Mason Blanford
CS-225: Discrete Structures in CS
Homework 2, Part 1
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Problem 32 (b, d)

Determine if the statements are true or false, giving counterexamples if false.

Let $\mathbf{R}$ be the domain of the predicate variable x.

(b) $x > 2 \Longrightarrow x^2 > 4$
(d) $x^2 > 4 \Longleftrightarrow \mid x \mid > 2$

(b) True: Any real number greater than 2 is greater than 4 when raised to the second power.

$$2.1^2 \text{ or } \frac{21}{10}^2 = 4.41 \text{ or } 4\frac{41}{100}$$

(d) False: Both predicates must have identical truth sets.

$$(-2) > 4 \text{ but } \mid -2 \mid \not> 2.$$

Problem 4(b, d)

Write an informal negation for these statements:

(b) All graphs are connected.
(d) Some estimates are accurate.

Answer:
(b) Some graphs are not connected.
(d) No estimates are accurate.

Problem 12

Determine if the proposed negation is correct. If not, write a correct negation.

Statement: The product of any irrational number and any rational number is irrational.
Proposed negation: The product of any irrational number and any rational number is rational.

Answer:
The proposed negation is not correct. The negation of a universal statement — e.g., the product of any irrational number and any rational number is irrational — would mean if only one product of an irrational and rational number is rational, then the statement is false.

Correct negation: There is at least one irrational number and one rational number whose product is rational.

Problem 29

Write the contrapositive, converse, and inverse of the statement. Determine the true and false statements, with counterexamples of false statements.

Statement: $\forall n \in \mathbf{Z}$, if n is prime then n is odd or $n = 2$.

Contrapositive: $\forall n \in \mathbf{Z}$, if it is the case that n is **neither** odd **nor** $n = 2$ then n is not prime.
Converse: $\forall n \in \mathbf{Z}$, if it is the case that n is odd or $n = 2$ then n is prime.
Inverse $\forall n \in \mathbf{Z}$, if n is not prime then it is the case that n is **neither** odd **nor** $n = 2$.

Answer:
The statement and its contrapositive are true.

The statement's converse and inverse are false. As a counterexample, let $n = 35$. Then n is odd and $n \neq 2$, but n is not prime because the numbers dividing 35 evenly are $\{1, 5, 7, 35\}$.

Problem 33

Write the contrapositive, converse, and inverse of the statement. Determine the true and false statements, with counterexamples of false statements.

Statement: If a function is differentiable then it is continuous.

Contrapositive: If a function is not continuous then it is not differentiable.
Converse: If a function is continuous then it is differentiable.
Inverse If a function is not differentiable then it is not continuous.

Answer:
The statement and its contrapositive are true.
The statement's converse and inverse are false. As a counterexample, let $f(x) = \mid x+1 \mid$.

$$\mid x+1 \mid = \begin{cases} x+1 & \text{if } x \geq -1 \\ -(x+1) & \text{if } x < -1 \end{cases} \tag{1}$$

As x approaches -1 from the right-hand side, the limit is 1.

$$\lim_{h\to 0^+} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0^+} \frac{f((x+1)+h)-f((x+1))}{h} = \lim_{h\to 0^+} \frac{(x+1)+h-(x+1)}{h}$$
$$= \lim_{h\to 0^+} \frac{x+1+h-x-1}{h} = \lim_{h\to 0^+} \frac{h}{h} = \lim_{h\to 0^+} 1$$

As x approaches -1 from the left-hand side, the limit is -1.

$$\lim_{h\to 0^-} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0^-} \frac{f(-((x+1)+h))-f(-(x+1))}{h} = \lim_{h\to 0^-} \frac{-x-1-h+x+1}{h}$$
$$= \lim_{h\to 0^-} \frac{-h}{h} = \lim_{h\to 0^-} -1$$

This function is continuous but not differentiable at -1, as the left- and right-side limits do not agree.

Problem 46

Rewrite without using the words "necessary" or "sufficient."

The negation of a $\forall$ statement is an $\exists$ statement, and the negation of an "if-then" statement is an "and" statement.

Statement:
Having a large income is not a necessary condition for a person to be happy.

Answer:
The above statement means, "If a person does not have a large income, then that person is not happy," or $s \to r$, where $s =$ "a person is happy" and $r =$ "has a large income."

The negation is $s \wedge \sim r$, or, "There is a person who does not have a large income and is happy."

Problem 48

Rewrite without using the words "necessary" or "sufficient."

The negation of a $\forall$ statement is an $\exists$ statement, and the negation of an "if-then" statement is an "and" statement.

Statement:
Being a polynomial is not a sufficient condition for a function to have a real root.

Answer:
The above statement means, "A function should not need to be a polynomial to have a real root," or $\sim (r \to s)$, where $r =$ "being a polynomial" and $s =$ "having a real root."

The negation of $\sim (r \to s)$ is $\sim r \vee s$, or, "There is a function that is not a polynomial, or it has a real root."

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Canvas Problem

Express each of the following English sentences in terms of $B(x)$, $W(x)$, $S(x)$ quantifiers and logical connectives. Assume the domain D is the set of all people.

Let $B(x)$, $W(x)$, and $S(x)$ be the predicates:
$B(x)$: x is a female
$W(x)$: x is a good athlete
$S(x)$: x is young

Statements:
(a) Not all young females are good athletes.
(b) A person is a good athlete only if it is the case that both she is a female and she is young.
(c) Some females are not good athletes.
(d) All good athletes are neither young nor are they female.
(e) Any person is a good athlete unless he/she is not young.

Answers:
(a) $\sim \forall x \in D, (B(x) \wedge S(x)) \rightarrow W(x)$ or $\exists x \in D$, such that $(B(x) \wedge S(x)) \wedge \sim W(x)$
(b) $\forall x \in D, W(x) \rightarrow (B(x) \wedge S(x))$ or $\forall x \in D, (\sim B(s) \wedge \sim S(x)) \rightarrow \sim W(x)$
(c) $\exists x \in D$ such that $B(x) \wedge \sim W(x)$
(d) $\forall x \in D, W(x) \wedge (\sim S(x) \wedge \sim B(x))$
(e) $\forall x \in D, \sim S(x) \rightarrow W(x)$