Power flow analysis

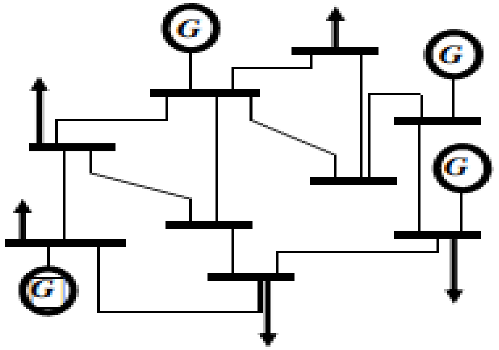
Lec_6

Power Flow Problem

- Computation of voltage magnitude and phase angle at each bus in the system
- In the process we also find
  - Real Power Flows
  - Reactive Power Flows
  - System Losses
- Starting point is the single line diagram of the system

Introduction to Load Flow Analysis

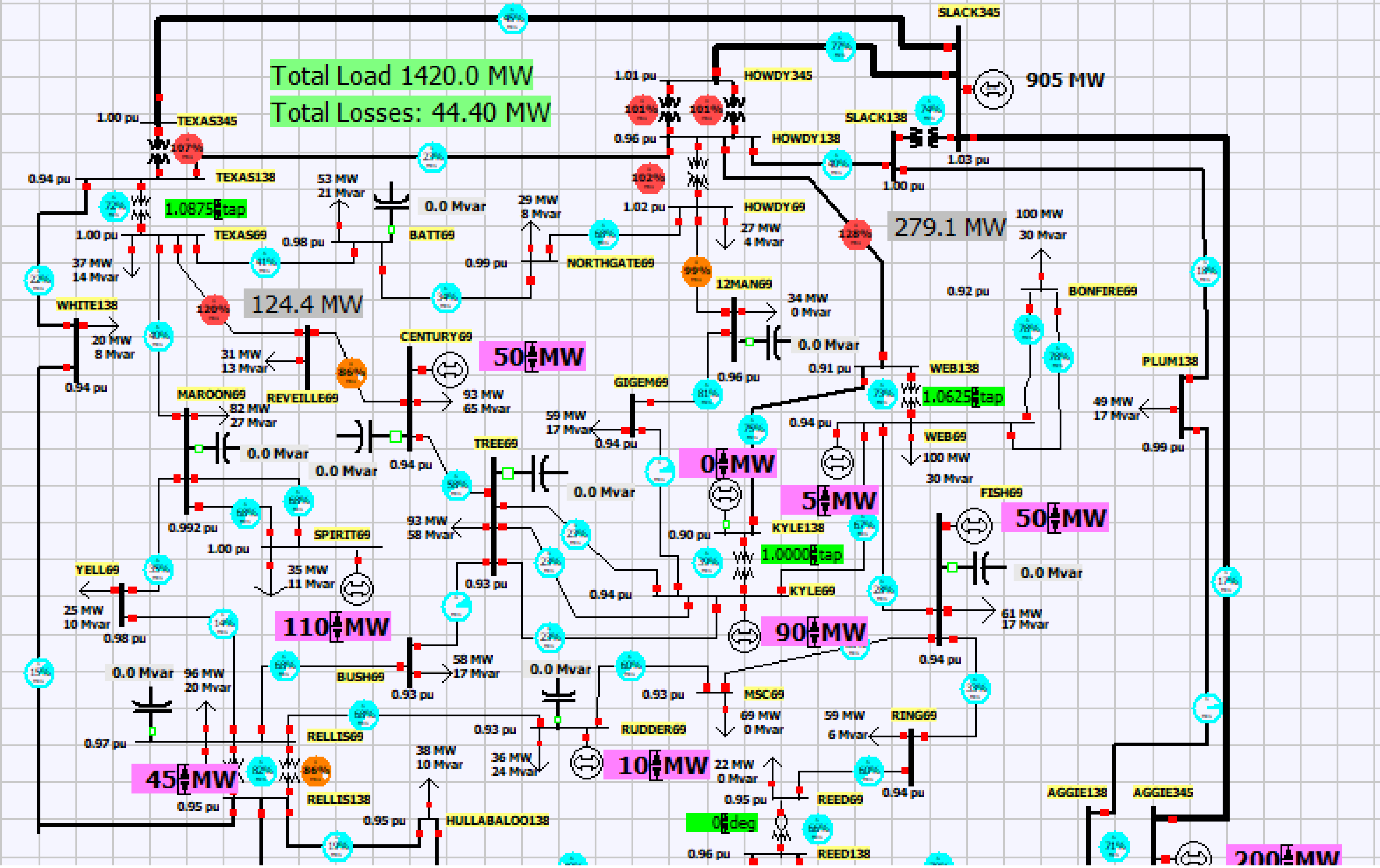
The power flow is the backbone of the power system *operation*, *analysis* and *design*. It is necessary for planning, operation, economic scheduling and exchange power between utilities.



The power flow is also required for many other applications such as *short-circuit* calculations, *transient stability* and *contingency analysis*.

For the network shown, there are some buses connected with the generators and other buses are connected to the loads.

Aggieland Power and Light



Introduction to Load Flow Analysis

The Real and Reactive power is known at each Load bus. The Generator Voltages are Also Specified at the generator buses.

The Transmission Lines interconnecting the buses have resistance and inductance. Therefore, the Electric Current flowing through the lines results in Electrical Losses.

The Generators in the System Must supply the Total Electrical Loads plus the Electrical Losses.

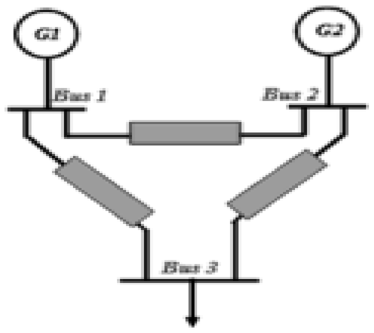
There are some constrains should be considered while running the system:

1. **The Generators Must Operate within their Generation Capabilities.**
2. **The Generators Must Deliver the required power at the Desired Voltage at the Loads.**
3. **There should be no bus voltage either above or below the specified Voltage operating limits.**
4. **There Should be no Over-Loading of equipment, including Transmission Lines and Transformers**

In Case of: an Equipment Over-Loaded Or Voltage-Limit Violation.

The Generation Schedule have to be adjusted and Power Flow in the transmission lines have to be Re-routed or Capacitor Banks have to be switched in order to bring the system into its Normal Operating Conditions.

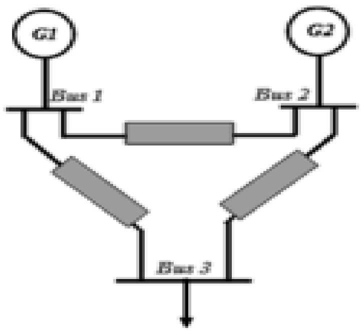
To Satisfy all the previous requirement for a *Reliable Power System Operation*, Power Flow Study is a MUST. The Power flow study is an essential part in power system Operation, Planning and Design.



Consider the three-bus power system. Generators (*G_1 and G_2*) are connected to the first two buses and an electric load is connected to the third bus.

The *real and reactive power demands* are known for the load bus (3). The *generator voltages* are also specified at bus 1 and bus 2.

The three transmission lines interconnecting the buses contain both *resistance and reactance*, thus currents flow through these lines results in *electrical-losses*.



The two generators (*G_1 and G_2*) must jointly supply the total *load requirements and the power losses* in the transmission lines.

The generators are constrained to operate within their *power generation capabilities*.

The generators are also constrained to deliver the required *power at the desired voltage* at the customer loads.

In addition, there should be *no over-loading* of the power system equipments including transmission lines and transformers.

Furthermore, there should be no bus voltage either above or below specified values of the *bus voltage operating limits*.

Solution for the *static operating condition* of a power system:

The *node voltage method* is commonly used for the power system analysis. The formulation of the network equations results in *complex linear equations* in terms of node currents.

In power systems, *powers are known rather than currents*. Thus, resulting equations in terms of power become *non-linear* and must be solved by *iterative techniques*.

These *non-linear equations* are known as *power flow equations* or *load flow equations*.

The *power flow programs* compute the *voltage magnitude* and *phase angle* at each bus bar in the system *under steady-state operation condition*.

These programs use the bus-voltage data to compute the *power flow in the network* and the *power losses* for all equipment and transmission lines.

Formulation of the Bus Admittance Matrix

The first step in developing the mathematical model describing the power flow in the network is the formulation of the bus admittance matrix.

The bus admittance matrix is an $n \times n$ matrix (where n is the number of buses in the system) constructed from the admittances of the equivalent circuit elements of the segments making up the power system.

Most system segments are represented by a combination of shunt elements (connected between a bus and the reference node) and series elements (connected between two system buses).

Node Analysis

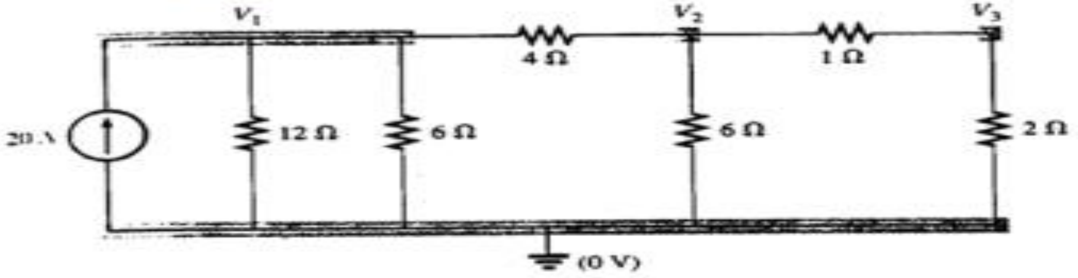


FIG. 8.58
Converting the voltage source to a current source and defining the nodes for the network of Fig. 8.57.

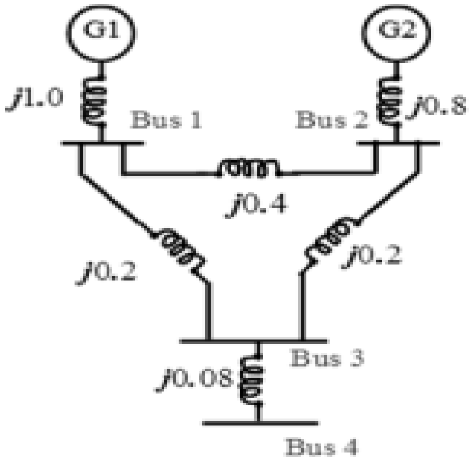
$$V_1:\quad \left(\frac{1}{12\,\Omega}+\frac{1}{6\,\Omega}+\frac{1}{4\,\Omega}\right)V_1-\left(\frac{1}{4\,\Omega}\right)V_2+\quad 0 \quad = 20\text{ V}$$
$$V_2:\quad \left(\frac{1}{4\,\Omega}+\frac{1}{6\,\Omega}+\frac{1}{1\,\Omega}\right)V_2-\left(\frac{1}{4\,\Omega}\right)V_1-\left(\frac{1}{1\,\Omega}\right)V_3 = 0$$
$$V_3:\quad \left(\frac{1}{1\,\Omega}+\frac{1}{2\,\Omega}\right)V_3-\left(\frac{1}{1\,\Omega}\right)V_2+\quad 0 \quad = 0$$

and

$$0.5V_1 \quad -0.25V_2 \quad +0 = 20$$
$$-0.25V_1 \quad +\frac{17}{12}V_2 \quad -1V_3 = 0$$

Example:

Formulate the *bus admittance matrix* for the network shown in the Figure. The *Impedance diagram* of the system is as indicated. Shunt elements are ignored.



Impedance diagram

Solution:

The *node voltage method* is commonly used for the power system analysis. Where,

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Or

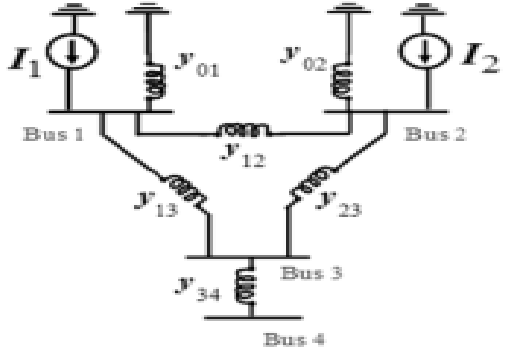
$$I_{bus} = [Y_{bus}]\, V_{bus}$$

The system can be represented in terms of its admittance elements as shown, where:

$$y_{ij} = \frac{1}{Z_{ij}}$$

$$y_{01} = \frac{1}{j1.0} = -j1.0 \qquad y_{12} = -j2.5$$

$$y_{02} = \frac{1}{j0.8} = -j1.25 \qquad y_{13} = y_{23} = -j5.0$$



Admittance diagram

Applying *KCL* at each node (bus), then

$$I_1 = (y_{01} + y_{12} + y_{13})V_1 - y_{12}V_2 - y_{13}V_3$$

$$I_2 = (y_{02} + y_{12} + y_{23})V_2 - y_{12}V_1 - y_{23}V_3$$

$$0 = (y_{31} + y_{32} + y_{34})V_3 - y_{31}V_1 - y_{32}V_2 - y_{34}V_4$$

$$0 = y_{34}V_4 - y_{34}V_3$$

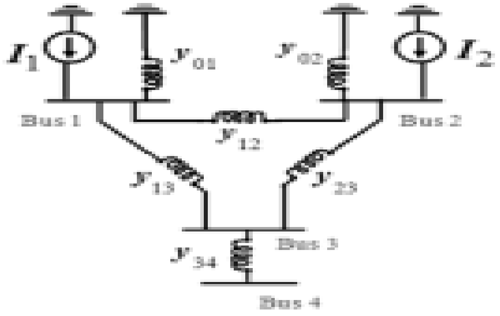
18:14

defined: $Y_{11} = y_{01} + y_{12} + y_{13}$ *and* $Y_{12} = Y_{21} = -y_{12}$ $Y_{13} = Y_{31} = -y_{13}$

$Y_{22} = y_{02} + y_{12} + y_{23}$ $Y_{23} = Y_{32} = -y_{23}$ $Y_{34} = Y_{43} = -y_{34}$

$Y_{33} = y_{31} + y_{32} + y_{34}$ $Y_{14} = Y_{41} = 0$ $Y_{42} = Y_{24} = 0$

$Y_{44} = y_{34}$



Then, the Node Voltage Equation is:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$$I_{bus} = [Y_{bus}]\, V_{bus} \qquad Or \qquad V_{bus} = [Y_{bus}^{-1}]\, I_{bus} = [Z_{bus}]\, I_{bus}$$

Substituting the values, then the bus admittance matrix of the network is:

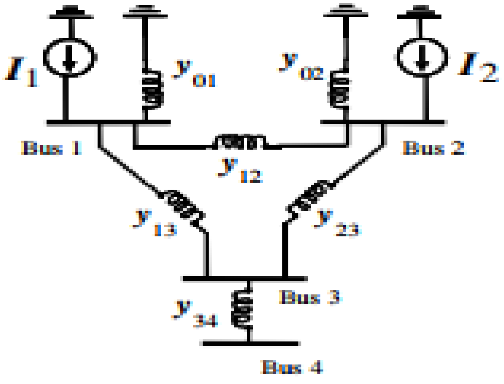
$$Y_{bus} = \begin{bmatrix} -j8.5 & j2.5 & j5.0 & 0 \\ j2.5 & -j8.75 & j5.0 & 0 \\ j5.0 & j5.0 & -j22.5 & j12.5 \\ 0 & 0 & j12.5 & -j12.5 \end{bmatrix}$$

18:14

<u>NOTE:</u>

Formulation of the bus admittance matrix follows three simple rules:

1. **The admittance of elements connected between node *k* and *reference* is added to the *(k, k)* entry of the admittance matrix.**



2. **The admittance of elements connected between nodes *j* and *k* is added to the *(j, j)* and *(k, k)* entries of the admittance matrix.**

3. **The negative of the admittance is added to the *(j, k)* and *(k, j)* entries of the admittance matrix.**

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} (y_{01}+y_{12}+y_{13}) & (-y_{12}) & (-y_{13}) & (0) \\ (-y_{21}) & (y_{02}+y_{21}+y_{23}) & (-y_{23}) & (0) \\ (-y_{31}) & (-y_{32}) & (y_{31}+y_{32}+y_{43}) & (-y_{34}) \\ (0) & (0) & (-y_{43}) & (y_{43}) \end{bmatrix}$$

18:14

Power Flow and Losses

After the iterative solution of the bus voltages (will be discussed later), power flows on the lines and power losses are computed.

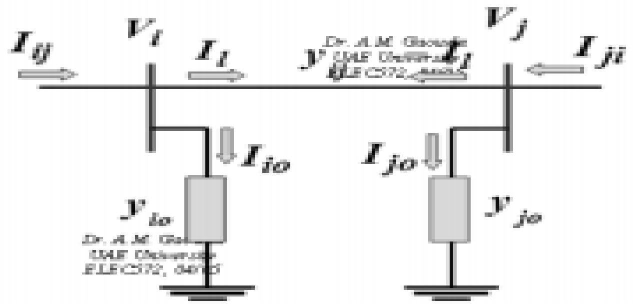
Consider the line connecting the two buses *i* and *j*. Using the bus voltages and the currents flowing in the lines then:

$$I_{ij} = I_l + I_{io}$$

$$I_{ij} = y_{ij}(V_i - V_j) + y_{io}V_i$$

$$I_{ji} = -I_l + I_{jo}$$

$$I_{ji} = y_{ij}(V_j - V_i) + y_{jo}V_j$$

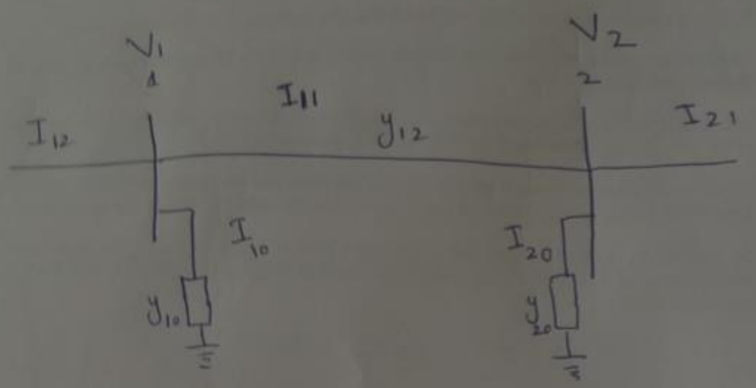


The *complex power flows* from bus *i* to bus *j* is:

$$S_{ij} = V_i \quad I_{ij}^*$$

And the *complex power flows* from bus *j* to bus *i* is:

$$S_{ji} = V_j \quad I_{ji}^*$$



$$I_{12} = I_{11} + I_{10}$$

$$= y_{12}(V_1 - V_2) + V_1 y_{10}$$

$$I_{21} = -I_{11} + I_{20}$$

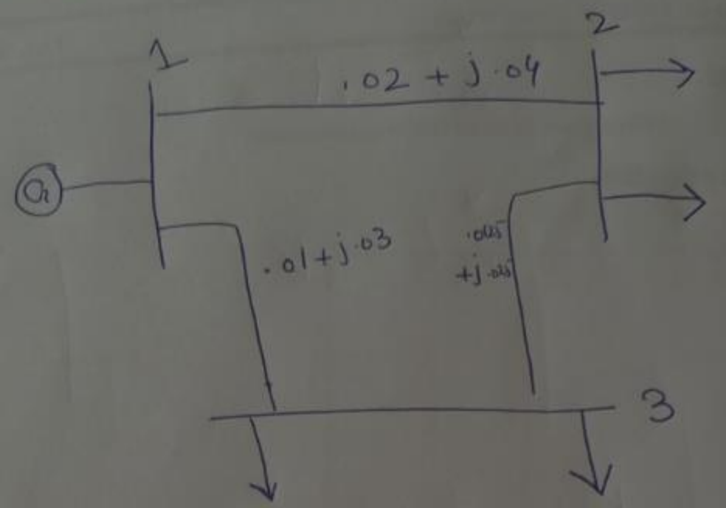
$$= y_{12}(V_2 - V_1) + V_2 y_{20}$$

$$S_{12} = V_1 \, I_{12}^*$$

$$S_{21} = V_2 \, I_{21}^*$$

$$\boxed{S_{L12} = S_{12} + S_{21}}$$

$$\boxed{S_{L13} = S_{13} + S_{31}}$$



$$y_{21} = y_{12} = \frac{1}{.02 + j.04} = 10 - 20j$$

$$y_{31} = y_{13} = \frac{1}{.01 + j.03} = 10 - j30$$

$$y_{32} = y_{23} = \frac{1}{.0125 + j.025} = 16 - j32$$

$$I_{12} = y_{12}(V_1 - V_2)$$

$$I_{13} = y_{13}(V_1 - V_3)$$

$$I_{23} = y_{23}(V_2 - V_3).$$

$$S_{12} = V_1 I_{12}^* \qquad S_{21} = V_2 I_{21}^*$$

$$S_{13} = V_1 I_{13}^* \qquad S_{31} = V_3 I_{31}^*$$

$$S_{23} = V_2 I_{23}^* \qquad S_{32} = V_3 I_{32}^*$$

$$S_{L\,12} = S_{12} + S_{21}$$

$$S_{L\,13} = S_{13} + S_{31}$$

$$S_{L\,23} = S_{23} + S_{32}$$

The *power losses* in the line connecting the two buses *i* and *j* is the algebraic sum of the power flows, i.e.;

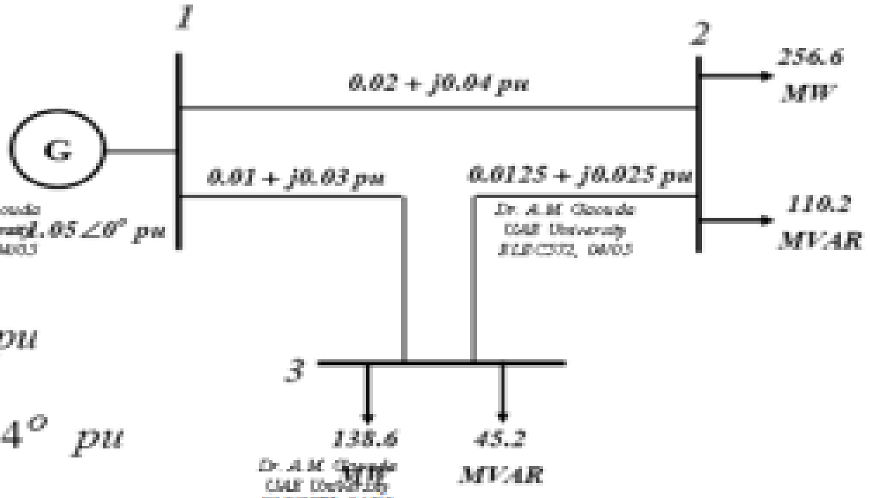
$$S_{L_{ij}} = S_{ij} + S_{ji}$$

EXAMPLE :

The line impedances are as indicated in per unit on 100MVA base. The bus voltages are:

$$V_2 = 0.9800 - j0.0600 = 0.98183\angle -3.5035^{o} \;\; pu$$

$$V_3 = 1.0000 - j0.0500 = 0.00125\angle -2.8624^{o} \;\; pu$$



Find the power flow and line losses.

SOLUTION :

$$V_2 = 0.9800 - j0.0600$$
$$V_3 = 1.0000 - j0.0500$$

$$V_1 = 1.05\angle 0^{\circ}\ pu$$

The line admittances are:

$$y_{12} = \frac{1}{0.02 + j0.04} = 10 - j20 = y_{21}$$

$$y_{13} = \frac{1}{0.01 + j0.03} = 10 - j30 = y_{31}$$

$$y_{23} = \frac{1}{0.0125 + j0.025} = 16 - j32 = y_{32}$$

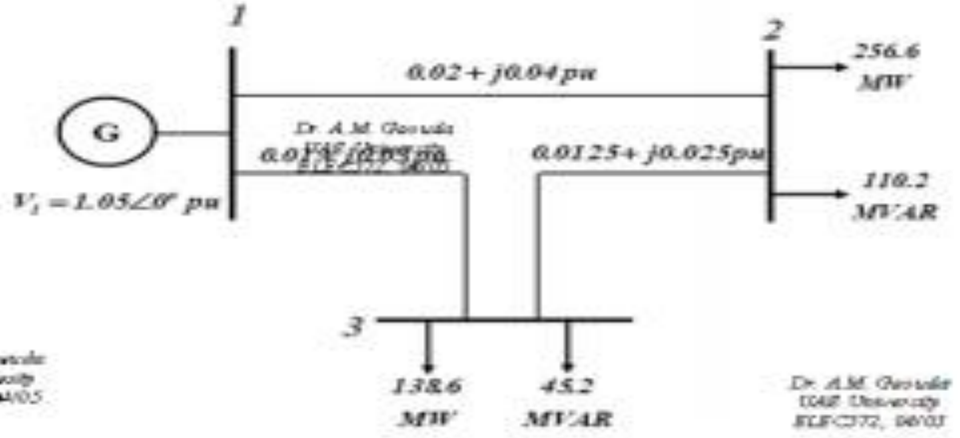
The line currents are:

$$I_{ij} = y_{ij}(V_i - V_j)$$

$$I_{12} = 1.8994 - j0.7997$$

$$I_{13} = 1.9995 - j0.9997$$

$$I_{23} = -0.6396 - j0.4798$$



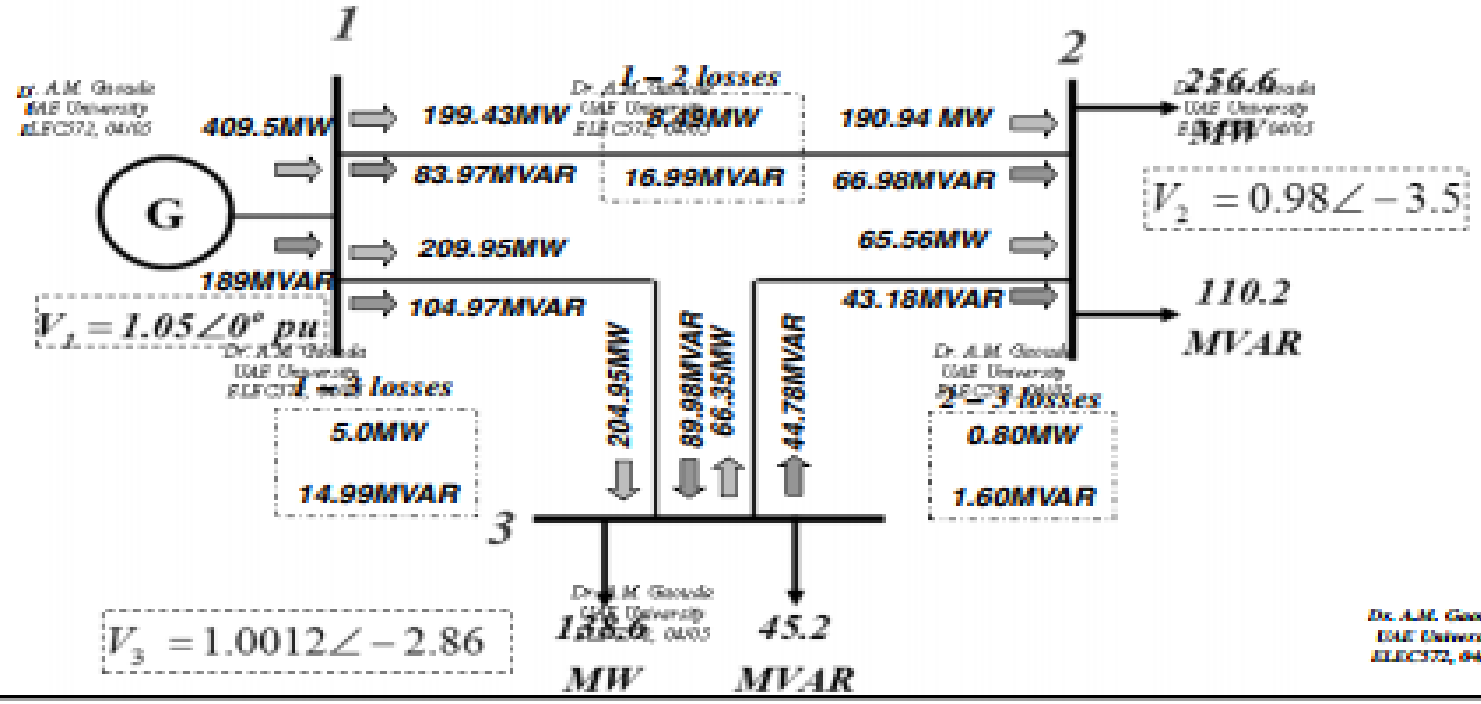
The lines power flow and line losses are:

$$S_{12} = V_1 \; I_{12}^* \qquad S_{21} = V_2 \; I_{21}^* \qquad S_{L12} = S_{12} + S_{21}$$

$S_{12} = 1.9943 + j\,0.8397$ $\quad$ $S_{21} = -\,1.9094 - j\,0.6698$ $\quad$ $S_{L12} = 0.0849 + j\,0.1699$

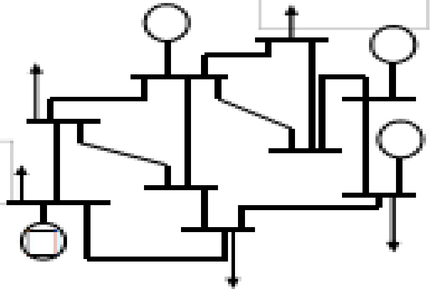
$S_{13} = 2.0995 + j1.0497$ $\quad$ $S_{31} = -2.0495 - j0.8998$ $\quad$ $S_{L13} = 0.0500 + j\,0.1499$

$S_{23} = -0.6556 - j0.4318$ $\quad$ $S_{32} = 0.6635 + j0.4478$ $\quad$ $S_{L23} = 0.0080 + j0.0160$



Load Flow Equations:

For an '*n*' bus system



$$I_{bus} = Y_{bus}\, V_{bus}$$

The current enters the i^{th} bus for an '*n*' bus system is:

$$I_i = Y_{i1} V_1 + Y_{i2} V_2 + \ldots\ldots + Y_{ii} V_i + \ldots Y_{in} V_n$$

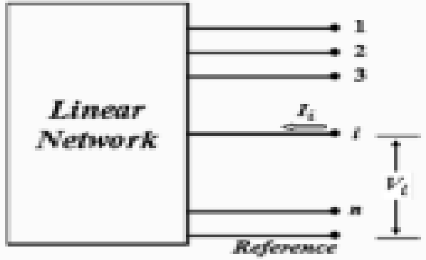
$$I_i = \sum_{p=1}^{n} Y_{ip} V_p$$

$$V_p = |V_p|^{\angle \delta_p}$$

$$Y_{ip} = |Y_{ip}|^{\angle \gamma_{ip}}$$

$$I_i = \sum_{p=1}^{n} |Y_{ip}|^{\angle \gamma_{ip}} |V_p|^{\angle \delta_p}$$

$$I_i = \sum_{p=1}^{n} |Y_{ip}||V_p|^{\angle(\delta_p + \gamma_{ip})}$$



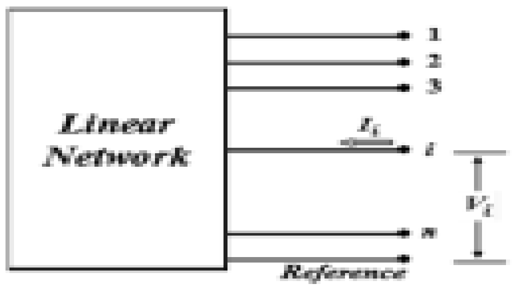
$$\begin{bmatrix} I_1 \\ \ldots \\ I_i \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & \ldots & Y_{1n} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ Y_{i1} & Y_{i2} & Y_{i3} & \ldots & Y_{in} \\ Y_{n1} & Y_{n2} & Y_{n3} & \ldots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ \ldots \\ V_i \\ V_n \end{bmatrix}$$

18:14

In power systems, power is known rather than currents. The complex power injected into the i^{th} bus is:

$$S_i = P_i + jQ_i = V_i \, I_i^*$$

$$S_i^* = P_i - jQ_i = V_i^* \, I_i$$



$$V_i^* = |V_i| \angle{-\delta_i} \qquad \textbf{and} \qquad I_i = \sum_{p=1}^{n} |Y_{ip}||V_p| \angle{(\delta_p + \gamma_{ip})}$$

$$S_i^* = P_i - jQ_i = |V_i| \angle{-\delta_i} \sum_{p=1}^{n} |Y_{ip}||V_p| \angle{(\delta_p + \gamma_{ip})}$$

$$S_i^* = |V_i| \sum_{p=1}^{n} |Y_{ip}||V_p| \angle{(\delta_p + \gamma_{ip} - \delta_i)}$$

$$S_i^* = P_i - jQ_i = |V_i| \sum_{p=1}^{n} |Y_{ip}||V_p| \angle{-(\delta_i - \delta_p - \gamma_{ip})}$$

$$P_i = |V_i| \sum_{p=1}^{n} |Y_{ip}||V_p| \cos(\delta_i - \delta_p - \gamma_{ip}) = P_{i,cal}$$

$$Q_i = |V_i| \sum_{p=1}^{n} |Y_{ip}||V_p| \sin(\delta_i - \delta_p - \gamma_{ip}) = Q_{i,cal}$$

Load Flow Equation:

$$\begin{bmatrix} I_1 \\ \vdots \\ I_i \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & \cdots & Y_{1n} \\ & & & & \\ Y_{i1} & Y_{i2} & Y_{i3} & \cdots & Y_{in} \\ Y_{n1} & Y_{n2} & Y_{3} & & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_i \\ V_n \end{bmatrix}$$

$$I_i = Y_{i1} V_1 + Y_{i2} V_2 + \cdots Y_{ii} V_i + Y_{in} V_n$$

$$I_i = \sum_{p=1}^{N} Y_{ip} V_p$$

$$V_p = |V_p| \angle \delta_p$$

$$Y_{ip} = |Y_{ip}| \angle \gamma_{ip}$$

$$I_i = \sum_{p=1}^{n} |Y_{ip}| \angle \gamma_{ip} \, |V_p| \angle \delta_p$$

$$I_i = \sum_{p=1}^{n} |Y_{ip}| \, |V_p| \angle (\delta_p + \gamma_{ip}) \longrightarrow (1)$$

$$S_i = V_i \, I_i^* \longrightarrow (A)$$

eq (1) in eq (A).

$$S_i = V_i \angle \delta_i \left(\sum_{p=1}^{n} |Y_{ip}| \, |V_p| \angle (\delta_p + \gamma_{ip}) \right)^*$$

$$S_i^* = V_i^* \, I_i$$

$$= |V_i| \angle -\delta_i \sum_{p=1}^{n} |Y_{ip}| \, |V_p| \angle \delta_p + \gamma_{ip}$$

$$S_i^* = |V_i| \sum_{p=1}^{n} |Y_{ip}||V_p| \angle -(\delta_i - \delta_p - \gamma_{ip})$$

$$S_i^* = P_i - jQ$$

$$P_i = |V_i| \sum_{p=1}^{n} |Y_{ip}||V_p| \cos(\delta_i - \delta_p - \gamma_{ip})$$

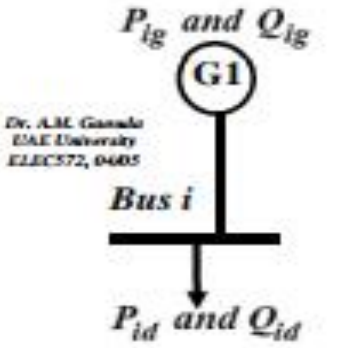
$$Q_i = |V_i| \sum_{p=1}^{n} |Y_{ip}||V_p| \sin(\delta_i - \delta_p - \gamma_{ip})$$

The two equations are known as the ***polar form*** of the ***load flow equations***. They provide ***the calculated value*** of ***net*** real power and ***net*** reactive power ***entering*** bus '*i*'.

Let P_{gi} be ***the scheduled value*** of the real power ***generated*** at bus '*i*'.

Let P_{di} be ***the scheduled value*** of the real power ***demand*** at bus '*i*'.

Let $P_{i,sch}$ be ***the net scheduled*** power ***injected*** at bus '*i*'.



$$P_{i,sch} = P_{gi} - P_{di}$$

ΔP_i *is the mismatch*

$$\Delta P_i = P_{i,sch} - P_{i,cal}$$

$$\Delta P_i = (P_{gi} - P_{di}) - P_{i,cal}$$

Likewise, for *the net scheduled* reactive power *injected* at bus '*i*'

$$Q_{i,sch} = Q_{gi} - Q_{di}$$

ΔQ_i *is the mismatch*

$$\Delta Q_i = Q_{i,sch} - Q_{i,cal}$$

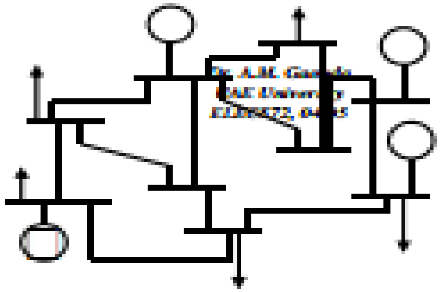
$$\Delta Q_i = (Q_{gi} - Q_{di}) - Q_{i,cal}$$

If the *calculated "net" values* match the *scheduled "net" values*, then the mismatch is zero and the *power balance condition* is:

$$\textit{Mismatch} = \textit{Scheduled values} - \textit{Calculated values} = 0$$

$$\Delta P_i = (P_{gi} - P_{di}) - P_{i,cal} = 0$$
$$\Delta Q_i = (Q_{gi} - Q_{di}) - Q_{i,cal} = 0$$

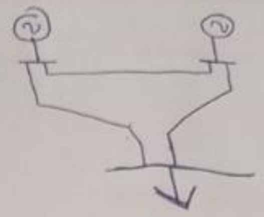
What is the Power Flow Problem?



For the network shown in the figure, there are *two power flow equations* can be written *for each bus*. The power flow solution is to solve these equations for values of *unknown voltages* that satisfy the *power balance condition* at each bus.

NOTE 1: If there is no scheduled value $P_{i,sch}$ for bus "*i*" then the mismatch can not be defined and there is *no requirement to satisfy the balance condition at this bus* while solving the power flow problem.

NOTE 2: Similarly, If there is no scheduled value $Q_{i,sch}$ for bus "*i*" then the mismatch can not be defined and there is *no requirement to satisfy the balance condition at this* bus while solving the power flow problem.



$\leftarrow \begin{cases} n = \text{bus} \\ P = \text{number of buses} \end{cases}$

$$P_i = V_i \sum_{P=1}^{n} |Y_{iP}| V_P \cos(\delta_i - \delta_P - \gamma_{iP}).$$

$$P_1 = V_1 \Big[|Y_{11}| V_1 \cos(\delta_1 - \delta_1 - \gamma_{11}) + |Y_{12}| |V_2| \cos(\delta_1 - \delta_2 - \gamma_{12}) + |Y_{13}| |V_3| \cos(\delta_1 - \delta_2 - \gamma_{13}) \Big].$$

$$Q_1 = V_1 \Big[|Y_{11}| V_1 \cos(\delta_1 - \delta_1 - \gamma_{11}) + |Y_{12}| V_2 \cos(\delta_1 - \delta_2 - \gamma_{12}) + |Y_{13}| |V_3| \cos(\delta_1 - \delta_3 - \gamma_{13}) \Big]$$

$$P_2 = |V_2| \Big[|Y_{21}||V_1| \cos(\delta_2 - \delta_1 - \gamma_{21})$$
$$+ |Y_{22}||V_2| \cos(\delta_2 - \delta_2 - \gamma_{22})$$
$$+ |Y_{23}||V_3| \cos(\delta_2 - \delta_3 - \gamma_{23}) \Big]$$

$$Q_2 = |V_2| \Big[|Y_{21}||V_1| \sin(\delta_2 - \delta_1 - \gamma_{21})$$
$$+ |Y_{22}||V_2| \sin(\delta_2 - \delta_2 - \gamma_{22})$$
$$+ |Y_{23}||V_3| \sin(\delta_2 - \delta_3 - \gamma_{23}) \Big]$$

$$P_3 = |V_3| \Big[|Y_{31}||V_1| \cos(\delta_3 - \delta_1 - \gamma_{31})$$
$$+ |Y_{32}||V_2| \cos(\delta_3 - \delta_2 - \gamma_{32})$$
$$+ |Y_{33}||V_3| \cos(\delta_3 - \delta_3 - \gamma_{33}) \Big]$$