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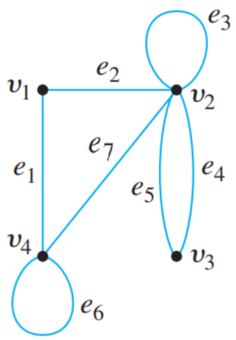
CS-225: Discrete Structures in CS
Assignment 9
Canvas Exercise Set
Page 1

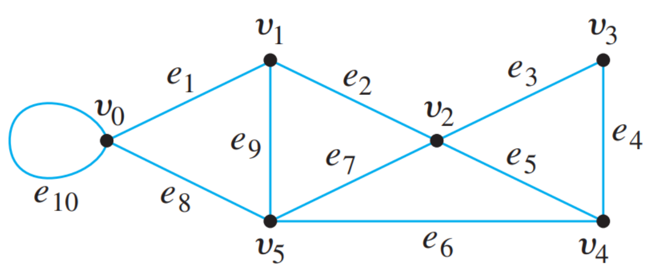
Canvas Problem

For each of the graphs, find:
(i) All edges incident on v2;
(ii) All vertices adjacent to v4;
(iii) All edges adjacent to e1;
(iv) All loops;
(v) All parallel edges; and
(vi) The degree of v2.

Graph 1:



Graph 2:



Answer (i): Edges incident on $v2$:

Graph 1: $e1, e2, e3, e4, e5$

Graph 2: $e1, e2, e3, e5$

Answer (ii): Vertices adjacent to $v4$:

Graph 1: $v1, v2$

Graph 2: $v2, v3, v5$

Answer (iii): Edges adjacent to $e1$:

Graph 1: $e2, e6, e7$

Graph 2: $e8, e9, e10,$

Answer (iv): Loops:

Graph 1: $e3, e6$

Graph 2: $e10$

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Canvas Problem (Continued)

Answer (iv): Parallel edges:

Graph 1: $e4, e5$

Graph 2: None

Answer (iv): Degree of $v2$:

Graph 1: 6

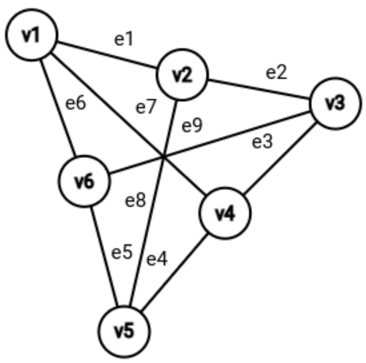
Graph 2: 4

Problem 13

Draw a graph with the specified properties, or explain why it doesn't exist.

Graph:
Simple graph with nine edges and all vertices of degree 3.

Answer:



Problem 18

Explain if there's a simple graph where each vertex has an even degree.

Answer:
The handshake theorem says, for a graph, the sum of a vertices' degrees equals twice the number of edges.

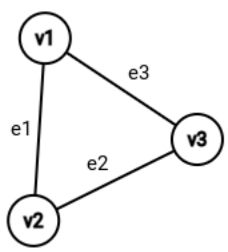
$$2 \cdot (\text{number of edges}) = \sum_{i=1}^{n} \deg(v_i)$$

Let:
$3 =$ the number of edges; and
$v_i = 3$ vertices with degree 2 each.

$$2 \cdot 3 = 2(3) \text{ by substitution}$$

$$6 = 6$$

This graph exists.



Problem 21c

For any integer $n \geq 5$, is there a simple graph of n vertices, all with different degrees?

Answer:
A simple graph:
has no parallel edges, meaning vertices share one edge maximum; and
has no loops, meaning a vertex doesn't link to itself.

This means a vertex can't have a degree above $n - 1$, where n is the number of vertices.

Let $n = 5$ vertices of degrees:
$v_1 = 0$
$v_2 = 1$
$v_3 = 2$
$v_4 = 3$
$v_5 = 4$

By the handshake theorem:

$$2 \cdot (\text{number of edges}) = v_1 + v_2 + v_3 + v_4 + v_5$$

$$2 \cdot (\text{number of edges}) = 0 + 1 + 2 + 3 + 4$$

$$2 \cdot (\text{number of edges}) = 10$$

$$(\text{number of edges}) = 5$$

For v_4, linking to $n - 1$ vertices contradicts vertices $v_i < v_4$ each linking to $n - 1$ vertices.

The supposition requires more than 5 edges.

There is no simple graph of 5 or more vertices with different degrees each.

Problems 23e, 23f

Recall how $K_{m,n}$ denotes a complete bipartite graph on (m, n) vertices.

For $K(m, n)$:
  (e) What is the total degree?
  (f) Find a formula for the number of edges.

Answer (e):
Bipartite graphs have two disjoint sets with no adjacent vertices, so it has m vertices of degree n and vice-versa.

For total degree of K_n and K_m separately:

$$\text{[total degree]}$$
$$= \text{[number of vertices]} \cdot \text{[degree of each vertex]}$$

Recall the handshake theorem [...]

$$2 \cdot (\text{number of edges}) = \sum_{i=1}^{n} \deg(v_i)$$

[...] where deg is the degree of each vertex, and v_i is the number of vertices.

For $K_{m,n}$ total degree:

$$\text{[total degree]} = [K_m \text{ total degree}] + [K_n \text{ total degree}]$$

Answer (f):
Use the handshake theorem, where [...]

$$2 \cdot deg(v_i) = [K_m \text{ total degree}] + [K_n \text{ total degree}]$$

[...] because $deg(v_i)$ is the total degree of only one disjoint set.

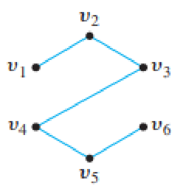
Let:
  $m = K_m$ total degree
  $n = K_n$ total degree
  e = number of edges

$$2 \cdot (\text{number of edges}) = \sum_{i=1}^{n} \deg(v_i)$$
$$2 \cdot e = m \cdot n \text{ by substitution}$$
$$e = \frac{mn}{2} \text{ by transposition}$$

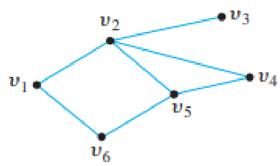
Problem 24c, Problem 24d

Redraw the graph if it's bipartite. If not, explain with contradiction.

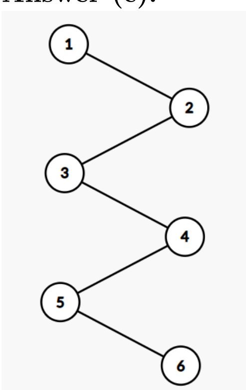
c.



d.



Answer (c):



Answer(d):
Put each vertex into two disjoint subsets A and B to prove (d) is bipartite.

By definition, bipartite graphs have no adjacent vertices in the same subset.

With $v3$ in set A, $v2$ is in set B, and each adjacent vertex alternates subsets. Vertices $v3$ and $v2$ have an adjacent edge, as do $v2$ and $v4$.

$$A = \{v3, v4\}$$

$$B = \{v2, \}$$

But $v5$ has an adjacent edge with $v4$ in set A and $v2$ in set B, creating a disjoint subset with adjacent vertices whether $v5$ is assigned to sets A or B.

So, (d) is not a bipartite graph.