

## Maximizing Profit: Analyzing Cost Efficiency in Mango Graham Float Production

Mango Graham Float is a classic Filipino dessert featuring layers of ripe mangoes, crushed graham crackers, and a smooth, creamy milk blend. This sophisticated treat expertly balances sweetness and texture, delivering a rich yet refreshing flavor profile that exemplifies Filipino culinary artistry.

### Problem statement

The objective of this optimization project is to maximize the profit from the production of Mango Graham Float, a dessert made from a combination of various ingredients. The production is constrained by the availability of these ingredients, each with a specific cost and required quantity per product. The ingredients and their respective details are as follows:

- *Mango*: 30 pcs available, unit cost of ₱25.00, 4 pcs required per product.
- *Graham Crackers*: 1000 g available, unit cost of ₱0.90, 200 g required per product.
- *Crushed Grahams*: 500 g available, unit cost of ₱1.00, 50 g required per product.
- *All-Purpose Cream*: 5000 ml available, unit cost of ₱0.45, 400 g required per product.
- *Condensed Milk*: 3000 g available, unit cost of ₱0.60, 300 g required per product.

### Assumptions

This project is based on key assumptions to simplify and clarify the profit-maximization analysis for Mango Graham Float production. We assume: (1) fixed availability of ingredients, (2) a linear relationship between ingredient use and output, (3) a constant selling price per unit, (4) no consideration of labor or overhead costs, and (5) focus on a single product. Additionally, production is limited by the most restrictive (binding) constraint. These assumptions define the boundaries of the optimization model.

1. Ingredient quantities are fixed and represent the maximum available; no restocking is allowed.
2. Production is linear—doubling output requires double the ingredients.
3. Selling price per unit is constant, with no discounts or price changes.
4. Labor, overhead, and transport costs are excluded—only ingredient costs are factored.
5. All products are identical in quality and composition.
6. Output is limited by the ingredient that runs out first (binding constraint).

### Algorithm/Design Approach

For this project, we'll be using the Simplex Method to solve the optimization problem. It's a well-known and effective approach used in linear programming to find the optimal solution. The method follows a step-by-step process that makes it easier to reach the best possible outcome based on the given constraints and objectives.

1. *Define the Decision Variables*: Let  $x$  be the number of Mango Graham Float units produced.
2. *Formulate the Objective Function*: The objective is to maximize profit where Revenue is the selling price per unit multiplied by  $x$ , and Cost is the total cost of ingredients used.
3. *Set Up the Constraints*: The constraints based on the available quantities of ingredients.
4. *Solve the Linear Program*: Using the Simplex Method, we will find the optimal value of  $x$  that maximizes profit while satisfying all constraints.

### Sample Calculations

This section shows how the Simplex Method is used to find the units to produce, based on available ingredients and cost constraints. It highlights how linear programming guides efficient, profit-maximizing decisions.

#### Problem Formulation:

*Decision Variable*:

$x$  = Number of Mango Graham Float units to produce

Objective Function:

Maximize  $Z = x$  (maximize production units)

*Ingredients Constraints*:

- Mango:  $4x \leq 30$
- Graham Crackers:  $200x \leq 1000$

- Crushed Graham:  $50x \leq 500$
- All Purpose Cream:  $400x \leq 5000$
- Condensed Milk:  $300x \leq 3000$
- Non-negativity:  $x \geq 0$

### Convert to Standard Form:

To use the Simplex Method, we need to convert all inequalities to equations by introducing slack variables:

- $4x + s_1 = 30$  (where  $s_1$  = unused mangoes)
- $200x + s_2 = 1000$  (where  $s_2$  = unused graham crackers)
- $50x + s_3 = 500$  (where  $s_3$  = unused crushed graham)
- $400x + s_4 = 5000$  (where  $s_4$  = unused all purpose cream)
- $300x + s_5 = 3000$  (where  $s_5$  = unused condensed milk)

The objective function becomes:

- Maximize  $Z - x = 0$  or equivalently:  $Z - x = 0$

### Initial Simplex Tableau

We set up the initial tableau with the objective function coefficients in the first row (with signs reversed), constraint coefficients in the body, and the constants on the right side:

Basic Variables	Z	x	s1	s2	s3	s4	s5	RHS
Z	1	-1	0	0	0	0	0	0
s1	0	4	1	0	0	0	0	30
s2	0	200	0	1	0	0	0	1000
s3	0	50	0	0	1	0	0	500
s4	0	400	0	0	0	1	0	5000
s5	0	300	0	0	0	0	1	3000

### First Iteration

*Step 1: Select the entering variable*

The entering variable is the one with the most negative coefficient in the objective row.

In this case,  $x$  has a coefficient of  $-1$ , so  $x$  enters the basis.

*Step 2: Select the leaving variable*

Compute the ratio of RHS to the corresponding positive coefficients in the entering variable's column:

- $s_1: 30 \div 4 = 7.5$
- $s_2: 1000 \div 200 = 5 \leftarrow$  Minimum ratio
- $s_3: 500 \div 50 = 10$
- $s_4: 5000 \div 400 = 12.5$
- $s_5: 3000 \div 300 = 10$

The minimum ratio is 5 for  $s_2$ , so  $s_2$  leaves the basis.

*Step 3: Perform row operations*

Pivot on the intersection of the entering column ( $x$ ) and the leaving row ( $s_2$ ).

Divide the  $s_2$  row by 200 to make the pivot element 1:

Basic Variables	Z	x	s1	s2	s3	s4	s5	RHS
Z	1	-1	0	0	0	0	0	0
s1	0	4	1	0	0	0	0	30
x	0	1	0	1/200	0	0	0	5
s3	0	50	0	0	1	0	0	500
s4	0	400	0	0	0	1	0	5000
s5	0	300	0	0	0	0	1	3000

Use row operations to eliminate  $x$  from all other rows:

1. Row Z = Row Z + 1  $\times$  Row x
2. Row s1 = Row s1 - 4  $\times$  Row x

3. Row s3 = Row s3 - 50 × Row x
4. Row s4 = Row s4 - 400 × Row x
5. Row s5 = Row s5 - 300 × Row x

This gives the new tableau:

Basic Variables	Z	x	s1	s2	s3	s4	s5	RHS
Z	1	0	0	1/200	0	0	0	5
s1	0	0	1	-4/200	0	0	0	10
x	0	1	0	1/200	0	0	0	5
s3	0	0	0	-50/200	1	0	0	250
s4	0	0	0	-400/200	0	1	0	3000
s5	0	0	0	-300/200	0	0	1	1500

Simplifying the fractions:

Basic Variables	Z	x	s1	s2	s3	s4	s5	RHS
Z	1	0	0	0.005	0	0	0	5
s1	0	0	1	-0.02	0	0	0	10
x	0	1	0	0.005	0	0	0	5
s3	0	0	0	-0.25	1	0	0	250
s4	0	0	0	-2	0	1	0	3000
s5	0	0	0	-1.5	0	0	1	1500

*Step 4: Check for optimality*

Since there are no more negative coefficients in the objective row, we have reached an optimal solution. The optimal value is  $Z = 5$  units of Mango Graham Float.

### Optimal Solution

- $x = 5$  (Produce 5 units of Mango Graham Float)
- $s1 = 10$  (10 mangoes remain unused)
- $s2 = 0$  (All graham crackers are used - binding constraint)
- $s3 = 250$  (250g of crushed graham remains unused)
- $s4 = 3000$  (3000ml of all purpose cream remains unused)
- $s5 = 1500$  (1500ml of condensed milk remains unused)

### Verification:

We can verify that this solution satisfies all constraints:

- Mango:  $4 \times 5 = 20 \leq 30$  ✓
- Graham Crackers:  $200 \times 5 = 1000 \leq 1000$  ✓ (Binding)
- Crushed Graham:  $50 \times 5 = 250 \leq 500$  ✓
- All Purpose Cream:  $400 \times 5 = 2000 \leq 5000$  ✓
- Condensed Milk:  $300 \times 5 = 1500 \leq 3000$  ✓

### Discussion of results

The application of the Simplex Method reveals that the maximum production capacity for Mango Graham Float is 5 units. The binding constraint in this scenario is the availability of Graham Crackers, which limits the production despite the availability of other ingredients.

This result aligns with the calculations performed by the program, confirming that the Simplex Method is effective for this single-product optimization problem. The identical results obtained from both the Simplex Method and the program's simpler approach validate the algorithm's efficiency in solving production planning issues.