$$\left[\begin{array}{l} MGF = E[e^{tx}] \\ \text{MGF of } N(\mu, \sigma^2) \\ \quad e^{\mu t + t^2\sigma^2/2} \\ \text{MGF of } N(0,1) \\ \quad e^{t^2/2} \end{array}\right]$$

• Exponential Multiplicative

$$y = \alpha e^{(\beta x + \varepsilon)} = \alpha e^{\beta x} e^{\varepsilon}$$

with $\varepsilon \overset{iid}{\sim} N(0, \sigma^2)$

$$E[y] = E[\alpha e^{\beta x} e^{\varepsilon}] = \alpha e^{\beta x} \underbrace{E[e^{\varepsilon}]}_{\substack{\text{MGF of } \varepsilon \\ \text{at } t=1}} = \alpha e^{\beta x} M_{\varepsilon}(1) = \alpha e^{\beta x} e^{\sigma^2/2} = \alpha e^{\beta x + \sigma^2/2} \checkmark$$

$$E[y^2] = E[(\alpha e^{\beta x} e^{\varepsilon})^2] = E[(\alpha e^{\beta x})^2 (e^{\varepsilon})^2] = (\alpha e^{\beta x})^2 E[(e^{\varepsilon})^2]$$

$$= (\alpha e^{\beta x})^2 E[e^{2\varepsilon}] = (\alpha e^{\beta x})^2 M_{\varepsilon}(2) = (\alpha e^{\beta x})^2 e^{4\sigma^2/2}$$

$$= (\alpha e^{\beta x})^2 e^{2\sigma^2} = (\alpha e^{\beta x + \sigma^2})^2 \checkmark$$

$$Var(y) = E[y^2] - E[y]^2 = (\alpha e^{\beta x + \sigma^2})^2 - (\alpha e^{\beta x + \sigma^2/2})^2$$

$$= (\alpha e^{\beta x})^2 e^{2\sigma^2} - (\alpha e^{\beta x})^2 e^{\sigma^2}$$

$$= (\alpha e^{\beta x})^2 (e^{2\sigma^2} - e^{\sigma^2})$$

$$\left\{\begin{array}{l} \Rightarrow E[y] = \alpha e^{\beta x + \sigma^2/2} \\ \Rightarrow Var(y) = (\alpha e^{\beta x})^2 (e^{2\sigma^2} - e^{\sigma^2}) \end{array}\right\}$$

so... if we set $Var(\varepsilon) = \sigma^2$

$$Var(y) = (\alpha e^{\beta x})^2 (e^{2\sigma^2} - e^{\sigma^2})$$

but we want data with different σ^2 to be roughly comparable... then

set $\alpha = \sqrt{1 / (e^{2\sigma^2} - e^{\sigma^2})}$

$$\Rightarrow Var(y) = \left(\sqrt{1/(e^{2\sigma^2} - e^{\sigma^2})}\, e^{\beta x}\right)^2 (e^{2\sigma^2} - e^{\sigma^2}) = e^{\beta x}$$

Consider σ_1^2 and σ_2^2 .. Then we want...

$$E[y \mid \sigma_1^2, xMax, \beta] = E[y \mid \sigma_2^2, xMax, \beta)$$

$$\Rightarrow \alpha e^{\beta xMax} e^{\sigma_1^2/2} = \alpha e^{\beta xMax} e^{\sigma_2^2/2}$$

$$\Rightarrow \alpha e^{\beta xMax} (e^{\sigma_1^2/2} - e^{\sigma_2^2/2}) = 0$$

$\Rightarrow$

$$E[y] = \alpha e^{\beta x} e^{\sigma^2/2}$$

but we want $E[y]$ to be about the same for diffrent σ^2 values...

so set $\alpha = 1 / e^{\sigma^2/2}$

$$\Rightarrow E[y] = 1/e^{\sigma^2/2}\, e^{\beta x} e^{\sigma^2/2} = e^{\beta x}$$