# Power system Stability

# Power System Stability

- In order to operate as an interconnected system all of the generators (and other synchronous machines) must remain in synchronism with one another.
- Loss of synchronism results in a condition in which no net power can be transferred between the machines.
- A system is said to be transiently unstable if following a disturbance one or more of the generators lose synchronism.

# Types of Disturbances

- Loss of generation
- Line switching
- Faults
- Sudden load changes

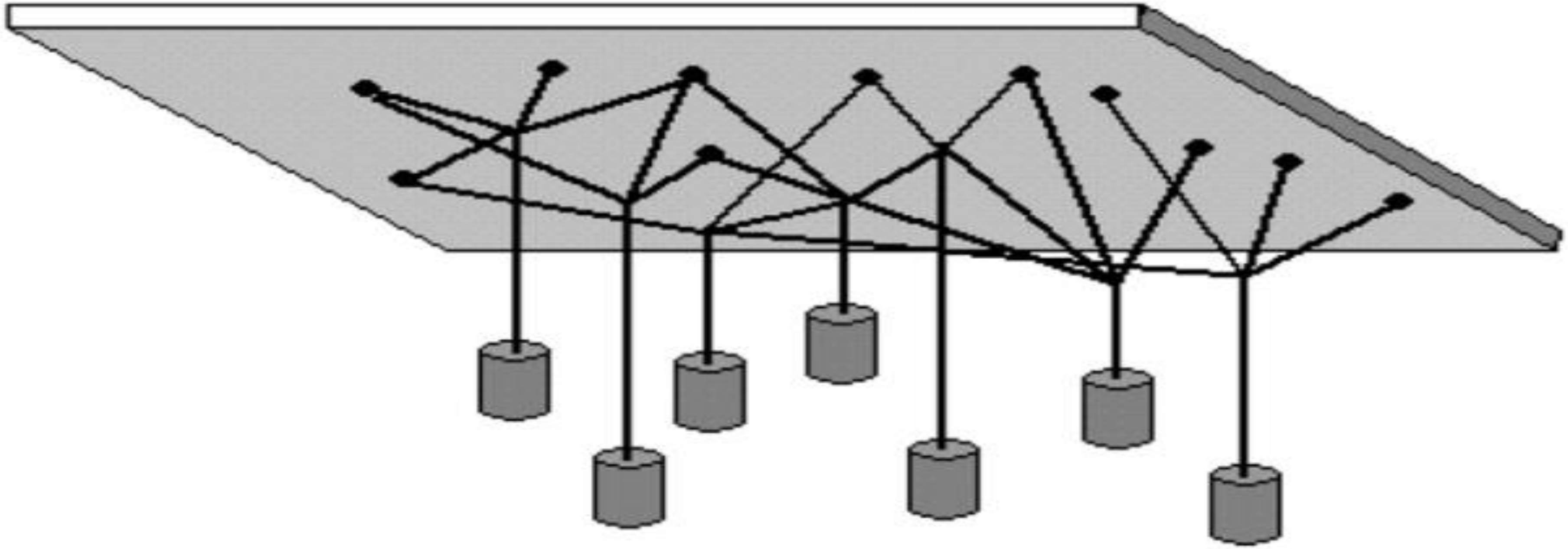
# Definition

- **Power system stability** refers to the ability of the synchronous machines to move from one steady-state operating point following a disturbance to another steady-state operating point, without losing synchronism.

# Power System Stability

- Stability problems are generally divided into two major categories:
  - Steady state stability.
  - Transient stability.
- Steady state stability refers to the ability of the power system to regain synchronism after small and slow disturbance like gradual power changes.
- Transient stability refers to the ability of the power system to regain synchronism after large and sudden disturbance like the occurrence of a fault.

# Mechanical Analogy of PSTS



# Power System Stability

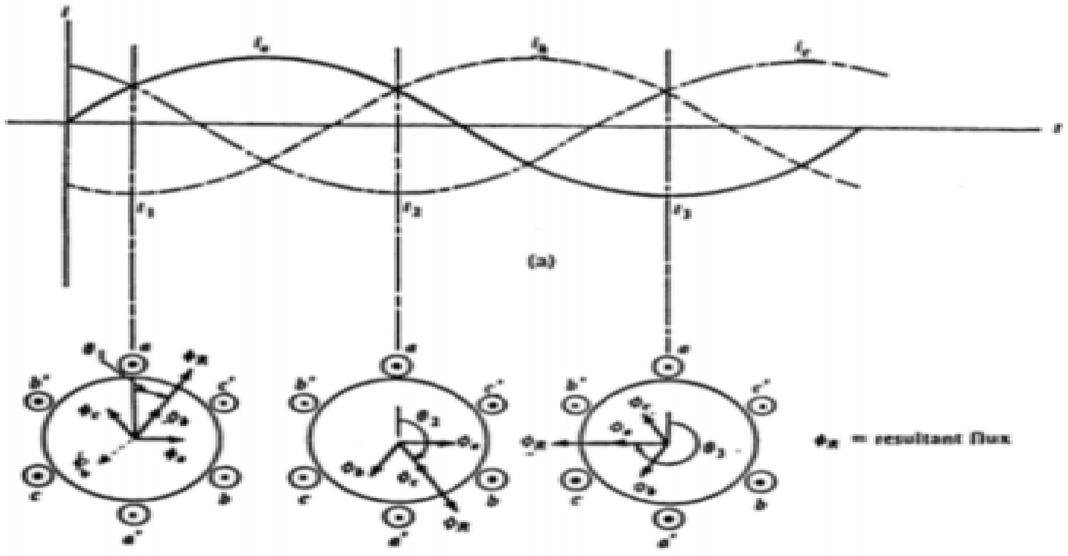
- Number of masses representing synchronous machines are interconnected by a network of elastic strings representing transmission lines.
- Assume the system is initially at rest with the net force on each string is below its break point.
- If one of the strings is cut, representing the loss of a transmission line, the masses will undergo transient oscillations and the forces on the strings will fluctuate.
- The system will either settle down to a new steady state operating point or additional strings will break and the system might collapse completely.

# Power System Transient Stability

- In order to operate as an interconnected system all of the generators must remain in synchronism with one another
  - synchronism requires that the rotors turn at exactly the same speed
- Loss of synchronism results in a condition in which no net power can be transferred between the machines
- A system is said to be transiently unstable if following a disturbance one or more of the generators lose synchronism

# Swing Equation

- Under normal operating conditions, the relative position of the rotor axis and the resultant magnetic field is fixed. This angle is called the power angle.



- During any disturbance, rotor will accelerate or decelerate with respect to the synchronously rotating air gap mmf.
- The equation describing the relative motion is called the swing equation.

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# Swing Equation

- Consider a synchronous generator developing an electromagnetic torque $T_e$ and running at the synchronous speed. If $T_m$ is the mechanical torque, then under steady state operation with losses being neglected:

$$T_m = T_e$$

- A deviation from steady state due to a disturbance results in an accelerating ($T_m > T_e$) or decelerating ($T_m < T_e$) torque $T_\alpha$ on the rotor:

$$T_\alpha = T_m - T_e$$

# Swing Equation

- If J is the moment of inertia of the rotor, from law's of rotation we have:

$$J\frac{d^2\theta_m}{dt^2} = T_m - T_e$$

Where $\theta_m$ is the angular displacement of the rotor with respect to the stationary reference axis of the stator.

- Since we are interested in the rotor speed relative to the synchronous speed, the angular reference is chosen relative to a synchronously rotating frame moving with constant angular velocity $\omega_{sm}$:

$$\theta_m = \omega_{sm}t + \delta_m$$

Where $\delta_m$ is the rotor position before disturbance.

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# Swing Equation

- Derivative of the previous equation results in:

$$\omega_m = \frac{d\theta_m}{dt} = \omega_{sm} + \frac{d\delta_m}{dt}$$

- And the rotor acceleration is: $$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2}$$

- So: $$J\frac{d^2\delta_m}{dt^2} = T_m - T_e$$

- Multiplying the previous equation with $\omega_m$:

$$J\omega_m \frac{d^2\delta_m}{dt^2} = \omega_m T_m - \omega_m T_e \quad \Longrightarrow \quad J\omega_m \frac{d^2\delta_m}{dt^2} = P_m - P_e$$

$$\therefore M\frac{d^2\delta_m}{dt^2} = P_m - P_e$$ Where M is called the inertia constant

# Swing Equation

- An other constant which is often used because its range of values for particular types of rotating machines is quite narrow is called normalized inertia constant H. It is related to M as follows:

$$H = \frac{1}{2} \cdot \frac{M\omega_{sm}}{S_{rated}}$$

- So the swing equation will be:

$$M \frac{d^2\delta_m}{dt^2} = \left( \frac{2HS_{rated}}{\omega_{sm}} \right) \frac{d^2\delta_m}{dt^2}$$

$$\frac{2H}{\omega_{sm}} \frac{d^2\delta_m}{dt^2} = P_{m(p.u.)} - P_{e(p.u.)}$$

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# Swing Equation

- Depending on the unit of the angle δ, the previous equation can be either:

$$\frac{H}{\pi.f}\frac{d^2\delta}{dt^2} = P_{m(p.u.)} - P_{e(p.u.)}$$

When δ is in electrical radian

OR

$$\frac{H}{180.f}\frac{d^2\delta}{dt^2} = P_{m(p.u.)} - P_{e(p.u.)}$$

When δ is in electrical degree

# Swing Equation

- Consider a system that consists of a single machine connected to an infinite bus through an external reactance.
- The electrical power output may be expressed as follows:

$$P_e = P_{\max} \sin \delta$$

Substituting the expression for the electrical power output in the swing equation will result in:

$$\frac{2H}{\omega_s} \frac{d^2\delta}{dt^2} + P_{\max} \sin \delta = P_m$$

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# Example 1

A 500 MVA, 20 kV, 60 Hz, four pole synchronous generator is connected to an infinite bus through a purely reactive network. The generator has an inertia constant H = 6 MJ/MVA and is delivering power of 1.0 per unit to the infinite bus at steady state. The maximum power that can be delivered is 2.5 per unit. A fault occurs that reduces the generator output power to zero, find:

a) The angular acceleration
b) The speed in rev/min at the end of 15 cycles.
c) The change in the power angle at the end of the 15 cycles.

# Example 1, Solution

At steady state $\Longrightarrow$ $P_m = P_e = 1.0$

Under fault conditions the accelerating torque is found as follows:

$$\alpha = \frac{d^2\delta}{dt^2} = \frac{\pi.f}{H}(P_m - P_e) \qquad \text{or} \qquad \alpha = \frac{180.f}{H}(P_m - P_e)$$

$$\Longrightarrow \quad \alpha = \frac{180.60}{6}(1-0) = 1800\,\text{elec. degrees/s}^2$$

$$\Longrightarrow \quad \alpha = \frac{2}{P}(1800) = 900\,\text{mech. degrees/s}^2$$

$$\Longrightarrow \quad \alpha = 900\left(\frac{60 \;\; s/\min}{360^\circ / rev}\right) = 150\,\text{rpm/s}$$

# Example 1, Solution

A 15-cycle interval is equivalent to a time interval of:

$$t = (15)(1/60) = 0.25 \text{ s}$$

The synchronous speed of the machine is found as follows

$$\omega_{sm} = \frac{120.f}{P} = \frac{120.60}{4} = 1800 \text{ rpm}$$

$$\omega_m = \omega_{sm} + \frac{d\delta_m}{dt} \Longrightarrow \omega_m = \omega_{sm} + \alpha t = 1800 + (150)(0.25) = 1837.5 \text{ rpm}$$

# Example 1, Solution

The machine is initially operating at $\delta_0$ which can be found as follows:

$$P_o = P_{\max} \sin \delta_o$$

Therefore the initial angle is:

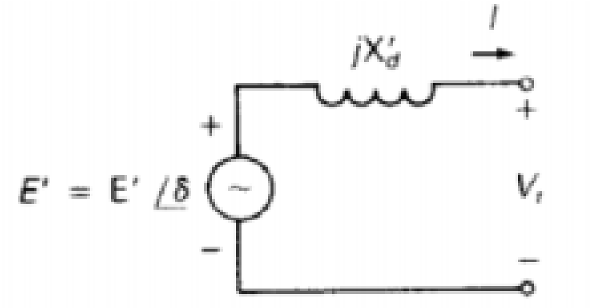
$$\delta_o = \sin^{-1}(P_o / P_{\max}) = \sin^{-1}(1/2.5) = 23.6^o$$

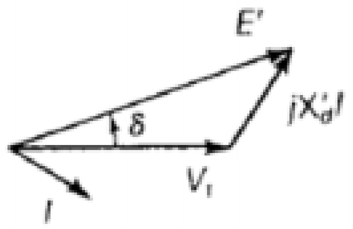
Integrating the following equation $\omega_m = \omega_{sm} + \alpha t$

$$\Longrightarrow \quad \delta = \delta_o + \frac{1}{2}\alpha t^2 = 79.83^o$$

# Simplified Synchronous Machine Model and System Equivalent

- The simplified model of a synchronous machine along with its phasor diagram is shown below:
- The model is based on the following assumptions:
  - The machine is operating under balanced three-phase positive sequence conditions.
  - Machine excitation is constant
  - Machines losses and saturation are neglected.

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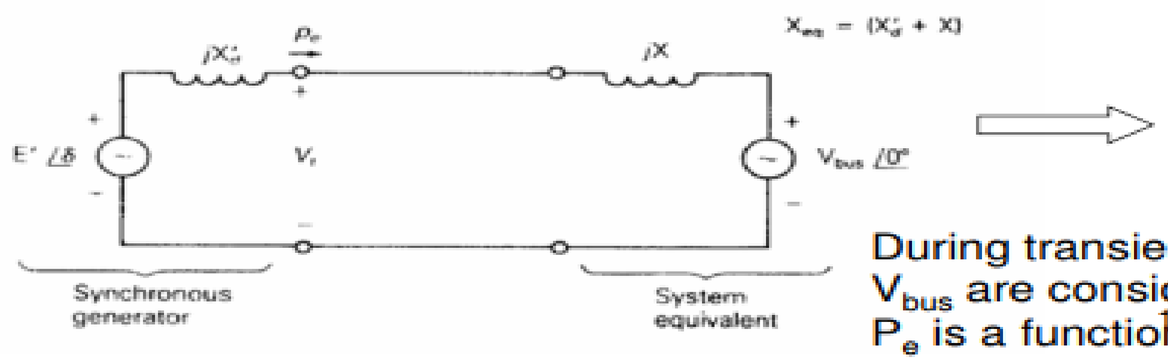
# Simplified Synchronous Machine Model and System Equivalent

- Each generator in the model is connected to a system consisting of transmission lines, transformers, loads and other machines.
- The system can be represented by an "infinite bus" behind a system of reactance.
- An infinite bus is an ideal voltage source that maintains constant voltage magnitude, constant phase and constant frequency.
- Typical system is shown in the following figure:

$$P_e = \frac{E' V_{bus}}{X_{eq}} \sin\delta$$



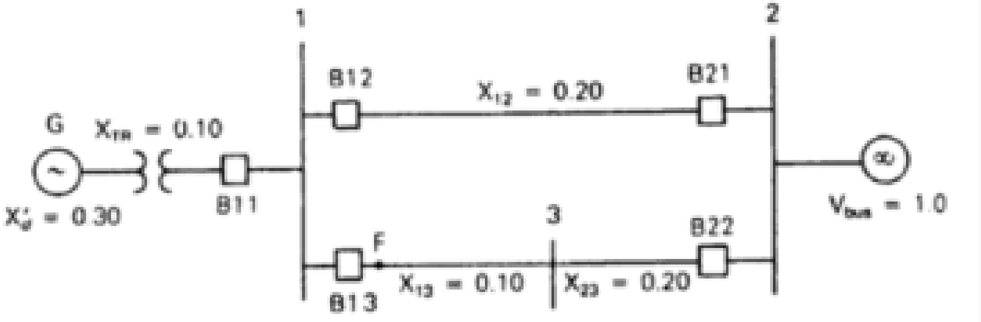

During transient disturbance, both E' and $V_{bus}$ are considered constant and hence $P_e$ is a function of the power angle.

# Example 2

The system shown below is a three phase, 60 Hz synchronous generator connected through a transformer and parallel transmission lines to an infinite bus. All reactances are given in per unit on a common system base. If the infinite bus receives 1.0 per unit real power at 0.95 p.f. lagging determine:

a) The internal voltage of the generator.

b) The equation for the electrical power delivered by the generator versus its power angle.



# Example 2, solution

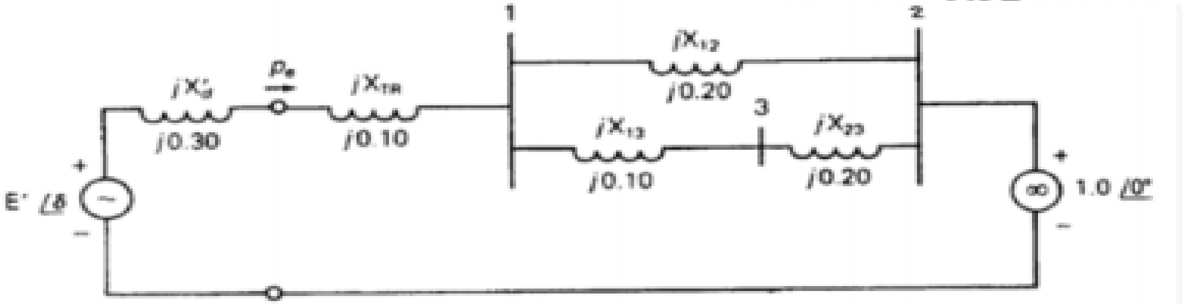
The equivalent reactance diagram is shown below:

$$X_{eq} = X'_d + X_{TR} + X_{12} \| (X_{13} + X_{23}) = 0.520 \quad per\ unit$$

$$I = \frac{P}{V_{bus}(p.f.)} \angle \text{-}\cos^{-1}(\text{p.f.}) = \frac{1}{1(0.95)} \angle \text{-}\cos^{-1}(0.95) = 1.053\angle\text{-}18.2 \quad per\ unit$$

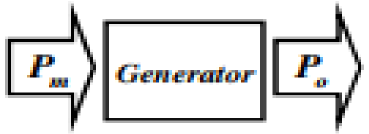
The machine internal voltage is:

$$E' = V_{bus} + jX_{eq} I = 1.28\angle 23.95 \quad per\ unit \quad \Longrightarrow \quad P_e = \frac{(1.28)(1.0)}{0.52} \sin\delta = 2.46 \sin\delta$$



# Transient Stability

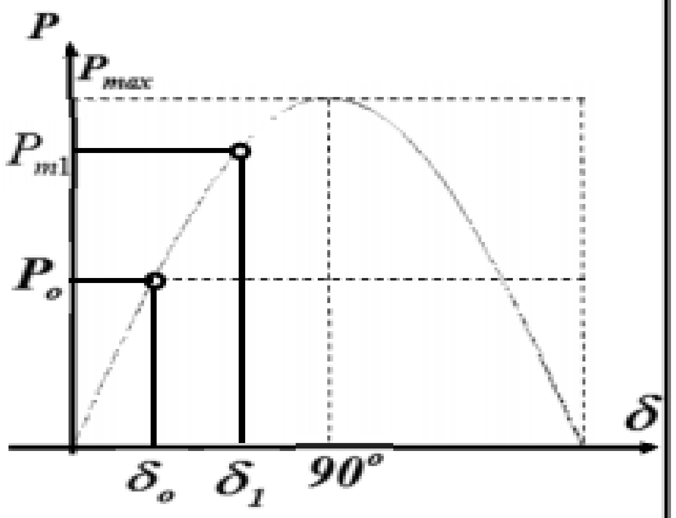
**Consider a power angle curve of a generator as shown in the Figure. Under steady-state operation:**




$$Mechnical\ \ input = Electrical\ \ output$$

$$P_m = P_o \quad at \quad \delta = \delta_o < 90^o$$

$$sychronous\ \ Speed$$




$At \quad t = t_o$

**If the generator mechanical input power is increased to *$P_{m1}$ to supply $P_{e1}$***

$At \quad t = t_{o+}$

**Due to the rotor inertia, the rotor angle $\delta$ can not adjust instantaneously.**

$$\delta = \delta_o$$

$$Mechnical\ \ Input = P_{m1} \quad \& \quad Output = P_o$$

$$P_{m1} > P_{o1}$$

**The rotor starts accelerating and the rotor angle $\delta$ starts increasing.**

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$$At \quad \delta = \delta_1$$

$$Mechnical \;\; Input\,(P_{m1}) = Electrical \;\; output\,(P_{e1})$$

**The rotor can not stop at $\delta = \delta_1$ due to its inertia and $\delta$ continuous to increase up to $\delta = \delta_2 > \delta_1$ .**

**There are two cases for $\delta$ swing:**

$Case\ 1: \quad \delta_2 < \delta_m$

$$Mechnical \;\; Input\,(P_{m1}) \; < \; Electrical \;\; output\,(P_{e1})$$

**The rotor starts decelerating and the rotor angle $\delta$ starts decreasing.**

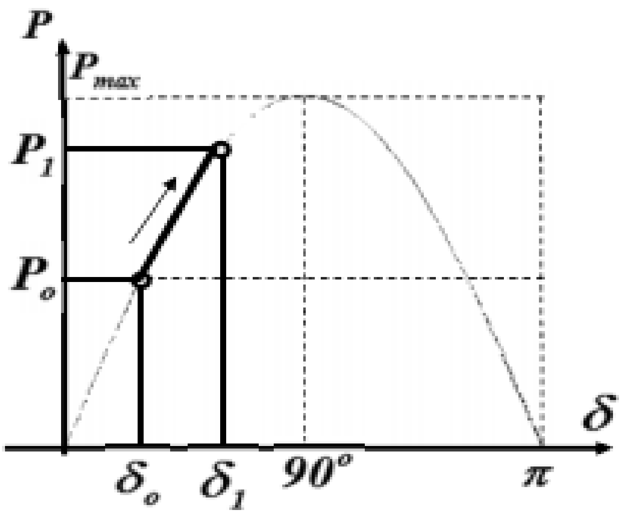
**The rotor angle will oscillate around $\delta_1$ and the system losses will damp this oscillation.**

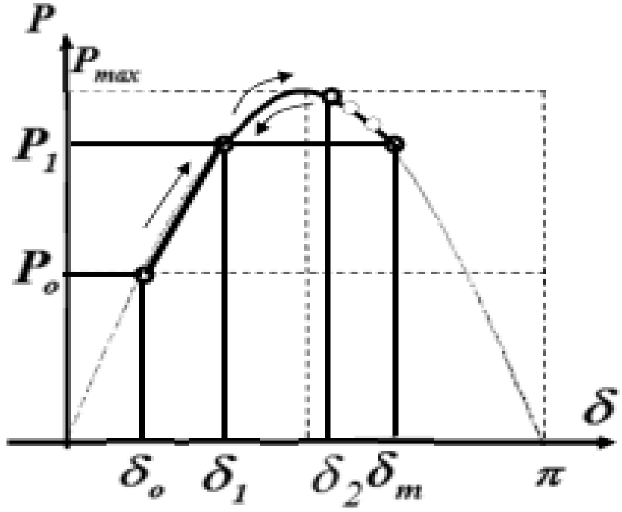
$$Mechnical \;\; Input(P_{m1}) = Electrical \;\; output(P_{e1})$$

$$Steady \;\; State \;\; and \;\; Synchronios \;\; Speed$$

**<u>NOTE</u>**

**The maximum swing of the rotor angle beyond $\delta_1$ depends on the amount of sudden increase in input.**

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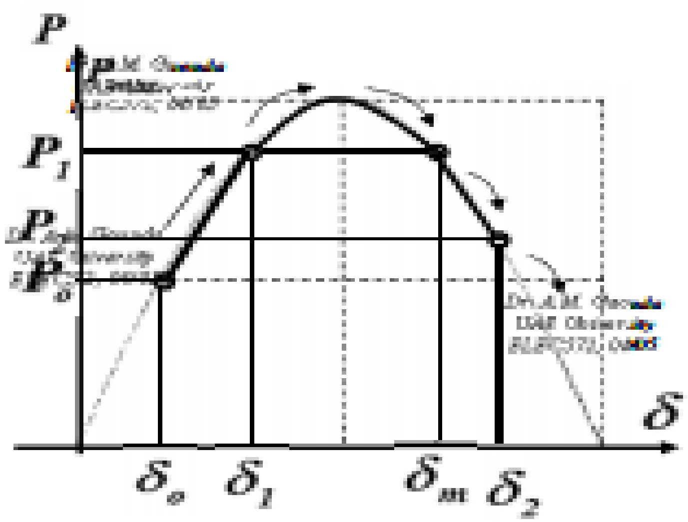
**Case 2 :** $\delta_2 > \delta_m$

$$Mechnical\ \ Input\,(P_{m1}) > \ Electrical\ \ output\,(P_{e2})$$




**The rotor starts accelerating and the rotor angle starts increasing.**

$$Mechnical\ \ Input\,(P_m) >> \ Electrical\ \ output\,(P_e)$$

**The rotor angle will further increase beyond** $\delta_2$ **and the system becomes unstable.**

## NOTES

**1 - The changes in the input are generally small, therefore can not cause transient stability problem.**

**2 - In other cases, the system are subject to transient changes (such as faults and switching or disconnecting tie lines) which may cause transient stability problem.** ***Equal Area Criteria*** **is used to study the transient stability problem.**

## Equal-Area for Transient Stability Assessment:

**Reminder**
**"Stability"**

**A power system is *stable* if it reaches another stable operating point after a disturbance**

- **In order to predict whether a particular system is stable after a disturbance it is necessary to solve the dynamic equation describing the behavior of the rotor angle immediately following an imbalance or disturbance to the system.**

- **The system is said to be *unstable* if the angle between any two machines tends to increase without limit.**

- **On the other hand, if under disturbance effects, the angles between every possible pair reach maximum value and decrease thereafter, the system is *stable*.**

**A simple method for determining stability is known as the *equal-area method.***

**Consider a power angle curve of a system working under steady-state condition where:**

$$Mechnical \ \ input = Electrical \ \ output$$

$$\delta = \delta_o$$

$$sychronous \ \ Speed$$

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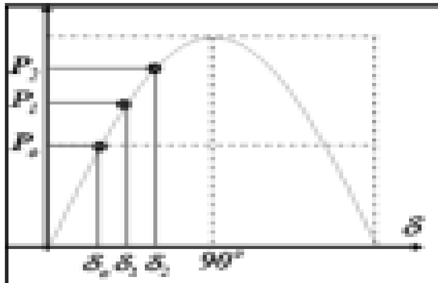


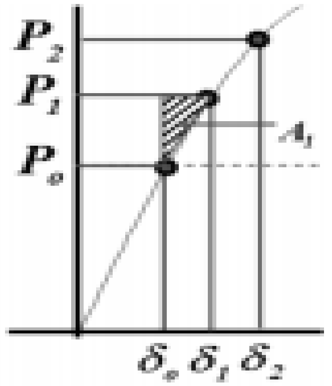


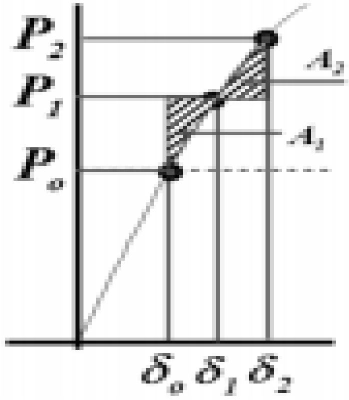


**If the mechanical input for the generator is increased.**

$$Mechnical\ \ Input\ \ = P_{m1}$$

$At\ \ \delta = \delta_o$

$$Mechnical\ \ Input\ \ > Electrical\ \ Output$$

**The rotor starts accelerating and the rotor angle starts increasing from $\delta = \delta_o$ to $\delta = \delta_1$ . During the time taken by the load angle to increase from $\delta_o$ to $\delta_1$ , the rotor absorbs kinetic energy equal the area defined as *A1* in the power angle curve.**

$At\ \ \delta = \delta_1$

$$Mechnical\ \ Input\ \ = Electrical\ \ Output$$

**The rotor has gained some energy that will drive its speed slightly beyond the synchronous speed. The rotor angle ( $\delta = \delta_1$ ) continues to increase until the rotor releases the kinetic energy stored in it ( $\delta = \delta_2$ ) . This energy is defined by area *A2*.**

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$$At \quad \delta = \delta_2$$

$$Mechnical \ Input \ < Electrical \ Output$$

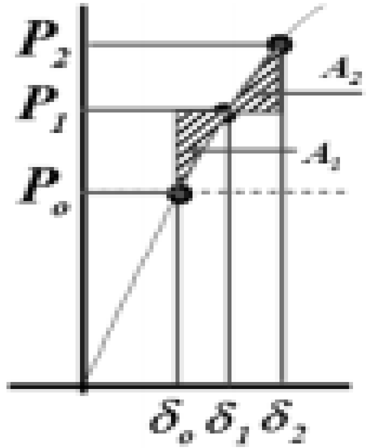
**The rotor starts decelerating and the rotor angle ( $\delta$ )starts decreasing to ( $\delta = \delta_1$ ). *Area 1 must equal to Area 2.***

**The area *A1* is:**

$$A_1 = \int_{\delta_o}^{\delta_1} P_1 \ d\delta - \int_{\delta_o}^{\delta_1} P_{max} \ sin(\delta) \ d\delta$$

$$A_1 = P_1(\delta_1 - \delta_o) - \left[ - P_{max} \cos(\delta) \right]_{\delta_o}^{\delta_1}$$

$$A_1 = P_1(\delta_1 - \delta_o) - P_{max}(\cos\delta_o - \cos\delta_1)$$




**The area *A2* is:**

$$A_2 = \int_{\delta_1}^{\delta_2} P_{max} \ sin(\delta) \ d\delta - \int_{\delta_1}^{\delta_2} P_1 \ d\delta$$

$$A_2 = P_{max}(\cos\delta_1 - \cos\delta_2) - P_1(\delta_2 - \delta_1)$$

**For stability, The area *A1 (energy stored)* = The area *A2 (energy released)***

$$P_1(\delta_1 - \delta_o) - P_{max}(\cos\delta_o - \cos\delta_1) = P_{max}(\cos\delta_1 - \cos\delta_2) - P_1(\delta_2 - \delta_1)$$

$$P_1\delta_1 - P_1\delta_o - P_{max}\cos\delta_o + P_{max}\cos\delta_1 = P_{max}\cos\delta_1 - P_{max}\cos\delta_2 - P_1\delta_2 + P_1\delta_1$$

$$P_{max}(\cos\delta_o - \cos\delta_2) = P_1(\delta_2 - \delta_o) \qquad \textbf{Eq. 1}$$

The angles are in radian.

**NOTE:**

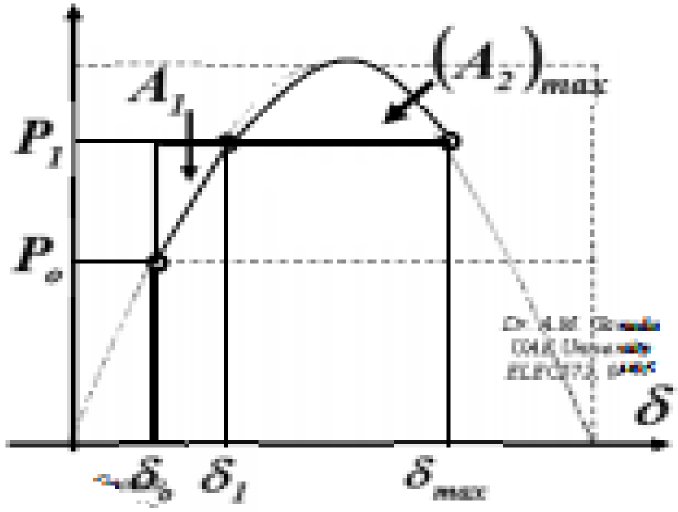
For a given ($\delta_o$), the maximum value of the swing is given by:

$$\delta_{max} = \pi - \delta_1$$

If the load angle ($\delta$) does not go beyond ($\delta_{max}$), then the rotor will swing back to equilibrium (steady-state condition) at ($\delta = \delta_1$).

As long as: $\delta < \delta_{max}$

The equal area condition is satisfied. The system remains within the stability limits.



# Example 3

Consider the system shown below:

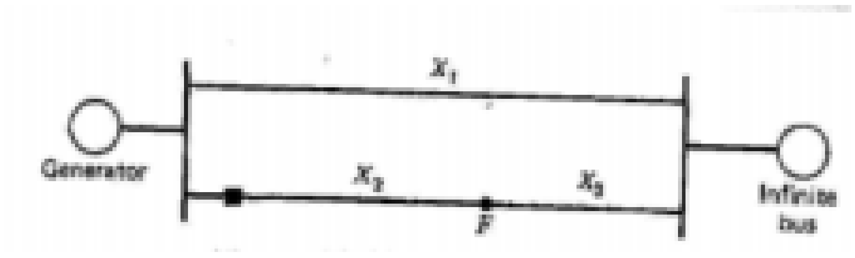
$X_1$=0.4, $X_2$=0.2, $X_3$=0.2, $X_d'$=0.2

E (generator internally generated voltage)=1.2

V (at the load) = 1.0

$P_m$ = 1.5

Suppose the system is working at steady state at $\delta_1$ when the breaker due to a fault at point F opens. Find $\delta_1$, $\delta_2$ (the power angle under the new condition) and $\delta_4$ (the maximum power angle when the breaker opens. Is the system stable or not?



# Example 3, solution

At $t = 0^-$

$$X_{tot} = \frac{(0.2+0.2)(0.4)}{.2+0.2+0.4} = 0.2$$

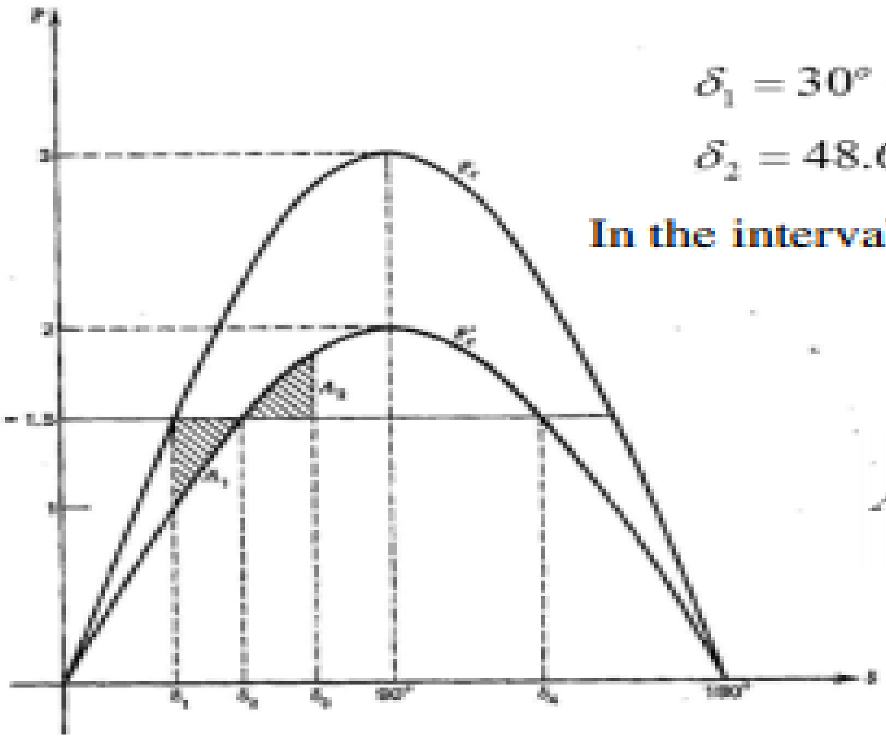
$$P_e = \frac{E \times V}{X} \sin \delta_1 = P_m = 1.5 \quad \Longrightarrow \quad P_e = \frac{1.2 \times 1.0}{0.2+0.2} \sin \delta_1 = 3.0 \times \sin \delta_1 = 1.5$$

$$\Longrightarrow \quad \delta_1 = 30^o$$

When the breaker opens:

$$P'_e = \frac{1.2 \times 1.0}{0.2+0.4} \sin \delta_2 = 2.0 \times \sin \delta_2 = 1.5 \quad \Longrightarrow \quad \delta_2 = 48.6^o \quad \Longrightarrow \quad \delta_4 = 131.4^o$$

# Example 3, solution



nge the angles to radian

$\delta_1 = 30^\circ = 0.524\,\text{rad}$

$\delta_2 = 48.6^\circ = 0.848\,\text{rad}$

$\delta_4 = 131.4^\circ = 2.293\,\text{rad}$

In the interval $\delta_1$- $\delta_2$, $P_m > P_e$ and the rotor is accelerating.

$$(P_m - P_e)d\delta$$

$$A_1 = \int_{0.524}^{0.848}(1.5 - 2\sin\delta)d\delta = 0.0773$$

The question is, will $A_2$ will be large enough before reaching $\delta_4$

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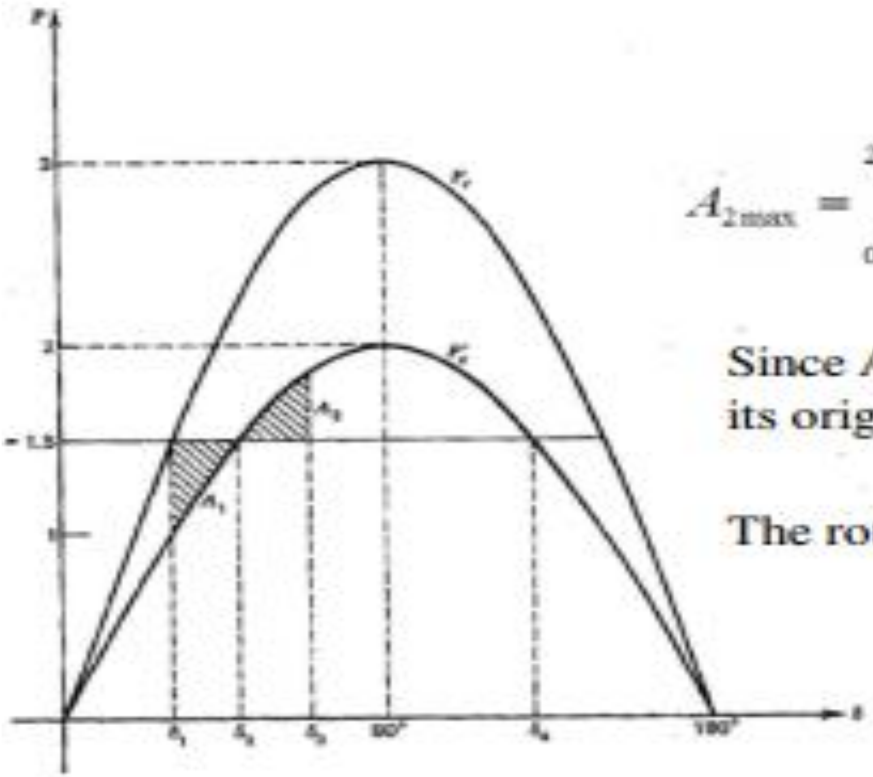
# Example 3, solution



$$(P_e - P_m)d\delta$$

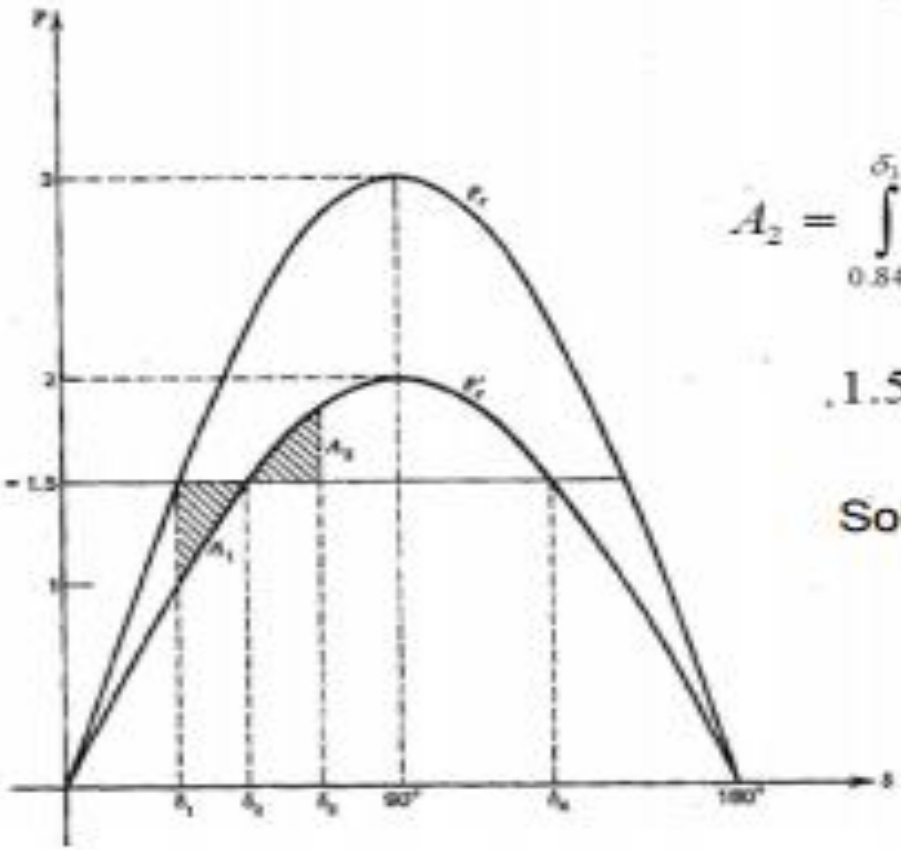
$$A_{2\max} = \int_{0.848}^{2.293} (2\sin\delta - 1.5)d\delta = 0.478$$

Since $A_{2max} > A_1$ the power angle will come back to its original state and hence the system is stable.

The rotor will stop before $\delta_4$.

To calculate the angle at which the rotor will stop to decelerate, $\delta_3$.

# Example 3, solution




$$P_e - P_m)d\delta$$

$$A_2 = \int_{0.848}^{\delta_3} (2\sin\delta - 1.5)d\delta = 0.0773$$

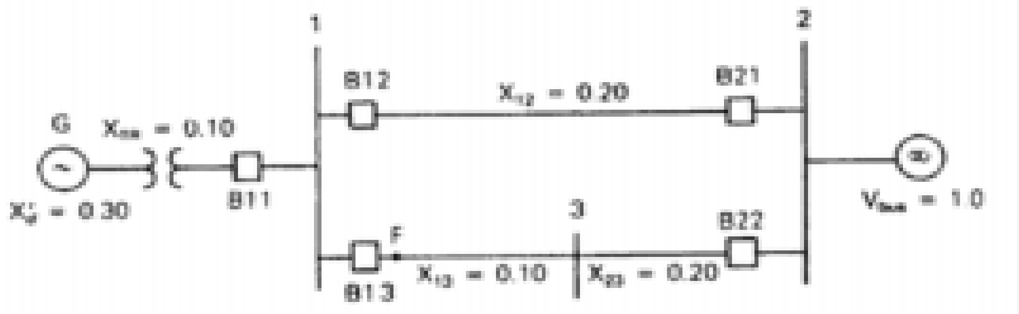
$$1.5\delta_3 + 2\cos\delta_3 = 2.518$$

Solving this non-linear equation:

$$\delta_3 = 1.218 \text{ rad}$$

# Example 4

The synchronous generator shown below is initially operating in the steady state condition given in example 2 when a temporary three phase-to-ground fault occurs near bus-1. Three cycles later the fault extinguishes by itself. Due to a relay mis-operation all circuit breakers remain closed. Determine whether stability is maintained or not.



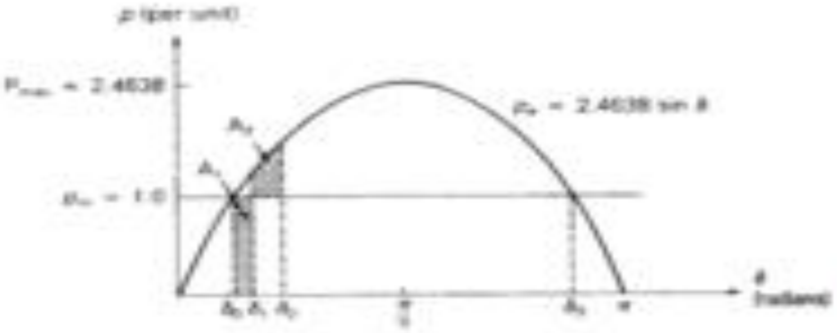
# Example 4, solution

Plots of $P_e$ and $P_m$ are shown below: $$P_e = \frac{(1.28)(1.0)}{0.52} \sin \delta = 2.46 \sin \delta$$

The initial torque angle is: $$\delta_o = 23.95^\circ = 0.4179 \ radian$$

When the short circuit occurs, $P_e$ drops to zero and the swing equation will be: $$\frac{2H}{\omega_{syn}} \frac{d^2\delta(t)}{dt^2} = P_{mp.u.}$$

Integrating twice $\Longrightarrow$ $$\delta(t) = \frac{\omega_{syn} P_{mp.u.}}{4H} t^2 + \delta_0$$



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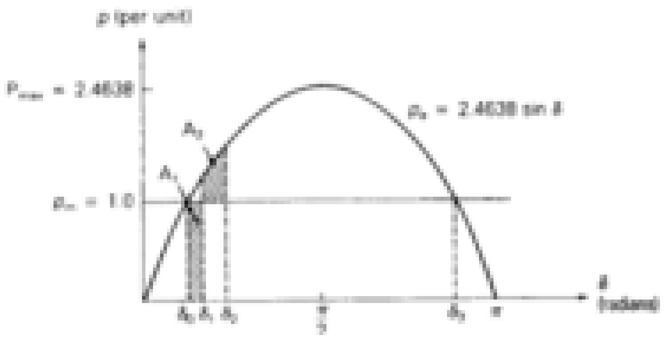
# Example 4, solution

At t = 3 cycles = 0.05 second $\Longrightarrow$ $\delta_1(0.05s) = \dfrac{\omega_{syn} P_{m\,p.u.}}{4H} t^2 + \delta_0 = 0.4964 = 28.44^0$

The acceleration area A1 is equal to: $A_1 = \int_{\delta_0}^{\delta_1} P_m d\delta = \int_{\delta_0}^{\delta_1} 1.0 d\delta = 0.4964 - 0.4179 = 0.0785$

At t=0.05 s the fault extinguishes and $P_e$ instantaneously increases from zero to sinusoidal curve and the torque angle continues to increase until the decelerating A2 equals A1.

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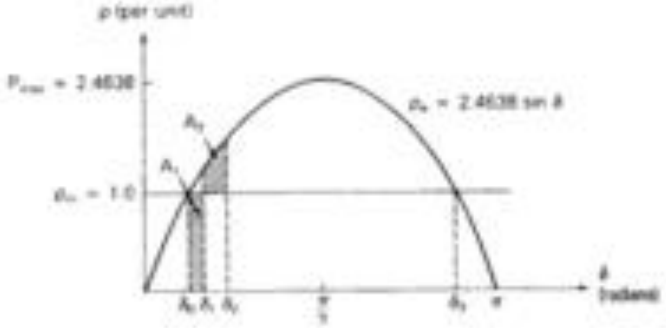
# Example 4, solution

$$A_2 = \int_{\delta_1}^{\delta_2} (P_{\max} \sin\delta - P_m) d\delta = \int_{0.4964}^{\delta_2} (2.4638 \sin\delta - 1) d\delta = A_1 = 0.0785$$

$$2.4638 \cos\delta_2 + \delta_2 = 0.0785 \Longrightarrow \delta_2 = 0.7003 = 40.12^{\circ}$$

Sine the torque angle is less than the maximum angle (180-23.95=156.05), the system is stable

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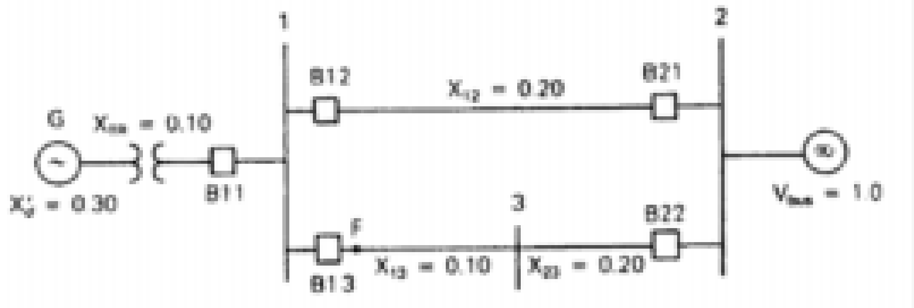
# Critical Clearing Time

- In the previous example the fault took 3 cycles before it cleared itself. What will happen if the fault was not cleared?
- This will lead to the following question: what is the longest allowed time for the fault after which the system will lose its stability?
- This time is called critical clearing time and will be discussed in the following example.

# Example 5

For the question in example 4, assume the short circuit was not cleared after 3 cycles, calculate the critical clearing time.

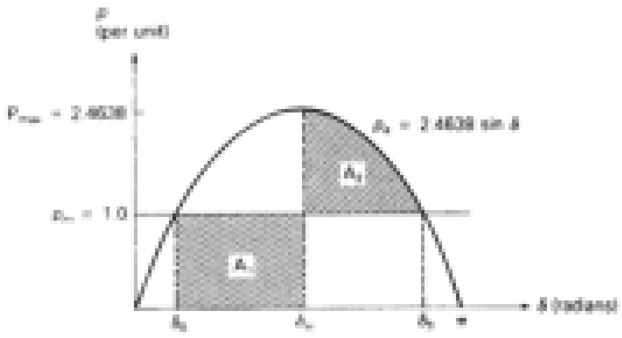


# Example 5, solution

- At the critical clearing angle the fault is extinguished.
- The power angle is then increases to a maximum of 180-23.95=156.05 = 2.7236 radian which gives the maximum decelerating area.
- Equating the accelerating and decelerating areas will result in:

$$A_1 = \int_{\delta_0}^{\delta_{cr}} P_m d\delta = A_2 = \int_{\delta_{cr}}^{\delta_2} (P_{\max} \sin\delta - P_m) d\delta \Longrightarrow \int_{0.4179}^{\delta_{cr}} 1.0 d\delta = A_2 = \int_{\delta_{cr}}^{2.7236} (2.4638 \sin\delta - 1.0) d\delta$$

$$\Longrightarrow \quad \delta_{cr} = 1.5489 = 88.74^0$$



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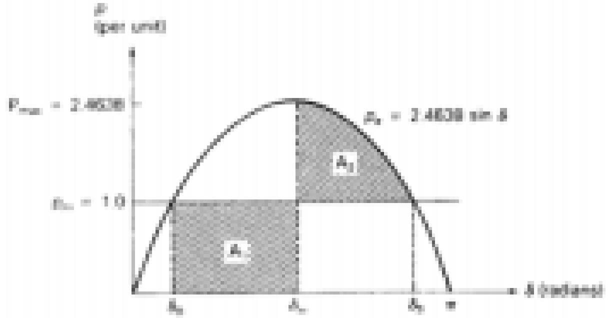
# Example 5, solution

From the solution to the swing equation given in the previous example:

$$\delta(t) = \frac{\omega_{syn} P_{mp.u.}}{4H} t^2 + \delta_0 \quad \Longrightarrow \quad t = \sqrt{\frac{4H}{\omega_{syn} P_{mp.u.}} (\delta(t) - \delta_0)}$$

$$\Longrightarrow \quad t_{cr} = \sqrt{\frac{4H}{\omega_{syn} P_{mp.u.}} (\delta_{cr} - \delta_0)} = 0.1897\, s = 11.38\, cycles$$

So if the fault is cleared before the critical clearing time (11.38 cycles), stability is maintained.



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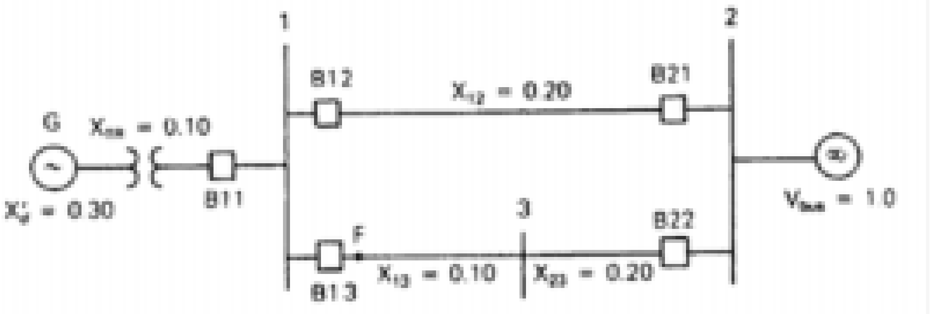
# Example 6

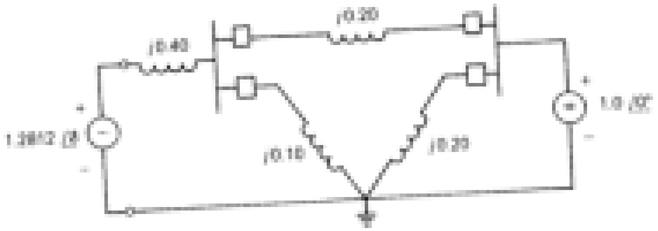
The synchronous generator shown below is initially operating in the steady state condition given in example 2 when a permanent three phase-to-ground fault occurs on bus 1-3 at bus-3. The fault is cleared by opening the circuit breaker at the ends of line 1-3 and line 2-3. These circuit breakers then remain open. Calculate the critical clearing angle.



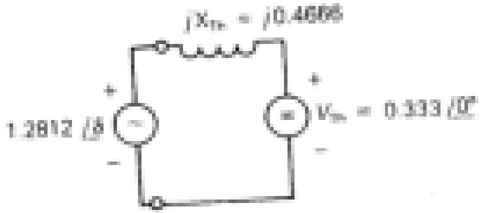
# Example 6, solution

From example 2: $P_{e1} = \dfrac{(1.28)(1.0)}{0.52}\sin\delta = 2.46\sin\delta$

The faulted network is shown below:



The Thevenin equivalent of the faulted network is:

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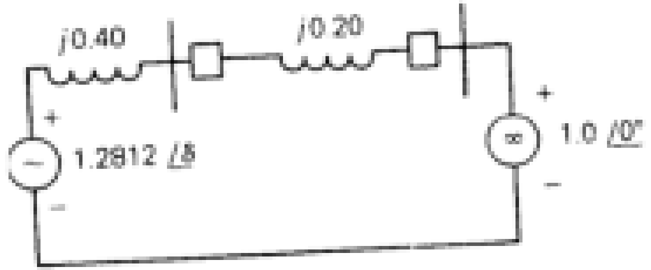
# Example 6, solution

So: $$P_{e2} = \frac{(1.28)(0.333)}{0.4666} \sin\delta = 0.915 \sin\delta$$

The post-fault network is shown below:

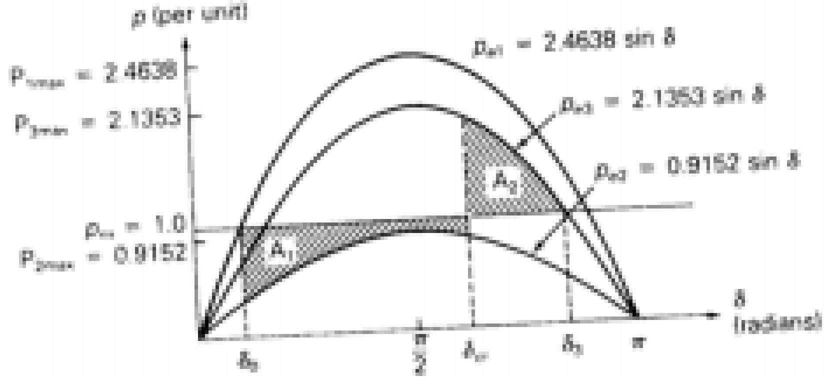



So, the post-fault electrical power delivered is:

$$P_{e3} = \frac{(1.28)(1.0)}{0.6} \sin\delta = 2.1353 \sin\delta$$

# Example 6, solution

The p-δ curves as well as the accelerating are A1 and decelerating area A2 corresponding to critical clearing are shown below:




$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - P_{2\max} \sin \delta) d\delta = A_2 = \int_{\delta_{cr}}^{\delta_3} (P_{3\max} \sin \delta - P_m) d\delta$$

$$\int_{0.4179}^{\delta_{cr}} (1 - 0.9152 \sin \delta) d\delta = \int_{\delta_{cr}}^{2.6542} (2.1353 \sin \delta - 1) d\delta \Rightarrow \delta_{cr} = 1.98 \text{ rad} = 113.5^\circ$$

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