<u>Model</u> $\underset{\sim}{Y} = X\underset{\sim}{\beta} + \underset{\sim}{\varepsilon}$

$\underset{\sim}{Y}$ and $\underset{\sim}{\varepsilon}$ are $n \times 1$

X is $n \times p$ $\underset{\sim}{\beta}$ is $p \times 1$.

New observation $\underset{\sim}{x}_0$ $(p \times 1)$

$$E(Y \mid \underset{\sim}{x}_0) = \hat{y}_0 = \underset{\sim}{x}_0' \hat{\underset{\sim}{\beta}}$$

$\operatorname{var}(aZ) = a^2 \cdot \operatorname{var}(Z)$

$\operatorname{var}(\hat{\underset{\sim}{\beta}}) = \sigma^2 (X'X)^{-1}$ [Given]

$\operatorname{var}(\underset{\sim}{a}'\underset{\sim}{Z}) = \underline{a}' \operatorname{var}(\underline{Z})\, \underline{a}$ [Rule]

$$\therefore \operatorname{var}(\hat{y}_0) = \operatorname{var}(\underline{x}_0' \hat{\underset{\sim}{\beta}})$$

$$= \underline{x}_0' \operatorname{var}(\hat{\underset{\sim}{\beta}})\, \underline{x}_0$$

$$= \sigma^2 \underset{\sim}{x}_0' (X'X)^{-1} \underset{\sim}{x}_0$$