Homework 8.

Amath 301
Beginning Scientific Computing

Due: 5/23/23 at 11:59pm to Gradescope

Directions:

For the **written exercises** of the homework, please do the following:

- Write your name, student number, and section at the top of your written exercises. A half point will be deducted if these are not included.
- Complete all written exercises to the best of your ability and as neatly as possible.
- For those who are writing their answers by hand, please scan your work and save it as a pdf file.
- You are encouraged to type your written homework using LaTeX. Doing so will earn you a half bonus point. Check out my LaTeX beginner document and overleaf.com if you are new to LaTeX.

For the **coding exercises** of the homework, please do the following:

- Create a MATLAB script titled `hw8.m` and divide your script into sections according to the various coding exercises.
- Make sure your final answers in each coding exercise are assigned to the correct variable names.
- Publish your script as a pdf file. You can do this by typing `publish('hw8.m','pdf')` in the Command Window.
- Attach your published code to the end of your written exercises. Submit your entire homework assignment (written + coding exercises) to Gradescope.

Pro-Tips:

- You can check many of your answers to the written exercises of this homework by having MATLAB, Wolfram Alpha, or Chat GPT do the calculations for you.
- Teamwork makes the dream work, but please make sure you write up your solutions.
- Don't wait until the last minute. ☺

Written Exercises

(12 points total)

Exercise 1. (Component Skill 8.1)

The hyperbolic cosine is defined as

$$\cosh(x) = \frac{e^x + e^{-x}}{2}.$$

(Pronounced "kawsh of x.") Calculate the fourth-degree Taylor polynomial of $\cosh(x)$ at $x = 0$.

Exercise 2. (Component Skill 8.2)

Consider the integral

$$\mathcal{I} = \int_0^{\frac{\pi}{2}} \sin(x)dx.$$

a. What is the exact value of $\mathcal{I}$? You don't have to show how you obtained this value.

b. Approximate $\mathcal{I}$ with the left-hand rule using $N = 4$ subintervals. Show how you set-up the left-hand rule. Write your answer in decimal form. Round to the nearest fourth decimal place.

c. Does your answer from (b) overestimate or underestimate the exact value of $\mathcal{I}$? Provide a brief justification as to why this is (besides that the calculations suggest that this is the case).

d. What is the maximum possible error of your answer for (b)? Show how you obtained this maximum possible error.

   Note: This question is asking about the maximum possible error, not the exact error. Use the global error estimate for the left-hand rule.

e. Approximate $\mathcal{I}$ with the right-hand rule using $N = 4$ subintervals. Show how you set-up the right-hand rule. Write your answer in decimal form. Round to the nearest fourth decimal place.

f. Does your answer from (e) overestimate or underestimate the exact value of $\mathcal{I}$? Provide a brief justification as to why this is (besides that the calculations suggest that this is the case).

g. What is the maximum possible error of your answer for (e)? Show how you obtained this maximum possible error.

   Note: This question is asking about the maximum possible error, not the exact error. Use the global error estimate for the right-hand rule.

h. Approximate $\mathcal{I}$ with the midpoint rule using $N = 4$ subintervals. Show how you set-up the midpoint rule. Write your answer in decimal form. Round to the nearest fourth decimal place.

i. What is the maximum possible error of your answer for (h)? Show how you obtained this maximum possible error.

   Note: This question is asking about the maximum possible error, not the exact error. Use the global error estimate for the midpoint rule.

Exercise 3. (Component Skills 8.3-8.4)

Consider an ellipse of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

The circumference of this ellipse is given by the integral

$$C = \int_0^{2\pi} \sqrt{a^2 \sin^2(\theta) + b^2 \cos^2(\theta)}\, d\theta.$$

This integral can be solved exactly if the ellipse is a circle. Then, a and b are equal to the radius of the circle r, and the circumference $C = 2\pi r$, as you learned in your grade school days. If a and b are not equal to each other, then we can't solve the integral above exactly.

a. Estimate C by the trapezoidal rule with $N = 4$ subintervals. Assume a and b are positive parameters. What you will find is an approximate formula for C as a function of a and b.

b. Estimate C by Simpson's 1/3 rule with $N = 4$ subintervals. Assume a and b are positive parameters. What you will find is an approximate formula for C as a function of a and b.

Coding Exercises

(8 points total)

Exercise 1. (Component Skills 8.2-8.4)

The birth weight of a newborn kitten is normally distributed with a mean of $\mu = 3.5$ ounces and a standard deviation of $\sigma = 0.7$ ounces. To compute the probability that a randomly selected newborn kitten is a heckin' chonk like Sally with weight between 4.5 and 6 ounces, you would need to compute the integral

$$P = \int_{4.5}^{6} \frac{e^{\frac{-(x-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} dx.$$

This integral cannot be evaluated exactly using any of the methods you learned in your calculus classes, so we will evaluate it using numerical integration.

a. Use the MATLAB built-in function `integral` to estimate the "exact" value of P. Assign `A1` to this value.

   Answer:

```
A1 =

   0.076386205819461
```

b. Use the midpoint rule to approximate P with $N = 2, 4, 8, 16$, and 32 subintervals. Assign the variable `A2` to a 5×1 column vector containing your approximations of P for each step size.

   Answer:

```
A2 =

   0.066354199839345
   0.073936935528133
   0.075777953024860
   0.076234400368788
   0.076348270617094
```

c. Assign the variable `A3` to a 5×1 column vector containing the errors of your approximations from (b). Define the error to be the difference in absolute value of the approximations obtained in (b) minus the exact value of the integral obtained in (a).

   Answer:

```
A3 =

   0.010032005980116
   0.002449270291329
   0.000608252794601
   0.000151805450673
   0.000037935202367
```

d. Use the trapesoidal rule to approximate P with $N = 2, 4, 8, 16$, and 32 subintervals. Assign the variable `A4` to a 5×1 column vector containing your approximations of P for each step size.

Note: Please use the MATLAB built-in function `trapz`.

Answer:

```
A4 =

   0.096178059707752
   0.081266129773549
   0.077601532650841
   0.076689742837850
   0.076462071603319
```

e. Assign the variable `A5` to a 5×1 column vector containing the errors of your approximations from (d). Define the error to be the difference in absolute value of the approximations obtained in (d) minus the exact value of the integral obtained in (a).

Answer:

```
A5 =

   0.019791853888291
   0.004879923954088
   0.001215326831379
   0.000303537018389
   0.000075865783858
```

f. Use Simpson's 1/3 rule to approximate P with $N = 2, 4, 8, 16$, and 32 subintervals. Assign the variable `A6` to a 5×1 column vector containing your approximations of P for each step size.

Answer:

```
A6 =

   0.076638921110098
   0.076295486462148
   0.076380000276605
   0.076385812900187
   0.076386181191809
```

g. Assign the variable `A7` to a 5×1 column vector containing the errors of your approximations from (f). Define the error to be the difference in absolute value of the approximations obtained in (f) minus the exact value of the integral obtained in (a).

Answer:

```
A7 =

   1.0e-03 *

   0.252715290636932
   0.090719357313584
   0.006205542856574
   0.000392919274464
   0.000024627652265
```

h. Create the a plot in MATLAB that meets the following specifications:

- Plot `A3` as a function of N. Instead of the usual plot command, use **`semilogy`** to create a plot with a logarithmic scale on the y-axis. Adjust the line width to 2. Use blue for the line color.
- Superimpose a plot of `A5` as a function of N. Instead of the usual plot command, use **`semilogy`** to create a plot with a logarithmic scale on the y-axis. Adjust the line width to 2. Use red for the line color.
- Superimpose a plot of `A7` as a function of N. Instead of the usual plot command, use **`semilogy`** to create a plot with a logarithmic scale on the y-axis. Adjust the line width to 2. Use black for the line color.
- Add a legend in the northeast corner of the graph. Use the label "Midpoint" for the first plot, the label "Trapezoid" for the second plot, and the label "Simpson" for the third plot.
- Set the domain of the plot to be $0 \leq N \leq 33$. Set the range of the plot to be $10^{-8} \leq y \leq 10^{-1}$. Set the font size to 10.
- Add the label "N" for the x-axis and the label "$\log_{10}$(error)" for the y-axis. Set the font sizes of these labels to 20. Add the title "Error of Numerical Integration Methods." Set the font size to 20.
- Turn on the grid and box.

Answer: