Lab Quiz

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R version

The R version used is `4.1.01`

Question 1

```
data = c(12.1, 13.2, 13.8, 13.5, 14.3, 15.2, 12.4, 14.5, 12.4, 11.7, 13.4, 12.8,
    11.6, 13.2, 11.1, 10.9)
m = 4
weights = c(1/(2 * m), rep(1/m, m - 1), 1/(2 * m))
filter(data, weights)
```

```
## Time Series:
## Start = 1
## End = 16
## Frequency = 1
##  [1]      NA      NA 13.4250 13.9500 14.0250 13.9750 13.8625 13.1875 12.8750
## [10] 12.7875 12.4750 12.5625 12.4625 11.9375      NA      NA
```

The first two and last two moving average values are: 13.4250, 13.9500, 12.4625, 11.9375

Question 2

We replace `data` with `diff(data)`

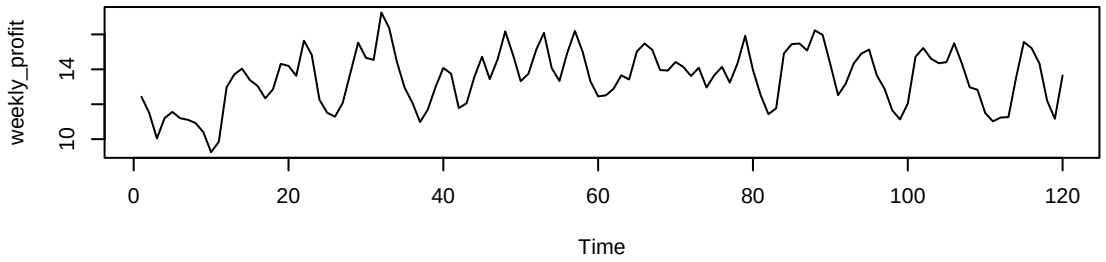
```
data = c(12.1, 13.2, 13.8, 13.5, 14.3, 15.2, 12.4, 14.5, 12.4, 11.7, 13.4, 12.8,
    11.6, 13.2, 11.1, 10.9)
m = 4
weights = c(1/(2 * m), rep(1/m, m - 1), 1/(2 * m))
filter(diff(data), weights)
```

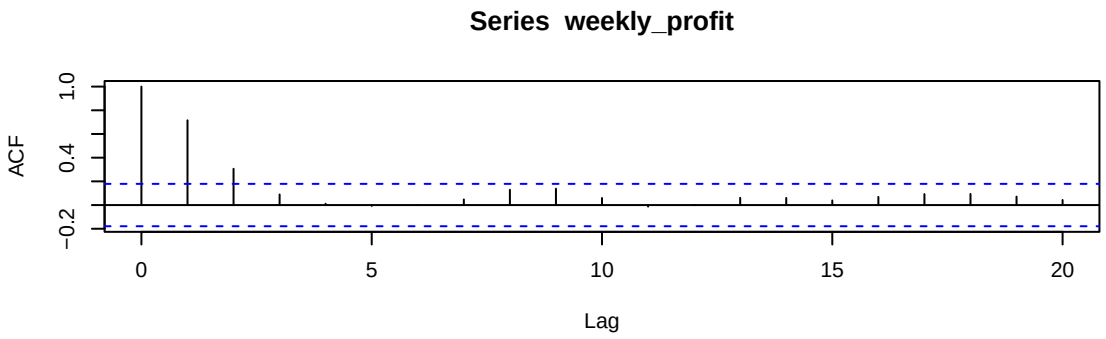
```
## Time Series:
## Start = 1
## End = 15
## Frequency = 1
##  [1]      NA      NA  0.5250  0.0750 -0.0500 -0.1125 -0.6750 -0.3125 -0.0875
## [10] -0.3125  0.0875 -0.1000 -0.5250      NA      NA
```

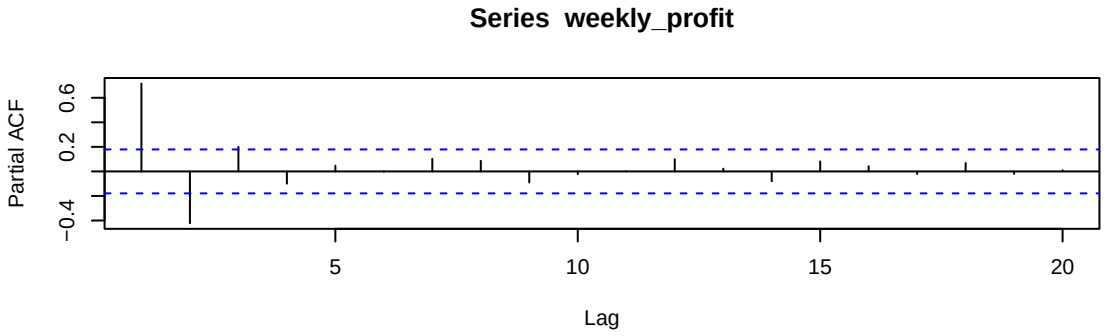
The first two and last two de-trended values are: 0.5250, 0.0750, -0.1, -0.525

```
weekly_profit = c(12.43, 11.51, 10.04, 11.21, 11.56, 11.2, 11.11, 10.92, 10.4, 9.25,
    9.85, 12.97, 13.71, 14.04, 13.39, 13.05, 12.34, 12.86, 14.31, 14.2, 13.63, 15.64,
    14.83, 12.26, 11.51, 11.29, 12.07, 13.8, 15.53, 14.66, 14.54, 17.25, 16.37, 14.43,
```

```
    12.95, 12.09, 10.98, 11.7, 12.99, 14.08, 13.75, 11.78, 12.05, 13.55, 14.72, 13.44,
    14.56, 16.17, 14.84, 13.32, 13.75, 15.12, 16.09, 14.07, 13.33, 14.93, 16.2, 15,
    13.32, 12.45, 12.51, 12.89, 13.66, 13.42, 15.02, 15.48, 15.11, 13.96, 13.93,
    14.42, 14.14, 13.62, 14.09, 12.96, 13.66, 14.14, 13.24, 14.33, 15.92, 13.96,
    12.52, 11.43, 11.77, 14.91, 15.44, 15.48, 15.08, 16.23, 15.97, 14.28, 12.52,
    13.19, 14.32, 14.9, 15.13, 13.65, 12.88, 11.65, 11.13, 12.04, 14.72, 15.22, 14.61,
    14.35, 14.41, 15.49, 14.32, 12.97, 12.83, 11.5, 11.02, 11.24, 11.26, 13.54, 15.57,
    15.21, 14.34, 12.21, 11.17, 13.65)
weekly_profit_mean = mean(weekly_profit)
weekly_profit_var = var(weekly_profit)
par(mfrow = c(3, 1))
ts.plot(weekly_profit)
rhos = acf(weekly_profit)
pis = pacf(weekly_profit)
```







We see that the time series seems stationary with constant mean 13.5 and variance 2.5955832. This is supported by a quickly decayiing ACF plot.

Question 4

We stored the ACF and PACF values of the time series from before:

```
rhos
```

```
##
## Autocorrelations of series 'weekly_profit', by lag
##
##      0      1      2      3      4      5      6      7      8      9     10
##  1.000  0.716  0.306  0.089  0.011 -0.007  0.000  0.048  0.129  0.138  0.060
##     11     12     13     14     15     16     17     18     19     20
## -0.013  0.000  0.061  0.060  0.039  0.068  0.093  0.094  0.071  0.043
```

```
pis
```

```
##
## Partial autocorrelations of series 'weekly_profit', by lag
##
##      1      2      3      4      5      6      7      8      9     10     11
##  0.716 -0.422  0.200 -0.100  0.047 -0.003  0.102  0.086 -0.091 -0.023  0.001
##     12     13     14     15     16     17     18     19     20
##  0.099  0.022 -0.080  0.081  0.042 -0.020  0.068 -0.020  0.009
```

- The first 5 ACF values at lags 1 to 5 are: 0.716, 0.306, 0.089, 0.011, -0.007
- The first 5 PACF values at lags 1 to 5 are: 0.716, -0.422, 0.200, -0.100, 0.047

Question 5

We are given the model

$$X_t = 0.7X_{t-1} + 0.8X_{t-3} + Z_t - 0.7Z_{t-1} + 0.3Z_{t-2} + 0.5Z_{t-3} \quad Z_t \sim NID(0, 1.7^2)$$

Which can be written as:

$$(1 - 0.7B - 0.8B^3)X_t = (1 - 0.7B + 0.3B^2 + 0.5B^3)Z_t$$

Hence:

- AR(3) polynomial is $a(\omega) = 1 - 0.7\omega - 0.8\omega^3$
- MA(3) polynomial is $b(\omega) = 1 - 0.7\omega + 0.3\omega^2 + 0.5\omega^3$

We find the modulus of the roots using `R`

```
# For the AR(3) polynomial
ar_roots = polyroot(c(1, -0.7, 0, 0.8))
abs(ar_roots)
```

```
## [1] 0.9645315 1.3436223 0.9645315
```

```
ma_roots = polyroot(c(1, -0.7, 0.3, 0.5))
abs(ma_roots)
```

```
## [1] 1.027231 1.895369 1.027231
```

- The ARMA(3,3) model **is not** stationary because not all of it's roots for the AR(3) polynomial have a modulus greater than 1
- The ARMA(3,3) model **is** invertible because all of it's roots for the MA(3) polynomial are greater than 1

Question 6

We simulate 600 values

```
set.seed(142)
de = 15 + arima.sim(model = list(ma = c(0.5, 0.9)), n = 600)[201:600]
fit = arima(de, order = c(0, 0, 2))
fit
```

```
##
## Call:
## arima(x = de, order = c(0, 0, 2))
##
## Coefficients:
##          ma1     ma2  intercept
##       0.5434  0.8761    15.1299
## s.e.  0.0235  0.0269     0.1161
##
## sigma^2 estimated as 0.9259:  log likelihood = -553.69,  aic = 1115.37
```

(i) Estimates are:

- $\hat{\mu} = 15.1299$
- $\hat{\beta}_1 = 0.5430$
- $\hat{\beta}_2 = 0.8761$
- $\hat{\sigma^2} = 0.9259$

(ii) The AIC is: 1115.37

Question 7

```
fit2 = arima(de, order = c(1, 0, 2))
fit2
```

```
##
## Call:
## arima(x = de, order = c(1, 0, 2))
##
## Coefficients:
##          ar1     ma1     ma2  intercept
##       0.0283  0.5372  0.8740    15.1296
## s.e.  0.0569  0.0270  0.0276     0.1191
##
## sigma^2 estimated as 0.9254:  log likelihood = -553.56,  aic = 1117.13
```

(i) Estimates are:

- $\hat{\mu} = 15.1296$
- $\hat{\beta}_1 = 0.5372$
- $\hat{\beta}_2 = 0.8740$
- $\hat{\sigma^2} = 0.9254$
- $\hat{\alpha_1} = 0.0283$

(ii) The AIC is: 1117.13

(iii) Since the AIC is lower for model in Q6 we prefer that one.

Question 8

```
fit3 = arima(de, order = c(2, 0, 1))
fit3
```

```
##
## Call:
## arima(x = de, order = c(2, 0, 1))
##
## Coefficients:
##           ar1     ar2     ma1  intercept
##       -0.0409  0.5148  0.4788    15.1201
## s.e.   0.0801  0.0512  0.0823     0.1548
##
## sigma^2 estimated as 1.225:  log likelihood = -608.49,  aic = 1226.99
```

- ar1 se = 0.0801 and t ratio = -0.5106
- ar2 se = 0.0512 and t ratio = 10.05
- ma1 se = 0.0823 and t ratio = 5.83
- intercept se = 0.1548 and t ratio = 100.07

Question 9

```
Box.test(resid(fit), lag = 12, type = "Ljung")
```

```
##
##  Box-Ljung test
##
## data:  resid(fit)
## X-squared = 4.5378, df = 12, p-value = 0.9717
```

X-squared = 4.5378 and p-value = 0.9717

Question 10

Didn't finish on time