THE UNIVERSITY OF SYDNEY

Mathematical Statistics, STAT3925/4025

Semester 1	Time Series Analysis: Computer Quiz 2 - Monday 23 May	2022

Time allowed: 60 minutes. There are 10 questions with equal weights.
This Computer Lab Quiz is open book and worth 6% towards your final mark.

Use/Execute appropriate R command/s to each question and find your answer.
Prepare a single pdf file with all outputs/answers/comments.
Submit your answer sheet/s through turnitin by due time.

R version used:...

1. **Use the data below to answer the questions Q1 and Q2.** A time series consists of weekly profit (in '000 of dollars) collected in the past 120 weeks of a small company. The manager wants to find the next five forecast values from this series.

   12.43 11.51 10.04 11.21 11.56 11.20 11.11 10.92 10.40 9.25 9.85 12.97 13.71 14.04 13.39 13.05 12.34 12.86
   14.31 14.20 13.63 15.64 14.83 12.26 11.51 11.29 12.07 13.80 15.53 14.66 14.54 17.25 16.37 14.43 12.95 12.09
   10.98 11.70 12.99 14.08 13.75 11.78 12.05 13.55 14.72 13.44 14.56 16.17 14.84 13.32 13.75 15.12 16.09 14.07
   13.33 14.93 16.20 15.00 13.32 12.45 12.51 12.89 13.66 13.42 15.02 15.48 15.11 13.96 13.93 14.42 14.14 13.62
   14.09 12.96 13.66 14.14 13.24 14.33 15.92 13.96 12.52 11.43 11.77 14.91 15.44 15.48 15.08 16.23 15.97 14.28
   12.52 13.19 14.32 14.90 15.13 13.65 12.88 11.65 11.13 12.04 14.72 15.22 14.61 14.35 14.41 15.49 14.32 12.97
   12.83 11.50 11.02 11.24 11.26 13.54 15.57 15.21 14.34 12.21 11.17 13.65

   (i) Dr Pin Bell believes that an MA(2) model would be the best for this data. Fit an MA(2) model to this data and write down the corresponding parameter estimates with their corresponding standard errors (s.e.).

   (ii) Find the next five predicted values for this series.

2. Refer to the fitted MA(2) and the next five predicted values in Q1.

   (i) Find 95% prediction intervals for the five predicted values in Q1(ii).

   (ii) Plot the next five predicted values together with their 95% bounds on the same plot.

3. Consider the MA(3) process given by

   $$X_t = Z_t - 0.6Z_{t-1} + 0.6Z_{t-2} + 0.4Z_{t-3}, \text{ where}\{Z_t\} \sim NID(0,1).$$

   (i) Draw the theoretical spectrum of this process.

   (ii) Inspect the plot in (i) and write down the approximate frequency where the spectrum is minimum. Any approximate interval for the frequency containing the minimum point is acceptable.

4. Use set.seed(341) to simulate 600 values from the MA(3) model in Q3.

   (i) By discarding the first 400 values, store the remaining values in $d4$ and draw the sample periodogram.

   (ii) Comment on the shape of the sample periodogram in (i) referring to the minimum of the theoretical spectrum in Q3.

5. Martin Robandan wants to see the theoretical spectrum of the ARMA(1,2) process given by

   $$X_t = 10 + 0.4X_{t-1} + Z_t - 0.5Z_{t-1} + 0.7Z_{t-2}, \quad \{Z_t\} \sim NID(0,1).$$

   (i) Draw the theoretical spectrum of this process.

   (ii) Write down the approximate frequencies where the spectrum has its maximum value, the minimum value and all points of inflection. Any approximate intervals for frequencies containing these points are accepted.

PTO for Q6-Q10

6. Let $(1-0.7B)(1-B)^{0.3}X_t = (1+0.5B)Z_t$, where $\{Z_t\} \sim WN(0,1)$ be an ARFIMA(1,0.3,1). It can be shown that its theoretical spectrum (pdf) $f_X(\omega) = \frac{1}{2\pi}\frac{1.25+\cos(\omega)}{1.49-1.4\cos(\omega)}\frac{1}{(2\sin(\omega/2))^{0.6}}$, $-\pi < \omega < \pi, \omega \neq 0$. Draw this pdf $f_X(\omega)$ for $0 < \omega < \pi$ and comment on its behaviour near the origin. *Hint:* Set up ω values using:

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o=seq(0.0001,pi, by=0.001)
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7. The chief statistician at the Bank of Hora Salli, Mr Ali Bada wants to look at the first five acf and pacf values generated by the fractional ARFIMA(2, 0.4,2) model given by

$$(1-0.3B+0.7B^2)(1-B)^{0.4} = (1+0.4B-0.6B^2)Z_t, \text{ where } \{Z_t\} \sim NID(0,1).$$

His assistant Peter Nidagannam generated 1200 values with set.seed(1205) and discarded the first 200 values.

(i) Write down clearly, the first five sample acf and pacf values that Peter will get.

(ii) Report the first five values of the sample periodogram.

8. Use **set.seed(8134)** to simulate 1500 values from the ARFIMA(1,0.4,1) model given by

$$(1-0.4B)(1-B)^{0.4} = (1-0.6B)Z_t, \text{ where } \{Z_t\} \sim NID(0,1).$$

(i) Discard the first 300 values from the series and store the remaining time series data in $d8$.

(ii) Now fit an ARFIMA(1,d,1) for the data $d8$ in (i) and report the corresponding estimates and their 95% confidence intervals for the parameters α, β, d in $(1-\alpha B)(1-B)^d = (1-\beta B)Z_t$.

9. Banku Hitinova, a PhD student of Professor Ukuno Kanava simulates 1800 values (**with set.seed(1456)**) from the Gaussian GARCH (1,1) process given by $X_t = \sigma_t \epsilon_t$; $\sigma_t^2 = 0.58 + 0.3X_{t-1}^2 + 0.4\sigma_{t-1}^2$, where $\{\epsilon_t\}$ follows iid $NID(0,1)$ distribution. After discarding the first 400 values John stores the data in $d9$.

(i) Write down the first three acf and pacf values of this sample.

(ii) Comment on the data in $d9$ and $(d9)^2$.

10. Refer the data $d9$ in Q9.

(i) Fit a GARCH(1,1) model to the data in $d9$. Report all parameter estimates and the p-values for Jarque Bera and Box-Ljung tests. Comment on the fitted GARCH(1,1) model based on these p-values.

(ii) Fit ARMA(1,1) and ARMA(0,2) models for the squared data in $(d9)^2$. Report all parameter estimates and the aic values. What is the best fit for the data in $(d9)^2$? Is this consistent with the theory?

THIS IS THE LAST QUESTION. END OF THE TEST