THE UNIVERSITY OF SYDNEY

Mathematical Statistics: STAT3925/STAT4025 - Semester 1 - 2022

Time Series Analysis : Problem Set - Week 8 (Tutorial and Computer Problems)

Attempt these questions before your class and discuss any issues with your tutor
Go to your assigned tutorial class/Lab and record your attendance

1. Suppose that $\{X_t\}$ is a stationary time series satisfying $X_t = \sum_{j=0}^{\infty} \psi_j Z_{t-j}$, where $\sum_{j=0}^{\infty} \psi_j^2 < \infty$ and $\{Z_t\} \sim WN(0,\ \sigma^2)$.

   (a) (i) Write down the ℓ-step-ahead forecast function, $\hat{X}_{t+\ell}$ for $X_{t+\ell}$ for all $\ell \geq 1$.
   (ii) Find the variance of the ℓ-step-ahead forecast error $Var(\epsilon_{t+\ell})$.

   (b) (i) Find the sdf $f_Z(\omega)$ of $\{Z_t\}$.
   (ii) Write down the sdf $f_X(\omega)$ of $\{X_t\}$.

2. Suppose the AR(2) model $X_t = * + X_{t-1} - 0.5X_{t-2} + Z_t$, $\{Z_t\} \sim NID(0, 4)$ has been fitted to a time series of length 84, where the value in * cannot be read. The last four observations of the series are 40.5, 31, 36, 28.5.

   (i) Show that this AR(2) is stationary.
   (ii) Given that $\mu = EX_t) = 36$, find this value in *.
   (iii) Forecast the value of this series at $t = 85$ and calculate a 90% FI for X_{85}.
   (iv) Find $\hat{x}_{86}$ (the 2-step ahead forecast from the origin $t = 84$) and its error variance.

3. (i) An insurance company uses the $ARIMA(1, 0, 3)$ model,

$$(I - 0.5B)X_t = 50 + Z_t - 0.20Z_{t-1} - 0.10Z_{t-3},$$

   where $\{Z_t\} \sim NID(0, 1)$, to forecast its claims by calendar quarter (ignoring any seasonal effect).

   (a) Write down the ℓ-step-ahead forecast function, $\hat{X}_{t+\ell}$ for $\ell = 1, 2, 3$, and show that for $\ell \geq 4$, $\hat{X}_{t+\ell}$ satisfies the recursion

$$\hat{X}_{t+\ell} = 50 + 0.5\hat{X}_{t+\ell-1}.$$

   (b) What happens to $\hat{X}_{t+\ell}$ as $\ell \to \infty$? What is the consequence of this result in long term forecasting?

   (ii) Suppose that the following data are available for quarterly claims for 2001 and 2002 in (i):

Year	Quarter	Actual	Forecast
2001	3	90	94
	4	98	100
2002	1	109	108

   Determine the claims forecast by the model for the second quarter of 2002 and provide a 95% forecast interval for the corresponding true value.

4. Suppose that $\{X_t\}$ is an MA(1) process given by $X_t = Z_t + 0.7Z_{t-1}$, where $\{Z_t\} \sim WN(0, 1.5^2)$.

   (i) Find $f_X(\omega)$ and $f_X^*(\omega)$.
   (ii) Sketch these functions $f_X(\omega)$ and $f_X^*(\omega)$ for $\omega \in (-\pi, \pi)$.

PTO for the Computer Exercise

Computer Exercise: Submit your answers to Q3 by 11.59 on Monday 26 April

1. Daily closing price of a certain stock (in Australian dollars) is given by

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218.45 218.98 219.48 219.96 220.42 220.90 221.40 221.94 222.54 223.21
223.93 224.67 225.44 226.19 226.95 227.73 228.53 229.36 230.21 231.08
231.96 232.82 233.66 234.52 235.38 236.26 237.16 238.06 238.96 239.86
240.74 241.59 242.42 243.25 244.06 244.86 245.63 246.40 247.16 247.96
248.78 249.65 250.53 251.42 252.32 253.24 254.18 255.12 256.06 256.97
257.86 258.72 259.55 260.35 261.13 261.88 262.60 263.29 263.95 264.57
265.17 265.74 266.29 266.85 267.41 267.98 268.57 269.20 269.87 270.59
271.37 272.21 273.09 273.98 274.88 275.78 276.69 277.61 278.54 279.51
280.52 281.58 282.70 283.86 285.04 286.23 287.41 288.58 289.74 290.89
292.06 293.23 294.40 295.59 296.79 297.98 299.15 300.26 301.33 302.35
303.33 304.29 305.20 306.09 306.95 307.82 308.69 309.57 310.46 311.38
312.32 313.29 314.27 315.27 316.29 317.35 318.44 319.54 320.63 321.71
322.76 323.80 324.82 325.82 326.82 327.82 328.83 329.83 330.84 331.88
332.95 334.08 335.24 336.41 337.57 338.73 339.85 340.94 342.03 343.11
344.19 345.27 346.34 347.39 348.46 349.53 350.59 351.67 352.75 353.83
354.91 355.99 357.07 358.12 359.15 360.13 361.10 362.04 362.97 363.88
364.78 365.69 366.63 367.57 368.54 369.51 370.53 371.59 372.69 373.84
375.03 376.23 377.45 378.68 379.92 381.17 382.44 383.70 384.97 386.22
387.46 388.69 389.93 391.18 392.46 393.77 395.11 396.49 397.91 399.36
400.84 402.32 403.80 405.27 406.72 408.15 409.58 411.02 412.47 413.94
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   (i) Store the above data in x.

   (ii) Obtain the tsplot and the acf for the data in x and comment on this time series.

   (iii) Since the series looks non-stationary, Dr Hora Jonka suggested to use a suitable differencing filter of the form $y_t = (1-B)^d x_t$ to make the series stationary. Investigate the series for $d = 1, = 2, \ldots$.

   (iv) What is your best value of d to form a stationary series?

   (v) Plot the tsplot, the acf and pacf of the transformed stationary series in (iv).

2. Consider the MA(1) given by $X_t = Z_t + 0.7Z_{t-1}$, $\{Z_t\} \sim NID(0, 1.5^2)$. Draw the theoretical spectrums f and f^* by setting:

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o=c(-32:32)/10   # to create a vector for omega OR o= seq(-pi,pi,0.1)
s2=1.5^2                # variance of the noise
f=(..................)/2*pi #take from Q4
plot(o,f, type="l", main="sdf of MA(1) with beta=0.7")
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3. Dr Jonka suggest to investigate the AR(1), AR(2), AR(3), MA(3), ARMA(1,2) models for your stationary series in Q1.

   (i) Fit all of the suggested models and report all parameter estimates and s.e. s for each model.

   (ii) Select the best possible model based on the AIC value. Write down this model in full.

   (iii) Perform the tsdiag for all models in (i). Check whether your conclusion in (ii) remains unchanged about the best choice of your proposed ARMA model.

   (iv) Find the next five forecast values and their se's from the best possible model based on aic.

   (v) Plot the next five forecast values and their se's in together on a plot.

   (vi) Plot the next five forecast values together with their 95% forecast bounds on the same plot.