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CS-225: Discrete Structures in CS
Assignment 4, Part 2
Exercise Set 6.2

Problem 14

Use an element argument to prove the statement. Assume all sets are subsets of universal set U.

Statement:
For all sets A and B, $A \cup (A \cap B) = A$.

Proof:
Suppose A and B are any sets.

To show $A \cup (A \cap B) = A$, we must show:
(1) $A \cup (A \cap B) \subseteq A$ and
(2) $A \subseteq A \cup (B \cap A)$.

(1) Proof for $A \cup (A \cap B) \subseteq A$:
Suppose x is any element in $A \cup (A \cap B) \subseteq A$ to show that $x \in A$.

Either (1.1) $x \in A$ or (1.2) $x \in A \cap B$ by the definition of union.

(1.1) As $x \in A$, clearly $x \in A$.
(1.2) As $x \in (A \cap B)$, $x \in A$ and $x \in B$ by the definition of intersection.

In both cases, $x \in A$, so by the definition of subset, $A \cup (A \cap B) \subseteq A$.

(2) Proof for $A \subseteq A \cup (B \cap A)$:
Suppose x is any element in $A \subseteq A \cup (B \cap A)$ to show that $x \in A \cup (B \cap A)$.

Since $x \in A$, then $x \in A$ or (2.1) $x \in (B \cap A)$ by the definition of union.

(2.1) Since $x \in A$, then $x \in A$ and $x \in B$ by the definition of intersection.

By the definition of subset, $A \subseteq A \cup (B \cap A)$.

Conclusion: By the definition of set equality, $A \cup (A \cap B) = A$, as both subset relations have been proven.

Problem 17

Use an element argument to prove the statement. Assume all sets are subsets of universal set U.

Statement:
For all sets A, B, and C, if $A \subseteq B$, then $A \cup C \subseteq B \cup C$.

Proof:
Suppose A, B, and C are any sets such that $A \subseteq B$. We must show $A \cup C \subseteq B \cup C$.

Let $x \in A \cup C$. By the definition of union, (1.1) $x \in A$ or (1.2) $x \in C$.

By the definition of union:
(1.1) Since $A \subseteq B$ and $x \in A$, then $x \in B$ and $x \in B \cup C$.
(1.2) Since $x \in C$, then $x \in B \cup C$.

By the definition of subset, $A \cup C \subseteq B \cup C$.

Problem 18

Use an element argument to prove the statement. Assume all sets are subsets of universal set U.

Statement:
For all sets A and B, if $A \subseteq B$, then $B^c \subseteq A^c$.

Proof:
Suppose A and B are any sets such that $A \subseteq B$. We must show $B^c \subseteq A^c$.

Let $x \in A^c$, meaning $x \notin A$ by the definition of complement.

Since $x \notin A$, by the definition of subset:
(1) $x \notin B$, so $x \in B^c$
(2) $x \in B^c$, so $x \in A^c$

So, $B^c \subseteq A^c$ by subset relations.

Problem 33

Write an algebraic proof for the statement, citing properties from Theorem 6.2.2 on page 394 for each step.

Statement:
For all sets A and B, $(A - B) \cap (A \cap B) = \emptyset$.

Proof:
Suppose A and B are any sets. We must show $(A - B) \cap (A \cap B) = \emptyset$.

$(A - B) \cap (A \cap B)$
$= (A \cap B^c) \cap (A \cap B)$ by **Set Difference Law**
$= [(A \cap B^c) \cap A] \cap B$ by **Associative Law**
$= [(A \cap A) \cap B^c] \cap B$ by **Commutative Law**
$= (A \cap B^c) \cap B$ by **Idempotent Law**
$= A \cap (B^c \cap B)$ by **Associative Law**
$= A \cap \emptyset$ by **Complement Law**
$= \emptyset$ by **Universal Bound Law**

Problem 40

Write an algebraic proof for the statement, citing properties from Theorem 6.2.2 on page 394 for each step.

Statement:
For all sets A, B, and C, $(A - B) - (B - C) = (A - B)$.

Proof:
Suppose A, B, and C are any sets. We must show $(A - B) - (B - C) = (A - B)$.

(1) Initial laws for $(A - B) - (B - C)$:
(1.1) $= (A \cap B^c) - (B \cap C^c)$ by **Set Difference Law**
(1.2) $= (A \cap B^c) \cap (B \cap C^c)^c$ by **Set Difference Law**
(1.3) $= (A \cap B^c) \cap (B^c \cup (C^c)^c)$ by **De Morgan's Law**
(1.4) $= (A \cap B^c) \cap (B^c \cup C)$ by **Double Complement Law**

(2) Let $(A \cap B^c) = A$:
(2.1) $(A \cap B^c) \cap (B^c \cup C) = A \cap (B^c \cup C)$ by substitution

(3) $A \cap (B^c \cup C) = (A \cap B^c) \cup (A \cap C)$ by **Distributive Law**

(4) Let $A = (A \cap B^c)$:
(4.1) $(A \cap B^c) \cup (A \cap C) = [(A \cap B^c) \cap B^c] \cup [(A \cap B^c) \cap C]$ by substitution

(5) Final laws for $[(A \cap B^c) \cap B^c] \cup [(A \cap B^c) \cap C]$:
(5.1) $= [A \cap (B^c \cap B^c)] \cup [(A \cap B^c) \cap C]$ by **Associative Law**
(5.2) $= (A \cap B^c) \cup [(A \cap B^c) \cap C]$ by **Idempotent Law**
(5.3) $= A \cap B^c$ by **Absorption Law**
(5.4) $= A - B$ by **Set Difference Law**