

Multiplicative Exponential:

$$y = \alpha e^{Bx+E} + \theta$$

where $E \sim N(0, \sigma^2)$

$$\begin{aligned} E[y] &= E[\alpha e^{Bx+E} + \theta] = \alpha e^{Bx} E[e^E] + \theta = \alpha e^{Bx} M_E(1) + \theta \\ &= \alpha e^{Bx + \sigma^2/2} + \theta \end{aligned}$$

Goal: $E[y]$ to be equal for:

+ Diff values of σ

+ $X_{\min} \rightarrow \text{min Mag}$

+ $X_{\max} \rightarrow \text{max Mag}$

we can adjust α and θ ...

Set:

$$\text{min Mag} = \alpha e^{Bx_{\min} + \sigma^2/2} + \theta$$

$$\text{max Mag} = \alpha e^{Bx_{\max} + \sigma^2/2} + \theta$$

$$\Rightarrow \text{min Mag} - \theta = \alpha e^{Bx_{\min} + \sigma^2/2}$$

$$\text{max Mag} = \left(\frac{\text{min Mag} - \theta}{e^{Bx_{\min} + \sigma^2/2}} \right) e^{Bx_{\max} + \sigma^2/2} + \theta$$

$$\Rightarrow \frac{\text{min Mag} - \theta}{e^{Bx_{\min} + \sigma^2/2}} = \alpha$$

$$= \frac{\text{min Mag}}{e^{Bx_{\min} + \sigma^2/2}} e^{Bx_{\max} + \sigma^2/2}$$

$$+ \theta (1 - e^{Bx_{\max} + \sigma^2/2})$$

$$\Rightarrow \text{max Mag} = \text{min Mag} \left(\frac{e^{Bx_{\max}}}{e^{Bx_{\min}}} \right) + \theta (1 - e^{Bx_{\max} + \sigma^2/2})$$

$$\Rightarrow \frac{\text{max Mag} - \text{min Mag} \left(\frac{e^{Bx_{\max}}}{e^{Bx_{\min}}} \right)}{(1 - e^{Bx_{\max} + \sigma^2/2})} = \theta$$

$$\Rightarrow \theta = \frac{y_{\max} - y_{\min} (e^{B[x_{\max} - x_{\min}]})}{(1 - e^{Bx_{\max} + \sigma^2/2})}$$

* This causes issues on the log scale...