### **Investment Simulation 3**

### Wei Mu

Introduction: This main purpose of this investment simulation is to utilize the Sharpe Single Index (Market) model to determine the (optimized) strategic weights of the portfolio. In order to get the prospective  $\alpha$ ,  $\beta$  and  $\sigma(e)$  for each stock required by Sharpe model, I need to first calculate the corresponding historical statistics to get estimates. To calculate historical estimates, I use realized monthly returns for the recent 60 months on the stocks, the market index and the risk-free security.

In this simulation, I use all the stocks picked in Investment Simulation 2. The tickers are as follows:

ALK HOG IPG LNT LUK PNW RE SNA TMK WU

For market returns, I use ^SP500TR (S&P 500 index) as data source.

I download monthly historical price data for all the stocks above from Yahoo! Finance on Nov 27, 2017. Data ranges from October 2012 to October 2017, which consists of 61 months. (Data for November 2017 are discarded.) I choose the columns with adjusted closing prices for the rest of the calculations. Then I calculate monthly returns using the adjusted prices:

$$\frac{p_t^*}{p_{t-1}^*} - 1$$

Then the monthly return ranges from November 2012 to October 2017, which consists of 60 months.

1	Α	С	D	Е	F	G	н	I	J	К	L	м	N	0	Р	0	R	S	Т	U	V	w
1		Adj Close				_		_			_		r	_	-	•		_			-	
2		ALK	HOG	IPG	LNT	LUK	PNW	RE	SNA	TMK	WU		ALK	HOG	IPG	LNT	LUK	PNW	RE	SNA	TMK	WU
3	October-12	\$20.15	\$42.19	\$9.61	\$18.05	\$20.22	\$42.81	\$97.95	\$72.91	\$33.08	\$10.74											
4	November-12	\$20.31	\$43.87	\$9.85	\$17.68	\$21.72	\$42.41	\$99.74	\$72.87	\$32.87	\$11.59		0.80%	3.98%	2.43%	-2.03%	7.40%	-0.93%	1.83%	-0.06%	-0.62%	7.93%
5	December-12	\$21.75	\$47.25	\$10.82	\$18.46	\$23.48	\$44.41	\$105.06	\$74.74	\$35.44	\$12.24		7.06%	7.70%	9.89%	4.40%	8.11%	4.71%	5.33%	2.57%	7.82%	5.55%
6	January-13	\$24.30	\$47.44	\$11.42	\$19.40	\$25.49	\$47.02	\$113.04	\$74.05	\$35.85	\$12.06		11.75%	0.40%	5.53%	5.12%	8.55%	5.87%	7.60%	-0.93%	1.16%	-1.41%
7	February-13	\$30.15	\$48.23	\$11.64	\$20.42	\$25.99	\$48.66	\$117.80	\$76.67	\$38.16	\$12.93		24.07%	1.67%	1.96%	5.22%	1.97%	3.49%	4.21%	3.54%	6.42%	7.20%
8	March-13	\$29.06	\$49.45	\$12.44	\$21.77	\$29.34	\$51.19	\$122.93	\$79.91	\$39.60	\$12.84		-3.63%	2.53%	6.84%	6.64%	12.88%	5.20%	4.35%	4.23%	3.80%	-0.67%
59	June-17	\$84.53	\$48.30	\$21.42	\$39.93	\$25.92	\$85.40	\$261.09	\$152.67	\$78.82	\$19.57		-5.05%	-9.90%	-11.52%	0.90%	-0.25%	1.84%	3.06%	-2.41%	3.43%	4.62%
60	July-17	7 \$74.04	\$46.65	\$19.96	\$42.43	\$23.58	\$89.26	\$251.23	\$146.11	\$76.83	\$18.75		-12.40%	-3.41%	-6.80%	6.27%	-9.03%	4.52%	-3.78%	-4.30%	-2.53%	-4.20%
61	August-17	\$75.93	\$47.84	\$20.79	\$41.27	\$25.14	\$83.89	\$227.26	\$148.23	\$79.94	\$19.02		2.54%	2.55%	4.17%	-2.74%	6.63%	-6.01%	-9.54%	1.45%	4.05%	1.48%
62	September-17	\$65.73	\$47.34	\$19.25	\$42.95	\$25.30	\$87.02	\$237.45	\$156.95	\$83.97	\$19.86		-13.43%	-1.04%	-7.41%	4.07%	0.63%	3.73%	4.48%	5.89%	5.04%	4.39%
63	October-17	\$65.06	\$47.52	\$18.58	\$44.61	\$25.67	\$90.01	\$223.07	\$160.45	\$85.74	\$19.48		-1.01%	0.38%	-3.48%	3.87%	1.46%	3.44%	-6.06%	2.22%	2.10%	-1.91%

The following table shows the first and last five months of adjusted closing prices and returns for the stocks and market security:

	Α	Y	AA
1		Adj Close	r
2		^SP500TR	^SP500TR
3	October-12	\$2,481.82	
4	November-12	\$2,504.44	0.91%
5	December-12	\$2,634.16	5.18%
6	January-13	\$2,669.92	1.36%
7	February-13	\$2,770.05	3.75%
8	March-13	\$2,823.42	1.93%
59	June-17	\$4,774.56	2.06%
60	July-17	\$4,789.18	0.31%
61	August-17	\$4,887.97	2.06%
62	September-17	\$5,002.03	2.33%
63	October-17	\$5,064.23	1.24%

I use 1-Month Treasury Constant Maturity Rate available at St. Louis Federal Reserve Bank website as the risk-free rate. When downloading the data, use Monthly as the Frequency and End of Period as the Aggregation Method. The screenshots are as follows:

Percent			▼ (
odify frequency:		Aggregation method:	
Monthly	т 📵	End of Period ▼	

The returns are realized in the following months. The downloaded data are shown in percent per year, so I order to convert the data to decimal return per month, I first divide the corresponding realized rate of returns by 100, and then divide the results by 12 to convert the numbers. The screenshots for the first and last five months for the risk-free security are as follows:

4	Α	В	С	D	Е	F	G
1	FRED Graph Observation	1S					
2	Federal Reserve Econon	nic Data					
3	Link: https://fred.stlouisfe	ed.org					
4	Help: https://fred.stlouisf	ed.org/help-faq					
5	Economic Research Divi	sion					
6	Federal Reserve Bank of	St. Louis					
7							
8	DGS1MO	1-Month Treasury Consta	nt Maturity Rate	e, Percent, Mo	onthly, Not Se	asonally Adjust	ed
9							
10	Frequency: Monthly						
11	observation_date	DGS1MO		realized		realized	
12	2012-10-01	0.09		Annual		Monthly	
13	2012-11-01	0.11	November-12	0.0009	<=B12/100	0.00007500	<=D13/12
14	2012-12-01	0.02	December-12	0.0011	<=B13/100	0.00009167	<=D14/12
15	2013-01-01	0.04	January-13	0.0002		0.00001667	
16	2013-02-01	0.07	February-13	0.0004		0.00003333	
17	2013-03-01	0.04	March-13	0.0007		0.00005833	
68	2017-06-01	0.84	June-17	0.0086		0.00071667	
69	2017-07-01	1.00	July-17	0.0084		0.00070000	
70	2017-08-01	0.95	August-17	0.0100		0.00083333	
71	2017-09-01	0.96	September-17	0.0095		0.00079167	
72	2017-10-01	0.99	October-17	0.0096	<=D71/100	0.00080000	<=E72/12

Only the realized monthly decimal returns per month (the F column) are used in the following calculations.

The next step is to calculate the  $\alpha$ ,  $\beta$ , and  $\sigma$  for each stock using historical returns obtained in the previous steps. The following screenshot is a table showing the inputs and results from MMTool for one of the stocks, ticker name "WU".



- In cell C14, enter the number of returns you have (or want to use) for the market index, risk-free security and the security/portfolio.
- Enter the dates, and the returns on the market, risk-free security, and the security/portfolio in tan cells in the appropriate columns in the block below
- The output for the excess return market model estimates is given in block J16:M18, and the output for total return market model estimates is given in block J20:M22.

14	N of	Returns	60		
15					
16	#	Month	r <sub>M</sub>	r <sub>f</sub>	r <sub>i</sub>
17	1	Nov 2012	0.91%	0.008%	7.93%
18	2	Dec 2012	5.18%	0.009%	5.55%
19	3	Jan 2013	1.36%	0.002%	-1.41%
20	4	Feb 2013	3.75%	0.003%	7.20%
21	5	Mar 2013	1.93%	0.006%	-0.67%
22	6	Apr 2013	2.34%	0.003%	10.60%
23	7	May 2013	-1.34%	0.003%	4.46%
24	8	Jun 2013	5.09%	0.003%	5.76%
25	9	Jul 2013	-2.90%	0.002%	-2.39%
26	10	Aug 2013	3.14%	0.003%	6.45%
72	56	Jun 2017	2.06%	0.072%	4.62%
73	57	Jul 2017	0.31%	0.070%	-4.20%
74	58	Aug 2017	2.06%	0.083%	1.48%
75	59	Sep 2017	2.33%	0.079%	4.39%
76	60	Oct 2017	1.24%	0.080%	-1.91%

6

7

8

9 10

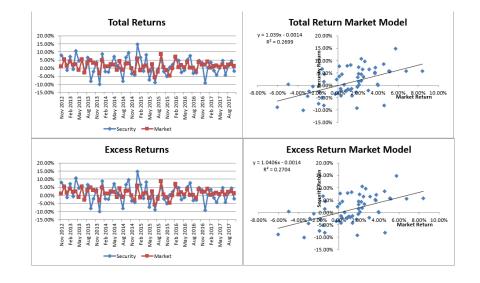
11 12

13

$R_i = r_i - r_f$	$R_M = r_M - r_f$
7.92%	0.90%
5.54%	5.17%
-1.41%	1.36%
7.20%	3.75%
-0.68%	1.92%
10.60%	2.34%
4.45%	-1.35%
5.76%	5.09%
-2.40%	-2.90%
6.44%	3.13%
4.54%	1.98%
-4.27%	0.24%
1.40%	1.98%
4.31%	2.25%
-1.99%	1.16%

Excess Re	eturn Mark	cet Model	Estimates
α	β	σ(e)	R Square
-0.14%	1.0406	4.67%	27.0%

Total Re	turn Mark	et Model E	stimates
а	b	s(e)	R Square
-0.14%	1.0390	4.67%	27.0%



For the inputs of the MMtool,  $r_{M}$  are the monthly returns of ^SP500TR.  $r_{f}$  are the monthly return for the risk-free security (1-Month Treasury Constant Maturity Rate).  $r_{i}$  are the monthly returns for each stock ("WU" in this example). Other data in the screenshot above are all output results. For Sharpe model, the data in the "Excess Return Market Model Estimates" should be used, and the results in "Excess Return Market Model Estimates" are dropped.

In the next step, I will now calculate the annualized prospective  $\alpha$ ,  $\beta$  and  $\sigma(e)$ .

First, I use the following formulas to annualize the historical quantities:

$$lpha_{Annual} = 12 * lpha_{Monthly}$$
  $eta_{Annual} = eta_{Monthly}$   $\sigma(e)_{Annual} = \sqrt{12} * \sigma(e)_{Monthly}$ 

Then I use the following formula to get prospective  $oldsymbol{eta}$ 

$$\beta_{prospective} = \lambda \beta_{Historical} + (1 - \lambda)$$

I assume  $\lambda = 0.6$ . It is arbitrary but reasonable, because  $\lambda$  is typically assumed to be in 0.6-0.7 range.

So

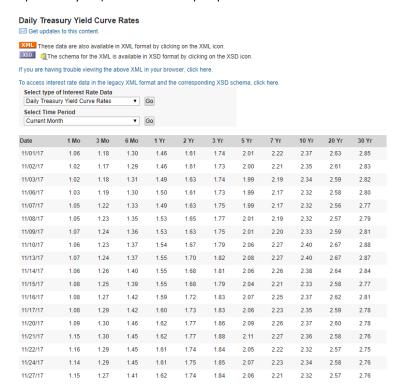
$$\beta_{prospective} = 0.6 \beta_{Historical} + 0.4$$

I assume the market is efficient, so set  $\alpha_{prospective} = 0$ . Based on the assumption that the risk of the stock from sources other than market is stationary over time, I set prospective  $\sigma(e_i)$  to be the same as historical ones.

The following screenshot shows the results of historical monthly, historical annual and prospective annual  $\alpha, \beta$  and  $\sigma(e)$ . Excel formulas are also included in the picture.

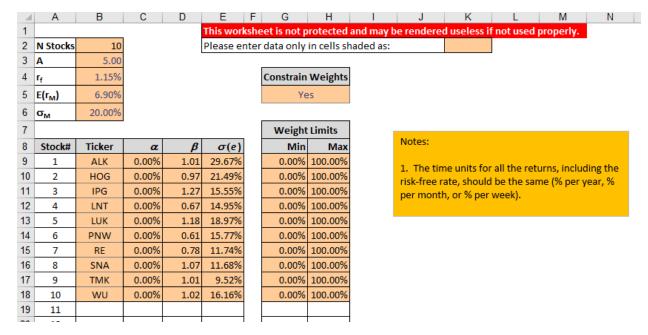
/_	Α	В	С	D	Ε	F	G	Н	I	J	K	L
1	ALK											
2	Excess Ret	urn Market	Model Estir	mates		annual				prospective (p	er year)	
3	alpha	beta	s(e)	R Square		alpha	beta	s(e)		alpha	beta	s(e)
4	1.11%	1.0101	8.56%	9.4%		0.13373	1.0101	0.296673		(	1.00603016	0.296673
5						<=A4*12	<=B4	<=SQRT(1	2)*C4		<=0.6*G4+0.4*1	<=H4
6	HOG											
7	Excess Ret	urn Market	Model Esti	mates		annual				prospective (p	er year)	
8	alpha	beta	s(e)	R Square		alpha	beta	s(e)		alpha	beta	s(e)
9	-0.00748	0.944241	0.062038	0.147197		-0.08979	0.944241	0.214905		(	0.966544362	0.214905
10												
11	IPG											
12	Excess Ret	urn Market	Model Estir	mates		annual				prospective (p	er year)	
13	alpha	beta	s(e)	R Square		alpha	beta	s(e)		alpha	beta	s(e)
14	-0.00509	1.454121	0.044881	0.438872		-0.06103	1.454121	0.155472		0	1.272472449	0.155472
15												
16	LNT											
17	Excess Ret	urn Market	Model Estir	mates		annual				prospective (p	er year)	
18	alpha	beta	s(e)	R Square		alpha	beta	s(e)		alpha	beta	s(e)
19	0.01051	0.453422	0.043154	0.076004		0.126121	0.453422	0.149489			0.672053118	0.149489
20												
21	LUK											
22	Excess Ret	urn Market	Model Estir	mates		annual				prospective (p	er year)	
23	alpha	beta	s(e)	R Square		alpha	beta	s(e)		alpha	beta	s(e)
24		1.295549	0.054767			-0.11807	1.295549				1.177329366	
25												
26	PNW											
27	Excess Ret	urn Market	Model Estir	mates		annual				prospective (p	er vear)	
28	alpha	beta	s(e)	R Square		alpha	beta	s(e)		alpha	beta	s(e)
29			0.045513				0.357272	0.15766				
30	0.00000		0.0.0020	0.0.00				0.20.00		_	0.02.00201	0.227.00
31	RE											
32		urn Market	Model Estir	mates		annual				prospective (p	er vear)	
33	alpha	beta	s(e)	R Square		alpha	beta	s(e)		alpha	beta	s(e)
34	0.006678		0.033886			0.080136		0.117385		a.p.i.a		
35	0.000070	0.05202	0.055000	0.205050		0.000150	0.05202	0.117,505			0.775211720	0.117505
36	SNA											
37		urn Market	Model Estir	matec		annual				prospective (p	or voar)	
38	alpha	beta	s(e)	R Square		alpha	beta	s(e)		alpha	beta	s(e)
30 39	-		0.033722	-			1.114942			аірпа		
39 40	0.000313	1.117572	0.033722	0.7700/1		0.000100	1.117572	0.110010		ļ ,	1.000303497	0.110010
41	ТМК											
41 42		urn Markot	Model Estir	matoc		annual				prospective (n	or voar)	
42 43	alpha	beta					beta	o/o \		prospective (p	beta	c/o \
			s(e)	R Square		alpha		s(e ) 0.095161		alpha		s(e )
44 45	0.004303	1.00943	0.027471	0.301304		0.051635	1.00943	0.093101			1.0056579	0.095101
16	I NAME I											
46	WU Evenes Bet	turn Made-t	Model Fett	mates		annual				prospective /-	or voor)	
47			Model Esti			annual	h - 4 -	-/- \		prospective (p	1.	-(- )
48	alpha	beta	s(e)	R Square		alpha	beta	s(e)		alpha	beta	s(e)
49	-0.00139	1.040648	0.046661	0.2/0391		-0.01666	1.040648	0.161638		(	1.024388527	0.161638

In the next step, I will now obtain the current risk-free rate. Look up the yield on a 1 Month Treasury security from the U.S. Department of Treasury website (https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yield). Use this as the prospective risk-free rate.



The screenshot is taken on 11/27/2017. I use 1 month treasury yield curve rates corresponding to the date 11/27/2017. The numbers are shown in percent, so the current risk-free rate is 1.15% per year.

In the next step, I will utilize the Sharpe Optimal Portfolio Selection spreadsheet.



All numbers in the input areas are precise numbers, although only two digits are shown.

Set the number of stock as 10 and risk-free rate as 1.15% according to the previous step. Risk-aversion coefficient A is useless in the Sharpe Optimal Portfolio Selection process so it is unchanged.

I use  $E(r_M) = r_f + MarketRiskPremium$  to estimate  $E(r_M)$ . According to Equity Market Risk Premium –

### Research Summary published by KPMG on 13 July, 2017

Stock Ticker Weights

(https://assets.kpmg.com/content/dam/kpmg/nl/pdf/2017/advisory/mrp-summary-july-2017.pdf), the market risk premium is recommend to be set as 5.75%, so I choose this number to estimate the current Market Risk Premium.  $r_f=1.15\%$  per year according to the previous step, so  $E(r_M)=1.15\%+5.75\%=6.90\%$  per year. Then I assume  $\sigma_M=20.00\%$  (reasonable because  $\sigma_M$  is typically around 20%)

Because the Virtual Stock Exchange for this simulation does not allow short-selling and borrowing, I set the weight constraints to be between 0% and 100%.

Then I click "Run the optimizer" on the "Models" worksheet. The results for "Case 4: Maximize Slope of the CAL" are:

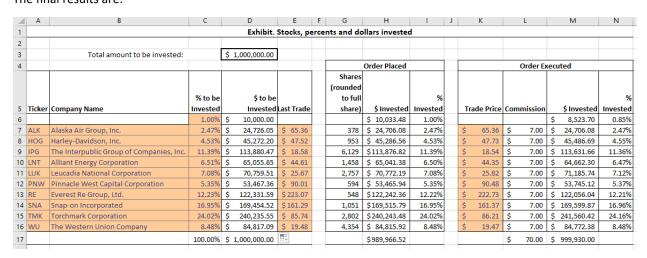
	HCKEI	WCIgitta			
1	ALK	2.50%			
2	HOG	4.57%			
3	IPG	11.50%			
4	LNT	6.57%			
5	LUK	7.15%			
6	PNW	5.40%			
7	RE	12.36%			
8	SNA	17.12%			
9	TMK	24.27%		φ	0
10	WU	8.57%			Efficient Frontier and Optimal Portfolios
11					Sharpe Model
12				12%	Sharpe Model
13					<u> </u>
14					
15				10% -	
16					
17					•
18				8% -	5
19					9 10 1
20					2
				E(r) 6%	
$\Sigma w_i$		100.00%			— Unconstrained Frontier
α <sub>p</sub>		0.00%			Unconstrained Global Minimum Variance     Constrained Global Minimum Variance
3 <sub>p</sub>		0.99		4% -	Maximize Expected Utility
σ(e <sub>p</sub> )		4.65%			Minimize Std Dev for Exp Ret
		6.83%			Maximize Exp Ret for Std Dev     Maximize CAL Slope
$E(R_p)$			Tangency Line	2% -	Minimize Std Dev for Beta
σ <sub>p</sub>		20.30%	0.00% 1.15%		Risk-free Rate
E(U)		-0.0347			—— Capital Allocation Line
Prob (r<0)		36.82%	20.30% 6.83%	0% -	
			30.46% 9.67%	01	5% 10% 15% 20% 25% 30% 35% 40%

These weights the portfolio weights for my stocks. In Sharpe Optimal Portfolio Selection process, the target is to find out the optimal portfolio weights that maximize the slope for Capital Allocation Line. The optimizer result is the one that maximize this slope within the constrains.

The final step is to calculate the number of shares, and record the orders placed and executed. First, set aside 1% (\$10,000) for transaction costs and to absorb the effect of fluctuations in prices. Then divide the remaining \$990,000 among the stocks according to the weights above. Using the weights obtained from the previous step, the percent to be invested can be calculated as:

% to be	
invested	
1%	
2.47%	<=weight*99%
4.53%	
11.39%	
6.51%	
7.08%	
5.35%	
12.23%	
16.95%	
24.02%	
8.48%	<=weight*99%
100%	
	1% 2.47% 4.53% 11.39% 6.51% 7.08% 5.35% 12.23% 16.95% 24.02% 8.48%

# The final results are:



Note: \$ to be Invested = % to be Invested \* Total amount to be invested

# In "Order Placed" section:

Because partial shares are not allowed, round the number of shares to the nearest integer.

Shares = ROUND(\$ to be Invested/Last Trade, 0)

\$ Invested = Last Trade\*Shares

% Invested = \$ Invested/Total amount to be invested

### In "Order Executed" section:

Trade price is the execution price for each stock.

\$ Invested = Trade Price\*Shares

% Invested = \$ Invested (in Order Executed section)/ Total amount to be invested

Conclusion: This investment simulation use Sharpe model to determine the optimal weights of the portfolio. The optimal weights are obtained by maximizing slope for the CAL (within constraints). Sharpe model use prospective  $\alpha, \beta$  and  $\sigma(e)$  for each stock, which can be obtained from historical estimates. Historical monthly return data over a long period for each stock, the historical and current risk-free rates, and  $E(r_M)$  and  $\sigma_M$  estimates are used in this process. By converting historical estimates to prospective estimates, the optimal weights can be finally determined using tools like Excel. The main outcome of the process, as stated above, is the optimal weights of the portfolio.