STAT 3023/3923/4023

Semester 2	Statistical Inference	2021

Computer Lab Week 3

1. (a) Generate 100 realisations of the sample variance of 10 independent $N(0,1)$ random variables and store them in `s2`.

   (b) Plot the histogram of `(10-1)*s2` and overlay it with the density function of the χ^2_9 distribution (use `dchisq`).

   (c) Repeat (a) and (b) with $n = 60$ independent $N(0,1)$ random variables. Overlay the histogram with both the density curve of χ^2_{n-1} and the density curve of $N(n-1, 2(n-1))$ (in two different colours). Comment on the fit.

   (d) For $n = 60$, compute $P((n-1)\cdot S^2 > 68)$ using both the exact distribution (χ^2_{n-1}) and the normal approximation. Compare the results.

2. (a) When two random variables (X, Y) follow a bivariate normal distribution, the covariance matrix Σ is defined as

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix},$$

   where ρ is the correlation, σ_1^2, σ_2^2 are the variances of X and Y respectively. Use `mvrnorm` from the MASS library (use `library(MASS)`) to generate 100 samples from a bivariate normal distribution with $\mu = (\mu_1, \mu_2)$ with $\mu_1 = 2$, $\mu_2 = 3$, and $\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 4 \end{pmatrix}$. Call the first column `x` and the second column `y`.

   (b) Plot the histogram of `x` and overlay it with the corresponding marginal normal density. Repeat for `y`. (Recall the marginal distribution of X is $N(\mu_1, \sigma_1^2)$.)

   (c) Produce a scatter plot of `x` and `y` (use `plot`). Compute the sample correlation coefficient (use `cor`) and compare with the population correlation ρ. (First work out ρ in the Σ given.)

3. (a) Generate 100 realisations of the minimum of 10 independent exponential(1) random variables. Note the rate parameter in `rexp` is defined as the reciprocal the expectation (check the density function in the help file `?rexp`).

   (b) Plot the histogram and overlay it with the density of exponential(1/10) (`rate=10`) distribution. Comment on the fit.