Synchronous machine transient behavior

Clo_3

8.3 SYNCHRONOUS MACHINE TRANSIENTS

The synchronous machine consists of three stator windings mounted on the stator, and one field winding mounted on the rotor. Two additional fictitious windings could be added to the rotor, one along the direct axis and one along the quadrature axis, which model the short-circuited paths of the damper windings. These windings are shown schematically in Figure 8.3.

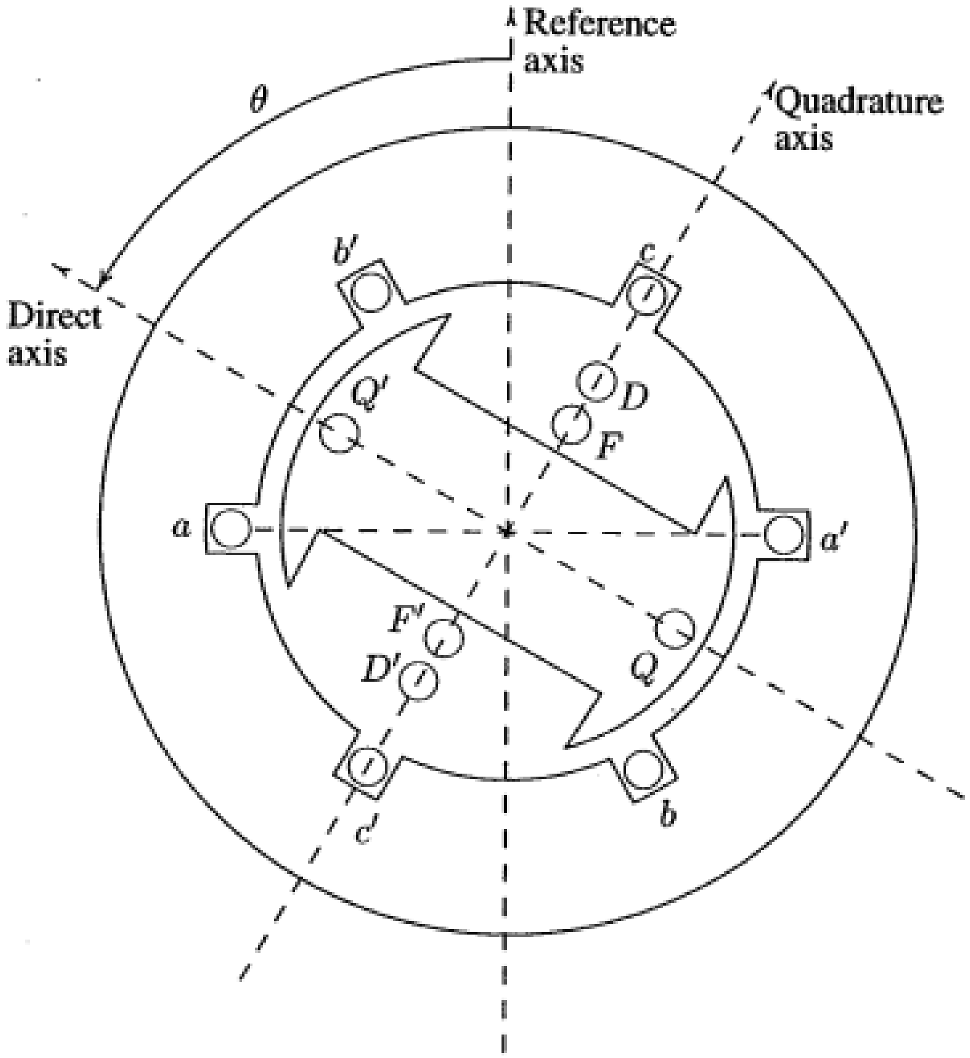

FIGURE 8.3
Schematic representation of a synchronous machine.

We shall assume a synchronously rotating reference frame (axis) rotating with the synchronous speed ω which will be along the axis of phase a at $t = 0$. If θ is the angle by which rotor direct axis is ahead of the magnetic axis of phase a, then

$$\theta = \omega t + \delta + \frac{\pi}{2} \tag{8.3}$$

where δ is the displacement of the quadrature axis from the synchronously rotating reference axis and $(\delta + \frac{\pi}{2})$ is the displacement of the direct axis.

In the classical method, the idealized synchronous machine is represented as a group of magnetically coupled circuits with inductances which depend on the angular position of the rotor. In addition, saturation is neglected and spatial distribution of armature mmf is assumed sinusoidal. The circuits are shown schematically in Figure 8.4.

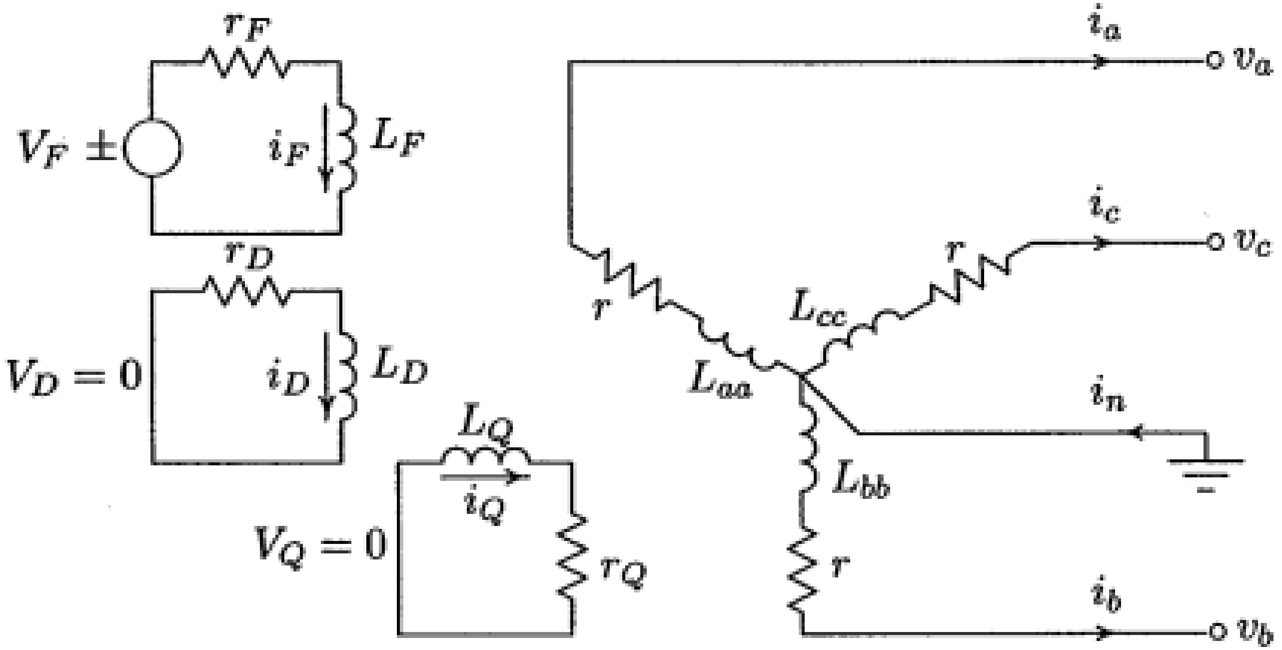

FIGURE 8.4
Schematic representation of mutually coupled circuits.

The stator currents are assumed to have a positive direction flowing out of the machine terminals. Since the machine is a generator, following the circuit passive sign convention, the voltage equation becomes

$$\begin{bmatrix} v_a \\ v_b \\ v_c \\ -v_F \\ 0 \\ 0 \end{bmatrix} = -\begin{bmatrix} r & 0 & 0 & 0 & 0 & 0 \\ 0 & r & 0 & 0 & 0 & 0 \\ 0 & 0 & r & 0 & 0 & 0 \\ 0 & 0 & 0 & r_F & 0 & 0 \\ 0 & 0 & 0 & 0 & r_D & 0 \\ 0 & 0 & 0 & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_Q \end{bmatrix} - \frac{d}{dt} \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_F \\ \lambda_D \\ \lambda_Q \end{bmatrix} \tag{8.4}$$

The above equation may be written in partitioned form as

$$\begin{bmatrix} \mathbf{v}_{abc} \\ \mathbf{v}_{FDQ} \end{bmatrix} = -\begin{bmatrix} \mathbf{R}_{abc} & 0 \\ 0 & \mathbf{R}_{FDQ} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abc} \\ \mathbf{i}_{FDQ} \end{bmatrix} - \frac{d}{dt} \begin{bmatrix} \lambda_{abc} \\ \lambda_{FDQ} \end{bmatrix} \tag{8.5}$$

where

$$\mathbf{v}_{FDQ} = \begin{bmatrix} -v_F \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{i}_{FDQ} = \begin{bmatrix} i_F \\ i_D \\ i_Q \end{bmatrix} \quad \lambda_{FDQ} = \begin{bmatrix} \lambda_F \\ \lambda_D \\ \lambda_Q \end{bmatrix} \quad \text{etc.} \tag{8.6}$$

The flux linkages are functions of self- and mutual inductances given by

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_F \\ \lambda_D \\ \lambda_Q \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{aF} & L_{aD} & L_{aQ} \\ L_{ba} & L_{bb} & L_{bc} & L_{bF} & L_{bD} & L_{bQ} \\ L_{ca} & L_{cb} & L_{cc} & L_{cF} & L_{cD} & L_{cQ} \\ L_{Fa} & L_{Fb} & L_{Fc} & L_{FF} & L_{FD} & L_{FQ} \\ L_{Da} & L_{Db} & L_{Dc} & L_{DF} & L_{DD} & L_{DQ} \\ L_{Qa} & L_{Qb} & L_{Qc} & L_{QF} & L_{QD} & L_{QQ} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_Q \end{bmatrix} \tag{8.7}$$

or in compact form we have

$$\begin{bmatrix} \lambda_{abc} \\ \lambda_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{SS} & \mathbf{L}_{SR} \\ \mathbf{L}_{RS} & \mathbf{L}_{RR} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abc} \\ \mathbf{i}_{FDQ} \end{bmatrix} \tag{8.8}$$

8.3.1 INDUCTANCES OF SALIENT-POLE MACHINES

The self-inductance of any stator coil varies periodically from a maximum (when the direct axis coincides with the coil magnetic axis) to a minimum (when the quadrature axis is in line with the coil magnetic axis). The self-inductance L_{aa}, for example, will be a maximum for $\theta = 0$, a minimum for $\theta = 90°$ and maximum, again for $\theta = 180°$, and so on. That is, L_{aa} has a period of 180° and can be represented approximately by cosines of second harmonics. Because of the rotor symmetry, the diagonal elements of the submatrix $\mathbf{L}_{SS}$ are represented as

$$\begin{aligned} L_{aa} &= L_s + L_m \cos 2\theta \\ L_{bb} &= L_s + L_m \cos 2(\theta - 2\pi/3) \\ L_{cc} &= L_s + L_m \cos 2(\theta + 2\pi/3) \end{aligned} \tag{8.9}$$

where θ is the angle between the direct axis and the magnetic axis of phase a, as shown in Figure 8.3. The mutual inductances between any two stator phases are also periodic functions of rotor angular position because of the rotor saliency. We can conclude from the symmetry considerations that the mutual inductance between phase a and b should have a negative maximum when the pole axis is lined up 30° behind phase a or 30° ahead of phase b, and a negative minimum when it is midway between the two phases. Thus, the variations of stator mutual inductances, i.e., the off-diagonal elements of the submatrix $\mathbf{L}_{SS}$ can be represented as follows.

$$\begin{aligned} L_{ab} = L_{ba} &= -M_s - L_m \cos 2(\theta + \pi/6) \\ L_{bc} = L_{cb} &= -M_s - L_m \cos 2(\theta - \pi/2) \\ L_{ca} = L_{ac} &= -M_s - L_m \cos 2(\theta + 5\pi/6) \end{aligned} \tag{8.10}$$

All the rotor self-inductances are constant since the effects of stator slots and saturation are neglected. They are represented with single subscript notation.

$$L_{FF} = L_F \quad L_{DD} = L_D \quad L_{QQ} = L_Q \tag{8.11}$$

The mutual inductance between any two circuits both in direct axis (or both in quadrature axis) is constant. The mutual inductance between any rotor direct axis circuit and quadrature axis circuit vanishes. Thus, we have

$$L_{FD} = L_{DF} = M_R \quad L_{FQ} = L_{QF} = 0 \quad L_{DQ} = L_{QD} = 0 \tag{8.12}$$

$$
\begin{aligned}
L_{aF} &= L_{Fa} = M_F \cos\theta \\
L_{bF} &= L_{Fb} = M_F \cos(\theta - 2\pi/3) \\
L_{cF} &= L_{Fc} = M_F \cos(\theta + 2\pi/3) \\
L_{aD} &= L_{Da} = M_D \cos\theta \\
L_{bD} &= L_{Db} = M_D \cos(\theta - 2\pi/3) \\
L_{cD} &= L_{Dc} = M_D \cos(\theta + 2\pi/3) \\
L_{aQ} &= L_{Qa} = M_Q \sin\theta \\
L_{bQ} &= L_{Qb} = M_Q \sin(\theta - 2\pi/3) \\
L_{cQ} &= L_{Qc} = M_Q \sin(\theta + 2\pi/3)
\end{aligned} \tag{8.13}
$$

8.4 THE PARK TRANSFORMATION

A great simplification can be made by transformation of stator variables from phases a, b, and c into new variables the frame of reference of which moves with the rotor. The transformation is based on the so called *two-axis theory*, which was pioneered by Blondel, Doherty, Nickle, and Park [20, 61].

The Park transformation for currents is as follows

$$\begin{bmatrix} i_0 \\ i_d \\ i_q \end{bmatrix} = \sqrt{2/3} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ \cos\theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ \sin\theta & \sin(\theta - 2\pi/3) & \sin(\theta + 2\pi/3) \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{8.14}$$

or, in matrix notation

$$\mathbf{i}_{0dq} = \mathbf{P}\mathbf{i}_{abc} \tag{8.15}$$

Similarly for voltages and flux linkages, we have

$$\mathbf{v}_{0dq} = \mathbf{P}\mathbf{v}_{abc} \tag{8.16}$$

$$\lambda_{0dq} = \mathbf{P}\lambda_{abc} \tag{8.17}$$

We now wish to transform the time-varying inductances to a rotor frame of reference with the original rotor quantities unaffected. Thus, in (8.17) we augment the $\mathbf{P}$ matrix with a 3×3 identity matrix $\mathbf{U}$ to get

$$\begin{bmatrix} \lambda_{0dq} \\ \lambda_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \lambda_{abc} \\ \lambda_{FDQ} \end{bmatrix} \tag{8.19}$$

or

$$\begin{bmatrix} \lambda_{abc} \\ \lambda_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \lambda_{odq} \\ \lambda_{FDQ} \end{bmatrix} \tag{8.20}$$

Substituting in (8.8), we get

$$\begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \lambda_{odq} \\ \lambda_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{SS} & \mathbf{L}_{SR} \\ \mathbf{L}_{RS} & \mathbf{L}_{RR} \end{bmatrix} \begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{odq} \\ \mathbf{i}_{FDQ} \end{bmatrix} \tag{8.21}$$

or

$$\begin{bmatrix} \lambda_{odq} \\ \lambda_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{L}_{SS} & \mathbf{L}_{SR} \\ \mathbf{L}_{RS} & \mathbf{L}_{RR} \end{bmatrix} \begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{odq} \\ \mathbf{i}_{FDQ} \end{bmatrix} \tag{8.22}$$

Substituting for $\mathbf{P}$, $\mathbf{P}^{-1}$ and the inductances given by (8.9)–(8.13), the above equation reduces to

$$\begin{bmatrix} \lambda_0 \\ \lambda_d \\ \lambda_q \\ \lambda_F \\ \lambda_D \\ \lambda_Q \end{bmatrix} = \begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F & KM_D & 0 \\ 0 & 0 & L_q & 0 & 0 & kM_Q \\ 0 & kM_F & 0 & L_F & M_R & 0 \\ 0 & kM_D & 0 & M_R & L_D & 0 \\ 0 & 0 & kM_Q & 0 & 0 & L_Q \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_D \\ i_Q \end{bmatrix} \tag{8.23}$$

where we have introduced the following new parameters

$$L_0 = L_s - 2M_s \tag{8.24}$$

$$L_d = L_s + M_s + \frac{3}{2}L_m \tag{8.25}$$

$$L_q = L_s + M_s - \frac{3}{2}L_m \tag{8.26}$$

and $k = \sqrt{3/2}$.

Transforming the stator-based currents ($\mathbf{i}_{abc}$) into rotor-based currents ($\mathbf{i}_{0dq}$), with rotor currents unaffected, we obtain

$$\begin{bmatrix} \mathbf{i}_{0dq} \\ \mathbf{i}_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abc} \\ \mathbf{i}_{FDQ} \end{bmatrix} \tag{8.27}$$

or

$$\begin{bmatrix} \mathbf{i}_{abc} \\ \mathbf{i}_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{odq} \\ \mathbf{i}_{FDQ} \end{bmatrix} \tag{8.28}$$

and similarly for voltages, we get

$$\begin{bmatrix} \mathbf{v}_{abc} \\ \mathbf{v}_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{v}_{odq} \\ \mathbf{v}_{FDQ} \end{bmatrix} \tag{8.29}$$

Substituting (8.20), (8.28), and (8.29) into (8.5), we get

$$\begin{aligned} \begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{v}_{0dq} \\ \mathbf{v}_{FDQ} \end{bmatrix} = - \begin{bmatrix} \mathbf{R}_{abc} & 0 \\ 0 & \mathbf{R}_{FDQ} \end{bmatrix} \begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{0dq} \\ \mathbf{i}_{FDQ} \end{bmatrix} \\ - \frac{d}{dt} \begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \lambda_{0dq} \\ \lambda_{FDQ} \end{bmatrix} \end{aligned} \tag{8.30}$$

or

$$\begin{bmatrix} \mathbf{v}_{0dq} \\ \mathbf{v}_{FDQ} \end{bmatrix} = -\begin{bmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{R}_{abc} & 0 \\ 0 & \mathbf{R}_{FDQ} \end{bmatrix} \begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{0dq} \\ \mathbf{i}_{FDQ} \end{bmatrix} - \begin{bmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \lambda_{0dq} \\ \lambda_{FDQ} \end{bmatrix} \tag{8.31}$$

Evaluating the first term, and obtaining the derivative of the second term in (8.31), yields

$$\begin{bmatrix} \mathbf{v}_{0dq} \\ \mathbf{v}_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{abc} & 0 \\ 0 & \mathbf{R}_{FDQ} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{0dq} \\ \mathbf{i}_{FDQ} \end{bmatrix} - \begin{bmatrix} \mathbf{P}\frac{d}{dt}\mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \lambda_{0dq} \\ \lambda_{FDQ} \end{bmatrix} - \frac{d}{dt} \begin{bmatrix} \lambda_{0dq} \\ \lambda_{FDQ} \end{bmatrix} \tag{8.32}$$