

Perception of exponentially increasing data displayed on a log scale

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ARTICLE HISTORY

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ABSTRACT

Log scales are often used to display data over several orders of magnitude within one graph. During the COVID-19 pandemic, we have seen both the benefits and the pitfalls of using log scales to display case counts, transmission rates, and outbreak regions. In this paper, we explore the use of linear and log scales to determine whether our ability to notice differences in the data is different when different scales are used. Our goal is to provide basic research to support the principles used to guide design decisions in scientific visualizations of exponential data. We found that displaying increasing exponential data on a log scale improved the accuracy of differentiating between two increasing exponential trends with more similar growth rates, particularly when identifying an exponential curve with a lower growth rate than others. When there was a larger discrepancy in exponential growth rates, participants accurately differentiated between the two curves on both the linear and log scale.

KEYWORDS

Exponential; Log; Visual Inference; Perception

1. Introduction and Background

Graphics are a useful tool for displaying and communicating information. (Vanderplas, Cook, and Hofmann 2020) Researchers include graphics to communicate their results in scientific publications and news sources rely on graphics to convey news stories to the public. May want to ask Heather about this one during the meeting (or by email)? During the onset of the novel coronavirus pandemic, we saw an influx of dashboards being developed to display case counts, transmission rates, and outbreak regions (Charlotte 2020). People began subscribing to news sources involved in graphically tracking the coronavirus as a direct result of these pieces of work and the need for continually updated information (Rost 2020); thus, gaining more exposure to the use of graphics. Many of these graphics helped guide decision makers to implement policies such as shut-downs or mandated mask wearing, but also facilitated communication with the public to increase compliance. With the increasing importance graphics play in our everyday lives, we must actively choose which of many possible graphics to draw, according to some set of design choices to ensure that our charts are effective, as suggested in Unwin (2020).

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When faced with data which spans several orders of magnitude, we must decide whether to show the data on its original scale (compressing the smaller magnitudes into relatively little area) or to transform the scale and alter the contextual appearance of the data. One common solution is to use a log scale transformation to display data over several orders of magnitude within one graph. Logarithms make multiplicative relationships additive, showing elasticities and other proportional changes, and also linearize power laws (Menge et al. 2018). When presenting log-scaled data, it is possible to use either untransformed scale labels (for example, values of 1, 10 and 100 are equally spaced along the axis) or log-transformed scale labels (for example, 0, 1, and 2, showing the corresponding powers of 10).

We have recently experienced the benefits and pitfalls of using log-scales as covid-19 dashboards displayed case count data on both the log and linear scale (Fagen-Ulmschneider 2020). In spring 2020, during the early stages of the coronavirus pandemic, there was a large magnitude discrepancy at a given time between different geographic regions (at all orders of magnitude - states and provinces as well as countries and continents). During this time, we saw the usefulness of log-scales showing case count curves for areas with few cases and areas with many cases within one chart. As the pandemic evolved, and the case counts were no longer spreading exponentially, graphs with linear scales seemed more effective at spotting early increases in case counts that signaled more localized outbreaks.

This is only one recent example of a situation in which both log and linear scales are useful for showing different aspects of the same data; there are long histories of using log scales to display results in ecology, psychophysics, engineering, and physics (Munroe 2013; Menge et al. 2018). [XXX it may be interesting to find papers objecting to the use of log scales in some other disciplines beyond ecology.](#)

Research suggests our perception and mapping of numbers to a number line is logarithmic at first, but transitions to a linear scale later in development, with formal mathematics education. This transition to linear scales occurs first in small numbers (e.g. 1-10) and then gradually expands to higher orders of magnitude; thus, the logarithmic intuition about numbers in children is often more noticeable on scales in the thousands to hundreds of thousands. (Varshney and Sun 2013; Siegler and Braithwaite 2017; Dehaene et al. 2008). If we perceive logarithmically by default, it is a natural (and presumably low-effort) way to display information and should be easy to read and understand/use. In fact, early studies explored the estimation and prediction of exponential growth, finding that growth is underestimated when presented both numerically and graphically but that numerical estimation is more accurate than graphical estimation for exponential curves. One way to improve estimation of increasing exponential trends other than changing the scale is through the use of providing immediate feedback (Mackinnon and Wearing 1991). While prior contextual knowledge or experience with exponential growth does not improve estimation, previous instruction on exponential growth reduces the underestimation by adjusting the initial starting value but not adjusting their perception of growth rate (Wagenaar and Sagaria 1975; Jones 1977). Estimation was shown to improve when subjects were presented with decreasing exponential functions (Timmers and Wagenaar 1977). Jones (1977, 1979) [and](#) Wagenaar and Timmers (1978) propose competing polynomial models for the perception and extrapolation of exponential series. It seems that estimation is a two-stage process: first, we identify the type of curve and direction and then, we use that information for prediction (Best, Smith, and Stubbs 2007).

Our inability to accurately predict exponential growth might also be addressed by log-transforming the data, however, this transformation introduces new complexities:

most readers are not mathematically sophisticated enough to intuitively understand logarithmic math and translate that back into real-world effects. In Menge et al. (2018), ecologists were surveyed to determine how often ecologists encounter log-scaled data and how well ecologists understand log-scaled data when they see it in the literature. Participants were presented two relationships displayed on linear-linear scales, log-log scales with untransformed values, or log-log scales with log-transformed values. Menge et al. (2018) propose three types of misconceptions participants encountered when presented data on log-log scales: 'hand-hold fallacy', 'Zeno's zero fallacy', and 'watch out for curves fallacies'.

The 'hand-hold fallacy' stems from the misconception that steeper slopes in log-log relationships are steeper slopes in linear-linear space. In fact, it is not only the slope that matters, but also the intercept and the location on the horizontal axis since a line in log-log space represents a power law in linear-linear space (i.e. linear extrapolation). Emerging from 'Zeno's zero fallacy' is the misconception that positively sloped lines in log-log space can imply a non-zero value of y when x is zero. This is never true as positively sloped lines in log-log space actually imply that $y = 0$ when $x = 0$. This misconception again is a result of linear extrapolation assuming that a line in log-log space represents a line instead of the power law in linear-linear space. The last misconception, 'watch out for curves fallacies' encompasses three faults: (1) lines in log-log space are lines in linear-linear space, (2) lines in log-log space curve upward in linear-linear space, and (3) curves in log-log space have the same curvature in linear-linear space. Linear extrapolation is again responsible for the first and third faults while the second fault is a result of error in thinking that log-log lines represent power laws (which are exponential relationships), and all exponential relationships curve upward; this is only true when the log-log slope is greater than 1. Menge et al. (2018) found that in each of these scenarios, participants were confident in their incorrect responses, indicating incorrect knowledge rather than a lack of knowledge. This affirms that our graphical design choices have a direct impact on the user's evaluation of the information.

In order to provide a set of principles to guide design choices, we must evaluate these design choices through the use of graphical tests. These tests may take many forms: identifying differences in graphs, reading information off of a chart accurately, using data to make correct real-world decisions, and predicting the next few observations. All of these types of tests require different levels of use and manipulation of the information presented in the chart. To lay a foundation for future exploration of the use of log scales, we begin with the most fundamental ability to identify differences in charts: this does not require that participants understand exponential growth, identify log scales, or have any mathematical training. Instead, we are simply testing the change in perceptual sensitivity resulting from visualization choices.

A statistic is a numerical function which summarizes the data; by this definition, graphs are visual statistics. To evaluate a graph, we have to run our statistic through a visual evaluation - a person. If two different methods of presenting data result in qualitatively different results when evaluated visually, then we can conclude that the visual statistics are significantly different. Recent graphical experiments have utilized statistical lineups to quantify the perception of graphical design choices (VanderPlas and Hofmann 2017; Hofmann et al. 2012; Loy, Follett, and Hofmann 2016). Statistical lineups provide an elegant way of combining perception and statistical hypothesis testing using graphical experiments (Wickham et al. 2010; Majumder, Hofmann, and Cook 2013; Vanderplas et al. n.d.). 'Lineups' are named after the 'police lineup' of criminal investigations where witnesses are asked to identify the criminal from a set

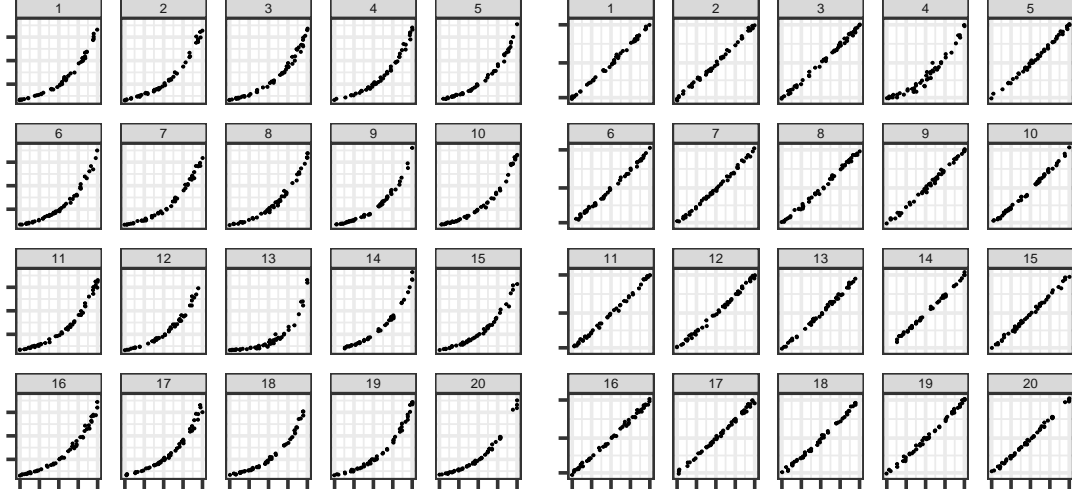


Figure 1. The plot on the left displays increasing exponential data on a linear scale with panel $(2 \times 5) + 3$ as the target. The plot on the right displays increasing exponential data on the log scale with panel 2×2 as the target.

of individuals. Similarly, a statistical lineup is a plot consisting of smaller panels; the viewer is asked to identify the plot of the real data from among a set of decoy null plots. A statistical lineup typically consists of 20 panels - 1 target panel and 19 null panels (Figure 1). If the viewer can identify the target panel embedded within the set of null panels, this suggests that the real data is visually distinct from data generated under the null model. Crowd sourcing websites such as Amazon Mechanical Turk, Reddit, and Prolific allow us to collect responses from multiple viewers. In this paper, we use statistical lineups to test our ability to differentiate between increasing exponential data with differing growth rates, using linear and log scales.

2. Methodology

2.1. Data Generation

In this study, both the target and null data sets were generated by simulating data from an exponential model; the models differ in the parameters selected for the null and target panels. In order to guarantee the simulated data spans the same range of values, we implemented a range constraint of $y \in [10, 100]$ and a domain constraint of $x \in [0, 20]$ with $N = 50$ points randomly assigned throughout the domain and mapped to the y-axis using the exponential model with the selected parameters. These constraints provide some assurance that participants who select the target plot are doing so because of their visual perception differentiating between curvature or slope rather than different starting or ending values.

We simulated data based on a 3-parameter exponential model with multiplicative errors. Use an actual equation to show the model with all parameters.

$$\dots \tag{1}$$

with parameters defined underneath (though without the numeric references, use nonnumber to get rid of the numeric references on the lines)

Table 1. Exponential parameter selection

	x_{Mid}	$\hat{\sigma}$	$\hat{\alpha}$	$\tilde{\alpha}$	$\hat{\beta}$	$\hat{\theta}$
Obvious Curvature	14.5	0.25	0.91	0.88	0.23	9.10
Noticable Curvature	13.0	0.12	6.86	6.82	0.13	3.14
Almost Linear	11.5	0.05	37.26	37.22	0.06	-27.26

The parameters α and θ are adjusted based on β and σ^2 to guarantee the range and domain constraints are met. The model generated $N = 50$ points $(x_i, y_i), i = 1, \dots, N$ where x and y have an increasing exponential relationship. The heuristic data generation procedure is provided in Appendix A.

The exponential model provides the base for this graphical experiment. We manipulate the midpoint, x_{mid} , and in turn the estimated parameters to control the amount of curvature present in the data and the error standard deviation, σ , to control the amount of deviation from the exponential curve. We selected three midpoints corresponding to difficulty levels easy (obvious curvature), medium (noticeable curvature), and hard (almost linear) along with a sensible choice of standard deviation, σ , as shown in Table 1. The midpoints and standard deviation combinations were chosen using a method described in VanderPlas and Hofmann (2017); additional details are available in Appendix B.

2.2. Lineup Setup

The lineup plots were generated by mapping simulating data corresponding to difficulty level A to a scatterplot while the null plots were generated by mapping simulated data corresponding to difficulty level B to a scatterplot. For example, a target plot with simulated data following an increasing exponential curve with obvious curvature is embedded within null plots with simulated data following an increasing exponential that is almost linear (i.e. Easy-Hard). By our constraints, the target plot and null plots will span a similar domain and range. There are a total of 6 (i.e. 3 choose 2) lineup parameter combinations. Two sets of each lineup parameter combination were simulated (total of 12 test datasets) and plotted on both the linear and the log scale (total of 24 test lineup plots). In addition, there are three parameter combinations which generate homogeneous "Rorschach" lineups, where all panels are from the same distribution. Each participant evaluated one of these lineups, but for simplicity, these evaluations are not described in this paper.

2.3. Study Design

Each participant was shown a total of thirteen lineup plots (twelve) test lineup plots and one Rorschach lineup plot). Participants were randomly assigned one of the two replicate datasets for each of the six unique lineup parameter combinations. For each assigned test dataset, the participant was shown the lineup plot corresponding to both the linear scale and the log scale. For the additional Rorschach lineup plot, participants were randomly assigned a one data set shown on either the linear or the log scale. The order of the thirteen lineup plots shown was randomized for each participant.

Participants above the age of majority were recruited from Reddit's Visualization

and Sample Size communities. Since participants recruited on Reddit were not compensated for their time, most participants have an interest in data visualization research. Previous literature suggests that prior mathematical knowledge or experience with exponential data is not associated with the outcome of graphical experiments (VanderPlas and Hofmann 2016). Participants completed the experiment using a Shiny applet (<https://shiny.srvanderplas.com/log-study/>).

Participants were shown a series of twelve test lineup plots and asked to identify the plot that was most different from the others. On each plot, participants were asked to justify their choice and provide their level of confidence in their choice. The goal of this experimental task is to test an individuals ability to perceptually differentiate exponentially increasing data with differing rates of change on both the linear and log scale. In Best, Smith, and Stubbs (2007), the authors explored whether discrimination between curve types is possible. They found that accuracy higher when nonlinear trends presented (e.g. it's hard to say something is linear, but easy to say that it isn't) and that accuracy is higher with low additive variability. XXX this may not be as relevant here, can you move it up to the lit review? XXX We hypothesize log scales should make it much easier to estimate the growth rate since we estimate slopes relatively accurately, resolving much of the difficulty with exponential estimation (Mosteller et al. 1981). XXX You don't have to state this explicitly XXX

3. Results

Participant recruitment through Reddit occurred over the course of two weeks during which 58 individuals completed 518 unique test lineup evaluations. Participants who completed fewer than 6 lineup evaluations were removed from the study (17 participants, 41 evaluations). The final dataset included a total of 41 participants and 477 lineup evaluations. Each plot was evaluated between 18 and 28 individuals (Mean: 21.77, SD: 2.29). In 67 Target plot identification was analyzed using the Glimmix Procedure in SAS 9.4. Each lineup plot evaluated was assigned a value based on the participant response (correct = 1, not correct = 0). Define Y_{ijkl} to be the event that participant l correctly identifies the target plot for dataset k with curvature j plotted on scale i . The binary response was analyzed using generalized linear mixed model following a binomial distribution with a logit link function following a row-column blocking design accounting for the variation due to participant and data set respectively, with a split plot design accounting for the same participant evaluating the same dataset on both the linear scale and log scale as shown in the Equation (1).

$$\text{logit } P(Y_{ijk}) = \eta + \delta_i + \gamma_j + \delta\gamma_{ij} + s_l + d_k + wp_{jkl} \quad (2)$$

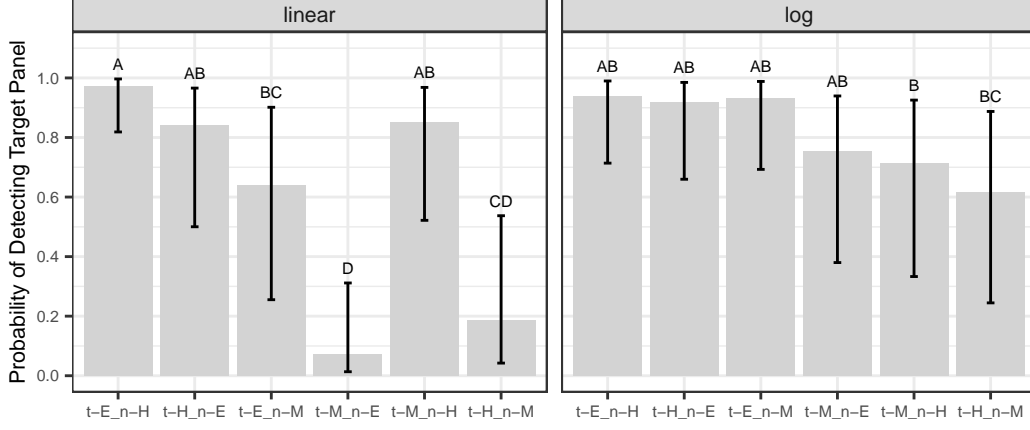


Figure 2. Least Squares Means

where

η is the baseline average probability of selecting the target plot.

δ_i is the effect of the log/linear scale.

γ_j is the effect of the curvature combination.

$\delta\gamma_{ij}$ is the two-way interaction effect of the scale and curvature.

$s_l \sim N(0, \sigma_{\text{participant}}^2)$, random effect for participant characteristics

$d_k \sim N(0, \sigma_{\text{dataset}}^2)$, random effect for data specific characteristics.

$wp_{jkl} \sim N(0, \sigma_{\text{whole-plot}}^2)$, random effect for the particular participant by dataset characteristics.

We assume that random effects for dataset, participant, and whole plot effects are independent. ADD MODEL DETAILS TO APPENDIX (e.g. ANOVA, p-values, pairwise comparisons, etc.)

The estimated probability of selecting the target plot for each of the curvature combinations on both the linear and log scale are shown in Figure 2. We see that detecting the target panel in a lineup consisting of a combination of easy (lots of curvature) and hard (almost linear) (i.e. Easy-Hard or Hard-Easy) is detected with higher probabilities on both the linear and log scale. This is to be expected since the easy and hard models have the largest difference in rates of exponential growth. When comparing models that have similar rates of change (e.g. Medium-Easy), we notice a slight decrease in accuracy, particularly when presented on the linear scale. It is worth noting that there is a more significant decrease in accuracy on the linear scale when comparing a target plot with a lower rate of change embedded in null plots with higher rates of change (e.g. Medium-Easy and Hard-Medium). This agrees with Best, Smith, and Stubbs (2007) whose results found that accuracy was higher when nonlinear trends were presented indicating that it is hard to say something is linear (i.e. something has less curvature), but easy to say that it isn't (i.e. something has more curvature). Overall, there are no significant differences in accuracy between curvature combinations when data is presented on a log scale indicating participants were consistent in their success of identifying the target panel on the log scale.

Figure 3 displays the estimated odds of successfully identifying the target panel on the log scale compared to the linear scale. From left to right, the (log) odds of

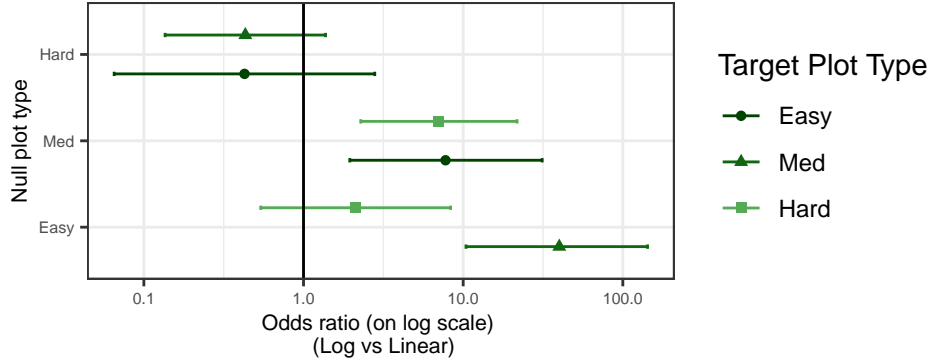


Figure 3. Odds Ratio's

successfully identifying the target panel on the log scale compared to the linear scale increase. As hypothesized, the scale had the least effect on selecting a target plot with lots of curvature (Easy) embedded in almost linear null plots (Hard) followed closely by a target plot with noticeable curvature (Medium) embedded in almost linear null plots (Hard) on both the linear and log scales. Can you revisit this with the modified figure? I think it's a bit more clear. While the log scale increased the estimated accuracy of detecting an almost linear target plot (Hard) embedded in obvious curvature plots (Easy), there is no significant evidence of a difference between the log and linear scale due to the large difference in exponential growth rates. We saw a significant increase in accuracy when exponential growth was plotted on the log scale when detecting a an almost linear target plot (Hard) embedded in null plots with noticeable curvature (Medium) and a target plot with obvious curvature (Easy) embeded in null plots with noticeable curvature (Medium). Displaying data on a log scale had the largest effect on selecting a target plot with noticeable curvature (Medium) embedded in null plots with obvious curvature (Hard).

Breaking it out this way, it's clear that the scale type doesn't matter if the curvature differences are large - comparisons between easy and hard null and target plots are not significantly different. But comparisons between slight curvature differences are almost all significantly more likely to find the target on the log scale (if I'm reading it correctly) - we should probably adjust the y-axis labels to make this clearer. The one exception is a target with medium curvature vs. a null plot with hard curvature, but I think that might just be the effect you cited from the other paper - it's easier to find a curve in a bunch of lines than a line in a bunch of curves

I would move the LSMeans plot output to the appendix, along with as much of the modeling detail as you can (or condense it into a smaller table).

4. Discussion and Conclusion

The overall goal of this research is to provide basic research to support the principles used to guide design decisions in scientific visualizations of exponential data. In this study, we explored the use of linear and log scales to determine whether our ability to notice differences in the data is different when different scales are used. Our results indicated that displaying increasing exponential data on a log scale improved the accuracy of differentiating between two increasing exponential trends with more similar growth rates, particularly when identifying an exponential curve with a lower growth

rate than others. When there was a larger discrepancy in exponential growth rates, participants accurately differentiated between the two curves on both the linear and log scale.

Further experimentation is necessary to test an individual's ability to make predictions for exponentially increasing data. Previous literature suggests that we tend to underestimate predictions of exponentially increasing data. (Jones 1979, 1977; Wagenaar and Timmers 1978). (Mosteller et al. 1981) designed and carried out an empirical investigation to explore properties of lines fitted by eye. The researchers found that students tended to fit the slope of the first principal component or major axis (the line that minimizes the sum of squares of perpendicular rather than vertical distances) and that students who gave steep slopes for one data set also tended to give steep slopes on the others. Interestingly, the individual-to-individual variability in slope and in intercept was near the standard error provided by least squares. A similar graphical task is used in the New York Times "You Draw It" page asking readers to test their knowledge by using their cursor to estimate values of a certain topic under different political administrations or over different years (CITE THIS). In addition to differentiation and prediction of exponentially increasing data, it is of interest to test an individual's ability to translate a graph of exponentially increasing data into real value quantities and extend their estimations by making comparisons. (Friel, Curcio, and Bright 2001) emphasize the importance of graph comprehension proposing that the graph construction plays a role in the ability to read and interpret graphs.

Supplementary Materials

- **Code:** R code to reproduce figures of the article as well as SAS code to fit models used in the article. (image-generator.R, R file; glmm-analysis-jsm-student-paper.sas, SAS file)
- **Data:** Anonymized responses from the Reddit study to investigate the use of logarithmic scales. Each line corresponds to one lineup evaluation by a participant. (jsm-student-paper.csv, csv file)

Acknowledgement(s)

All data collection has been conducted with approval from the University of Nebraska - Lincoln Institutional Review Board (UNL IRB). [Add the approval number if you're going to mention this, otherwise you can remove it](#)

Appendix A. Data Generation Procedure

Algorithm 2.1.1: Parameter Estimation

Input Parameters: domain $x \in [0, 20]$, range $y \in [10, 100]$, midpoint x_{mid} .

Output: estimated model parameters $\hat{\alpha}, \hat{\beta}, \hat{\theta}$

1. Determine the $y = -x$ line scaled to fit the assigned domain and range.
2. Map the values $x_{mid} - 0.1$ and $x_{mid} + 0.1$ to the $y = -x$ line for two additional points.
3. From the set points (x_k, y_k) for $k = 1, 2, 3, 4$, obtain the coefficients from the linear model $\ln(y_k) = b_0 + b_1 x_k$ to obtain starting values - $\alpha_0 = e^{b_0}, \beta_0 = b_1, \theta_0 = 0.5 \cdot \min(y)$
4. Using the 'nls()' function from the 'stats' package in Rstudio and the starting parameter values - $\alpha_0, \beta_0, \theta_0$ - fit the nonlinear model, $y_k = \alpha \cdot e^{\beta \cdot x_k} + \theta$ to obtain estimated parameter values - $\hat{\alpha}, \hat{\beta}, \hat{\theta}$.

Algorithm 2.1.2: Exponential Simulation

Input Parameters: sample size $N = 50$, estimated parameters $\hat{\alpha}, \hat{\beta}$, and $\hat{\theta}, \sigma$ standard deviation from the exponential curve.

Output Parameters: N points, in the form of vectors \mathbf{x} and \mathbf{y} .

1. Generate $\tilde{x}_j, j = 1, \dots, N \cdot \frac{3}{4}$ as a sequence of evenly spaced points in $[0, 20]$. This ensures the full domain of x is used, fulfilling the constraints of spanning the same domain and range for each parameter combination.
2. Obtain $\tilde{x}_i, i = 1, \dots, N$ by sampling $N = 50$ values from the set of \tilde{x}_j values. This guarantees some variability and potential clustering in the exponential growth curve disrupting the perception due to continuity of points.
3. Obtain the final x_i values by jittering \tilde{x}_i .
4. Calculate $\tilde{\alpha} = \frac{\hat{\alpha}}{e^{\sigma^2/2}}$. This ensures that the range of simulated values for different standard deviation parameters has an equal expected value for a given rate of change due to the non-constant variance across the domain.
5. Generate $y_i = \tilde{\alpha} \cdot e^{\hat{\beta}x_i + e_i} + \hat{\theta}$ where $e_i \sim N(0, \sigma^2)$.

Appendix B. Parameter Selection

For each level of difficulty, we simulated 1000 data sets of (x_{ij}, y_{ij}) points for $i = 1, \dots, 50$ and $j = 1 \dots 10$. Each generated x_i point from *Algorithm 2.1.2* was replicated 10 times.

Then the lack of fit statistic (LOF) was computed for each simulated dataset by calculating the deviation of the data from a linear line. Plotting the density curves of the LOF statistics for each level of difficulty choice allows us to evaluate the ability of differentiating between the difficulty levels and thus detecting the target plot. In Figure B1, we can see the densities of each of the three difficulty levels. While the LOF statistic provides us a numerical value for discriminating between the difficulty levels, we cannot directly relate this to the perceptual discriminability; it serves primarily as an approximation to ensure that we are testing parameters at several distinct levels of difficulty.

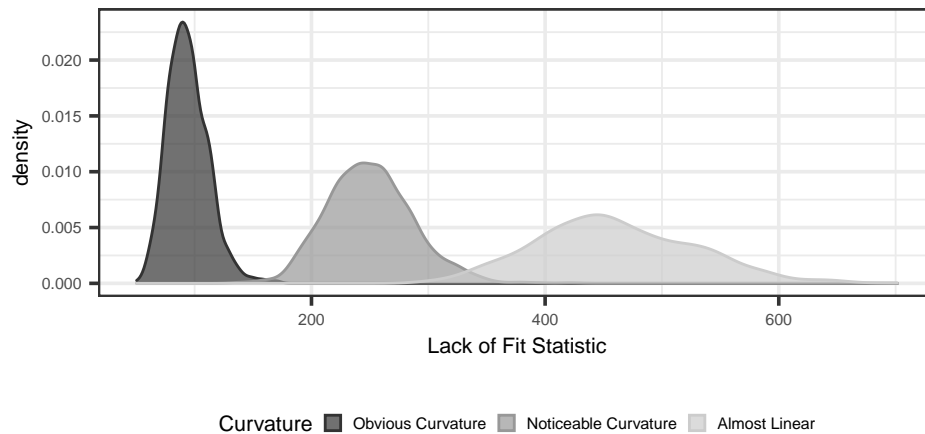


Figure B1. Density plot of the lack of fit statistic showing separation of difficulty levels: obvious curvature, noticeable curvature, and almost linear.

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