqrt[8]{\frac{3818}{1}} = 2.8037$$

Using the given polynomial, the roots are found to be -0.2469, 1.4441 and 2.8037.

2.19 Given $f(x) = x \sin x - 1$, we observe the $f(0) = -1.0$, $f(2) = 0.81859$. Using the method of false position, various approximations to the root are given as

n	x_{n+1}	$f(x_{n+1})$
2	1.09975	−0.02002
3	1.12124	0.00983
4	1.11416	0.563×10^{-5}

2.20 $(x^2 - 1.5x + 4.3)(x^2 - 4.2x + 16)$

2.21 $(x^2 + 5x + 2)(2x^2 - 3x + 7)$

2.22 $x = 0.5086, \quad y = 0.8411$

Exact solution is, $x = 0.5$, $y = 0.866$

2.23 $x = 0.5005$, $y = 0.9993$

Exact solution is $x = 0.5$, $y = 1.0$.