This is a program to compute the $\mathbb{F}_2[\rho]$-module structure on the E_2-term of the Borel C_2-equivariant Adams spectral sequence for the C_2-equivariant sphere:

$$E_2^{s,t,w} = \mathrm{Ext}^{s,t,w}_{\mathcal{A}^h_{*,*}}\left((H_{C_2})^h_{*,*}\right) \cong \begin{cases} \mathrm{Ext}^{s-1,t-w-1}_{\mathcal{A}_*}\left(H_*\Sigma\mathbb{P}^{-w-1}_{-\infty}\right), & w > 0; \\ \mathrm{Ext}^{s-1,t-w-1}_{\mathcal{A}_*}\left(H_*\mathbb{P}^{\infty}_{-w}\right) \oplus \mathrm{Ext}^{s,t-w-1}_{\mathcal{A}_*}\left(H_*S^{-1}\right), & w \leq 0. \end{cases}$$

The file `AAHSS_without_-1.cpp` uses the Curtis algorithm to compute the Lambda complex of $\Sigma\mathbb{P}^{-w-1}_{-\infty}$ for $w > 0$, where

$$H_*\Sigma\mathbb{P}^{-w-1}_{-\infty} = \begin{cases} \mathbb{F}_2, & * \geq -w \text{ and } * \neq -1, \\ 0, & \text{otherwise.} \end{cases}$$

After inputting a positive integer w as the weight, and a positive integer t as the total degree, you will get an output of all permanent cycles and differentials between elements from different cells in the Lambda complex, ordered by the homological degree s.

For example, when you input $w = 6$ and $t = 12$, you will get the output, a part of which is listed as follows:

```
homological degree s=3 (topological degree 9, coweight 3):
3 0 0 <-- 4 0
1 1 1
-3 3 3 <-- 1 3
-5 5 3 <-- -3 7

homological degree s=4 (topological degree 8, coweight 2):
-5 5 1 1 <-- -4 6 1
-6 2 3 3 <-- 0 2 1
```

The line

```
1 1 1
```

implies that there is an element

$$h_1^2[1] \in \mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\Sigma\mathbb{P}^{-7}_{-\infty}\right),$$

which maps to the nontrivial element

$$h_1^2[1] \in \mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\Sigma\mathbb{P}^{-2}_{-\infty}\right).$$

The line

```
-6 2 3 3 <-- 0 2 1
```

implies that there is an element

$$h_0h_2[0] \in \mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\Sigma\mathbb{P}^{-6}_{-\infty}\right),$$

which maps to the nontrivial element

$$h_0h_2[0] \in \mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\Sigma\mathbb{P}^{-2}_{-\infty}\right),$$

but cannot be lifted to $\mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\Sigma\mathbb{P}^{-7}_{-\infty}\right)$.
The line

```
-5 5 1 1 <-- -4 6 1
```

implies that there is an element

$$h_0h_3[-4] \in \mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\Sigma\mathbb{P}^{-5}_{-\infty}\right),$$

which maps to 0 in $\mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\Sigma\mathbb{P}^{-4}_{-\infty}\right)$, and cannot be lifted to $\mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\Sigma\mathbb{P}^{-6}_{-\infty}\right)$.

The file `AAHSS_with_-1.cpp` computes the Lambda complex of $\mathbb{P}^{\infty}_{-w}$.

For example, when you input $w = 6$ and $t = 12$, you will get the output, a part of which is listed as follows:

```
homological degree s=3 (topological degree 9, coweight 3):
3 0 0 <-- 4 0
-3 3 3 <-- 1 3
-5 5 3 <-- -3 7

homological degree s=4 (topological degree 8, coweight 2):
-1 1 1 1 <-- 0 2 1
-5 5 1 1 <-- -4 6 1
-6 2 3 3 <-- 1 1 1
```

The line

```
-1 1 1 1 <-- 0 2 1
```

implies that there is an element

$$h_0h_2[0] \in \mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\mathbb{P}^{\infty}_{0}\right),$$

which maps to 0 in $\mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\mathbb{P}^{\infty}_{1}\right)$.

The line

```
-6 2 3 3 <-- 1 1 1
```

implies that there is an element

$$h_1^2[1] \in \mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\mathbb{P}^{\infty}_{1}\right),$$

which can be lifted to

$$h_1^2[1] \in \mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\mathbb{P}^{\infty}_{0}\right),$$

and maps to 0 in $\mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\mathbb{P}^{\infty}_{2}\right)$.

To understand the ρ-multiplication map

$$\mathrm{Ext}^{3,7,1}_{\mathcal{A}^h_{*,*}}\left((H_{C_2})^h_{*,*}\right) \to \mathrm{Ext}^{3,6,0}_{\mathcal{A}^h_{*,*}}\left((H_{C_2})^h_{*,*}\right),$$

we need to extend the range by inputting $w = 7$ and $t = 13$ to get the partial output from `AAHSS_without_-1.cpp`:

```
homological degree s=3 (topological degree 10, coweight 3):
3 0 0 <-- 4 0
-3 3 3 <-- 1 3
-5 5 3 <-- -3 7

homological degree s=4 (topological degree 9, coweight 2):
-5 5 1 1 <-- -4 6 1
-6 2 3 3 <-- 0 2 1
-7 3 3 3 <-- 1 1 1
```

and the partial output from `AAHSS_with_-1.cpp`:

```
homological␣degree␣s=3␣(topological␣degree␣10,␣coweight␣3):
3␣0␣0␣<--␣4␣0
-3␣3␣3␣<--␣1␣3
-5␣5␣3␣<--␣-3␣7

homological␣degree␣s=4␣(topological␣degree␣9,␣coweight␣2):
-1␣1␣1␣1␣<--␣0␣2␣1
-5␣5␣1␣1␣<--␣-4␣6␣1
-6␣2␣3␣3␣<--␣1␣1␣1
-7␣3␣3␣3
```

Then we can see that $h_0h_2[0]$ maps to $c_0[-6]$ under the map

$$\mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\Sigma\mathbb{P}^{-2}_{-\infty}\right) \to \varprojlim_k \mathrm{Ext}^{3,5}_{\mathcal{A}_*}\left(H_*\mathbb{P}^{-2}_{-k}\right) \to \varprojlim_k \mathrm{Ext}^{3,5}_{\mathcal{A}_*}\left(H_*\mathbb{P}^{-1}_{-k}\right).$$

On the other hand, $c_0[-6]$ is the image of $h_1^2[1]$ under the map

$$\mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\mathbb{P}^{\infty}_{0}\right) \to \varprojlim_k \mathrm{Ext}^{3,5}_{\mathcal{A}_*}\left(H_*\mathbb{P}^{-1}_{-k}\right).$$

Therefore, $h_0h_2[0]$ maps to $h_1^2[1]$ under the upper map in the square

$$\begin{tikzcd}
\mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\Sigma\mathbb{P}^{-2}_{-\infty}\right) \arrow[r] \arrow[d, "\cong"'] & \mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\mathbb{P}^{\infty}_{0}\right) \oplus \mathrm{Ext}^{3,5}_{\mathcal{A}_*}\left(H_*S^{-1}\right) \arrow[d, "\cong"] \\
\mathrm{Ext}^{3,7,1}_{\mathcal{A}^h_{*,*}}\left(\left(H_{C_2}\right)^h_{*,*}\right) \arrow[r, "\cdot\rho"] & \mathrm{Ext}^{3,6,0}_{\mathcal{A}^h_{*,*}}\left(\left(H_{C_2}\right)^h_{*,*}\right).
\end{tikzcd}$$

We can also see that $h_1^2[1]$ maps to $h_2^3[-7]$ under the map

$$\mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\Sigma\mathbb{P}^{-2}_{-\infty}\right) \to \varprojlim_k \mathrm{Ext}^{3,5}_{\mathcal{A}_*}\left(H_*\mathbb{P}^{-2}_{-k}\right) \to \varprojlim_k \mathrm{Ext}^{3,5}_{\mathcal{A}_*}\left(H_*\mathbb{P}^{-1}_{-k}\right).$$

On the other hand, $h_2^3[-7]$ is not in the image of the map

$$\mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\mathbb{P}^{\infty}_{0}\right) \to \varprojlim_k \mathrm{Ext}^{3,5}_{\mathcal{A}_*}\left(H_*\mathbb{P}^{-1}_{-k}\right).$$

Therefore, it maps to a nontrivial element under the map

$$\varprojlim_k \mathrm{Ext}^{3,5}_{\mathcal{A}_*}\left(H_*\mathbb{P}^{-1}_{-k}\right) \to \varprojlim_k \mathrm{Ext}^{3,5}_{\mathcal{A}_*}\left(H_*\mathbb{P}^{\infty}_{-k}\right) \cong \mathrm{Ext}^{3,5}_{\mathcal{A}_*}\left(H_*S^{-1}\right),$$

which has to be $h_1^3[-1]$. Therefore, $h_1^2[1]$ maps to $h_1^3[-1]$ under the upper map in the square

$$\begin{tikzcd}
\mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\Sigma\mathbb{P}^{-2}_{-\infty}\right) \arrow[r] \arrow[d, "\cong"'] & \mathrm{Ext}^{2,5}_{\mathcal{A}_*}\left(H_*\mathbb{P}^{\infty}_{0}\right) \oplus \mathrm{Ext}^{3,5}_{\mathcal{A}_*}\left(H_*S^{-1}\right) \arrow[d, "\cong"] \\
\mathrm{Ext}^{3,7,1}_{\mathcal{A}^h_{*,*}}\left(\left(H_{C_2}\right)^h_{*,*}\right) \arrow[r, "\cdot\rho"] & \mathrm{Ext}^{3,6,0}_{\mathcal{A}^h_{*,*}}\left(\left(H_{C_2}\right)^h_{*,*}\right).
\end{tikzcd}$$