Problem 34

Five sit at a round table.
How many different, non-rotated seatings are possible?

Calculate the number of permutations with the multiplication rule.

$$n!$$

Here, n is the number of objects, and $n = 5$.

But in a circular permutation, each permutation has n rotations.

$$\frac{n!}{n} = (n-1)! \text{ by cancellation}$$

$$= \frac{5!}{5} \text{ by substitution}$$

$$= \frac{\cancel{5} \cdot 4 \cdot 3 \cdot 2 \cdot 1}{\cancel{5}} \text{ by cancellation}$$

$$= 4 \cdot 3 \cdot 2 \cdot 1 \text{ or } 4!$$

$$= 24 \text{ by products of integers}$$

There are 4! or 24 possible permutations by calculating $\frac{n!}{n}$ or $(n-1)!$, where $n = 5$.

Problem 39d

How many six-letter strings are in the word [...]

$$ALGORITHM$$

[...] with O and R as the first two letters?

There are nine distinct letters in the word "ALGORITHM."

Make OR a fixed unit, leaving seven distinct letters [...]

A, L, G, I, T, H, and M

[...] possible for the remaining four letters of possible six-letter strings.

Calculate the r-permutation of a set of $n = 7$ elements, with $r = 4$ elements selected.

$$P(7,4) = \frac{n!}{(n-r)!}$$

$$= \frac{7!}{(7-4)!} \text{ by substitution}$$

$$= \frac{7!}{3!} \text{ by differences of integers}$$

$$= \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!}} \text{ by cancellation}$$

$$= 7 \cdot 6 \cdot 5 \cdot 4$$

$$= 840 \text{ by products of integers}$$

Answer: There are 840 possible permutations.

Problem 20(a-c)

$$MILLIMICRON$$

Find the number of distinguishable strings:
(a) For all letters;
(b) Starting with M and ending with N; and
(c) Containing the units CR and ON

Answer (a):
There are 11 letters. Let these variables equal these frequencies:
n_1 for M: 2 ;
n_2 for I: 3;
n_3 for L: 2; and
n_4, n_5, n_6, and n_7 for C, R, O, and N: 1

Find the permutations for the sets of indistinguishable objects.

Let $n = 11$, the number of characters in each distinguishable string.

$$\frac{n!}{n_1! \cdot n_2! \cdot n_3! \cdot n_4! \cdot n_5! \cdot n_6! \cdot n_7!} = \frac{11!}{2! \cdot 3! \cdot 2! \cdot 1! \cdot 1! \cdot 1! \cdot 1!} \text{ by substitution}$$

$$\frac{11!}{(2 \cdot 1) \cdot (3 \cdot 2 \cdot 1) \cdot (2 \cdot 1) \cdot 1 \cdot 1 \cdot 1 \cdot 1} \text{ by products of integers}$$

$$\frac{11!}{2 \cdot 6 \cdot 2} \text{ by products of integers}$$

$$= \frac{11!}{24} \text{ or } \frac{39,916,800}{24} \text{ by products of integers}$$

$$= 1,663,200 \text{ by quotients of integers}$$

Answer (a): There are $1,663,200$ distinguishable strings.

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Problem 20(a-c) (Continued)

Answer (b):
Any distinguishable string must start with M and end with N, leaving $11 - 2 = 9$ possible characters.

I, L, L, I, M, I, C, R, and O

Remember:
$n_1 = 2$, the frequency of M. But all strings start with M, so $n_1 = 2 - 1 = 1$.
$n_7 = 1$, the frequency N. But all strings end with N, and $n_7 \not> 1$, so it's not computed.

$$\frac{n!}{n_1! \cdot n_2! \cdot n_3! \cdot n_4! \cdot n_5! \cdot n_6!} = \frac{9!}{1! \cdot 3! \cdot 2! \cdot 1! \cdot 1! \cdot 1!} \text{ by substitution}$$

$$\frac{9!}{1 \cdot (3 \cdot 2 \cdot 1) \cdot (2 \cdot 1) \cdot 1 \cdot 1 \cdot 1} \text{ by products of integers}$$

$$= \frac{9!}{6 \cdot 2} = \frac{9!}{12} \text{ or } \frac{362,880}{12} \text{ by products of integers}$$

$$= 30,240 \text{ by quotients of integers}$$

Answer: There are $30,240$ distinguishable strings.

Answer(c):
Make n_4 and n_5 the frequencies of units CR and ON: $n_4 = 1$ and $n_5 = 1$, with n_6 and n_7 as null.

So, $11 - 2 = 9$ units are possible for any distinguishable string.

M, I, L, L, I, M, I, CR, and ON

$$\frac{n!}{n_1! \cdot n_2! \cdot n_3! \cdot n_4! \cdot n_5!} = \frac{9!}{2! \cdot 3! \cdot 2! \cdot 1! \cdot 1!} \text{ by substitution}$$

$$= \frac{9!}{(2 \cdot 1) \cdot (3 \cdot 2 \cdot 1) \cdot (2 \cdot 1) \cdot 1 \cdot 1} \text{ by products of integers}$$

$$\frac{9!}{2 \cdot 6 \cdot 2} = \frac{9!}{24} \text{ or 362,88024 by products of integers}$$

$$= 15,120 \text{ by quotients of integers}$$

Answer: There are $15,120$ distinguishable strings.

Canvas Problem 1

Five men and eight women sit at a round table.
How many seating permutations exist if no men sit together?

Remember:

$$\frac{n!}{n} = (n-1)!$$

[...] computes circular r-permutations with repetition, as each n permutation has n rotations.

Calculate the possible seating permutations for eight women.

$$P(7,7) = (n-1)! = (8-1)! \text{ by substitution}$$

$$= 7! \text{ or } 5,040 \text{ seating permutations for the women}$$

Five men remain, with eight possible seats.

Find the possible combinations for r men to sit at n seats, where $r = 5$ and $n = 8$.

$$C(8,5) = \frac{n!}{r!(n-r)!} = \frac{8!}{5!(8-5)!} \text{ by substitution}$$

$$= \frac{8!}{5! \cdot 3!} \text{ or } \frac{8!}{120 \cdot 6} \text{ by differences of integers}$$

$$= \frac{8!}{720} \text{ or } \frac{40,320}{720} \text{ by products of integers}$$

$$= 56 \text{ by quotients of integers}$$

There are $n!$ possible ways to seat the five men, not $(n-1)!$: Each rotation produces a unique permutation.

$$P(5,5) = n! = 5! = 120 \text{ seating permutations for the men}$$

Find all possible permutations by the product rule for counting:

$$P(7,7) \cdot C(8,5) \cdot P(5,5)$$

$$= 7! \cdot 56 \cdot 5! \text{ by substitution}$$

$$= 33,868,800 \text{ by products of integers}$$

Answer: There are $33,868,800$ permutations where no men sit together.

Canvas Problem 2

Find the number of distinguishable six-letter strings for the word [...]

$$PALINDROME$$

[...] when the first two letters are PA or the last two letters are ME.

Let:

$PA =$ the set of strings starting with P, then A
$ME =$ the set of strings ending with M, then E
$(PA \cup ME) =$ the set of strings starting with PA **or** ending with ME
$(PA \cap ME)$ the set of strings starting with PA **and** ending with ME

Find $(PA \cup ME)$.

Sets PA and ME each have eight remaining characters for four possible spaces.

L, I, N, D, R, O, M, and E, or

P, A, L, I, N, D, R, and O

$$PA \text{ and } ME = P(8,4) \cdot 2 = \frac{n!}{(n-r)!} \cdot 2$$

$$= \frac{8!}{(8-4)!} \cdot 2 \text{ by substitution}$$

$$= \frac{8!}{4!} \cdot 2 \text{ or } \frac{40,320}{24} \cdot 2 \text{ by differences of integers}$$

$$= 1,680 \cdot 2 = 3,360 \text{ by products of integers}$$

There are $3,360$ permutations, but by the inclusion-exclusion principle:

$$(PA \cup ME) = PA + ME - (PA \cap ME)$$

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Canvas Problem 2 (Continued)

Six possible characters compose two spots in six-letter strings starting with PA and ending with ME.

$$\text{L, I, N, D, R, and O}$$

$$(PA \cap ME) = P(6,2) = \frac{n!}{(n-r)!}$$

$$= \frac{6!}{(6-2)!} \text{ by substitution}$$

$$= \frac{6!}{3!} \text{ or } \frac{720}{6} \text{ by products of integers}$$

$$= 120 \text{ by quotients of integers}$$

There are 120 distinguishable six-letter strings starting in PA and ending in ME.

$$(PA \cup ME) = PA + ME - (PA \cap ME)$$

$$= 3,360 - 120 \text{ by substitution}$$

$$= 3,240 \text{ by differences of integers}$$

Answer: There are $3,240$ distinguishable six-letter strings starting in PA or ending in ME.

Canvas Problem 3

Find the number of strings with the letters in the word [...]

$$SALESPERSONS$$

[...] without consecutive S's.

There are four S's and eight non-S letters, including two E's.

$$\{A, L, E, P, E, R, O, N\}, \{S, S, S, S\}$$

Find the permutations with distinguishable objects for the eight non-S letters.

$$\frac{8!}{2!}, \text{ as there are two } E\text{'s and one of every other letter}$$

$$\frac{8!}{2!} = 20,160 \text{ possible permutations}$$

There are nine spaces to insert four S's: $C(9, 4)$.

$$= \frac{9!}{4!(9-4)!} \text{ by substitution}$$

$$= \frac{9!}{4! \cdot 5!} \text{ by differences of integers}$$

$$= 126 \text{ by quotients of integers}$$

Find the number of possible strings.

$$20,160 \cdot 126 = 2,540,160 \text{ by products of integers}$$

Answer: There are $2,540,160$ possible strings.