STAT3023 Statiscal Inference

Lab Week 6

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Suppose $\boldsymbol{X} = (X_1, ..., Xn)$ is a vector of iid RVs with common PDF $f_\theta(.)$ where:

$$f_\theta(x) = \frac{1}{\theta} g\left(\frac{x}{\theta}\right)$$

for a known PDF $g(.)$ which possesses a continuous derivative. The family $\mathcal{F} = \{f_\theta(.) : \theta > 0\}$ is thus a *scale family* and θ is a *scale parameter*, like the standard deviation in the normal family. The Cramer-Rao Lower Bound for variance of an unbiased estimator of θ in such a family based on n iid observations is given by:

$$\frac{1}{nI_\theta} \text{ where } I_\theta = \frac{J(g) - 1}{\theta^2} \text{ and } J(g) = \int \frac{[xg'(x)]^2}{g(x)} \mathrm{d}x$$

We shall study what happens when

$$g(x) = \frac{1}{\pi(1 + x^2)}$$

is the Cauchy density (same as Students-t with 1 degree of freedom, is also the density of the ratio of two independent $N(0, 1)$ random variables); note that the quartiles of $g(.)$ are ± 1, and also that neither the mean nor the variance exist! We shall consider two estimators of θ based on $\boldsymbol{X}$:

- $\widehat{\theta}_{\text{IQR}}(\boldsymbol{X}) = \frac{\text{IQR}(\boldsymbol{X})}{2}$
- $\widehat{\theta}_{\text{MLE}}(\boldsymbol{X})$, the maximum likelihood estimator (obtained numerically using R)

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