A multiagent based road pricing approach for urban traffic management

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Abstract. Traffic is a social system composed by different interacting entities, and its optimization is not a trivial task. When drivers and infrastructure co-adapt to deal with the varying demand and infrastructure changes, respectively, centralized traffic optimization approaches face many difficulties. This work presents a multiagent based approach that uses variable road pricing to improve traffic efficiency. While infrastructure updates roads prices to cope with the varying demand, drivers try to adapt themselves to the road network changes in order to minimize their costs. Drivers have different preferences, caring either about their travel time (being hasty) or credit expenditure (being economic). Results shows that the proposed road pricing approach benefits the hasty drivers, while more sophisticated pricing update policies need to be developed in order to create better alternatives for economic drivers.

1 Introduction

Traffic is a social system where millions of people with different ages, lifestyles, needs and other characteristics mingle everyday. Traffic optimization is a subject that presents challenging issues to authorities, traffic engineers and researchers.

Ideally, the different drivers' characteristics should be taken into account when a traffic optimization method is being designed or tested. Agent-based modeling and simulation is suitable for this need, as traffic entities can be modeled as intelligent agents, having their own decision making processes and individual characteristics.

Regarding control, transportation systems are dynamic and are composed of a large number of interacting entities. According to [1], centralized approaches for managing such systems need a deep knowledge on the underlying domain and can be computationally intractable as well as face difficulties on responding to changes. The adoption of distributed, agent-based control systems is a promising alternative in this context.

This work presents a road pricing approach to traffic management. To cope with the varying demand, roads' prices are updated according to certain policies adopted by link manager agents. The main goal is to provide an incentive to driver agents so that they spread themselves through the road network, leaving the fastest roads to the ones that need or are willing to pay for using it.

The approach of road pricing is studied in the literature and it is used in real world as the survey done in [2] shows. Also, in [3], some works that investigate this approach are reviewed. The authors conclude that congestion tolls are useful as they make each driver internalize the costs it imposes to the others and to the road network when acting greedily. In the real world, the city of London can be highlighted as a successful case of congestion pricing [4].

The remainder of this document is organized as follows: Section 2 introduces basic concepts and presents the traffic model that will be used throughout this paper. Section 3 discusses related work done in this field. Section 4 presents the road pricing approach for traffic management whose results are discussed in Section 5. Finally, Section 6 concludes the paper and presents opportunities for further study.

2 Basic concepts and traffic model

This section introduces basic concepts and presents the traffic model used in the present work.

A road network can be modeled as a graph, with a set of nodes, which represent the intersections, and links among these nodes. The link weight represents a form of cost associated with the link, such as travel time, length or a monetary cost associated with a toll.

A driver's trip consists of a set of links, forming a route between his origin and destination (OD pair) among the available routes.

Traffic flow is defined by the number of vehicles that use a network link in a given period of time. Capacity is understood as the number of traffic units that a link supports in a given instant of time. Load is understood as the demand generated on a link at a given moment. When the demand reaches the link's maximum capacity, congestion is formed.

One of the most common cost function that relates link's attributes (capacity, free-flow travel time) and traffic flow is shown on Eq. (1) [5].

$$t_j(v) = f_j[1 + \alpha \left(\frac{v}{c_j}\right)^{\beta}] \ . \tag{1}$$

In this function, t_j is the travel time on link j applied to the number of vehicles in it (v), c_j is the link's capacity, f_j is the free-flow travel time on link j and α and β are calibration parameters. This will be the travel time function used throughout the present work.

3 Related work

In this section, we review works proposing traffic management methods and road pricing approaches, showing their contributions, similarities and limitations that are tackled in the present work.

Vasirani and Ossowski [6] present a market-based approach to control multiple intersections in the road network. Driver agents purchase reservations to cross the intersections from the intersection manager agents. Drivers can cross intersections for free too, but in this case, they must wait for the intersection to be empty. The authors present a learning mechanism for the intersection managers with the goal of maximizing the global revenue. Different driver preferences are analyzed: there are time-based agents and price-based agents, who try to minimize the travel time and credit expenditure, respectively.

The work by Vasirani and Ossowski [6] assumes the existence of fully autonomous vehicles, that obey the market rules: they pay for reservations or stop at intersections until they get one for free. The present work does not have this drivers movement restriction: drivers can use any network link anytime without having to wait for a reservation, as the present approach is not based on reservations. Also, as we don't assume the existence of fully autonomous vehicles, the approach presented here is suitable for shorter term implementation in real world.

Regarding road pricing, Arnott et. al. [7] concludes that traffic efficiency improves when tolls are used. In [7], the toll value is traffic-dependent, but it is calculated in a centralized way. Also, the work analyses the choice of departure times of drivers. The focus on the present work is to present decentralized pricing calculation methods and to assess its effects in drivers route choice, instead of departure times.

In [8] different toll schemes are proposed and drivers route choice behavior is analysed. The work discusses what kind of toll pricing information should be given to the user and the usefulness of letting the user know the toll price in advance or just in the collection moment. The work assumes the existence of a control center with perfect information on the traffic network state, which is not the case in the present work.

Bazzan and Junges [9] present a study on how drivers' decision making can be affected by using congestion tolls. A control center provides drivers with a cost estimation for a certain route. Driver agents update their knowledge base with available information of the routes and the utility received in the past episodes. The work shows that congestion tolls are useful to align private utility with a global optimum, but this is done in a centralized way.

A decentralized, agent-based approach to road pricing is shown in [10], where the authors compare the performance of autonomous links and centralized control on capacity expansion of a highway network. Competitive autonomous links adjust their prices in order to maximize their profit. Also, they can invest on their own expansion in order to increase capacity, thus attracting more vehicles and increasing profit. This scheme is compared to a centralized approach where a government entity has global information and takes decisions regarding prices adjustment and capacity expansion. The authors conclude that, compared to the government entity, autonomous links generate more revenue and provide higher road capacity, thus allowing drivers to achieve higher speeds. The drawback is that road prices are higher, increasing the costs for drivers.

The study done in [10] does not consider driver preferences and has its focus on how highway expansion would affect traffic pattern. In the present work, links will have fixed capacity and driver preferences will be considered.

A detailed study of the effects of congestion tolls is found in [11]. The authors perform a large-scale microsimulation in the city of Zurich. Citizens are modeled as agents with activity plans for working, shopping, leisure and education through the day. Agents can also plan the mode (car or public transportation), departure time and route choice (if driving a car) for each activity. The agents have different utility functions rating travel delays and early or late arrival for each activity. The authors present a fixed city toll that is applied in the afternoon rush hours. Experimental results show that agents not only change the afternoon but also the morning activity plans when the toll is introduced.

The work done in [11] presents a contribution on the effects of a fixed toll system on citizens daily activities in a large-scale urban scenario. The focus of the present work is different: we want to assess how a variable road pricing approach can benefit drivers regarding their route choices, assuming that mode and departure time were already chosen. Also, we study drivers and infrastructure co-adaptation, that is, drivers gain experience by interacting with the road network and this infrastructure must respond to changes in the traffic patterns by adjusting road network prices.

In the literature, most works confirm that road pricing can enhance traffic efficiency. Many works present centralized approaches for road pricing. Also, in many cases, driver and infrastructure co-adaptation is not assessed. The contribution of the present work lies on the proposal of a decentralized road pricing approach that takes into account different drivers preferences and the evaluation of co-adaptation of drivers and road network infrastructure.

4 Approach and scenario

In the proposed approach, drivers need to pay credits to traverse any network link. Driver agents will try to minimize their individual cost functions while link manager agents will adjust link prices in order to provide an incentive for drivers to distribute themselves over the road network.

We assume the existence of an infrastructure that makes drivers pay credits whenever they enter a network link. Vehicles have an identification device that communicates with the road infrastructure without the need of reducing the vehicle speed for toll collection. Such infrastructure would be similar to Singapore's electronic road pricing [12] that exists in certain roads of the state-city. We assume that such electronic toll collector exists in every network link of our studied scenario.

4.1 Driver agents

Each driver d is modeled as an agent whose goal is to minimize a cost function over its route R_d^* given by $z(R_d^*)$, defined in Eq. (2). In this equation, $t'_{d,j}$ and

$p'_{d,j}$ are the travel time and price that driver d knows for link j, respectively. The ρ_d coefficient adjusts whether driver d will prefer to minimize travel time (ρ_d closer to 1) or credit expenditure (ρ_d closer to 0).

$$z(R_d^*) = \sum_{j \in R_d^*} (\rho_d) t'_{d,j} + (1 - \rho_d) p'_{d,j} \ . \tag{2}$$

Drivers have local knowledge, that is, they only know prices and travel times of the links they traversed. Also, these informations about a link are updated only when the driver traverses the link. For the unknown links, drivers estimate travel time as if the link is with half occupancy and price as the half of maximum price. Drivers' knowledge persists along trips, thus they learn about the traffic network by exploring it.

Algorithm 1 describes the drivers initialization and route, travel time, and credits expenditure calculation procedures that are called in Alg. 3. In practice, the route calculation procedure consists of using a shortest path algorithm with the z value of a link (Eq. 2) as its weight. The probability distribution for ρ_d selection is a parameter for the drivers' initialization procedure as well as the probability distribution for selecting the origin and destination nodes.

4.2 Link manager agents

Every network link has its respective manager agent. The price of a given link j ($p_j \in [0 : 100]$) is initialized in proportion to its capacity. At the end of each trip, it is adjusted according to different policies. In the present work, three policies are tested:

- Fixed: no adjustment will be made. Links' prices will remain as initialized throughout all trips.
- Incremental: if a given link's occupancy is higher than the average of its alternatives (the outbound links of the given link's origin node), price is incremented by 10 units. If it is lower, price is decremented by 10 units.
- Greedy: if a given link's occupancy is higher than the average of its alternatives, the given link's price receives the highest value of its alternatives, plus 10 units. The inverse occurs if the occupancy is lower.

The goal of price adjustment is to make an uncongested link more attractive than its congested alternatives. Algorithm 2 describes the initialization and price adjustment procedures of the link manager agents. These procedures are called in Alg. 3.

Algorithm 1 Driver agents

▷ L is the set of road network links, D is the set of drivers
▷ Π is the probability distribution for the selection of ρ_d
▷ Ω is the origins and destinations probability distribution
▷ ρ_d is the driver d's preference coefficient
▷ $\hat{n}_d$ and $\check{n}_d$ are the origin and destination nodes of driver d
▷ $t'_{d,l}$ and $p'_{d,l}$ are the travel time and price that driver d knows for link l
▷ R^*_d is the route (a set of links) of driver d
▷ $R_{x,y}$ is a possible route between nodes x and y. $\mathbb{R}_{x,y}$ is the set of all routes between x and y
▷ t_l, p_l, c_l and v_l are the travel time (Eq. 1), price, capacity and the number of vehicles on link l
▷ $\hat{t}_{R^*_d}$ and $\hat{p}_{R^*_d}$ are the travel time and credits expenditure on route R^*_d

```
procedure InitializeDrivers(Π, Ω)
    for all d ∈ D do
        (n̂_d, ň_d) ← select_OD(Ω)        ▷ Select origin and destination according to Ω
        ρ_d ← select_ρ(Π)                 ▷ Select ρ_d according to Π
        for all l ∈ L do
            t'_{d,l} ← t_l(c_l/2)         ▷ Estimates travel time using Eq. 1
            p'_{d,l} ← 50                 ▷ Estimates price (50 is half of the maximum price)
        end for
    end for
end procedure

procedure CalculateRoutes
    for all d ∈ D do
        R*_d ← arg min_{R_{n̂_d,ň_d} ∈ ℝ_{n̂_d,ň_d}} z(R_{n̂_d,ň_d})   ▷ Finds the minimum z (Eq. 2) route
    end for
end procedure

procedure CalculateTravelTimes
    for all d ∈ D do
        t̂_{R*_d} ← 0                      ▷ Initializes the route travel time
        for all l ∈ R*_d do
            t'_{d,l} ← t_l(v_l)           ▷ Updates knowledge base (travel time given by Eq. 1)
            t̂_{R*_d} ← t̂_{R*_d} + t'_{d,l}  ▷ Updates total travel time on route R_d
        end for
    end for
end procedure

procedure CalculateCreditsExpenditure
    for all d ∈ D do
        p̂_{R*_d} ← 0                      ▷ Initializes the route credits expenditure
        for all l ∈ R*_d do
            p'_{d,l} ← p_l                ▷ Updates knowledge base with the price of link l
            p̂_{R*_d} ← p̂_{R*_d} + p'_{d,l}  ▷ Updates credits expenditure on route R*_d
        end for
    end for
end procedure
```

Algorithm 2 Link manager agents

```
▷ L is the set of road network links
▷ p_l, c_l and v_l are the price, capacity and the number of vehicles on link l

procedure InitializeLinks
    for all l ∈ L do
        p_l ← 100 × c_l/(max_{j∈L} c_j)    ▷ Initial price is proportional to link's capacity
    end for
end procedure

▷ For the price adjustment procedures, l is the link to have its price adjusted and O
is the set of alternative links to l
procedure AdjustRoadsPrices(policy)
    for all l ∈ L do
        O ← {j ∈ L | j starts in the same node as l }
        if v_l/c_l > average_{j∈O}(v_j/c_j) then
            IncrementPrice(l, O, policy)
        else if v_l/c_l < average_{j∈O}(v_j/c_j) then
            DecrementPrice(l ,O, policy)
        end if
    end for
end procedure

procedure IncrementPrice(l, O, policy)
    if policy = GREEDY then
        p_l ← max_{j∈O}(p_j) + 10
    else if policy = INCREMENTAL then
        p_l ← p_l + 10
    end if
    if p_l > 100 then
        p_l ← 100                           ▷ Keeps p_l within upper limit
    end if
end procedure

procedure DecrementPrice(l, O, policy)
    if policy = GREEDY then
        p_l ← min_{j∈O}(p_j) − 10
    else if policy = INCREMENTAL then
        p_l ← p_l − 10
    end if
    if p_l < 0 then
        p_l ← 0                             ▷ Keeps p_l within lower limit
    end if
end procedure
```

4.3 Multiagent based road pricing approach

The procedure that executes the multiagent based road pricing approach is given by Alg. 3. At first, driver and link variables are initialized. Then, for η iterations, drivers travel from their origins to their destinations.

Algorithm 3 Multiagent based road pricing

```
procedure EXPERIMENT(η, Π, Ω, policy)
    ▷ η is the number of iterations
    ▷ Π is the probability distribution for the selection of ρ_d
    ▷ Ω is the origins and destinations probability distribution
    ▷ The policy parameter can be either FIXED, GREEDY or INCREMENTAL
    ▷ L is the set of road network links, D is the set of drivers
    ▷ D̃ is the set of drivers that finished their trips
    ▷ ǹ_d, ṅ_d, n̂_d, and ň_d are the previous, current, origin and destination nodes of driver d
    ▷ v_e is the number of vehicles on link e

    InitializeDrivers(Π, Ω)                          ▷ This procedure is in Alg. 1
    InitializeLinks()                                ▷ This procedure is in Alg. 2
    i ← 0                                            ▷ Starts the iterations counter
    while i < η do                                   ▷ Executes η iterations
        CalculateRoutes()                            ▷ This procedure is in Alg. 1
        for all d ∈ D do
            ṅ_d ← n̂_d                                ▷ Sets current node as origin to start the trip
        end for
        D̃ ← ϕ
        repeat
            for all d ∈ D − D̃ do
                ǹ_d ← ṅ_d
                ṅ_d ← next(ṅ_d,R*_d)                 ▷ ṅ_d becomes the next node on route R*_d
                e ← link_between(ǹ_d,ṅ_d)            ▷ Finds current link for driver d
                v_e ← v_e + 1                        ▷ Increment number of vehicles on current link
                if ṅ_d = ň_d then
                    D̃ ← D̃ ∪ {d}                      ▷ Add driver to the set of drivers with finished trips
                end if
            end for
        until D − D̃ = ϕ                              ▷ Iteration lasts until all drivers finish their trips
        CalculateTravelTimes()                       ▷ This procedure is in Alg. 1
        CalculateCreditsExpenditure()                ▷ This procedure is in Alg. 1
        AdjustRoadsPrices(policy)                    ▷ This procedure is in Alg. 2
        i ← i + 1                                    ▷ Increase iterations counter
    end while
end procedure
```

At each iteration, when all drivers arrive at their destinations, travel time and credits expenditure are calculated. Also, link managers adjust the link prices.

Within travel time and credits expenditure calculation procedures, drivers also update their knowledge base. The updated information is used for route calculation on future trips.

4.4 Scenario

The abstract traffic scenario studied in this work contains 36 nodes connected by 60 one-way links, as shown in Fig. 1. Drivers can turn to two directions in each node. This is a complex scenario from the point of view of route choice, as the number of possible routes between two locations is high.

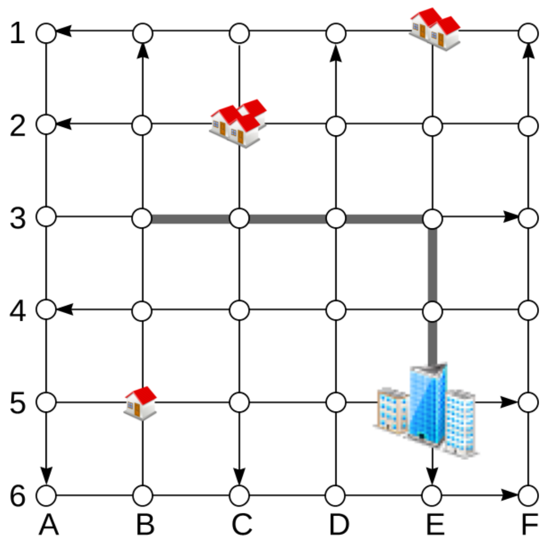


Fig. 1. Abstract grid scenario with the main origins (nodes B5, C2 and E1) and the main destination (node E5). Arrows show the streets' directions. Thicker lines represent the main links.

The scenario settings are the same as in [13]: three nodes have higher probabilities of being origins of trips, and one node has a high probability of being a destination of trips, as Table 1 shows. In this table, PO means the probability of the node be an origin and PD is the probability of the node be a destination. This is to simulate a real-world feature: the existence of main residential areas (nodes with higher chance of being origins) and a business center (the node with high chance of being a destination).

Regarding the road network capacity, there are main links, i.e., links whose capacity is higher than the others, namely, 45 vehicles. The remaining links are assigned with the capacity of 15 vehicles. The main links are those between the nodes B3 to E3 and E3 to E5 (thicker lines on Fig. 1). With this feature,

Node	PO	PD
E5	2.67%	60%
B5	3%	1.15%
C2	4%	1.15%
E1	5%	1.15%
Every other node	2.67%	1.15%

Table 1. Origins and destinations probability distribution (Ω for Alg. 3).

the abstract 6x6 scenario becomes more realistic, as roads capacities are not homogeneous.

The pricing policies performance is evaluated based on two metrics: average travel time and average credits expenditure. Values will be measured for each driver type (different values of ρ_d in Eq. 2).

5 Results and discussion

In the experiments, $|D| = 300$ drivers populate the road network. Prior experimentation has shown that this number yields reasonable load in the road network. That is, with fewer drivers, traffic management is not needed as congestions seldom occur. On the other hand, with more drivers, no traffic management method would be effective as the alternative routes for a driver are congested.

Each experiment was executed 10 times. Each execution consists in $\eta = 50$ iterations for Alg. 3. The measured values for each iteration were obtained as the average of the 10 executions. The bar plots (Fig. 3) show the average and standard deviation for the last 10 iterations, when drivers and link managers adaptation stabilizes (the standard deviation is small). This means that drivers are less prone to change routes and link managers tend to keep prices unchanged.

For the travel time equation (1), f_j is set to 5 time units for all links, $\alpha = 1$ and $\beta = 2$ making travel time increase quadratically with the number of drivers. These values were also used in other works with the same traffic model, such as [14] and [15].

In this study, drivers have either $\rho_d = 1$ or $\rho_d = 0$. These will be called hasty and economic drivers, respectively. The hasty drivers try to minimize their travel times regardless of the amount they have to pay for using the road network. The economic drivers try to minimize their credit expenditure regardless of the travel time needed to complete their trips.

Regarding the parameter Π in Alg. 3, for the tests with no pricing, $\rho_d = 1$ is selected with probability 1, so all drivers will be hasty. For the other tests, $\rho_d = 1$ or $\rho_d = 0$ are selected with probability 0.5 each.

Travel time values are normalized to be kept in the same range of the link price (from 0 to 100 time units for each link).

5.1 Drivers learning to commute

Figure 2 shows how drivers learn to commute when there is no road pricing. In this situation, all drivers choose their routes greedily regarding travel times. Drivers spread themselves over the road network (straight line with travel time = 68 time units, lower than the initial value of 77 time units) after an exploration phase (when travel time rises to 412 time units).

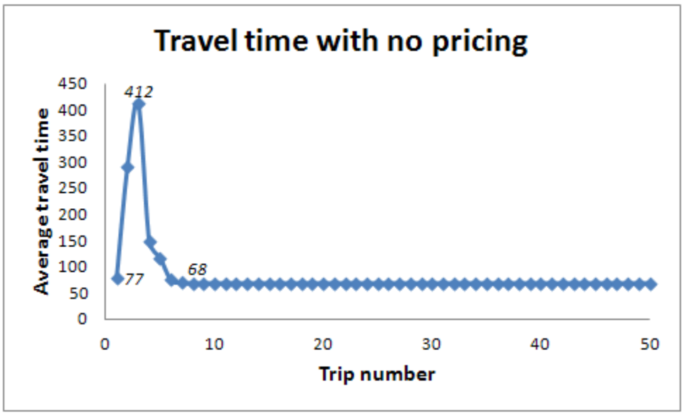


Fig. 2. Average travel time of drivers with no road pricing. Labels are the travel time values (77 time units in the beginning, 412 at peak and 68 after stabilization).

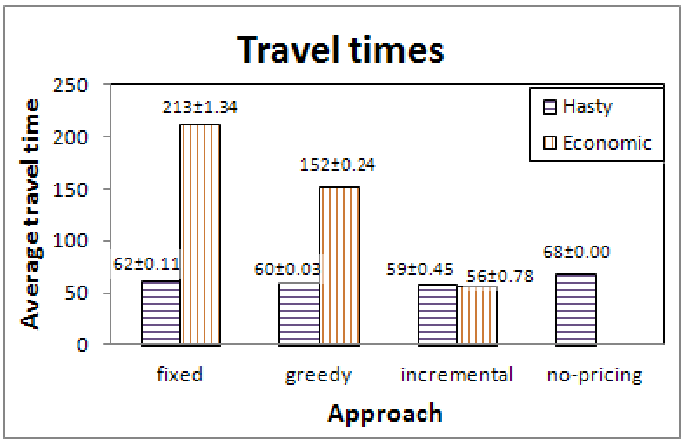
5.2 Evaluating price update policies

Figures 3(a) and 3(b) show the average drivers' travel time and credits expenditure for each price update policy, respectively. All price update policies resulted in lower travel times for the hasty drivers compared to no road pricing.

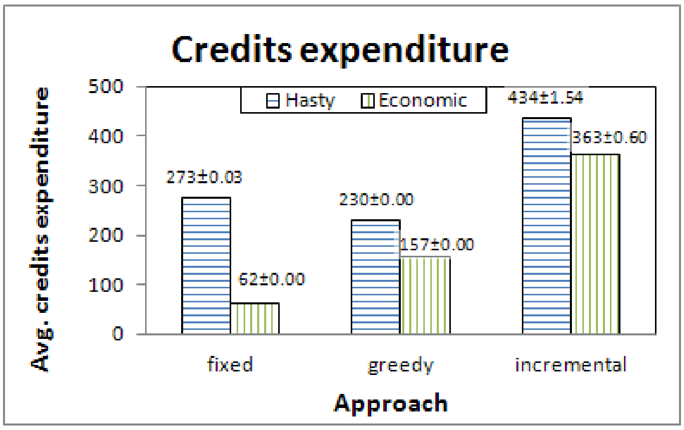
Comparing hasty and economic drivers performances, the incremental update policy was unfair with the hasty drivers: even paying more credits, they achieved higher travel times than the economic ones. Also, with the incremental policy, all drivers spent more credits than with the other policies. This is good only from the infrastructure manager point of view, as revenue is higher. Even so, with the incremental update policy, travel times were lower than the ones achieved with other policies. This means that drivers spread themselves over the road network in a better way compared to other policies.

The greedy price update policy resulted in interesting outcomes: it was fair, since hasty drivers paid more and achieved lower travel times. Also, economic drivers achieved lower travel times compared to when pricing is fixed, although they spent more credits.

Looking at the cost functions as a whole, the price update policies have yielded worse performance for the economic drivers than fixed pricing. With fixed pricing, economic drivers achieved the lowest cost functions (z from Eq. 2), since they only regard the credits expenses.



(a)



(b)

Fig. 3. Average travel time (a), and credit expenditure (b) of drivers for each price update policy. Labels are the travel time and spent credits value with standard deviation, in each plot.

In general, fixed and greedy policies were harmful for the economic drivers as they got their travel times highly increased. The economic drivers achieved their best travel times with incremental policy, but their credits expenditure were the highest compared to economic drivers in other policies.

This shows that a better pricing strategy must be developed to improve the performance of hasty drivers as well as leave good alternative routes for the economic drivers. This is not a trivial task in a complex scenario with many origins and destinations. Also, an issue is the fact that drivers adapt themselves to changes in the road network, making this a non-stationary environment for the link managers.

6 Conclusions and future work

In this work we have presented a multiagent based road pricing approach for urban traffic management, modeling drivers and link managers as intelligent agents. Results showed that drivers preferences and their adaptation to the road network is an important factor on traffic optimization and this should be taken into account when designing traffic management methods. Modeling drivers and infrastructure co-adaptation is not a trivial task but usually pays off. Other works, such as [16, 17], confirm this statement.

The present work opened opportunities for further investigation about multiagent based road pricing as a traffic management method. More sophisticated schemes for road price update need to be developed in order to cope with the varying demand in a way that it results in lower travel times for hasty drivers and cheaper trips for economic drivers. Also, drivers other than economic or hasty, adopting a mixed strategy, with ρ_d between 1 and 0, could be considered in future studies.

Another interesting point for investigation would be the existence of competitive link management companies, where each company would manage a portion of a complex road network. This would be an extension of the work done by Vasirani and Ossowski [1]. In their work, the authors investigated whether two competitive companies managing two links in parallel could learn the optimal pricing policy calculated analytically.

Also, future works could investigate not only the drivers route planning but the trip planning as a whole, that is, departure times and route choice when a road pricing scheme is being used.

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