Problem 12b

Hexadecimal numbers have sixteen hexadecimal digits:

$$0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F$$

[...] and are denoted by the subscript 16 — e.g., $9A2D_{16}$ *and* $BC54_{16}$.

How many hexadecimal numbers [...]
begin with a digit 4 *through* D,
end with a digit 2 *through* E, *and*
are 6 *digits long?*

Answer:
Use the multiplication rule.

Make $N(F)$ the number of first digit in 4 through D possibilities.

$$N(F) = 1 \cdot 10$$

$$N(F) = 10 \text{ possibilities}$$

Make $N(M)$ the four middle digits of any digit possibilities.

$$N(M) = 4 \cdot 16$$

$$N(M) = 64 \text{ possibilities}$$

Make $N(L)$ the last digit in 2 through E possibilities.

$$N(L) = 1 \cdot 13$$

$$N(L) = 13 \text{ possibilities}$$

Make $N(H)$ the number of hexadecimal numbers possible.

$$N(H) = N(F) \cdot N(M) \cdot N(L)$$

$$N(H) = 10 \cdot 64 \cdot 13 =$$

$$8,320 \text{ hexadecimal numbers possible}$$

Problem 17d

How many odd integers $1,000$ *through* $9,999$ *have distinct digits?*

Answer:
Use the multiplication rule.

Make $N(F)$ the number of first digit possibilities. It isn't 0 or the final digit, which must be odd.

$$N(F) = 1 \cdot (10 - 2) = 1 \cdot 8$$

$$N(F) = 8 \text{ possibilities}$$

Make $N(M)$ the number of two middle digit possibilities.
One isn't the first or last digit.
One isn't the first, last, or other middle digit:

$$N(M) = [1 \cdot (10 - 2)] \cdot [1 \cdot (10 - 3)]$$

$$N(M) = (1 \cdot 8) \cdot (1 \cdot 7)$$

$$N(M) = 56 \text{ possibilities}$$

Make $N(L)$ the number of last digit possibilities. It's one of five odd integers.

$$N(L) = 1 \cdot 5$$

$$N(L) = 5 \text{ possibilities}$$

Make $N(D)$ the number of distinct digits possible.

$$N(D) = N(F) \cdot N(M) \cdot N(L)$$

$$= 8 \cdot 56 \cdot 5$$

$$= 2,240 \text{ odd integers with distinct digits possible}$$

Problem 18c

For an ATM keypad, how many four-digit numeric sequences have no repeated digit?

Answer:
Use the multiplication rule.

Make $N(F)$ the number of first digit possibilities. It's any digit.

$$N(F) = 1 \cdot 10$$

$$N(F) = 10 \text{ possibilities}$$

Make $N(S)$ the number of second digit possibilities. It's not the first.

$$N(S) = 1 \cdot (10 - 1) = 1 \cdot 9$$

$$N(S) = 9 \text{ possibilities}$$

Make $N(T)$ the number of third-digit possibilities. It's not the first or second.

$$N(T) = 1 \cdot (10 - 2) = 1 \cdot 8$$

$$N(T) = 8 \text{ possibilities}$$

Make $N(R)$ the number of fourth-digit possibilities. It's not the first, second or third.

$$N(R) = 1 \cdot (10 - 3) = 1 \cdot 7$$

$$N(T) = 7 \text{ possibilities}$$

Make $N(A)$ the number of all possibilities.

$$N(F) \cdot N(S) \cdot N(T) \cdot N(R)$$

$$= 10 \cdot 9 \cdot 8 \cdot 7 =$$

$$= 5,040 \text{ four-digit numeric sequences possible}$$

And for any-digit numeric sequence:

$$10! = 3,628,800 \text{ any-digit numeric sequences possible}$$

Problem 28

Determine the number of inner-loop iterations. All variables are positive integers.

Assume $a \leq b$ and $c \leq d$.

for $i := a$ ***to*** b
    for $j := c$ ***to*** d
        statements (none branch outside loop)
    next j
next i

Answer:
Use the multiplication rule.

Outer loop for i:

$$(b + 1) - a \text{ iterations}$$

Inner loop for j:

$$(d + 1) - c \text{ iterations}$$

$$[(b + 1) - a] \cdot [(d + 1) - c] \text{ inner-loop iterations}$$

Problem 5a

How many five-digit integers $10,000$ *through* $99,999$ *are divisible by* 5*?*

Answer:
Use the multiplication rule.

Make $N(F)$ the number of first digit possibilities. It isn't zero.

$$N(F) = 1 \cdot (10 - 1) = 1 \cdot 9$$

$$N(F) = 9 \text{ possibilities}$$

Make $N(M)$ the number of middle digit possibilities. They're any digit.

$$N(M) = (1 \cdot 10) \cdot (1 \cdot 10) \cdot (1 \cdot 10)$$

$$N(M) = 10 \cdot 10 \cdot 10$$

$$= 1,000 \text{ possibilities}$$

Make $N(L)$ the number of last digit possibilities. It's zero or five.

$$N(L) = 1 \cdot (10 - 8) = 1 \cdot 2$$

$$M(L) = 2 \text{ possibilities}$$

Make $N(A)$ the number of all possibilities.

$$N(A) = N(F) \cdot N(M) \cdot N(L)$$

$$N(A) = 9 \cdot 1,000 \cdot 2 =$$

$$N(A) = 18,000 \text{ five-digit integers divisible by 5}$$

Problem 23c

How many integers 1 *through* 1,000 *aren't multiples of* 4 *or multiples of* 7*?*

Find the integers that are multiples of 4 or 7 and subtract the difference.

Let:
A = the set of integers that are multiples of 4
B = the set of integers that are multiples of 7
$A \cap B$ = the set of integers that are multiples of 28

Multiply by the largest possible number to find the number of integers that are multiples.

$$4 \cdot 1 = 4 \ldots 4 \cdot 2 = 8 \ldots 4 \cdot 3 = 12 \ldots 4 \cdot k = 1,000$$

The integer 1,000 is divisible by 4.

$$N(A) = 1,000 = 4k$$

$$k = 250 \text{ integers that are multiples of 4}$$

The closest number to 1,000 that's a multiple of 7 is 994.

$$N(B) = 994 = 7k$$

$$k = 142 \text{ integers that are multiples of 7}$$

The closest number to 1,000 that's a multiple of 28 is 980.

$$N(A \cap B) = 980 = 28k$$

$$k = 35 \text{ integers that are multiples of 28}$$

Subtracting set $A \cup B$ to avoid repeat answers:

$$N(A \cup B) = N(A) + N(B) - N(A \cap B)$$

$$= 250 + 142 - 35$$

$$= 357 \text{ integers that are multiples of 4 or 7}$$

There are 1,000 integers 1 through 1,000, and 357 are multiples of 4 or 7.

Use the difference rule.

$$1,000 - 357 =$$

$$643 \text{ integers that aren't multiples of 4 or 7}$$

Problems 29f, 29g

Solve:
(f) What is the dotted decimal form of the IP address for a computer in a Class C network?
(g) How many host IDs can there be for a Class C network?

Answer (f):
Range for the network ID part of a Class C IP address:

11000000 00000000 00000000 to 11011111 11111111 11111111

Convert binary to decimal:

$$11111111_2 = (1 \cdot 2^7) + (1 \cdot 2^6) + (1 \cdot 2^5) + (1 \cdot 2^4) + (1 \cdot 2^3) + (1 \cdot 2^2) + (1 \cdot 2^1) + (1 \cdot 2^0)$$

$$11111111_2 = 128_{10} + 64_{10} + 32_{10} + 16_{10} + 8_{10} + 4_{10} + 2_{10} + 2_{10}$$

$$11111111_2 = 255_{10}$$

$$11011111_2 = (1 \cdot 2^7) + (1 \cdot 2^6) + ... + (1 \cdot 2^4) + (1 \cdot 2^3) + (1 \cdot 2^2) + (1 \cdot 2^1) + (1 \cdot 2^0)$$

$$11011111_2 = 128_{10} + 64_{10} + 16_{10} + 8_{10} + 4_{10} + 2_{10} + 1_{10}$$

$$11011111_2 = 223_{10}$$

$$11000000_2 = (1 \cdot 2^7) + (1 \cdot 2^6) + (0 \cdot 2^5)...(0 \cdot 2^0) = 128_{10} + 64_{10}$$

$$11000000_2 = 192_{10}$$

$$00000000_2 = 0_{10}$$

Let $w.x.y.z$ be the dotted decimal form of a Class C IP address where [...]

$$192 \leq w \leq 223$$

$$0 \leq x \leq 255$$

$$0 \leq y \leq 255, \text{ and}$$

$$0 < z < 255, \text{ as host IDs aren't all 0's or all 1's}$$

Answer(g):
Class C network host IDs have 8 bits of 0 or 1. IDs with all 0's or 1's aren't possible, so subtract two possibilities.

$$2^8 - 2 = 256 - 2 =$$

254 host IDs possible

Problem 33f

A college survey on students' academic interests and achievements asked them to check all statements true for them.

Statement 1: "I was on the Dean's list last term."
Statement 2: "I belong to an academic club, such as the math club or the Spanish club."
Statement 3: "I am majoring in at least two subjects."

Out of 100 students:
$N(A) =$ *28 checked Statement 1*
$N(B) =$ *26 checked Statement 2*
$N(C) =$ *14 checked Statement 3*
$N(A \cap B) =$ *8 checked Statements 1 and 2*
$N(A \cap C) =$ *4 checked Statements 1 and 3*
$N(B \cap C) =$ *3 checked Statements 2 and 3*
$N(A \cap B \cap C) =$ *2 checked all three Statements*

Problem:
(f) How many students checked only Statement 2?

Answer:
Use the inclusion-exclusion rule.

Eight students checked Statements 1 and 2.

$$N(A \cap B) - N(A \cap B \cap C) = 8 - 2$$

$$= 6 \text{ students who only checked Statements 1 and 2}$$

Three students checked Statements 2 and 3.

$$N(A \cap C) - N(A \cap B \cap C) = 3 - 2$$

$$= 1 \text{ student who only checked Statements 2 and 3}$$

Twenty-six students checked Statement 2.

$$N(B) - 2 - 6 - 1$$

$$= 26 - 2 - 6 - 1$$

$$= 17 \text{ students who only checked Statement 2}$$

Problem 34d

A study gave 50 subjects three drugs A, B, and C for headache pain, and the subjects reported relief.
$N(A) =$ 21 from drug A
$N(B) =$ 21 from drug B
$N(C) =$ 31 from drug C
$N(A \cap B) =$ 9 from drugs A and B
$N(A \cap C) =$ 14 from drugs A and C
$N(B \cap C) =$ 15 from drugs B and C
$N(A \cup B \cup C) =$ 41 from at least one of the drugs

How many got relief from drug A only?

Answer:
Make $A \cap B \cap C$ the set of subjects who got relief from all three drugs.

Forty-one subjects took at least one of the drugs.

$$N(A \cup B \cup C) = N(A) + N(B) + N(C) - N(A \cap B) - N(A \cap C) - N(B \cap C) + N(A \cap B \cap C)$$
$$41 = 21 + 21 + 31 - 9 - 14 - 15 + N(A \cap B \cap C)$$
$$41 = 35 + N(A \cap B \cap C)$$
$$N(A \cap B \cap C) = 6 \text{ subjects who got relief from all three drugs}$$

Nine felt relief from drugs A and B.

$$N(A \cap B) - N(A \cap B \cap C) = 9 - 6 =$$
$$3 \text{ subjects who got relief from only drugs } A \text{ and } B$$

Fourteen felt relief from drugs A and C.

$$N(A \cap C) - N(A \cap B \cap C) = 14 - 6$$
$$8 \text{ subjects who got relief from only drugs } A \text{ and } C$$

Fifteen felt relief from drugs B and C.

$$N(B \cap C) - N(A \cap B \cap C) = 15 - 6$$
$$9 \text{ subjects who got relief from only drugs } B \text{ and } C$$

Twenty-one got relief from drug A.

$$N(A) - 3 - 8 - 9 =$$
$$21 - 3 - 8 - 9 =$$
$$1 \text{ one subject who got relief only from drug } A$$

Canvas Problem

How many integers 1 *through* 1,000 *are multiples of* 6 *or* 8*?*

Answer:
Let:
$A =$ the set of integers that are multiples of 6
$B =$ the set of integers that are multiples of 8
$A \cap B =$ the set of integers that are multiples of 48

Multiply by the largest possible integer to find the number of integers that are multiples.

$$6 \cdot 1 = 6 \text{ ... } 6 \cdot 2 = 12 \text{ ... } 6 \cdot 3 = 18 \text{ ... } 6 \cdot k = 1,000$$

The closest integer to 1,000 that's a multiple of 6 is 996.

$$N(A) \equiv 996 = 6k$$

$$k = 166 \text{ integers that are multiples of 6}$$

The integer 1,000 is divisible by 8.

$$N(B) \equiv 1,000 = 8k$$

$$k = 125 \text{ integers that are multiples of 8}$$

The closest integer to 1,000 that's a multiple of 48 is 960.

$$N(A \cap B) \equiv 960 = 48k$$

$$k = 20 \text{ integers that are multiples of 48}$$

Subtracting $N(A \cap B)$ to avoid repeat answers:

$$N(A \cup B) = N(A) + N(B) - N(A \cap B)$$

$$= 166 + 125 - 20$$

$$= 271 \text{ integers that are multiples of 6 or 8}$$