

EE 242 Spring 2023

HW 6

- $t \in \mathbb{R}$ and $n \in \mathbb{Z}$.
- Point for each problem is shown in the right.
- $x(t) \leftrightarrow X(\omega)$ denotes that $X(\omega)$ is the Fourier transform of $x(t)$.
- Lectures: 27-30.

1. Find the Fourier transform of the following signals. [3 X (6+2+2)]

a) $x(t) = \delta(t) + 2 \delta(t + 3) + 2 \delta(t - 3)$

b) $x(t) = 2 \sin(2t) - \cos^2(\pi t)$

c) $x(t) = 2 e^{-3t} u(t) - 4 e^{5t} u(-t)$

Also draw the spectrum $|X(\omega)|$ and $\angle X(\omega)$ for each of the signals.

2. Prove the following properties: [2 X 10]

a) $x(t) e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0)$

b) $(-jt)^n x(t) \leftrightarrow \frac{d^n X(\omega)}{d\omega^n}$

[Hint: Successively differentiate the analysis equation with respect to ω]

3. Using 2(b) property in above show the following where $a > 0$: [7]

$$t e^{-at} u(t) \leftrightarrow \frac{1}{(a + j\omega)^2}$$

4. Suppose the Fourier transform of a signal $x(t)$ is given by [8]

$$X(\omega) = e^{-2|\omega|}$$

Find the energy of the signal between the frequency band $-3 < \omega < 3$. Use Parseval's relation.

5. Suppose the Fourier transform of a signal $x(t)$ is given by [2 X 10]

$$X(\omega) = \frac{j\omega - 1}{(3 - \omega^2) + j4\omega}$$

Using the properties of the Fourier transform, find the Fourier transform expressions of the following signals.

a) $x(-2t + 1)$

b) $x(t) * x(t - 1)$. (* denotes convolution)

6. Compute the convolution of the following pairs of signal $x(t)$ and $h(t)$ in time-domain by using the convolution property of the Fourier transform. (Do not compute the convolution directly in the time-domain) [15]

$$x(t) = e^{-3t} u(t)$$

$$h(t) = e^{5t} u(-t)$$

Note, you already have obtained the Fourier transform of the above signals in Problem 1(c).