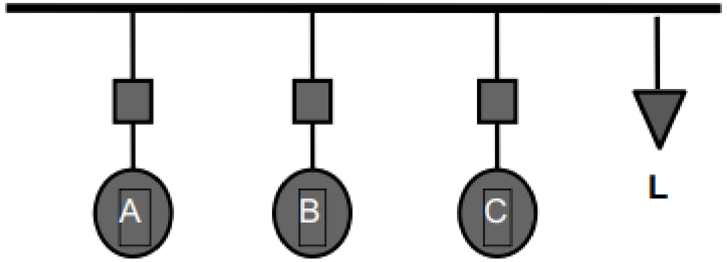
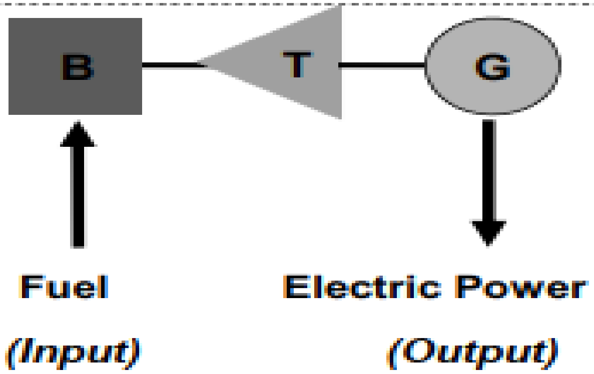
Optimization Techniques and Economic Dispatch

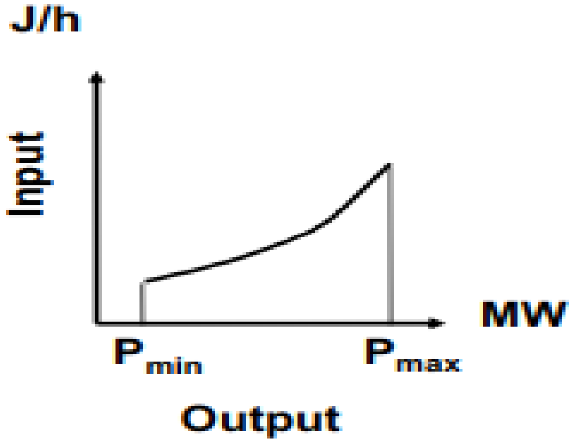
Economic Dispatch Problem



- Several generating units serving the load
- What share of the load should each generating unit produce?
- Consider the limits of the generating units
- Ignore the limits of the network

Characteristics of the Generating Units

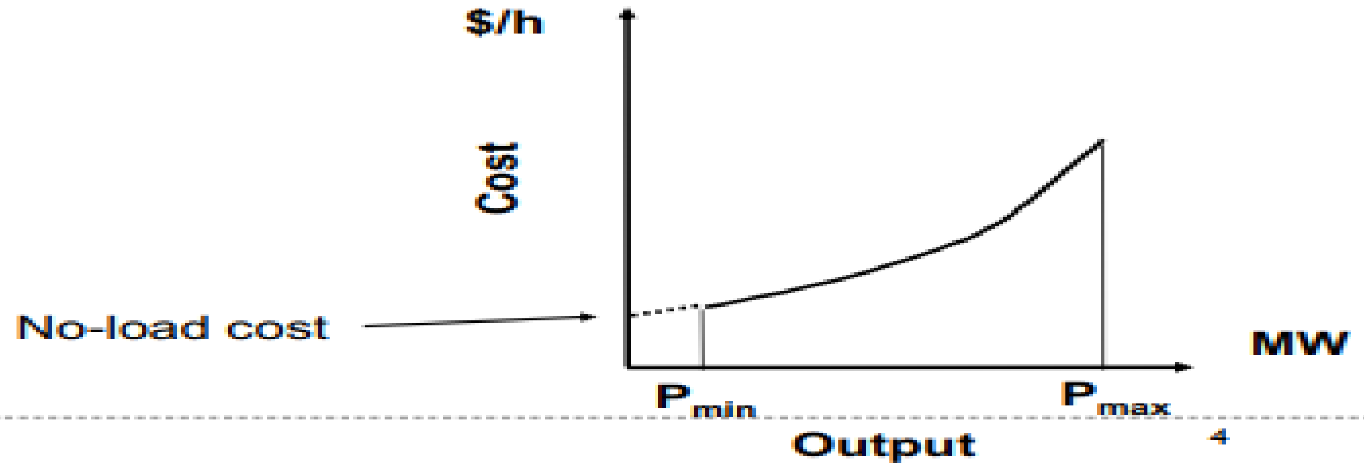




- Thermal generating units
- Consider the running costs only
- Input / Output curve
    - Fuel vs. electric power
- Fuel consumption measured by its energy content
- Upper and lower limit on output of the generating unit

Cost Curve

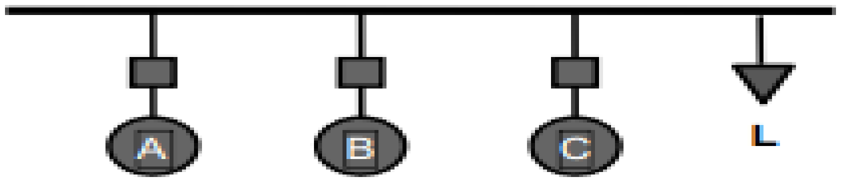
- Multiply fuel input by fuel cost
- No-load cost
  - Cost of keeping the unit running if it could produce zero MW



Mathematical formulation

- Objective function

$$C = C_A(P_A) + C_B(P_B) + C_C(P_C)$$



- Constraints
  - Load / Generation balance:
  - Unit Constraints:

$$L = P_A + P_B + P_C$$

$$P_A^{\min} \le P_A \le P_A^{\max}$$
$$P_B^{\min} \le P_B \le P_B^{\max}$$
$$P_C^{\min} \le P_C \le P_C^{\max}$$

This is an *optimization problem*

Objective

- Most engineering activities have an objective:
  - Achieve the best possible design
  - Achieve the most economical operating conditions
- This objective is usually quantifiable
- Examples:
  - minimize cost of building a transformer
  - minimize cost of supplying power
  - minimize losses in a power system
  - maximize profit from a bidding strategy

Optimization Problem

- What value should the decision variables take so that

$$F = f\left(x_1, x_2, x_3, \ldots x_n\right)$$

is minimum or maximum?

7.2.1 CONSTRAINED PARAMETER OPTIMIZATION: EQUALITY CONSTRAINTS

This type of problem arises when there are functional dependencies among the parameters to be chosen. The problem is to minimize the cost function

$$f(x_1, x_2, \ldots, x_n) \tag{7.6}$$

subject to the equality constraints

$$g_i(x_1, x_2, \ldots, x_n) = 0 \qquad i = 1, 2, \ldots, k \tag{7.7}$$

Such problems may be solved by the *Lagrange multiplier* method. This provides an augmented cost function by introducing k-vector λ of undetermined quantities. The unconstrained cost function becomes

$$\mathcal{L} = f + \sum_{i=1}^{k} \lambda_i g_i \tag{7.8}$$

The resulting necessary conditions for constrained local minima of $\mathcal{L}$ are the following:

$$\frac{\partial \mathcal{L}}{\partial x_i} = \frac{\partial f}{\partial x_i} + \sum_{i=1}^{k} \lambda_i \frac{\partial g_i}{\partial x_i} = 0 \tag{7.9}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_i} = g_i = 0 \tag{7.10}$$

Note that Equation (7.10) is simply the original constraints.

Example 7.2

Use the Lagrange multiplier method for solving constrained parameter optimizations to determine the minimum distance from origin of the xy plane to a circle described by

$$(x-8)^2 + (y-6)^2 = 25$$

The minimum distance is obtained by minimization of the distance square, given by

$$f(x, y) = x^2 + y^2$$

The *MATLAB* **plot** command is used to plot the circle as shown in Figure 7.1.

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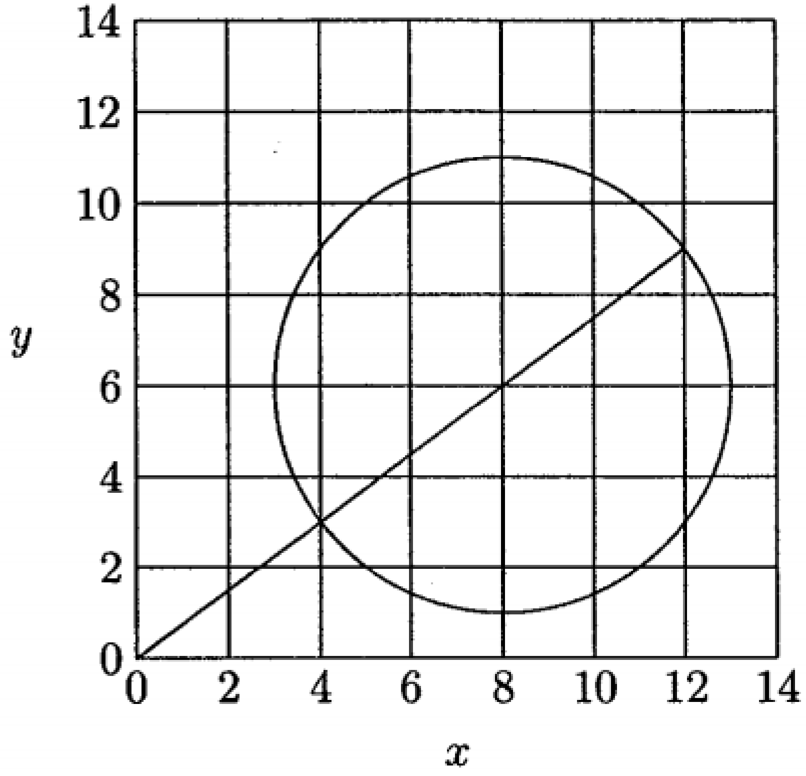

From this graph, clearly the minimum distance is 5, located at point (4, 3).

Now let us use Lagrange multiplier to minimize $f(x, y)$ subject to the constraint described by the circle equation. Forming the Lagrange function, we obtain

$$\mathcal{L} = x^2 + y^2 + \lambda[(x - 8)^2 + (y - 6)^2 - 25]$$

The necessary conditions for extrema are

$$\frac{\partial \mathcal{L}}{\partial x} = 2x + \lambda(2x - 16) = 0 \quad \text{or} \quad 2x(\lambda + 1) = 16\lambda$$

$$\frac{\partial \mathcal{L}}{\partial y} = 2y + \lambda(2y - 12) = 0 \quad \text{or} \quad 2y(\lambda + 1) = 12\lambda$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = (x - 8)^2 + (y - 6)^2 - 25 = 0$$

The solution of the above three equations will provide optimal points. In this problem, a direct solution can be obtained as follows:

Eliminating λ from the first two equations results in

$$y = \frac{3}{4}x$$

Substituting for y in the third equation yields

$$\frac{25}{16}x^2 - 25x + 75 = 0$$

The solutions of the above quadratic equations are $x = 4$ and $x = 12$. Thus, the corresponding extrema are at points (4, 3) with $\lambda = 1$, and (12, 9) with $\lambda = -3$. From Figure 7.1, it is clear that the minimum distance is at point (4, 3) and the maximum distance is at point (12, 9). To distinguish these points, the second derivatives are obtained and the Hessian matrices evaluated at these points are formed. The matrix with positive eigenvalues is a positive definite matrix and the parameters correspond to the minimum point.

OPERATING COST OF A THERMAL PLANT

The factors influencing power generation at minimum cost are operating efficiencies of generators, fuel cost, and transmission losses. The most efficient generator in the system does not guarantee minimum cost as it may be located in an area where fuel cost is high. Also, if the plant is located far from the load center, transmission losses may be considerably higher and hence the plant may be overly uneconomical. Hence, the problem is to determine the generation of different plants such that the total operating cost is minimum. The operating cost plays an important role in the economic scheduling and are discussed here.

The input to the thermal plant is generally measured in Btu/h, and the output is measured in MW. A simplified input-output curve of a thermal unit known as *heat-rate* curve is given in Figure 7.3(a). Converting the ordinate of heat-rate

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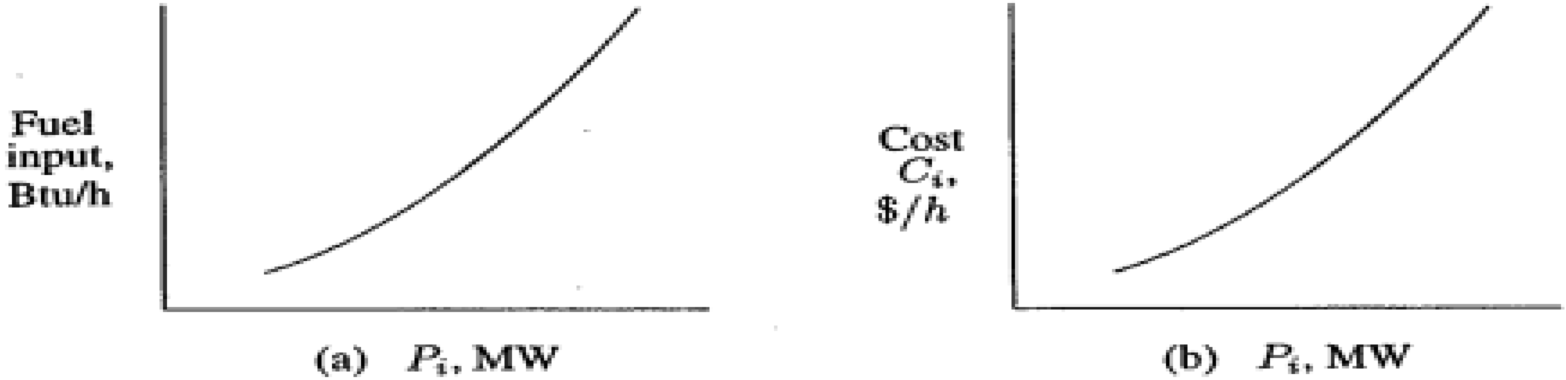

FIGURE 7.3
(a) Heat-rate curve. (b) Fuel-cost curve.

curve from Btu/h to \$/h results in the *fuel-cost* curve shown in Figure 7.3(b). In all practical cases, the fuel cost of generator i can be represented as a quadratic function of real power generation

$$C_i = \alpha_i + \beta_i P_i + \gamma_i P_i^2 \tag{7.21}$$

An important characteristic is obtained by plotting the derivative of the fuel-cost curve versus the real power. This is known as the *incremental fuel-cost* curve shown in Figure 7.4.

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$$\frac{dC_i}{dP_i} = 2\gamma_i P_i + \beta_i \tag{7.22}$$

The incremental fuel-cost curve is a measure of how costly it will be to produce the next increment of power. The total operating cost includes the fuel cost, and the cost of labor, supplies and maintenance. These costs are assumed to be a fixed percentage of the fuel cost and are generally included in the incremental fuel-cost curve.

Since transmission losses are neglected, the total demand P_D is the sum of all generation. A cost function C_i is assumed to be known for each plant. The problem is to find the real power generation for each plant such that the objective function (i.e., total production cost) as defined by the equation

$$\begin{aligned} C_t &= \sum_{i=1}^{n_g} C_i \\ &= \sum_{i=1}^{n} \alpha_i + \beta_i P_i + \gamma_i P_i^2 \end{aligned} \tag{7.23}$$

is minimum, subject to the constraint

$$\sum_{i=1}^{n_g} P_i = P_D \tag{7.24}$$

where C_t is the total production cost, C_i is the production cost of ith plant, P_i is the generation of ith plant, P_D is the total load demand, and n_g is the total number of dispatchable generating plants.

A typical approach is to augment the constraints into objective function by using the Lagrange multipliers

$$\mathcal{L} = C_t + \lambda \left(P_D - \sum_{i=1}^{n_g} P_i \right) \tag{7.25}$$

The minimum of this unconstrained function is found at the point where the partials of the function to its variables are zero.

$$\frac{\partial \mathcal{L}}{\partial P_i} = 0 \tag{7.26}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \tag{7.27}$$

First condition, given by (7.26), results in

$$\frac{\partial C_t}{\partial P_i} + \lambda(0 - 1) = 0$$

Since

$$C_t = C_1 + C_2 + \cdots + C_{n_g}$$

then

$$\frac{\partial C_t}{\partial P_i} = \frac{dC_i}{dP_i} = \lambda$$

and therefore the condition for optimum dispatch is

$$\frac{dC_i}{dP_i} = \lambda \qquad i = 1, \ldots, n_g \tag{7.28}$$

or

$$\beta_i + 2\gamma_i P_i = \lambda \tag{7.29}$$

Second condition, given by (7.27), results in

$$\sum_{i=1}^{n_g} P_i = P_D \tag{7.30}$$

Equation (7.30) is precisely the equality constraint that was to be imposed. In summary, when losses are neglected with no generator limits, for most economic operation, all plants must operate at equal incremental production cost while satisfying the equality constraint given by (7.30). In order to find the solution, (7.29) is solved for P_i

$$P_i = \frac{\lambda - \beta_i}{2\gamma_i} \tag{7.31}$$

The relations given by (7.31) are known as the *coordination equations*. They are functions of λ. An analytical solution can be obtained for λ by substituting for P_i in (7.30), i.e.,

$$\sum_{i=1}^{n_g} \frac{\lambda - \beta_i}{2\gamma_i} = P_D \tag{7.32}$$

or

$$\lambda = \frac{P_D + \sum_{i=1}^{n_g} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{n_g} \frac{1}{2\gamma_i}} \tag{7.33}$$

The value of λ found from (7.33) is substituted in (7.31) to obtain the optimal scheduling of generation.

The solution for economic dispatch neglecting losses was found analytically. However when losses are considered the resulting equations as seen in Section 7.6 are nonlinear and must be solved iteratively. Thus, an iterative procedure is introduced here and (7.31) is solved iteratively. In an iterative search technique, starting with two values of λ, a better value of λ is obtained by extrapolation, and the process is continued until ΔP_i is within a specified accuracy. However, as mentioned earlier, a rapid solution is obtained by the use of the gradient method. To do this, (7.32) is written as

$$f(\lambda) = P_D \tag{7.34}$$

Expanding the left-hand side of the above equation in Taylor's series about an operating point $\lambda^{(k)}$, and neglecting the higher-order terms results in

$$f(\lambda)^{(k)} + \left(\frac{df(\lambda)}{d\lambda}\right)^{(k)} \Delta\lambda^{(k)} = P_D \tag{7.35}$$

or

$$\Delta\lambda^{(k)} = \frac{\Delta P^{(k)}}{\left(\frac{df(\lambda)}{d\lambda}\right)^{(k)}}$$

$$= \frac{\Delta P^{(k)}}{\sum\left(\frac{dP_i}{d\lambda}\right)^{(k)}} \qquad (7.36)$$

or

$$\Delta\lambda^{(k)} = \frac{\Delta P^{(k)}}{\sum \frac{1}{2\gamma_i}} \qquad (7.37)$$

and therefore,

$$\lambda^{(k+1)} = \lambda^{(k)} + \Delta\lambda^{(k)} \qquad (7.38)$$

where

$$\Delta P^{(k)} = P_D - \sum_{i=1}^{n_g} P_i^{(k)} \qquad (7.39)$$

The process is continued until $\Delta P^{(k)}$ is less than a specified accuracy.

Example 7.4

The fuel-cost functions for three thermal plants in \$/h are given by

$$C_1 = 500 + 5.3P_1 + 0.004P_1^2$$
$$C_2 = 400 + 5.5P_2 + 0.006P_2^2$$
$$C_3 = 200 + 5.8P_3 + 0.009P_3^2$$

where P_1, P_2, and P_3 are in MW. The total load, P_D, is 800 MW. Neglecting line losses and generator limits, find the optimal dispatch and the total cost in \$/h

(a) by analytical method using (7.33)

(b) by graphical demonstration.

(c) by iterative technique using the gradient method.

(a) From (7.33), λ is found to be

$$\lambda = \frac{800 + \frac{5.3}{0.008} + \frac{5.5}{0.012} + \frac{5.8}{0.018}}{\frac{1}{0.008} + \frac{1}{0.012} + \frac{1}{0.018}}$$

$$= \frac{800 + 1443.0555}{263.8889} = 8.5 \quad \$/\text{MWh}$$

Substituting for λ in the coordination equation, given by (7.31), the optimal dispatch is

$$P_1 = \frac{8.5 - 5.3}{2(0.004)} = 400.0000$$
$$P_2 = \frac{8.5 - 5.5}{2(0.006)} = 250.0000$$
$$P_3 = \frac{8.5 - 5.8}{2(0.009)} = 150.0000$$

(c) For the numerical solution using the gradient method, assume the initial value of $\lambda^{(1)} = 6.0$. From coordination equations, given by (7.31), P_1, P_2, and P_3 are

$$P_1^{(1)} = \frac{6.0 - 5.3}{2(0.004)} = 87.5000$$
$$P_2^{(1)} = \frac{6.0 - 5.5}{2(0.006)} = 41.6667$$
$$P_3^{(1)} = \frac{6.0 - 5.8}{2(0.009)} = 11.1111$$

Since $P_D = 800$ MW, the error ΔP from (7.39) is

$$\Delta P^{(1)} = 800 - (87.5 + 41.6667 + 11.1111) = 659.7222$$

From (7.37)

$$\Delta\lambda^{(1)} = \frac{659.7222}{\frac{1}{2(0.004)} + \frac{1}{2(0.006)} + \frac{1}{2(0.009)}} = \frac{659.7222}{263.8888} = 2.5$$

Therefore, the new value of λ is

$$\lambda^{(2)} = 6.0 + 2.5 = 8.5$$

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Continuing the process, for the second iteration, we have

$$P_1^{(2)} = \frac{8.5 - 5.3}{2(0.004)} = 400.0000$$
$$P_2^{(2)} = \frac{8.5 - 5.5}{2(0.006)} = 250.0000$$
$$P_3^{(2)} = \frac{8.5 - 5.8}{2(0.009)} = 150.0000$$

and

$$\Delta P^{(2)} = 800 - (400 + 250 + 150) = 0.0$$

Since $\Delta P^{(2)} = 0$, the equality constraint is met in two iterations. Therefore, the optimal dispatch are

$$P_1 = 400 \quad \text{MW}$$

$$\begin{aligned} P_2 &= 250 \quad \text{MW} \\ P_3 &= 150 \quad \text{MW} \\ \hat{\lambda} &= 8.5 \quad \$/\text{MWh} \end{aligned}$$

and the total fuel cost is

$$\begin{aligned} C_t = {} & 500 + 5.3(400) + 0.004(400)^2 + 400 + 5.5(250) + 0.006(250)^2 \\ & +200 + 5.8(150) + 0.009(150)^2 = 6,682.5 \quad \$/\text{h} \end{aligned}$$

7.5 ECONOMIC DISPATCH NEGLECTING LOSSES AND INCLUDING GENERATOR LIMITS

The Kuhn-Tucker conditions complement the Lagrangian conditions to include the inequality constraints as additional terms. The necessary conditions for the optimal dispatch with losses neglected becomes

$$
\begin{aligned}
\frac{dC_i}{dP_i} &= \lambda \qquad \text{for} \quad P_{i(min)} < P_i < P_{i(max)} \\
\frac{dC_i}{dP_i} &\leq \lambda \qquad \text{for} \quad P_i = P_{i(max)} \\
\frac{dC_i}{dP_i} &\geq \lambda \qquad \text{for} \quad P_i = P_{i(min)}
\end{aligned}
\tag{7.41}
$$

Example 7.6

Find the optimal dispatch and the total cost in \$/h for the thermal plants of Example 7.4 when the total load is 975 MW with the following generator limits (in MW):

$$200 \le P_1 \le 450$$
$$150 \le P_2 \le 350$$
$$100 \le P_3 \le 225$$

Assume the initial value of $\lambda^{(1)} = 6.0$. From coordination equations given by (7.31), P_1, P_2, and P_3 are

$$P_1^{(1)} = \frac{6.0 - 5.3}{2(0.004)} = 87.5000$$
$$P_2^{(1)} = \frac{6.0 - 5.5}{2(0.006)} = 41.6667$$
$$P_3^{(1)} = \frac{6.0 - 5.8}{2(0.009)} = 11.1111$$

$$\Delta P^{(1)} = 975 - (87.5 + 41.6667 + 11.1111) = 834.7222$$

From (7.37)

$$\Delta\lambda^{(1)} = \frac{834.7222}{\frac{1}{2(0.004)} + \frac{1}{2(0.006)} + \frac{1}{2(0.009)}} = \frac{834.7222}{263.8888} = 3.1632$$

Therefore, the new value of λ is

$$\lambda^{(2)} = 6.0 + 3.1632 = 9.1632$$

Continuing the process, for the second iteration, we have

$$P_1^{(2)} = \frac{9.1632 - 5.3}{2(0.004)} = 482.8947$$

$$P_2^{(2)} = \frac{9.1632 - 5.5}{2(0.006)} = 305.2632$$

$$P_3^{(2)} = \frac{9.1632 - 5.8}{2(0.009)} = 186.8421$$

and

$$\Delta P^{(2)} = 975 - (482.8947 + 305.2632 + 186.8421) = 0.0$$

Since $\Delta P^{(2)} = 0$, the equality constraint is met in two iterations. However, P_1 exceeds its upper limit. Thus, this plant is pegged at its upper limit. Hence $P_1 = 450$ and is kept constant at this value. Thus, the new imbalance in power is

$$\Delta P^{(2)} = 975 - (450 + 305.2632 + 186.8421) = 32.8947$$

From (7.37)

$$\Delta\lambda^{(2)} = \frac{32.8947}{\frac{1}{2(0.006)} + \frac{1}{2(0.009)}} = \frac{32.8947}{138.8889} = 0.2368$$

Therefore, the new value of λ is

$$\lambda^{(3)} = 9.1632 + 0.2368 = 9.4$$

For the third iteration, we have

$$P_1^{(3)} = 450$$
$$P_2^{(3)} = \frac{9.4 - 5.5}{2(0.006)} = 325$$
$$P_3^{(3)} = \frac{9.4 - 5.8}{2(0.009)} = 200$$

and

$$\Delta P^{(3)} = 975 - (450 + 325 + 200) = 0.0$$

$\Delta P^{(3)} = 0$, and the equality constraint is met and P_2 and P_3 are within their limits. Thus, the optimal dispatch is

$\Delta P^{(3)} = 0$, and the equality constraint is met and P_2 and P_3 are within their limits. Thus, the optimal dispatch is

$$\begin{aligned} P_1 &= 450 \quad \text{MW} \\ P_2 &= 325 \quad \text{MW} \\ P_3 &= 200 \quad \text{MW} \\ \hat{\lambda} &= 9.4 \quad \$/\text{MWh} \end{aligned}$$

and the total fuel cost is

$$\begin{aligned} C_t = {} & 500 + 5.3(450) + 0.004(450)^2 + 400 + 5.5(325) + 0.006(325)^2 \\ & + 200 + 5.8(200) + 0.009(200)^2 = 8,236.25 \quad \$/\text{h} \end{aligned}$$

7.6 ECONOMIC DISPATCH INCLUDING LOSSES

When transmission distances are very small and load density is very high, transmission losses may be neglected and the optimal dispatch of generation is achieved with all plants operating at equal incremental production cost. However, in a large interconnected network where power is transmitted over long distances with low load density areas, transmission losses are a major factor and affect the optimum dispatch of generation. One common practice for including the effect of transmission losses is to express the total transmission loss as a quadratic function of the generator power outputs. The simplest quadratic form is

$$P_L = \sum_{i=1}^{n_g} \sum_{j=1}^{n_g} P_i B_{ij} P_j \tag{7.42}$$

A more general formula containing a linear term and a constant term, referred to as *Kron's loss formula*, is

$$P_L = \sum_{i=1}^{n_g} \sum_{j=1}^{n_g} P_i B_{ij} P_j + \sum_{i=1}^{n_g} B_{0i} P_i + B_{00} \tag{7.43}$$

The coefficients B_{ij} are called *loss coefficients* or *B-coefficients*. B-coefficients are assumed constant, and reasonable accuracy can be expected provided the actual operating conditions are close to the base case where the B-constants were computed. There are various ways of arriving at a loss equation. A method for obtaining these B-coefficients is presented in Section 7.7.

The economic dispatching problem is to minimize the overall generating cost C_i, which is the function of plant output

$$C_t = \sum_{i=1}^{n_g} C_i$$
$$= \sum_{i=1}^{n} \alpha_i + \beta_i P_i + \gamma_i P_i^2 \tag{7.44}$$

$i=1$

subject to the constraint that generation should equal total demands plus losses, i.e.,

$$\sum_{i=1}^{n_g} P_i = P_D + P_L \tag{7.45}$$

satisfying the inequality constraints, expressed as follows:

$$P_{i(min)} \leq P_i \leq P_{i(max)} \qquad i = 1, \ldots, n_g \tag{7.46}$$

where $P_{i(min)}$ and $P_{i(max)}$ are the minimum and maximum generating limits, respectively, for plant i.

Using the Lagrange multiplier and adding additional terms to include the inequality constraints, we obtain

$$\mathcal{L} = C_t + \lambda(P_D + P_L - \sum_{i=1}^{n_g} P_i) + \sum_{i=1}^{n_g} \mu_{i(max)}(P_i - P_{i(max)}) +$$
$$\sum_{i=1}^{n_g} \mu_{i(min)}(P_i - P_{i(min)}) \qquad (7.47)$$

The constraints should be understood to mean the $\mu_{i(max)} = 0$ when $P_i < P_{i(max)}$ and that $\mu_{i(min)} = 0$ when $P_i > P_{i(min)}$. In other words, if the constraint is not violated, its associated μ variable is zero and the corresponding term in (7.47) does not exist. The constraint only becomes active when violated. The minimum of this unconstrained function is found at the point where the partials of the function to its variables are zero.

$$\frac{\partial \mathcal{L}}{\partial P_i} = 0 \tag{7.48}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \tag{7.49}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{i(max)}} = P_i - P_{i(max)} = 0 \tag{7.50}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{i(min)}} = P_i - P_{i(min)} = 0 \tag{7.51}$$

Equations (7.50) and (7.51) imply that P_i should not be allowed to go beyond its limit, and when P_i is within its limits $\mu_{i(min)} = \mu_{i(max)} = 0$ and the Kuhn-Tucker function becomes the same as the Lagrangian one. First condition, given by (7.48), results in

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$$\frac{\partial C_t}{\partial P_i} + \lambda(0 + \frac{\partial P_L}{\partial P_i} - 1) = 0$$

Since

$$C_t = C_1 + C_2 + \cdots + C_{n_g}$$

then

$$\frac{\partial C_t}{\partial P_i} = \frac{dC_i}{dP_i}$$

and therefore the condition for optimum dispatch is

$$\frac{dC_i}{dP_i} + \lambda\frac{\partial P_L}{\partial P_i} = \lambda \qquad i = 1, \ldots, n_g \tag{7.52}$$

The term $\frac{\partial P_L}{\partial P_i}$ is known as the incremental transmission loss. Second condition, given by (7.49), results in

$$\sum_{i=1}^{n_g} P_i = P_D + P_L \tag{7.53}$$

Equation (7.53) is precisely the equality constraint that was to be imposed.

Classically, Equation (7.52) is rearranged as

$$\left(\frac{1}{1 - \frac{\partial P_L}{\partial P_i}} \right) \frac{dC_i}{dP_i} = \lambda \qquad i = 1, \ldots, n_g \tag{7.54}$$

or

$$L_i \frac{dC_i}{dP_i} = \lambda \qquad i = 1, \ldots, n_g \tag{7.55}$$

where L_i is known as the *penalty factor* of plant i and is given by

$$L_i = \frac{1}{1 - \frac{\partial P_L}{\partial P_i}} \tag{7.56}$$

Hence, the effect of transmission loss is to introduce a penalty factor with a value that depends on the location of the plant. Equation (7.55) shows that the minimum cost is obtained when the incremental cost of each plant multiplied by its penalty factor is the same for all plants.

The incremental production cost is given by (7.22), and the incremental transmission loss is obtained from the loss formula (7.43) which yields

$$\frac{\partial P_L}{\partial P_i} = 2\sum_{j=1}^{n_g} B_{ij}P_j + B_{0i} \tag{7.57}$$

Substituting the expression for the incremental production cost and the incremental transmission loss in (7.52) results in

$$\beta_i + 2\gamma_i P_i + 2\lambda \sum_{j=1}^{n_g} B_{ij}P_j + B_{0i}\lambda = \lambda$$

$$\beta_i + 2\gamma_i P_i + 2\lambda \sum_{j=1}^{n_g} B_{ij} P_j + B_{0i}\lambda = \lambda$$

or

$$\left(\frac{\gamma_i}{\lambda} + B_{ii}\right) P_i + \sum_{\substack{j=1 \\ j\neq i}}^{n_g} B_{ij} P_j = \frac{1}{2}\left(1 - B_{0i} - \frac{\beta_i}{\lambda}\right) \tag{7.58}$$

Extending (7.58) to all plants results in the following linear equations in matrix form

$$\begin{bmatrix} \frac{\gamma_1}{\lambda} + B_{11} & B_{12} & \cdots & B_{1n_g} \\ B_{21} & \frac{\gamma_2}{\lambda} + B_{22} & \cdots & B_{2n_g} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n_g 1} & B_{n_g 2} & \cdots & \frac{\gamma_{n_g}}{\lambda} + B_{n_g n_g} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_{n_g} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 - B_{01} - \frac{\beta_1}{\lambda} \\ 1 - B_{02} - \frac{\beta_2}{\lambda} \\ \vdots \\ 1 - B_{0n_g} - \frac{\beta_{n_g}}{\lambda} \end{bmatrix} \tag{7.59}$$

or in short form

$$\boldsymbol{EP} = \boldsymbol{D} \tag{7.60}$$

To find the optimal dispatch for an estimated value of $\lambda^{(1)}$, the simultaneous linear equation given by (7.60) is solved. In *MATLAB* use the command $\mathbf{P} = \mathbf{E}\backslash\mathbf{D}$.

Then the iterative process is continued using the gradient method. To do this, from (7.58), P_i at the kth iteration is expressed as

$$P_i^{(k)} = \frac{\lambda^{(k)}(1 - B_{0i}) - \beta_i - 2\lambda^{(k)} \sum_{j \neq i} B_{ij} P_j^{(k)}}{2(\gamma_i + \lambda^{(k)} B_{ii})} \tag{7.61}$$

Substituting for P_i from (7.61) in (7.53) results in

$$\sum_{i=1}^{n_g} \frac{\lambda^{(k)}(1 - B_{0i}) - \beta_i - 2\lambda^{(k)} \sum_{j \neq i} B_{ij} P_j^{(k)}}{2(\gamma_i + \lambda^{(k)} B_{ii})} = P_D + P_L^{(k)} \tag{7.62}$$

or

$$f(\lambda)^{(k)} = P_D + P_L^{(k)} \tag{7.63}$$

Expanding the left-hand side of the above equation in Taylor's series about an operating point $\lambda^{(k)}$, and neglecting the higher-order terms results in

$$f(\lambda)^{(k)} + \left(\frac{df(\lambda)}{d\lambda}\right)^{(k)} \Delta\lambda^{(k)} = P_D + P_L^{(k)} \tag{7.64}$$

or

$$\begin{aligned} \Delta\lambda^{(k)} &= \frac{\Delta P^{(k)}}{\left(\frac{df(\lambda)}{d\lambda}\right)^{(k)}} \\ &= \frac{\Delta P^{(k)}}{\sum \left(\frac{dP_i}{d\lambda}\right)^{(k)}} \end{aligned} \tag{7.65}$$

where

$$\sum_{i=1}^{n_g} \left(\frac{\partial P_i}{\partial \lambda}\right)^{(k)} = \sum_{i=1}^{n_g} \frac{\gamma_i(1 - B_{0i}) + B_{ii}\beta_i - 2\gamma_i \sum_{j \neq i} B_{ij} P_j^{(k)}}{2(\gamma_i + \lambda^{(k)} B_{ii})^2} \tag{7.66}$$

and therefore,

and therefore,

$$\lambda^{(k+1)} = \lambda^{(k)} + \Delta\lambda^{(k)} \tag{7.67}$$

where

$$\Delta P^{(k)} = P_D + P_L^{(k)} - \sum_{i=1}^{n_g} P_i^{(k)} \tag{7.68}$$

The process is continued until $\Delta P^{(k)}$ is less than a specified accuracy.

If an approximate loss formula expressed by

$$P_L = \sum_{i=1}^{n_g} B_{ii} P_i^2 \tag{7.69}$$

is used, $B_{ij} = 0$, $B_{00} = 0$, and solution of the simultaneous equation given by (7.61) reduces to the following simple expression

$$P_i^{(k)} = \frac{\lambda^{(k)} - \beta_i}{2(\gamma_i + \lambda^{(k)} B_{ii})} \tag{7.70}$$

and (7.66) reduces to

$$\sum_{i=1}^{n_g} \left(\frac{\partial P_i}{\partial \lambda} \right)^{(k)} = \sum_{i=1}^{n_g} \frac{\gamma_i + B_{ii}\beta_i}{2(\gamma_i + \lambda^{(k)} B_{ii})^2} \tag{7.71}$$

Example 7.7

The fuel cost in \$/h of three thermal plants of a power system are

$$C_1 = 200 + 7.0P_1 + 0.008P_1^2 \quad \$/\text{h}$$
$$C_2 = 180 + 6.3P_2 + 0.009P_2^2 \quad \$/\text{h}$$
$$C_3 = 140 + 6.8P_3 + 0.007P_3^2 \quad \$/\text{h}$$

where P_1, P_2, and P_3 are in MW. Plant outputs are subject to the following limits

$$10 \ \text{MW} \leq 85 \ \text{MW}$$
$$10 \ \text{MW} \leq 80 \ \text{MW}$$
$$10 \ \text{MW} \leq 70 \ \text{MW}$$

For this problem, assume the real power loss is given by the simplified expression

$$P_{L(pu)} = 0.0218P_{1(pu)}^2 + 0.0228P_{2(pu)}^2 + 0.0179P_{3(pu)}^2$$

where the loss coefficients are specified in per unit on a 100-MVA base. Determine the optimal dispatch of generation when the total system load is 150 MW.

In the cost function P_i is expressed in MW. Therefore, the real power loss in terms of MW generation is

$$P_L = \left[0.0218\left(\frac{P_1}{100}\right)^2 + 0.0228\left(\frac{P_2}{100}\right)^2 + 0.0179\left(\frac{P_3}{100}\right)^2\right] \times 100 \quad \text{MW}$$
$$= 0.000218P_1^2 + 0.000228P_2^2 + 0.000179P_3^2 \quad \text{MW}$$

For the numerical solution using the gradient method, assume the initial value of $\lambda^{(1)} = 8.0$. From coordination equations, given by (7.70), $P_1^{(1)}$, $P_2^{(1)}$, and $P_3^{(1)}$ are

$$P_1^{(1)} = \frac{8.0 - 7.0}{2(0.008 + 8.0 \times 0.000218)} = 51.3136 \quad \text{MW}$$
$$P_2^{(1)} = \frac{8.0 - 6.3}{2(0.009 + 8.0 \times 0.000228)} = 78.5292 \quad \text{MW}$$
$$P_3^{(1)} = \frac{8.0 - 6.8}{2(0.007 + 8.0 \times 0.000179)} = 71.1575 \quad \text{MW}$$

The real power loss is

$$P_L^{(1)} = 0.000218(51.3136)^2 + 0.000228(78.5292)^2 + 0.000179(71.1575)^2 = 2.886$$

Since $P_D = 150$ MW, the error $\Delta P^{(1)}$ from (7.68) is

$$\Delta P^{(1)} = 150 + 2.8864 - (51.3136 + 78.5292 + 71.1575) = -48.1139$$

From (7.71)

$$\sum_{i=1}^{3} \left(\frac{\partial P_i}{\partial \lambda}\right)^{(1)} = \frac{0.008 + 0.000218 \times 7.0}{2(0.008 + 8.0 \times 0.000218)^2} + \frac{0.009 + 0.000228 \times 6.3}{2(0.009 + 8.0 \times 0.000228)^2}$$
$$+ \frac{0.007 + 0.000179 \times 6.8}{2(0.007 + 8.0 \times 0.000179)^2} = 152.4924$$

From (7.65)

$$\Delta\lambda^{(1)} = \frac{-48.1139}{152.4924} = -0.31552$$

Therefore, the new value of λ is

$$\lambda^{(2)} = 8.0 - 0.31552 = 7.6845$$

Continuing the process, for the second iteration, we have

$$P_1^{(2)} = \frac{7.6845 - 7.0}{2(0.008 + 7.6845 \times 0.000218)} = 35.3728 \quad \text{MW}$$

$$P_2^{(2)} = \frac{7.6845 - 6.3}{2(0.009 + 7.6845 \times 0.000228)} = 64.3821 \quad \text{MW}$$

$$P_3^{(2)} = \frac{7.6845 - 6.8}{2(0.007 + 7.6845 \times 0.000179)} = 52.8015 \quad \text{MW}$$

The real power loss is

$$P_L^{(2)} = 0.000218(35.3728)^2 + 0.000228(64.3821)^2 + 0.000179(52.8015)^2 = 1.717$$

Since $P_D = 150$ MW, the error $\Delta P^{(2)}$ from (7.68) is

$$\Delta P^{(2)} = 150 + 1.7169 - (35.3728 + 64.3821 + 52.8015) = -0.8395$$

From (7.71)

$$\sum_{i=1}^{3}\left(\frac{\partial P_i}{\partial \lambda}\right)^{(2)} = \frac{0.008 + 0.000218 \times 7.0}{2(0.008 + 7.684 \times 0.000218)^2} + \frac{0.009 + 0.000228 \times 6.3}{2(0.009 + 7.684 \times 0.000228)^2}$$
$$+ \frac{0.007 + 0.000179 \times 6.8}{2(0.007 + 7.6845 \times 0.000179)^2} = 154.588$$

From (7.65)

$$\Delta\lambda^{(2)} = \frac{-0.8395}{154.588} = -0.005431$$

Therefore, the new value of λ is

$$\lambda^{(3)} = 7.6845 - 0.005431 = 7.679$$

For the third iteration, we have

$$P_1^{(3)} = \frac{7.679 - 7.0}{2(0.008 + 7.679 \times 0.000218)} = 35.0965 \quad \text{MW}$$

$$P_2^{(3)} = \frac{7.679 - 6.3}{2(0.009 + 7.679 \times 0.000228)} = 64.1369 \quad \text{MW}$$

$$P_3^{(3)} = \frac{7.679 - 6.8}{2(0.007 + 7.679 \times 0.000179)} = 52.4834 \quad \text{MW}$$

The real power loss is

$$P_L^{(3)} = 0.000218(35.0965)^2 + 0.000228(64.1369)^2 + 0.000179(52.4834)^2 = 1.699$$

Since $P_D = 150$ MW, the error $\Delta P^{(3)}$ from (7.68) is

$$\Delta P^{(3)} = 150 + 1.6995 - (35.0965 + 64.1369 + 52.4834) = -0.01742$$

From (7.71)

$$\sum_{i=1}^{3}\left(\frac{\partial P_i}{\partial \lambda}\right)^{(3)} = \frac{0.008 + 0.000218 \times 7.0}{2(0.008 + 7.679 \times 0.000218)^2} + \frac{0.009 + 0.000228 \times 6.3}{2(0.009 + 7.679 \times 0.000228)^2}$$
$$+ \frac{0.007 + 0.000179 \times 6.8}{2(0.007 + 7.679 \times 0.000179)^2} = 154.624$$

From (7.65)

$$\Delta\lambda^{(3)} = \frac{-0.01742}{154.624} = -0.0001127$$

Therefore, the new value of λ is

$$\lambda^{(4)} = 7.679 - 0.0001127 = 7.6789$$

Since $\Delta\lambda^{(3)}$, is small the equality constraint is met in four iterations, and the optimal dispatch for $\lambda = 7.6789$ are

$$\begin{aligned}
P_1^{(4)} &= \frac{7.6789 - 7.0}{2(0.008 + 7.679 \times 0.000218)} = 35.0907 \quad \mathrm{MW} \\
P_2^{(4)} &= \frac{7.6789 - 6.3}{2(0.009 + 7.679 \times 0.000228)} = 64.1317 \quad \mathrm{MW} \\
P_3^{(4)} &= \frac{7.6789 - 6.8}{2(0.007 + 7.679 \times 0.000179)} = 52.4767 \quad \mathrm{MW}
\end{aligned}$$

The real power loss is

$$P_L^{(4)} = 0.000218(35.0907)^2 + 0.000228(64.1317)^2 + 0.000179(52.4767)^2 = 1.699$$

and the total fuel cost is

$$\begin{aligned}
C_t = {} & 200 + 7.0(35.0907) + 0.008(35.0907)^2 + 180 + 6.3(64.1317) + \\
& 0.009(64.1317)^2 + 140 + 6.8(52.4767) + 0.007(52.4767)^2 = 1592.65 \ \$/\mathrm{h}
\end{aligned}$$