Mason Blanford
CS-225: Discrete Structures in CS
Homework 1, Part 1
Exercise Set 2.1

Problem 5b, 5d

Indicate if the sentences are statements.

Problems:
(b) She is a mathematics major.
(d) $x = 2^6$

Answers:
(b) Yes, this is a statement, as it states something true.
(d) This is not a statement, as the finality of x decides if it's true or not.

Problem 8c

Write symbolically.

Problem:
John is neither healthy, wealthy, nor wise.

Let:
$h =$ "John is healthy,"
$w =$ "John is wealthy," and
$s =$ "John is wise."

Answer:
$(\sim h \wedge \sim w \wedge \sim s)$

Problem 9c

Write symbolically.

Problem:
$10 > x \geq 5$

Let:
$p = x > 5$,
$q = x = 5$, and
$r = 10 > x$

Answer:
$r \wedge (p \vee q)$

Problem 10e

Write symbolically.

Problem:
Either DATAENDFLAG is on or it is the case that both ERROR equals 0 and SUM is less than 1,000.

Let:
p be the statement "DATAENDFLAG is off,"
q the statement "ERROR equals 0," and
r the statement "SUM is less than 1,000."

Answer:
$\sim p \vee (q \wedge r)$.

Problem 29

Use De Morgan's laws to write negations.

Problem:
This computer program has a logical error in the first ten lines, or it is being run with an incomplete data set.

Answer:
This computer program does not have a logical error in the first ten lines, and it is not being run with an incomplete data set.

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Problem 35

Let x be a particular real number, and use De Morgan's laws to write negations.

Problem:
$x \leq -1$ or $x > 1$

Answer:
$-1 < x \leq 1$

Problem 39

Write negations.

Let *numorders* and *numinstock* be particular values.

Problem:
($numorders < 50$ and $numinstock > 300$) or ($50 \leq numorders < 75$ and $numinstock > 500$)

Answer:
This statement's logical form is $(p \wedge q) \vee ((r \wedge s) \wedge t)$, so its negation has the form:

$\sim ((p \wedge q) \vee ((r \wedge s) \wedge t))$
$\equiv \sim (p \wedge q) \wedge \sim ((r \wedge s) \wedge t)$
$\equiv (\sim p \vee \sim q) \wedge (\sim (r \wedge s) \vee \sim t)$
$\equiv (\sim p \vee \sim q) \wedge ((\sim r \vee \sim s) \vee \sim t)$

Thus, a negation for this statement is:
($numorders \geq 50$ or $numinstock \leq 300$) and (($numorders < 50$ or $numorders \geq 75$) or $numinstock \leq 500$)

Problem 43

Use a truth table to establish if the statement is a tautology or contradiction.

Problem:
$(\sim p \vee q) \vee (p \wedge \sim q)$

Answer:
It is a tautology:

p	q	$\sim p$	$\sim q$	$(\sim p \vee q)$	$(p \wedge \sim q)$	$(\sim p \vee q) \vee (p \wedge \sim q)$
T	T	F	F	T	F	**T**
T	F	F	T	T	T	**T**
F	T	T	F	T	F	**T**
F	F	T	T	T	F	**T**

Problem 49

Give the steps for logical equivalence.

Problem:
$(p \vee \sim q) \wedge (\sim p \vee \sim q)$

Answer:
$(p \vee \sim q) \wedge (\sim p \vee \sim q)$
$\equiv (\sim q \vee p) \wedge (\sim q \vee \sim p)$ by commutative law
$\equiv \sim q \vee (p \wedge \sim p)$ by distributive law
$\equiv \sim q \vee \mathbf{c}$ by negation law
$\equiv \sim q$ by identity law

Therefore, $(p \vee \sim q) \wedge (\sim p \vee \sim q) \equiv \sim q$.

Problem 54

Write the logical equivalence.

Problem:
$(p \wedge (\sim (\sim p \vee q))) \vee (p \wedge q) \equiv p$

Answer:
$(p \wedge (\sim (\sim p \vee q))) \vee (p \wedge q)$
$\equiv (p \ \wedge (p \wedge \sim q)) \vee (p \wedge q)$ by De Morgan's law
$\equiv (\sim q \wedge (p \wedge p)) \vee (p \wedge q)$ by associative law
$\equiv (\sim q \wedge p) \vee (p \wedge q)$ by idempotent law
$\equiv (p \wedge \sim q) \vee (p \wedge q)$ by commutative law
$\equiv p \wedge (\sim q \vee q)$ by distributive law
$\equiv p \wedge \mathbf{t}$ by negation law
$\equiv p$ by identity law

Therefore, $(p \wedge (\sim (\sim p \vee q))) \vee (p \wedge q) \equiv p$.

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Canvas Problem

Write the logical equivalence::

Problem:
$((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (\sim p \wedge q) \equiv \sim p$

Answer:
$((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (\sim p \wedge q)$
$\equiv (\sim p \wedge (q \vee \sim q)) \vee (\sim p \wedge q)$ by distributive law
$\equiv (\sim p \wedge \mathbf{t}) \vee (\sim p \wedge q)$ by negation law
$\equiv \sim p \vee (\sim p \wedge q)$ by identity law
$\equiv \sim p$ by absorption law

Therefore, $((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (\sim p \wedge q) \equiv \sim p$.