Q1.
b=garch.sim(alpha=c(0.7, 0.4), beta=0.5, n = 500)
d1=b[201:500]
(i) These plots show that the data in d1 look random
(ii) These plots show that the data in d1^2 look serially correlated

Q2, Q3 – Repeat Q1

Q4.
(i)
garch(d1, order=c(0,1))
garch(d1, order=c(0,2))
garch(d1, order=c(1,1))
garch(d1, order=c(2,1))

GARCH(1,1) looks better than the other fits. Your answer may be different as I did not use a seed your result may be different.

(ii)
arima(d1^2, order = c(1, 0, 0))
arima(d1^2, order = c(2, 0, 0))
arima(d1^2, order = c(1, 0, 1))
arima(d1^2, order = c(2, 0, 2))

ARMA(1,1) looks better than the other fits. Your answer may be different as I did not use a seed your result may be different.

Q5.

(i) d= data-mean(data)
ts.plot(d), acf(d), pacf(d) show that the data look random.

d5=scan()

m1=garchFit(~garch(1,0),data=d5)

```
Final Estimate of the Negative LLH:
 LLH:  332.4596    norm LLH:  1.662298
       mu      omega     alpha1
10.0641054  0.8160422  0.6847046

R-optimhess Difference Approximated Hessian Matrix:
                mu      omega      alpha1
mu     -211.5801562  -1.003108   0.3808157
omega    -1.0031081 -65.301454 -29.7203763
alpha1    0.3808157 -29.720376 -49.7063042
```

```
attr(,"time")
Time difference of 0.002166033 secs

--- END OF TRACE ---


Time to Estimate Parameters:
 Time difference of 0.01169991 secs

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 0), data = d5)

Mean and Variance Equation:
 data ~ garch(1, 0)
<environment: 0x7fa8ee20ad30>
 [data = d5]


Summary(m1)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 0), data = d5)

Mean and Variance Equation:
 data ~ garch(1, 0)
<environment: 0x7fa8eeeabd68>
 [data = d5]

Conditional Distribution:
 norm

Coefficient(s):
      mu     omega    alpha1
10.06411   0.81604   0.68470

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu      10.06411     0.06875  146.378  < 2e-16 ***
```

```
omega   0.81604    0.14506    5.626 1.85e-08 ***
alpha1  0.68470    0.16626    4.118 3.82e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 -332.4596    normalized:  -1.662298

Description:
 Mon May 16 22:20:02 2022 by user:


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  0.104691  0.9490009
 Shapiro-Wilk Test  R    W      0.9955609 0.828483
 Ljung-Box Test     R    Q(10)  10.2359   0.4200464
 Ljung-Box Test     R    Q(15)  12.72242  0.6237286
 Ljung-Box Test     R    Q(20)  20.09918  0.4517411
 Ljung-Box Test     R^2  Q(10)  15.95943  0.1007996
 Ljung-Box Test     R^2  Q(15)  18.60537  0.2321764
 Ljung-Box Test     R^2  Q(20)  27.20672  0.1295476
 LM Arch Test       R    TR^2   15.93026  0.1944513

Information Criterion Statistics:
     AIC      BIC      SIC     HQIC
3.354596 3.404071 3.354155 3.374618
```

All P-values are large. Thus this fitted model is satisfactory.

Repeat this for other models.
GARCH(1,1) looks the best for this data.

ALTERNATIVE

```
g1=garch(d, order=c(0,1))

summary(g1)

Call:
garch(x = a, order = c(0, 1))

Model:
GARCH(0,1)
```

```
Residuals:
    Min       1Q   Median       3Q      Max
-2.77776 -0.70136  0.09428  0.65142  2.86480

Coefficient(s):
   Estimate  Std. Error  t value Pr(>|t|)
a0   0.8337     0.1333    6.255 3.97e-10 ***
a1   0.6812     0.1865    3.653 0.000259 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Diagnostic Tests:
     Jarque Bera Test

data:  Residuals
X-squared = 0.090302, df = 2, p-value = 0.9559


     Box-Ljung test

data:  Squared.Residuals
X-squared = 1.8713, df = 1, p-value = 0.1713
```

```
g2=garch(d, order=c(1,1))

summary(g2)

Call:
garch(x = a, order = c(1, 1))

Model:
GARCH(1,1)

Residuals:
    Min       1Q   Median       3Q      Max
-2.67015 -0.59961  0.08337  0.56258  2.07003

Coefficient(s):
   Estimate  Std. Error  t value Pr(>|t|)
a0 1.779e+00   7.155e-01    2.486   0.0129 *
a1 2.111e-01   9.765e-02    2.162   0.0306 *
b1 7.532e-15   2.754e-01    0.000   1.0000
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Diagnostic Tests:
```

```
	Jarque Bera Test

data:  Residuals
X-squared = 2.006, df = 2, p-value = 0.3668


	Box-Ljung test

data:  Squared.Residuals
X-squared = 7.9246, df = 1, p-value = 0.004877
```

****

```
g3=garch(d, order=c(1,2))

summary(g3)

Call:
garch(x = a, order = c(1, 2))

Model:
GARCH(1,2)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.76431 -0.58835  0.09166  0.55101  1.94566 

Coefficient(s):
   Estimate  Std. Error  t value Pr(>|t|)
a0 1.683e+00   1.270e+00    1.326    0.185
a1 1.594e-01   1.038e-01    1.537    0.124
a2 1.165e-01   1.958e-01    0.595    0.552
b1 2.275e-15   6.668e-01    0.000    1.000

Diagnostic Tests:
	Jarque Bera Test

data:  Residuals
X-squared = 2.735, df = 2, p-value = 0.2547


	Box-Ljung test

data:  Squared.Residuals
X-squared = 9.8921, df = 1, p-value = 0.00166
```

****

```
g4=garch(d, order=c(2,1))

summary(g4)

Call:
garch(x = a, order = c(2, 1))

Model:
GARCH(2,1)

Residuals:
    Min       1Q   Median       3Q      Max
-2.69889 -0.61240  0.09099  0.57946  2.11699

Coefficient(s):
   Estimate  Std. Error  t value Pr(>|t|)
a0 1.680e+00   7.701e-01    2.182   0.0291 *
a1 2.114e-01   9.508e-02    2.223   0.0262 *
b1 6.535e-03   2.898e-01    0.023   0.9820
b2 3.034e-15   3.380e-01    0.000   1.0000
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Diagnostic Tests:
     Jarque Bera Test

data:  Residuals
X-squared = 2.0782, df = 2, p-value = 0.3538


     Box-Ljung test

data:  Squared.Residuals
X-squared = 7.3499, df = 1, p-value = 0.006707

************

g5=garch(d, order=c(2,2))

summary(g5)

Call:
garch(x = a, order = c(2, 2))

Model:
GARCH(2,2)
```

```
Residuals:
    Min       1Q   Median       3Q      Max 
-2.79412 -0.59868  0.09417  0.56012  1.98694 

Coefficient(s):
   Estimate  Std. Error  t value Pr(>|t|)
a0 1.583e+00   1.413e+00    1.120    0.263
a1 1.609e-01   1.045e-01    1.540    0.124
a2 1.176e-01   3.222e-01    0.365    0.715
b1 2.739e-03   1.650e+00    0.002    0.999
b2 9.154e-15   8.984e-01    0.000    1.000

Diagnostic Tests:
      Jarque Bera Test

data:  Residuals 
X-squared = 2.561, df = 2, p-value = 0.2779


      Box-Ljung test

data:  Squared.Residuals 
X-squared = 9.2966, df = 1, p-value = 0.002296
```

Comment: ARCH(1) looks a better fit than the others.

****

(iii)

```
f1=arima(d^2,order=c(1,0,0))
> f1

Call:
arima(x = d^2, order = c(1, 0, 0))

Coefficients:
        ar1  intercept
     0.3562     2.0000
s.e.  0.0659     0.3522

sigma^2 estimated as 10.34:  log likelihood = -517.49,  aic = 1038.97

> f2=arima(d^2,order=c(1,0,1))
> f2
```

```
Call:
arima(x = d^2, order = c(1, 0, 1))

Coefficients:
         ar1      ma1  intercept
      0.8372  -0.5861     1.9755
s.e.  0.0767   0.1146     0.5513

sigma^2 estimated as 9.749:  log likelihood = -511.64,  aic = 1029.28

> f3=arima(d^2,order=c(2,0,1))
> f3

Call:
arima(x = d^2, order = c(2, 0, 1))

Coefficients:
         ar1      ar2      ma1  intercept
      0.8441  -0.0043  -0.5918     1.9734
s.e.  0.2024   0.1180   0.1897     0.5527

sigma^2 estimated as 9.749:  log likelihood = -511.64,  aic = 1031.28

> f4=arima(d^2,order=c(2,0,2))
> f4

Call:
arima(x = d^2, order = c(2, 0, 2))

Coefficients:
         ar1      ar2      ma1     ma2  intercept
      1.2397  -0.3385  -0.9899  0.2402     1.9750
s.e.  7.3962   6.1931   7.3937  4.3313     0.5494

sigma^2 estimated as 9.749:  log likelihood = -511.64,  aic = 1033.28

> f5=arima(d^2,order=c(1,0,2))
> f5

Call:
arima(x = a^2, order = c(1, 0, 2))

Coefficients:
         ar1      ma1      ma2  intercept
      0.8391  -0.5866  -0.0033     1.9754
s.e.  0.0912   0.1145   0.0886     0.5526
```

sigma^2 estimated as 9.749: log likelihood = -511.64, aic = 1031.28

Comment: ARMA(1,1) looks like a better fit than the others.

NOTE: The conclusion is slightly different.