

# Class 15: Inference and simulation I

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June 11, 2018



# General

# Announcements

- Questions for Reading 13 due on **June 13** by 9:00am: **Introductory Statistics with Randomization and Simulation**
  - From chapter 2: section 2.4 through to the end of section 2.5
  - From chapter 4: section 4.5 (skip 4.5.3)
- Homework 3 due date pushed back to **11:59pm on Wednesday, June 13th**
- Midterm reports and presentation slides due at the start of class on Thursday, June 14th

# Midterm project: rough drafts

- Whatever you have, push it to Github.
- If the questions are spread out in the various group member branches, let me know by posting this in your Slack channels for your midterm group
- I will be ending class early so that I can go and review these immediately after class, I will leave feedback in a *Pull Request* and post in your group's Slack channel when I am done

# Data and drawing conclusions: limits set by experimental sampling

# Populations and samples

**Research question:** Can people become better, more efficient runners on their own, merely by running?

PHYS ED | AUGUST 29, 2012, 12:01 AM | 21 Comments

## Finding Your Ideal Running Form

By GRETCHEN REYNOLDS



David De Lossy/Getty Images

Source: <http://well.blogs.nytimes.com/2012/08/29/finding-your-ideal-running-form>

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**Question:** Population to which results can be generalized?

**Answer:** Adult women, if the data are randomly sampled

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- It was concluded that "smoking is a complex human behavior, by its nature difficult to study, confounded by human variability."
- In time researchers were able to examine larger samples of cases (smokers), and trends showing that smoking has negative health impacts became much clearer.



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- Taking a census may be more complex than sampling.

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- If your spoonful comes only from the surface and the salt is collected at the bottom of the pot, what you tasted is probably not representative of the whole pot.
- If you first stir the soup thoroughly before you taste, your spoonful will more likely be representative of the whole pot.

# Sampling bias

- **Non-response:** If only a small fraction of the randomly sampled people choose to respond to a survey, the sample may no longer be representative of the population.
- **Voluntary response:** Occurs when the sample consists of people who volunteer to respond because they have strong opinions on the issue. Such a sample will also not be representative of the population.

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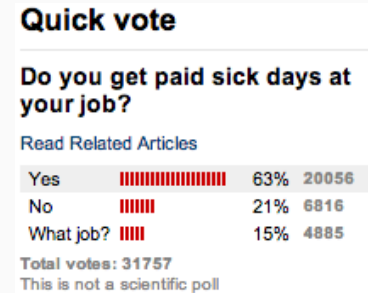
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- **Convenience sample:** Individuals who are easily accessible are more likely to be included in the sample.



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In 1936, Landon sought the Republican presidential nomination opposing the re-election of FDR.

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- The magazine was completely discredited because of the poll, and was soon discontinued.

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These groups had incomes well above the national average of the day (remember, this is Great Depression era) which resulted in lists of voters far more likely to support Republicans than a truly **typical** voter of the time, i.e. the sample was not representative of the American population at the time.

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- The Literary Digest election poll was based on a sample size of 2.4 million, which is huge, but since the sample was **biased**, the sample did not yield an accurate prediction.
- Back to the soup analogy: If the soup is not well stirred, it doesn't matter how large a spoon you have, it will still not taste right. If the soup is well stirred, a small spoon will suffice to test the soup.

**"Correlation does not imply causation"**

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- To identify the explanatory variable in a pair of variables, identify which of the two is suspected of affecting the other:
- explanatory variable  response variable
- Labeling variables as explanatory and response does not guarantee the relationship between the two is actually causal, even if there is an association identified between the two variables. We use these labels only to keep track of which variable we suspect affects the other.

# Observational studies and experiments

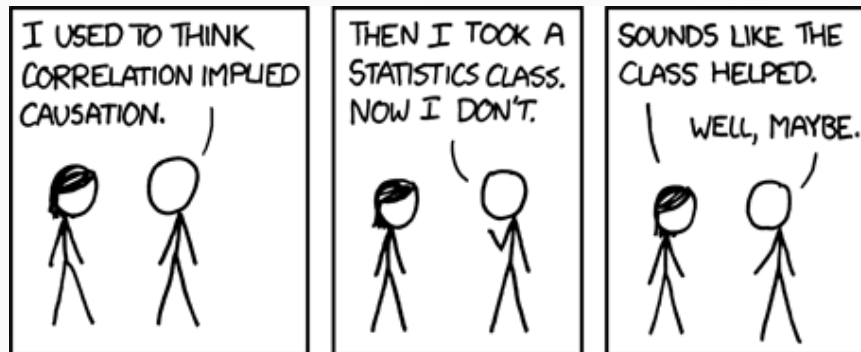
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- **Experiment:** Researchers randomly assign subjects to various treatments in order to establish causal connections between the explanatory and response variables.
- If you're going to walk away with one thing from the last few weeks of this class, let it be "correlation does not imply causation".



Source: <http://xkcd.com/552/>

# Case study: Gender discrimination

# Study description and data

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*This is an example of an experiment*

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# Practice

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1. If we were to repeat the experiment we will definitely see that more female files get promoted. This was a fluke.
2. Promotion is dependent on gender, males are more likely to be promoted, and hence there is gender discrimination against women in promotion decisions.
3. The difference in the proportions of promoted male and female files is due to chance, this is not evidence of gender discrimination against women in promotion decisions.
4. Women are less qualified than men, and this is why fewer females get promoted.

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Promotion and gender are **dependent**, there is gender discrimination, observed difference in proportions is not due to chance. → **Alternative hypothesis**

# A trial as a hypothesis test

- As a process, hypothesis testing is analogous to a court trial
- $H_0$ : Defendant is innocent
- $H_A$ : Defendant is guilty
- We then present the evidence – collect data.



Image from [http://www.nwherald.com/\\_internal/cimg!0/oo1il4sf8zzaqbboq25oewvbg99wpot](http://www.nwherald.com/_internal/cimg!0/oo1il4sf8zzaqbboq25oewvbg99wpot)

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- Ultimately we must make a decision. How unlikely is unlikely?



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- In a hypothesis test, the burden of proof is on the unusual claim.
- The null hypothesis is the ordinary state of affairs, so it's the alternative hypothesis that we consider unusual and for which we must gather evidence.

# Recap: hypothesis testing framework

- We start with a null hypothesis ( $H_0$ ) that represents the status quo
- We also have an alternative hypothesis ( $H_A$ ) that represents our research question, i.e. what we're testing for
- We conduct a hypothesis test under the assumption that the null hypothesis is true, either via simulation or theoretical methods
  - If the test results suggest that the data do not provide convincing evidence for the alternative hypothesis, we stick with the null hypothesis
  - If they do, then we reject the null hypothesis in favor of the alternative

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- If results from the simulations based on the **chance model** look like the data, then we can determine that the difference between the proportions of promoted files between males and females was simply **due to chance** (promotion and gender are independent).
- If the results from the simulations based on the chance model do not look like the data, then we can determine that the difference between the proportions of promoted files between males and females was not due to chance, but **due to an actual effect of gender** (promotion and gender are dependent).

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  - Consider aces as face cards
  - Set aside the jokers
  - Take out 3 aces → there are exactly 13 face cards left in the deck (face cards: A, K, Q, J)
  - Take out a number card → there are exactly 35 number (non-face) cards left in the deck (number cards: 2-10)

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2. Shuffle the cards and deal them into two groups of size 24, representing males and females



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1. Let a face card represent *not promoted* and a non-face card represent *promoted*
  - Consider aces as face cards
  - Set aside the jokers
  - Take out 3 aces → there are exactly 13 face cards left in the deck (face cards: A, K, Q, J)
  - Take out a number card → there are exactly 35 number (non-face) cards left in the deck (number cards: 2-10)
2. Shuffle the cards and deal them into two groups of size 24, representing males and females
3. Count and record how many files in each group are promoted (number cards)

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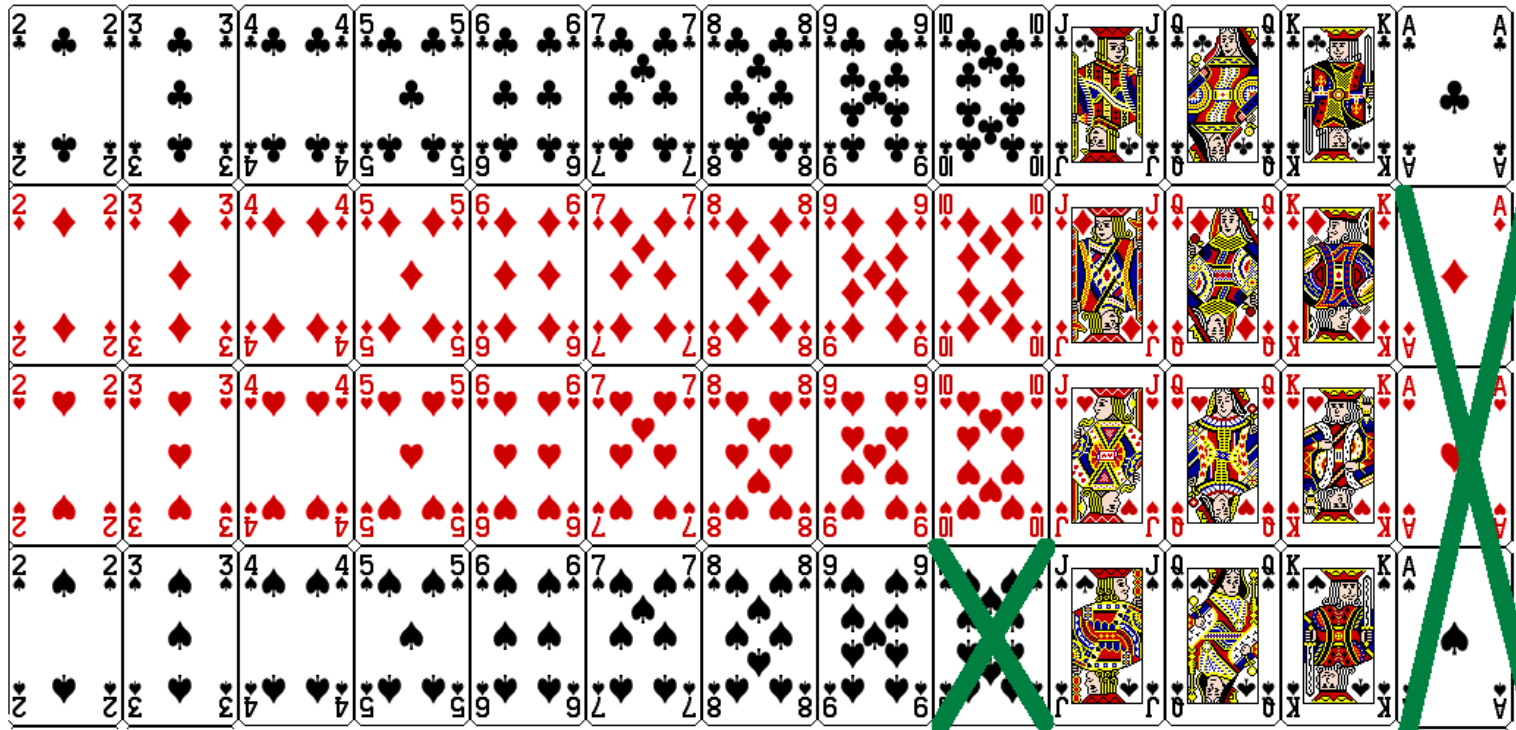
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5. Repeat steps 2 – 4 many times

# Step 1

35 number (non-face) cards

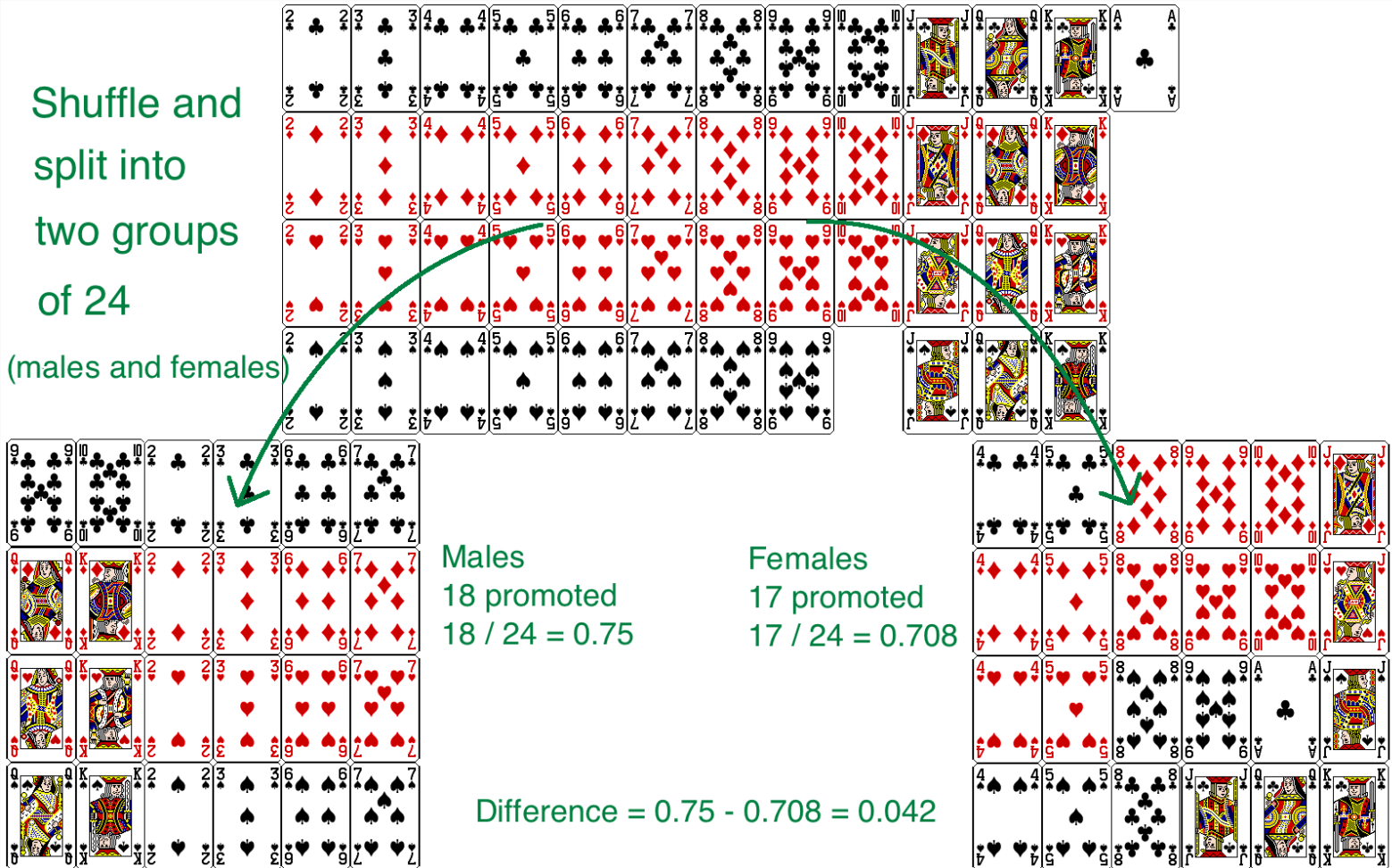
13 face cards



# Step 2

Shuffle and  
split into  
two groups  
of 24

(males and females)



# Credits

These slides were adapted from the chapter 1 and chapter 3 [OpenIntro Statistics slides](#) developed by Mine Çetinkaya-Rundel and made available under the [CC BY-SA 3.0 license](#).