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Perception of exponentially increasing data displayed on a log scale

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ABSTRACT
Log scales are often used to display data over several orders of magnitude within one graph. During the COVID pandemic, we've seen both the benefits and the pitfalls of using log scales to display data. This paper aims to...

KEYWORDS
Exponential; Log; Visual Inference; Perception

1. Introduction and Background

- Why Graphics? (communication to the public, technological advances, need for research on graphics)

Graphics are a useful tool for displaying and communicating information. Researchers include graphics to communicate their results in scientific publications and news sources rely on graphics to convey news stories to the public. During the onset of the novel coronavirus - covid19 - pandemic, we saw an influx of dashboards being developed to display case counts, transmission rates, and outbreak regions (Charlotte 2020). As a result, people began subscribing to news sources involved in graphically tracking the coronavirus (example John Burn-Murdoch Financial Times - CITE THIS) and gaining more exposure to the use of graphics. Many of these graphics helped guide decision makers to implement policies such as shut-downs or mandated mask wearing. Better software has meant easier and more flexible drawing, consistent themes, and higher standards. Consequentially, we must develop a set of principles to help us actively choose which of many possible graphics to draw (Unwin 2020).

- Introduce Log Scales (what are they used for, where are they used (ecological data, covid, etc.))

When faced with data which spans several orders of magnitude, we must decide whether to show the data on its original scale (compressing the smaller magnitudes into relatively little area) or to transform the scale and alter the contextual appearance of the data. [EXAMPLE HERE] One common graphical display choice is the use of

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log scales used to display data over several orders of magnitude within one graph. Logarithms convert multiplicative relationships to additive ones, providing an elegant way to span many orders of magnitude, to show elasticities and other proportional changes, and to linearize power laws (Menge et al. 2018). When presenting log-scaled data, it is possible to use either untransformed values (for example, values of 1, 10 and 100 are equally spaced along the axis) or log-transformed values (for example, 0, 1, and 2). We have recently experienced the benefits and pitfalls of using log-scales as covid-19 dashboards displayed case count data on both the log and linear scale (Fagen-Ulmschneider 2020). INSERT BENEFITS AND PITFALLS OF LOG SCALES HERE. While COVID-19 is the most well known example, log-scales have been used to display data in ecological research, etc. PUT OTHER AREAS HERE.

- Previous exponential (log/linear scale) studies (literature review).

Previous research suggests our perception and mapping of numbers to a numberline proceeds logrithmically at first and transitions to linear later in development. The logarithmic mapping is more noticible in larger numbers as the transition to linear mapping occurs first for small numbers in young children and later for larger numbers (Varshney and Sun 2013; Siegler and Braithwaite 2017; Dehaene et al. 2008). Early studies explored the estimation and prediction of exponential growth. Findings indicate that growth is underestimated when presented both numerically and graphically but that numerical estimation is more accurate than graphical estimation for exponential curves. While prior contextual knowledge or experience with exponential growth does not improve estimation, previous instruction on exponential growth reduces the underestimation by adjusting the initial starting value but not adjusting their perception of growth parameter (Wagenaar and Sagaria 1975; Jones 1977). Estimation was shown to improve when subjects were presented with decreasing exponential functions (Timmers and Wagenaar 1977). Jones (1977), Wagenaar and Timmers (1978), and Jones (1979) propose competing polynomial models for the perception and extrapolation of exponential series. We hypothesize that estimation is a two-stage process. First, we identify the type of curve and direction and then use that information for prediction (Best, Smith, and Stubbs 2007).

This paper aims to investigate the use of log scales to display exponentially increasing data. We hypothesize log scales should make it much easier to estimate the growth paramter since we estimate slopes relatively accurately, resolving much of the difficulty with exponential estimation (Mosteller et al. 1981). In Menge et al. (2018), ecologists were surveyed to determine how often ecologists encounter log-scaled data and how well ecologists understand log-scaled data. Participants were presented two relationships displayed on linear-linear scales, log-log scales with untransformed values, or log–log scales with log-transformed values. Menge et al. (2018) propose three types of misconceptions participants encountered when presented data on log-log scales: 'hand-hold fallacy', 'Zeno's zero fallacy', and 'watch out for curves fallacies'.

- Visual Inference (what is it? how do we use it? etc.) *needs lots of work yet*

Recent graphical experiments have utilized statistical lineups to quantify the perception of graphical design choices(VanderPlas and Hofmann 2017). Statistical lineups provide an elegant way of combining perception and statistical signficiance by valdiating the findings from a graphical experiment (Buja et al. 2009; Wickham et al. 2010; Hofmann et al. 2012; Majumder, Hofmann, and Cook 2013; VanderPlas and Hofmann 2017). 'Lineups' are named after the 'police lineup' of criminal investigations where witnesses are asked to identify the criminal from a set of individuals. Similarly, a statistical lineup is a plot consisting of smaller panels of plots in which the viewer is asked to identify the plot of the real data from among a set of decoys, the null plots. A statistical lineup typically consists of 20 panels - 1 target panel and 19 null panels (INSERT EXAMPLE). If the viewer can identify the target panel embeded within the set of null panels, this suggests that the real data is interesting or has unqiue properties. Crowd sourcing websites such as Amazon Mechanical Turk, Reddit, and Proflic allow us to collect responses from multiple viewers. VanderPlas et al. (n.d.) provides an approach for calculating visual p-values utilizing a 'rorschach' lineup which consists soley of null panels.

- What is new in this paper.

In this paper, we use statistical lineups to test human subjects ability to differentiate between increasing exponential data with differing growth rates displayed on both the linear scale and log scale.

2. Data Generation

The most common type of lineup used in graphical experiments is a standard lineup containing one "target" dataset embeded within a set of null datasets. One way to generate the null datasets when working with real data is through the use of permutation. In this study, the target dataset was generated by model A while the null datasets were generated by model B.

2.1. *Exponential Model*

2.2. *Parameter Selection*

- Use of lack of fit statistic.
- Mapping parameter selections to what we see visually.
- Curvature (Easy/Medium/Hard)

3. Study Design

3.1. *Lineup Setup*

3.2. *Participant Recruitment*

Participants above age 19 were recruited from Reddit. GIVE SUMMARY DESCRIPTIVE STATISTICS OF PARTICIPANT DEMOGRAPHICS.

3.3. *Task Description*

Participants were shown a series of twelve test lineup plots and asked to identify the plot that was different. On each plot, participants were asked to justify their choice and provide their level of confidence in their choice. The goal of this experimental task is to test an individuals ability to perceptually differentiate exponentially increasing data with differing rates of change on both the linear and log scale. In Best, Smith, and Stubbs (2007), the authors expored whether descrimination between curve types is possible. They found that accuracy higher when nonlinear trends presented (e.g. it's hard to say something is linear, but easy to say that it isn't) and that accuracy higher with low additive variability.

4. Results

4.1. *Effect of Curvature*

4.2. *Effect of Variability*

4.3. *Linear vs Log*

4.4. *Participant Reasoning*

5. Discussion

5.1. *Conclusion*

5.2. *Future Research*

- What we learned from lineups but what we still want to learn. In this study, we discovered that differentiation between data following exponentially increasing trends with differing growth rates is ...
- You draw it
    - (Mosteller et al. 1981) designed and carried out an empirical investigation to explore properties of lines fitted by eye. The researchers found that students tended to fit the slope of the first principal component or major axis (the line that minimizes the sum of squares of perpendicular rather than vertical distances) and that students who gave steep slopes for one data set also tended to give steep slopes on the others. Interestingly, the individual-to-individual variability in slope and in intercept was near the standard error provided by least squares for the four data sets.
    - The goal of this task is to test an individuals ability to make predictions for exponentially increasing data.
    - Previous literature suggests that we tend to underestimate predictions of exponentially increasing data.(Jones 1979, 1977; Wagenaar and Timmers 1978)
    - The idea for this task was inspired by the New York Times "You Draw It" page which is fun to check out.
- Estimation
    - This tests an individuals ability to translate a graph of exponentially increasing data into real value quantities. We then ask individuals to extend their estimates by making comparisons across levels of the independent variable.
    - (Friel, Curcio, and Bright 2001) emphasize the importance of graph comprehension proposing that the graph construction plays a role in the ability to read and interpret graphs.

Supplementary Materials

Acknowledgement(s)

References

Best, Lisa A., Laurence D. Smith, and D. Alan Stubbs. 2007. "Perception of Linear and Nonlinear Trends: Using Slope and Curvature Information to Make Trend Discriminations." *Perceptual and Motor Skills* 104 (3): 707–721. Publisher: SAGE Publications Inc, Accessed 2020-07-06. https://doi.org/10.2466/pms.104.3.707-721.

Buja, Andreas, Dianne Cook, Heike Hofmann, Michael Lawrence, Eun-Kyung Lee, Deborah F. Swayne, and Hadley Wickham. 2009. "Statistical inference for exploratory data analysis and model diagnostics." *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* 367 (1906): 4361–4383. Accessed 2020-10-06. https://royalsocietypublishing.org/doi/10.1098/rsta.2009.0120.

Charlotte, Lisa. 2020. "You've informed the public with visualizations about the coronavirus. Thank you." Jul. https://blog.datawrapper.de/datawrapper-effect-corona/.

Dehaene, Stanislas, Véronique Izard, Elizabeth Spelke, and Pierre Pica. 2008. "Log or Linear? Distinct Intuitions of the Number Scale in Western and Amazonian Indigene Cultures." *Science* 320 (5880): 1217–1220. 00651 Publisher: American Association for the Advancement of Science Section: Report, Accessed 2020-05-19. https://science.sciencemag.org/content/320/5880/1217.

Fagen-Ulmschneider, Wade. 2020. "91-DIVOC." http://91-divoc.com/pages/covid-visualization/.

Friel, Susan N., Frances R. Curcio, and George W. Bright. 2001. "Making Sense of Graphs: Critical Factors Influencing Comprehension and Instructional Implications." *Journal for Research in Mathematics Education* 32 (2): 124. Accessed 2020-05-29. https://www.jstor.org/stable/749671?origin=crossref.

Hofmann, Heike, Lendie Follett, Mahbubul Majumder, and Dianne Cook. 2012. "Graphical Tests for Power Comparison of Competing Designs." *IEEE Transactions on Visualization and Computer Graphics* 18 (12): 2441–2448. Accessed 2020-04-06. http://ieeexplore.ieee.org/document/6327249/.

Jones, Gregory V. 1977. "Polynomial perception of exponential growth." *Perception & Psychophysics* 21 (2): 197–198. Accessed 2020-06-25. http://link.springer.com/10.3758/BF03198726.

Jones, Gregory V. 1979. "A generalized polynomial model for perception of exponential series." *Perception & Psychophysics* 25 (3): 232–234. Accessed 2020-05-19. http://link.springer.com/10.3758/BF03202992.

Majumder, Mahbubul, Heike Hofmann, and Dianne Cook. 2013. "Validation of Visual Statistical Inference, Applied to Linear Models." *Journal of the American Statistical Association* 108 (503): 942–956. Accessed 2020-02-28. http://www.tandfonline.com/doi/abs/10.1080/01621459.2013.808157.

Menge, Duncan N. L., Anna C. MacPherson, Thomas A. Bytnerowicz, Andrew W. Quebbeman, Naomi B. Schwartz, Benton N. Taylor, and Amelia A. Wolf. 2018. "Logarithmic scales in ecological data presentation may cause misinterpretation." *Nature Ecology & Evolution* 2 (9): 1393–1402. Accessed 2020-08-18. http://www.nature.com/articles/s41559-018-0610-7.

Mosteller, Frederick, Andrew F. Siegel, Edward Trapido, and Cleo Youtz. 1981. "Eye Fitting Straight Lines." *The American Statistician* 35 (3): 150–152. Accessed 2020-05-29. http://www.tandfonline.com/doi/abs/10.1080/00031305.1981.10479335.

Siegler, Robert S., and David W. Braithwaite. 2017. "Numerical Development." *Annual Review of Psychology* 68 (1): 187–213. Accessed 2020-05-19. http://www.annualreviews.org/doi/10.1146/annurev-psych-010416-044101.

Timmers, Han, and Willem A. Wagenaar. 1977. "Inverse statistics and misperception of exponential growth." *Perception & Psychophysics* 21 (6): 558–562. Accessed 2020-07-06. http://link.springer.com/10.3758/BF03198737.

Unwin, Anthony. 2020. "Why is Data Visualization Important? What is Important in Data Visualization?" *Harvard Data Science Review* Accessed 2020-04-27. https://hdsr.mitpress.mit.edu/pub/zok97i7p.

VanderPlas, Susan, and Heike Hofmann. 2017. "Clusters Beat Trend!? Testing Feature Hierarchy in Statistical Graphics." *Journal of Computational and Graphical Statistics* 26 (2): 231–242. Accessed 2020-02-28. https://www.tandfonline.com/doi/full/10.1080/10618600.2016.1209116.

VanderPlas, Susan, Christian Rottger, Dianne Cook, and Heike Hofmann. n.d. "Statistical Significance Calculations for Scenarios in Visual Inference." Preprint, Accessed 2020-09-29. https://github.com/srvanderplas/visual-inference-alpha.

Varshney, Lav R., and John Z. Sun. 2013. "Why do we perceive logarithmically?" *Significance* 10 (1): 28–31. Accessed 2020-05-07. http://doi.wiley.com/10.1111/j.1740-9713.2013.00636.x.

Wagenaar, W. A., and H. Timmers. 1978. "Extrapolation of exponential time series is not enhanced by having more data points." *Perception & Psychophysics* 24 (2): 182–184. Accessed 2020-05-19. http://link.springer.com/10.3758/BF03199548.

Wagenaar, William A., and Sabato D. Sagaria. 1975. "Misperception of exponential growth." *Perception & Psychophysics* 18 (6): 416–422. Accessed 2020-07-07. http://link.springer.com/10.3758/BF03204114.

Wickham, Hadley, Dianne Cook, Heike Hofmann, and Andreas Buja. 2010. "Graphical inference for infovis." *IEEE Transactions on Visualization and Computer Graphics* 16 (6): 973–979.