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EE 242, Assignment 4

1.

Computing the convolution $x(t) * h(t)$:

1. $x(t)$: $$\mathbf{x}(t) = \begin{cases}

   ```
   1, & -1 \le t < 0 \\
   1 - t, & 0 \le t \le 1 \\
   0, & \text{otherwise}
   ```

   \end{cases}$$

2. $h(t)$: $$\mathbf{h}(t) = \begin{cases}

   ```
   1, & -2 \le t \le 2 \\
   0, & \text{otherwise}
   ```

   \end{cases}$$

Convolution formula:

$$(\mathbf{x} * \mathbf{h})(\tau) = \int_{-\infty}^{\infty} \mathbf{x}(t)\mathbf{h}(\tau - t)dt$$

Considering the different cases for the convolution:

- For $\tau < -3$, the functions do not overlap, so the convolution is 0.
- For $-3 \le \tau < -2$:

$$(\mathbf{x} * \mathbf{h})(\tau) = \int_{\tau+2}^{0} \mathbf{x}(t)\mathbf{h}(\tau - t)dt$$

$$(\mathbf{x} * \mathbf{h})(\tau) = \int_{\tau+2}^{0} dt$$

$$(\mathbf{x} * \mathbf{h})(\tau) = \tau + 2$$

- For $-2 \le \tau < -1$:

$$(\mathbf{x} * \mathbf{h})(\tau) = \int_{\tau+2}^{1} \mathbf{x}(t)\mathbf{h}(\tau - t)dt$$

$$(\mathbf{x} * \mathbf{h})(\tau) = \int_{\tau+2}^{1} dt$$

$$(\mathbf{x} * \mathbf{h})(\tau) = 3 + \tau$$

- For $-1 \leq \tau < 0$:

$$(\mathbf{x} * \mathbf{h})(\tau) = \int_{0}^{\tau+2} \mathbf{x}(t)\mathbf{h}(\tau - t)dt$$

$$(\mathbf{x} * \mathbf{h})(\tau) = \int_{0}^{\tau+2} dt$$

$$(\mathbf{x} * \mathbf{h})(\tau) = \tau + 2$$

- For $0 \leq \tau < 1$:

$$(\mathbf{x} * \mathbf{h})(\tau) = \int_{\tau-2}^{1} (1 - t)\mathbf{h}(\tau - t)dt$$

$$(\mathbf{x} * \mathbf{h})(\tau) = \int_{\tau-2}^{1} (1 - t)dt$$

$$(\mathbf{x} * \mathbf{h})(\tau) = \frac{1}{2}(3 - \tau)(1 + \tau)

2.

We can analyze the given impulse responses:

a) $h[n] = (2)^n * u[n-2]u[n+2]$

(i) Causal: Since the impulse response is 0 for n < 2 and non-zero for n ≥ 2, the system is causal.

(ii) Stable: The impulse response has an exponential growth term (2)^n, which makes the system unstable.

b) $h[n] = (n * (0.5)^n) * u[n-1]$

(i) Causal: Since the impulse response is 0 for n < 1 and non-zero for n ≥ 1, the system is causal.

(ii) Stable: The impulse response has an exponential decay term (0.5)^n, which makes the system stable.

c) $h(t) = sin(t+1)u(t-1) + cos(t-1)u(-t+1)$

(i) Causal: Since the impulse response is 0 for t < 1, the system is causal.

(ii) Stable: Both sine and cosine functions are bounded between -1 and 1, so the system is stable.

d) $h(t) = e^{-5|t|}$

(i) Causal: The impulse response is non-zero for all t, so the system is non-causal.

(ii) Stable: The impulse response has an exponential decay term e^{-5|t|}, which makes the system stable.

3.

We can analyze the given impulse responses and determine their stability by converting them to polar form:

a) $h[n] = \left(\frac{1}{2} + j\frac{2}{3}\right)^n u[n]$

In polar form: $h[n] = \rho^n e^{j\theta n} u[n]$

where $\rho = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{2}{3}\right)^2} = \frac{\sqrt{13}}{6}$ and $\theta = \tan^{-1}\left(\frac{2}{3}\right)$

Since $\rho < 1$ and $u[n]$ is causal, the system is **stable**.

b) $h[n] = \left(\frac{1}{2} + j\frac{2}{3}\right)^n u[-n]$

In polar form: $h[n] = \rho^n e^{j\theta n} u[-n]$

where ρ and θ are the same as in part (a).

Since $\rho < 1$ and $u[-n]$ is non-causal, the system is **unstable**.

c) $h(t) = e^{\left(\frac{1}{2} - j\frac{2}{3}\right)t} u(t)$

In polar form: $h(t) = e^{(\frac{1}{2}t)} e^{-j\frac{2}{3}t} u(t)$

Since the exponential term $e^{\frac{1}{2}t}$ has a positive exponent and $u(t)$ is causal, the system is **unstable**.

d) $h(t) = e^{\left(\frac{1}{2} - j\frac{2}{3}\right)t} u(-t)$

In polar form: $h(t) = e^{(\frac{1}{2}t)} e^{-j\frac{2}{3}t} u(-t)$

Since the exponential term $e^{\frac{1}{2}t}$ has a positive exponent and $u(-t)$ is non-causal, the system is **unstable**.