Problem 20b, 20c, 20e, 20g

Write negations for the statements.

Problems:
(b) If today is New Year's Eve, then tomorrow is January.
(c) If the decimal expansion of r is terminating, then r is rational.
(e) If x is nonnegative, then x is positive or x is 0.
(g) If n is divisible by 6, then n is divisible by 2 and n is divisible by 3.

Answers:
(b) Today is New Year's Eve, and tomorrow is not January.
(c) The decimal expansion of r is terminating, and r is not rational.
(e) The variable x is nonnegative, and x is neither positive nor 0.
(g) The variable n is divisible by 6, and n is neither divisible by 2 nor divisible by 3.

Problem 27

Use a truth table to verify or refute the statement.

The converse and inverse of a conditional statement are logically equivalent to each other.

Conditional: $p \to q$
Converse: $q \to p$
Inverse: $\sim p \to \sim q$

p	q	$\sim p$	$\sim q$	$p \to q$	$q \to p$	$\sim p \to \sim q$
T	T	F	F	T	**T**	**T**
T	F	F	T	F	**T**	**T**
F	T	T	F	T	**F**	**F**
F	F	T	T	T	**T**	**T**

The converse $q \to p$ and inverse $\sim p \to \sim q$ have equivalent truth values, proving logical equivalence. The two are logically equivalent to each other despite not being logically equivalent to the conditional $p \to q$.

Problem 31

If P and Q are logically equivalent, $P \longleftrightarrow Q$ is a tautology.

Use $\longleftrightarrow$ to convert the logical equivalence to a tautology, and verify with a truth table.

Problem:
$p \to (q \to r) \equiv (p \wedge q) \to r$

Answer:
$p \to (q \to r) \longleftrightarrow (p \wedge q) \to r$

p	q	r	$p \wedge q$	$q \to r$	$p \to (q \to r)$	$(p \wedge q) \to r$	$p \to (q \to r) \longleftrightarrow (p \wedge q) \to r$
T	T	T	T	T	T	T	**T**
T	T	F	T	F	F	F	**T**
T	F	T	F	T	T	T	**T**
T	F	F	F	T	T	T	**T**
F	T	T	F	T	T	T	**T**
F	T	F	F	F	T	T	**T**
F	F	T	F	T	T	T	**T**
F	F	F	F	T	T	T	**T**

The statement $p \to (q \to r) \longleftrightarrow (p \wedge q) \to r$ is a tautology, as all of its truth values are T.

Problem 39

Rewrite in if-then form.

If r and s are statements, [r unless s] means [if $\sim s$ then r].

Problem:
This door will not open unless a security code is entered.

Answer:
If a security code is not entered, this door will not open.

Problem 45

Rewrite in if-then form.

The statement "a sufficient condition for s is r" means r is a sufficient condition for s.
$r \rightarrow s$

The statement "a necessary condition for s is r" means r is a necessary condition for s.
$\sim r \rightarrow \sim s$

Problem:
A necessary condition for this computer program to be correct is that it not produce error messages during translation.

Answer:
If this computer program produces error messages during translation, then it is not correct.

Mason Blanford
CS-225: Discrete Structures in CS
Homework 1, Part 2
Exercise Set. 2.2
Page 6

Problem 46c, 46d, 46e, 46f

Assume the problem statement is true, and explain if the if-then statements are true or false.

The statement "a sufficient condition for s is r" means r is a sufficient condition for s.
$r \to s$

The statement "a necessary condition for s is r" means r is a necessary condition for s.
$\sim r \to \sim s$

Problems:
If compound X is boiling, then its temperature must be at least 150°C.

(c) Compound X will boil only if its temperature is at least 150°C.
(d) If compound X is not boiling, then its temperature is less than 150°C.
(e) A necessary condition for compound X to boil is that its temperature be at least 150°C.
(f) A sufficient condition for compound X to boil is that its temperature be at least 150°C

Answers:
$r =$ "compound X is boiling"
$s =$ "its temperature must be at least 150°C"
$r \to s$

(c) This statement is the biconditional $r \longleftrightarrow s$ of the given statement, so it isn't necessarily true. For example, this statement remains false as compound X is not boiling but the temperature is at least 150°C.

(d) This statement is the inverse $\sim r \to \sim s$ of the given statement, so it isn't necessarily true. For example, this statement remains true as compound X is boiling but the temperature isn't at least 150°C.

(e) This statement is the contrapositive $\sim s \to \sim r$ of the given statement, so it must be true.

(f) This statement is the converse $q \to p$ of the given statement, so it isn't necessarily true. For example, this remains false even as the temperature is at least 150°C.

Mason Blanford
CS-225: Discrete Structures in CS
Homework 1, Part 2
Exercise Set. 2.2
Page 7

Problem 50

(a) Rewrite without using the symbols $\to$ and $\longleftrightarrow$.
$p \to q \equiv \sim p \vee q$
$p \longleftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$

(b) Rewrite using only $\wedge$ and $\sim$.
$p \vee q \equiv \sim (\sim p \wedge \sim q)$

Problem:
$(p \to (q \to r)) \longleftrightarrow ((p \wedge q) \to r)$

Answer (a):
$(p \to (q \to r)) \longleftrightarrow ((p \wedge q) \to r)$
$\equiv (\sim p \vee (\sim q \vee r)) \longleftrightarrow (\sim (p \wedge q) \vee r)$
$\equiv [\sim (\sim p \vee (\sim q \vee r)) \vee (\sim (p \wedge q) \vee r)] \wedge [\sim (\sim (p \wedge q) \vee r) \vee (\sim p \vee (\sim q \vee r))]$
$\equiv [(\sim (\sim p) \wedge \sim (\sim q \vee r)) \vee ((\sim p \vee \sim q) \vee r)] \wedge [(\sim (\sim (p \wedge q))) \wedge \sim r) \vee (\sim p \vee (\sim q \vee r))]$ by De Morgan's law
$\equiv [(\sim (\sim p) \wedge (q \wedge \sim r)) \vee ((\sim p \vee \sim q) \vee r)] \wedge [(\sim (\sim (p \wedge q))) \wedge \sim r) \vee (\sim p \vee (\sim q \vee r))]$ by De Morgan's law
$\equiv [(p \wedge (q \wedge \sim r)) \vee ((\sim p \vee \sim q) \vee r)] \wedge [((p \wedge q) \wedge \sim r) \vee (\sim p \vee (\sim q \vee r))]$ by double negative law

Answer (b):
$[(p \wedge (q \wedge \sim r)) \vee ((\sim p \vee \sim q) \vee r)] \wedge [((p \wedge q) \wedge \sim r) \vee (\sim p \vee (\sim q \vee r))]$
$\equiv [\ \sim (\sim (p \wedge (q \wedge \sim r)) \wedge \sim ((\sim p \vee \sim q) \vee r))] \wedge [\sim (\sim ((p \wedge q) \wedge \sim r) \wedge \sim (\sim p \vee (\sim q \vee r)))]$
$\equiv [\ \sim (\sim (p \wedge (q \wedge \sim r)) \wedge \sim (\sim (\sim (\sim p \vee \sim q) \wedge \sim r)))] \wedge [\sim (\sim ((p \wedge q) \wedge \sim r) \wedge \sim (\sim (\sim (\sim p) \wedge \sim (\sim q \vee r))))]$
$\equiv [\ \sim (\sim (p \wedge (q \wedge \sim r)) \wedge \sim (\sim (\sim (\sim (\sim (\sim p) \wedge \sim (\sim q))) \wedge \sim r)))] \wedge [\sim (\sim ((p \wedge q) \wedge \sim r) \wedge \sim (\sim (\sim (\sim p) \wedge \sim (\sim (\sim (\sim q) \wedge \sim r)))))]$
$\equiv [\ \sim (\sim (p \wedge (q \wedge \sim r)) \wedge \sim (\sim (\sim (\sim (p\ \wedge\ q)) \wedge \sim r)))] \wedge [\sim (\sim ((p \wedge q) \wedge \sim r) \wedge \sim (\sim (p \wedge \sim (\sim (q \wedge \sim r)))))]$ by double negative law
$\equiv [\ \sim (\sim (p \wedge (q \wedge \sim r)) \wedge \sim (\sim ((p\ \wedge\ q) \wedge \sim r)))] \wedge [\sim (\sim ((p \wedge q) \wedge \sim r) \wedge \sim (\sim (p\ \wedge (q \wedge \sim r))))]$ by double negative law
$\equiv [\ \sim (\sim (p \wedge (q \wedge \sim r))\ \wedge ((p\ \wedge\ q) \wedge \sim r))] \wedge [\sim (\sim ((p \wedge q) \wedge \sim r) \wedge (p \wedge (q \wedge \sim r)))]$ by double negative law
$\equiv [\ \sim (\sim (p \wedge (q \wedge \sim r))\ \wedge ((p\ \wedge\ q) \wedge \sim r))] \wedge [\sim (\sim ((p \wedge q) \wedge \sim r) \wedge (p \wedge (q \wedge \sim r)))]$ by double negative law