Chi-square distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim \chi^2(n)$ is used to indicate that the random variable X has the chi-square distribution with positive integer parameter n, which is known as the degrees of freedom. A chi-square random variable X with n degrees of freedom has probability density function

$$f(x) = \frac{x^{n/2-1}e^{-x/2}}{2^{n/2}\Gamma(n/2)} \qquad x > 0,$$

for $n = 1, 2, \ldots$. The chi-square distribution is used for inference concerning observations drawn from an exponential population and in determining the critical values for the chi-square goodness-of-fit test. The probability density function with $n = 1$, 2, and 3 is illustrated below.

The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = \frac{\Gamma(n/2, x/2)}{\Gamma(n/2)} \qquad x > 0.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = 1 - \frac{\Gamma(n/2, x/2)}{\Gamma(n/2)} \qquad x > 0.$$

The hazard function $h(x) = f(x)/S(x)$ and the cumulative hazard function $H(x) = -\ln S(x)$ can be written in terms of the gamma and incomplete gamma functions. The inverse distribution function of X can't be expressed in closed form (except when $n = 2$). The mode of X is

$$\max\{n-2, 0\} \qquad n > 2.$$

The moment generating function of X is

$$M(t) = E\left[e^{tX}\right] = (1-2t)^{-n/2} \qquad t < 1/2.$$

The characteristic function of X is

$$\phi(t) = E\left[e^{itX}\right] = (1-2it)^{-n/2} \qquad t < 1/2.$$

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = n \qquad V[X] = 2n \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \sqrt{8/n} \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = 3 + \frac{12}{n}.$$

APPL verification: The APPL statements

```
X := ChiSquareRV(n);
CDF(X);
SF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the cumulative distribution function, survivor function, population mean, variance, skewness, kurtosis, and moment generating function.