Lab 02B: Week 6

Contents

The **specific aims** of this lab are:

- practice using the sign test, Wilcoxon signed-rank and Wilcoxon rank-sum test
- develop an understanding of when is best to use non-parametric and parametric tests presented for testing means
- learn how to check the normality assumption

The unit **learning outcomes** addressed are:

- LO1 Formulate domain/context specific questions and identify appropriate statistical analysis.
- LO3 Construct, interpret and compare numerical and graphical summaries of different data types including large and/or complex data sets.
- LO5 Identify, justify and implement appropriate parametric or non-parametric statistical tests.
- LO8 Create a reproducible report to communicate outcomes using a programming language.

1 Group work

Answer the following questions in small groups.

Five sets of identical twins were selected at random from a population of identical twins. One child was selected at random from each pair to form an "experimental group." These five children were sent to school. The other five children were kept at home as a control group. At the end of the school year the following IQ scores were obtained.

Pair	Experimental group	Control group

Pair	Experimental group	Control group
1	110	112
2	125	120
3	139	128
4	142	135
5	127	126

1. Which of the tests that we have considered so far in DATA2002 can be applied here? Why might you use one instead of the other?
2. If you were to proceed with a Wilcoxon signed rank test, calculate the test statistic.
3. Under the null hypothesis of no difference in the mean IQ score between the experimental and control group, what is the expected value of the Wilcoxon signed rank test statistic?
4. If you had to guess which of the following p-values looks most reasonable for this example?

    a. 0.0001

    b. 0.9663

    c. 0.0938

    d. 0.4367

    Don't perform the test in R, just think about the observed test statistic, the expected test statistic and the sample size.

2 Questions

2.1 Drug abuse and IQ

In a study of drug abuse in a suburban area, investigators found that the median IQ of arrested abusers who were 16 years of age or older was 107. The following table show the IQs of a random sample of 15 persons from another suburban area.

Subject	1	2	3	4	5	6	7	8	9
IQ	100	90	135	108	107	119	127	109	105

```
iq = c(100, 90, 135, 108, 107, 119, 127, 109, 105)
```

Check for normality in the data. Using the Wilcoxon signed-rank test can the researchers conclude that the mean IQ of arrested abusers who are 16 or older from the population of interest is higher than 107? Try calculating the test statistic **by hand** and use the normal approximation to identify the correct p-value from the options below. Confirm your calculations with R.

a. $P(Z < -1.77) = 0.04$

b. $P(Z > -0.77) = 0.78$

c. $P(Z < -0.77) = 0.22$

d. $P(Z > -1.77) = 0.96$

2.2 Weight gain

The weight of 5 pigs on diet X and 5 pigs on diet Y are

Diet X: {12, 16, 16, 12, 10} and diet Y: {30, 12, 24, 32, 24}.

Test if there is a difference in weight using the Wilcoxon rank-sum test. What is the corresponding p-value?

a. $P(Z < -2.02) = 0.02$

b. $2P(Z < -2.02) = 0.04$

c. $P(Z < -0.43) = 0.33$

d. $2P(Z > -0.43) = 0.66$

```
wdat = data.frame(
  weight = c(12, 16, 16, 12, 10, 30, 12, 24, 32, 24),
  diet = rep(c("X","Y"), each = 5)
)
```

2.3 How fast can you type?

Ten executive assistants were selected at random from among the executive assistants at a large university. The typing speed (number of words per minute) was recorded for each executive assistant on two different brands of computer keyboards. The following results were obtained.

Executive Assistant	Brand A	Brand B
Amy	74	72
Bruce	80	86
Carol	68	72
Dion	74	70
Ellen	86	85
Fred	75	73
Gwen	78	72

Executive Assistant	Brand A	Brand B
Hugh	69	65
Ingrid	76	79
Kenneth	77	75

```
x = data.frame(
  ea = c("Amy", "Bruce", "Carol", "Dion",
                "Ellen", "Fred", "Gwen", "Hugh",
                "Ingrid", "Kenneth"),
  brand_a = c(74, 80, 68, 74, 86, 75, 78, 69, 76, 77),
  brand_b = c(72, 86, 72, 70, 85, 73, 72, 65, 79, 75)
)
```

a. Add a column called `diff` to the data frame `x` that equals the difference between the Brand A speeds and the Brand B speeds. Also add a column `ranks` that has the ranks of the absolute values of the differences.

b. Generate diagnostic plots to ascertain whether or not the differences are normally distributed. Discuss, with reference to the diagnostic plots, why you think the differences can or cannot be reasonably assumed to follow a normal distribution.

c. Perform the **sign test** to determine if these data provide enough evidence at the 5% significance level to infer that the brands differ with respect to typing speed.

d. Perform the **Wilcoxon signed-rank test** at the 5% level of significance.

e. Use R to calculate the p-value for the paired t-test. Does the paired t-test come to the same decision as the sign test and Wilcoxon signed-rank test?

f. Which test is better, the **sign test**, **Wilcoxon signed-rank test** or **paired t-test**? Why?

2.4 Rats teaching rats

From a group of nine rats available for a study of transfer of learning, five were selected at random and were trained to imitate leader rats in a maze. They were then placed together with four untrained control rats in a situation where imitation of the leaders enable them to avoid receiving an electric shock. The results (the number of trials required to obtain ten correct responses in ten consecutive trials) were as follows:

Trained rats: {78, 64, 75, 45, 82} and Controls: {110, 70, 53, 51}.

Test if there is a difference in the number of trials required between the trained rats and the controls using the Wilcoxon rank-sum test given the following probabilities:

```
data.frame(x = 0:10, p = round(pwilcox(0:10, m = 4, n = 5), 4))
```

```
    x      p
1   0 0.0079
2   1 0.0159
3   2 0.0317
4   3 0.0556
5   4 0.0952
6   5 0.1429
7   6 0.2063
8   7 0.2778
9   8 0.3651
10  9 0.4524
11 10 0.5476
```

3 For practice after the computer lab

Read sections 14.1, 14.2 and 14.3 from Larsen and Marx (2012). You can look at questions 14.2.1, 14.2.2, 14.2.6, 14.2.7, 14.2.8, 14.2.9, 14.2.11, 14.3.1, 14.3.3, 14.3.4, 14.3.5, 14.3.6, 14.3.7, 14.3.8, 14.3.9 and 14.3.10.

3.1 Thinking about the WSR test statistic

When performing a Wilcoxon signed-rank test, in the event that there are no ties, you can calculate the exact p-value by deriving the exact distribution. Let W^+ be the Wilcoxon signed-rank distribution. When $n = 3$, find all the probabilities $P(W^+ \leq w^+)$ for $w^+ = 0, 1, 2, \ldots$ [Hint: How many ways can you get each possible w^+ outcome?] Check your answers with the `psignrank()` function.

References

Champely, Stephane. 2020. *Pwr: Basic Functions for Power Analysis*. https://CRAN.R-project.org/package=pwr.

Larsen, Richard J., and Morris L. Marx. 2012. *An Introduction to Mathematical Statistics and Its Applications*. 5th ed. Boston, MA: Prentice Hall.

Wickham, Hadley, Mara Averick, Jennifer Bryan, Winston Chang, Lucy D'Agostino McGowan, Romain François, Garrett Grolemund, et al. 2019. "Welcome to the tidyverse." *Journal of Open Source Software* 4 (43): 1686. https://doi.org/10.21105/joss.01686.