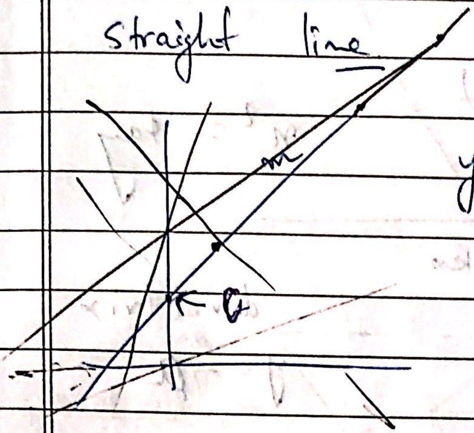
Coordinate geometry

Straight line



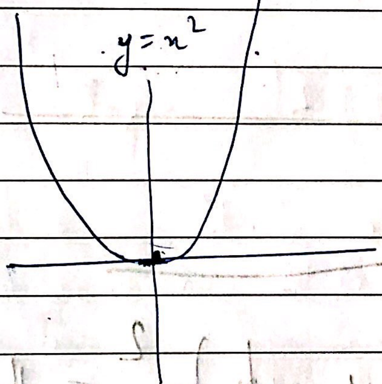
$$y = mx + c$$

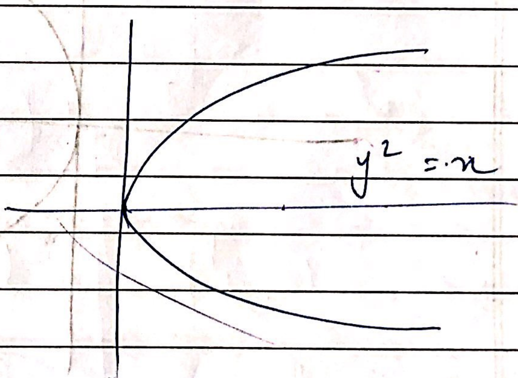
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Distance between two points on a line $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

Parabola

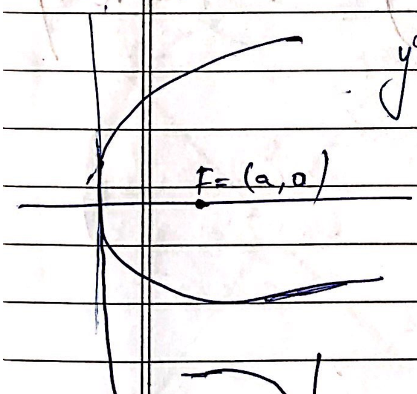
- any point is at an equal distance from
  - focus (fixed point)
  - straight line (directrix)

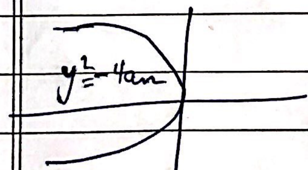


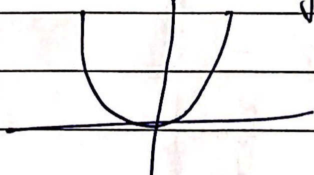


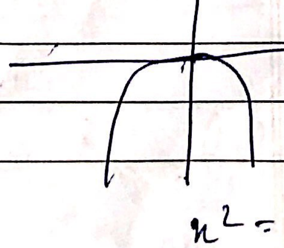
$y^2 = 4ax$ | $x^2 = 4ay$

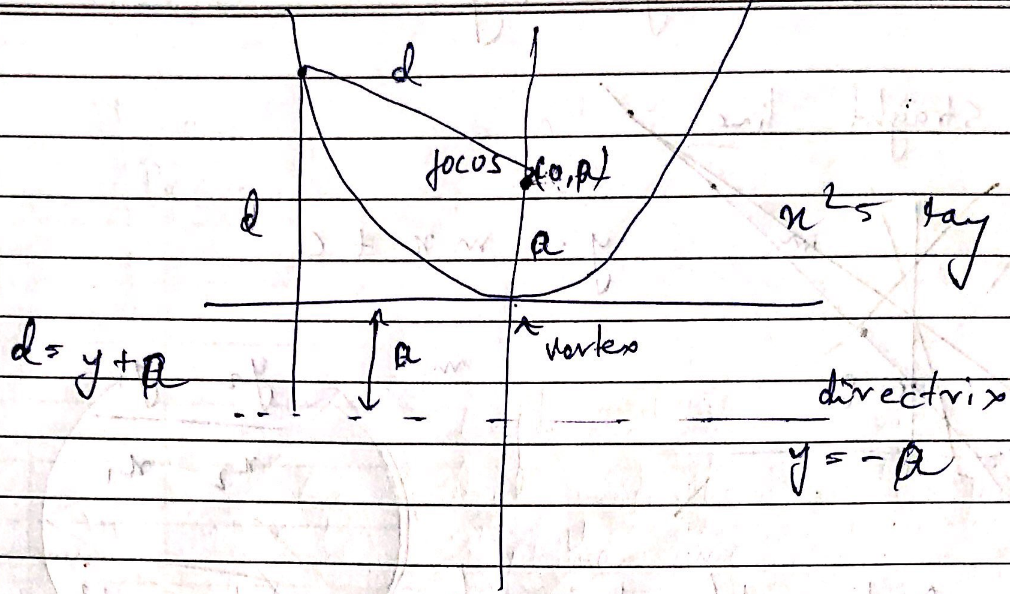
a: distance from the origin to the focus







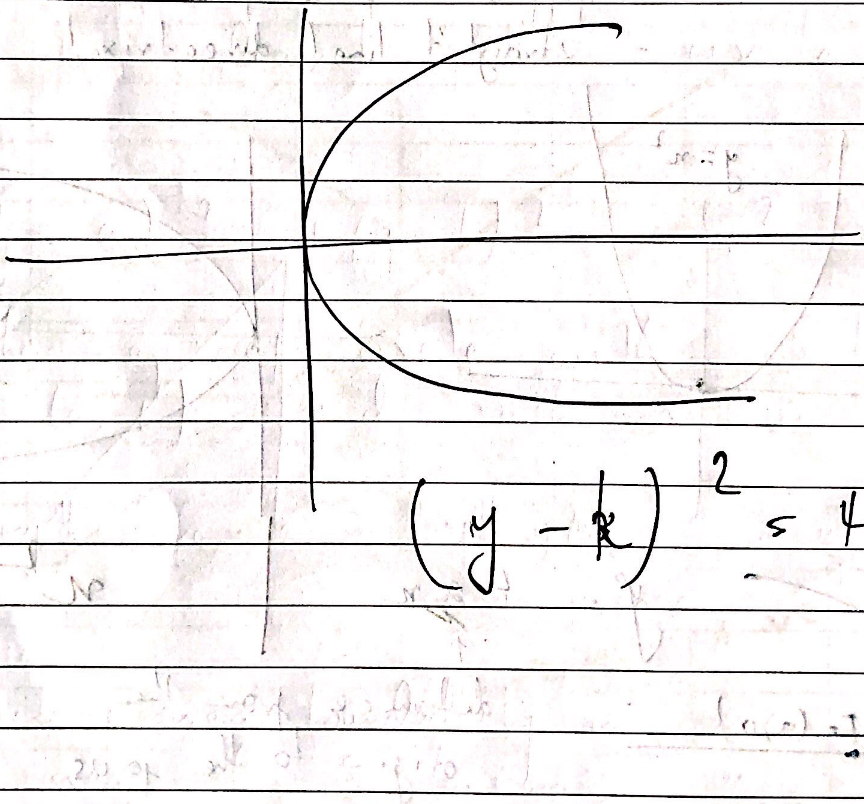




$x^2 = 4ay$

Shifting the vertex from the origin to (h, k)

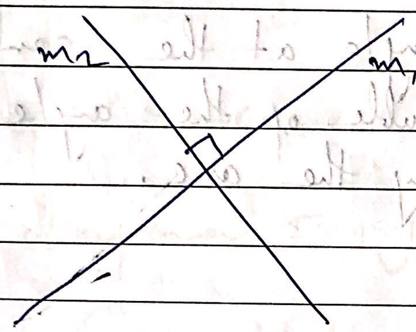
$$(x-h)^2 = 4a(y-k),$$



$$(y-k)^2 = 4a(x-h)$$

Straight lines

perpendicular lines have negative reciprocal slopes that is, the product of the two slopes is -1.



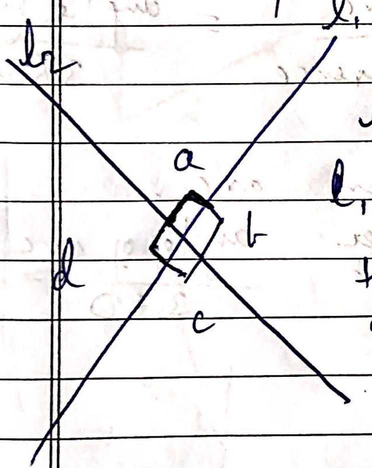
$m_1 \times m_2 = -1$

Distance between two points - (x_1, y_1) and (x_2, y_2)

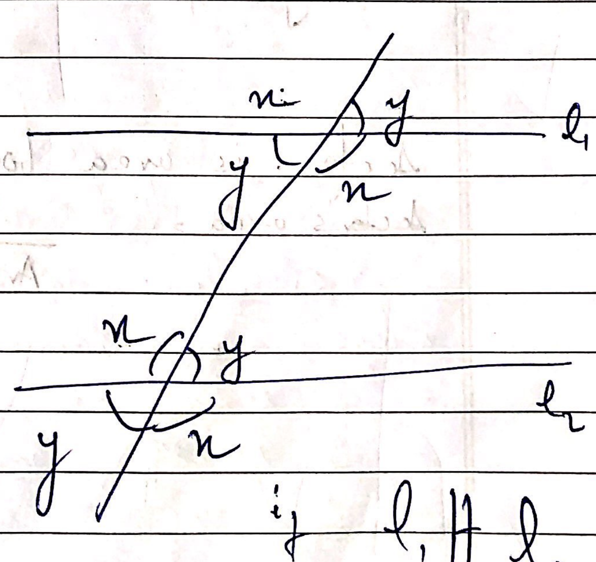
$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{(Rise over run)}$$

~~line of symmetry : $y = x$~~



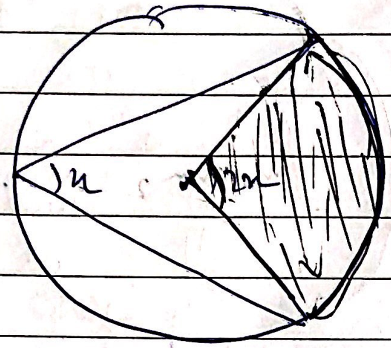
if, $l_1 \perp l_2$ then $a, b, c, d = 90$



if $l_1 \parallel l_2$ (parallel)

Circles

- Area $= \pi r^2$
- Perimeter $= 2\pi r$



Angle at the center is double of the angle formed by the arc.

- A triangle inscribed in a circle can be a right angled triangle only if the hypotenuse is a the diameter of the circle

$$(x-a)^2 + (y-b)^2 = r^2$$

is the eq. of circle with center at (a, b) and with radius $r > 0$.

For length of arc: $\frac{\text{length of arc}}{\text{circumference}} = \frac{\text{angle by arc}}{360}$

Sector is area bounded by an arc.

Sector's area: $\frac{\text{Sector's area}}{\text{Area of circle}} = \frac{\text{angle of arc}}{360}$

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Cylinders & other 3D figures

Cylinder

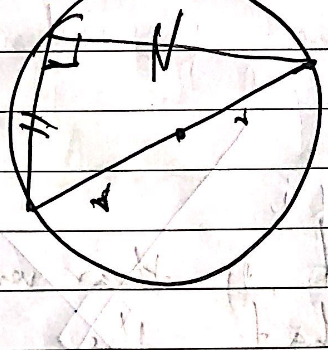
- Surface area: $2\pi r h + 2\pi r^2$
- Volume = $\pi r^2 h$

Cuboid

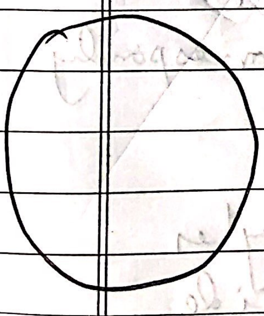
- Volume = lwh
- Surface area = $2(lw + lh + wh)$
- Diagonal = $\sqrt{l^2 + b^2 + h^2}$

Cube

- Volume = a^3
- Surface area = $6a^2$
- Diagonal = $\sqrt{3}\, a$



$(5, 6)$



$(x-5)^2 + (y-6)^2 = r^2$

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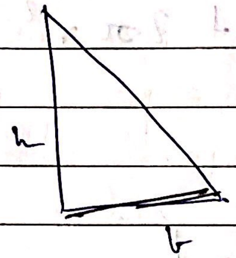
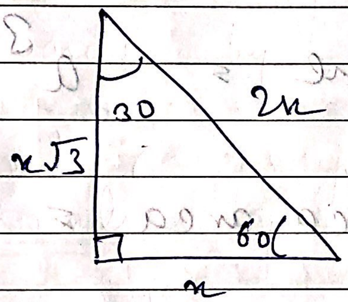
Triangles.



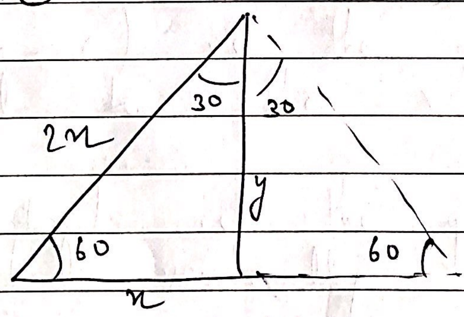
Area = $\frac{1}{2} h b$

30 - 60 - 90 rule

In a Δ 30°, 60°, 90 angles following must be the length of sides opposite to them.



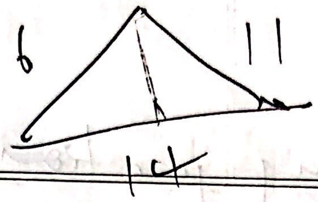
it comes from half of an equilateral triangle



Congruent Triangles.

Two triangles are congruent if their vertices can be matched up so that the corresponding angles and the corresponding sides are congruent.

- 3 sides are congruent
- 2 sides and included angles
- 2 angles and included side

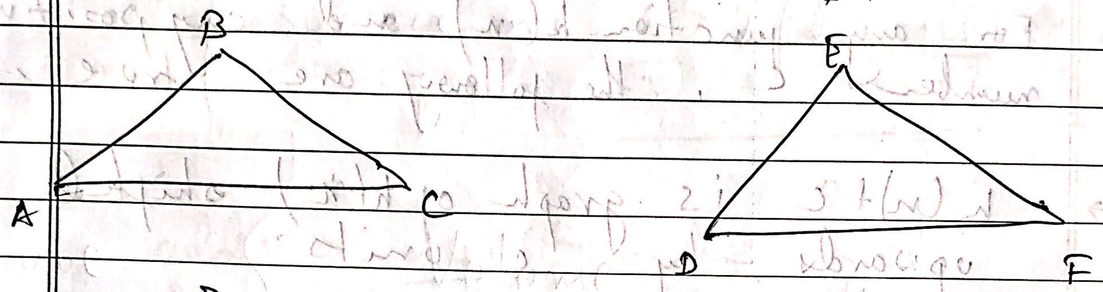


Similar triangles:

- Two triangles have the same shape but not necessarily the same size.

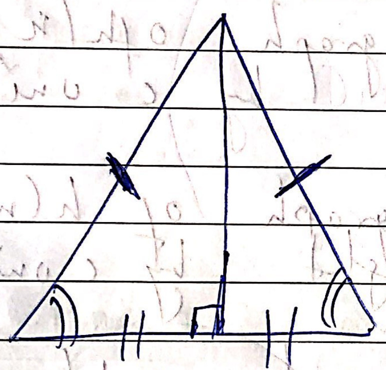
For ABC & DEF if they are similar

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$



$$\Rightarrow \frac{AB}{BC} = \frac{DE}{DF}$$

Isosceles triangle



Area of Δ

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$

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Graphing functions.

For any function h the graph of $y = -h(x)$ is the reflection of the graph of $y = h(x)$ about x-axis.

For any function $h(x)$ and any positive number c, the following are true.

→ $h(x)+c$ is graph of $h(x)$ shifted upwards by c units.

→ $h(x)-c$ is graph of $h(x)$ shifted downwards by c units.

→ $h(x+c)$ is the graph of $h(x)$ shifted to the left by c units.

→ $h(x-c)$ is the graph of $h(x)$ shifted to the right by c units.

→ $c\,h(x)$ is the graph of $h(x)$ streched vertically by the factor of c if $c > 1$.

→ $c\,h(x)$ is the graph of $h(x)$ shrunk vertically by a factor of c if $0 < c < 1$

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Averages, Weighted Average, Median, Mode.

$$\text{Mean} = \frac{\sum_{i=1}^{n} a_i}{n} \quad \left(\frac{\text{sum of all numbers}}{n} \right)$$

Median
- sort the numbers in ascending order.

if n is even: $\dfrac{\frac{n}{2}^{th} \text{ number} + \frac{n+1}{2}^{th} \text{ number}}{2}$

odd: $\frac{n+1}{2}^{th}$ number

For eg. 1, 2, 3, 4, 5, 6

median = $\frac{3+4}{2}$ = 3.5

or. 1, 2, 3, 4, 5

median = 3

Mode: count the frequency. Mode is the number with highest frequency

Standard deviation = $\sqrt{\dfrac{\sum (x - \bar{x})^2}{n}}$

$\bar{x}$ ← mean

n ← numbers

x ← individual number

It measures the spread of data about the mean value.

* Note: If you have a set of numbers and each number is increased by the same amount (say 2) the standard deviation will remain same.

* for mult/div SD is mult/div by same factor

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Calc standard deviation:

Eg. 1, 2, 5, 8.

$\bar{x}$ (mean) = $\frac{1+2+5+8}{4} = 4$

Standard deviation =

$$\sqrt{\frac{(1-4)^2 + (2-4)^2 + (5-4)^2 + (8-4)^2}{4}}$$

Now suppose numbers are ~~3, 4,~~

3, 4, 7, 10

Standard deviation will be same as above. Notice that each number has increased by 2.*

For grouped frequency distribution

Mean = $\sum \frac{\text{midpoint of group} \times \text{frequency}}{\text{total numbers}}$

Mode = $L + w \times \frac{(f_m - f_{m-1})}{(f_m - f_{m-1}) + (f_m - f_{m+1})}$ ← frequency of the group

lower class boundary

width of the group.

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For eg:

1 - 4	1
5 - 8	3
8 - 11	7
11 - 14	14
~~14~~ - 17	11

group (11 - 14) has the highest frequency. so the mode lies between 11 - 14. Lets calculate accurate value.

$L = 10.5$ (any thing above 10.5 will be rounded off to 11)

$w = 14 - 11 = 3$

$f_m = 14$ (frequency of mode group)

$f_{m-1} = 7$

$f_{m+1} = 11$

$$\text{Mode} = 10.5 + \frac{3 \times (14-7)}{(14-7) + (14-11)}$$

$$= 10.5 + \frac{3 \times 7}{7 + 3}$$

$$= 10.5 + \frac{21}{10} = 12.6$$

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$$\text{Median} = L + \frac{\frac{n}{2} - B}{G} \times w$$

L : lower class boundary

n : total no. of values

B : commulative frequency of the groups before median.

G : frequency of median group.

w : width

~~to~~ Taking the above example itself, Median group will be (8 – 11) as its the middle group.

$L = 7.5$

$w = 3$

$n = 36$ (total no. of values)

$G = 7$ (median group frequency)

$B = 1 + 3$ (commulative of frequencies
$= 4$ before median group)

$$\text{Median} = 7.5 + \frac{\frac{36}{2} - 4}{7}$$

$= 7.5 + 2$ ~~14/7~~

$= 9.5$

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Normal distribution.

Mean = Median = Mode.

- 68 % values are within 1 SD of mean
- 95 % values are within 2SD of mean
- 99.7 % values are within 3SD of mean

Standard scores.

- number of standard deviations from the mean is called standard score or sigma or z-score.

z-score of value v = $\frac{v - \text{mean}}{SD}$

Standarize: converting a distribution in terms of z-scores.

$$z = \frac{x - \mu}{\sigma}$$

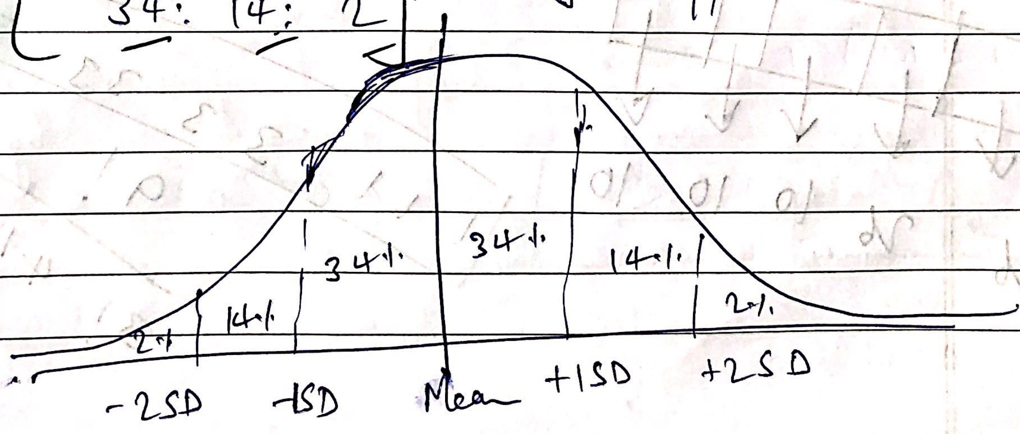
mean

value to be standarized

σ (standard deviation)

Use the following approxes.

[34 : 14 : 2]



$\frac{5!}{3! \times 2!}$ $\quad {}^5C_3 \quad {}^5C_2 \quad$ $\frac{5!}{2! \times 3!}$

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Box-and-whisker plot & quartiles.

- 2nd quartile : median
- 1st quartile : median of numbers before 2nd quartile
- 3rd quartile : median of numbers after 2nd quartile

For eg. 7, 3, 14, 9, 7, 8, 12

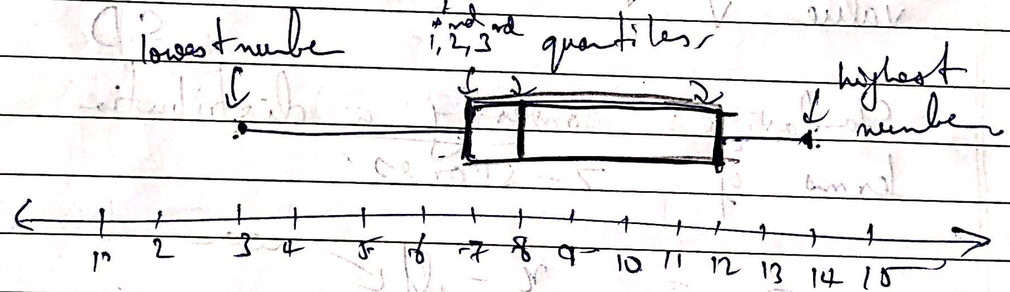
Q_2

sort → 1, 3, 4, 7, 7, 8, 8, 9, 12, 14

7 → 1st quartile

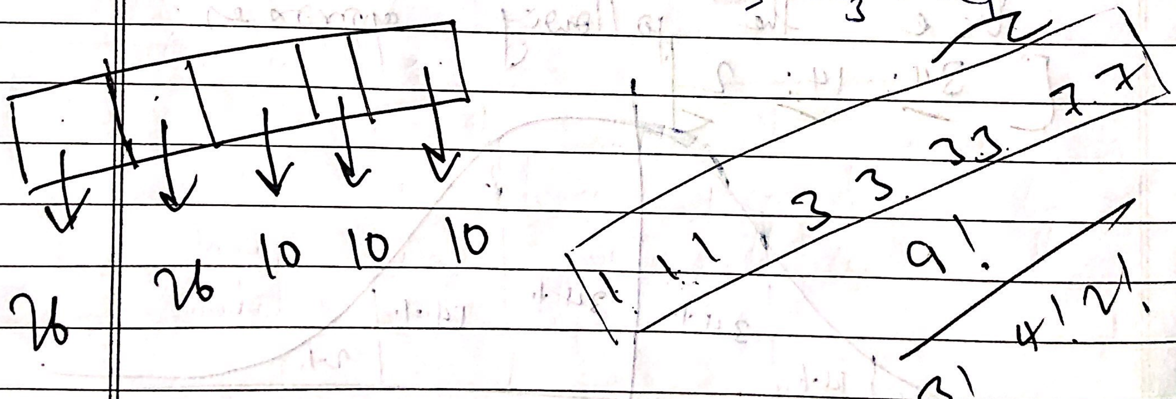
8 → 2nd quartile

12 → 3rd quartile



inter quartile range = 3rd quartile − 1st quartile

$= Q_3 - Q_1$



Permutations and Combinations

Permutations

number of all permutations of n things, taken r at a time,

$$^nP_r = \frac{n!}{(n-r)!}$$

If there are n subjects of which p_1 are alike of one kind, p_2 of another kind ... p_r of rth kind such that $p_1 + p_2 + \cdots p_r = n$

number of permutations of n objects is $=$

$$\frac{n!}{(p_1!)(p_2!)(p_r!)}$$

Combinations

Each of the groups or selections which can be formed by taking some or all of a number of objects is called a combination.

$$^nC_r = \frac{n!}{(r!)(n-r)!}$$

$^nC_n = 1$, $^nC_0 = 1$, $\boxed{^nC_r = {}^nC_{n-r}}$

Multiplication theorem.
if an operation can be performed in m different ways and following operation can be performed in n different ways then the two operation can be performed in
$m \times n$ different ways.

Addition theorem.
$operation_1 = m$ ways
$operation_2$ is an independant operation that can be performed in n different ways then either of the two operations can be performed in $(m+n)$ ways.

Factorial.
$n! = n(n-1)(n-2) \ldots 1$

$0! = 1 \qquad 1! = 1$

Permutation.
they are different arrangements of a given number of things by taking some or all at a time.

Combinations.
each of the different groups or ~~sect.~~ selections formed by taking some or all of a number of objects.

Note: if the order is important, problem is related to permutation.
if the order is not important, problem is related to combination

Number of permutations of n distinct things taken all at a time = $n!$

$^nP_0 = 1$ & $^nP_r = 0$ for $r > n$

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~~Algebra~~ Arithmetics

LCM : least common multiple

GCD / GCF / HCF : greatest common factor.

- HCF of given fractions = $\frac{\text{HCF of numerator}}{\text{LCM of denominator}}$

- LCM of given fractions = $\frac{\text{LCM of numerator}}{\text{HCF of denominator}}$

- Product of 2 numbers = HCF × LCM

- HCF of given numbers always divides the LCM,

- largest number which divides x, y, z to leave same remainder =

HCF of (y − x), (z − y), (z − x)

- largest number which divides x, y, z to leave remainder a, b, c =

HCF of (x − a), (y − b), (z − c)

- least number when divided by x, y, z and leaves remainder R in each cases

(LCM of x, y, z) + R

Algebra.

Quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Inequalities.

Rule 1:
when the same constant is added or substracted from both sides of an inequality the direction of pass inequality is preserved.

Rule 2: When both sides of the inequality is multiplied or divided by the same nonzero constant, the direction of the inequality is preserved if the constant is positive but the direction is reversed if the constant is negative

Functions.

Domain of a function is the set of all permissible inputs.

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Interest :

SI :

$$A = P\left(1 + \frac{rt}{100}\right)$$

CI

- annual interest rate of r percent, compounded annually,

$$A = P\left(1 + \frac{r}{100}\right)^{t}$$

- annual interest rate of r percent, compounded n times a year

$$A = P\left(1 + \frac{r}{100n}\right)^{nt}$$

AP, GP, HP.

n^{th} term $=$ $a + (n-1)d$

Sum of n terms $=$ $\frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{n}{2}\left[a + a_n\right]$

If 3 quantities are in AP then middle one is mean of other two.

$b = \frac{a+c}{2}$

GP.

$a, ar, ar^2, ar^3 \ldots$

$T_n = ar^{n-1}$

$r = \frac{T_n}{T_{n-1}}$

$S_n = \frac{a(r^n - 1)}{r-1}$

$S_{n \to \infty} = \frac{a}{1-r}$ $\{0 < r < 1$

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If three quantities are in GP then the middle one is

$$b = \sqrt{ac}$$

HP

$$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}$$

$$T_n = \frac{1}{a+(n-1)d}$$

If three terms in HP then

$$b = \frac{2ac}{a+c}$$

Sum of 1st $n/n^2/n^3$ natural numbers =

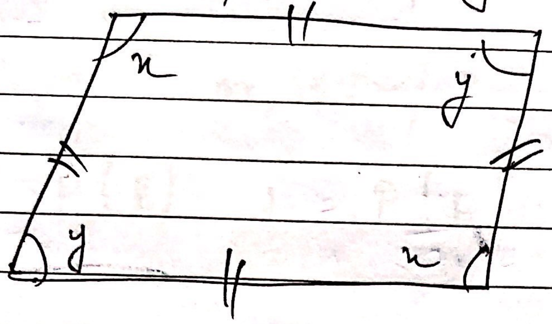
$$\sum n = \frac{n(n+1)}{2}$$

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Quadrilaterals.

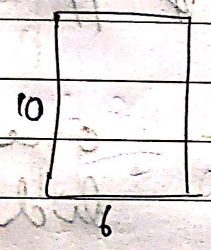
Parallelogram: opposite sides are congruent and opposite angles are congruent



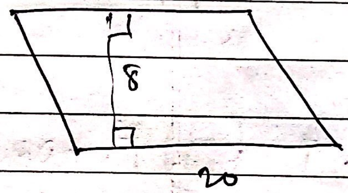
$(n-2)180$

Area of all parallelograms including rectangles and squares is

$A = bh$



$A = 60$

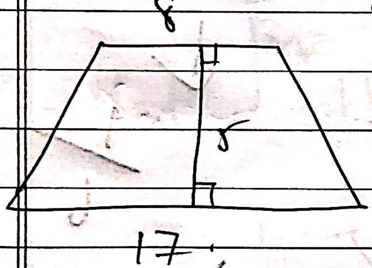


(8×20)

$A = 160$

Area of trapezoid

$A = \frac{1}{2}(b_1 + b_2)h$



$\frac{1}{2}(8+17) \times 5$

$= 125/2$

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Sets.

$$|A \cup B| = |A| + |B| - |A \cap B|$$

atleast one happered + atleast one didn't happen

$= 1 -$ one didn't happen

$\frac{1}{6} \times \frac{1}{6}$

1 - happen

even, odd, 4

$\downarrow$ $\downarrow$ $\downarrow$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{6}$

$1 - \left\{\frac{1}{2} \times \frac{1}{2} \times \frac{5}{6}\right\}$

$\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{6}\right)$

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Probability

for two events E and F.

P(E or F, or both)

= P(E) + P(F) − P(both E and F)

if E & F are exclusive

$1 = \underline{P(E) + P(F)}$ as $P(E \text{ and } F) = 0$

for independent events E & F.

$P(E \text{ and } F) = P(E)\,P(F)$

<u>Atleast 1 event occurs</u> =

1 − none of the event occurs.

for E, F, G probability that atleast one of them occurs

$= 1 - P(\overline{E})\,P(\overline{F})\,P(\overline{G})$