Problem 1

Find explicit formulas for sequences of the form $a_1, a_2, a_3, ...$ with the given terms.

(a) $\frac{1}{5}, \frac{3}{20}, \frac{5}{80}, \frac{7}{320}, \frac{9}{1280}, ...$

(b) $0, -\frac{1}{2}, \frac{2}{3}, -\frac{3}{4}, \frac{4}{5}, -\frac{5}{6}, ...$

Answer:

(a) $a_n = \frac{a_1+d(n-1)}{a_1(r^{n-1})} = \frac{1+2(n-1)}{5(4^{n-1})} = \frac{1+2n-2}{5(4^{n-1})} = \frac{2n-1}{5(4^{n-1})}$, where n is an integer and $n \geq 1$.

(b) $a_n = \frac{(-1^n)n}{n+1}$, where n is an integer and $n \geq 0$

Problem 2

Solve the following summation with the telescoping sum technique.

$\sum_{k=1}^{n} \frac{2k+1}{(k^2(k+1)^2)} = \sum_{k=1}^{n} \frac{A}{k^2} + \frac{B}{(k+1)^2}$

$A = \frac{2(0)+1}{(0+1)^2} = 1$ and $B = \frac{2(-1)+1}{-1^2} = -1$

$\sum_{k=1}^{n} \frac{A}{k^2} + \frac{B}{(k+1)^2} = \sum_{k=1}^{n} \frac{1}{k^2} - \frac{1}{(k+1)^2}$

$= (\frac{1}{1} - \frac{1}{4}) + (\frac{1}{4} - \frac{1}{9}) + (\frac{1}{9} - \frac{1}{16}) + ... + (\frac{1}{n-1} - \frac{1}{n}) + (\frac{1}{n} - \frac{1}{n+1})$

$= 1 - \frac{1}{n+1}$

Problem 3

Solve the summation.

$\sum_{i=33}^{500}(5i - \frac{7}{2}) = 5\sum_{i=33}^{500} i - \sum_{i=33}^{500} \frac{7}{2}$

$= (5 \cdot \frac{500(500+1)}{2}) - (\frac{7}{2} \cdot (500 - 33 + 1))$

$= (5 \cdot \frac{500 \cdot 501}{2}) - (\frac{7}{2} \cdot 468)$

$= 626,250 - 1,638 = 624,612$

Problem 4

Solve the summation.

$\sum_{j=0}^{200}(20j^2 - (-20)^j) = 20\sum_{j=1}^{200} j^2 + \sum_{j=0}^{200} 20^j$

$= (20 \cdot \frac{200(200+1)((2 \cdot 200)+1)}{6}) + (\frac{20^{200+1}-1}{20-1})$

$= (20 \cdot \frac{200 \cdot 201 \cdot 401}{6}) + (\frac{20^{201}-1}{19})$

$= 53,734,000 + (\frac{20^{201}-1}{19})$

Problem 5

$4\sum_{k=1}^{15}(4k^2+7)+3\sum_{k=1}^{15}(15k^2-9)$

$=(4\sum_{k=1}^{15}4k^2+4\sum_{k=1}^{15}7)+(3\sum_{k=1}^{15}15k^2+3\sum_{k=1}^{15}9)$

$=((4\cdot 4)\sum_{k=1}^{15}k^2+4\sum_{k=1}^{15}7)+((3\cdot 15)\sum_{k=1}^{15}k^2+3\sum_{k=1}^{15}9)$

$=[(16\cdot\frac{15(15+1)((15\cdot 2)+1)}{6})+((4\cdot 7)\cdot(15-1+1))]+[(45\cdot\frac{15(15+1)((15\cdot 2)+1)}{6})+((3\cdot 9)\cdot(15-1+1))]$

$=[(16\cdot\frac{15\cdot 16\cdot 31}{6})+(28\cdot 15)]+[(45\cdot\frac{15\cdot 16\cdot 31}{6})+(27\cdot 15)]$

$=[(16\cdot\frac{7{,}440}{6})+420]+[(45\cdot\frac{7{,}440}{6})+405]$

$=(19,840+420)+(55,800+405)=20,260+56,205=76,465$