

Red Wine Exploration by Matthew Mason

This report explores a dataset concerning red variants of vinho verde, a Portuguese wine. The dataset relates various chemical and physical measures to a subjective measure of quality for each wine.

Citation:

P. Cortez, A. Cerdeira, F. Almeida, T. Matos and J. Reis. Modeling wine preferences by data mining from physicochemical properties. In Decision Support Systems, Elsevier, 47(4):547-553. ISSN: 0167-9236.

Available at:

[@Elsevier] <http://dx.doi.org/10.1016/j.dss.2009.05.016>

[Pre-press (pdf)] <http://www3.dsi.uminho.pt/pcortez/winequality09.pdf>

[bib] <http://www3.dsi.uminho.pt/pcortez/dss09.bib>

Description of variables

The following description is taken from the reference cited above.

1. fixed acidity: most acids involved with wine are fixed or nonvolatile (do not evaporate readily)
 2. volatile acidity: the amount of acetic acid in wine, which at too high of levels can lead to an unpleasant, vinegar taste
 3. citric acid: found in small quantities, citric acid can add ‘freshness’ and flavor to wines
 4. residual sugar: the amount of sugar remaining after fermentation stops, it’s rare to find wines with less than 1 gram/liter and wines with greater than 45 grams/liter are considered sweet
 5. chlorides: the amount of salt in the wine
 6. free sulfur dioxide: the free form of SO₂ exists in equilibrium between molecular SO₂ (as a dissolved gas) and bisulfite ion; it prevents microbial growth and the oxidation of wine
 7. total sulfur dioxide: amount of free and bound forms of SO₂; in low concentrations, SO₂ is mostly undetectable in wine, but at free SO₂ concentrations over 50 ppm, SO₂ becomes evident in the nose and taste of wine
 8. density: the density is close to that of water depending on the percent alcohol and sugar content
 9. pH: describes how acidic or basic a wine is on a scale from 0 (very acidic) to 14 (very basic); most wines are between 3-4 on the pH scale
 10. sulphates: a wine additive which can contribute to sulfur dioxide gas (SO₂) levels, which acts as an antimicrobial and antioxidant
 11. alcohol: the percent alcohol content of the wine
- Output variable (based on sensory data):
12. quality (score between 0 and 10)

Univariate Plots Section

We begin with two tabular summaries of the data.

```
## 'data.frame': 1599 obs. of 13 variables:
##   $ X           : int  1 2 3 4 5 6 7 8 9 10 ...
##   $ fixed.acidity : num  7.4 7.8 7.8 11.2 7.4 7.4 7.9 7.3 7.8 7.5 ...
##   $ volatile.acidity : num  0.7 0.88 0.76 0.28 0.7 0.66 0.6 0.65 0.58 0.5 ...
##   $ citric.acid    : num  0 0 0.04 0.56 0 0 0.06 0 0.02 0.36 ...
##   $ residual.sugar : num  1.9 2.6 2.3 1.9 1.9 1.8 1.6 1.2 2 6.1 ...
##   $ chlorides      : num  0.076 0.098 0.092 0.075 0.076 0.075 0.069 0.065 0.073 0.071 ...
##   $ free.sulfur.dioxide : num  11 25 15 17 11 13 15 15 9 17 ...
##   $ total.sulfur.dioxide: num  34 67 54 60 34 40 59 21 18 102 ...
##   $ density         : num  0.998 0.997 0.997 0.998 0.998 ...
##   $ pH              : num  3.51 3.2 3.26 3.16 3.51 3.51 3.3 3.39 3.36 3.35 ...
##   $ sulphates       : num  0.56 0.68 0.65 0.58 0.56 0.56 0.46 0.47 0.57 0.8 ...
##   $ alcohol          : num  9.4 9.8 9.8 9.8 9.4 9.4 9.4 10 9.5 10.5 ...
##   $ quality          : int  5 5 5 6 5 5 5 7 7 5 ...

##             X      fixed.acidity  volatile.acidity  citric.acid
## Min.    : 1.0    Min.    : 4.60    Min.    :0.1200    Min.    :0.000
## 1st Qu.: 400.5  1st Qu.: 7.10    1st Qu.:0.3900    1st Qu.:0.090
## Median  : 800.0  Median  : 7.90    Median  :0.5200    Median  :0.260
## Mean    : 800.0  Mean    : 8.32    Mean    :0.5278    Mean    :0.271
## 3rd Qu.:1199.5  3rd Qu.: 9.20    3rd Qu.:0.6400    3rd Qu.:0.420
## Max.   :1599.0  Max.   :15.90    Max.   :1.5800    Max.   :1.000
## 
##   residual.sugar  chlorides  free.sulfur.dioxide
##   Min.    : 0.900  Min.    :0.01200  Min.    : 1.00
##   1st Qu.: 1.900  1st Qu.:0.07000  1st Qu.: 7.00
##   Median  : 2.200  Median  :0.07900  Median  :14.00
##   Mean    : 2.539  Mean    :0.08747  Mean    :15.87
##   3rd Qu.: 2.600  3rd Qu.:0.09000  3rd Qu.:21.00
##   Max.   :15.500  Max.   :0.61100  Max.   :72.00
## 
##   total.sulfur.dioxide  density      pH      sulphates
##   Min.    : 6.00      Min.    :0.9901  Min.    :2.740  Min.    :0.3300
##   1st Qu.: 22.00     1st Qu.:0.9956  1st Qu.:3.210  1st Qu.:0.5500
##   Median  : 38.00     Median :0.9968  Median :3.310  Median :0.6200
##   Mean    : 46.47     Mean    :0.9967  Mean    :3.311  Mean    :0.6581
##   3rd Qu.: 62.00     3rd Qu.:0.9978  3rd Qu.:3.400  3rd Qu.:0.7300
##   Max.   :289.00     Max.   :1.0037  Max.   :4.010  Max.   :2.0000
## 
##   alcohol        quality
##   Min.    : 8.40  Min.    :3.000
##   1st Qu.: 9.50  1st Qu.:5.000
##   Median  :10.20  Median  :6.000
##   Mean    :10.42  Mean    :5.636
##   3rd Qu.:11.10  3rd Qu.:6.000
##   Max.   :14.90  Max.   :8.000
## 
##             vars   n   mean      sd median trimmed   mad   min
##   $ X           1 1599 800.00 461.74 800.00 800.00 593.04 1.00
##   $ fixed.acidity 2 1599  8.32  1.74   7.90   8.15  1.48 4.60
##   $ volatile.acidity 3 1599  0.53  0.18   0.52   0.52  0.18 0.12
##   $ citric.acid   4 1599  0.27  0.19   0.26   0.26  0.25 0.00
##   $ residual.sugar 5 1599  2.54  1.41   2.20   2.26  0.44 0.90
##   $ chlorides      6 1599  0.09  0.05   0.08   0.08  0.01 0.01
```

```

## free.sulfur.dioxide    7 1599  15.87 10.46 14.00 14.58 10.38 1.00
## total.sulfur.dioxide  8 1599  46.47 32.90 38.00 41.84 26.69 6.00
## density                9 1599   1.00  0.00  1.00  1.00  0.00 0.99
## pH                      10 1599   3.31  0.15  3.31  3.31  0.15 2.74
## sulphates               11 1599   0.66  0.17  0.62  0.64  0.12 0.33
## alcohol                 12 1599  10.42  1.07 10.20 10.31  1.04 8.40
## quality                  13 1599   5.64  0.81  6.00  5.59  1.48 3.00
##                                max      range skew kurtosis      se
## X                         1599.00 1598.00 0.00   -1.20 11.55
## fixed.acidity            15.90   11.30 0.98    1.12 0.04
## volatile.acidity         1.58    1.46 0.67    1.21 0.00
## citric.acid              1.00    1.00 0.32   -0.79 0.00
## residual.sugar           15.50   14.60 4.53   28.49 0.04
## chlorides                 0.61    0.60 5.67   41.53 0.00
## free.sulfur.dioxide     72.00   71.00 1.25    2.01 0.26
## total.sulfur.dioxide   289.00  283.00 1.51    3.79 0.82
## density                  1.00    0.01 0.07    0.92 0.00
## pH                        4.01    1.27 0.19    0.80 0.00
## sulphates                2.00    1.67 2.42   11.66 0.00
## alcohol                  14.90   6.50 0.86    0.19 0.03
## quality                   8.00    5.00 0.22    0.29 0.02

```

We see all variables have values that are amenable to binwidths of 1, 0.1, 0.01, or 0.001. We should be able to visualize all the data in the histograms without needing to zoom or transform scales.

The only variable with a minimum value of 0 is citric acid. Below is a table of the values of citric acid.

```

##
##    0 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.1 0.11 0.12 0.13 0.14
##  132   33   50   30   29   20   24   22   33   30   35   15   27   18   21
## 0.15 0.16 0.17 0.18 0.19 0.2 0.21 0.22 0.23 0.24 0.25 0.26 0.27 0.28 0.29
##  19   9   16   22   21   25   33   27   25   51   27   38   20   19   21
## 0.3 0.31 0.32 0.33 0.34 0.35 0.36 0.37 0.38 0.39 0.4 0.41 0.42 0.43 0.44
##  30   30   32   25   24   13   20   19   14   28   29   16   29   15   23
## 0.45 0.46 0.47 0.48 0.49 0.5 0.51 0.52 0.53 0.54 0.55 0.56 0.57 0.58 0.59
##  22   19   18   23   68   20   13   17   14   13   12   8   9   9   8
## 0.6 0.61 0.62 0.63 0.64 0.65 0.66 0.67 0.68 0.69 0.7 0.71 0.72 0.73 0.74
##  9   2   1   10   9   7   14   2   11   4   2   1   1   3   4
## 0.75 0.76 0.78 0.79   1
##  1   3   1   1   1

```

We see there are 132 values for 0 for citric acid, which is a high count relative to the other values. However, we don't see a discontinuity at low values.

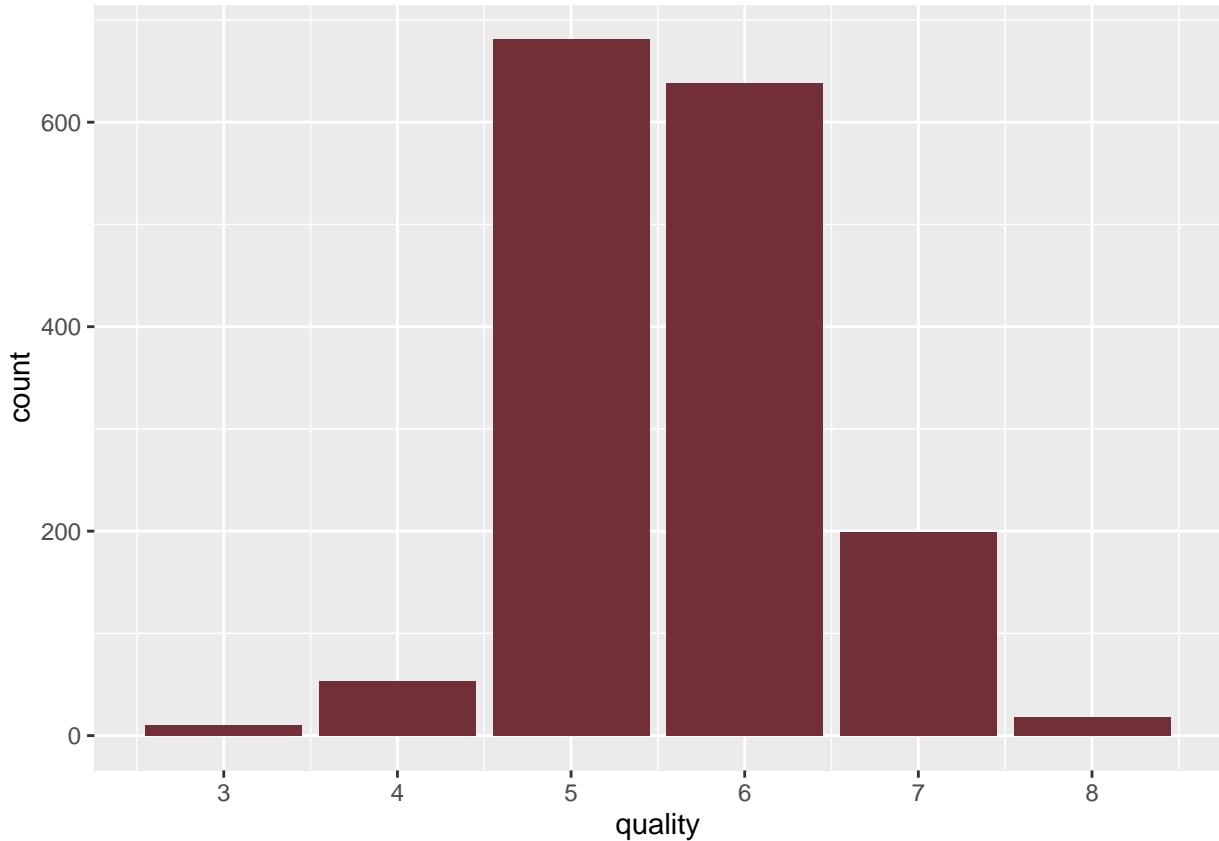
Let's check for NAs.

```
sum(is.na(wine))
```

```
## [1] 0
```

No NAs in the dataset.

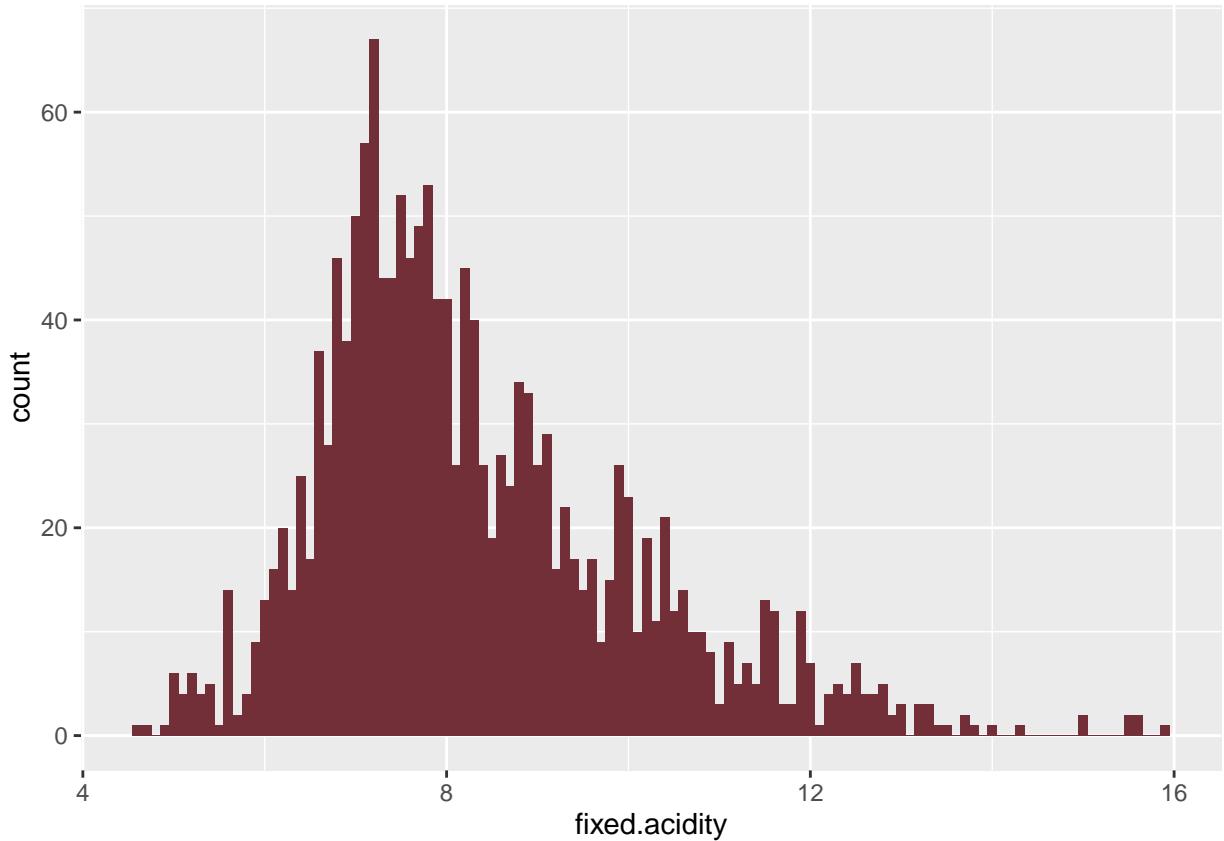
The quality ratings are discrete values. Let's plot a bar graph of the quality ratings.



```
##  
##   3   4   5   6   7   8  
## 10  53 681 638 199  18  
## [1] 0.8248906
```

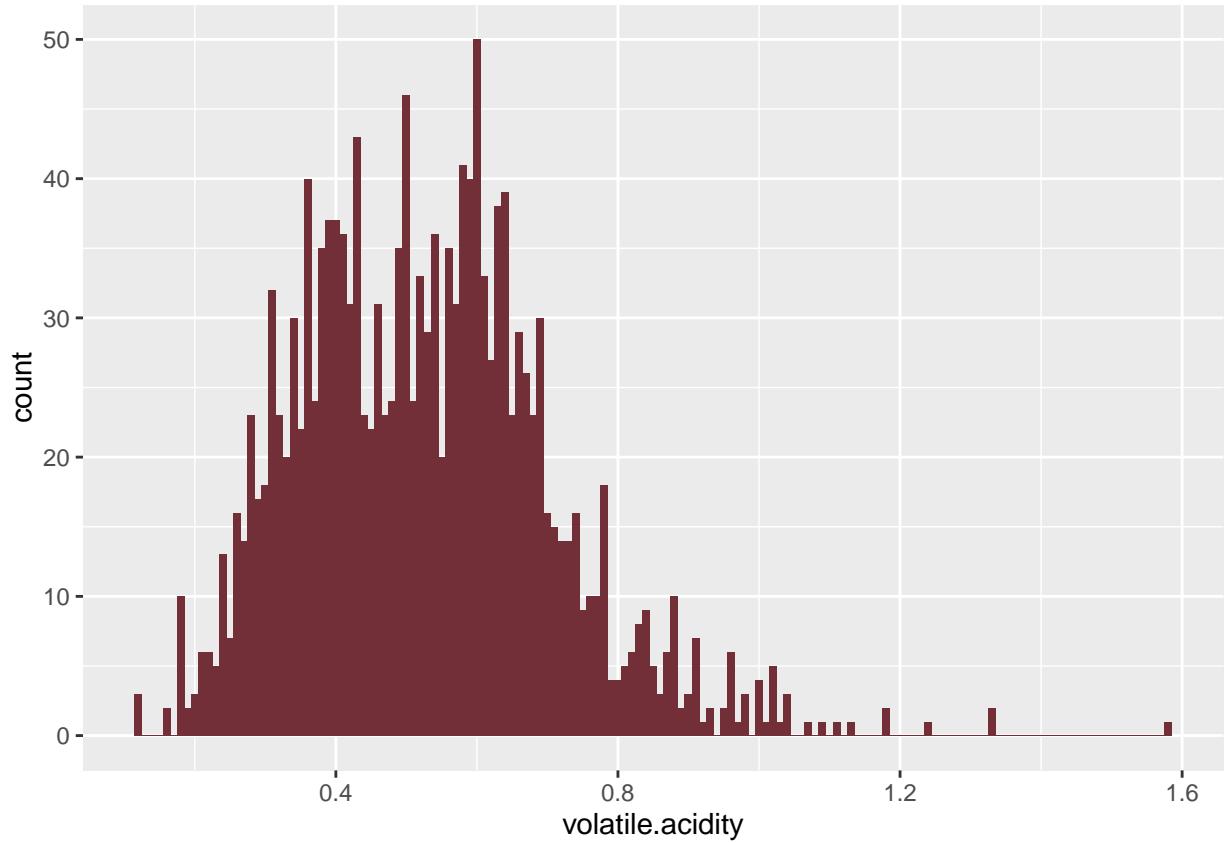
The distribution of quality ratings is unimodal but narrow. 82% of quality ratings are either 5 or 6. It will be interesting to see how strongly the chemical measures correlate with quality ratings given the high number of average ratings.

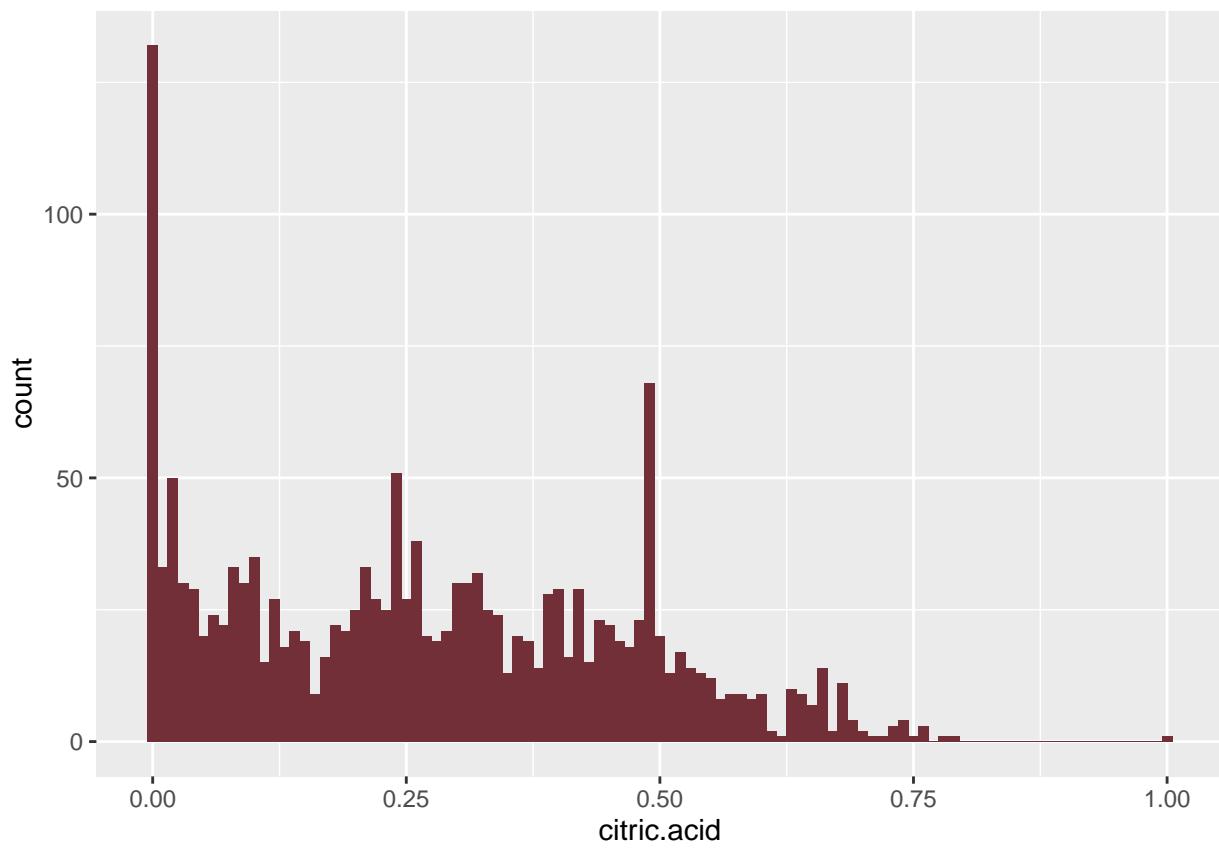
Let's look at the distributions of the other variables.

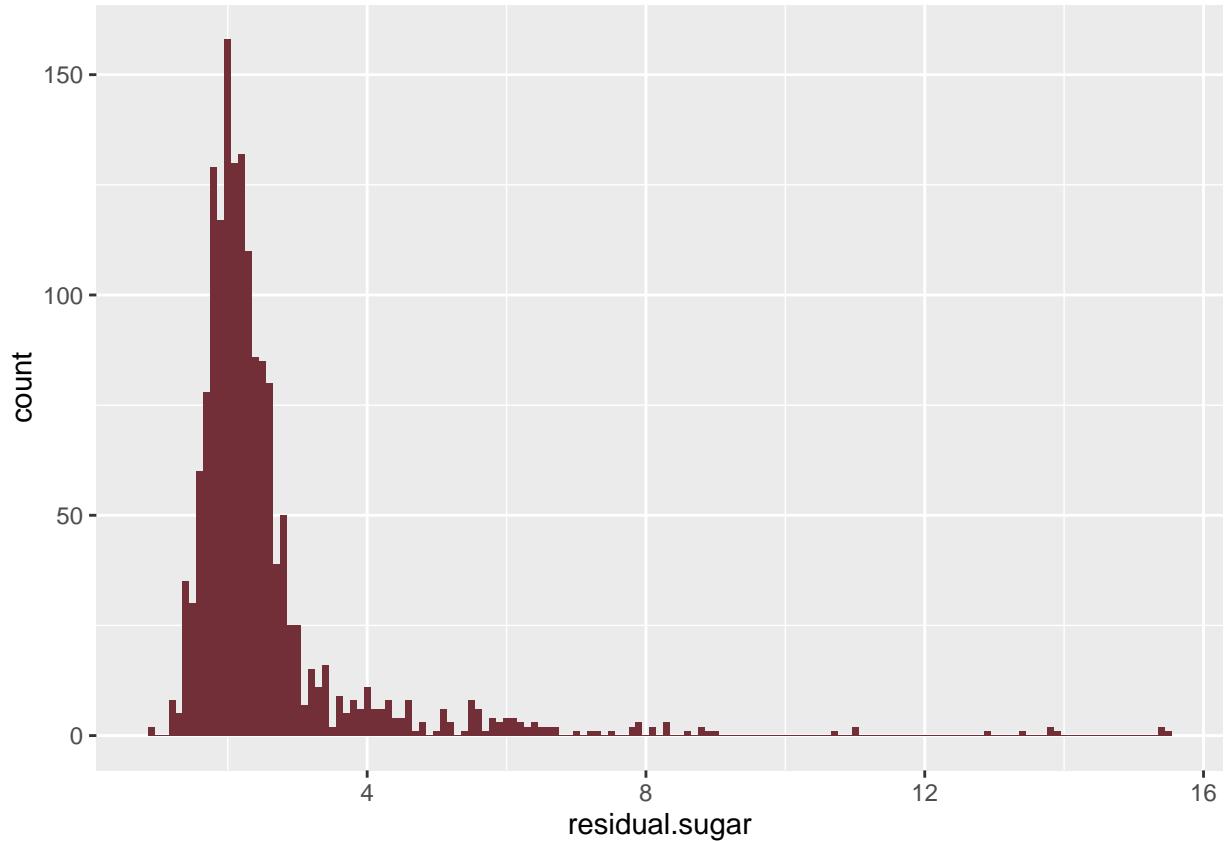


We see from the above graph that fixed acidity is distributed over a broad range.

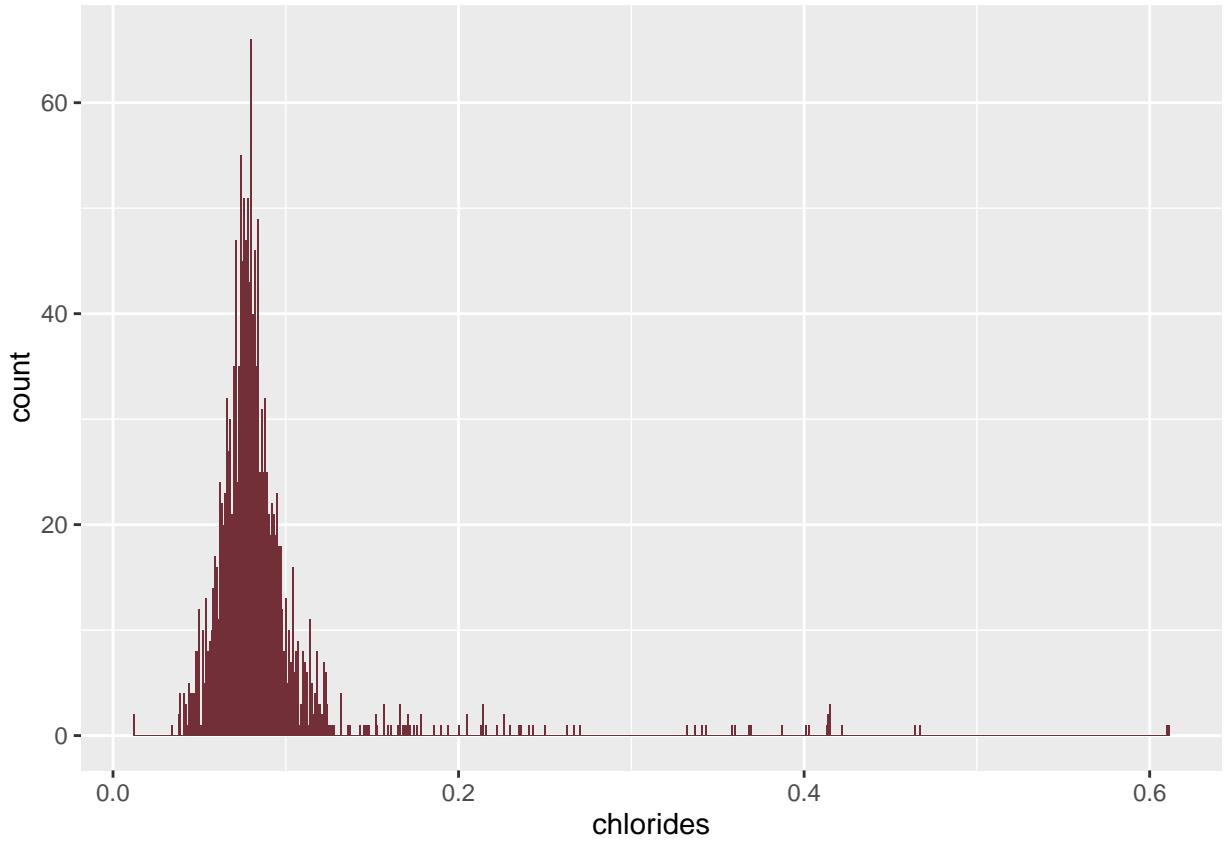
Many of the following graphs also have outliers. It will be interesting to compare the outliers to quality.

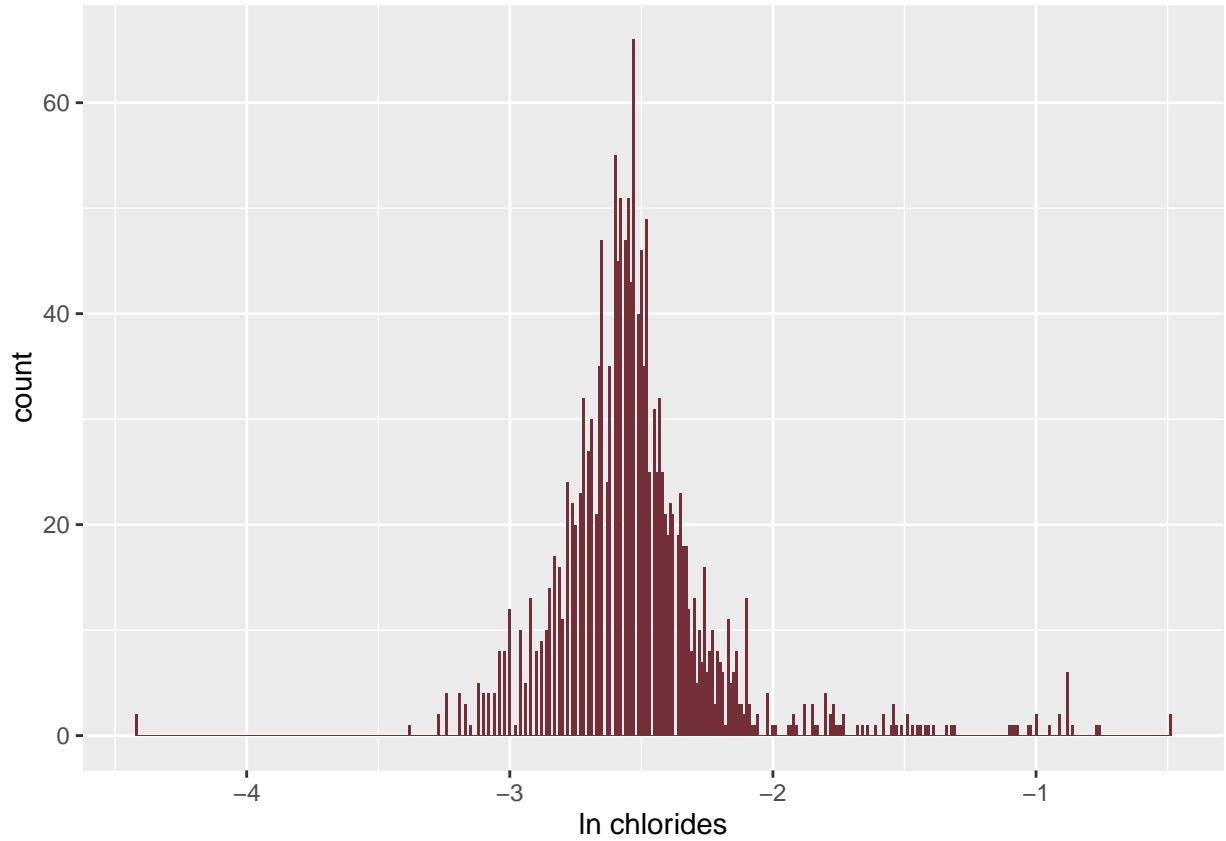


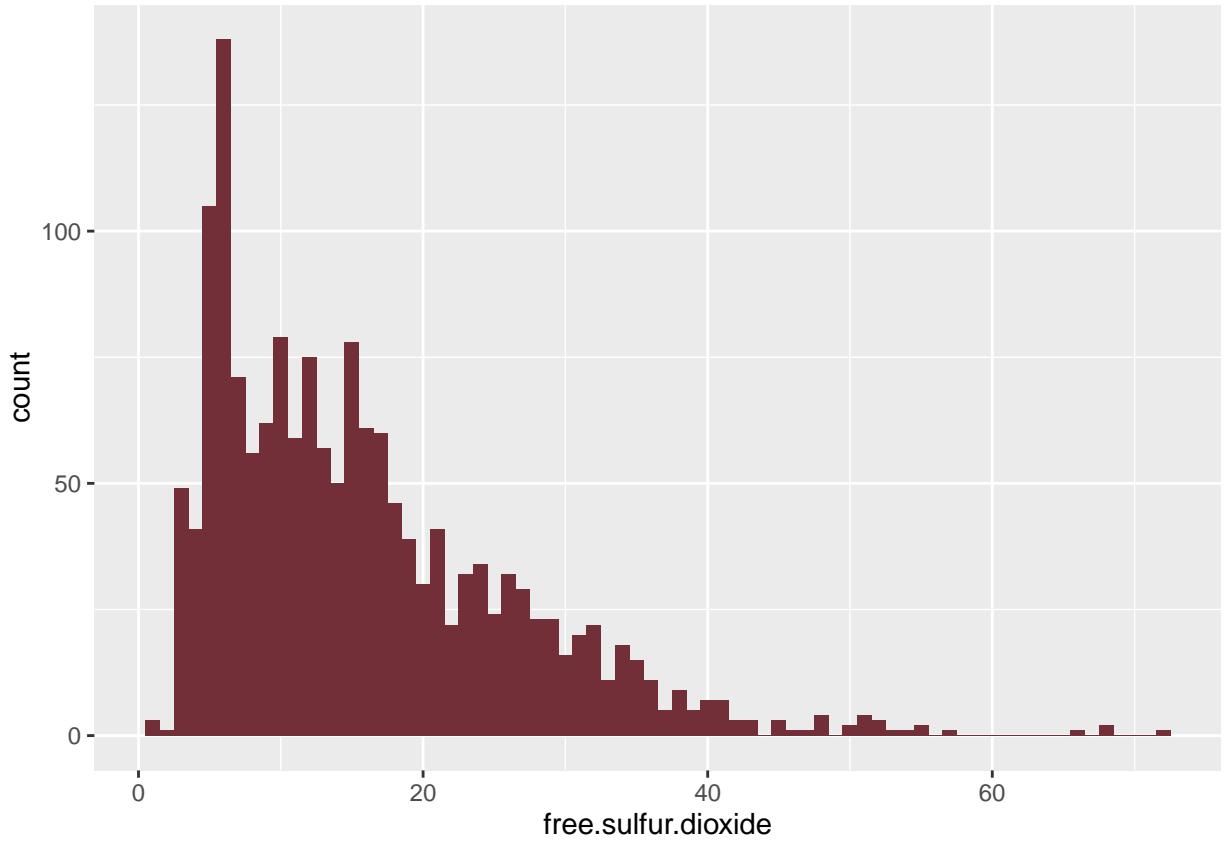




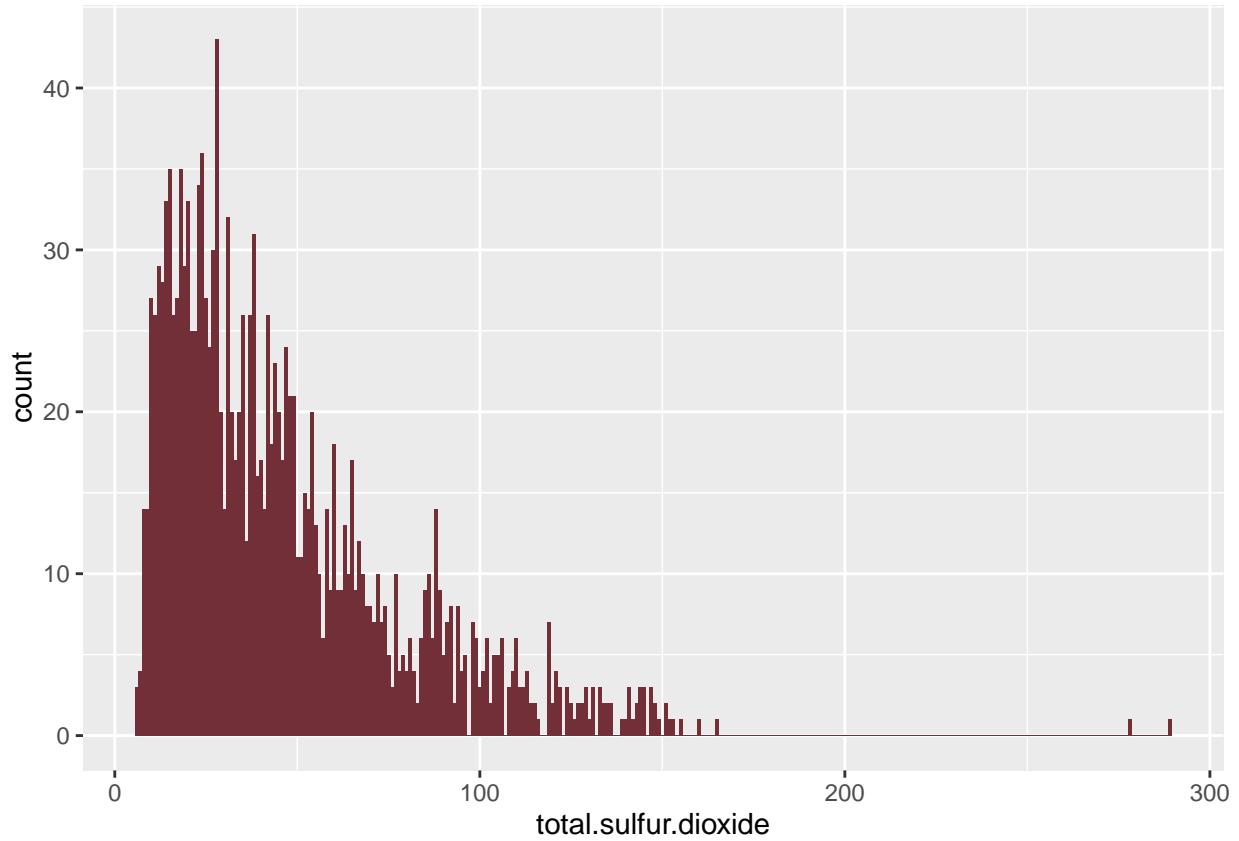
Residual sugar looks like it might be right-skewed, but we found that the correlation coefficient of a log transform of residual sugar with the quality variable was not significant.

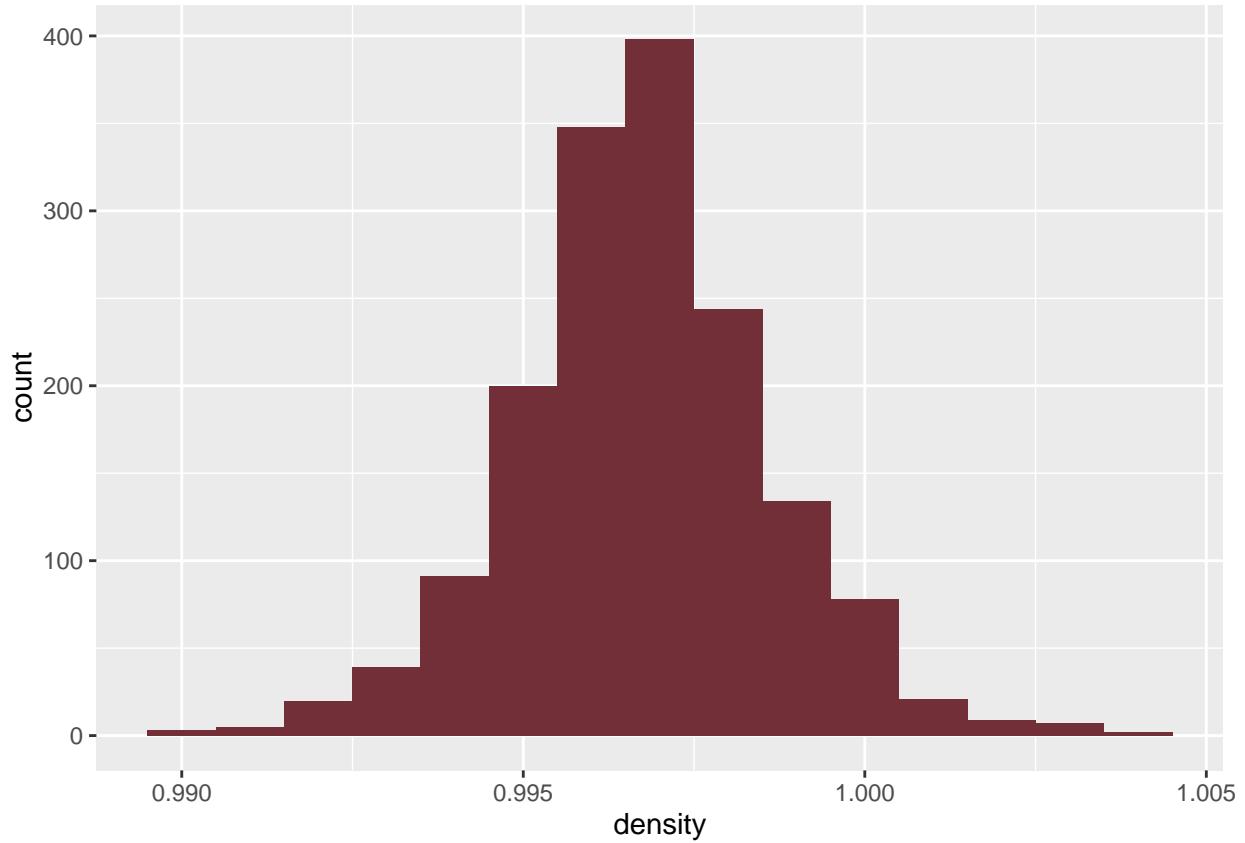


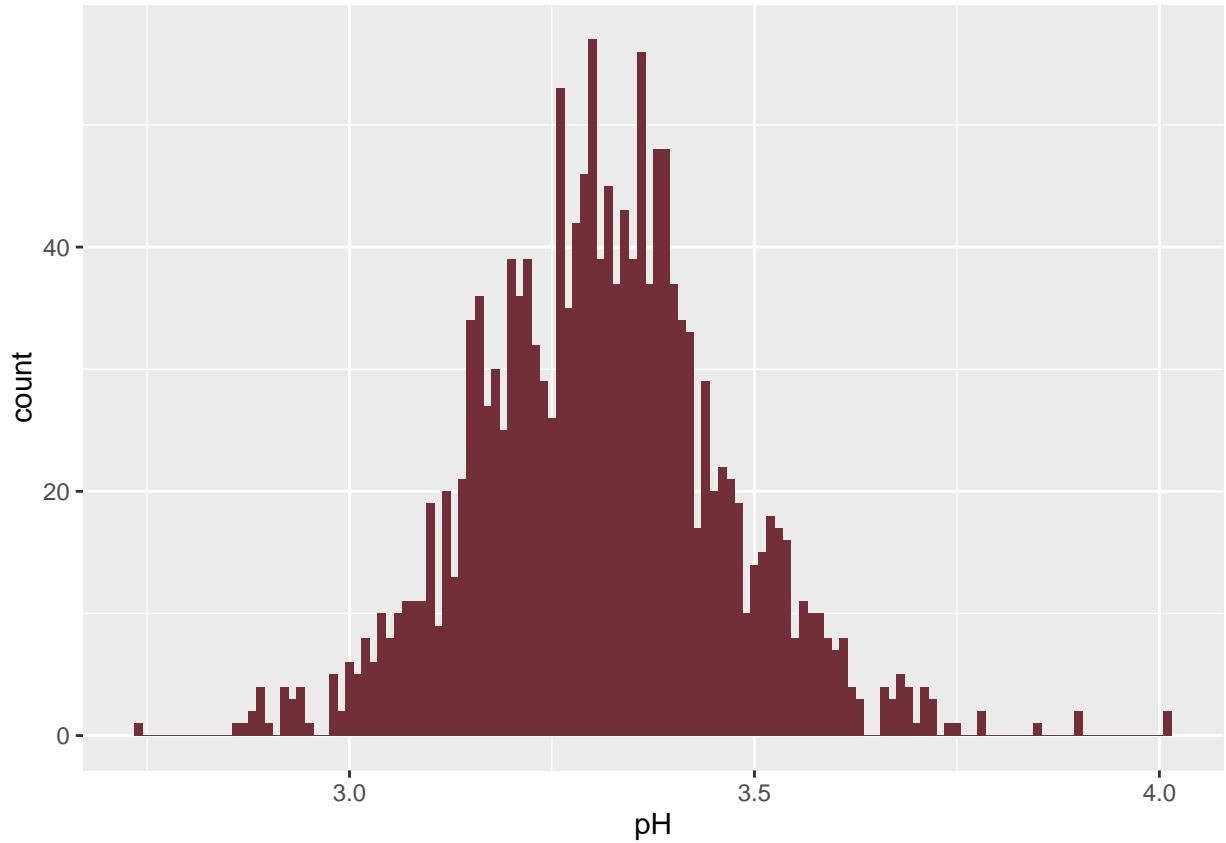


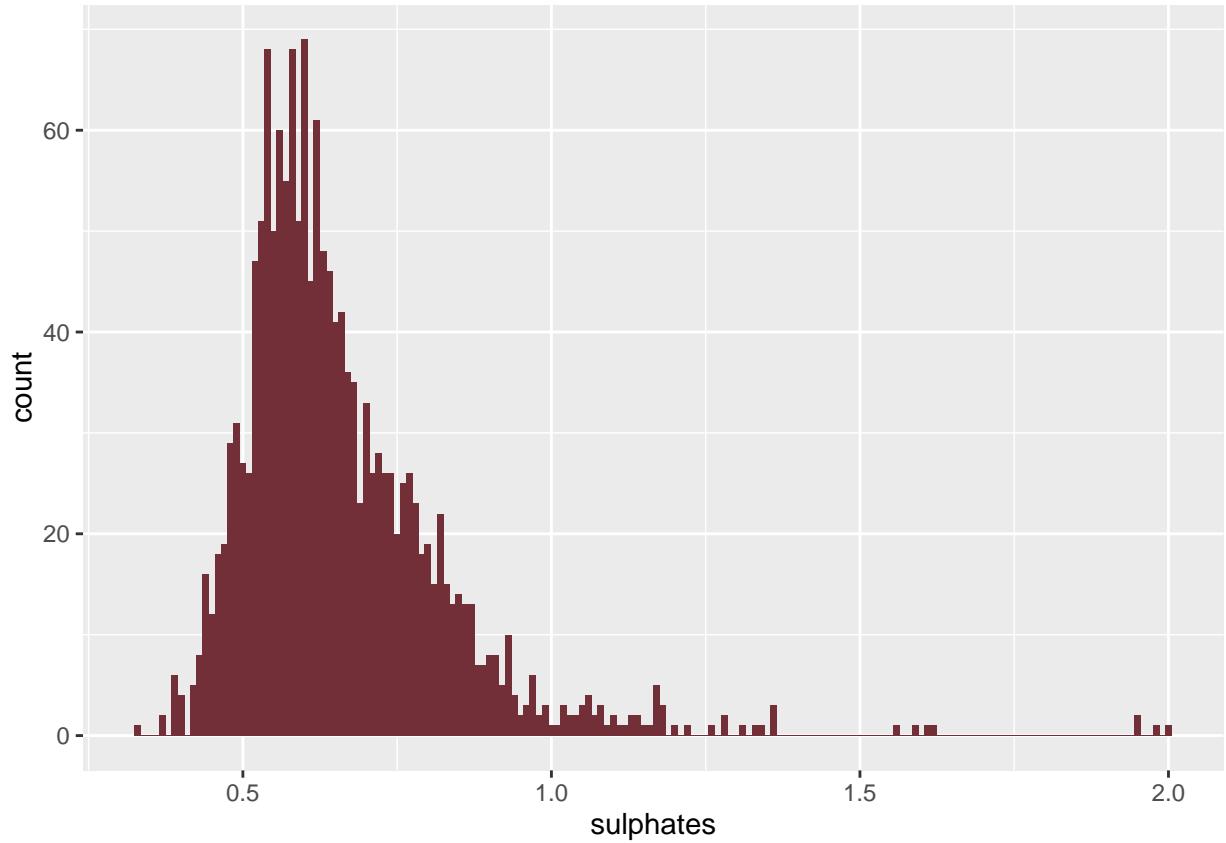


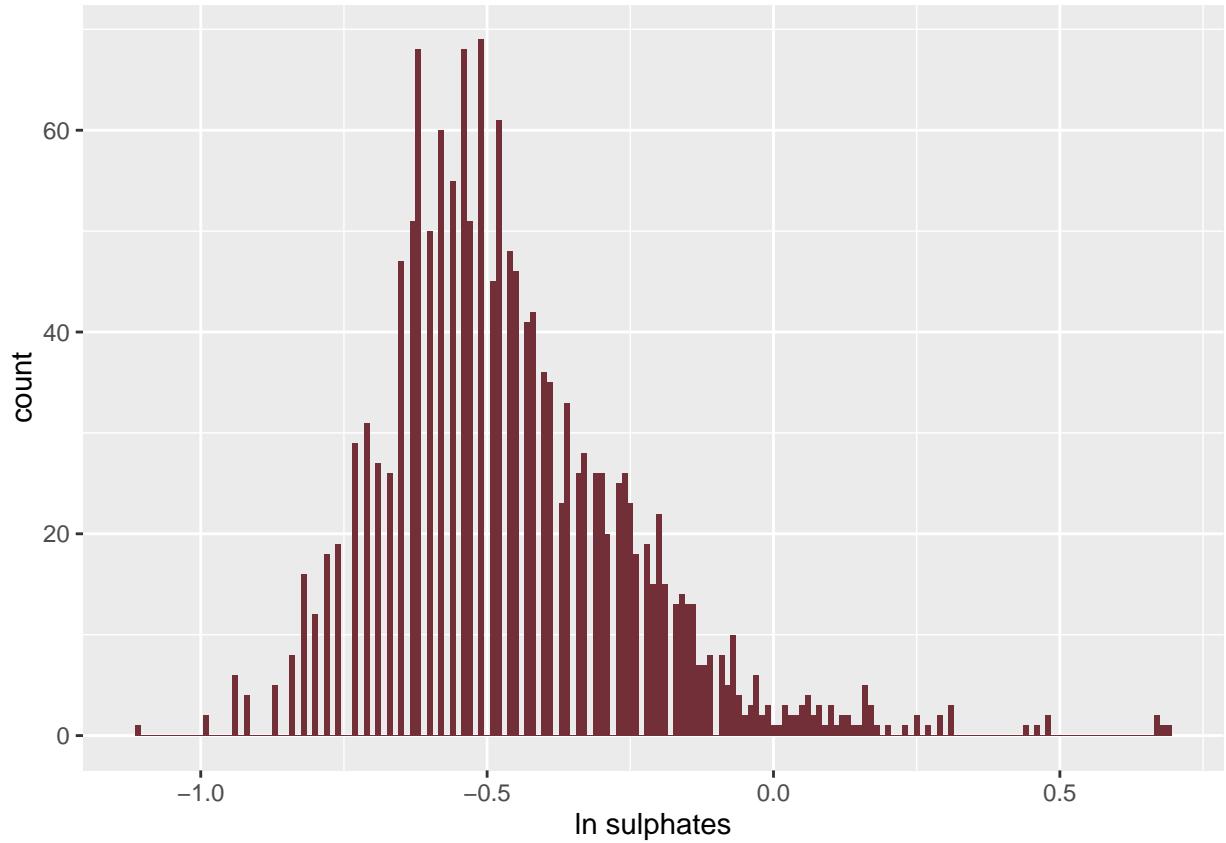
We found that a log transform of free and total sulfur dioxide did not provide improved correlation coefficients with quality when compared to the untransformed variables.

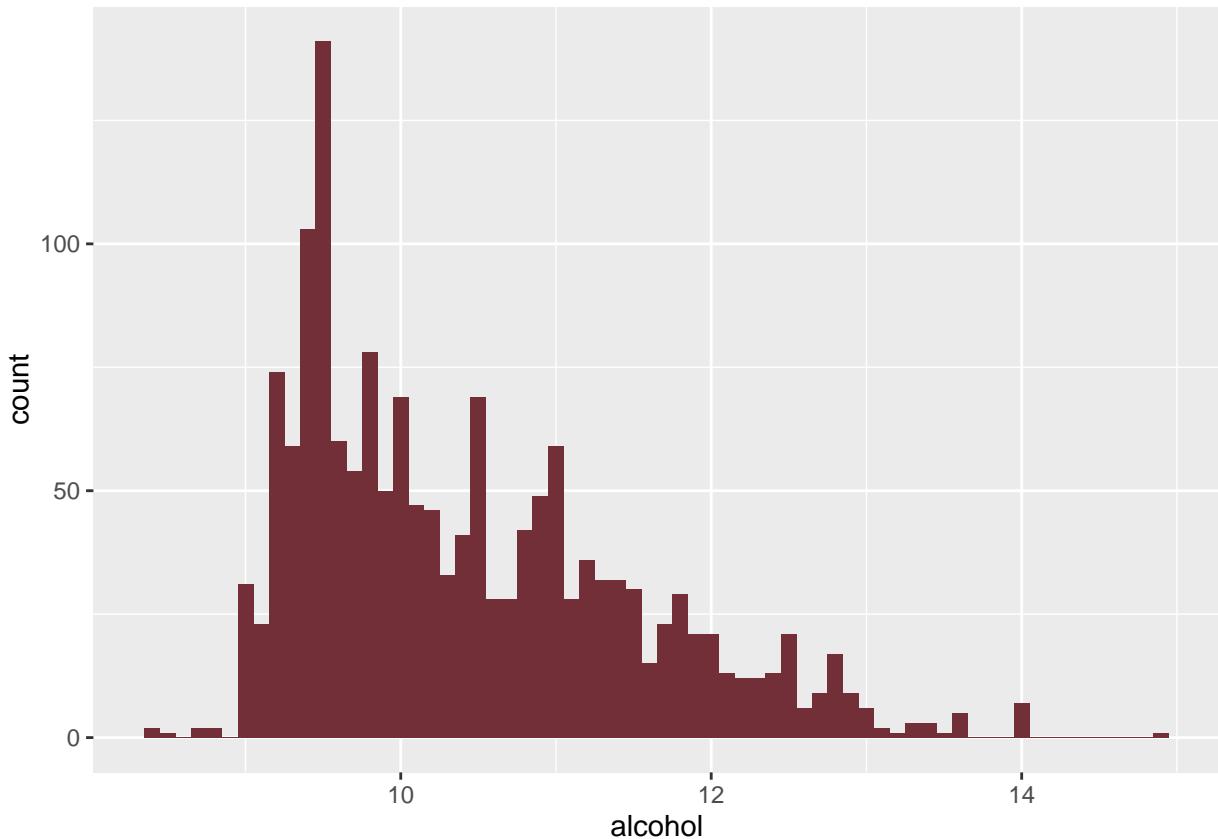












Univariate Analysis

What is the structure of your dataset?

There are 1599 observations, each having values for twelve features. Of the twelve features, eleven are chemical measures and one is a subjective quality measure. The eleven chemical measures are continuous variables that actually take discrete values depending on the number of significant figures used for the measure.

Other observations:

- Most of the chemical measures have a roughly unimodal distribution, and some have long tails.
- There is a large number of observations with a citric acid value of 0.49.
- The average quality measure is 5.636, and 82% of quality measures are either 5 or 6.

What is/are the main feature(s) of interest in your dataset?

The main feature of interest is quality. Any other features that correlate strongly with quality will be of interest.

What other features in the dataset do you think will help support your investigation into your feature(s) of interest?

We can make a priori comments regarding an anticipated relation between a chemical measure and the quality measure. For example, a component of volatile acidity is acetic acid, which can result in a vinegar taste. However, the quality measure is subjective. Therefore, the predictive value of each chemical measure is what matters.

We therefore applied log transforms to some of the predictors to provide a more normal distribution. Here, we plotted the log transforms of only chlorides and sulfates. Log transforms of free and total sulfur dioxide actually worsened the correlation coefficients with quality. A log transform of residual sugar was not significantly correlated with quality.

Did you create any new variables from existing variables in the dataset?

The log transforms of chlorides and sulphates might be considered new variables. Further, a linear combination of some features may be interesting after we have explored correlation.

Of the features you investigated, were there any unusual distributions?

Did you perform any operations on the data to tidy, adjust, or change the form of the data? If so, why did you do this?

As previously noted, we have applied a natural logarithm to two predictors.

Bivariate Plots Section

Following is a matrix of correlation coefficients (Pearson's r), followed by their p-values.

```
##      fix.H    VA citric sugar      Cl Cl.ln f.SO2 t.SO2 dense     pH     SO4
## fix.H  1.00 -0.26  0.67  0.11  0.09  0.18 -0.15 -0.11  0.67 -0.68  0.18
## VA     -0.26  1.00 -0.55  0.00  0.06  0.11 -0.01  0.08  0.02  0.23 -0.26
## citric  0.67 -0.55  1.00  0.14  0.20  0.18 -0.06  0.04  0.36 -0.54  0.31
## sugar   0.11  0.00  0.14  1.00  0.06  0.10  0.19  0.20  0.36 -0.09  0.01
## Cl      0.09  0.06  0.20  0.06  1.00  0.91  0.01  0.05  0.20 -0.27  0.37
## Cl.ln   0.18  0.11  0.18  0.10  0.91  1.00  0.00  0.06  0.35 -0.28  0.28
## f.SO2   -0.15 -0.01 -0.06  0.19  0.01  0.00  1.00  0.67 -0.02  0.07  0.05
## t.SO2   -0.11  0.08  0.04  0.20  0.05  0.06  0.67  1.00  0.07 -0.07  0.04
## dense   0.67  0.02  0.36  0.36  0.20  0.35 -0.02  0.07  1.00 -0.34  0.15
## pH      -0.68  0.23 -0.54 -0.09 -0.27 -0.28  0.07 -0.07 -0.34  1.00 -0.20
## SO4     0.18 -0.26  0.31  0.01  0.37  0.28  0.05  0.04  0.15 -0.20  1.00
## SO4.ln   0.20 -0.30  0.33  0.02  0.31  0.24  0.05  0.01  0.17 -0.15  0.98
## ETOH    -0.06 -0.20  0.11  0.04 -0.22 -0.30 -0.07 -0.21 -0.50  0.21  0.09
## qual    0.12 -0.39  0.23  0.01 -0.13 -0.18 -0.05 -0.19 -0.17 -0.06  0.25
##      SO4.ln  ETOH  qual
## fix.H   0.20 -0.06  0.12
## VA      -0.30 -0.20 -0.39
## citric  0.33  0.11  0.23
## sugar   0.02  0.04  0.01
## Cl      0.31 -0.22 -0.13
## Cl.ln   0.24 -0.30 -0.18
## f.SO2   0.05 -0.07 -0.05
## t.SO2   0.01 -0.21 -0.19
```

```

## dense    0.17 -0.50 -0.17
## pH      -0.15  0.21 -0.06
## S04     0.98  0.09  0.25
## S04.ln   1.00  0.14  0.31
## ETOH    0.14  1.00  0.48
## qual    0.31  0.48  1.00
##
## n= 1599
##
##
## P
##      fix.H  VA      citric sugar Cl      Cl.ln f.S02 t.S02 dense
## fix.H      0.0000 0.0000 0.0000 0.0002 0.0000 0.0000 0.0000 0.0000
## VA        0.0000      0.0000 0.9389 0.0142 0.0000 0.6747 0.0022 0.3788
## citric   0.0000 0.0000      0.0000 0.0000 0.0000 0.0147 0.1555 0.0000
## sugar    0.0000 0.9389 0.0000      0.0262 0.0000 0.0000 0.0000 0.0000
## Cl       0.0002 0.0142 0.0000 0.0262      0.0000 0.8241 0.0581 0.0000
## Cl.ln    0.0000 0.0000 0.0000 0.0000      0.0000 0.9301 0.0196 0.0000
## f.S02    0.0000 0.6747 0.0147 0.0000 0.8241 0.9301      0.0000 0.3805
## t.S02    0.0000 0.0022 0.1555 0.0000 0.0581 0.0196 0.0000      0.0044
## dense   0.0000 0.3788 0.0000 0.0000 0.0000 0.0000 0.3805 0.0044
## pH      0.0000 0.0000 0.0000 0.0006 0.0000 0.0000 0.0049 0.0078 0.0000
## S04     0.0000 0.0000 0.0000 0.8252 0.0000 0.0000 0.0389 0.0860 0.0000
## S04.ln   0.0000 0.0000 0.0000 0.5222 0.0000 0.0000 0.0547 0.5952 0.0000
## ETOH    0.0136 0.0000 0.0000 0.0926 0.0000 0.0000 0.0055 0.0000 0.0000
## qual    0.0000 0.0000 0.0000 0.5832 0.0000 0.0000 0.0428 0.0000 0.0000
##      pH     S04    S04.ln ETOH   qual
## fix.H  0.0000 0.0000 0.0000 0.0136 0.0000
## VA     0.0000 0.0000 0.0000 0.0000 0.0000
## citric 0.0000 0.0000 0.0000 0.0000 0.0000
## sugar  0.0006 0.8252 0.5222 0.0926 0.5832
## Cl     0.0000 0.0000 0.0000 0.0000 0.0000
## Cl.ln  0.0000 0.0000 0.0000 0.0000 0.0000
## f.S02  0.0049 0.0389 0.0547 0.0055 0.0428
## t.S02  0.0078 0.0860 0.5952 0.0000 0.0000
## dense  0.0000 0.0000 0.0000 0.0000 0.0000
## pH     0.0000 0.0000 0.0000 0.0000 0.0210
## S04    0.0000      0.0000 0.0002 0.0000
## S04.ln 0.0000 0.0000      0.0000 0.0000
## ETOH   0.0000 0.0002 0.0000      0.0000
## qual   0.0210 0.0000 0.0000 0.0000

```

Let's start by examining the correlation coefficients for quality.

```

##                  quality
## quality           1.00
## alcohol          0.48
## sulphates        0.25
## citric.acid     0.23
## fixed.acidity   0.12
## X                0.07
## residual.sugar  0.01
## free.sulfur.dioxide -0.05
## pH               -0.06
## chlorides        -0.13

```

```

## density           -0.17
## total.sulfur.dioxide -0.19
## volatile.acidity   -0.39

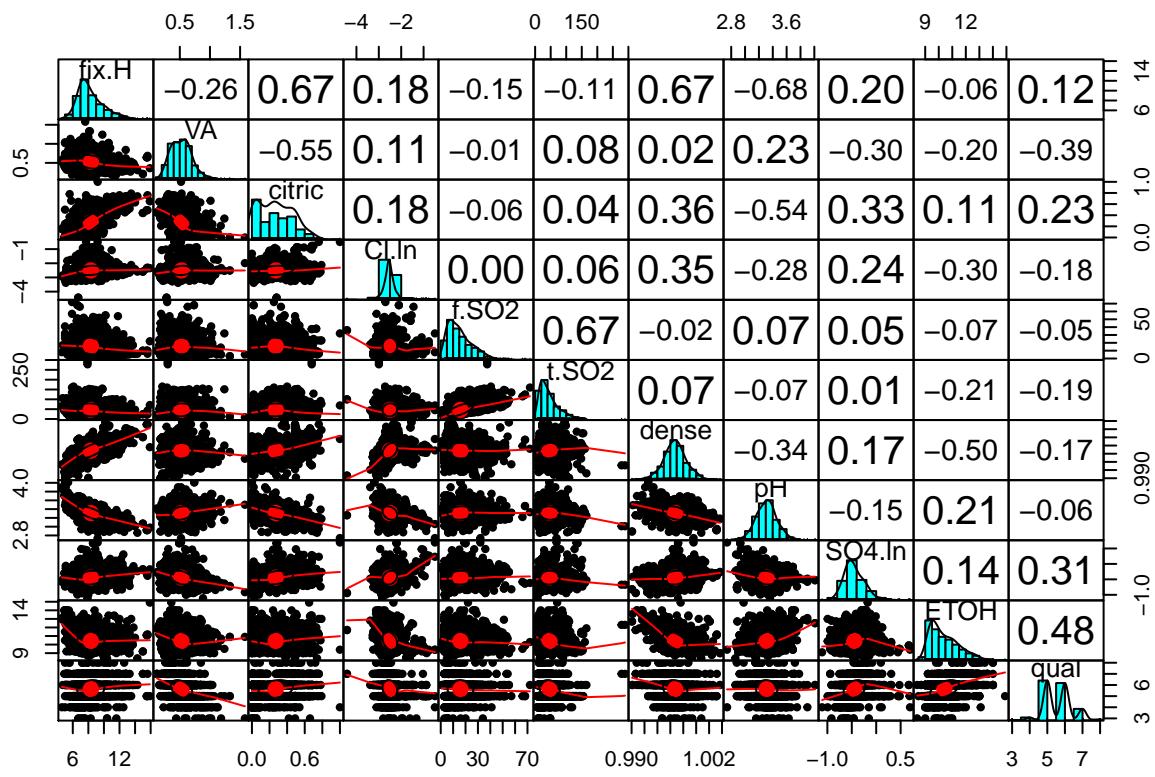
## 
## Pearson's product-moment correlation
##
## data: wine$quality and log(wine$chlorides)
## t = -7.1508, df = 1597, p-value = 1.308e-12
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
## -0.2232336 -0.1282260
## sample estimates:
##       cor
## -0.17614

## 
## Pearson's product-moment correlation
##
## data: wine$quality and log(wine$sulphates)
## t = 12.967, df = 1597, p-value < 2.2e-16
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
## 0.2636092 0.3523323
## sample estimates:
##       cor
## 0.3086419

```

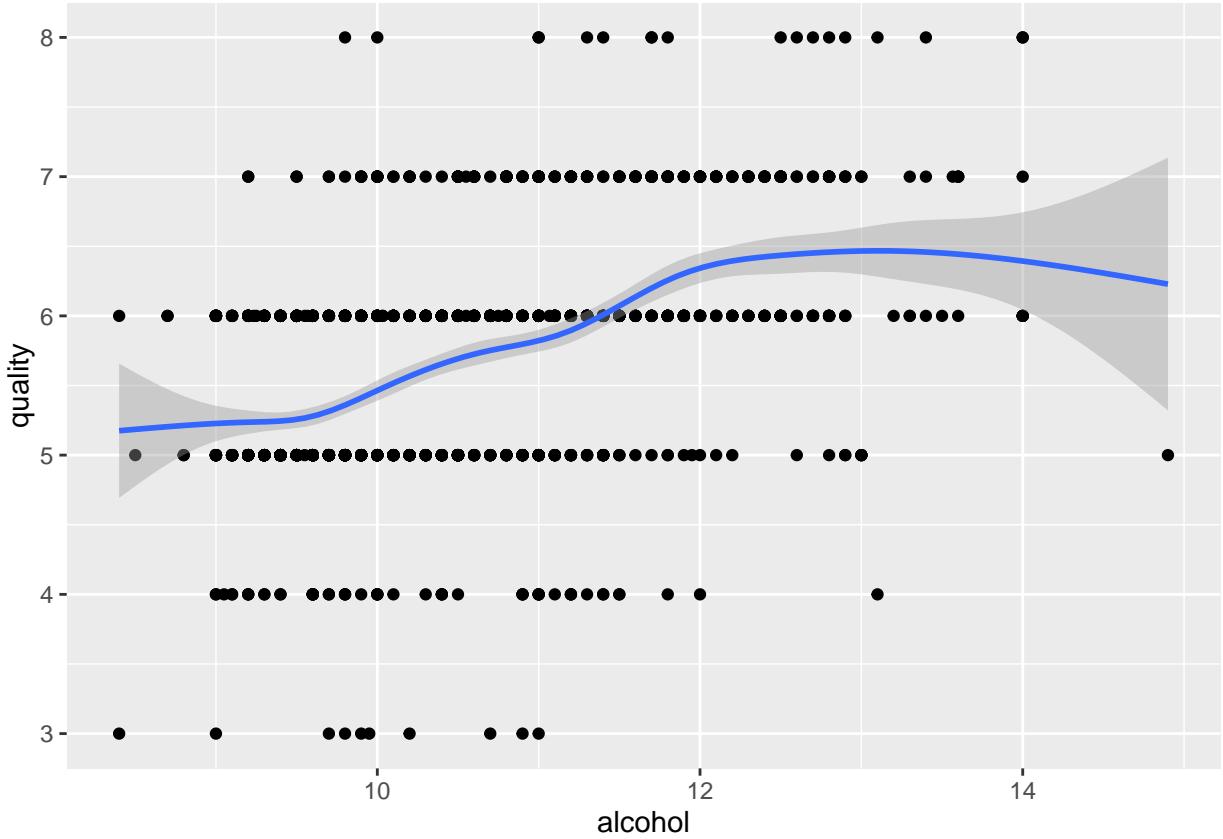
From the p-value table, we see there is no significant correlation between quality and residual sugar ($p < 0.05$). Log transformations of chlorides and sulphates provide minuscule p-values and improved correlation coefficients.

Let's subset the data to exclude residual sugar and X. Note we log-transformed chlorides and sulphates in the following panel.



We will now plot the variables that had significant correlation with quality.

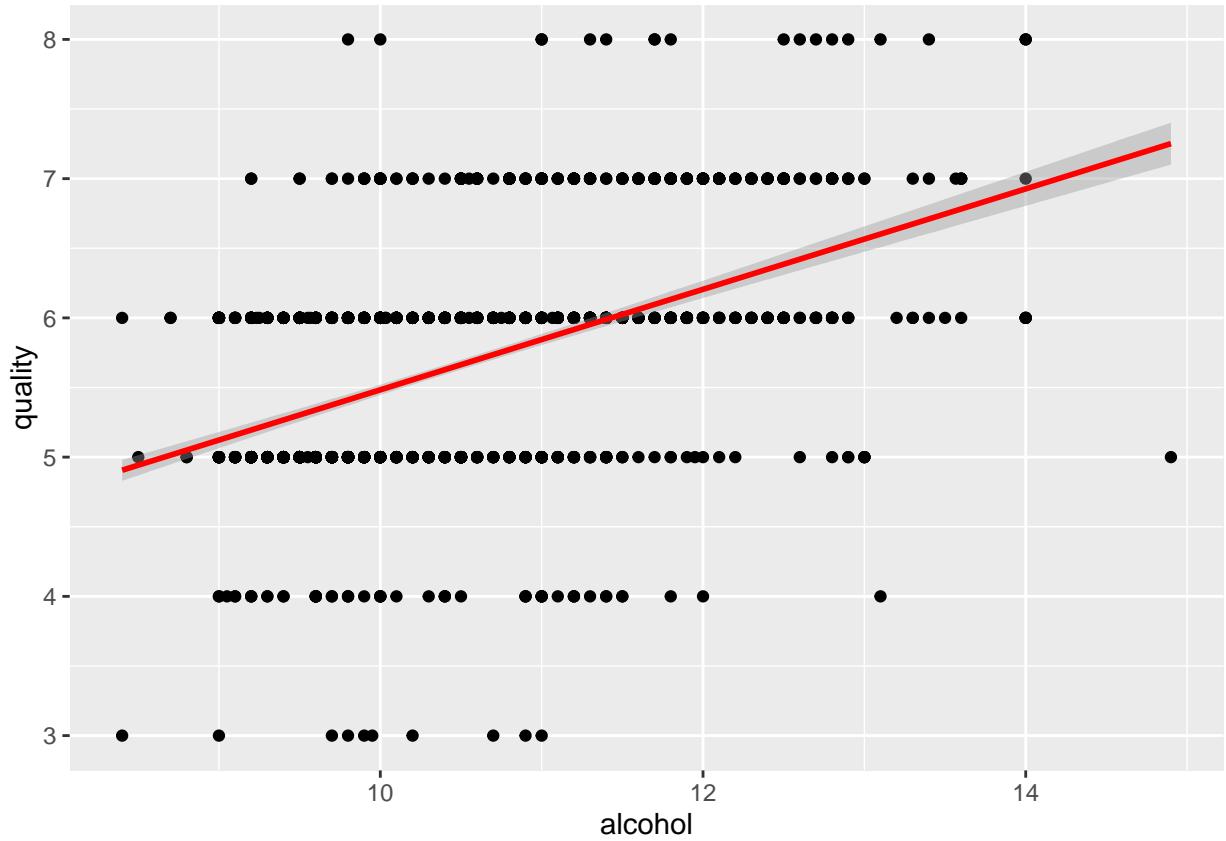
The plots include a 95% confidence interval and are followed by plots of the residuals.

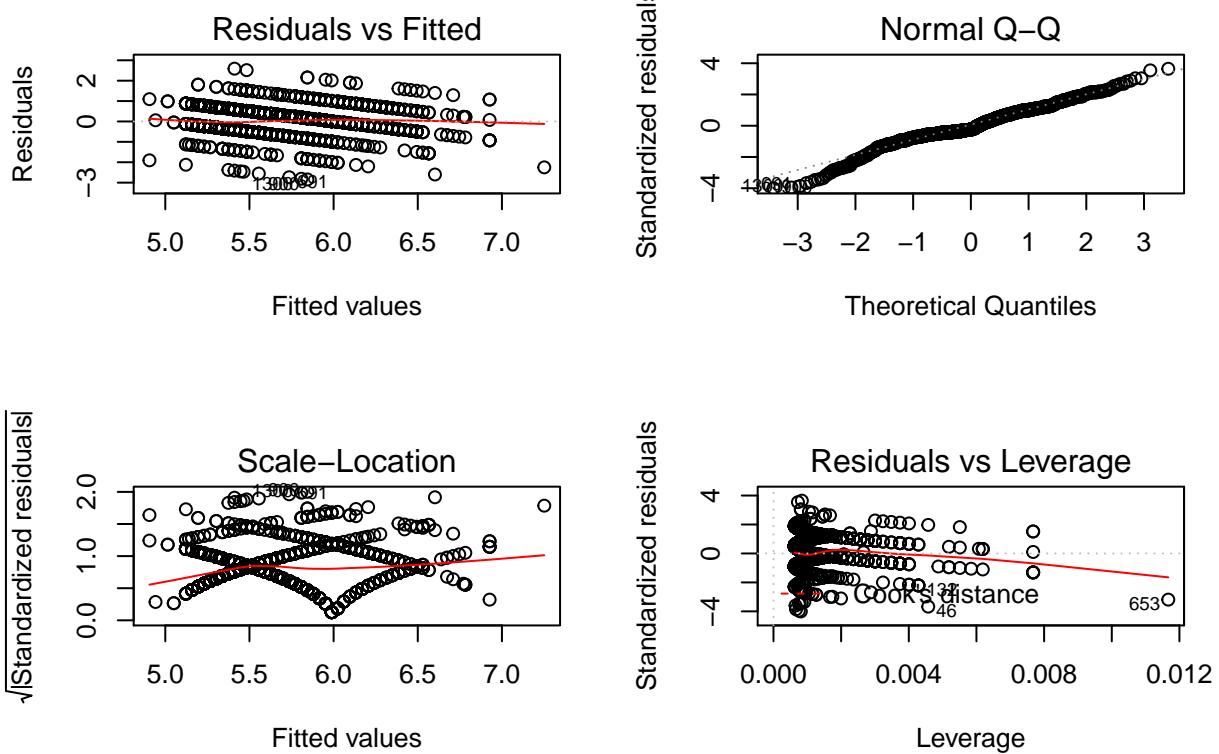


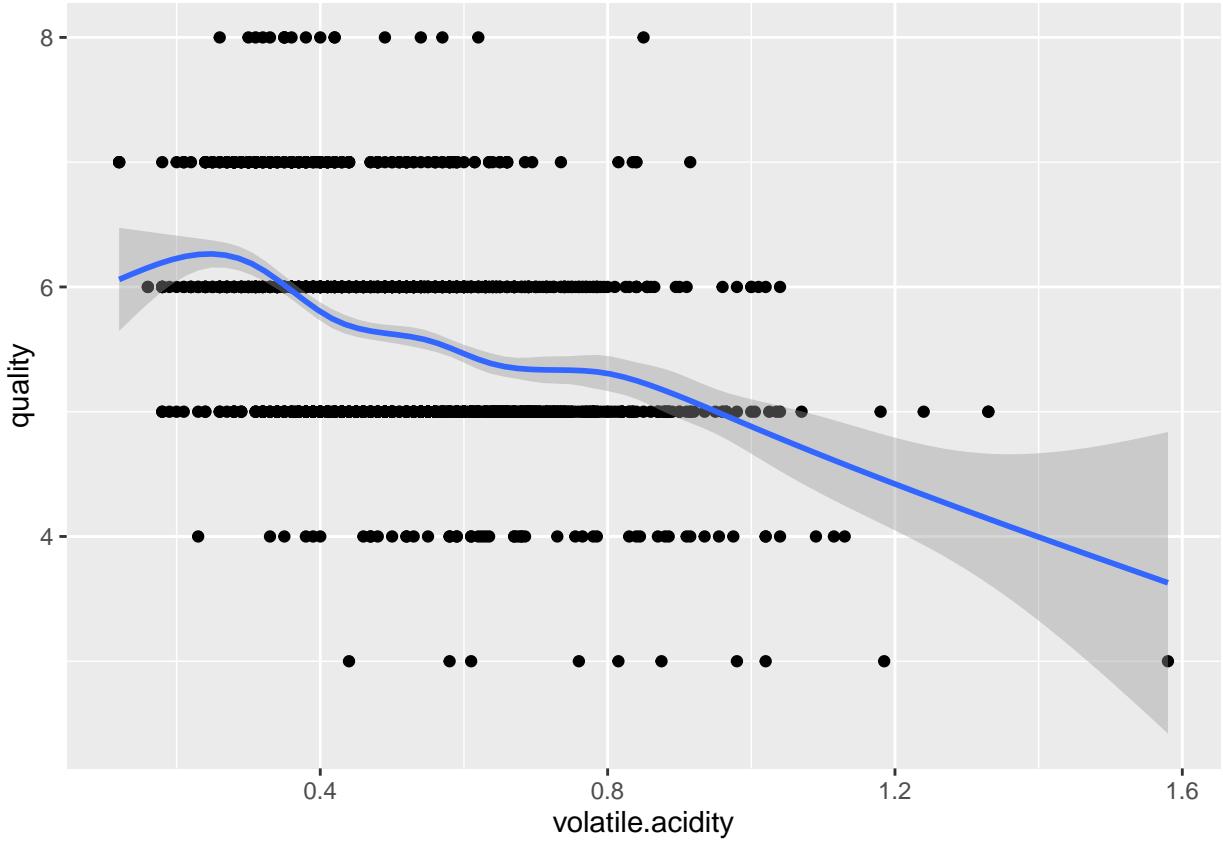
```

## Correlation coefficient: 0.476166
##
## Call:
## lm(formula = quality ~ alcohol, data = wine)
##
## Residuals:
##    Min     1Q   Median     3Q    Max 
## -2.8442 -0.4112 -0.1690  0.5166  2.5888 
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 1.87497   0.17471   10.73 <2e-16 ***
## alcohol      0.36084   0.01668   21.64 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7104 on 1597 degrees of freedom
## Multiple R-squared:  0.2267, Adjusted R-squared:  0.2263 
## F-statistic: 468.3 on 1 and 1597 DF,  p-value: < 2.2e-16

```



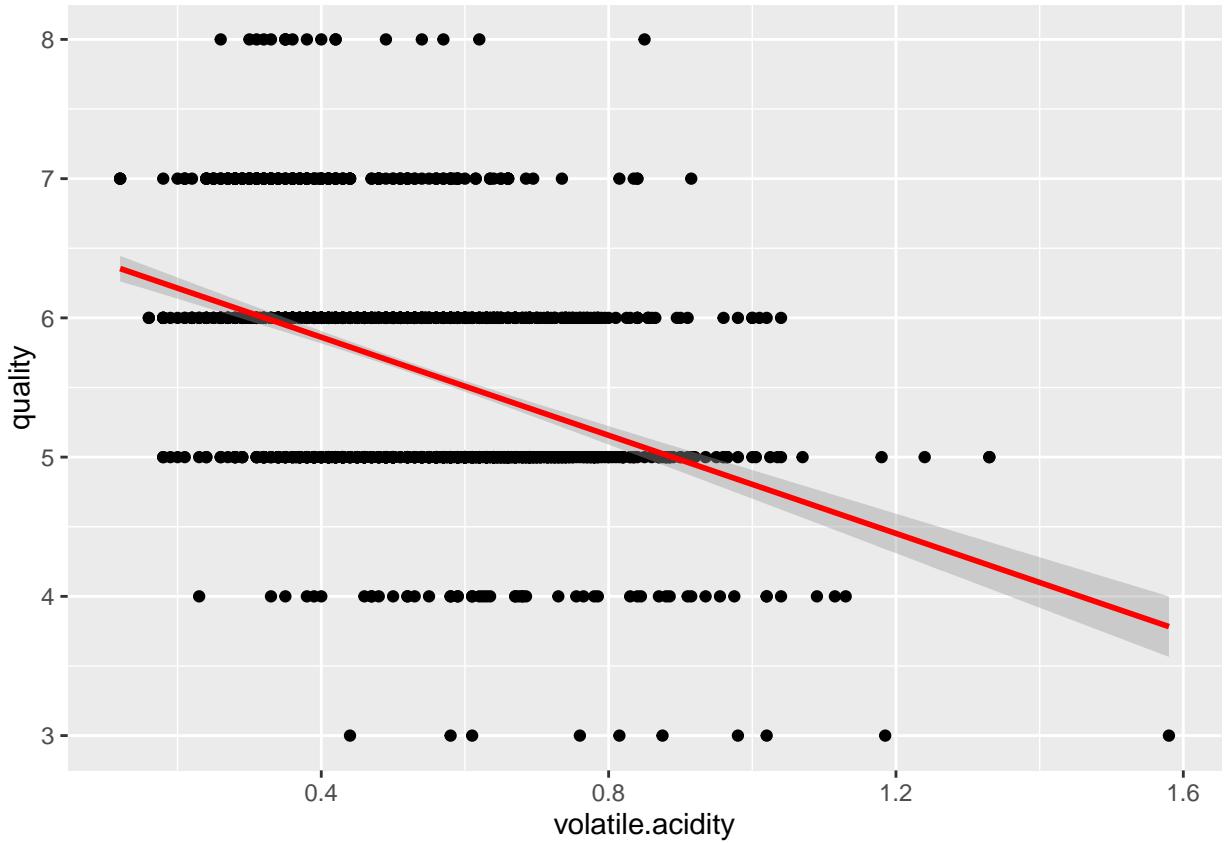


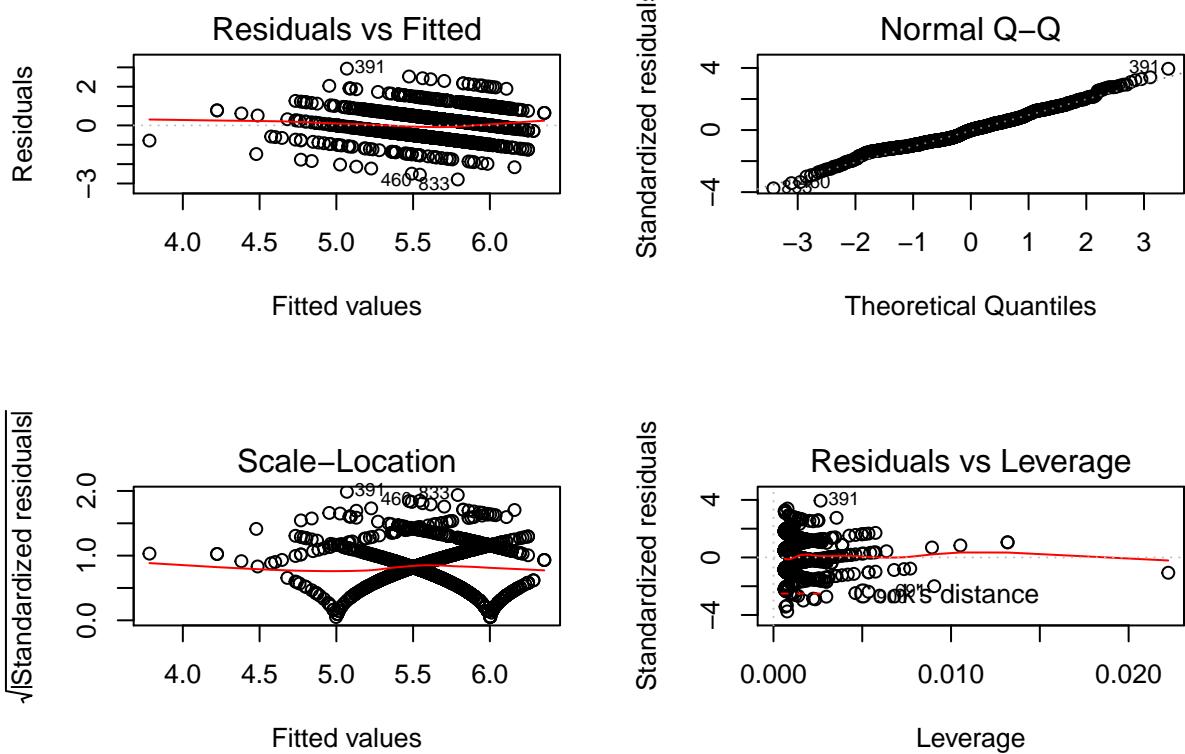


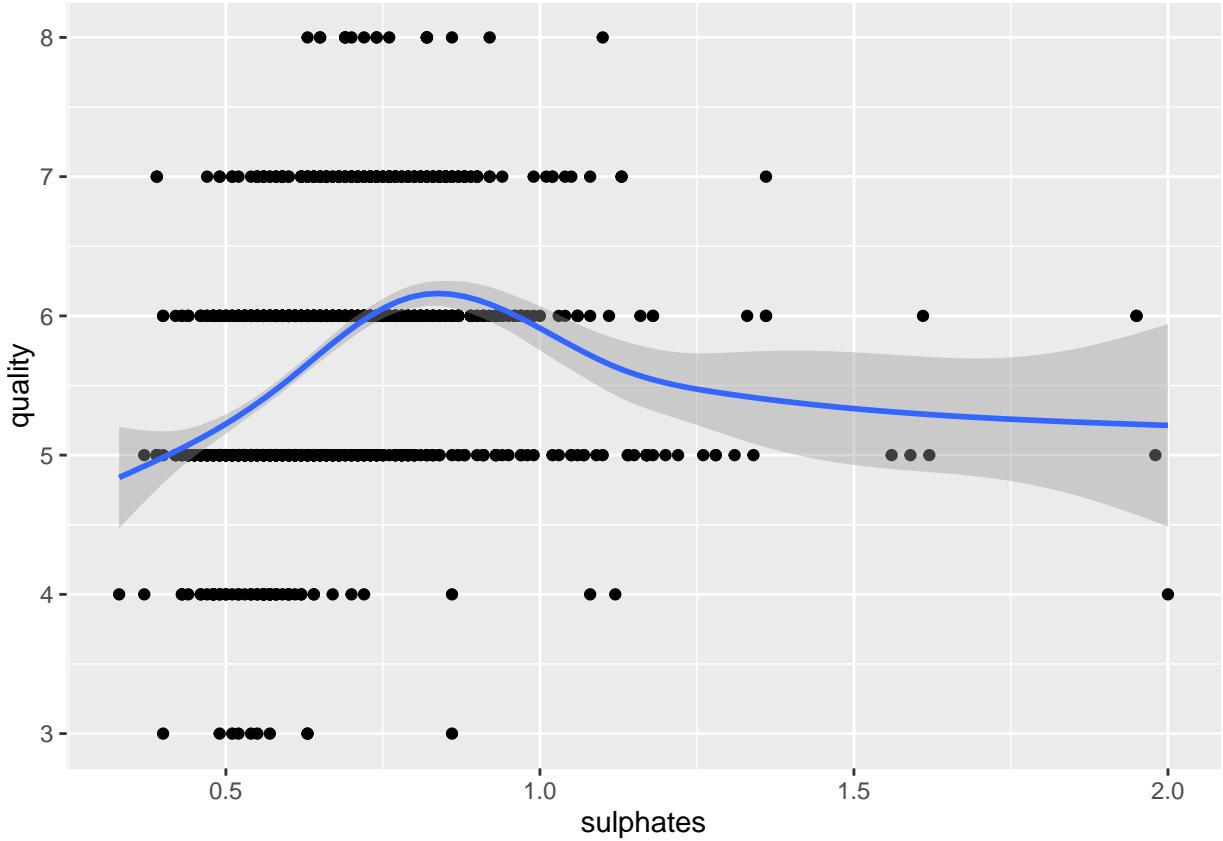
```

## Correlation coefficient: -0.390558
##
## Call:
## lm(formula = quality ~ volatile.acidity, data = wine)
##
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -2.79071 -0.54411 -0.00687  0.47350  2.93148 
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 6.56575   0.05791 113.39 <2e-16 ***
## volatile.acidity -1.76144   0.10389 -16.95 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7437 on 1597 degrees of freedom
## Multiple R-squared:  0.1525, Adjusted R-squared:  0.152 
## F-statistic: 287.4 on 1 and 1597 DF,  p-value: < 2.2e-16

```



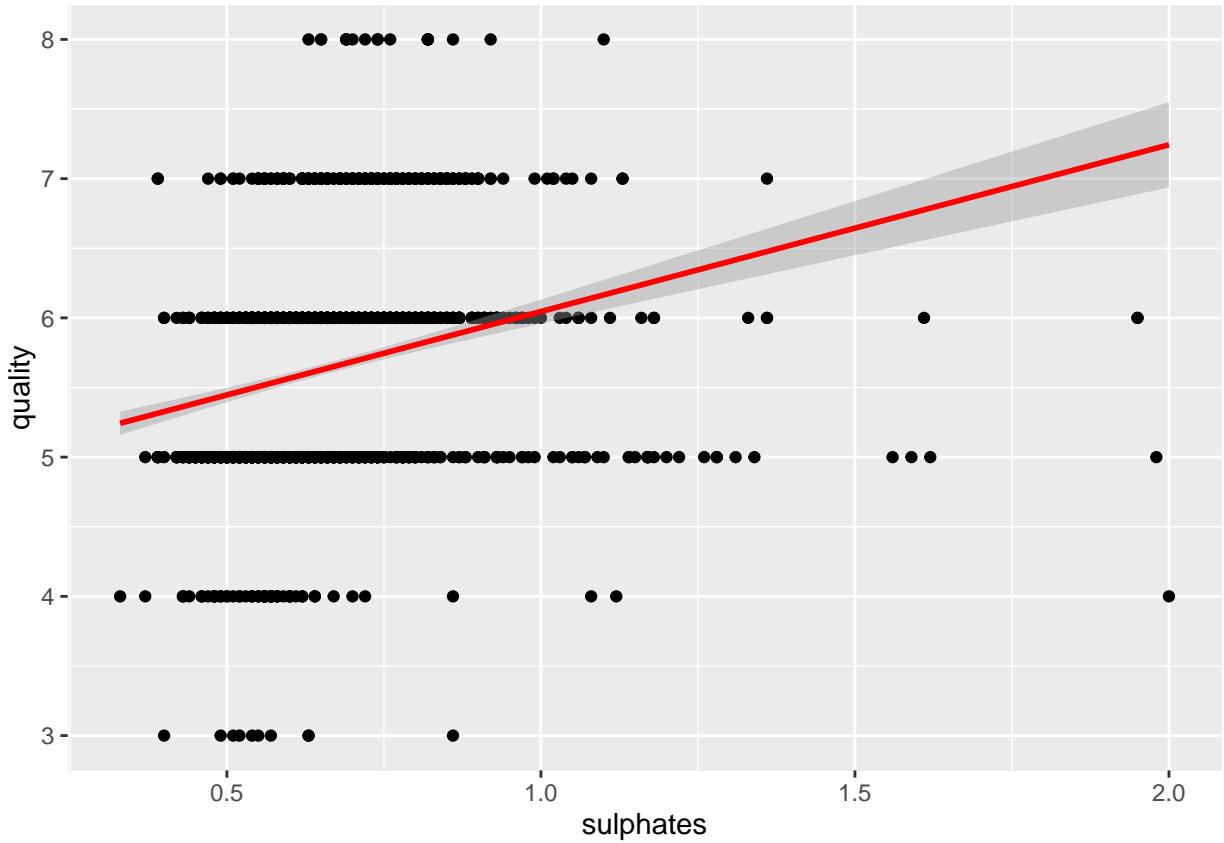


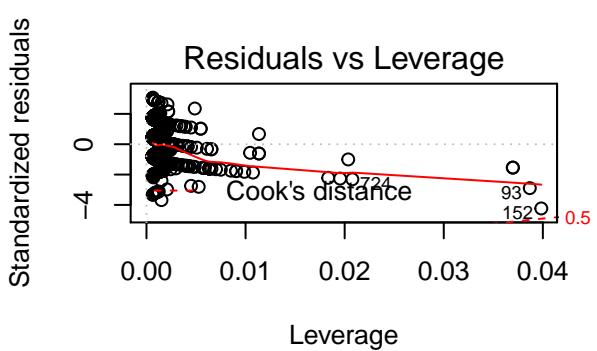
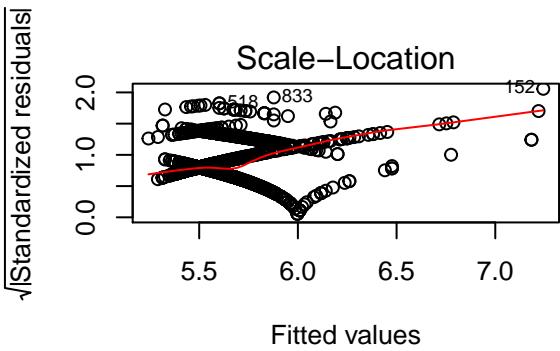
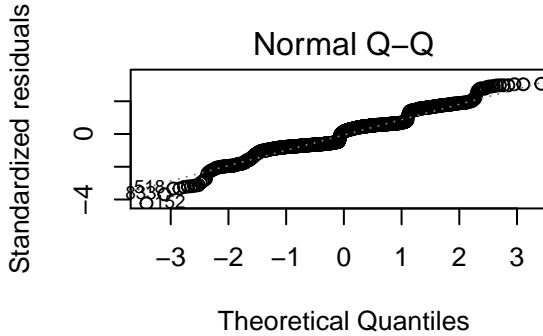
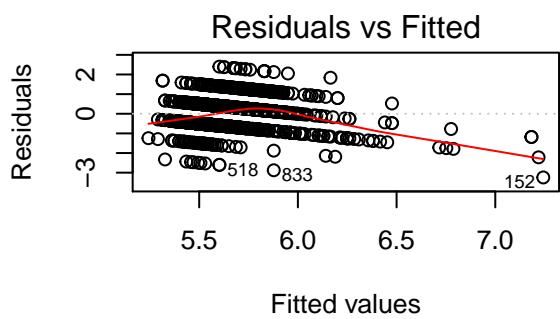


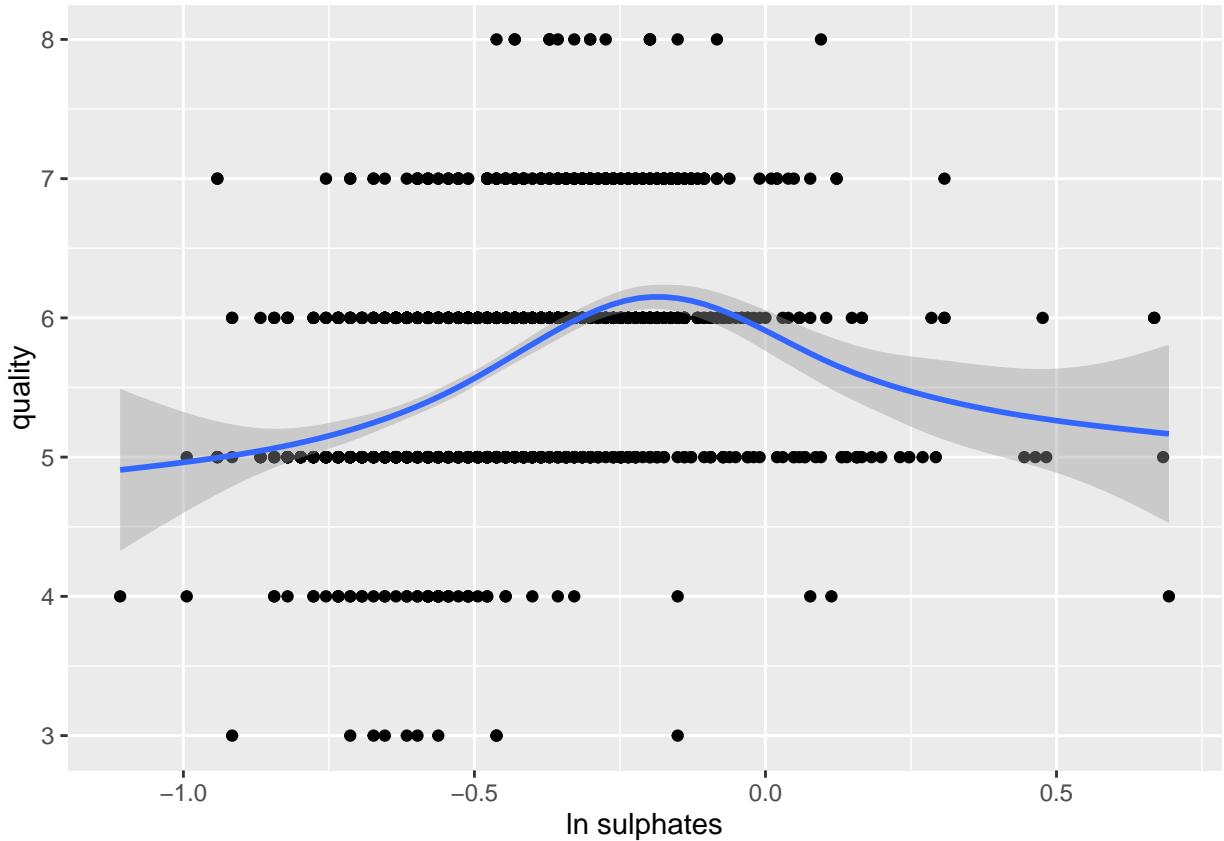
```

## Correlation coefficient: 0.251397
##
## Call:
## lm(formula = quality ~ sulphates, data = wine)
##
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -3.2432 -0.5424  0.1102  0.4456  2.3977 
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 4.84775   0.07842   61.82 <2e-16 ***
## sulphates   1.19771   0.11539   10.38 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7819 on 1597 degrees of freedom
## Multiple R-squared:  0.0632, Adjusted R-squared:  0.06261 
## F-statistic: 107.7 on 1 and 1597 DF,  p-value: < 2.2e-16

```



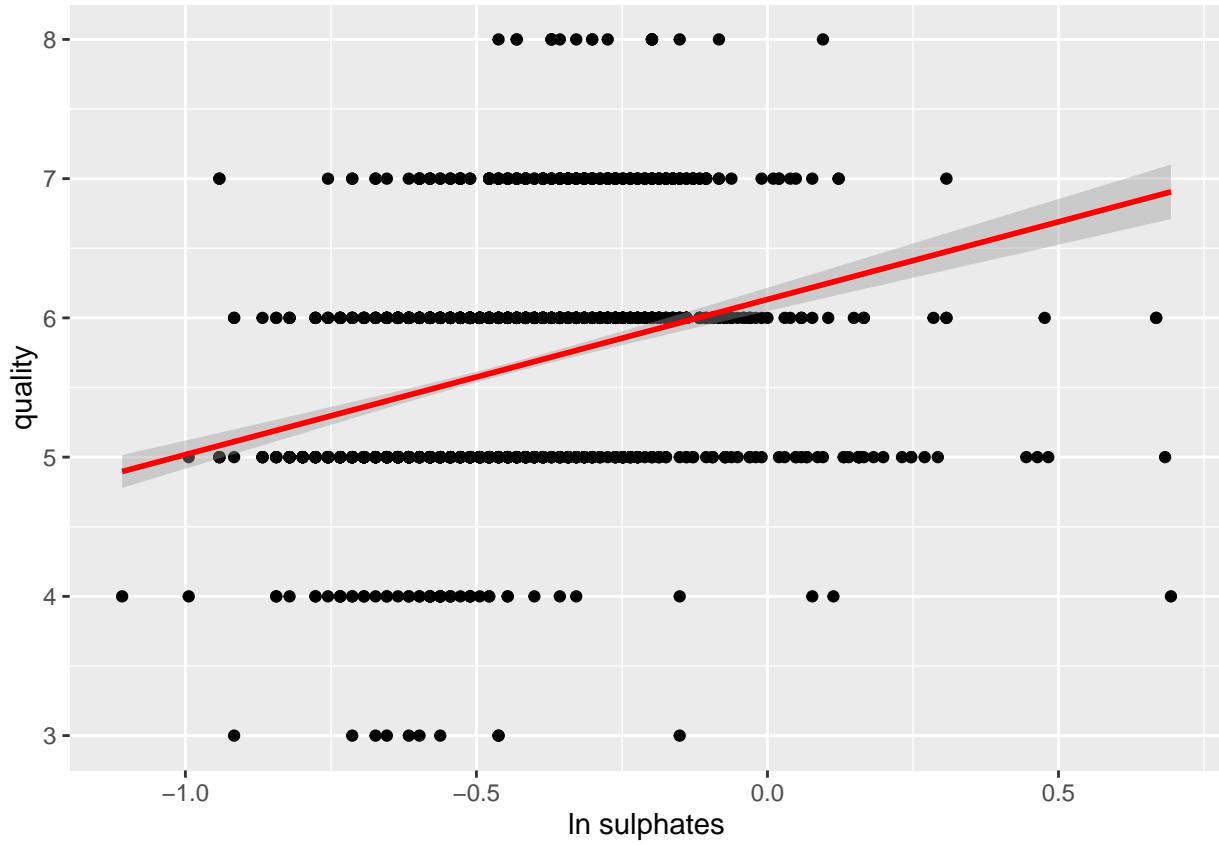


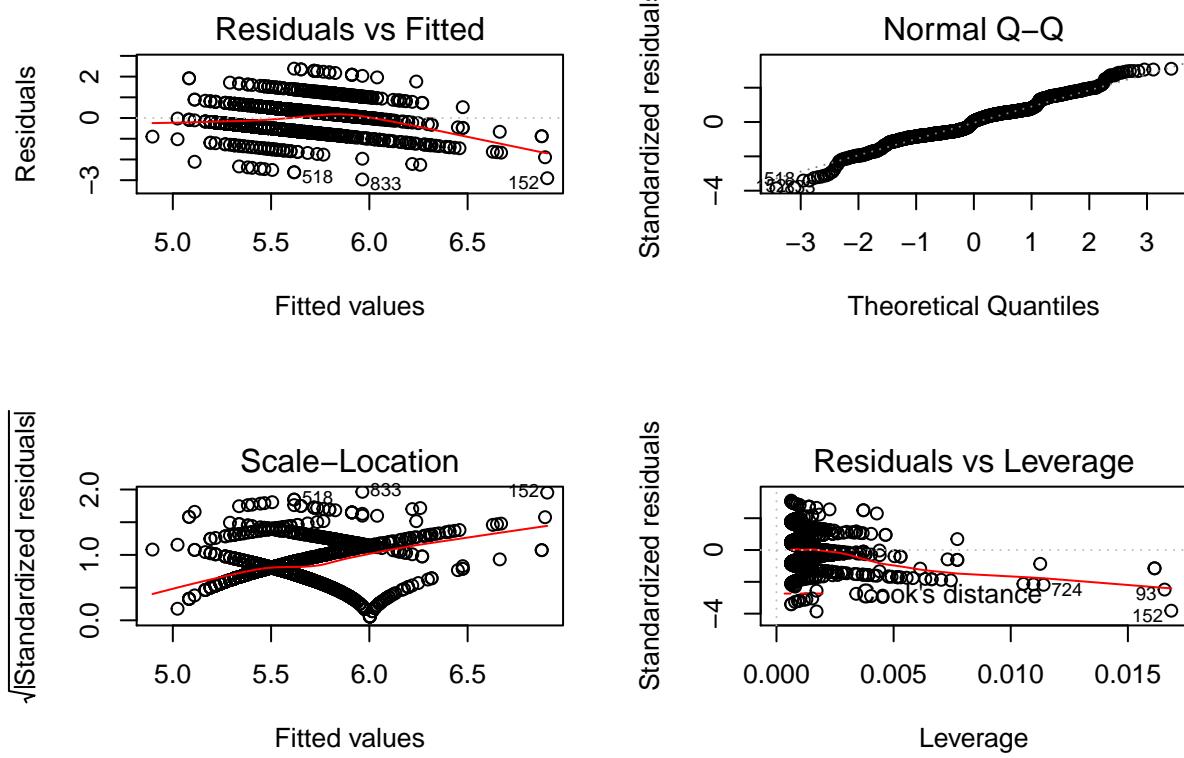


```

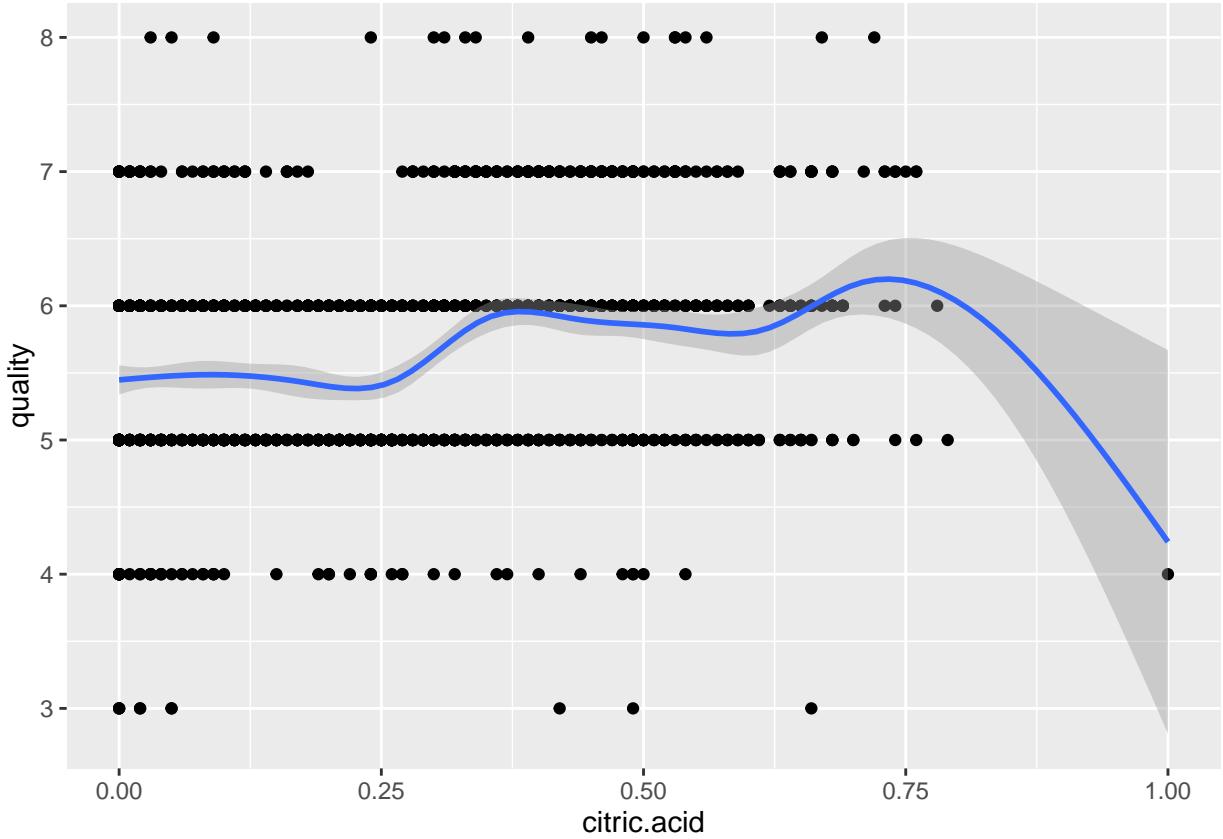
## Correlation coefficient: 0.308642
##
## Call:
## lm(formula = quality ~ log(sulphates), data = wine)
##
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -2.96428 -0.52513  0.02283  0.45581  2.38268 
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 6.13243   0.04283 143.17 <2e-16 ***
## log(sulphates) 1.11488   0.08598 12.97 <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7684 on 1597 degrees of freedom
## Multiple R-squared: 0.09526, Adjusted R-squared: 0.09469 
## F-statistic: 168.1 on 1 and 1597 DF, p-value: < 2.2e-16

```





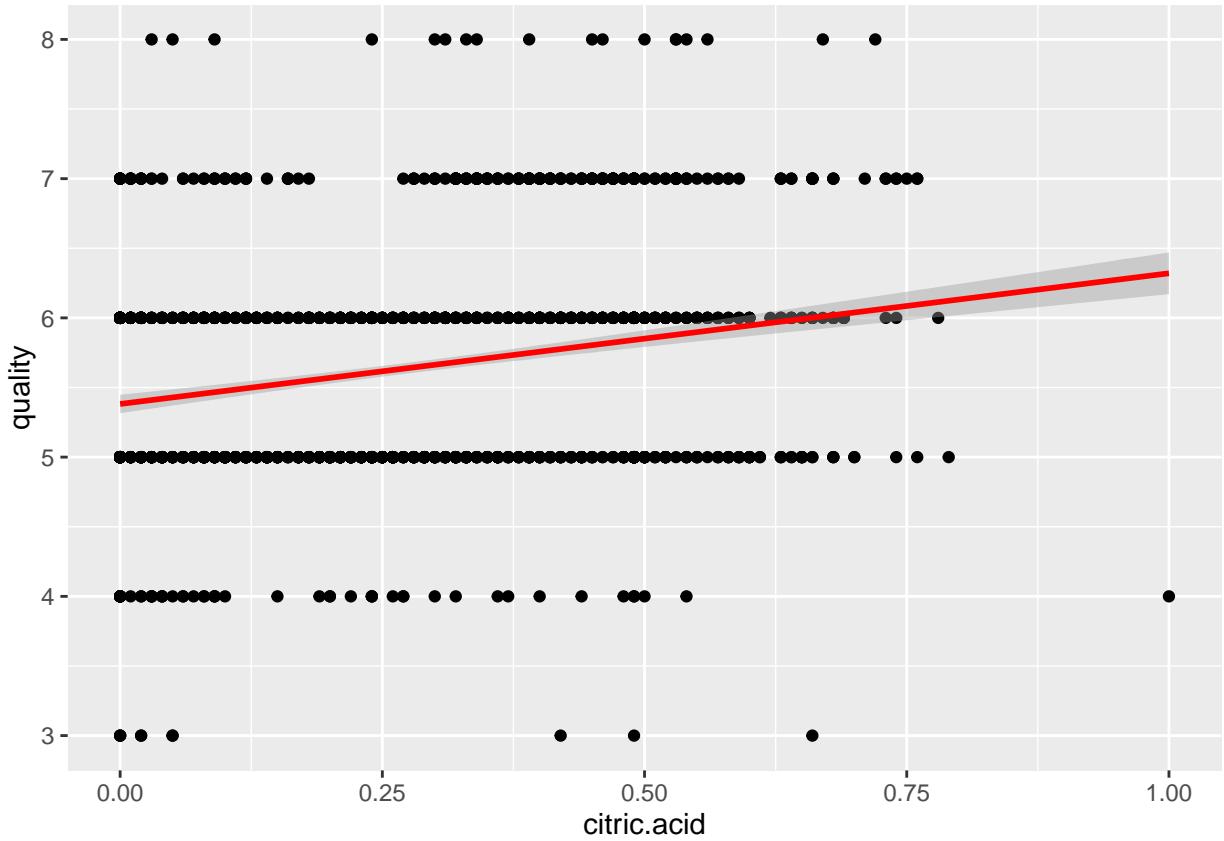
As expected, the residuals plot is improved by log-transforming sulphates.

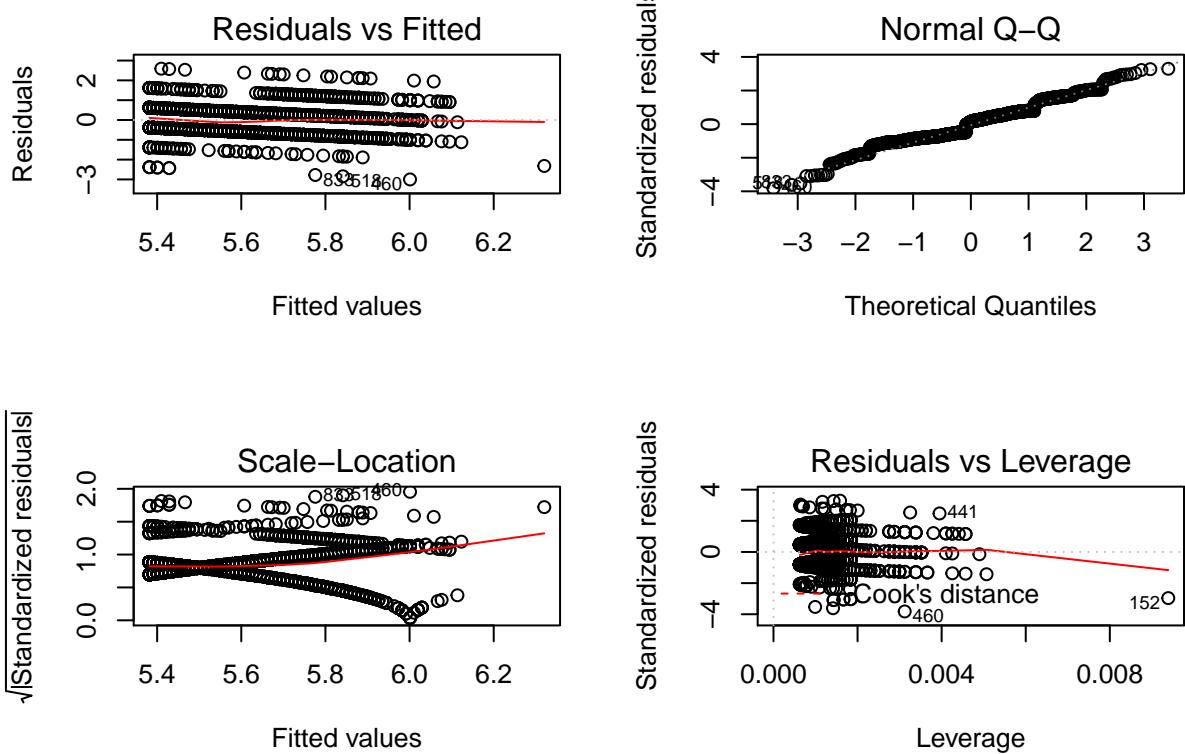


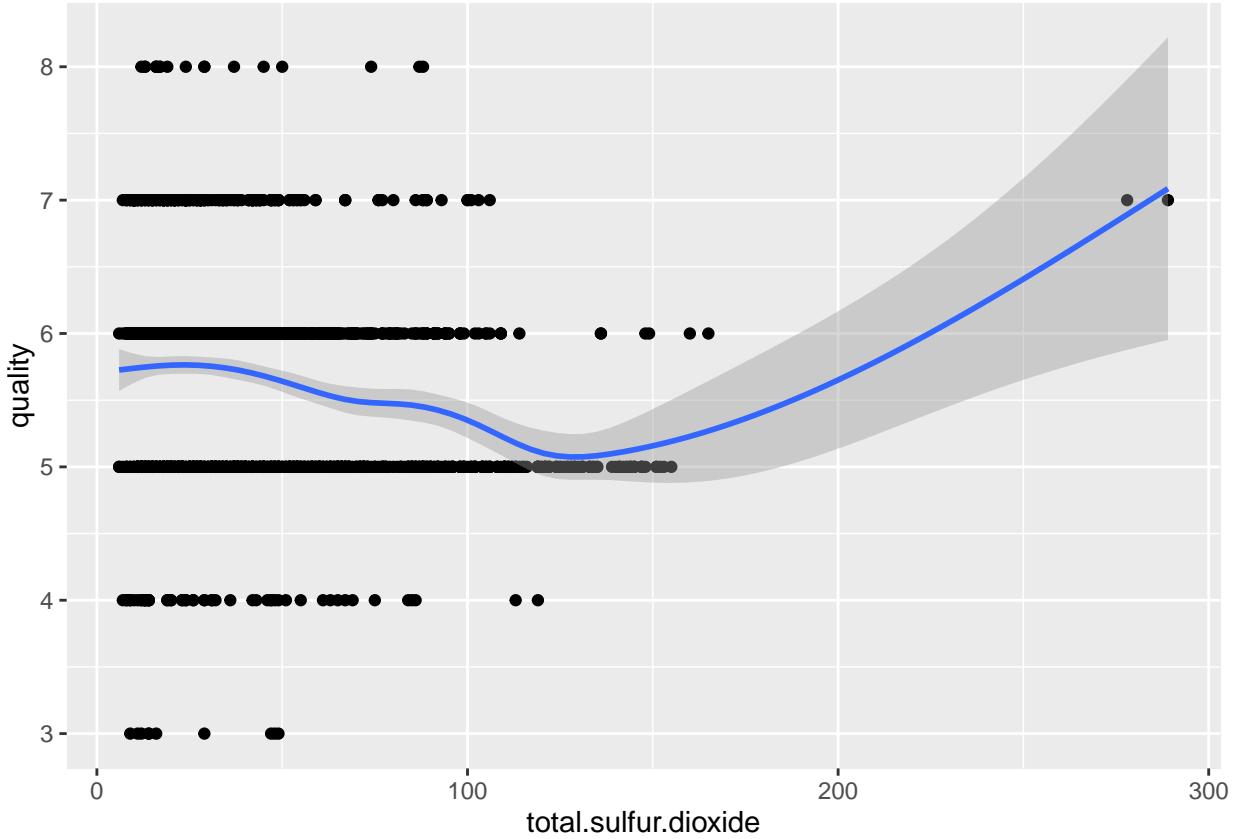
```

## Correlation coefficient: 0.226373
##
## Call:
## lm(formula = quality ~ citric.acid, data = wine)
##
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -3.0011 -0.5976  0.1021  0.5057  2.5901 
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 5.38172   0.03372 159.610 <2e-16 ***
## citric.acid 0.93845   0.10104   9.288 <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7869 on 1597 degrees of freedom
## Multiple R-squared: 0.05124,    Adjusted R-squared: 0.05065 
## F-statistic: 86.26 on 1 and 1597 DF, p-value: < 2.2e-16

```



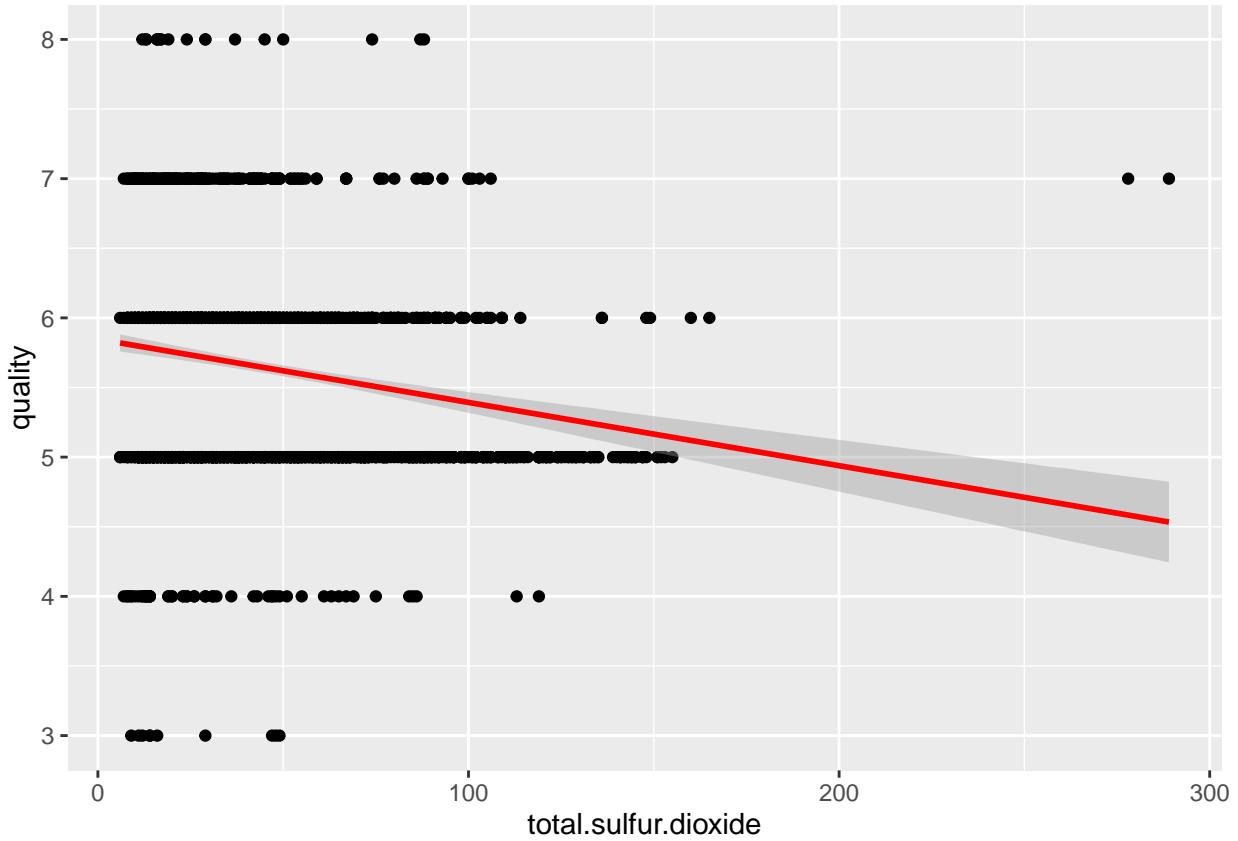


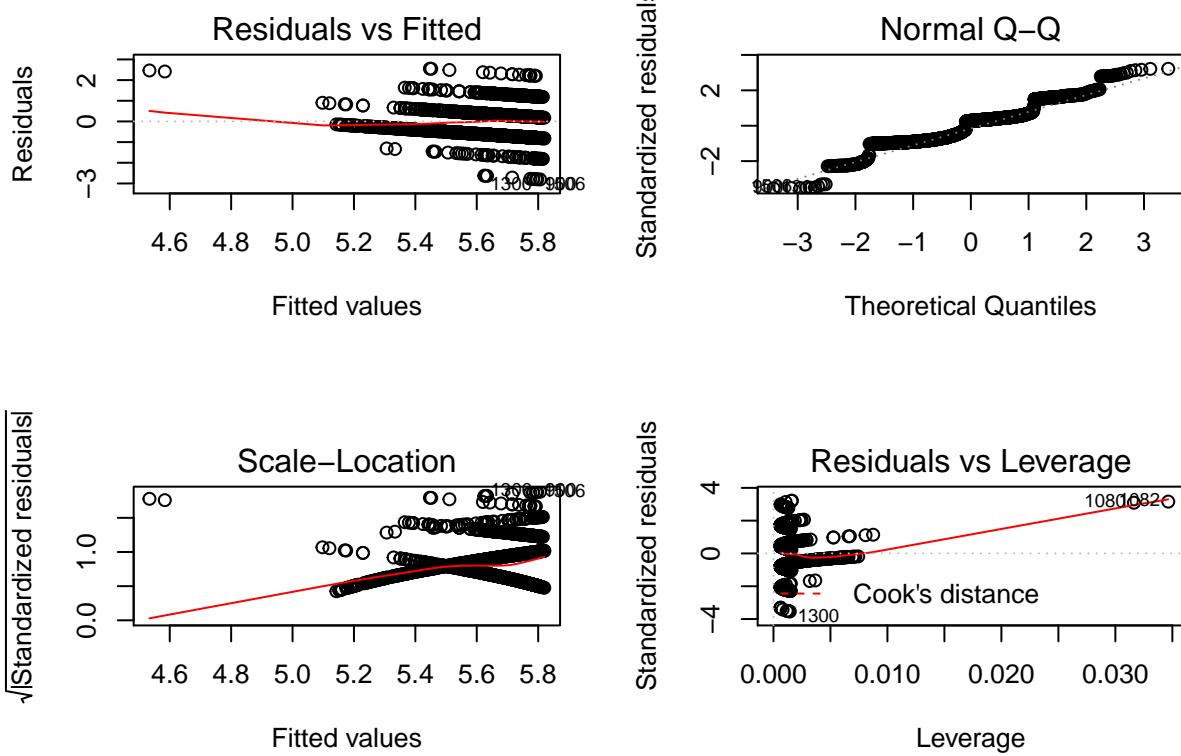


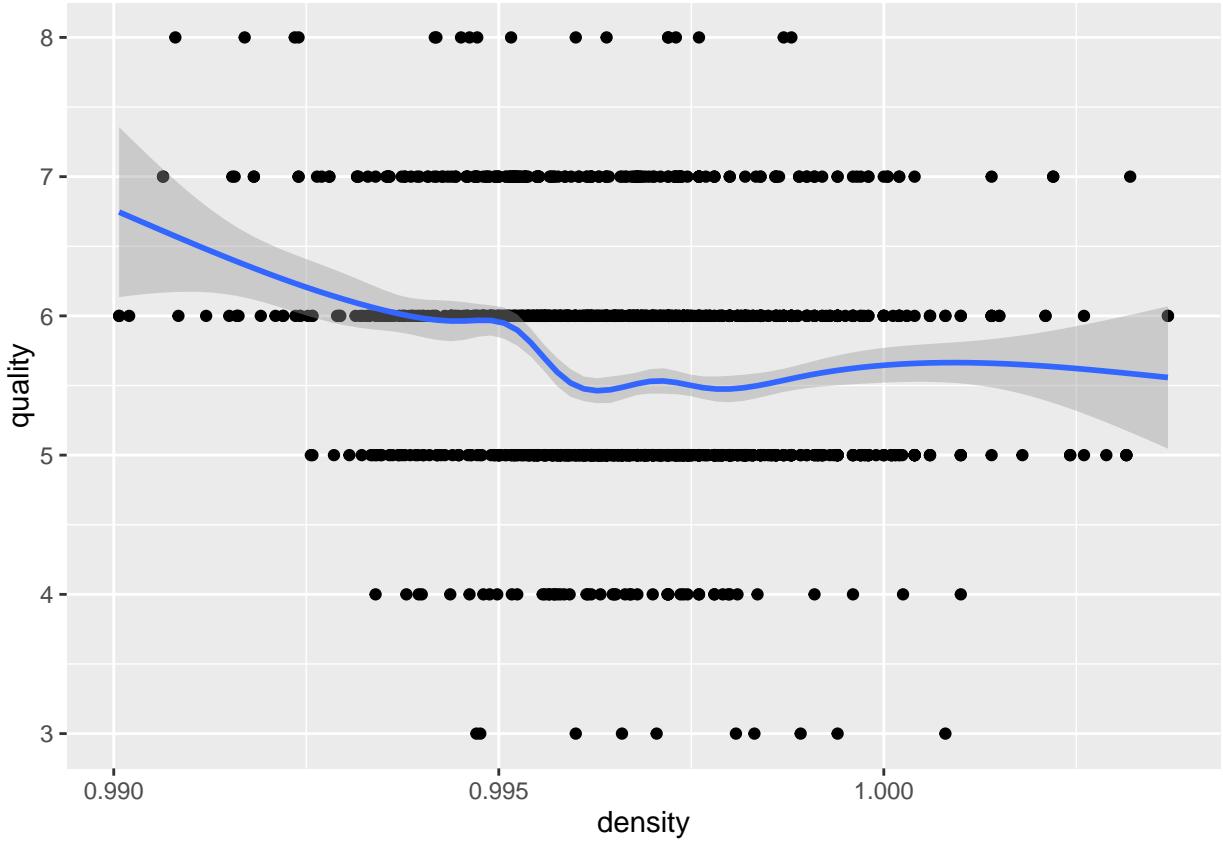
```

## Correlation coefficient: -0.185100
##
## Call:
## lm(formula = quality ~ total.sulfur.dioxide, data = wine)
##
## Residuals:
##    Min     1Q   Median     3Q    Max 
## -2.8063 -0.6336  0.2164  0.3800  2.5527 
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 5.8471792  0.0343670 170.140 < 2e-16 ***
## total.sulfur.dioxide -0.0045442  0.0006037 -7.527 8.62e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7939 on 1597 degrees of freedom
## Multiple R-squared:  0.03426,    Adjusted R-squared:  0.03366 
## F-statistic: 56.66 on 1 and 1597 DF,  p-value: 8.622e-14

```



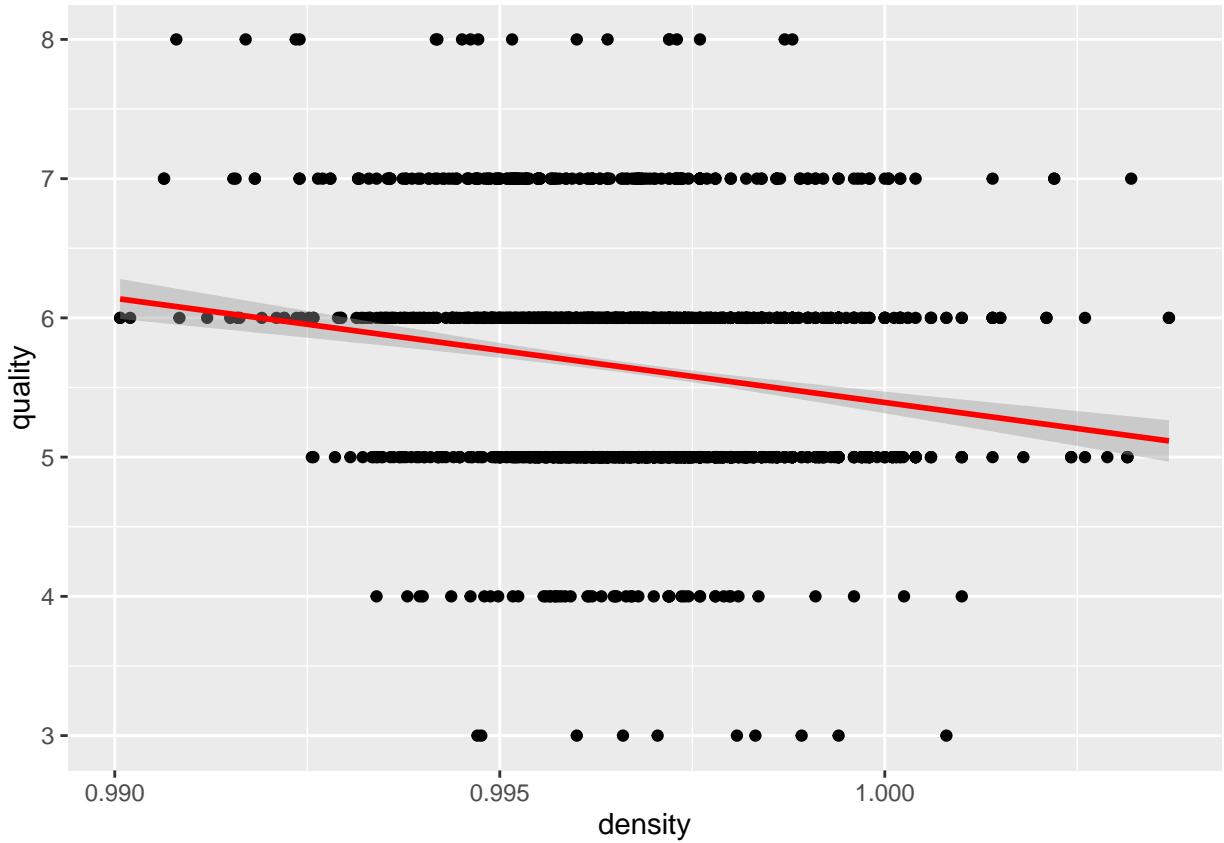


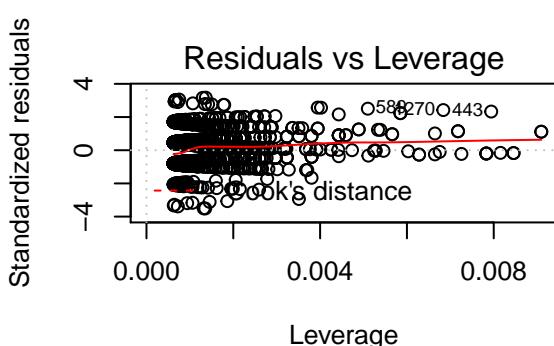
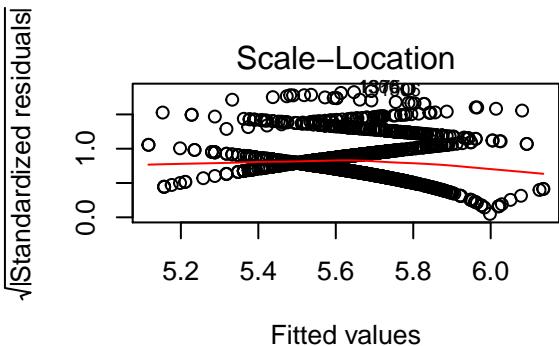
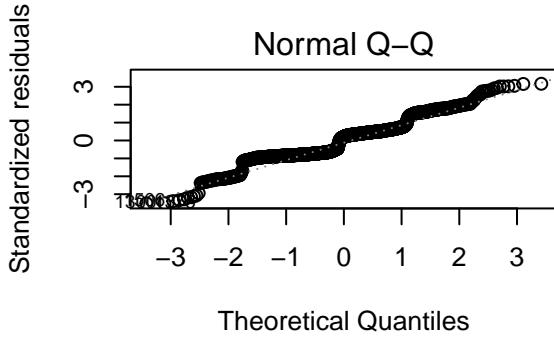
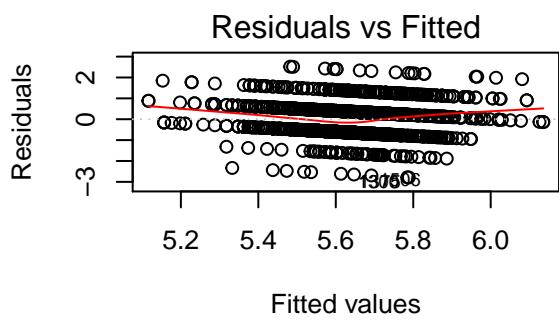


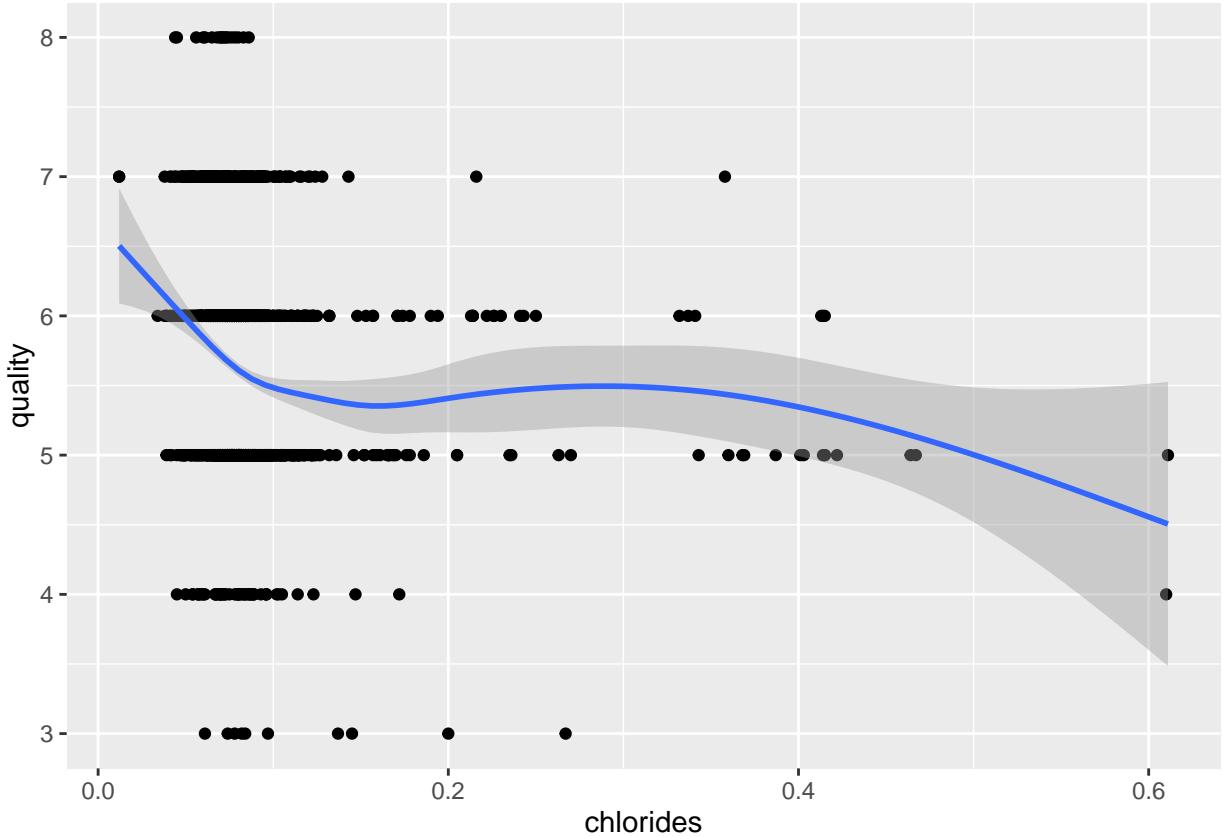
```

## Correlation coefficient: -0.174919
##
## Call:
## lm(formula = quality ~ density, data = wine)
##
## Residuals:
##    Min     1Q   Median     3Q    Max 
## -2.7885 -0.6216  0.1554  0.4271  2.5177 
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 80.24      10.51   7.636 3.83e-14 ***
## density     -74.85      10.54  -7.100 1.87e-12 ***
## ---        
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7954 on 1597 degrees of freedom
## Multiple R-squared:  0.0306, Adjusted R-squared:  0.02999 
## F-statistic: 50.41 on 1 and 1597 DF,  p-value: 1.875e-12

```



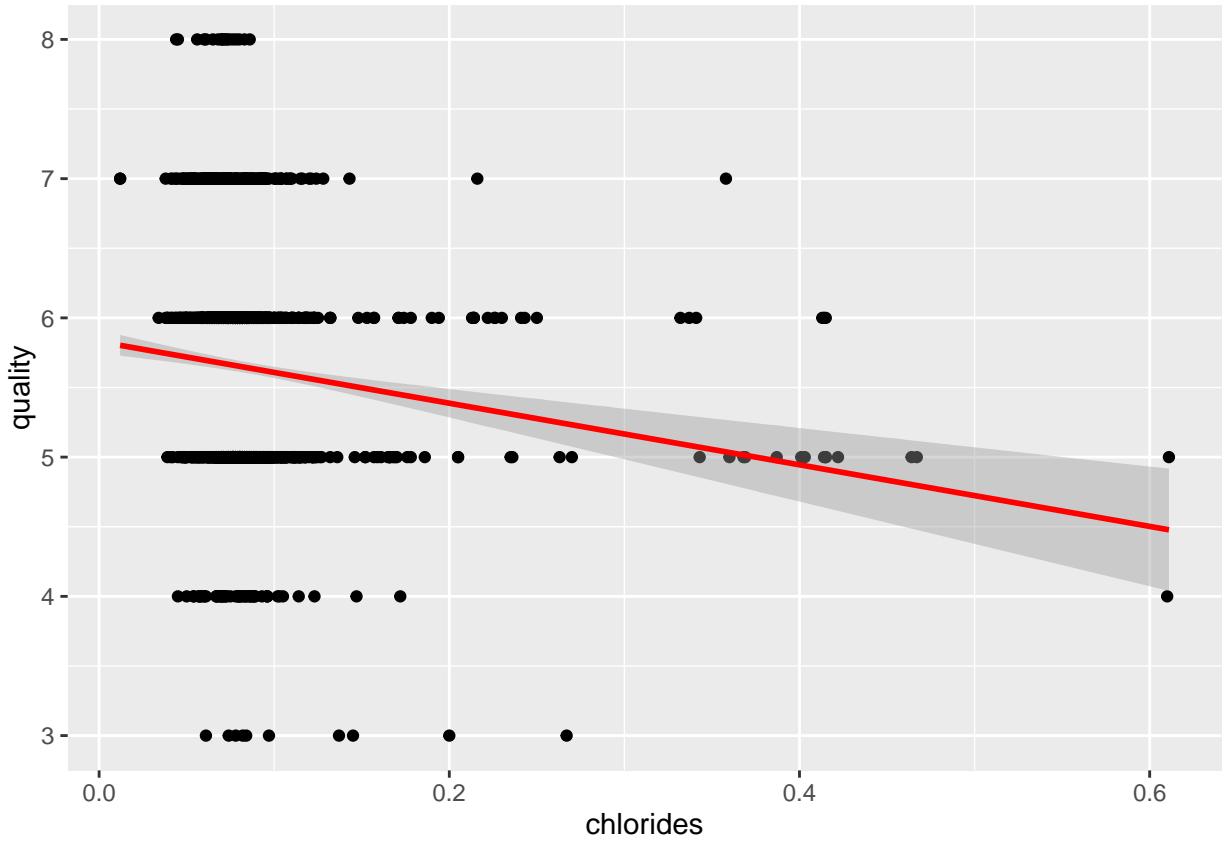


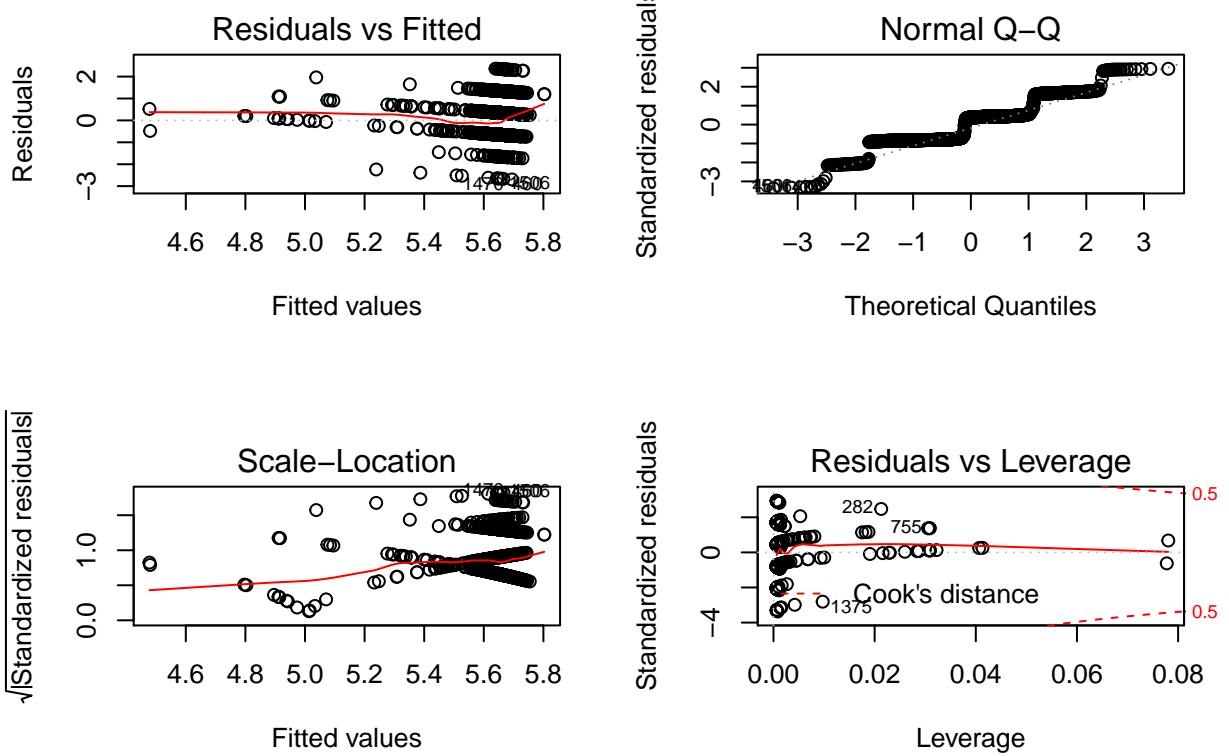


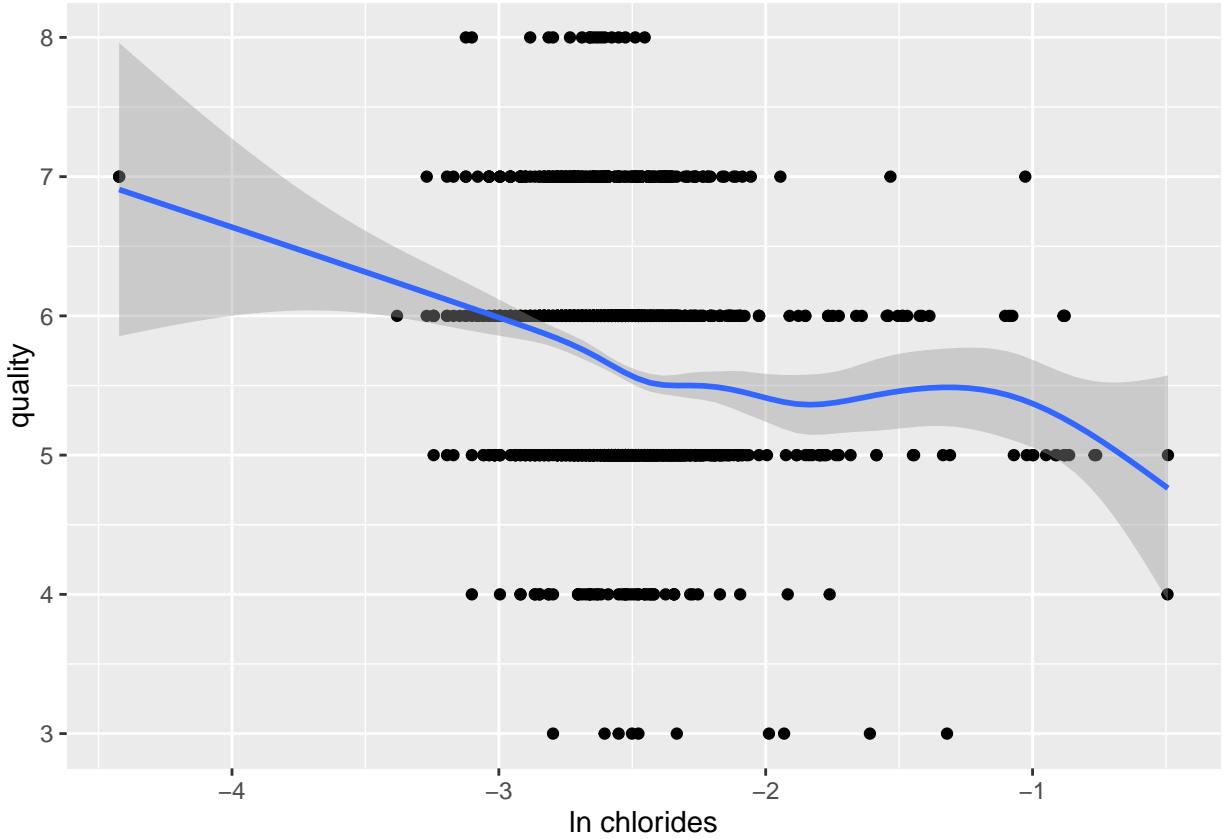
```

## Correlation coefficient: -0.128907
##
## Call:
## lm(formula = quality ~ chlorides, data = wine)
##
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -2.6946 -0.6503  0.3010  0.3607  2.3607 
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 5.82948   0.04229 137.852 < 2e-16 ***
## chlorides   -2.21184   0.42578  -5.195 2.31e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.8011 on 1597 degrees of freedom
## Multiple R-squared:  0.01662,    Adjusted R-squared:  0.016 
## F-statistic: 26.99 on 1 and 1597 DF,  p-value: 2.313e-07

```



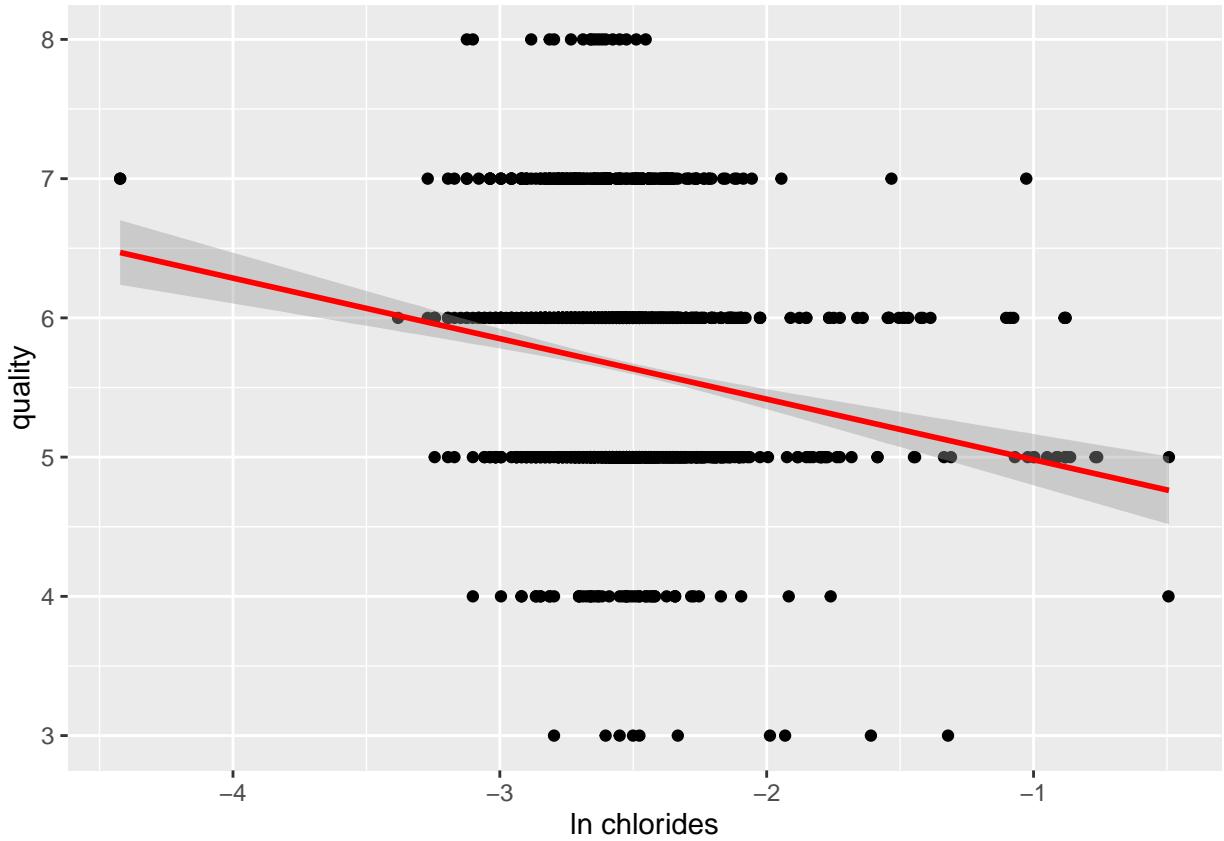


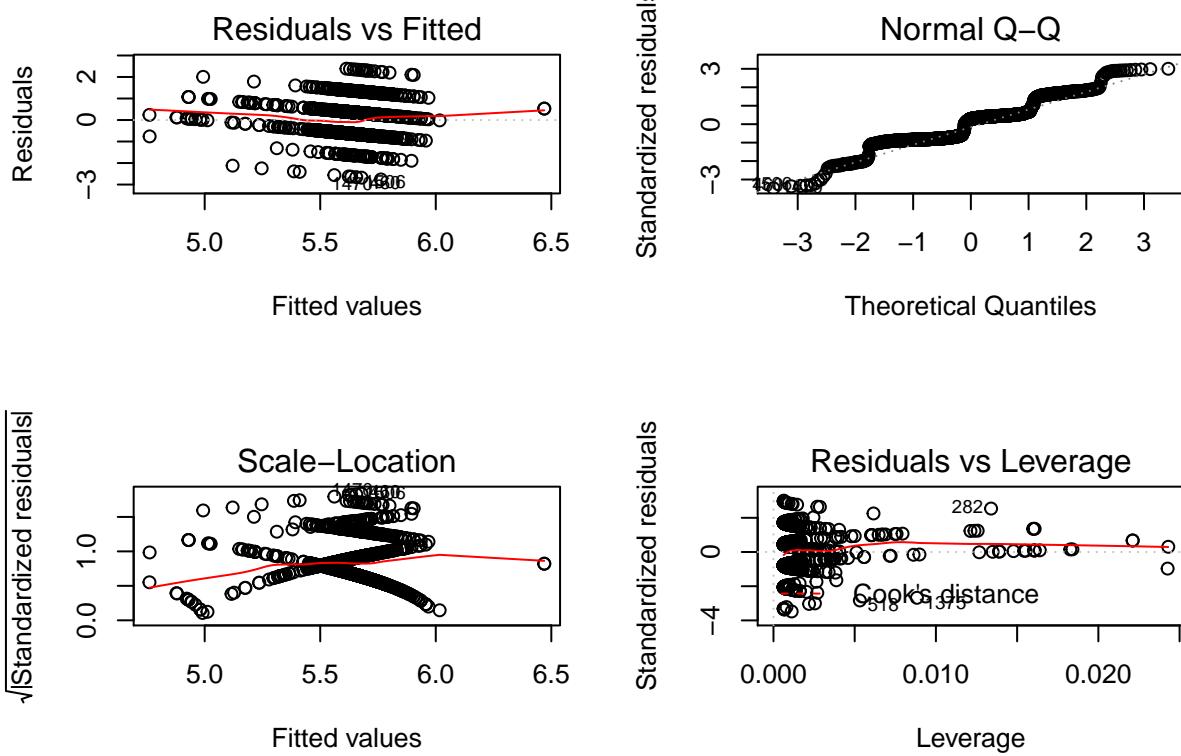


```

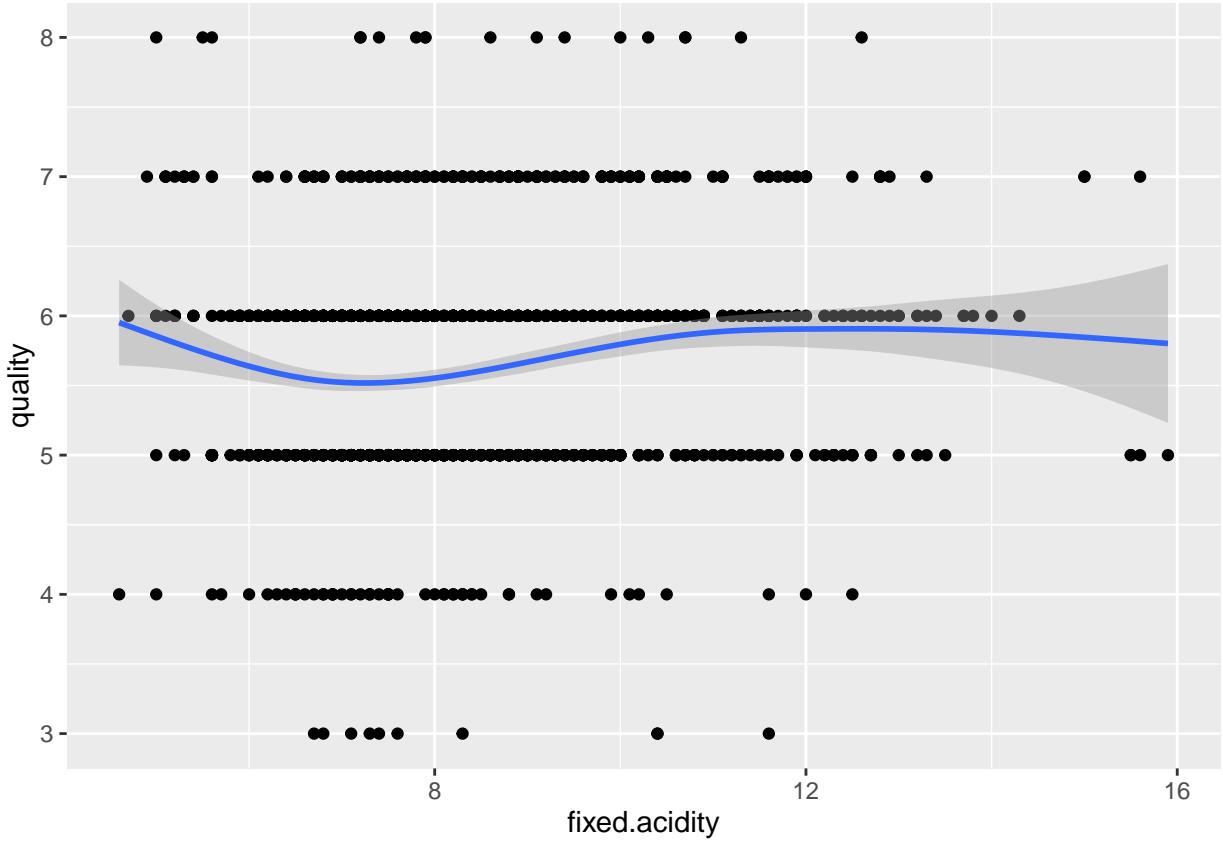
## Correlation coefficient: -0.176140
##
## Call:
## lm(formula = quality ~ log(chlorides), data = wine)
##
## Residuals:
##    Min     1Q   Median     3Q    Max 
## -2.7627 -0.6394  0.2228  0.3841  2.3866 
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 4.54686   0.15361  29.601 < 2e-16 ***
## log(chlorides) -0.43471   0.06079 -7.151 1.31e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7952 on 1597 degrees of freedom
## Multiple R-squared:  0.03103,    Adjusted R-squared:  0.03042 
## F-statistic: 51.13 on 1 and 1597 DF,  p-value: 1.308e-12

```





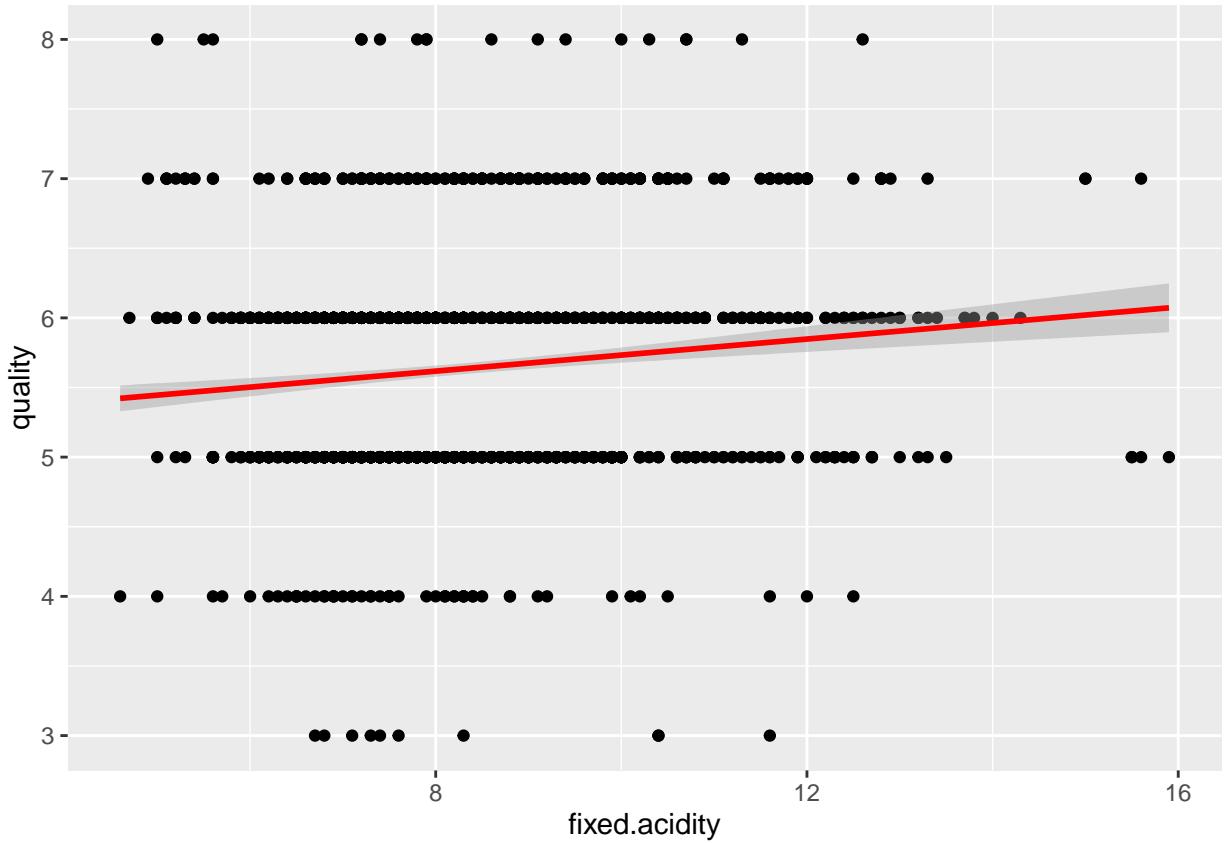
We see the fanning in the residuals plot is reduced by log-transforming chlorides.

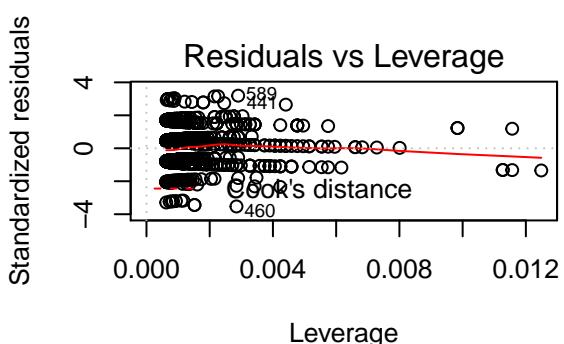
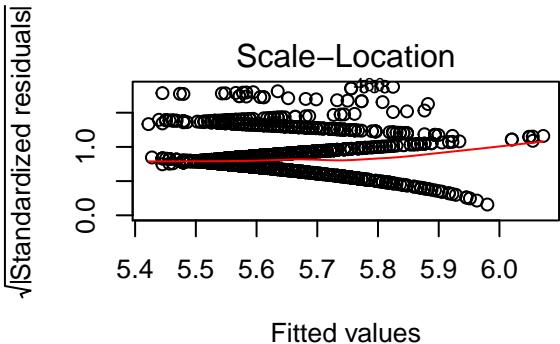
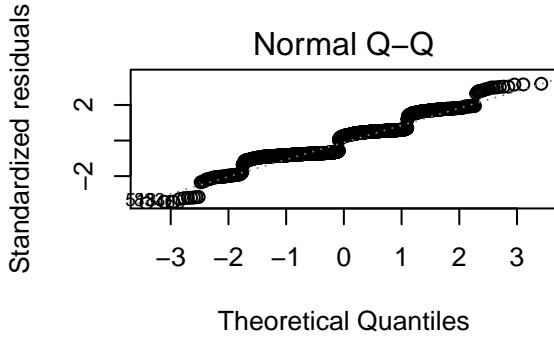
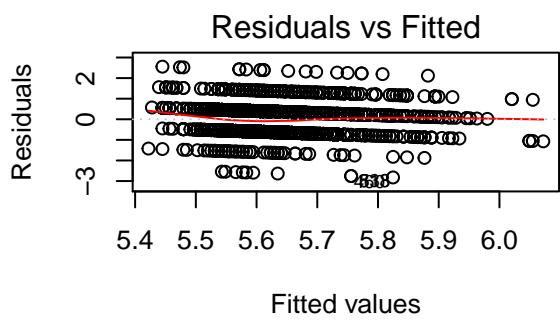


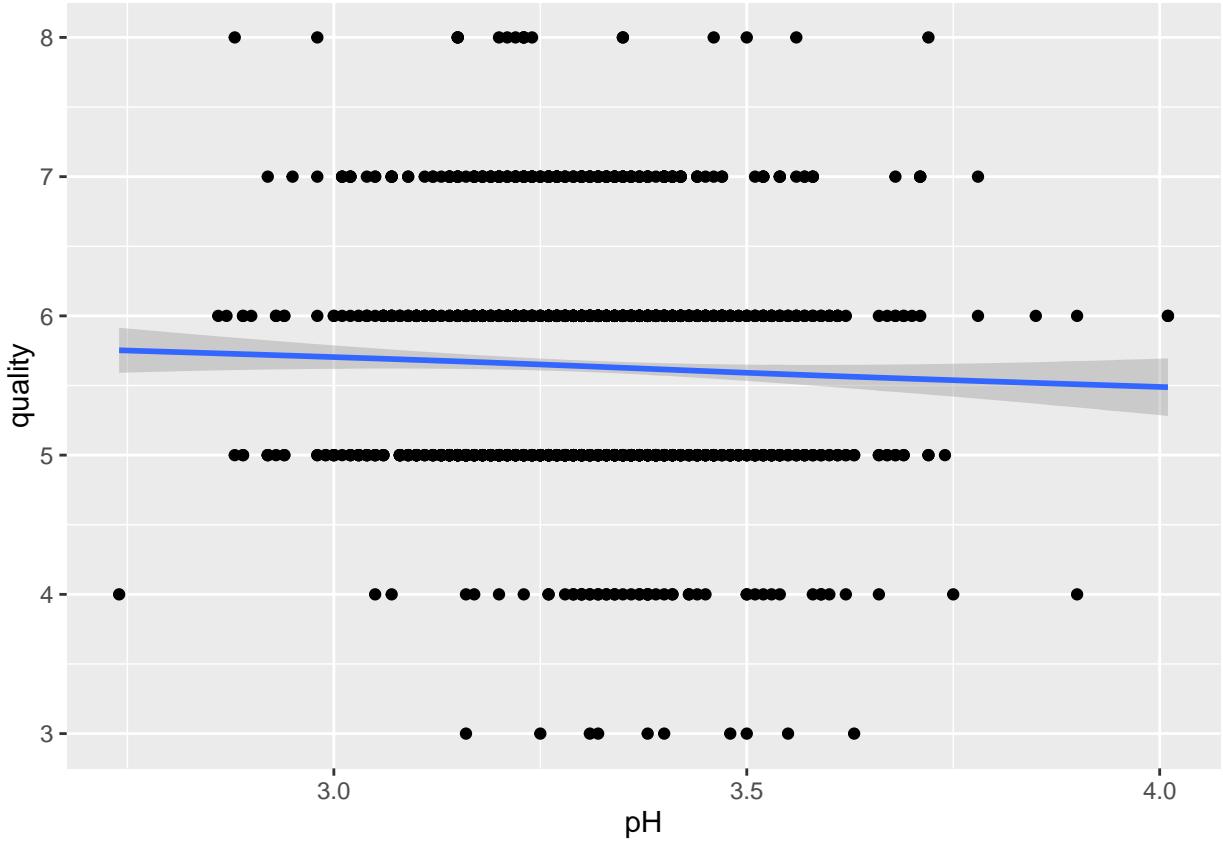
```

## Correlation coefficient:  0.124052
##
## Call:
## lm(formula = quality ~ fixed.acidity, data = wine)
##
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -2.8248 -0.6061  0.1925  0.4341  2.5550 
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 5.15732   0.09789  52.684 < 2e-16 ***
## fixed.acidity 0.05754   0.01152   4.996 6.5e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.8016 on 1597 degrees of freedom
## Multiple R-squared:  0.01539,    Adjusted R-squared:  0.01477 
## F-statistic: 24.96 on 1 and 1597 DF,  p-value: 6.496e-07

```



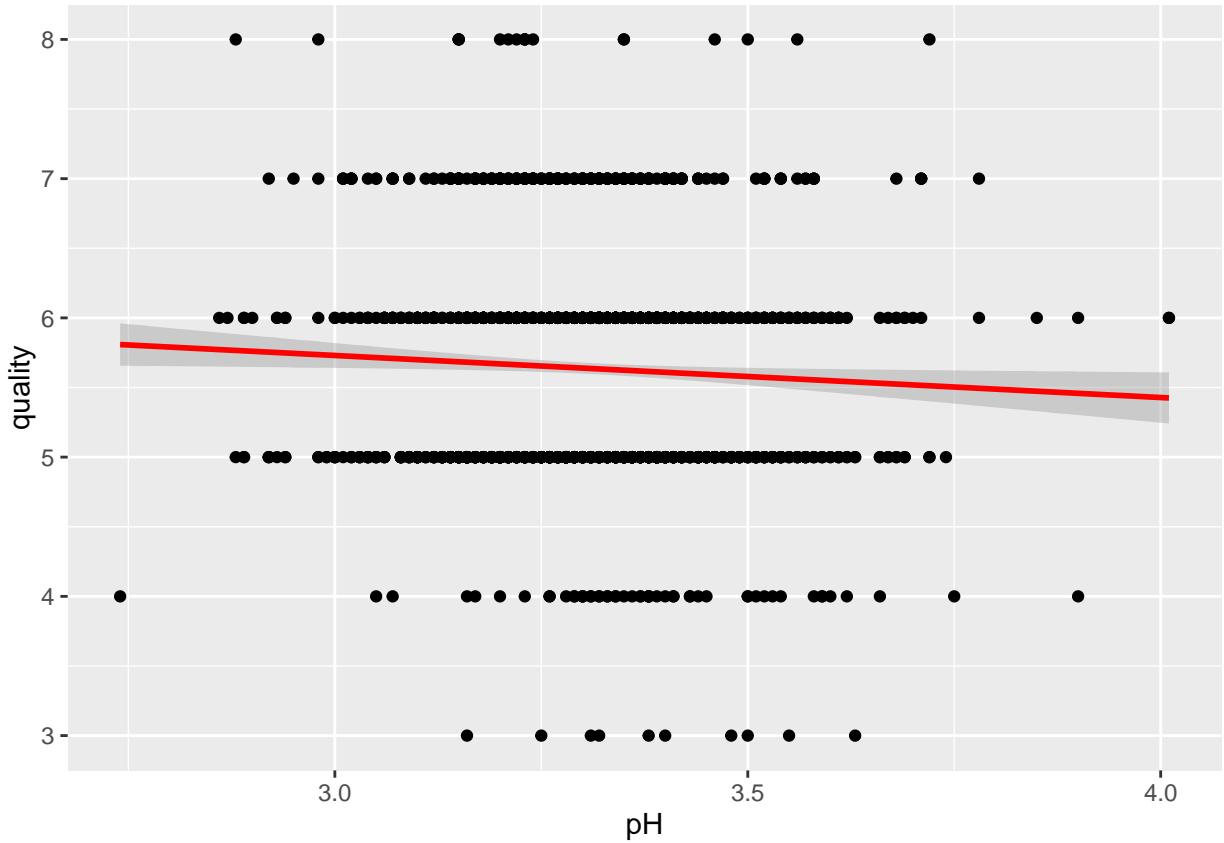


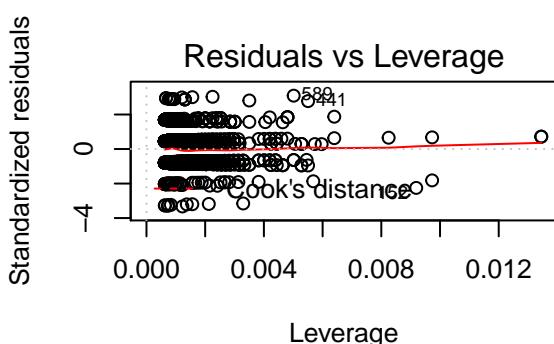
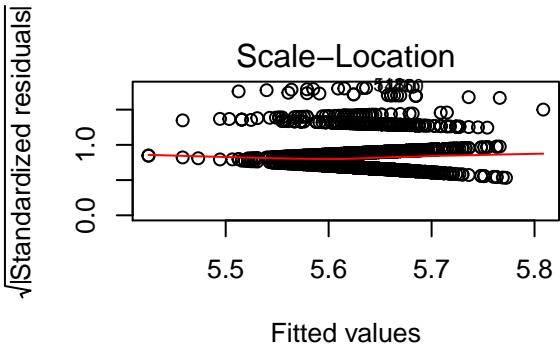
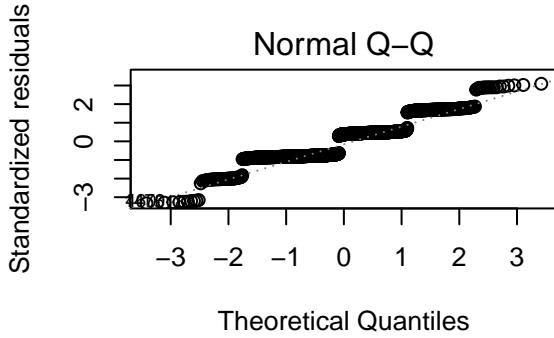
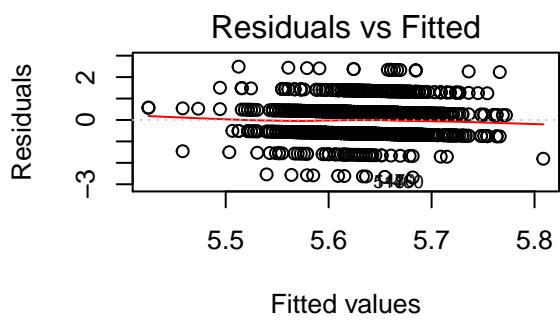


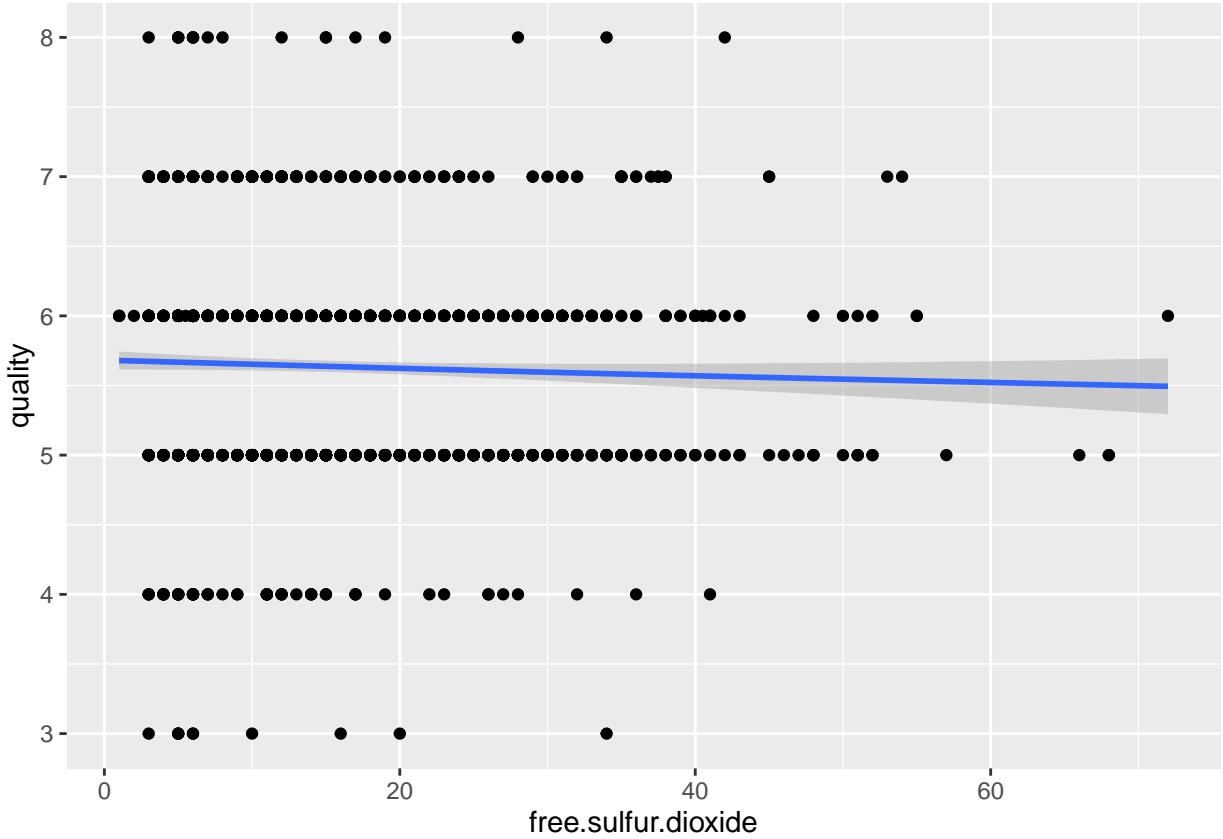
```

## Correlation coefficient: -0.057731
##
## Call:
## lm(formula = quality ~ pH, data = wine)
##
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -2.6817 -0.6394  0.3032  0.3878  2.4874 
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 6.6359    0.4332 15.320 <2e-16 ***
## pH          -0.3020    0.1307 -2.311   0.021 *  
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.8065 on 1597 degrees of freedom
## Multiple R-squared:  0.003333, Adjusted R-squared:  0.002709 
## F-statistic:  5.34 on 1 and 1597 DF,  p-value: 0.02096

```



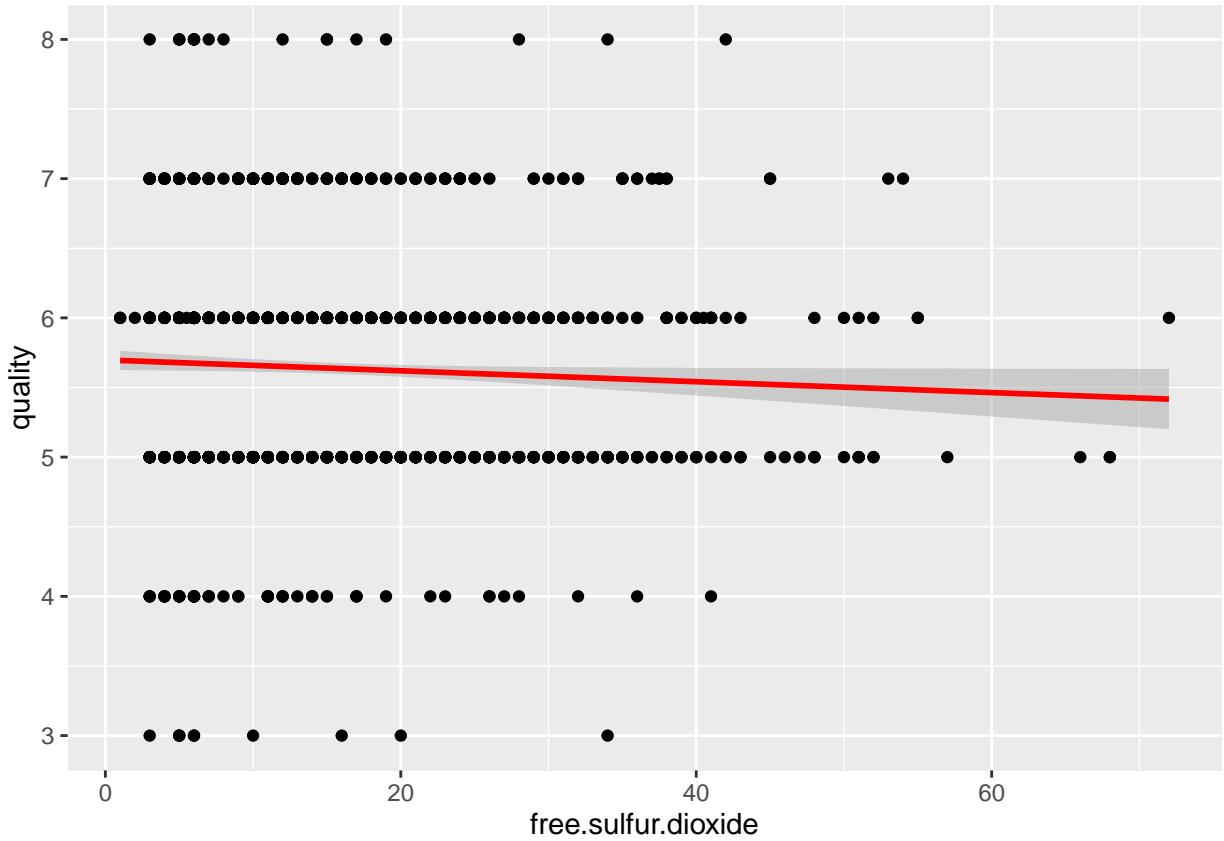


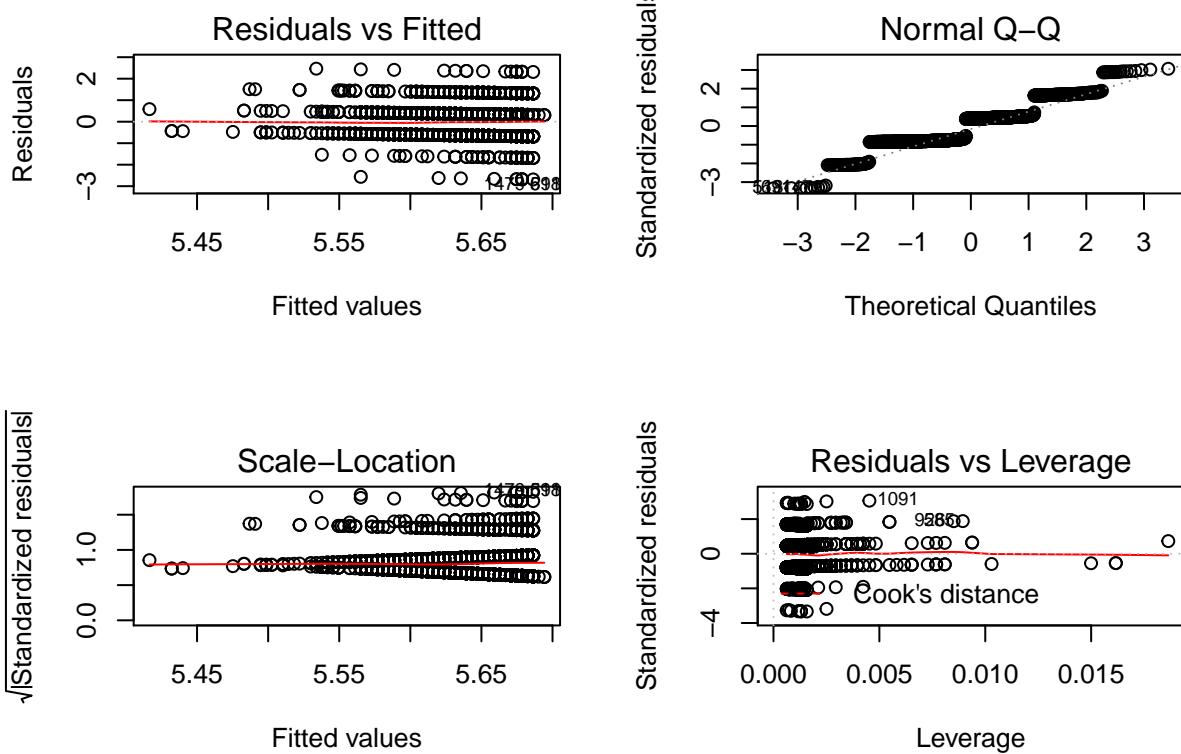


```

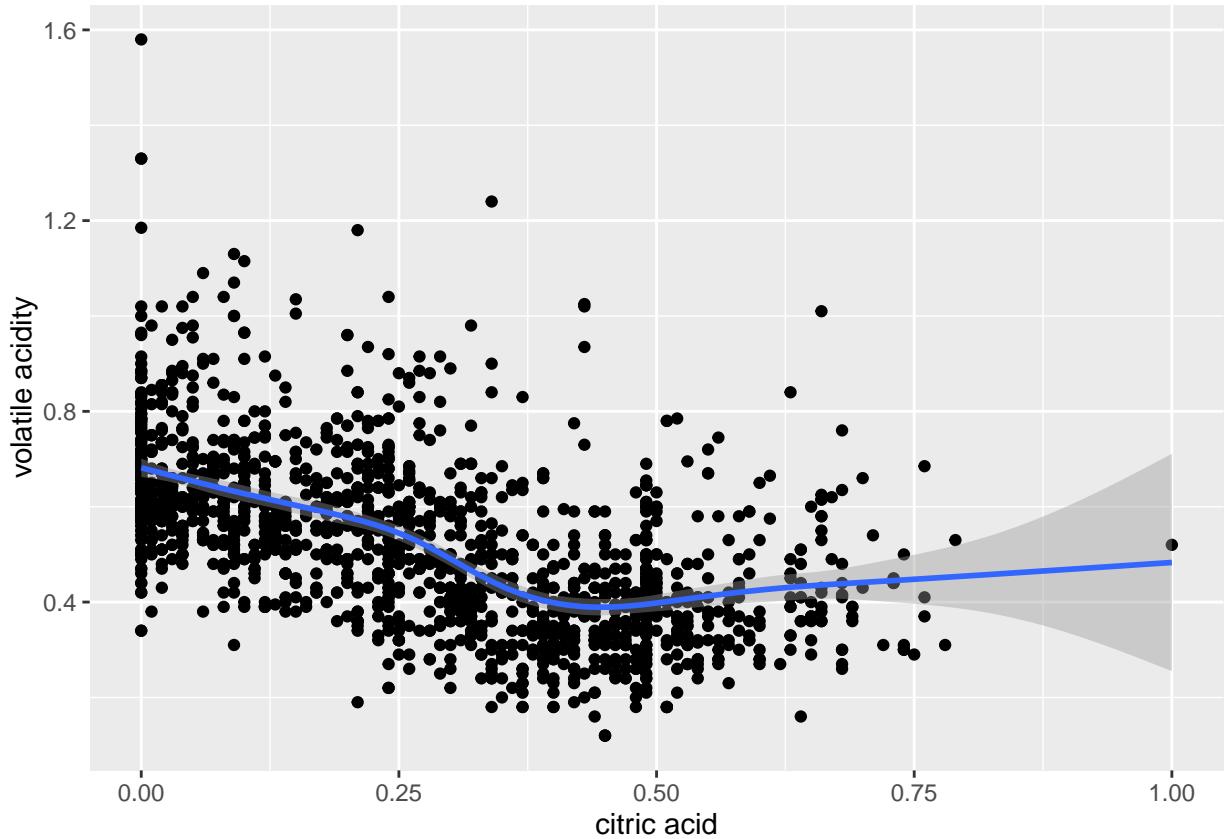
## Correlation coefficient: -0.050656
##
## Call:
## lm(formula = quality ~ free.sulfur.dioxide, data = wine)
##
## Residuals:
##    Min     1Q   Median     3Q    Max 
## -2.6864 -0.6394  0.3215  0.3762  2.4661 
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 5.698107  0.036678 155.357 <2e-16 ***
## free.sulfur.dioxide -0.003911  0.001929 -2.027  0.0428 *  
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.8068 on 1597 degrees of freedom
## Multiple R-squared:  0.002566, Adjusted R-squared:  0.001941 
## F-statistic: 4.109 on 1 and 1597 DF,  p-value: 0.04283

```

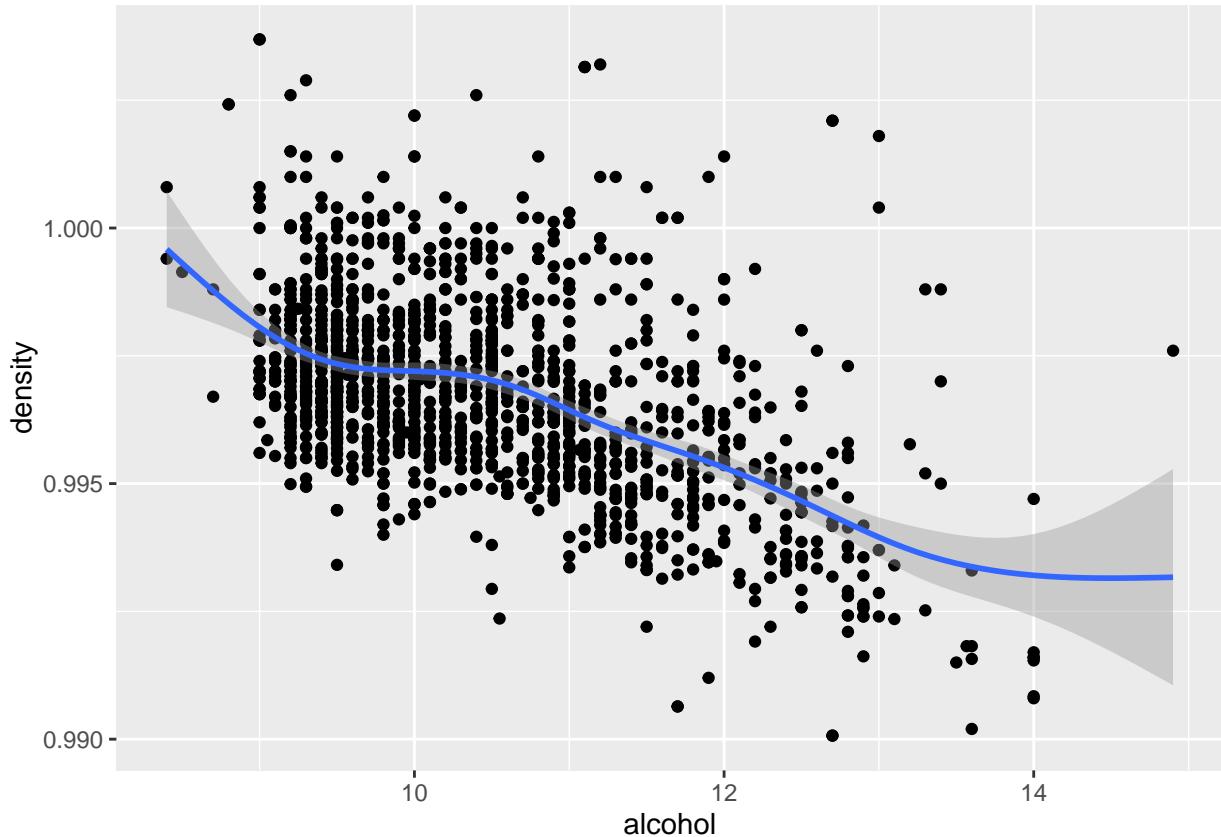




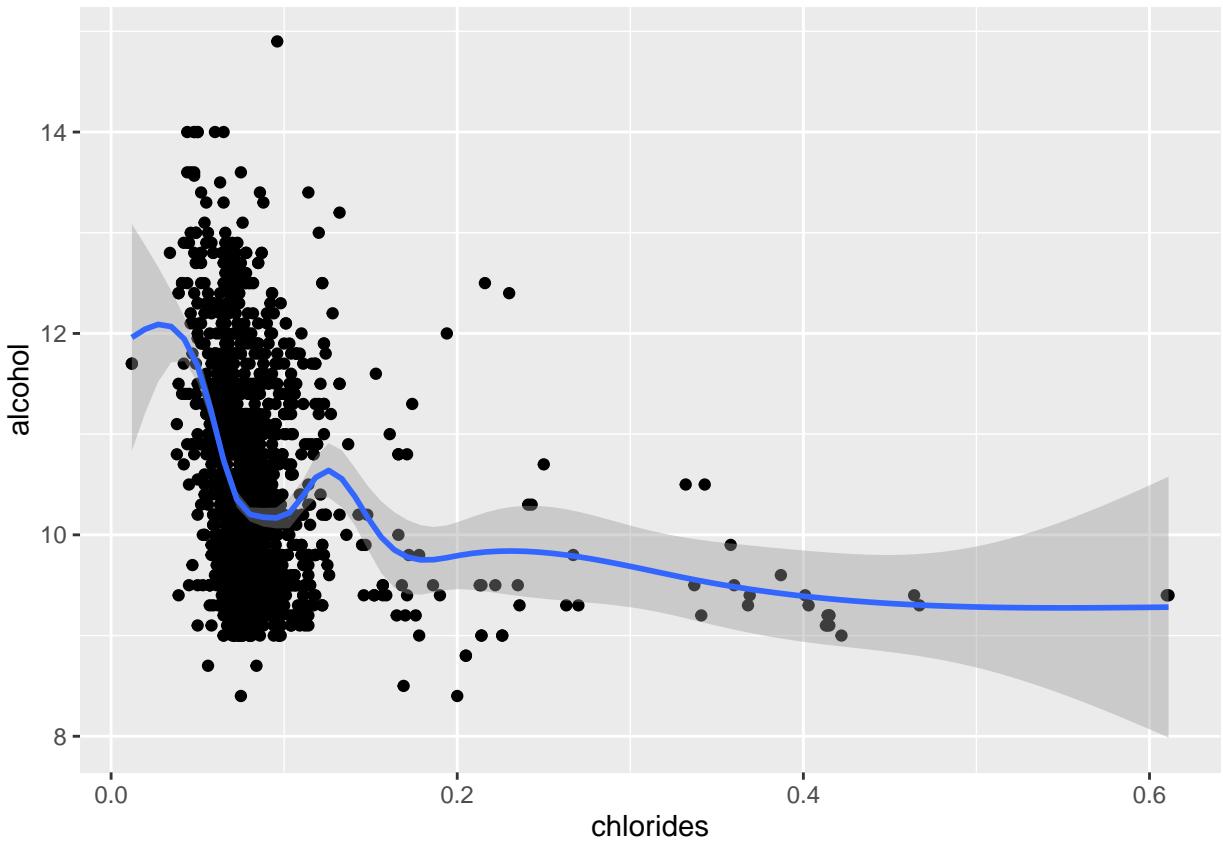
There are a few combinations of variables other than quality which have interesting non-linearities. Let's plot them in order by correlation coefficient.



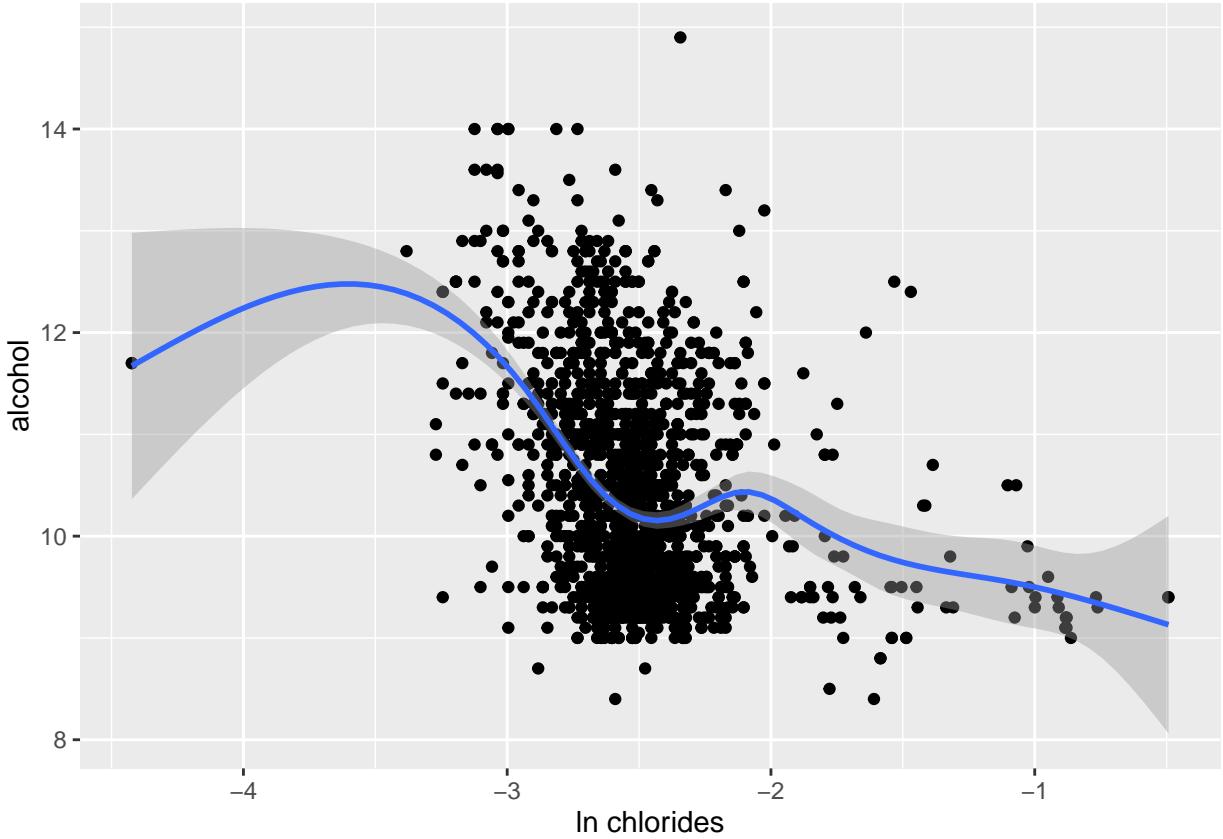
```
##  
## Pearson's product-moment correlation  
##  
## data: wine$citric.acid and wine$volatile.acidity  
## t = -26.489, df = 1597, p-value < 2.2e-16  
## alternative hypothesis: true correlation is not equal to 0  
## 95 percent confidence interval:  
## -0.5856550 -0.5174902  
## sample estimates:  
## cor  
## -0.5524957
```



```
##  
## Pearson's product-moment correlation  
##  
## data: wine$alcohol and wine$density  
## t = -22.838, df = 1597, p-value < 2.2e-16  
## alternative hypothesis: true correlation is not equal to 0  
## 95 percent confidence interval:  
## -0.5322547 -0.4583061  
## sample estimates:  
## cor  
## -0.4961798
```

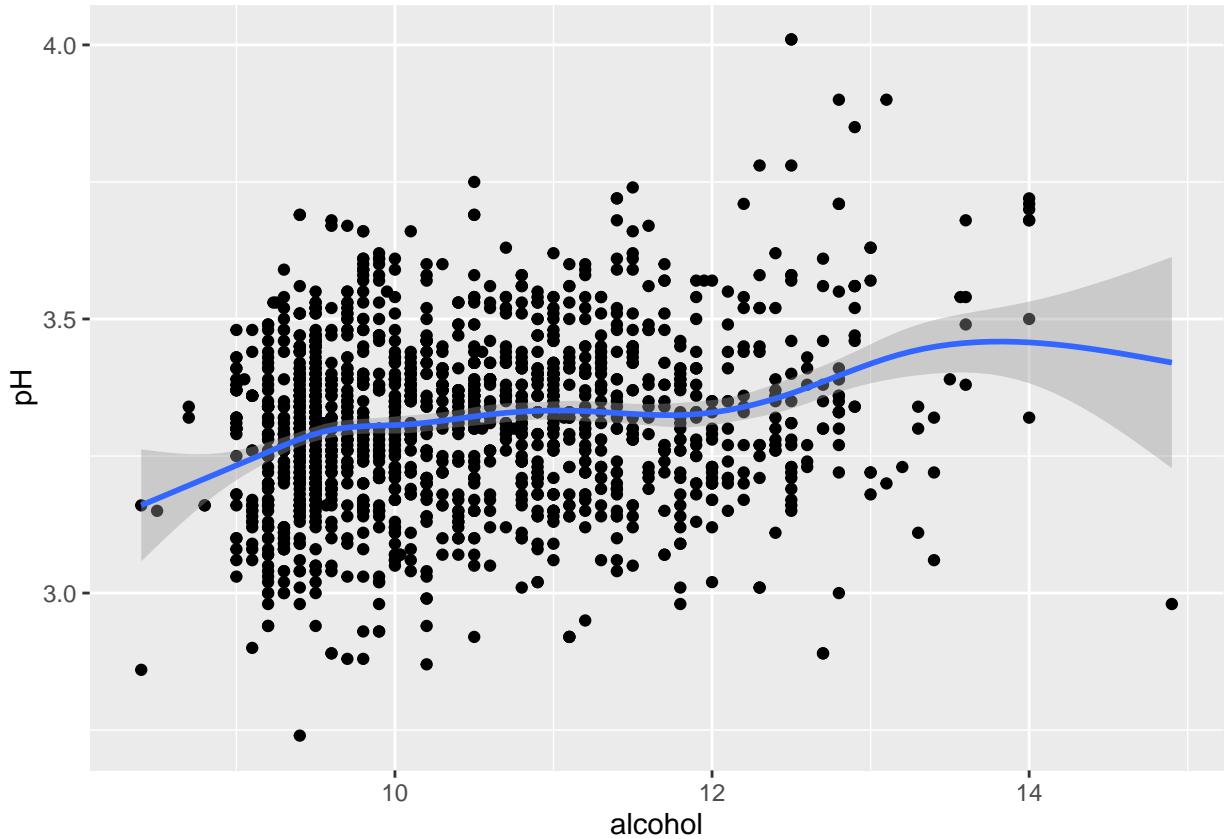


```
##  
## Pearson's product-moment correlation  
##  
## data: wine$chlorides and wine$alcohol  
## t = -9.0617, df = 1597, p-value < 2.2e-16  
## alternative hypothesis: true correlation is not equal to 0  
## 95 percent confidence interval:  
## -0.2672644 -0.1740057  
## sample estimates:  
## cor  
## -0.2211405
```

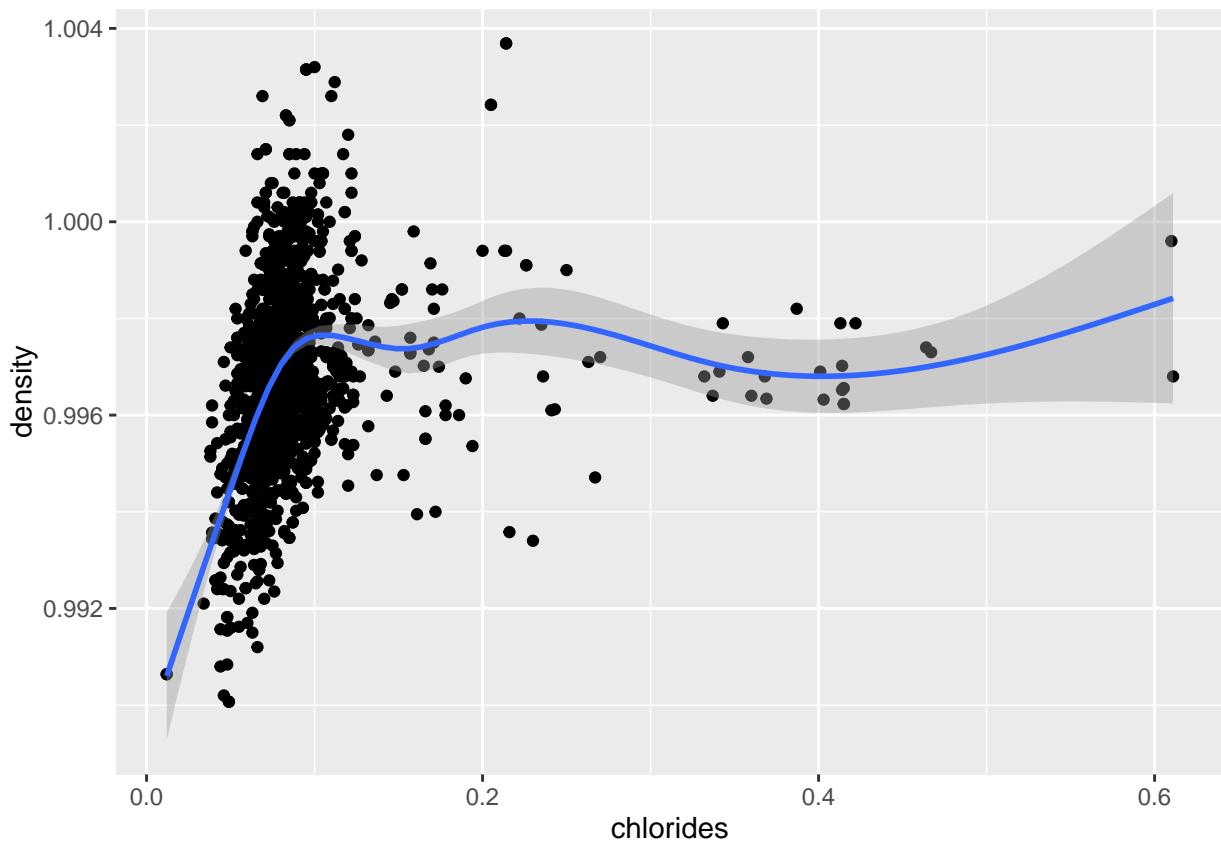


```
##
## Pearson's product-moment correlation
##
## data: log(wine$chlorides) and wine$alcohol
## t = -12.75, df = 1597, p-value < 2.2e-16
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
## -0.3477997 -0.2587960
## sample estimates:
##       cor
## -0.303961
```

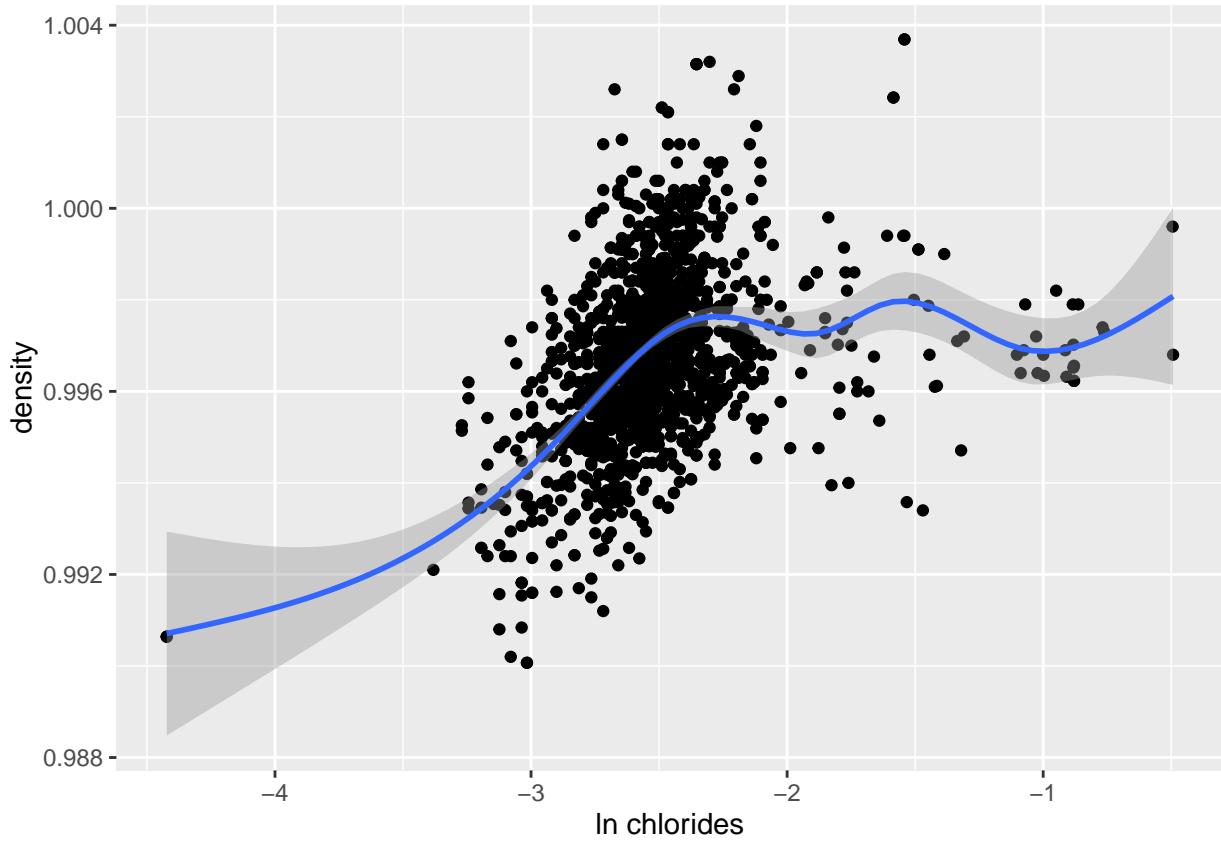
The distribution and correlation coefficient improve by log-transforming chlorides.



```
##  
## Pearson's product-moment correlation  
##  
## data: wine$alcohol and wine$pH  
## t = 8.397, df = 1597, p-value < 2.2e-16  
## alternative hypothesis: true correlation is not equal to 0  
## 95 percent confidence interval:  
## 0.1582061 0.2521123  
## sample estimates:  
## cor  
## 0.2056325
```

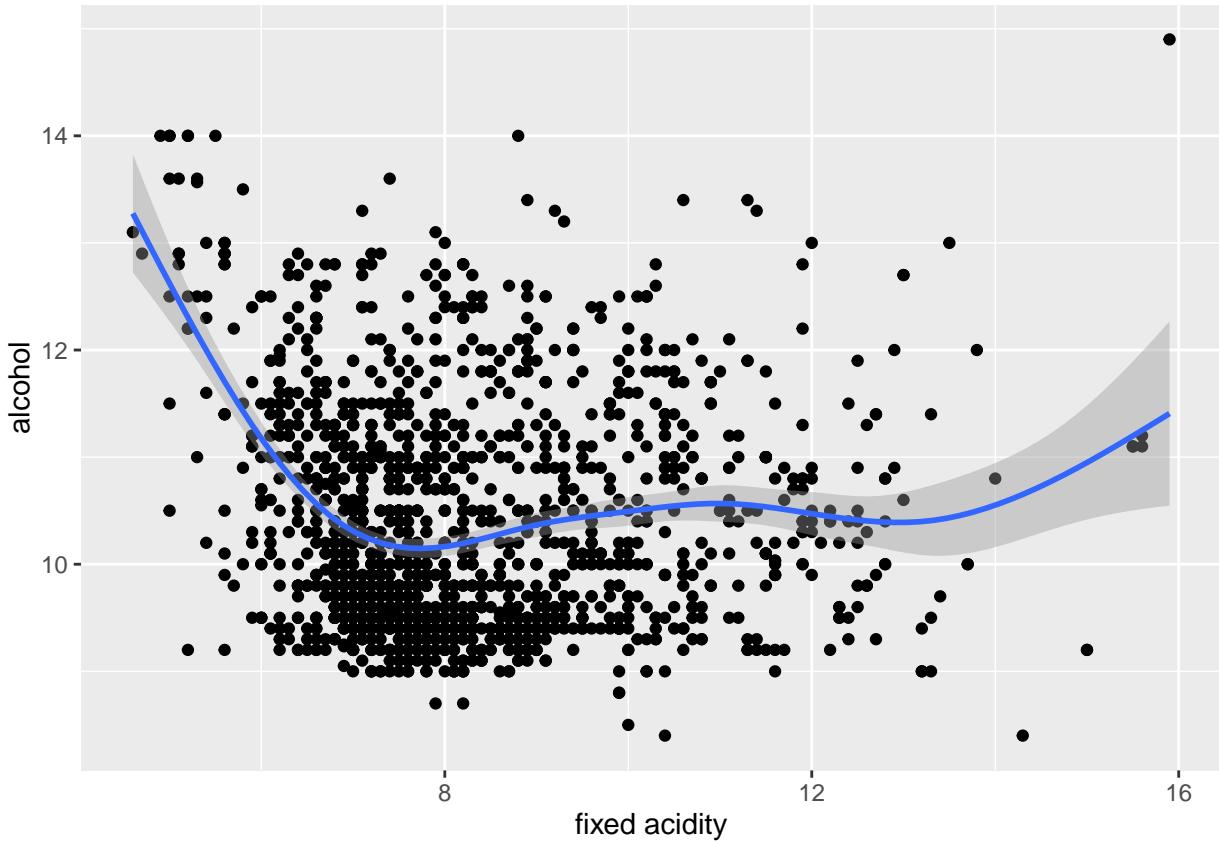


```
##  
## Pearson's product-moment correlation  
##  
## data: wine$chlorides and wine$density  
## t = 8.1842, df = 1597, p-value = 5.541e-16  
## alternative hypothesis: true correlation is not equal to 0  
## 95 percent confidence interval:  
## 0.1531171 0.2472220  
## sample estimates:  
## cor  
## 0.2006323
```



```
##
## Pearson's product-moment correlation
##
## data: log(wine$chlorides) and wine$density
## t = 15.026, df = 1597, p-value < 2.2e-16
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  0.3082352 0.3941595
## sample estimates:
##        cor
## 0.3519385
```

Again, we see improved distribution and correlation coefficient by log-transforming chlorides.



```
##
## Pearson's product-moment correlation
##
## data: wine$fixed.acidity and wine$alcohol
## t = -2.4691, df = 1597, p-value = 0.01365
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
## -0.11035580 -0.01268548
## sample estimates:
## cor
## -0.06166827
```

Bivariate Analysis

Talk about some of the relationships you observed in this part of the investigation. How did the feature(s) of interest vary with other features in the dataset?

The features having significant correlation with quality are as follows.

Feature	Correlation Coefficient with Quality
alcohol	0.48
volatile acidity	-0.39
ln sulphates	0.31

Feature	Correlation Coefficient with Quality
sulphates	0.25
citric acid	0.23
total sulfur dioxide	-0.19
ln chlorides	-0.18
density	-0.17
chlorides	-0.13
fixed acidity	0.12
pH	-0.06
free sulfur dioxide	-0.05

The plots of quality against predictors generally reflect these correlation coefficients. We used general additive models (GAMs) for the nonlinear bivariate plots.

We see a stationary point for sulphates when using a GAM.

Did you observe any interesting relationships between the other features (not the main feature(s) of interest)?

Most pairs of predictors have significant correlation. Some of these pairs have noticeable non-linearity.

In the multivariate section, we will consider which of the pairs of predictors should be represented as interaction terms when performing multiple regression.

What was the strongest relationship you found?

Several variable pairs had correlation coefficients higher than the highest pairing of predictor with quality. (See following chart.)

Feature 1	Feature 2	Correlation Coefficient
fixed acidity	pH	-0.68
citric acid	fixed acidity	0.67
density	fixed acidity	0.67
citric acid	volatile acidity	-0.55
citric acid	pH	-0.54
free sulfur dioxide	total sulfur dioxide	0.67
alcohol	density	-0.5

Multivariate Plots Section

Tip: Now it's time to put everything together. Based on what you found in the bivariate plots section, create a few multivariate plots to investigate more complex interactions between variables. Make sure that the plots that you create here are justified by the plots you explored in the previous section. If you plan on creating any mathematical models, this is the section where you will do that.

Let's find a multivariate model using best subset selection.

We know R^2 will increase as we add predictors to the model. However, if we add too many predictors, we may overfit the model. We can use adjusted R^2 , C_p , and the Bayesian information criterion (BIC) to guide

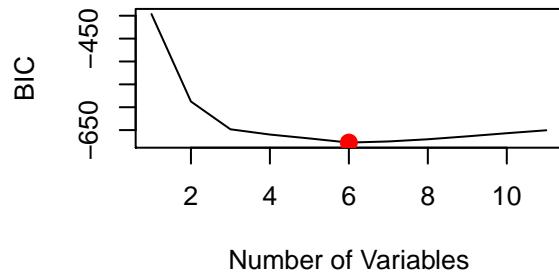
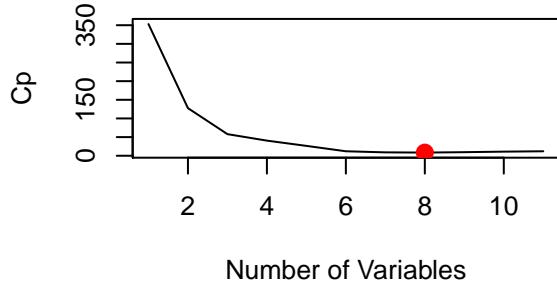
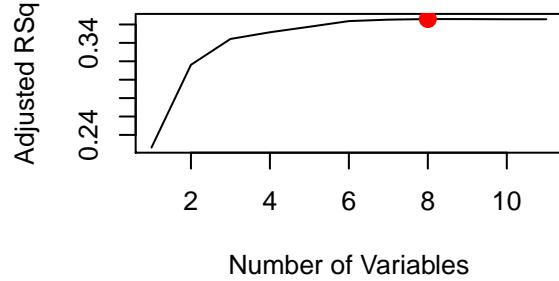
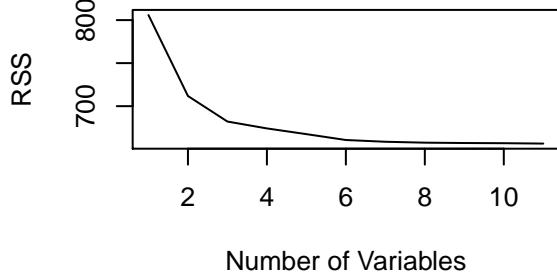
our selection of an appropriate number of predictors to avoid overfitting.

```

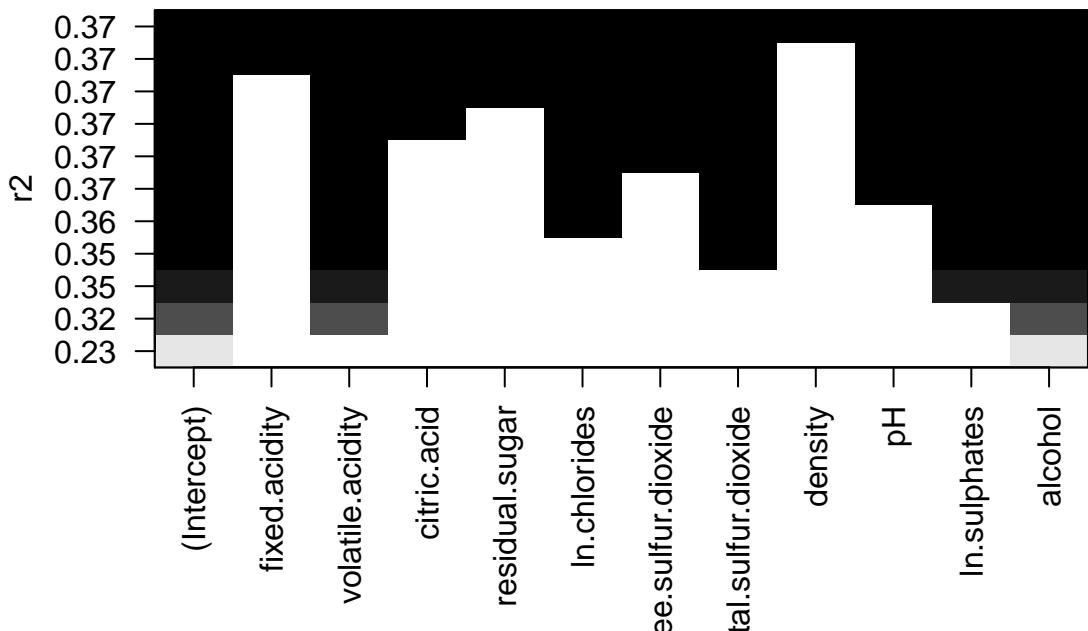
## Subset selection object
## Call: regsubsets.formula(quality ~ ., wine, nvmax = 11)
## 11 Variables (and intercept)
##          Forced in    Forced out
## fixed.acidity      FALSE      FALSE
## volatile.acidity   FALSE      FALSE
## citric.acid       FALSE      FALSE
## residual.sugar    FALSE      FALSE
## ln.chlorides       FALSE      FALSE
## free.sulfur.dioxide FALSE      FALSE
## total.sulfur.dioxide FALSE      FALSE
## density           FALSE      FALSE
## pH                FALSE      FALSE
## ln.sulphates      FALSE      FALSE
## alcohol           FALSE      FALSE
## 1 subsets of each size up to 11
## Selection Algorithm: exhaustive
##          fixed.acidity volatile.acidity citric.acid residual.sugar
## 1  ( 1 )   " "          " "          " "          " "
## 2  ( 1 )   " "          "*"         " "          " "
## 3  ( 1 )   " "          "**"        " "          " "
## 4  ( 1 )   " "          "**"        " "          " "
## 5  ( 1 )   " "          "**"        " "          " "
## 6  ( 1 )   " "          "**"        " "          " "
## 7  ( 1 )   " "          "**"        " "          " "
## 8  ( 1 )   " "          "**"        "*"         " "
## 9  ( 1 )   " "          "**"        "**"        "*"
## 10 ( 1 )   **"        "*"         "**"        "*"
## 11 ( 1 )   **"        "*"         "**"        "**"
##          ln.chlorides free.sulfur.dioxide total.sulfur.dioxide density
## 1  ( 1 )   " "          " "          " "          " "
## 2  ( 1 )   " "          " "          " "          " "
## 3  ( 1 )   " "          " "          " "          " "
## 4  ( 1 )   " "          " "          "*"         " "
## 5  ( 1 )   **"        " "          "**"        " "
## 6  ( 1 )   **"        " "          "**"        " "
## 7  ( 1 )   **"        " "          "**"        " "
## 8  ( 1 )   **"        " "          "**"        " "
## 9  ( 1 )   **"        " "          "**"        " "
## 10 ( 1 )   **"        " "          "**"        " "
## 11 ( 1 )   **"        " "          "**"        "*"
##          pH    ln.sulphates alcohol
## 1  ( 1 )   " "  " "      "*" 
## 2  ( 1 )   " "  " "      "*" 
## 3  ( 1 )   " "  "*"      "*" 
## 4  ( 1 )   " "  "*"      "*" 
## 5  ( 1 )   " "  "*"      "*" 
## 6  ( 1 )   **"  **"      "*" 
## 7  ( 1 )   **"  **"      "*" 
## 8  ( 1 )   **"  **"      "*" 
## 9  ( 1 )   **"  **"      "*" 

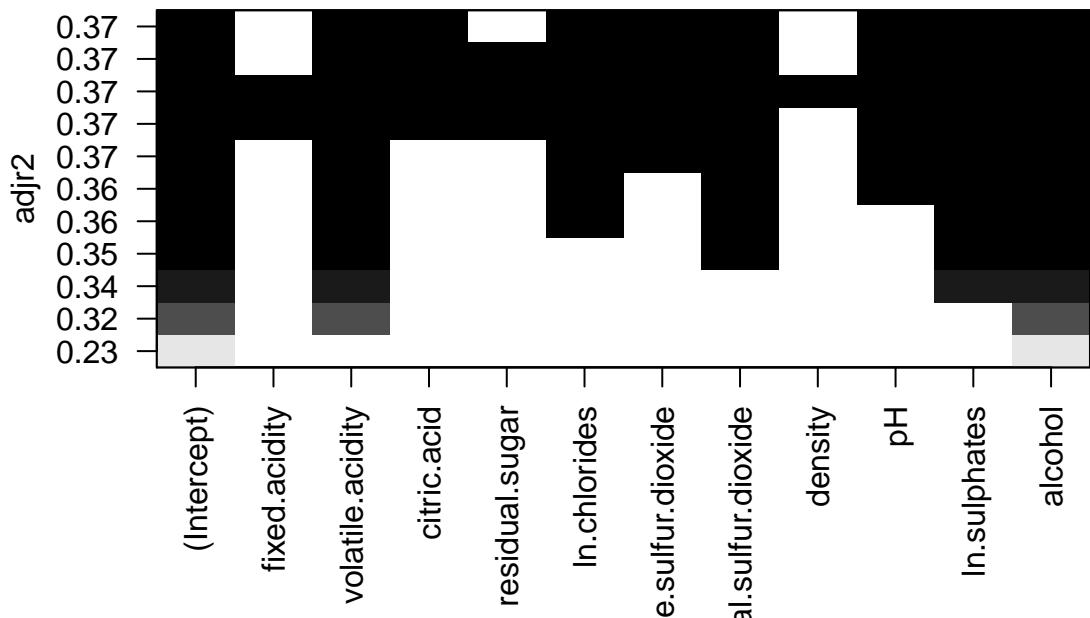
```

```
## 10  ( 1 ) "*" "*"      "*"  
## 11  ( 1 ) "*" "*"      "*"
```



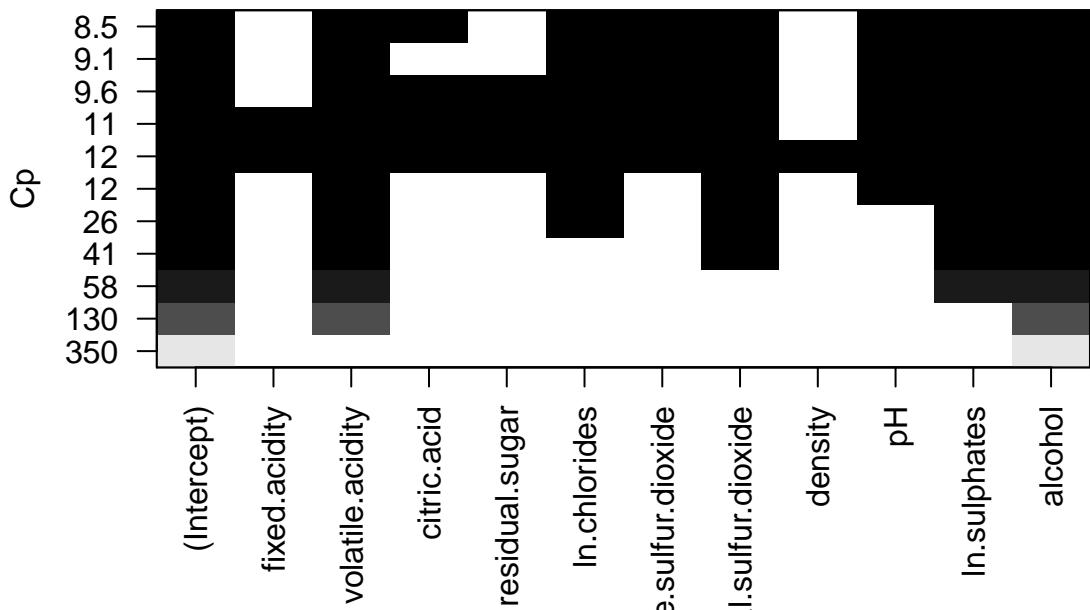
We see a model having six or eight variables would be appropriate.



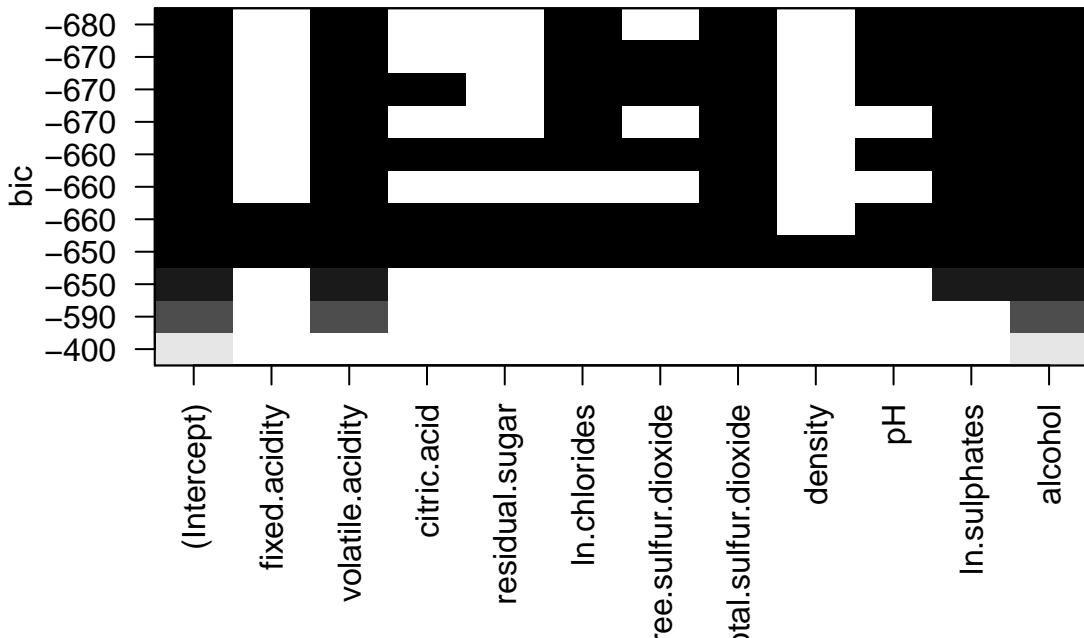


The eight-variable model includes the following variables:

1. volatile acidity
2. citric acid
3. ln chlorides
4. free sulfur dioxide
5. total sulfur dioxide
6. pH
7. ln sulphates
8. alcohol



C_8 is minimal.



The six-variable model minimizing BIC is the same as the eight-variable model, except citric acid and free sulfur dioxide are no longer present.

Following are adjusted R^2 , C_p , and BIC values for six-, seven-, and eight-variable models.

```
## [1] 0.3636935 0.3652585 0.3658819
## [1] 12.014545 9.086078 8.522699
## [1] -677.2429 -674.8080 -670.0075
```

We next create interaction terms by multiplying each of the eleven predictors by the ten other predictors, resulting in sixty-six predictors comprising the eleven predictors and fifty-five predictors of the form $(X_1 \times X_2)$. We then perform best subset selection. We limit the search space to models having at most nine predictors. The algorithm requires less than ten minutes on an i7-6700k in RStudio.

```
## Reordering variables and trying again:
## Subset selection object
## Call: regsubsets.formula(quality ~ ., wine.interactions, nvmax = 8,
##   really.big = T)
## 67 Variables  (and intercept)
##                                     Forced in    Forced out
## fixed.acidity                      FALSE      FALSE
## volatile.acidity                    FALSE      FALSE
## citric.acid                        FALSE      FALSE
## residual.sugar                     FALSE      FALSE
## ln.chlorides                       FALSE      FALSE
## free.sulfur.dioxide                 FALSE      FALSE
## total.sulfur.dioxide                FALSE      FALSE
```

## density	FALSE	FALSE
## pH	FALSE	FALSE
## ln.sulphates	FALSE	FALSE
## alcohol	FALSE	FALSE
## `fixed.acidity:volatile.acidity`	FALSE	FALSE
## `fixed.acidity:citric.acid`	FALSE	FALSE
## `fixed.acidity:residual.sugar`	FALSE	FALSE
## `fixed.acidity:ln.chlorides`	FALSE	FALSE
## `fixed.acidity:free.sulfur.dioxide`	FALSE	FALSE
## `fixed.acidity:total.sulfur.dioxide`	FALSE	FALSE
## `fixed.acidity:density`	FALSE	FALSE
## `fixed.acidity:pH`	FALSE	FALSE
## `fixed.acidity:ln.sulphates`	FALSE	FALSE
## `fixed.acidity:alcohol`	FALSE	FALSE
## `volatile.acidity:citric.acid`	FALSE	FALSE
## `volatile.acidity:residual.sugar`	FALSE	FALSE
## `volatile.acidity:ln.chlorides`	FALSE	FALSE
## `volatile.acidity:free.sulfur.dioxide`	FALSE	FALSE
## `volatile.acidity:total.sulfur.dioxide`	FALSE	FALSE
## `volatile.acidity:density`	FALSE	FALSE
## `volatile.acidity:pH`	FALSE	FALSE
## `volatile.acidity:ln.sulphates`	FALSE	FALSE
## `volatile.acidity:alcohol`	FALSE	FALSE
## `citric.acid:residual.sugar`	FALSE	FALSE
## `citric.acid:ln.chlorides`	FALSE	FALSE
## `citric.acid:free.sulfur.dioxide`	FALSE	FALSE
## `citric.acid:total.sulfur.dioxide`	FALSE	FALSE
## `citric.acid:density`	FALSE	FALSE
## `citric.acid:pH`	FALSE	FALSE
## `citric.acid:ln.sulphates`	FALSE	FALSE
## `citric.acid:alcohol`	FALSE	FALSE
## `residual.sugar:ln.chlorides`	FALSE	FALSE
## `residual.sugar:free.sulfur.dioxide`	FALSE	FALSE
## `residual.sugar:total.sulfur.dioxide`	FALSE	FALSE
## `residual.sugar:density`	FALSE	FALSE
## `residual.sugar:pH`	FALSE	FALSE
## `residual.sugar:ln.sulphates`	FALSE	FALSE
## `residual.sugar:alcohol`	FALSE	FALSE
## `ln.chlorides:free.sulfur.dioxide`	FALSE	FALSE
## `ln.chlorides:total.sulfur.dioxide`	FALSE	FALSE
## `ln.chlorides:density`	FALSE	FALSE
## `ln.chlorides:pH`	FALSE	FALSE
## `ln.chlorides:ln.sulphates`	FALSE	FALSE
## `ln.chlorides:alcohol`	FALSE	FALSE
## `free.sulfur.dioxide:total.sulfur.dioxide`	FALSE	FALSE
## `free.sulfur.dioxide:density`	FALSE	FALSE
## `free.sulfur.dioxide:pH`	FALSE	FALSE
## `free.sulfur.dioxide:ln.sulphates`	FALSE	FALSE
## `free.sulfur.dioxide:alcohol`	FALSE	FALSE
## `total.sulfur.dioxide:density`	FALSE	FALSE
## `total.sulfur.dioxide:pH`	FALSE	FALSE
## `total.sulfur.dioxide:ln.sulphates`	FALSE	FALSE
## `total.sulfur.dioxide:alcohol`	FALSE	FALSE
## `density:pH`	FALSE	FALSE

```

## `density:ln.sulphates`          FALSE    FALSE
## `density:alcohol`              FALSE    FALSE
## `pH:ln.sulphates`             FALSE    FALSE
## `pH:alcohol`                  FALSE    FALSE
## `ln.sulphates:alcohol`        FALSE    FALSE
## `(Intercept)`                 FALSE    FALSE
## 1 subsets of each size up to 9
## Selection Algorithm: exhaustive
##           `(Intercept)` fixed.acidity volatile.acidity citric.acid
## 1 ( 1 ) " "      " "      " "      " "
## 2 ( 1 ) " "      " "      " "      " "
## 3 ( 1 ) " "      " "      " "      " "
## 4 ( 1 ) " "      " "      " "      " "
## 5 ( 1 ) " "      " "      " "      " "
## 6 ( 1 ) " "      " "      " "      " "
## 7 ( 1 ) " "      " "      " "      " "
## 8 ( 1 ) " "      " "      " "      " "
## 9 ( 1 ) " "      " "      " "      " "
##           residual.sugar ln.chlorides free.sulfur.dioxide
## 1 ( 1 ) " "      " "      " "
## 2 ( 1 ) " "      " "      " "
## 3 ( 1 ) " "      " "      " "
## 4 ( 1 ) " "      " "      " "
## 5 ( 1 ) " "      " "      " "
## 6 ( 1 ) " "      " "      " "
## 7 ( 1 ) " "      " "      " "
## 8 ( 1 ) " "      " "      " "
## 9 ( 1 ) " "      " "      " "
##           total.sulfur.dioxide density pH  ln.sulphates alcohol
## 1 ( 1 ) " "      " "      " "      " "
## 2 ( 1 ) " "      " "      " "      "*"
## 3 ( 1 ) " "      " "      " "      "*"
## 4 ( 1 ) " "      " "      " "      "*"
## 5 ( 1 ) " "      " "      " "      "*"
## 6 ( 1 ) " "      " "      " "      " "
## 7 ( 1 ) " "      " "      " "      "*"
## 8 ( 1 ) " "      " "      " "      "*"
## 9 ( 1 ) " "      " "      " "      "*"
##           `fixed.acidity:volatile.acidity` `fixed.acidity:citric.acid`
## 1 ( 1 ) " "      " "
## 2 ( 1 ) " "      " "
## 3 ( 1 ) " "      " "
## 4 ( 1 ) " "      " "
## 5 ( 1 ) " "      " "
## 6 ( 1 ) " "      " "
## 7 ( 1 ) " "      " "
## 8 ( 1 ) " "      " "
## 9 ( 1 ) " "      " "
##           `fixed.acidity:residual.sugar` `fixed.acidity:ln.chlorides`
## 1 ( 1 ) " "      " "
## 2 ( 1 ) " "      " "
## 3 ( 1 ) " "      " "
## 4 ( 1 ) " "      " "
## 5 ( 1 ) " "      " "

```

```

## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `fixed.acidity:free.sulfur.dioxide`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `fixed.acidity:total.sulfur.dioxide` `fixed.acidity:density`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `fixed.acidity:pH` `fixed.acidity:ln.sulphates`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `fixed.acidity:alcohol` `volatile.acidity:citric.acid`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `volatile.acidity:residual.sugar` `volatile.acidity:ln.chlorides`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "

```

```

##          `volatile.acidity:free.sulfur.dioxide`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `volatile.acidity:total.sulfur.dioxide`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) "*"
## 9  ( 1 ) "*"
##          `volatile.acidity:density` `volatile.acidity:pH`
## 1  ( 1 ) " "           " "
## 2  ( 1 ) " "           " "
## 3  ( 1 ) " "           "*"
## 4  ( 1 ) " "           "*"
## 5  ( 1 ) " "           "*"
## 6  ( 1 ) "*"          " "
## 7  ( 1 ) " "           "*"
## 8  ( 1 ) " "           "*"
## 9  ( 1 ) " "           "*"
##          `volatile.acidity:ln.sulphates` `volatile.acidity:alcohol`
## 1  ( 1 ) " "           " "
## 2  ( 1 ) "*"          " "
## 3  ( 1 ) " "           " "
## 4  ( 1 ) " "           " "
## 5  ( 1 ) " "           " "
## 6  ( 1 ) " "           " "
## 7  ( 1 ) " "           " "
## 8  ( 1 ) " "           " "
## 9  ( 1 ) " "           " "
##          `citric.acid:residual.sugar` `citric.acid:ln.chlorides`
## 1  ( 1 ) " "           " "
## 2  ( 1 ) " "           " "
## 3  ( 1 ) " "           " "
## 4  ( 1 ) " "           " "
## 5  ( 1 ) " "           " "
## 6  ( 1 ) " "           " "
## 7  ( 1 ) " "           "*"
## 8  ( 1 ) " "           "*"
## 9  ( 1 ) " "           "*"
##          `citric.acid:free.sulfur.dioxide`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "

```

```

## 4 ( 1 ) " "
## 5 ( 1 ) " "
## 6 ( 1 ) " "
## 7 ( 1 ) " "
## 8 ( 1 ) " "
## 9 ( 1 ) " "
##          `citric.acid:total.sulfur.dioxide` `citric.acid:density`
## 1 ( 1 ) " "                                " "
## 2 ( 1 ) " "                                " "
## 3 ( 1 ) " "                                " "
## 4 ( 1 ) "*"                               " "
## 5 ( 1 ) " "                                " "
## 6 ( 1 ) " "                                " "
## 7 ( 1 ) " "                                " "
## 8 ( 1 ) " "                                " "
## 9 ( 1 ) " "                                " "
##          `citric.acid:pH` `citric.acid:ln.sulphates` `citric.acid:alcohol`
## 1 ( 1 ) " "                                " "                                " "
## 2 ( 1 ) " "                                " "                                " "
## 3 ( 1 ) " "                                " "                                " "
## 4 ( 1 ) " "                                " "                                " "
## 5 ( 1 ) " "                                " "                                " "
## 6 ( 1 ) " "                                " "                                " "
## 7 ( 1 ) "*"                               " "                                " "
## 8 ( 1 ) "*"                               " "                                " "
## 9 ( 1 ) "*"                               " "                                " "
##          `residual.sugar:ln.chlorides`
## 1 ( 1 ) " "
## 2 ( 1 ) " "
## 3 ( 1 ) " "
## 4 ( 1 ) " "
## 5 ( 1 ) " "
## 6 ( 1 ) " "
## 7 ( 1 ) " "
## 8 ( 1 ) " "
## 9 ( 1 ) " "
##          `residual.sugar:free.sulfur.dioxide`
## 1 ( 1 ) " "
## 2 ( 1 ) " "
## 3 ( 1 ) " "
## 4 ( 1 ) " "
## 5 ( 1 ) " "
## 6 ( 1 ) " "
## 7 ( 1 ) " "
## 8 ( 1 ) " "
## 9 ( 1 ) "*"
##          `residual.sugar:total.sulfur.dioxide` `residual.sugar:density`
## 1 ( 1 ) " "                                " "
## 2 ( 1 ) " "                                " "
## 3 ( 1 ) " "                                " "
## 4 ( 1 ) " "                                " "
## 5 ( 1 ) " "                                " "
## 6 ( 1 ) " "                                " "
## 7 ( 1 ) " "                                " "

```

```

## 8  ( 1 ) " "
## 9  ( 1 ) " "
##      `residual.sugar:pH` `residual.sugar:ln.sulphates`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##      `residual.sugar:alcohol` `ln.chlorides:free.sulfur.dioxide`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##      `ln.chlorides:total.sulfur.dioxide` `ln.chlorides:density`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##      `ln.chlorides:pH` `ln.chlorides:ln.sulphates`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) "*"
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##      `ln.chlorides:alcohol` `free.sulfur.dioxide:total.sulfur.dioxide`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) "*"
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##      `free.sulfur.dioxide:density` `free.sulfur.dioxide:pH`
## 1  ( 1 ) " "

```

```

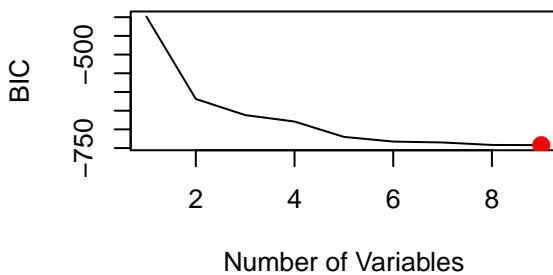
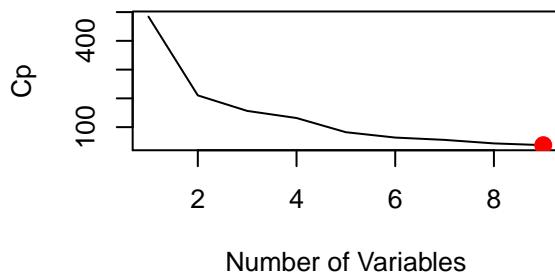
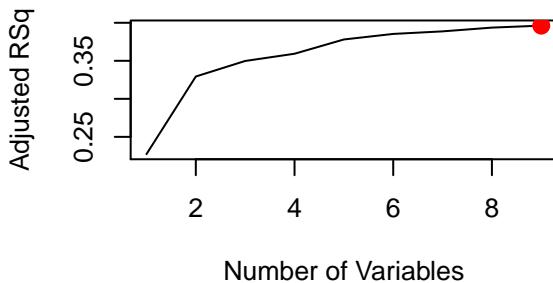
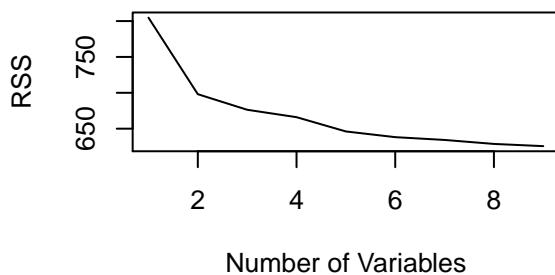
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `free.sulfur.dioxide:ln.sulphates` `free.sulfur.dioxide:alcohol`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `total.sulfur.dioxide:density` `total.sulfur.dioxide:pH`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) "*"
## 7  ( 1 ) "*"
## 8  ( 1 ) "*"
## 9  ( 1 ) "*"
##          `total.sulfur.dioxide:ln.sulphates`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) "*"
## 6  ( 1 ) "*"
## 7  ( 1 ) "*"
## 8  ( 1 ) "*"
## 9  ( 1 ) "*"
##          `total.sulfur.dioxide:alcohol` `density:pH`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `density:ln.sulphates` `density:alcohol` `pH:ln.sulphates`
## 1  ( 1 ) " "      "*"      " "
## 2  ( 1 ) " "      " "      " "
## 3  ( 1 ) " "      " "      " "
## 4  ( 1 ) " "      " "      "*"
## 5  ( 1 ) " "      " "      " "

```

```

## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `pH:alcohol` `ln.sulphates:alcohol`
## 1  ( 1 ) " "      " "
## 2  ( 1 ) " "      " "
## 3  ( 1 ) " "      "*"
## 4  ( 1 ) " "      " "
## 5  ( 1 ) " "      "*"
## 6  ( 1 ) " "      " "
## 7  ( 1 ) " "      "*"
## 8  ( 1 ) " "      "*"
## 9  ( 1 ) " "      "*"

```



We see a model having nine variables is the best among the models in the search space. This indicates that a model including more than nine predictors may be even better.

Following are adjusted R^2 , C_p , and BIC values for models containing interaction terms. Each row contains values for models having five, six, seven, eight, or nine variables.

```

## [1] 0.3780543 0.3853514 0.3887213 0.3936566 0.3962572
## [1] 82.32153 63.70033 55.63115 43.37950 37.39539
## [1] -720.1172 -732.6156 -735.0340 -741.6247 -742.1263

```

We could generate more models. Exponential running time constrained the size of the search space in best subset selection. We can try forward stepwise selection and backward stepwise selection if we want to consider

a deeper search space as they both have running time linear in the number of variables.

```
## Reordering variables and trying again:  
##  
## Subset selection object  
## Call: regsubsets.formula(quality ~ ., wine.interactions, method = "forward")  
## 67 Variables  (and intercept)  
##  
## fixed.acidity      Forced in    Forced out  
## volatile.acidity   FALSE        FALSE  
## citric.acid       FALSE        FALSE  
## residual.sugar    FALSE        FALSE  
## ln.chlorides      FALSE        FALSE  
## free.sulfur.dioxide FALSE        FALSE  
## total.sulfur.dioxide FALSE        FALSE  
## density           FALSE        FALSE  
## pH                FALSE        FALSE  
## ln.sulphates     FALSE        FALSE  
## alcohol          FALSE        FALSE  
## `fixed.acidity:volatile.acidity`  
## `fixed.acidity:citric.acid`  
## `fixed.acidity:residual.sugar`  
## `fixed.acidity:ln.chlorides`  
## `fixed.acidity:free.sulfur.dioxide`  
## `fixed.acidity:total.sulfur.dioxide`  
## `fixed.acidity:density`  
## `fixed.acidity:pH`  
## `fixed.acidity:ln.sulphates`  
## `fixed.acidity:alcohol`  
## `volatile.acidity:citric.acid`  
## `volatile.acidity:residual.sugar`  
## `volatile.acidity:ln.chlorides`  
## `volatile.acidity:free.sulfur.dioxide`  
## `volatile.acidity:total.sulfur.dioxide`  
## `volatile.acidity:density`  
## `volatile.acidity:pH`  
## `volatile.acidity:ln.sulphates`  
## `volatile.acidity:alcohol`  
## `citric.acid:residual.sugar`  
## `citric.acid:ln.chlorides`  
## `citric.acid:free.sulfur.dioxide`  
## `citric.acid:total.sulfur.dioxide`  
## `citric.acid:density`  
## `citric.acid:pH`  
## `citric.acid:ln.sulphates`  
## `citric.acid:alcohol`  
## `residual.sugar:ln.chlorides`  
## `residual.sugar:free.sulfur.dioxide`  
## `residual.sugar:total.sulfur.dioxide`  
## `residual.sugar:density`  
## `residual.sugar:pH`  
## `residual.sugar:ln.sulphates`  
## `residual.sugar:alcohol`  
## `ln.chlorides:free.sulfur.dioxide`  
## `ln.chlorides:total.sulfur.dioxide`
```

```

## `ln.chlorides:density`          FALSE   FALSE
## `ln.chlorides:pH`              FALSE   FALSE
## `ln.chlorides:ln.sulphates`    FALSE   FALSE
## `ln.chlorides:alcohol`         FALSE   FALSE
## `free.sulfur.dioxide:total.sulfur.dioxide` FALSE   FALSE
## `free.sulfur.dioxide:density`  FALSE   FALSE
## `free.sulfur.dioxide:pH`       FALSE   FALSE
## `free.sulfur.dioxide:ln.sulphates` FALSE   FALSE
## `free.sulfur.dioxide:alcohol`  FALSE   FALSE
## `total.sulfur.dioxide:density` FALSE   FALSE
## `total.sulfur.dioxide:pH`      FALSE   FALSE
## `total.sulfur.dioxide:ln.sulphates` FALSE   FALSE
## `total.sulfur.dioxide:alcohol`  FALSE   FALSE
## `density:pH`                  FALSE   FALSE
## `density:ln.sulphates`        FALSE   FALSE
## `density:alcohol`             FALSE   FALSE
## `pH:ln.sulphates`            FALSE   FALSE
## `pH:alcohol`                 FALSE   FALSE
## `ln.sulphates:alcohol`        FALSE   FALSE
## `(Intercept)`                 FALSE   FALSE

## 1 subsets of each size up to 9
## Selection Algorithm: forward
##           `(Intercept)` fixed.acidity volatile.acidity citric.acid
## 1 ( 1 ) " "          " "          " "          " "
## 2 ( 1 ) " "          " "          " "          " "
## 3 ( 1 ) " "          " "          " "          " "
## 4 ( 1 ) " "          " "          " "          " "
## 5 ( 1 ) " "          " "          " "          " "
## 6 ( 1 ) " "          " "          " "          " "
## 7 ( 1 ) " "          " "          " "          " "
## 8 ( 1 ) " "          " "          " "          " "
## 9 ( 1 ) " "          " "          " "          " "

##           residual.sugar ln.chlorides free.sulfur.dioxide
## 1 ( 1 ) " "          " "          " "
## 2 ( 1 ) " "          " "          " "
## 3 ( 1 ) " "          " "          " "
## 4 ( 1 ) " "          " "          " "
## 5 ( 1 ) " "          " "          " "
## 6 ( 1 ) " "          " "          " "
## 7 ( 1 ) " "          " "          " "
## 8 ( 1 ) " "          " "          " "
## 9 ( 1 ) " "          " "          " "

##           total.sulfur.dioxide density pH ln.sulphates alcohol
## 1 ( 1 ) " "          " "          " "          " "          " "
## 2 ( 1 ) " "          " "          " "          " "          " "
## 3 ( 1 ) " "          " "          " "          " "          " "
## 4 ( 1 ) " "          " "          " "          " "          " "
## 5 ( 1 ) " "          " "          " "          " "          " "
## 6 ( 1 ) " "          " "          " "          " "          " "
## 7 ( 1 ) " "          " "          " "          " "          " "
## 8 ( 1 ) " "          " "          " "          " "          " "
## 9 ( 1 ) " "          " "          " "          " "          " "

##           `fixed.acidity:volatile.acidity` `fixed.acidity:citric.acid`
## 1 ( 1 ) " "          " "

```

```

## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `fixed.acidity:residual.sugar` `fixed.acidity:ln.chlorides`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `fixed.acidity:free.sulfur.dioxide`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `fixed.acidity:total.sulfur.dioxide` `fixed.acidity:density`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) "*"
## 5  ( 1 ) "*"
## 6  ( 1 ) "*"
## 7  ( 1 ) "*"
## 8  ( 1 ) "*"
## 9  ( 1 ) "*"
##          `fixed.acidity:pH` `fixed.acidity:ln.sulphates`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `fixed.acidity:alcohol` `volatile.acidity:citric.acid`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "

```

```

## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `volatile.acidity:residual.sugar` `volatile.acidity:ln.chlorides`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `volatile.acidity:free.sulfur.dioxide`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `volatile.acidity:total.sulfur.dioxide`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) "*"
##          `volatile.acidity:density` `volatile.acidity:pH`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) "*"
##          `volatile.acidity:ln.sulphates` `volatile.acidity:alcohol`
## 1  ( 1 ) " "
## 2  ( 1 ) "*"
## 3  ( 1 ) "*"
## 4  ( 1 ) "*"
## 5  ( 1 ) "*"
## 6  ( 1 ) "*"
## 7  ( 1 ) "*"
## 8  ( 1 ) "*"
## 9  ( 1 ) "*"

```

```

##          `citric.acid:residual.sugar` `citric.acid:ln.chlorides`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `citric.acid:free.sulfur.dioxide`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `citric.acid:total.sulfur.dioxide` `citric.acid:density`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `citric.acid:pH` `citric.acid:ln.sulphates` `citric.acid:alcohol`
## 1  ( 1 ) " "           " "           " "
## 2  ( 1 ) " "           " "           " "
## 3  ( 1 ) " "           " "           " "
## 4  ( 1 ) " "           " "           " "
## 5  ( 1 ) " "           " "           " "
## 6  ( 1 ) " "           " "           " "
## 7  ( 1 ) " "           " "           " "
## 8  ( 1 ) " "           " "           " "
## 9  ( 1 ) " "           " "           " "
##          `residual.sugar:ln.chlorides`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `residual.sugar:free.sulfur.dioxide`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "

```

```

## 4 ( 1 ) " "
## 5 ( 1 ) " "
## 6 ( 1 ) " "
## 7 ( 1 ) " "
## 8 ( 1 ) " "
## 9 ( 1 ) " "
## `residual.sugar:total.sulfur.dioxide` `residual.sugar:density`
## 1 ( 1 ) " " " "
## 2 ( 1 ) " " " "
## 3 ( 1 ) " " " "
## 4 ( 1 ) " " " "
## 5 ( 1 ) " " " "
## 6 ( 1 ) " " " "
## 7 ( 1 ) " " " "
## 8 ( 1 ) " " " "
## 9 ( 1 ) " " " "
## `residual.sugar:pH` `residual.sugar:ln.sulphates`
## 1 ( 1 ) " " " "
## 2 ( 1 ) " " " "
## 3 ( 1 ) " " " "
## 4 ( 1 ) " " " "
## 5 ( 1 ) " " " "
## 6 ( 1 ) " " " "
## 7 ( 1 ) " " " "
## 8 ( 1 ) " " " "
## 9 ( 1 ) " " " "
## `residual.sugar:alcohol` `ln.chlorides:free.sulfur.dioxide`
## 1 ( 1 ) " " " "
## 2 ( 1 ) " " " "
## 3 ( 1 ) " " " "
## 4 ( 1 ) " " " "
## 5 ( 1 ) " " " "
## 6 ( 1 ) " " " "
## 7 ( 1 ) " " " "
## 8 ( 1 ) " " " "
## 9 ( 1 ) " " " "
## `ln.chlorides:total.sulfur.dioxide` `ln.chlorides:density`
## 1 ( 1 ) " " " "
## 2 ( 1 ) " " " "
## 3 ( 1 ) " " " "
## 4 ( 1 ) " " " "
## 5 ( 1 ) " " " "
## 6 ( 1 ) " " " "
## 7 ( 1 ) " " " "
## 8 ( 1 ) " " " "
## 9 ( 1 ) " " " "
## `ln.chlorides:pH` `ln.chlorides:ln.sulphates`
## 1 ( 1 ) " " " "
## 2 ( 1 ) " " " "
## 3 ( 1 ) " " " "
## 4 ( 1 ) " " " "
## 5 ( 1 ) " " " "
## 6 ( 1 ) " " " "
## 7 ( 1 ) " " " "

```

```

## 8 ( 1 ) " "
## 9 ( 1 ) " "
## `ln.chlorides:alcohol` `free.sulfur.dioxide:total.sulfur.dioxide`
## 1 ( 1 ) " "
## 2 ( 1 ) " "
## 3 ( 1 ) " "
## 4 ( 1 ) " "
## 5 ( 1 ) " "
## 6 ( 1 ) " "
## 7 ( 1 ) " "
## 8 ( 1 ) " "
## 9 ( 1 ) " "
## `free.sulfur.dioxide:density` `free.sulfur.dioxide:pH`
## 1 ( 1 ) " "
## 2 ( 1 ) " "
## 3 ( 1 ) " "
## 4 ( 1 ) " "
## 5 ( 1 ) " "
## 6 ( 1 ) " "
## 7 ( 1 ) " "
## 8 ( 1 ) " "
## 9 ( 1 ) " "
## `free.sulfur.dioxide:ln.sulphates` `free.sulfur.dioxide:alcohol`
## 1 ( 1 ) " "
## 2 ( 1 ) " "
## 3 ( 1 ) " "
## 4 ( 1 ) " "
## 5 ( 1 ) " "
## 6 ( 1 ) " "
## 7 ( 1 ) " "
## 8 ( 1 ) " "
## 9 ( 1 ) " "
## `total.sulfur.dioxide:density` `total.sulfur.dioxide:pH`
## 1 ( 1 ) " "
## 2 ( 1 ) " "
## 3 ( 1 ) " "
## 4 ( 1 ) " "
## 5 ( 1 ) " "
## 6 ( 1 ) " "
## 7 ( 1 ) " "
## 8 ( 1 ) " "
## 9 ( 1 ) " "
## `total.sulfur.dioxide:ln.sulphates`
## 1 ( 1 ) " "
## 2 ( 1 ) " "
## 3 ( 1 ) " "
## 4 ( 1 ) " "
## 5 ( 1 ) "*"
## 6 ( 1 ) "*"
## 7 ( 1 ) "*"
## 8 ( 1 ) "*"
## 9 ( 1 ) "*"
## `total.sulfur.dioxide:alcohol` `density:pH`
## 1 ( 1 ) " "

```

```

## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `density:ln.sulphates` `density:alcohol` `pH:ln.sulphates`
## 1  ( 1 ) " "      "*"      " "
## 2  ( 1 ) " "      "*"      " "
## 3  ( 1 ) " "      "*"      " "
## 4  ( 1 ) " "      "*"      " "
## 5  ( 1 ) " "      "*"      " "
## 6  ( 1 ) " "      "*"      "*"
## 7  ( 1 ) " "      "*"      "*"
## 8  ( 1 ) " "      "*"      "*"
## 9  ( 1 ) " "      "*"      "*"
##          `pH:alcohol` `ln.sulphates:alcohol`
## 1  ( 1 ) " "      " "
## 2  ( 1 ) " "      " "
## 3  ( 1 ) " "      " "
## 4  ( 1 ) " "      " "
## 5  ( 1 ) " "      " "
## 6  ( 1 ) " "      " "
## 7  ( 1 ) " "      " "
## 8  ( 1 ) " "      " "
## 9  ( 1 ) " "      " "
## [1] 0.3659452 0.3741626 0.3813019 0.3858620 0.3877112
## [1] 114.82254 93.71238 75.51981 64.26113 60.27545
## [1] -689.2845 -703.7700 -715.7432 -721.2003 -719.6512
## Reordering variables and trying again:
## Subset selection object
## Call: regsubsets.formula(quality ~ ., wine.interactions, method = "backward")
## 67 Variables  (and intercept)
##                                     Forced in  Forced out
## fixed.acidity                      FALSE     FALSE
## volatile.acidity                    FALSE     FALSE
## citric.acid                        FALSE     FALSE
## residual.sugar                     FALSE     FALSE
## ln.chlorides                       FALSE     FALSE
## free.sulfur.dioxide                FALSE     FALSE
## total.sulfur.dioxide               FALSE     FALSE
## density                            FALSE     FALSE
## pH                                 FALSE     FALSE
## ln.sulphates                      FALSE     FALSE
## alcohol                            FALSE     FALSE
## `fixed.acidity:volatile.acidity`   FALSE     FALSE
## `fixed.acidity:citric.acid`        FALSE     FALSE
## `fixed.acidity:residual.sugar`     FALSE     FALSE
## `fixed.acidity:ln.chlorides`       FALSE     FALSE

```

```

## `fixed.acidity:free.sulfur.dioxide`      FALSE  FALSE
## `fixed.acidity:total.sulfur.dioxide`     FALSE  FALSE
## `fixed.acidity:density`                 FALSE  FALSE
## `fixed.acidity:pH`                     FALSE  FALSE
## `fixed.acidity:ln.sulphates`            FALSE  FALSE
## `fixed.acidity:alcohol`                FALSE  FALSE
## `volatile.acidity:citric.acid`         FALSE  FALSE
## `volatile.acidity:residual.sugar`       FALSE  FALSE
## `volatile.acidity:ln.chlorides`         FALSE  FALSE
## `volatile.acidity:free.sulfur.dioxide`  FALSE  FALSE
## `volatile.acidity:total.sulfur.dioxide` FALSE  FALSE
## `volatile.acidity:density`              FALSE  FALSE
## `volatile.acidity:pH`                  FALSE  FALSE
## `volatile.acidity:ln.sulphates`         FALSE  FALSE
## `volatile.acidity:alcohol`              FALSE  FALSE
## `citric.acid:residual.sugar`           FALSE  FALSE
## `citric.acid:ln.chlorides`             FALSE  FALSE
## `citric.acid:free.sulfur.dioxide`      FALSE  FALSE
## `citric.acid:total.sulfur.dioxide`     FALSE  FALSE
## `citric.acid:density`                 FALSE  FALSE
## `citric.acid:pH`                      FALSE  FALSE
## `citric.acid:ln.sulphates`             FALSE  FALSE
## `citric.acid:alcohol`                 FALSE  FALSE
## `residual.sugar:ln.chlorides`          FALSE  FALSE
## `residual.sugar:free.sulfur.dioxide`   FALSE  FALSE
## `residual.sugar:total.sulfur.dioxide` FALSE  FALSE
## `residual.sugar:density`               FALSE  FALSE
## `residual.sugar:pH`                   FALSE  FALSE
## `residual.sugar:ln.sulphates`          FALSE  FALSE
## `residual.sugar:alcohol`               FALSE  FALSE
## `ln.chlorides:free.sulfur.dioxide`    FALSE  FALSE
## `ln.chlorides:total.sulfur.dioxide`   FALSE  FALSE
## `ln.chlorides:density`                FALSE  FALSE
## `ln.chlorides:pH`                     FALSE  FALSE
## `ln.chlorides:ln.sulphates`            FALSE  FALSE
## `ln.chlorides:alcohol`                FALSE  FALSE
## `free.sulfur.dioxide:total.sulfur.dioxide` FALSE  FALSE
## `free.sulfur.dioxide:density`          FALSE  FALSE
## `free.sulfur.dioxide:pH`               FALSE  FALSE
## `free.sulfur.dioxide:ln.sulphates`    FALSE  FALSE
## `free.sulfur.dioxide:alcohol`          FALSE  FALSE
## `total.sulfur.dioxide:density`        FALSE  FALSE
## `total.sulfur.dioxide:pH`              FALSE  FALSE
## `total.sulfur.dioxide:ln.sulphates`   FALSE  FALSE
## `total.sulfur.dioxide:alcohol`         FALSE  FALSE
## `density:pH`                         FALSE  FALSE
## `density:ln.sulphates`                FALSE  FALSE
## `density:alcohol`                     FALSE  FALSE
## `pH:ln.sulphates`                    FALSE  FALSE
## `pH:alcohol`                         FALSE  FALSE
## `ln.sulphates:alcohol`                FALSE  FALSE
## `(Intercept)`                        FALSE  FALSE
## 1 subsets of each size up to 9
## Selection Algorithm: backward

```

```

##          `(Intercept)` fixed.acidity volatile.acidity citric.acid
## 1      ( 1 ) " "      " "      " "      " "
## 2      ( 1 ) " "      " "      " "      " "
## 3      ( 1 ) " "      " "      " "      " "
## 4      ( 1 ) " "      " "      " "      " "
## 5      ( 1 ) " "      " "      " "      " "
## 6      ( 1 ) " "      " "      " "      " "
## 7      ( 1 ) " "      " "      " "      " "
## 8      ( 1 ) " "      " "      " "      " "
## 9      ( 1 ) " "      " "      " "      " "
##          residual.sugar ln.chlorides free.sulfur.dioxide
## 1      ( 1 ) " "      " "      " "
## 2      ( 1 ) " "      " "      " "
## 3      ( 1 ) " "      " "      " "
## 4      ( 1 ) " "      " "      " "
## 5      ( 1 ) " "      " "      " "
## 6      ( 1 ) " "      " "      " "
## 7      ( 1 ) " "      " "      " "
## 8      ( 1 ) " "      " "      "*" "
## 9      ( 1 ) " "      " "      "*" "
##          total.sulfur.dioxide density pH ln.sulphates alcohol
## 1      ( 1 ) " "      " "      " "      " "
## 2      ( 1 ) " "      " "      " "      " "
## 3      ( 1 ) " "      " "      " "      " "
## 4      ( 1 ) " "      " "      " "      " "
## 5      ( 1 ) " "      " "      " "      " "
## 6      ( 1 ) " "      " "      " "      " "
## 7      ( 1 ) " "      " "      " "      " "
## 8      ( 1 ) " "      " "      " "      " "
## 9      ( 1 ) "*"      " "      " "      " "
##          `fixed.acidity:volatile.acidity` `fixed.acidity:citric.acid`
## 1      ( 1 ) " "      " "
## 2      ( 1 ) " "      " "
## 3      ( 1 ) " "      " "
## 4      ( 1 ) " "      " "
## 5      ( 1 ) " "      " "
## 6      ( 1 ) " "      " "
## 7      ( 1 ) " "      " "
## 8      ( 1 ) " "      " "
## 9      ( 1 ) " "      " "
##          `fixed.acidity:residual.sugar` `fixed.acidity:ln.chlorides`
## 1      ( 1 ) " "      " "
## 2      ( 1 ) " "      " "
## 3      ( 1 ) " "      " "
## 4      ( 1 ) " "      " "
## 5      ( 1 ) " "      " "
## 6      ( 1 ) " "      " "
## 7      ( 1 ) " "      " "
## 8      ( 1 ) " "      " "
## 9      ( 1 ) " "      " "
##          `fixed.acidity:free.sulfur.dioxide`
## 1      ( 1 ) " "
## 2      ( 1 ) " "
## 3      ( 1 ) " "

```

```

## 4 ( 1 ) " "
## 5 ( 1 ) " "
## 6 ( 1 ) " "
## 7 ( 1 ) " "
## 8 ( 1 ) " "
## 9 ( 1 ) " "
##           `fixed.acidity:total.sulfur.dioxide` `fixed.acidity:density`
## 1 ( 1 ) " "                      " "
## 2 ( 1 ) " "                      " "
## 3 ( 1 ) " "                      " "
## 4 ( 1 ) " "                      " "
## 5 ( 1 ) " "                      " "
## 6 ( 1 ) " "                      " "
## 7 ( 1 ) " "                      " "
## 8 ( 1 ) " "                      " "
## 9 ( 1 ) " "                      " "
##           `fixed.acidity:pH` `fixed.acidity:ln.sulphates`
## 1 ( 1 ) " "                      " "
## 2 ( 1 ) " "                      " "
## 3 ( 1 ) " "                      " "
## 4 ( 1 ) " "                      " "
## 5 ( 1 ) " "                      " "
## 6 ( 1 ) " "                      " "
## 7 ( 1 ) " "                      " "
## 8 ( 1 ) " "                      " "
## 9 ( 1 ) " "                      " "
##           `fixed.acidity:alcohol` `volatile.acidity:citric.acid`
## 1 ( 1 ) " "                      " "
## 2 ( 1 ) " "                      " "
## 3 ( 1 ) " "                      " "
## 4 ( 1 ) " "                      " "
## 5 ( 1 ) " "                      " "
## 6 ( 1 ) " "                      " "
## 7 ( 1 ) " "                      " "
## 8 ( 1 ) " "                      " "
## 9 ( 1 ) " "                      " "
##           `volatile.acidity:residual.sugar` `volatile.acidity:ln.chlorides`
## 1 ( 1 ) " "                      " "
## 2 ( 1 ) " "                      " "
## 3 ( 1 ) " "                      " "
## 4 ( 1 ) " "                      " "
## 5 ( 1 ) " "                      " "
## 6 ( 1 ) " "                      " "
## 7 ( 1 ) " "                      " "
## 8 ( 1 ) " "                      " "
## 9 ( 1 ) " "                      " "
##           `volatile.acidity:free.sulfur.dioxide`
## 1 ( 1 ) " "
## 2 ( 1 ) " "
## 3 ( 1 ) " "
## 4 ( 1 ) " "
## 5 ( 1 ) " "
## 6 ( 1 ) " "
## 7 ( 1 ) " "

```

```

## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `volatile.acidity:total.sulfur.dioxide`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) "*"
## 8  ( 1 ) "*"
## 9  ( 1 ) "*"
##          `volatile.acidity:density` `volatile.acidity:pH`
## 1  ( 1 ) " "          " "
## 2  ( 1 ) "*"         " "
## 3  ( 1 ) "*"         " "
## 4  ( 1 ) "*"         " "
## 5  ( 1 ) "*"         " "
## 6  ( 1 ) "*"         " "
## 7  ( 1 ) "*"         " "
## 8  ( 1 ) "*"         " "
## 9  ( 1 ) "*"         " "
##          `volatile.acidity:ln.sulphates` `volatile.acidity:alcohol`
## 1  ( 1 ) " "          " "
## 2  ( 1 ) " "          " "
## 3  ( 1 ) " "          " "
## 4  ( 1 ) " "          " "
## 5  ( 1 ) " "          " "
## 6  ( 1 ) " "          " "
## 7  ( 1 ) " "          " "
## 8  ( 1 ) " "          " "
## 9  ( 1 ) " "          " "
##          `citric.acid:residual.sugar` `citric.acid:ln.chlorides`
## 1  ( 1 ) " "          " "
## 2  ( 1 ) " "          " "
## 3  ( 1 ) " "          " "
## 4  ( 1 ) " "          " "
## 5  ( 1 ) " "          " "
## 6  ( 1 ) " "          " "
## 7  ( 1 ) " "          " "
## 8  ( 1 ) " "          " "
## 9  ( 1 ) " "          " "
##          `citric.acid:free.sulfur.dioxide`
## 1  ( 1 ) " "
## 2  ( 1 ) " "
## 3  ( 1 ) " "
## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
##          `citric.acid:total.sulfur.dioxide` `citric.acid:density`
## 1  ( 1 ) " "          " "

```

```

## 2 ( 1 ) " "
## 3 ( 1 ) " "
## 4 ( 1 ) " "
## 5 ( 1 ) " "
## 6 ( 1 ) " "
## 7 ( 1 ) " "
## 8 ( 1 ) " "
## 9 ( 1 ) " "
##           `citric.acid:pH` `citric.acid:ln.sulphates` `citric.acid:alcohol`
## 1 ( 1 ) " "          " "          " "
## 2 ( 1 ) " "          " "          " "
## 3 ( 1 ) " "          " "          " "
## 4 ( 1 ) " "          " "          " "
## 5 ( 1 ) " "          " "          " "
## 6 ( 1 ) " "          " "          " "
## 7 ( 1 ) " "          " "          " "
## 8 ( 1 ) " "          " "          " "
## 9 ( 1 ) " "          " "          " "
##           `residual.sugar:ln.chlorides`
## 1 ( 1 ) " "
## 2 ( 1 ) " "
## 3 ( 1 ) " "
## 4 ( 1 ) " "
## 5 ( 1 ) " "
## 6 ( 1 ) " "
## 7 ( 1 ) " "
## 8 ( 1 ) " "
## 9 ( 1 ) " "
##           `residual.sugar:free.sulfur.dioxide`
## 1 ( 1 ) " "
## 2 ( 1 ) " "
## 3 ( 1 ) " "
## 4 ( 1 ) " "
## 5 ( 1 ) " "
## 6 ( 1 ) " "
## 7 ( 1 ) " "
## 8 ( 1 ) " "
## 9 ( 1 ) " "
##           `residual.sugar:total.sulfur.dioxide` `residual.sugar:density`
## 1 ( 1 ) " "          " "
## 2 ( 1 ) " "          " "
## 3 ( 1 ) " "          " "
## 4 ( 1 ) " "          " "
## 5 ( 1 ) " "          " "
## 6 ( 1 ) " "          " "
## 7 ( 1 ) " "          " "
## 8 ( 1 ) " "          " "
## 9 ( 1 ) " "          " "
##           `residual.sugar:pH` `residual.sugar:ln.sulphates`
## 1 ( 1 ) " "          " "
## 2 ( 1 ) " "          " "
## 3 ( 1 ) " "          " "
## 4 ( 1 ) " "          " "
## 5 ( 1 ) " "

```

```

## 6  ( 1 ) " "          " "
## 7  ( 1 ) " "          " "
## 8  ( 1 ) " "          " "
## 9  ( 1 ) " "          " "
##           `residual.sugar:alcohol` `ln.chlorides:free.sulfur.dioxide`
## 1  ( 1 ) " "          " "
## 2  ( 1 ) " "          " "
## 3  ( 1 ) " "          " "
## 4  ( 1 ) " "          " "
## 5  ( 1 ) " "          " "
## 6  ( 1 ) " "          "*"
## 7  ( 1 ) " "          "*"
## 8  ( 1 ) " "          "*"
## 9  ( 1 ) " "          "*"
##           `ln.chlorides:total.sulfur.dioxide` `ln.chlorides:density`
## 1  ( 1 ) " "          " "
## 2  ( 1 ) " "          " "
## 3  ( 1 ) " "          " "
## 4  ( 1 ) " "          " "
## 5  ( 1 ) " "          " "
## 6  ( 1 ) " "          " "
## 7  ( 1 ) " "          " "
## 8  ( 1 ) " "          " "
## 9  ( 1 ) " "          " "
##           `ln.chlorides:pH` `ln.chlorides:ln.sulphates`
## 1  ( 1 ) " "          " "
## 2  ( 1 ) " "          " "
## 3  ( 1 ) " "          " "
## 4  ( 1 ) " "          " "
## 5  ( 1 ) " "          " "
## 6  ( 1 ) " "          " "
## 7  ( 1 ) " "          " "
## 8  ( 1 ) " "          " "
## 9  ( 1 ) " "          " "
##           `ln.chlorides:alcohol` `free.sulfur.dioxide:total.sulfur.dioxide`
## 1  ( 1 ) " "          " "
## 2  ( 1 ) " "          " "
## 3  ( 1 ) " "          " "
## 4  ( 1 ) " "          " "
## 5  ( 1 ) " "          " "
## 6  ( 1 ) " "          " "
## 7  ( 1 ) " "          " "
## 8  ( 1 ) " "          " "
## 9  ( 1 ) " "          " "
##           `free.sulfur.dioxide:density` `free.sulfur.dioxide:pH`
## 1  ( 1 ) " "          " "
## 2  ( 1 ) " "          " "
## 3  ( 1 ) " "          " "
## 4  ( 1 ) " "          " "
## 5  ( 1 ) " "          " "
## 6  ( 1 ) " "          " "
## 7  ( 1 ) " "          " "
## 8  ( 1 ) " "          " "
## 9  ( 1 ) " "          " "

```

```

##          `free.sulfur.dioxide:ln.sulphates` `free.sulfur.dioxide:alcohol` 
## 1  ( 1 ) " "                      " "
## 2  ( 1 ) " "                      " "
## 3  ( 1 ) " "                      " "
## 4  ( 1 ) " "                      " "
## 5  ( 1 ) " "                      " "
## 6  ( 1 ) " "                      " "
## 7  ( 1 ) " "                      " "
## 8  ( 1 ) " "                      " "
## 9  ( 1 ) " "                      " "
##          `total.sulfur.dioxide:density` `total.sulfur.dioxide:pH` 
## 1  ( 1 ) " "                      " "
## 2  ( 1 ) " "                      " "
## 3  ( 1 ) " "                      " "
## 4  ( 1 ) " "                      " "
## 5  ( 1 ) "*"                     " "
## 6  ( 1 ) "*"                     " "
## 7  ( 1 ) "*"                     " "
## 8  ( 1 ) "*"                     " "
## 9  ( 1 ) "*"                     " "
##          `total.sulfur.dioxide:ln.sulphates` 
## 1  ( 1 ) " " 
## 2  ( 1 ) " " 
## 3  ( 1 ) " " 
## 4  ( 1 ) " " 
## 5  ( 1 ) " " 
## 6  ( 1 ) " " 
## 7  ( 1 ) " " 
## 8  ( 1 ) " " 
## 9  ( 1 ) " " 
##          `total.sulfur.dioxide:alcohol` `density:pH` 
## 1  ( 1 ) " "                      " "
## 2  ( 1 ) " "                      " "
## 3  ( 1 ) " "                      " "
## 4  ( 1 ) " "                      " "
## 5  ( 1 ) " "                      " "
## 6  ( 1 ) " "                      " "
## 7  ( 1 ) " "                      " "
## 8  ( 1 ) " "                      " "
## 9  ( 1 ) " "                      " "
##          `density:ln.sulphates` `density:alcohol` `pH:ln.sulphates` 
## 1  ( 1 ) " "                      "*"        " "
## 2  ( 1 ) " "                      "*"        " "
## 3  ( 1 ) " "                      "*"        "*"
## 4  ( 1 ) "*"                     "*"        "*"
## 5  ( 1 ) "*"                     "*"        "*"
## 6  ( 1 ) "*"                     "*"        "*"
## 7  ( 1 ) "*"                     "*"        "*"
## 8  ( 1 ) "*"                     "*"        "*"
## 9  ( 1 ) "*"                     "*"        "*"
##          `pH:alcohol` `ln.sulphates:alcohol` 
## 1  ( 1 ) " "                      " "
## 2  ( 1 ) " "                      " "
## 3  ( 1 ) " "                      " "

```

```

## 4  ( 1 ) " "
## 5  ( 1 ) " "
## 6  ( 1 ) " "
## 7  ( 1 ) " "
## 8  ( 1 ) " "
## 9  ( 1 ) " "
## [1] 0.3636313 0.3687240 0.3748090 0.3816577 0.3828507
## [1] 121.03329 108.30057 92.92524 75.52422 73.28846
## [1] -683.4596 -689.9345 -699.0499 -710.2912 -707.0080
##
## Call:
## lm(formula = quality ~ poly(alcohol, 5) + poly(volatile.acidity,
##      5) + poly(ln.sulphates, 5) + poly(citric.acid, 5) + poly(total.sulfur.dioxide,
##      5) + poly(density, 5) + poly(ln.chlorides, 5) + poly(fixed.acidity,
##      5) + poly(pH, 5) + poly(free.sulfur.dioxide, 5) + poly(residual.sugar,
##      5), data = wine)
##
## Residuals:
##      Min      1Q Median      3Q     Max
## -2.58348 -0.38124 -0.01934  0.39453  2.11170
##
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)
## (Intercept)                 5.63602   0.01567 359.666 < 2e-16 ***
## poly(alcohol, 5)1          10.28498   1.32902  7.739 1.80e-14 ***
## poly(alcohol, 5)2         -0.16211   0.76675 -0.211  0.83259
## poly(alcohol, 5)3         -1.92263   0.74062 -2.596  0.00952 **
## poly(alcohol, 5)4         -1.14866   0.74808 -1.535  0.12487
## poly(alcohol, 5)5          0.47296   0.74028  0.639  0.52298
## poly(volatile.acidity, 5)1 -6.54350   0.92731 -7.056 2.57e-12 ***
## poly(volatile.acidity, 5)2 -0.24013   0.71303 -0.337  0.73633
## poly(volatile.acidity, 5)3 -1.01968   0.64571 -1.579  0.11450
## poly(volatile.acidity, 5)4 -0.32106   0.65976 -0.487  0.62659
## poly(volatile.acidity, 5)5  0.27325   0.64625  0.423  0.67248
## poly(ln.sulphates, 5)1     8.21214   0.80830 10.160 < 2e-16 ***
## poly(ln.sulphates, 5)2    -4.11566   0.77500 -5.311 1.25e-07 ***
## poly(ln.sulphates, 5)3     0.05073   0.72421  0.070  0.94416
## poly(ln.sulphates, 5)4     1.62516   0.72143  2.253  0.02442 *
## poly(ln.sulphates, 5)5     0.87394   0.68433  1.277  0.20177
## poly(citric.acid, 5)1     -1.98558   1.23380 -1.609  0.10775
## poly(citric.acid, 5)2     0.99638   0.86512  1.152  0.24961
## poly(citric.acid, 5)3     0.53025   1.03412  0.513  0.60820
## poly(citric.acid, 5)4     -1.30723   1.00591 -1.300  0.19395
## poly(citric.acid, 5)5     -1.36090   0.74328 -1.831  0.06730 .
## poly(total.sulfur.dioxide, 5)1 -4.66213   1.18340 -3.940 8.53e-05 ***
## poly(total.sulfur.dioxide, 5)2     0.96016   0.84191  1.140  0.25427
## poly(total.sulfur.dioxide, 5)3     0.89955   0.76900  1.170  0.24228
## poly(total.sulfur.dioxide, 5)4     0.44546   0.72279  0.616  0.53779
## poly(total.sulfur.dioxide, 5)5     -0.55412   0.67060 -0.826  0.40876
## poly(density, 5)1            -3.40976   1.91186 -1.783  0.07471 .
## poly(density, 5)2            1.36859   1.03628  1.321  0.18681
## poly(density, 5)3            0.31069   0.95772  0.324  0.74567

```

```

## poly(density, 5)4          0.89110  0.84923  1.049  0.29420
## poly(density, 5)5          1.19202  0.77953  1.529  0.12643
## poly(ln.chlorides, 5)1     -2.45563  0.83799 -2.930  0.00344 **
## poly(ln.chlorides, 5)2     -0.41124  0.84804 -0.485  0.62779
## poly(ln.chlorides, 5)3     -1.33563  0.76963 -1.735  0.08287 .
## poly(ln.chlorides, 5)4     0.79402  0.78660  1.009  0.31293
## poly(ln.chlorides, 5)5     -1.71509  0.74406 -2.305  0.02130 *
## poly(fixed.acidity, 5)1    2.36567  1.93189  1.225  0.22094
## poly(fixed.acidity, 5)2    -1.83778  1.02206 -1.798  0.07236 .
## poly(fixed.acidity, 5)3    -0.75373  0.98014 -0.769  0.44201
## poly(fixed.acidity, 5)4    -0.45051  0.87504 -0.515  0.60673
## poly(fixed.acidity, 5)5    -1.52746  0.77222 -1.978  0.04810 *
## poly(pH, 5)1               -3.51244  1.30173 -2.698  0.00705 **
## poly(pH, 5)2               -0.70354  0.88837 -0.792  0.42851
## poly(pH, 5)3               -0.15072  0.84651 -0.178  0.85871
## poly(pH, 5)4               0.73429  0.89093  0.824  0.40996
## poly(pH, 5)5               0.23185  0.94864  0.244  0.80695
## poly(free.sulfur.dioxide, 5)1 1.93920  1.10241  1.759  0.07877 .
## poly(free.sulfur.dioxide, 5)2 -0.53875  0.84490 -0.638  0.52379
## poly(free.sulfur.dioxide, 5)3 1.93094  0.73444  2.629  0.00865 **
## poly(free.sulfur.dioxide, 5)4 -0.55901  0.68585 -0.815  0.41516
## poly(free.sulfur.dioxide, 5)5  0.16298  0.68028  0.240  0.81069
## poly(residual.sugar, 5)1    0.64924  0.96908  0.670  0.50299
## poly(residual.sugar, 5)2    -0.68548  0.87965 -0.779  0.43594
## poly(residual.sugar, 5)3    -0.60983  0.75345 -0.809  0.41843
## poly(residual.sugar, 5)4    -0.76856  0.72693 -1.057  0.29055
## poly(residual.sugar, 5)5    1.36550  0.68348  1.998  0.04591 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 0.6266 on 1543 degrees of freedom
## Multiple R-squared:  0.4187, Adjusted R-squared:  0.3979
## F-statistic:  20.2 on 55 and 1543 DF,  p-value: < 2.2e-16

```

A linear model comprising the fifth-degree polynomials of the log-transformed variables and the other nine original predictors results in an adjusted R^2 of 0.3979. However, some of these polynomials have p-values over 0.05.

Let's fit a linear model where we include the log-transformed variables ln.sulphates and ln.chlorides and the highest-degree polynomial ($d \leq 5$) having a p-value less than 0.05 for each of the other nine predictors. Note this model will not include citric acid or density.

```

## 
## Call:
## lm(formula = quality ~ poly(alcohol, 3) + volatile.acidity +
##      poly(ln.sulphates, 4) + total.sulfur.dioxide + poly(ln.chlorides,
##      5) + poly(fixed.acidity, 5) + pH + poly(free.sulfur.dioxide,
##      3) + poly(residual.sugar, 5), data = wine)
## 
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.49207 -0.38637 -0.02231  0.41817  2.02970
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)

```

```

## (Intercept) 8.8264592 0.5622489 15.698 < 2e-16 ***
## poly(alcohol, 3)1 12.4961718 0.7893134 15.832 < 2e-16 ***
## poly(alcohol, 3)2 0.1776200 0.6948740 0.256 0.79828
## poly(alcohol, 3)3 -1.7819382 0.6681835 -2.667 0.00774 **
## volatile.acidity -0.7974847 0.1024135 -7.787 1.24e-14 ***
## poly(ln.sulphates, 4)1 7.2856471 0.7409176 9.833 < 2e-16 ***
## poly(ln.sulphates, 4)2 -4.1652708 0.7340748 -5.674 1.66e-08 ***
## poly(ln.sulphates, 4)3 -0.1880934 0.6676312 -0.282 0.77819
## poly(ln.sulphates, 4)4 1.2363896 0.6730242 1.837 0.06639 .
## total.sulfur.dioxide -0.0035193 0.0007514 -4.684 3.06e-06 ***
## poly(ln.chlorides, 5)1 -3.1996839 0.7603324 -4.208 2.72e-05 ***
## poly(ln.chlorides, 5)2 -0.1438857 0.7273924 -0.198 0.84322
## poly(ln.chlorides, 5)3 -1.5566785 0.6696959 -2.324 0.02023 *
## poly(ln.chlorides, 5)4 0.6078748 0.6942536 0.876 0.38139
## poly(ln.chlorides, 5)5 -1.6207619 0.6728395 -2.409 0.01612 *
## poly(fixed.acidity, 5)1 -1.3746240 1.0388312 -1.323 0.18595
## poly(fixed.acidity, 5)2 -1.3769143 0.7119302 -1.934 0.05329 .
## poly(fixed.acidity, 5)3 -0.0654365 0.6811769 -0.096 0.92348
## poly(fixed.acidity, 5)4 -0.5603795 0.7022541 -0.798 0.42501
## poly(fixed.acidity, 5)5 -1.6203307 0.6584475 -2.461 0.01397 *
## pH -0.7870387 0.1695316 -4.642 3.73e-06 ***
## poly(free.sulfur.dioxide, 3)1 1.9548878 0.9033659 2.164 0.03061 *
## poly(free.sulfur.dioxide, 3)2 -0.4237379 0.6988166 -0.606 0.54436
## poly(free.sulfur.dioxide, 3)3 1.8427286 0.6534345 2.820 0.00486 **
## poly(residual.sugar, 5)1 0.2359469 0.6985237 0.338 0.73557
## poly(residual.sugar, 5)2 0.2525009 0.6921508 0.365 0.71531
## poly(residual.sugar, 5)3 -0.5623577 0.6618234 -0.850 0.39562
## poly(residual.sugar, 5)4 -0.1779414 0.6420164 -0.277 0.78169
## poly(residual.sugar, 5)5 1.6039378 0.6415296 2.500 0.01251 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6288 on 1570 degrees of freedom
## Multiple R-squared: 0.4043, Adjusted R-squared: 0.3936
## F-statistic: 38.05 on 28 and 1570 DF, p-value: < 2.2e-16

```

The adjusted R^2 decreases from 0.3979 to 0.3936, but most of the p-values are below 0.05. I wonder which model will fare better during cross-validation.

Next we determine a model including fifth-degree polynomials of the two log-transformed predictors and sixty-four variables, including the other nine predictors and all fifty-five interaction terms.

```

## 
## Call:
## lm(formula = quality ~ poly(alcohol, 5) + poly(volatile.acidity,
## 5) + poly(ln.sulphates, 5) + poly(citric.acid, 5) + poly(total.sulfur.dioxide,
## 5) + poly(density, 5) + poly(ln.chlorides, 5) + poly(fixed.acidity,
## 5) + poly(pH, 5) + poly(free.sulfur.dioxide, 5) + poly(residual.sugar,
## 5) + poly(`fixed.acidity:volatile.acidity`, 5) + poly(`fixed.acidity:citric.acid`,
## 5) + poly(`fixed.acidity:residual.sugar`, 5) + poly(`fixed.acidity:ln.chlorides`,
## 5) + poly(`fixed.acidity:free.sulfur.dioxide`, 5) + poly(`fixed.acidity:total.sulfur.dioxide`,
## 5) + poly(`fixed.acidity:density`, 5) + poly(`fixed.acidity:pH`,
## 5) + poly(`fixed.acidity:ln.sulphates`, 5) + poly(`fixed.acidity:alcohol`,
## 5) + poly(`volatile.acidity:citric.acid`, 5) + poly(`volatile.acidity:residual.sugar`,
## 5) + poly(`volatile.acidity:ln.chlorides`, 5) + poly(`volatile.acidity:free.sulfur.dioxide`,
## 5)

```

```

## 5) + poly(`volatile.acidity:total.sulfur.dioxide`, 5) + poly(`volatile.acidity:density`,
## 5) + poly(`volatile.acidity:pH`, 5) + poly(`volatile.acidity:ln.sulphates`,
## 5) + poly(`volatile.acidity:alcohol`, 5) + poly(`citric.acid:residual.sugar`,
## 5) + poly(`citric.acid:ln.chlorides`, 5) + poly(`citric.acid:free.sulfur.dioxide`,
## 5) + poly(`citric.acid:total.sulfur.dioxide`, 5) + poly(`citric.acid:density`,
## 5) + poly(`citric.acid:pH`, 5) + poly(`citric.acid:ln.sulphates`,
## 5) + poly(`citric.acid:alcohol`, 5) + poly(`residual.sugar:ln.chlorides`,
## 5) + poly(`residual.sugar:free.sulfur.dioxide`, 5) + poly(`residual.sugar:total.sulfur.dioxide`,
## 5) + poly(`residual.sugar:density`, 5) + poly(`residual.sugar:pH`,
## 5) + poly(`residual.sugar:ln.sulphates`, 5) + poly(`residual.sugar:alcohol`,
## 5) + poly(`ln.chlorides:free.sulfur.dioxide`, 5) + poly(`ln.chlorides:total.sulfur.dioxide`,
## 5) + poly(`ln.chlorides:density`, 5) + poly(`ln.chlorides:pH`,
## 5) + poly(`ln.chlorides:ln.sulphates`, 5) + poly(`ln.chlorides:alcohol`,
## 5) + poly(`free.sulfur.dioxide:total.sulfur.dioxide`, 5) +
## poly(`free.sulfur.dioxide:density`, 5) + poly(`free.sulfur.dioxide:pH`,
## 5) + poly(`free.sulfur.dioxide:ln.sulphates`, 5) + poly(`free.sulfur.dioxide:alcohol`,
## 5) + poly(`total.sulfur.dioxide:density`, 5) + poly(`total.sulfur.dioxide:pH`,
## 5) + poly(`total.sulfur.dioxide:ln.sulphates`, 5) + poly(`total.sulfur.dioxide:alcohol`,
## 5) + poly(`density:pH`, 5) + poly(`density:ln.sulphates`,
## 5) + poly(`density:alcohol`, 5) + poly(`pH:ln.sulphates`,
## 5) + poly(`pH:alcohol`, 5) + poly(`ln.sulphates:alcohol`,
## 5), data = wine.interactions)
##
## Residuals:
##      Min       1Q     Median      3Q      Max
## -1.9290 -0.3152 -0.0064  0.3286  1.9988
##
## Coefficients:
##                               Estimate Std. Error
## (Intercept)                5.636e+00  1.430e-02
## poly(alcohol, 5)1          -2.044e+06  1.842e+06
## poly(alcohol, 5)2           1.160e+05  1.044e+05
## poly(alcohol, 5)3          -1.031e+04  9.266e+03
## poly(alcohol, 5)4           1.186e+03  1.047e+03
## poly(alcohol, 5)5          -1.678e+02  1.424e+02
## poly(volatile.acidity, 5)1   -1.125e+04  1.713e+04
## poly(volatile.acidity, 5)2    1.589e+03  3.582e+03
## poly(volatile.acidity, 5)3   -2.356e+03  2.031e+03
## poly(volatile.acidity, 5)4   -5.817e+02  5.513e+02
## poly(volatile.acidity, 5)5   -8.904e+02  4.790e+02
## poly(ln.sulphates, 5)1        1.662e+03  3.301e+03
## poly(ln.sulphates, 5)2        4.329e+03  3.433e+03
## poly(ln.sulphates, 5)3        4.339e+03  2.959e+03
## poly(ln.sulphates, 5)4        2.016e+03  1.538e+03
## poly(ln.sulphates, 5)5        5.905e+02  4.711e+02
## poly(citric.acid, 5)1         -8.191e+03  3.764e+03
## poly(citric.acid, 5)2          2.471e+03  1.439e+03
## poly(citric.acid, 5)3         -1.468e+03  7.727e+02
## poly(citric.acid, 5)4          1.008e+02  4.157e+02
## poly(citric.acid, 5)5         -4.260e+02  3.119e+02
## poly(total.sulfur.dioxide, 5)1   2.867e+03  9.664e+03
## poly(total.sulfur.dioxide, 5)2   6.717e+03  1.146e+04
## poly(total.sulfur.dioxide, 5)3   5.400e+03  1.034e+04
## poly(total.sulfur.dioxide, 5)4   1.626e+03  2.343e+03

```

## poly(total.sulfur.dioxide, 5)5	6.633e+02	1.031e+03
## poly(density, 5)1	1.153e+06	4.595e+05
## poly(density, 5)2	-1.876e+03	7.425e+02
## poly(density, 5)3	8.199e+00	3.142e+00
## poly(density, 5)4	-1.913e+00	1.909e+00
## poly(density, 5)5	1.713e+00	1.573e+00
## poly(ln.chlorides, 5)1	1.888e+05	4.745e+04
## poly(ln.chlorides, 5)2	4.397e+04	1.108e+04
## poly(ln.chlorides, 5)3	1.426e+04	3.572e+03
## poly(ln.chlorides, 5)4	5.160e+03	1.256e+03
## poly(ln.chlorides, 5)5	1.371e+03	3.285e+02
## poly(fixed.acidity, 5)1	-3.791e+04	1.201e+05
## poly(fixed.acidity, 5)2	4.247e+03	1.626e+04
## poly(fixed.acidity, 5)3	-7.493e+02	3.585e+03
## poly(fixed.acidity, 5)4	1.023e+02	8.017e+02
## poly(fixed.acidity, 5)5	-1.532e+01	1.918e+02
## poly(pH, 5)1	2.938e+07	1.132e+07
## poly(pH, 5)2	-1.119e+06	4.313e+05
## poly(pH, 5)3	7.575e+04	2.914e+04
## poly(pH, 5)4	-5.876e+03	2.292e+03
## poly(pH, 5)5	5.140e+02	1.865e+02
## poly(free.sulfur.dioxide, 5)1	-7.420e+03	6.528e+03
## poly(free.sulfur.dioxide, 5)2	1.841e+03	2.070e+03
## poly(free.sulfur.dioxide, 5)3	-5.313e+02	1.434e+03
## poly(free.sulfur.dioxide, 5)4	2.184e+02	5.578e+02
## poly(free.sulfur.dioxide, 5)5	-1.497e+02	4.380e+02
## poly(residual.sugar, 5)1	1.355e+04	1.398e+04
## poly(residual.sugar, 5)2	1.193e+04	1.266e+04
## poly(residual.sugar, 5)3	7.958e+03	7.136e+03
## poly(residual.sugar, 5)4	2.119e+03	3.268e+03
## poly(residual.sugar, 5)5	3.739e+02	1.038e+03
## poly(`fixed.acidity:volatile.acidity`, 5)1	-1.888e+02	1.525e+02
## poly(`fixed.acidity:volatile.acidity`, 5)2	4.409e+01	3.714e+01
## poly(`fixed.acidity:volatile.acidity`, 5)3	-1.867e+01	1.535e+01
## poly(`fixed.acidity:volatile.acidity`, 5)4	3.885e+00	4.862e+00
## poly(`fixed.acidity:volatile.acidity`, 5)5	-7.764e+00	3.697e+00
## poly(`fixed.acidity:citric.acid`, 5)1	-1.296e+02	4.294e+01
## poly(`fixed.acidity:citric.acid`, 5)2	4.280e+01	1.785e+01
## poly(`fixed.acidity:citric.acid`, 5)3	-7.859e+00	9.873e+00
## poly(`fixed.acidity:citric.acid`, 5)4	3.632e+00	5.379e+00
## poly(`fixed.acidity:citric.acid`, 5)5	-1.011e+00	2.645e+00
## poly(`fixed.acidity:residual.sugar`, 5)1	3.196e+02	3.162e+02
## poly(`fixed.acidity:residual.sugar`, 5)2	3.966e+02	3.804e+02
## poly(`fixed.acidity:residual.sugar`, 5)3	2.155e+02	2.166e+02
## poly(`fixed.acidity:residual.sugar`, 5)4	8.332e+01	9.653e+01
## poly(`fixed.acidity:residual.sugar`, 5)5	1.542e+01	2.759e+01
## poly(`fixed.acidity:ln.chlorides`, 5)1	1.069e+02	6.429e+02
## poly(`fixed.acidity:ln.chlorides`, 5)2	2.890e+01	1.491e+02
## poly(`fixed.acidity:ln.chlorides`, 5)3	9.097e+00	5.119e+01
## poly(`fixed.acidity:ln.chlorides`, 5)4	6.671e+00	1.566e+01
## poly(`fixed.acidity:ln.chlorides`, 5)5	5.070e-01	6.171e+00
## poly(`fixed.acidity:free.sulfur.dioxide`, 5)1	-8.905e+01	6.499e+01
## poly(`fixed.acidity:free.sulfur.dioxide`, 5)2	2.286e+01	2.169e+01
## poly(`fixed.acidity:free.sulfur.dioxide`, 5)3	-5.502e+00	9.812e+00

## poly(`fixed.acidity:free.sulfur.dioxide` , 5)4	4.192e+00	5.204e+00
## poly(`fixed.acidity:free.sulfur.dioxide` , 5)5	-2.900e+00	2.964e+00
## poly(`fixed.acidity:total.sulfur.dioxide` , 5)1	3.095e+01	8.555e+01
## poly(`fixed.acidity:total.sulfur.dioxide` , 5)2	-3.012e+01	6.770e+01
## poly(`fixed.acidity:total.sulfur.dioxide` , 5)3	-2.117e+01	7.573e+01
## poly(`fixed.acidity:total.sulfur.dioxide` , 5)4	-7.943e+00	2.326e+01
## poly(`fixed.acidity:total.sulfur.dioxide` , 5)5	-3.438e+00	1.068e+01
## poly(`fixed.acidity:density` , 5)1	4.198e+04	1.159e+05
## poly(`fixed.acidity:density` , 5)2	-4.837e+03	1.582e+04
## poly(`fixed.acidity:density` , 5)3	8.792e+02	3.513e+03
## poly(`fixed.acidity:density` , 5)4	-1.455e+02	7.918e+02
## poly(`fixed.acidity:density` , 5)5	2.044e+01	1.860e+02
## poly(`fixed.acidity:pH` , 5)1	-7.410e+02	5.887e+03
## poly(`fixed.acidity:pH` , 5)2	7.235e+01	6.567e+02
## poly(`fixed.acidity:pH` , 5)3	-1.265e+01	1.177e+02
## poly(`fixed.acidity:pH` , 5)4	1.042e+01	2.105e+01
## poly(`fixed.acidity:pH` , 5)5	4.709e-01	5.943e+00
## poly(`fixed.acidity:ln.sulphates` , 5)1	2.728e+01	2.618e+01
## poly(`fixed.acidity:ln.sulphates` , 5)2	5.438e+00	1.520e+01
## poly(`fixed.acidity:ln.sulphates` , 5)3	3.920e+00	1.240e+01
## poly(`fixed.acidity:ln.sulphates` , 5)4	-2.226e-01	8.688e+00
## poly(`fixed.acidity:ln.sulphates` , 5)5	-3.723e+00	3.483e+00
## poly(`fixed.acidity:alcohol` , 5)1	-2.916e+03	1.473e+03
## poly(`fixed.acidity:alcohol` , 5)2	3.995e+02	1.999e+02
## poly(`fixed.acidity:alcohol` , 5)3	-1.468e+02	7.605e+01
## poly(`fixed.acidity:alcohol` , 5)4	3.014e+01	1.508e+01
## poly(`fixed.acidity:alcohol` , 5)5	-1.217e+01	7.296e+00
## poly(`volatile.acidity:citric.acid` , 5)1	2.394e+00	1.333e+01
## poly(`volatile.acidity:citric.acid` , 5)2	2.134e+00	6.100e+00
## poly(`volatile.acidity:citric.acid` , 5)3	-4.449e-01	3.647e+00
## poly(`volatile.acidity:citric.acid` , 5)4	8.464e-01	2.278e+00
## poly(`volatile.acidity:citric.acid` , 5)5	-1.788e+00	1.888e+00
## poly(`volatile.acidity:residual.sugar` , 5)1	6.371e+01	4.192e+01
## poly(`volatile.acidity:residual.sugar` , 5)2	1.493e+01	1.365e+01
## poly(`volatile.acidity:residual.sugar` , 5)3	4.448e+01	2.186e+01
## poly(`volatile.acidity:residual.sugar` , 5)4	2.435e+01	9.321e+00
## poly(`volatile.acidity:residual.sugar` , 5)5	1.228e+01	6.162e+00
## poly(`volatile.acidity:ln.chlorides` , 5)1	-7.562e+01	9.445e+01
## poly(`volatile.acidity:ln.chlorides` , 5)2	-2.260e+01	2.378e+01
## poly(`volatile.acidity:ln.chlorides` , 5)3	-6.052e+00	1.106e+01
## poly(`volatile.acidity:ln.chlorides` , 5)4	-7.379e+00	4.885e+00
## poly(`volatile.acidity:ln.chlorides` , 5)5	3.740e+00	2.972e+00
## poly(`volatile.acidity:free.sulfur.dioxide` , 5)1	-3.399e+01	1.803e+01
## poly(`volatile.acidity:free.sulfur.dioxide` , 5)2	1.158e+01	7.065e+00
## poly(`volatile.acidity:free.sulfur.dioxide` , 5)3	-5.945e+00	8.216e+00
## poly(`volatile.acidity:free.sulfur.dioxide` , 5)4	8.116e-01	3.841e+00
## poly(`volatile.acidity:free.sulfur.dioxide` , 5)5	-5.952e-01	3.135e+00
## poly(`volatile.acidity:total.sulfur.dioxide` , 5)1	5.329e+01	1.810e+01
## poly(`volatile.acidity:total.sulfur.dioxide` , 5)2	-1.555e+01	6.993e+00
## poly(`volatile.acidity:total.sulfur.dioxide` , 5)3	7.342e+00	4.807e+00
## poly(`volatile.acidity:total.sulfur.dioxide` , 5)4	-4.570e+00	2.671e+00
## poly(`volatile.acidity:total.sulfur.dioxide` , 5)5	2.311e+00	2.069e+00
## poly(`volatile.acidity:density` , 5)1	1.170e+04	1.716e+04
## poly(`volatile.acidity:density` , 5)2	-1.675e+03	3.584e+03

## poly(`volatile.acidity:density` , 5)3	2.429e+03	2.028e+03
## poly(`volatile.acidity:density` , 5)4	5.821e+02	5.484e+02
## poly(`volatile.acidity:density` , 5)5	9.149e+02	4.785e+02
## poly(`volatile.acidity:pH` , 5)1	-5.790e+02	5.729e+02
## poly(`volatile.acidity:pH` , 5)2	1.165e+02	1.249e+02
## poly(`volatile.acidity:pH` , 5)3	-8.294e+01	6.774e+01
## poly(`volatile.acidity:pH` , 5)4	8.618e+00	1.752e+01
## poly(`volatile.acidity:pH` , 5)5	-2.430e+01	1.654e+01
## poly(`volatile.acidity:ln.sulphates` , 5)1	-2.226e+01	9.049e+00
## poly(`volatile.acidity:ln.sulphates` , 5)2	-1.030e+01	1.428e+01
## poly(`volatile.acidity:ln.sulphates` , 5)3	-9.577e+00	1.954e+01
## poly(`volatile.acidity:ln.sulphates` , 5)4	-4.913e+00	9.471e+00
## poly(`volatile.acidity:ln.sulphates` , 5)5	5.398e-01	5.830e+00
## poly(`volatile.acidity:alcohol` , 5)1	1.652e+02	2.990e+02
## poly(`volatile.acidity:alcohol` , 5)2	-4.024e+01	6.174e+01
## poly(`volatile.acidity:alcohol` , 5)3	1.158e+01	4.346e+01
## poly(`volatile.acidity:alcohol` , 5)4	-4.266e+00	1.061e+01
## poly(`volatile.acidity:alcohol` , 5)5	8.122e+00	1.048e+01
## poly(`citric.acid:residual.sugar` , 5)1	8.939e+00	5.345e+01
## poly(`citric.acid:residual.sugar` , 5)2	4.840e-01	1.151e+02
## poly(`citric.acid:residual.sugar` , 5)3	1.826e+01	8.319e+01
## poly(`citric.acid:residual.sugar` , 5)4	2.006e+00	2.575e+01
## poly(`citric.acid:residual.sugar` , 5)5	1.971e+00	8.357e+00
## poly(`citric.acid:ln.chlorides` , 5)1	-4.677e+01	2.638e+01
## poly(`citric.acid:ln.chlorides` , 5)2	-1.767e+01	1.063e+01
## poly(`citric.acid:ln.chlorides` , 5)3	-6.632e+00	6.836e+00
## poly(`citric.acid:ln.chlorides` , 5)4	-8.097e-01	4.115e+00
## poly(`citric.acid:ln.chlorides` , 5)5	-7.988e+00	2.823e+00
## poly(`citric.acid:free.sulfur.dioxide` , 5)1	2.482e+00	7.089e+00
## poly(`citric.acid:free.sulfur.dioxide` , 5)2	7.290e+00	4.401e+00
## poly(`citric.acid:free.sulfur.dioxide` , 5)3	-1.117e+00	4.844e+00
## poly(`citric.acid:free.sulfur.dioxide` , 5)4	4.661e-01	3.327e+00
## poly(`citric.acid:free.sulfur.dioxide` , 5)5	-4.256e-01	2.516e+00
## poly(`citric.acid:total.sulfur.dioxide` , 5)1	5.456e+01	2.947e+02
## poly(`citric.acid:total.sulfur.dioxide` , 5)2	9.627e+01	5.762e+02
## poly(`citric.acid:total.sulfur.dioxide` , 5)3	3.728e+01	1.918e+02
## poly(`citric.acid:total.sulfur.dioxide` , 5)4	5.167e+00	4.563e+01
## poly(`citric.acid:total.sulfur.dioxide` , 5)5	4.785e+00	1.890e+01
## poly(`citric.acid:density` , 5)1	8.304e+03	3.815e+03
## poly(`citric.acid:density` , 5)2	-2.465e+03	1.458e+03
## poly(`citric.acid:density` , 5)3	1.417e+03	7.761e+02
## poly(`citric.acid:density` , 5)4	-1.073e+02	4.224e+02
## poly(`citric.acid:density` , 5)5	3.987e+02	3.108e+02
## poly(`citric.acid:pH` , 5)1	-1.124e+02	1.125e+02
## poly(`citric.acid:pH` , 5)2	-1.460e+01	4.483e+01
## poly(`citric.acid:pH` , 5)3	2.391e+01	2.252e+01
## poly(`citric.acid:pH` , 5)4	-1.196e+01	1.079e+01
## poly(`citric.acid:pH` , 5)5	1.093e+01	7.770e+00
## poly(`citric.acid:ln.sulphates` , 5)1	1.631e+01	9.655e+00
## poly(`citric.acid:ln.sulphates` , 5)2	6.793e+01	2.851e+01
## poly(`citric.acid:ln.sulphates` , 5)3	5.733e+01	2.471e+01
## poly(`citric.acid:ln.sulphates` , 5)4	1.839e+01	8.532e+00
## poly(`citric.acid:ln.sulphates` , 5)5	2.263e+00	3.846e+00
## poly(`citric.acid:alcohol` , 5)1	5.036e+01	5.039e+01

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## poly(`citric.acid:alcohol`, 5)2          -7.383e+00  1.990e+01
## poly(`citric.acid:alcohol`, 5)3          8.319e+00  1.107e+01
## poly(`citric.acid:alcohol`, 5)4          -7.010e+00  5.436e+00
## poly(`citric.acid:alcohol`, 5)5          -1.339e+00  2.849e+00
## poly(`residual.sugar:ln.chlorides`, 5)1   3.131e+02  1.190e+02
## poly(`residual.sugar:ln.chlorides`, 5)2   1.402e+01  1.080e+02
## poly(`residual.sugar:ln.chlorides`, 5)3   1.044e+02  9.270e+01
## poly(`residual.sugar:ln.chlorides`, 5)4   -1.062e+01  4.363e+01
## poly(`residual.sugar:ln.chlorides`, 5)5   3.099e+01  2.011e+01
## poly(`residual.sugar:free.sulfur.dioxide`, 5)1 -9.376e+01  1.830e+02
## poly(`residual.sugar:free.sulfur.dioxide`, 5)2 -1.043e+02  1.973e+02
## poly(`residual.sugar:free.sulfur.dioxide`, 5)3 -8.066e+01  1.251e+02
## poly(`residual.sugar:free.sulfur.dioxide`, 5)4 -4.381e+01  5.808e+01
## poly(`residual.sugar:free.sulfur.dioxide`, 5)5 -1.618e+01  2.050e+01
## poly(`residual.sugar:total.sulfur.dioxide`, 5)1 3.342e+02  4.419e+02
## poly(`residual.sugar:total.sulfur.dioxide`, 5)2 4.534e+02  5.815e+02
## poly(`residual.sugar:total.sulfur.dioxide`, 5)3 2.676e+02  3.184e+02
## poly(`residual.sugar:total.sulfur.dioxide`, 5)4 1.088e+02  1.124e+02
## poly(`residual.sugar:total.sulfur.dioxide`, 5)5 4.285e+01  3.204e+01
## poly(`residual.sugar:density`, 5)1         -1.467e+04  1.403e+04
## poly(`residual.sugar:density`, 5)2         -1.230e+04  1.260e+04
## poly(`residual.sugar:density`, 5)3         -8.436e+03  7.182e+03
## poly(`residual.sugar:density`, 5)4         -2.263e+03  3.299e+03
## poly(`residual.sugar:density`, 5)5         -4.512e+02  1.050e+03
## poly(`residual.sugar:pH`, 5)1             1.245e+03  6.094e+02
## poly(`residual.sugar:pH`, 5)2             7.278e+01  4.202e+02
## poly(`residual.sugar:pH`, 5)3             3.812e+02  2.712e+02
## poly(`residual.sugar:pH`, 5)4             7.378e+01  1.091e+02
## poly(`residual.sugar:pH`, 5)5             8.230e+01  4.468e+01
## poly(`residual.sugar:ln.sulphates`, 5)1    3.179e+01  5.523e+01
## poly(`residual.sugar:ln.sulphates`, 5)2    -9.568e+01  1.112e+02
## poly(`residual.sugar:ln.sulphates`, 5)3    8.311e+01  1.046e+02
## poly(`residual.sugar:ln.sulphates`, 5)4    -4.438e+01  4.651e+01
## poly(`residual.sugar:ln.sulphates`, 5)5    2.097e+01  2.620e+01
## poly(`residual.sugar:alcohol`, 5)1          -1.460e+02  2.443e+02
## poly(`residual.sugar:alcohol`, 5)2          -2.902e+01  1.018e+02
## poly(`residual.sugar:alcohol`, 5)3          -6.337e+01  9.261e+01
## poly(`residual.sugar:alcohol`, 5)4          -1.081e+01  3.212e+01
## poly(`residual.sugar:alcohol`, 5)5          -3.363e+00  1.571e+01
## poly(`ln.chlorides:free.sulfur.dioxide`, 5)1 -1.571e+01  4.864e+01
## poly(`ln.chlorides:free.sulfur.dioxide`, 5)2 -4.715e+00  1.628e+01
## poly(`ln.chlorides:free.sulfur.dioxide`, 5)3  3.034e+00  1.348e+01
## poly(`ln.chlorides:free.sulfur.dioxide`, 5)4 -5.922e+00  6.906e+00
## poly(`ln.chlorides:free.sulfur.dioxide`, 5)5  3.569e-01  5.816e+00
## poly(`ln.chlorides:total.sulfur.dioxide`, 5)1 3.554e+02  4.183e+02
## poly(`ln.chlorides:total.sulfur.dioxide`, 5)2 -8.837e+02  1.028e+03
## poly(`ln.chlorides:total.sulfur.dioxide`, 5)3  4.836e+02  5.630e+02
## poly(`ln.chlorides:total.sulfur.dioxide`, 5)4 -1.151e+02  1.239e+02
## poly(`ln.chlorides:total.sulfur.dioxide`, 5)5  4.240e+01  4.074e+01
## poly(`ln.chlorides:density`, 5)1            -1.863e+05  4.760e+04
## poly(`ln.chlorides:density`, 5)2            -4.327e+04  1.108e+04
## poly(`ln.chlorides:density`, 5)3            -1.396e+04  3.529e+03
## poly(`ln.chlorides:density`, 5)4            -5.063e+03  1.246e+03
## poly(`ln.chlorides:density`, 5)5            -1.338e+03  3.201e+02

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## poly(`ln.chlorides:pH`, 5)1          3.499e+02  1.571e+03
## poly(`ln.chlorides:pH`, 5)2          1.011e+02  3.971e+02
## poly(`ln.chlorides:pH`, 5)3          7.120e+01  1.258e+02
## poly(`ln.chlorides:pH`, 5)4          1.187e+01  4.089e+01
## poly(`ln.chlorides:pH`, 5)5          1.674e+01  1.422e+01
## poly(`ln.chlorides:ln.sulphates`, 5)1 4.234e+00  1.460e+01
## poly(`ln.chlorides:ln.sulphates`, 5)2 -7.415e-01  1.515e+01
## poly(`ln.chlorides:ln.sulphates`, 5)3 -4.532e+00  1.871e+01
## poly(`ln.chlorides:ln.sulphates`, 5)4  2.716e-01  1.115e+01
## poly(`ln.chlorides:ln.sulphates`, 5)5 -5.203e+00  7.213e+00
## poly(`ln.chlorides:alcohol`, 5)1       -3.148e+03  1.029e+03
## poly(`ln.chlorides:alcohol`, 5)2       -8.098e+02  2.676e+02
## poly(`ln.chlorides:alcohol`, 5)3       -2.930e+02  9.481e+01
## poly(`ln.chlorides:alcohol`, 5)4       -6.676e+01  2.573e+01
## poly(`ln.chlorides:alcohol`, 5)5       -4.947e+01  1.415e+01
## poly(`free.sulfur.dioxide:total.sulfur.dioxide`, 5)1 -4.781e+01  3.926e+01
## poly(`free.sulfur.dioxide:total.sulfur.dioxide`, 5)2 -3.467e+01  5.240e+01
## poly(`free.sulfur.dioxide:total.sulfur.dioxide`, 5)3 -4.668e+01  4.373e+01
## poly(`free.sulfur.dioxide:total.sulfur.dioxide`, 5)4 -1.839e+01  2.066e+01
## poly(`free.sulfur.dioxide:total.sulfur.dioxide`, 5)5 -1.324e+01  8.617e+00
## poly(`free.sulfur.dioxide:density`, 5)1 7.552e+03  6.626e+03
## poly(`free.sulfur.dioxide:density`, 5)2 -1.898e+03  2.095e+03
## poly(`free.sulfur.dioxide:density`, 5)3 5.322e+02  1.455e+03
## poly(`free.sulfur.dioxide:density`, 5)4 -2.336e+02  5.628e+02
## poly(`free.sulfur.dioxide:density`, 5)5 1.447e+02  4.402e+02
## poly(`free.sulfur.dioxide:pH`, 5)1      -1.892e+02  2.133e+02
## poly(`free.sulfur.dioxide:pH`, 5)2      6.281e+01  6.717e+01
## poly(`free.sulfur.dioxide:pH`, 5)3      -4.203e+00  3.932e+01
## poly(`free.sulfur.dioxide:pH`, 5)4      2.103e+01  1.575e+01
## poly(`free.sulfur.dioxide:pH`, 5)5      8.714e-01  9.921e+00
## poly(`free.sulfur.dioxide:ln.sulphates`, 5)1 -2.891e+01  6.686e+00
## poly(`free.sulfur.dioxide:ln.sulphates`, 5)2 -1.684e+01  6.624e+00
## poly(`free.sulfur.dioxide:ln.sulphates`, 5)3 -1.630e+01  8.872e+00
## poly(`free.sulfur.dioxide:ln.sulphates`, 5)4 -7.105e+00  5.047e+00
## poly(`free.sulfur.dioxide:ln.sulphates`, 5)5 -3.221e+00  3.691e+00
## poly(`free.sulfur.dioxide:alcohol`, 5)1 1.606e+02  9.315e+01
## poly(`free.sulfur.dioxide:alcohol`, 5)2 -4.362e+01  3.059e+01
## poly(`free.sulfur.dioxide:alcohol`, 5)3 1.330e+01  1.474e+01
## poly(`free.sulfur.dioxide:alcohol`, 5)4 -9.236e+00  6.698e+00
## poly(`free.sulfur.dioxide:alcohol`, 5)5 4.155e+00  5.019e+00
## poly(`total.sulfur.dioxide:density`, 5)1 -2.635e+03  9.637e+03
## poly(`total.sulfur.dioxide:density`, 5)2 -5.559e+03  1.110e+04
## poly(`total.sulfur.dioxide:density`, 5)3 -4.577e+03  1.016e+04
## poly(`total.sulfur.dioxide:density`, 5)4 -1.423e+03  2.305e+03
## poly(`total.sulfur.dioxide:density`, 5)5 -5.903e+02  1.019e+03
## poly(`total.sulfur.dioxide:pH`, 5)1 2.544e+02  2.980e+02
## poly(`total.sulfur.dioxide:pH`, 5)2 -3.134e+02  2.613e+02
## poly(`total.sulfur.dioxide:pH`, 5)3 -1.697e+02  2.932e+02
## poly(`total.sulfur.dioxide:pH`, 5)4 -7.093e+01  7.694e+01
## poly(`total.sulfur.dioxide:pH`, 5)5 -1.483e+01  3.210e+01
## poly(`total.sulfur.dioxide:ln.sulphates`, 5)1 5.673e+00  1.793e+01
## poly(`total.sulfur.dioxide:ln.sulphates`, 5)2 5.591e+01  3.717e+01
## poly(`total.sulfur.dioxide:ln.sulphates`, 5)3 -2.449e+01  3.874e+01
## poly(`total.sulfur.dioxide:ln.sulphates`, 5)4 1.521e+01  1.638e+01

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```

## poly(`total.sulfur.dioxide:ln.sulphates`, 5)5      -1.090e+01  7.735e+00
## poly(`total.sulfur.dioxide:alcohol`, 5)1      -4.631e+02  4.014e+02
## poly(`total.sulfur.dioxide:alcohol`, 5)2      -5.886e+02  8.748e+02
## poly(`total.sulfur.dioxide:alcohol`, 5)3      -3.684e+02  4.481e+02
## poly(`total.sulfur.dioxide:alcohol`, 5)4      -6.421e+01  9.895e+01
## poly(`total.sulfur.dioxide:alcohol`, 5)5      -4.833e+01  4.124e+01
## poly(`density:pH`, 5)1      -2.899e+07  1.117e+07
## poly(`density:pH`, 5)2      1.087e+06  4.190e+05
## poly(`density:pH`, 5)3      -7.225e+04  2.780e+04
## poly(`density:pH`, 5)4      5.513e+03  2.151e+03
## poly(`density:pH`, 5)5      -4.776e+02  1.724e+02
## poly(`density:ln.sulphates`, 5)1      -1.885e+03  3.337e+03
## poly(`density:ln.sulphates`, 5)2      -4.390e+03  3.452e+03
## poly(`density:ln.sulphates`, 5)3      -4.387e+03  2.976e+03
## poly(`density:ln.sulphates`, 5)4      -2.042e+03  1.544e+03
## poly(`density:ln.sulphates`, 5)5      -6.022e+02  4.741e+02
## poly(`density:alcohol`, 5)1      2.041e+06  1.812e+06
## poly(`density:alcohol`, 5)2      -1.145e+05  1.015e+05
## poly(`density:alcohol`, 5)3      1.011e+04  8.952e+03
## poly(`density:alcohol`, 5)4      -1.171e+03  1.016e+03
## poly(`density:alcohol`, 5)5      1.661e+02  1.400e+02
## poly(`pH:ln.sulphates`, 5)1      2.279e+02  8.695e+01
## poly(`pH:ln.sulphates`, 5)2      4.387e+01  4.790e+01
## poly(`pH:ln.sulphates`, 5)3      2.206e+01  2.761e+01
## poly(`pH:ln.sulphates`, 5)4      6.403e+00  1.826e+01
## poly(`pH:ln.sulphates`, 5)5      7.336e+00  9.850e+00
## poly(`pH:alcohol`, 5)1      -1.901e+04  3.178e+04
## poly(`pH:alcohol`, 5)2      1.570e+03  2.613e+03
## poly(`pH:alcohol`, 5)3      -1.950e+02  3.275e+02
## poly(`pH:alcohol`, 5)4      2.410e+01  4.658e+01
## poly(`pH:alcohol`, 5)5      -4.502e+00  7.201e+00
## poly(`ln.sulphates:alcohol`, 5)1      7.478e+00  3.846e+01
## poly(`ln.sulphates:alcohol`, 5)2      -3.065e-01  3.064e+01
## poly(`ln.sulphates:alcohol`, 5)3      1.182e+01  2.574e+01
## poly(`ln.sulphates:alcohol`, 5)4      1.984e+01  1.565e+01
## poly(`ln.sulphates:alcohol`, 5)5      9.422e+00  5.916e+00
##
## (Intercept)          394.059  < 2e-16 ***
## poly(alcohol, 5)1     -1.109  0.26743
## poly(alcohol, 5)2      1.111  0.26667
## poly(alcohol, 5)3     -1.113  0.26608
## poly(alcohol, 5)4      1.133  0.25757
## poly(alcohol, 5)5     -1.179  0.23879
## poly(volatile.acidity, 5)1    -0.657  0.51161
## poly(volatile.acidity, 5)2      0.444  0.65740
## poly(volatile.acidity, 5)3     -1.160  0.24624
## poly(volatile.acidity, 5)4     -1.055  0.29163
## poly(volatile.acidity, 5)5     -1.859  0.06324 .
## poly(ln.sulphates, 5)1      0.504  0.61460
## poly(ln.sulphates, 5)2      1.261  0.20752
## poly(ln.sulphates, 5)3      1.467  0.14274
## poly(ln.sulphates, 5)4      1.311  0.19007
## poly(ln.sulphates, 5)5      1.253  0.21027
## poly(citric.acid, 5)1     -2.176  0.02971 *

```

## poly(citric.acid, 5)2	1.717	0.08627	.
## poly(citric.acid, 5)3	-1.900	0.05768	.
## poly(citric.acid, 5)4	0.243	0.80839	
## poly(citric.acid, 5)5	-1.366	0.17224	
## poly(total.sulfur.dioxide, 5)1	0.297	0.76678	
## poly(total.sulfur.dioxide, 5)2	0.586	0.55808	
## poly(total.sulfur.dioxide, 5)3	0.522	0.60175	
## poly(total.sulfur.dioxide, 5)4	0.694	0.48770	
## poly(total.sulfur.dioxide, 5)5	0.643	0.52022	
## poly(density, 5)1	2.509	0.01222	*
## poly(density, 5)2	-2.527	0.01164	*
## poly(density, 5)3	2.610	0.00917	**
## poly(density, 5)4	-1.002	0.31644	
## poly(density, 5)5	1.089	0.27625	
## poly(ln.chlorides, 5)1	3.979	7.33e-05	***
## poly(ln.chlorides, 5)2	3.968	7.66e-05	***
## poly(ln.chlorides, 5)3	3.993	6.90e-05	***
## poly(ln.chlorides, 5)4	4.109	4.24e-05	***
## poly(ln.chlorides, 5)5	4.172	3.22e-05	***
## poly(fixed.acidity, 5)1	-0.315	0.75245	
## poly(fixed.acidity, 5)2	0.261	0.79400	
## poly(fixed.acidity, 5)3	-0.209	0.83450	
## poly(fixed.acidity, 5)4	0.128	0.89844	
## poly(fixed.acidity, 5)5	-0.080	0.93634	
## poly(pH, 5)1	2.595	0.00958	**
## poly(pH, 5)2	-2.595	0.00957	**
## poly(pH, 5)3	2.599	0.00945	**
## poly(pH, 5)4	-2.564	0.01047	*
## poly(pH, 5)5	2.757	0.00593	**
## poly(free.sulfur.dioxide, 5)1	-1.137	0.25591	
## poly(free.sulfur.dioxide, 5)2	0.889	0.37391	
## poly(free.sulfur.dioxide, 5)3	-0.371	0.71103	
## poly(free.sulfur.dioxide, 5)4	0.392	0.69545	
## poly(free.sulfur.dioxide, 5)5	-0.342	0.73256	
## poly(residual.sugar, 5)1	0.969	0.33251	
## poly(residual.sugar, 5)2	0.942	0.34633	
## poly(residual.sugar, 5)3	1.115	0.26501	
## poly(residual.sugar, 5)4	0.648	0.51680	
## poly(residual.sugar, 5)5	0.360	0.71871	
## poly(`fixed.acidity:volatile.acidity`, 5)1	-1.238	0.21588	
## poly(`fixed.acidity:volatile.acidity`, 5)2	1.187	0.23547	
## poly(`fixed.acidity:volatile.acidity`, 5)3	-1.216	0.22403	
## poly(`fixed.acidity:volatile.acidity`, 5)4	0.799	0.42432	
## poly(`fixed.acidity:volatile.acidity`, 5)5	-2.100	0.03594	*
## poly(`fixed.acidity:citric.acid`, 5)1	-3.018	0.00259	**
## poly(`fixed.acidity:citric.acid`, 5)2	2.398	0.01661	*
## poly(`fixed.acidity:citric.acid`, 5)3	-0.796	0.42618	
## poly(`fixed.acidity:citric.acid`, 5)4	0.675	0.49971	
## poly(`fixed.acidity:citric.acid`, 5)5	-0.382	0.70226	
## poly(`fixed.acidity:residual.sugar`, 5)1	1.011	0.31233	
## poly(`fixed.acidity:residual.sugar`, 5)2	1.043	0.29727	
## poly(`fixed.acidity:residual.sugar`, 5)3	0.995	0.32003	
## poly(`fixed.acidity:residual.sugar`, 5)4	0.863	0.38819	
## poly(`fixed.acidity:residual.sugar`, 5)5	0.559	0.57620	

## poly(`fixed.acidity:ln.chlorides` , 5)1	0.166	0.86792
## poly(`fixed.acidity:ln.chlorides` , 5)2	0.194	0.84629
## poly(`fixed.acidity:ln.chlorides` , 5)3	0.178	0.85898
## poly(`fixed.acidity:ln.chlorides` , 5)4	0.426	0.67010
## poly(`fixed.acidity:ln.chlorides` , 5)5	0.082	0.93453
## poly(`fixed.acidity:free.sulfur.dioxide` , 5)1	-1.370	0.17086
## poly(`fixed.acidity:free.sulfur.dioxide` , 5)2	1.054	0.29211
## poly(`fixed.acidity:free.sulfur.dioxide` , 5)3	-0.561	0.57504
## poly(`fixed.acidity:free.sulfur.dioxide` , 5)4	0.806	0.42067
## poly(`fixed.acidity:free.sulfur.dioxide` , 5)5	-0.978	0.32810
## poly(`fixed.acidity:total.sulfur.dioxide` , 5)1	0.362	0.71758
## poly(`fixed.acidity:total.sulfur.dioxide` , 5)2	-0.445	0.65643
## poly(`fixed.acidity:total.sulfur.dioxide` , 5)3	-0.280	0.77986
## poly(`fixed.acidity:total.sulfur.dioxide` , 5)4	-0.341	0.73283
## poly(`fixed.acidity:total.sulfur.dioxide` , 5)5	-0.322	0.74761
## poly(`fixed.acidity:density` , 5)1	0.362	0.71730
## poly(`fixed.acidity:density` , 5)2	-0.306	0.75985
## poly(`fixed.acidity:density` , 5)3	0.250	0.80241
## poly(`fixed.acidity:density` , 5)4	-0.184	0.85427
## poly(`fixed.acidity:density` , 5)5	0.110	0.91252
## poly(`fixed.acidity:pH` , 5)1	-0.126	0.89985
## poly(`fixed.acidity:pH` , 5)2	0.110	0.91229
## poly(`fixed.acidity:pH` , 5)3	-0.107	0.91442
## poly(`fixed.acidity:pH` , 5)4	0.495	0.62064
## poly(`fixed.acidity:pH` , 5)5	0.079	0.93685
## poly(`fixed.acidity:ln.sulphates` , 5)1	1.042	0.29744
## poly(`fixed.acidity:ln.sulphates` , 5)2	0.358	0.72055
## poly(`fixed.acidity:ln.sulphates` , 5)3	0.316	0.75206
## poly(`fixed.acidity:ln.sulphates` , 5)4	-0.026	0.97956
## poly(`fixed.acidity:ln.sulphates` , 5)5	-1.069	0.28527
## poly(`fixed.acidity:alcohol` , 5)1	-1.979	0.04798 *
## poly(`fixed.acidity:alcohol` , 5)2	1.999	0.04580 *
## poly(`fixed.acidity:alcohol` , 5)3	-1.931	0.05376 .
## poly(`fixed.acidity:alcohol` , 5)4	1.998	0.04589 *
## poly(`fixed.acidity:alcohol` , 5)5	-1.668	0.09553 .
## poly(`volatile.acidity:citric.acid` , 5)1	0.180	0.85747
## poly(`volatile.acidity:citric.acid` , 5)2	0.350	0.72654
## poly(`volatile.acidity:citric.acid` , 5)3	-0.122	0.90292
## poly(`volatile.acidity:citric.acid` , 5)4	0.372	0.71031
## poly(`volatile.acidity:citric.acid` , 5)5	-0.947	0.34400
## poly(`volatile.acidity:residual.sugar` , 5)1	1.520	0.12885
## poly(`volatile.acidity:residual.sugar` , 5)2	1.094	0.27436
## poly(`volatile.acidity:residual.sugar` , 5)3	2.035	0.04210 *
## poly(`volatile.acidity:residual.sugar` , 5)4	2.612	0.00911 **
## poly(`volatile.acidity:residual.sugar` , 5)5	1.993	0.04644 *
## poly(`volatile.acidity:ln.chlorides` , 5)1	-0.801	0.42350
## poly(`volatile.acidity:ln.chlorides` , 5)2	-0.951	0.34201
## poly(`volatile.acidity:ln.chlorides` , 5)3	-0.547	0.58425
## poly(`volatile.acidity:ln.chlorides` , 5)4	-1.510	0.13120
## poly(`volatile.acidity:ln.chlorides` , 5)5	1.258	0.20854
## poly(`volatile.acidity:free.sulfur.dioxide` , 5)1	-1.885	0.05967 .
## poly(`volatile.acidity:free.sulfur.dioxide` , 5)2	1.639	0.10154
## poly(`volatile.acidity:free.sulfur.dioxide` , 5)3	-0.724	0.46943
## poly(`volatile.acidity:free.sulfur.dioxide` , 5)4	0.211	0.83270

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## poly(`volatile.acidity:free.sulfur.dioxide`, 5)5      -0.190  0.84946
## poly(`volatile.acidity:total.sulfur.dioxide`, 5)1     2.944  0.00330 **
## poly(`volatile.acidity:total.sulfur.dioxide`, 5)2     -2.223  0.02638 *
## poly(`volatile.acidity:total.sulfur.dioxide`, 5)3      1.527  0.12695
## poly(`volatile.acidity:total.sulfur.dioxide`, 5)4     -1.711  0.08729 .
## poly(`volatile.acidity:total.sulfur.dioxide`, 5)5      1.117  0.26405
## poly(`volatile.acidity:density`, 5)1                 0.682  0.49547
## poly(`volatile.acidity:density`, 5)2                 -0.467  0.64031
## poly(`volatile.acidity:density`, 5)3                 1.197  0.23139
## poly(`volatile.acidity:density`, 5)4                 1.061  0.28869
## poly(`volatile.acidity:density`, 5)5                 1.912  0.05607 .
## poly(`volatile.acidity:pH`, 5)1                  -1.011  0.31233
## poly(`volatile.acidity:pH`, 5)2                  0.933  0.35115
## poly(`volatile.acidity:pH`, 5)3                 -1.224  0.22101
## poly(`volatile.acidity:pH`, 5)4                  0.492  0.62288
## poly(`volatile.acidity:pH`, 5)5                 -1.469  0.14219
## poly(`volatile.acidity:ln.sulphates`, 5)1        -2.460  0.01402 *
## poly(`volatile.acidity:ln.sulphates`, 5)2        -0.721  0.47082
## poly(`volatile.acidity:ln.sulphates`, 5)3        -0.490  0.62409
## poly(`volatile.acidity:ln.sulphates`, 5)4        -0.519  0.60403
## poly(`volatile.acidity:ln.sulphates`, 5)5        0.093  0.92624
## poly(`volatile.acidity:alcohol`, 5)1            0.552  0.58084
## poly(`volatile.acidity:alcohol`, 5)2            -0.652  0.51465
## poly(`volatile.acidity:alcohol`, 5)3            0.266  0.78994
## poly(`volatile.acidity:alcohol`, 5)4            -0.402  0.68774
## poly(`volatile.acidity:alcohol`, 5)5            0.775  0.43857
## poly(`citric.acid:residual.sugar`, 5)1          0.167  0.86721
## poly(`citric.acid:residual.sugar`, 5)2          0.004  0.99664
## poly(`citric.acid:residual.sugar`, 5)3          0.220  0.82627
## poly(`citric.acid:residual.sugar`, 5)4          0.078  0.93791
## poly(`citric.acid:residual.sugar`, 5)5          0.236  0.81361
## poly(`citric.acid:ln.chlorides`, 5)1           -1.773  0.07641 .
## poly(`citric.acid:ln.chlorides`, 5)2           -1.661  0.09687 .
## poly(`citric.acid:ln.chlorides`, 5)3           -0.970  0.33211
## poly(`citric.acid:ln.chlorides`, 5)4           -0.197  0.84403
## poly(`citric.acid:ln.chlorides`, 5)5           -2.829  0.00474 **
## poly(`citric.acid:free.sulfur.dioxide`, 5)1        0.350  0.72633
## poly(`citric.acid:free.sulfur.dioxide`, 5)2        1.657  0.09784 .
## poly(`citric.acid:free.sulfur.dioxide`, 5)3        -0.231  0.81761
## poly(`citric.acid:free.sulfur.dioxide`, 5)4        0.140  0.88859
## poly(`citric.acid:free.sulfur.dioxide`, 5)5        -0.169  0.86571
## poly(`citric.acid:total.sulfur.dioxide`, 5)1       0.185  0.85318
## poly(`citric.acid:total.sulfur.dioxide`, 5)2       0.167  0.86734
## poly(`citric.acid:total.sulfur.dioxide`, 5)3       0.194  0.84587
## poly(`citric.acid:total.sulfur.dioxide`, 5)4       0.113  0.90986
## poly(`citric.acid:total.sulfur.dioxide`, 5)5       0.253  0.80019
## poly(`citric.acid:density`, 5)1                  2.177  0.02969 *
## poly(`citric.acid:density`, 5)2                 -1.691  0.09116 .
## poly(`citric.acid:density`, 5)3                  1.825  0.06818 .
## poly(`citric.acid:density`, 5)4                 -0.254  0.79947
## poly(`citric.acid:density`, 5)5                  1.283  0.19980
## poly(`citric.acid:pH`, 5)1                  -0.999  0.31775
## poly(`citric.acid:pH`, 5)2                 -0.326  0.74473
## poly(`citric.acid:pH`, 5)3                  1.062  0.28845

```

## poly(`citric.acid:pH` , 5)4	-1.109	0.26775
## poly(`citric.acid:pH` , 5)5	1.407	0.15978
## poly(`citric.acid:ln.sulphates` , 5)1	1.689	0.09144 .
## poly(`citric.acid:ln.sulphates` , 5)2	2.383	0.01733 *
## poly(`citric.acid:ln.sulphates` , 5)3	2.320	0.02049 *
## poly(`citric.acid:ln.sulphates` , 5)4	2.156	0.03130 *
## poly(`citric.acid:ln.sulphates` , 5)5	0.588	0.55644
## poly(`citric.acid:alcohol` , 5)1	0.999	0.31786
## poly(`citric.acid:alcohol` , 5)2	-0.371	0.71071
## poly(`citric.acid:alcohol` , 5)3	0.751	0.45267
## poly(`citric.acid:alcohol` , 5)4	-1.290	0.19745
## poly(`citric.acid:alcohol` , 5)5	-0.470	0.63839
## poly(`residual.sugar:ln.chlorides` , 5)1	2.631	0.00861 **
## poly(`residual.sugar:ln.chlorides` , 5)2	0.130	0.89677
## poly(`residual.sugar:ln.chlorides` , 5)3	1.126	0.26043
## poly(`residual.sugar:ln.chlorides` , 5)4	-0.243	0.80778
## poly(`residual.sugar:ln.chlorides` , 5)5	1.541	0.12351
## poly(`residual.sugar:free.sulfur.dioxide` , 5)1	-0.512	0.60849
## poly(`residual.sugar:free.sulfur.dioxide` , 5)2	-0.529	0.59720
## poly(`residual.sugar:free.sulfur.dioxide` , 5)3	-0.645	0.51920
## poly(`residual.sugar:free.sulfur.dioxide` , 5)4	-0.754	0.45075
## poly(`residual.sugar:free.sulfur.dioxide` , 5)5	-0.789	0.43006
## poly(`residual.sugar:total.sulfur.dioxide` , 5)1	0.756	0.44959
## poly(`residual.sugar:total.sulfur.dioxide` , 5)2	0.780	0.43574
## poly(`residual.sugar:total.sulfur.dioxide` , 5)3	0.840	0.40085
## poly(`residual.sugar:total.sulfur.dioxide` , 5)4	0.968	0.33298
## poly(`residual.sugar:total.sulfur.dioxide` , 5)5	1.337	0.18132
## poly(`residual.sugar:density` , 5)1	-1.046	0.29584
## poly(`residual.sugar:density` , 5)2	-0.977	0.32892
## poly(`residual.sugar:density` , 5)3	-1.175	0.24035
## poly(`residual.sugar:density` , 5)4	-0.686	0.49283
## poly(`residual.sugar:density` , 5)5	-0.430	0.66744
## poly(`residual.sugar:pH` , 5)1	2.043	0.04126 *
## poly(`residual.sugar:pH` , 5)2	0.173	0.86252
## poly(`residual.sugar:pH` , 5)3	1.405	0.16013
## poly(`residual.sugar:pH` , 5)4	0.676	0.49915
## poly(`residual.sugar:pH` , 5)5	1.842	0.06572 .
## poly(`residual.sugar:ln.sulphates` , 5)1	0.576	0.56500
## poly(`residual.sugar:ln.sulphates` , 5)2	-0.861	0.38955
## poly(`residual.sugar:ln.sulphates` , 5)3	0.794	0.42714
## poly(`residual.sugar:ln.sulphates` , 5)4	-0.954	0.34016
## poly(`residual.sugar:ln.sulphates` , 5)5	0.800	0.42370
## poly(`residual.sugar:alcohol` , 5)1	-0.597	0.55028
## poly(`residual.sugar:alcohol` , 5)2	-0.285	0.77570
## poly(`residual.sugar:alcohol` , 5)3	-0.684	0.49391
## poly(`residual.sugar:alcohol` , 5)4	-0.337	0.73650
## poly(`residual.sugar:alcohol` , 5)5	-0.214	0.83051
## poly(`ln.chlorides:free.sulfur.dioxide` , 5)1	-0.323	0.74673
## poly(`ln.chlorides:free.sulfur.dioxide` , 5)2	-0.290	0.77210
## poly(`ln.chlorides:free.sulfur.dioxide` , 5)3	0.225	0.82197
## poly(`ln.chlorides:free.sulfur.dioxide` , 5)4	-0.858	0.39131
## poly(`ln.chlorides:free.sulfur.dioxide` , 5)5	0.061	0.95108
## poly(`ln.chlorides:total.sulfur.dioxide` , 5)1	0.849	0.39579
## poly(`ln.chlorides:total.sulfur.dioxide` , 5)2	-0.860	0.38991

## poly(`ln.chlorides:total.sulfur.dioxide` , 5)3	0.859	0.39047
## poly(`ln.chlorides:total.sulfur.dioxide` , 5)4	-0.929	0.35325
## poly(`ln.chlorides:total.sulfur.dioxide` , 5)5	1.041	0.29824
## poly(`ln.chlorides:density` , 5)1	-3.913	9.58e-05 ***
## poly(`ln.chlorides:density` , 5)2	-3.904	9.94e-05 ***
## poly(`ln.chlorides:density` , 5)3	-3.957	8.02e-05 ***
## poly(`ln.chlorides:density` , 5)4	-4.062	5.16e-05 ***
## poly(`ln.chlorides:density` , 5)5	-4.180	3.12e-05 ***
## poly(`ln.chlorides:pH` , 5)1	0.223	0.82374
## poly(`ln.chlorides:pH` , 5)2	0.255	0.79904
## poly(`ln.chlorides:pH` , 5)3	0.566	0.57148
## poly(`ln.chlorides:pH` , 5)4	0.290	0.77155
## poly(`ln.chlorides:pH` , 5)5	1.177	0.23928
## poly(`ln.chlorides:ln.sulphates` , 5)1	0.290	0.77193
## poly(`ln.chlorides:ln.sulphates` , 5)2	-0.049	0.96097
## poly(`ln.chlorides:ln.sulphates` , 5)3	-0.242	0.80861
## poly(`ln.chlorides:ln.sulphates` , 5)4	0.024	0.98057
## poly(`ln.chlorides:ln.sulphates` , 5)5	-0.721	0.47082
## poly(`ln.chlorides:alcohol` , 5)1	-3.059	0.00227 **
## poly(`ln.chlorides:alcohol` , 5)2	-3.026	0.00253 **
## poly(`ln.chlorides:alcohol` , 5)3	-3.091	0.00204 **
## poly(`ln.chlorides:alcohol` , 5)4	-2.595	0.00957 **
## poly(`ln.chlorides:alcohol` , 5)5	-3.495	0.00049 ***
## poly(`free.sulfur.dioxide:total.sulfur.dioxide` , 5)1	-1.218	0.22347
## poly(`free.sulfur.dioxide:total.sulfur.dioxide` , 5)2	-0.662	0.50832
## poly(`free.sulfur.dioxide:total.sulfur.dioxide` , 5)3	-1.067	0.28601
## poly(`free.sulfur.dioxide:total.sulfur.dioxide` , 5)4	-0.890	0.37357
## poly(`free.sulfur.dioxide:total.sulfur.dioxide` , 5)5	-1.536	0.12475
## poly(`free.sulfur.dioxide:density` , 5)1	1.140	0.25461
## poly(`free.sulfur.dioxide:density` , 5)2	-0.906	0.36501
## poly(`free.sulfur.dioxide:density` , 5)3	0.366	0.71469
## poly(`free.sulfur.dioxide:density` , 5)4	-0.415	0.67811
## poly(`free.sulfur.dioxide:density` , 5)5	0.329	0.74239
## poly(`free.sulfur.dioxide:pH` , 5)1	-0.887	0.37547
## poly(`free.sulfur.dioxide:pH` , 5)2	0.935	0.34995
## poly(`free.sulfur.dioxide:pH` , 5)3	-0.107	0.91488
## poly(`free.sulfur.dioxide:pH` , 5)4	1.335	0.18204
## poly(`free.sulfur.dioxide:pH` , 5)5	0.088	0.93002
## poly(`free.sulfur.dioxide:ln.sulphates` , 5)1	-4.324	1.65e-05 ***
## poly(`free.sulfur.dioxide:ln.sulphates` , 5)2	-2.543	0.01111 *
## poly(`free.sulfur.dioxide:ln.sulphates` , 5)3	-1.837	0.06645 .
## poly(`free.sulfur.dioxide:ln.sulphates` , 5)4	-1.408	0.15940
## poly(`free.sulfur.dioxide:ln.sulphates` , 5)5	-0.873	0.38301
## poly(`free.sulfur.dioxide:alcohol` , 5)1	1.724	0.08502 .
## poly(`free.sulfur.dioxide:alcohol` , 5)2	-1.426	0.15410
## poly(`free.sulfur.dioxide:alcohol` , 5)3	0.902	0.36718
## poly(`free.sulfur.dioxide:alcohol` , 5)4	-1.379	0.16813
## poly(`free.sulfur.dioxide:alcohol` , 5)5	0.828	0.40788
## poly(`total.sulfur.dioxide:density` , 5)1	-0.273	0.78453
## poly(`total.sulfur.dioxide:density` , 5)2	-0.501	0.61671
## poly(`total.sulfur.dioxide:density` , 5)3	-0.450	0.65257
## poly(`total.sulfur.dioxide:density` , 5)4	-0.617	0.53702
## poly(`total.sulfur.dioxide:density` , 5)5	-0.579	0.56246
## poly(`total.sulfur.dioxide:pH` , 5)1	0.854	0.39348

```

## poly(`total.sulfur.dioxide:pH`, 5)2           -1.199  0.23057
## poly(`total.sulfur.dioxide:pH`, 5)3           -0.579  0.56283
## poly(`total.sulfur.dioxide:pH`, 5)4           -0.922  0.35676
## poly(`total.sulfur.dioxide:pH`, 5)5           -0.462  0.64423
## poly(`total.sulfur.dioxide:ln.sulphates`, 5)1    0.316  0.75179
## poly(`total.sulfur.dioxide:ln.sulphates`, 5)2    1.504  0.13277
## poly(`total.sulfur.dioxide:ln.sulphates`, 5)3   -0.632  0.52733
## poly(`total.sulfur.dioxide:ln.sulphates`, 5)4    0.928  0.35347
## poly(`total.sulfur.dioxide:ln.sulphates`, 5)5   -1.409  0.15908
## poly(`total.sulfur.dioxide:alcohol`, 5)1          -1.154  0.24877
## poly(`total.sulfur.dioxide:alcohol`, 5)2          -0.673  0.50116
## poly(`total.sulfur.dioxide:alcohol`, 5)3          -0.822  0.41119
## poly(`total.sulfur.dioxide:alcohol`, 5)4          -0.649  0.51650
## poly(`total.sulfur.dioxide:alcohol`, 5)5          -1.172  0.24144
## poly(`density:pH`, 5)1             -2.595  0.00957 **
## poly(`density:pH`, 5)2             2.595  0.00957 **
## poly(`density:pH`, 5)3             -2.599  0.00945 **
## poly(`density:pH`, 5)4             2.563  0.01049 *
## poly(`density:pH`, 5)5             -2.770  0.00569 **
## poly(`density:ln.sulphates`, 5)1      -0.565  0.57214
## poly(`density:ln.sulphates`, 5)2      -1.272  0.20367
## poly(`density:ln.sulphates`, 5)3      -1.474  0.14068
## poly(`density:ln.sulphates`, 5)4      -1.322  0.18635
## poly(`density:ln.sulphates`, 5)5      -1.270  0.20427
## poly(`density:alcohol`, 5)1            1.126  0.26027
## poly(`density:alcohol`, 5)2            -1.128  0.25948
## poly(`density:alcohol`, 5)3            1.129  0.25905
## poly(`density:alcohol`, 5)4            -1.152  0.24938
## poly(`density:alcohol`, 5)5            1.187  0.23560
## poly(`pH:ln.sulphates`, 5)1          2.622  0.00886 **
## poly(`pH:ln.sulphates`, 5)2          0.916  0.35994
## poly(`pH:ln.sulphates`, 5)3          0.799  0.42439
## poly(`pH:ln.sulphates`, 5)4          0.351  0.72596
## poly(`pH:ln.sulphates`, 5)5          0.745  0.45653
## poly(`pH:alcohol`, 5)1              -0.598  0.54988
## poly(`pH:alcohol`, 5)2              0.601  0.54792
## poly(`pH:alcohol`, 5)3              -0.596  0.55158
## poly(`pH:alcohol`, 5)4              0.517  0.60493
## poly(`pH:alcohol`, 5)5              -0.625  0.53192
## poly(`ln.sulphates:alcohol`, 5)1     0.194  0.84586
## poly(`ln.sulphates:alcohol`, 5)2     -0.010  0.99202
## poly(`ln.sulphates:alcohol`, 5)3     0.459  0.64604
## poly(`ln.sulphates:alcohol`, 5)4     1.268  0.20506
## poly(`ln.sulphates:alcohol`, 5)5     1.592  0.11152
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5719 on 1268 degrees of freedom
## Multiple R-squared:  0.602, Adjusted R-squared:  0.4985
## F-statistic: 5.813 on 330 and 1268 DF, p-value: < 2.2e-16

```

We now have adjusted R^2 of 0.4985, but this model might not generalize well.

Let's select the highest-degree ($d \leq 5$) polynomials having a p-value less than 0.05.

```
##
```

```

## Call:
## lm(formula = quality ~ citric.acid + poly(density, 3) + poly(ln.chlorides,
##   5) + poly(pH, 5) + poly(`fixed.acidity:volatile.acidity`,
##   5) + poly(`fixed.acidity:citric.acid`, 2) + poly(`fixed.acidity:alcohol`,
##   4) + poly(`volatile.acidity:residual.sugar`, 5) + poly(`volatile.acidity:total.sulfur.dioxide`,
##   2) + `volatile.acidity:ln.sulphates` + poly(`citric.acid:ln.chlorides`,
##   5) + `citric.acid:density` + poly(`citric.acid:ln.sulphates`,
##   4) + `residual.sugar:ln.chlorides` + `residual.sugar:pH` +
##   poly(`ln.chlorides:density`, 5) + poly(`ln.chlorides:alcohol`,
##   5) + poly(`free.sulfur.dioxide:ln.sulphates`, 2) + poly(`density:pH`,
##   5) + `pH:ln.sulphates`, data = wine.interactions)
##
## Residuals:
##      Min       1Q     Median      3Q      Max
## -2.40207 -0.35744 -0.03154  0.40845  1.94300
##
## Coefficients:
##                               Estimate Std. Error
## (Intercept)                5.602e+00  4.047e-01
## citric.acid                 1.719e+02  8.390e+01
## poly(density, 3)1           5.454e+04  2.643e+05
## poly(density, 3)2          -8.523e+01  4.275e+02
## poly(density, 3)3           1.055e+00  1.509e+00
## poly(ln.chlorides, 5)1      7.278e+04  2.608e+04
## poly(ln.chlorides, 5)2      1.696e+04  6.121e+03
## poly(ln.chlorides, 5)3      5.517e+03  1.942e+03
## poly(ln.chlorides, 5)4      2.239e+03  7.149e+02
## poly(ln.chlorides, 5)5      5.941e+02  1.653e+02
## poly(pH, 5)1                 1.362e+06  6.496e+06
## poly(pH, 5)2                 -5.186e+04  2.474e+05
## poly(pH, 5)3                  3.500e+03  1.673e+04
## poly(pH, 5)4                 -2.889e+02  1.314e+03
## poly(pH, 5)5                  9.832e+00  1.096e+02
## poly(`fixed.acidity:volatile.acidity`, 5)1    -8.275e+00  3.052e+00
## poly(`fixed.acidity:volatile.acidity`, 5)2    -8.373e-01  1.061e+00
## poly(`fixed.acidity:volatile.acidity`, 5)3     1.033e+00  7.689e-01
## poly(`fixed.acidity:volatile.acidity`, 5)4    -1.111e+00  6.827e-01
## poly(`fixed.acidity:volatile.acidity`, 5)5    -5.733e-01  6.808e-01
## poly(`fixed.acidity:citric.acid`, 2)1        -2.341e+01  1.310e+01
## poly(`fixed.acidity:citric.acid`, 2)2        1.021e+01  2.566e+00
## poly(`fixed.acidity:alcohol`, 4)1            1.167e+01  5.235e+00
## poly(`fixed.acidity:alcohol`, 4)2            -4.948e+00  9.229e-01
## poly(`fixed.acidity:alcohol`, 4)3            -2.078e+00  7.366e-01
## poly(`fixed.acidity:alcohol`, 4)4            1.342e+00  7.609e-01
## poly(`volatile.acidity:residual.sugar`, 5)1   1.747e+00  3.854e+00
## poly(`volatile.acidity:residual.sugar`, 5)2   -1.957e+00  1.372e+00
## poly(`volatile.acidity:residual.sugar`, 5)3   3.755e-01  1.069e+00
## poly(`volatile.acidity:residual.sugar`, 5)4   4.234e-01  8.357e-01
## poly(`volatile.acidity:residual.sugar`, 5)5   -4.092e-01  7.586e-01
## poly(`volatile.acidity:total.sulfur.dioxide`, 2)1 -3.667e+00  1.108e+00
## poly(`volatile.acidity:total.sulfur.dioxide`, 2)2  2.417e+00  8.115e-01
## `volatile.acidity:ln.sulphates`                  -2.669e-01  4.575e-01
## poly(`citric.acid:ln.chlorides`, 5)1            6.507e+00  7.743e+00
## poly(`citric.acid:ln.chlorides`, 5)2            -1.703e+00  1.586e+00

```

```

## poly(`citric.acid:ln.chlorides`, 5)3      -3.593e-01 8.035e-01
## poly(`citric.acid:ln.chlorides`, 5)4      1.390e+00 7.066e-01
## poly(`citric.acid:ln.chlorides`, 5)5      -1.307e+00 7.002e-01
## `citric.acid:density`                     -1.695e+02 8.456e+01
## poly(`citric.acid:ln.sulphates`, 4)1      -3.219e+00 2.190e+00
## poly(`citric.acid:ln.sulphates`, 4)2      1.835e+00 2.693e+00
## poly(`citric.acid:ln.sulphates`, 4)3      5.669e+00 2.315e+00
## poly(`citric.acid:ln.sulphates`, 4)4      2.980e+00 1.071e+00
## `residual.sugar:ln.chlorides`              7.633e-02 4.638e-02
## `residual.sugar:pH`                       5.053e-02 4.248e-02
## poly(`ln.chlorides:density`, 5)1          -7.257e+04 2.601e+04
## poly(`ln.chlorides:density`, 5)2          -1.686e+04 6.080e+03
## poly(`ln.chlorides:density`, 5)3          -5.442e+03 1.916e+03
## poly(`ln.chlorides:density`, 5)4          -2.210e+03 7.047e+02
## poly(`ln.chlorides:density`, 5)5          -5.798e+02 1.619e+02
## poly(`ln.chlorides:alcohol`, 5)1           -1.238e+01 7.623e+00
## poly(`ln.chlorides:alcohol`, 5)2           4.550e+00 6.234e+00
## poly(`ln.chlorides:alcohol`, 5)3           -6.605e+00 7.420e+00
## poly(`ln.chlorides:alcohol`, 5)4           2.724e+00 3.676e+00
## poly(`ln.chlorides:alcohol`, 5)5           -5.782e+00 3.936e+00
## poly(`free.sulfur.dioxide:ln.sulphates`, 2)1 -3.313e+00 1.145e+00
## poly(`free.sulfur.dioxide:ln.sulphates`, 2)2 -8.418e-01 9.262e-01
## poly(`density:pH`, 5)1                    -1.344e+06 6.411e+06
## poly(`density:pH`, 5)2                    5.039e+04 2.404e+05
## poly(`density:pH`, 5)3                    -3.339e+03 1.596e+04
## poly(`density:pH`, 5)4                    2.721e+02 1.234e+03
## poly(`density:pH`, 5)5                    -6.673e+00 1.017e+02
## `pH:ln.sulphates`                         4.949e-01 1.103e-01
## t value Pr(>|t|)                          13.842 < 2e-16 ***
## (Intercept)                                2.048 0.040693 *
## citric.acid                                 0.206 0.836535
## poly(density, 3)1                           -0.199 0.841982
## poly(density, 3)2                           0.699 0.484501
## poly(density, 3)3                           2.790 0.005334 **
## poly(ln.chlorides, 5)1                      2.771 0.005647 **
## poly(ln.chlorides, 5)2                      2.841 0.004553 **
## poly(ln.chlorides, 5)3                      3.133 0.001765 **
## poly(ln.chlorides, 5)4                      3.594 0.000336 ***
## poly(ln.chlorides, 5)5                      0.210 0.834012
## poly(pH, 5)1                               -0.210 0.834017
## poly(pH, 5)2                               0.209 0.834248
## poly(pH, 5)3                               -0.220 0.826003
## poly(pH, 5)4                               0.090 0.928506
## poly(pH, 5)5                               -2.712 0.006772 **
## poly(`fixed.acidity:volatile.acidity`, 5)1 -0.789 0.430293
## poly(`fixed.acidity:volatile.acidity`, 5)2  1.343 0.179496
## poly(`fixed.acidity:volatile.acidity`, 5)3 -1.627 0.104025
## poly(`fixed.acidity:volatile.acidity`, 5)4 -0.842 0.399804
## poly(`fixed.acidity:volatile.acidity`, 5)5 -1.786 0.074243 .
## poly(`fixed.acidity:citric.acid`, 2)1      3.980 7.22e-05 ***
## poly(`fixed.acidity:citric.acid`, 2)2      2.228 0.026001 *
## poly(`fixed.acidity:alcohol`, 4)1           -5.361 9.52e-08 ***
## poly(`fixed.acidity:alcohol`, 4)2           -2.821 0.004854 **

```

```

## poly(`fixed.acidity:alcohol`, 4)4          1.764 0.077936 .
## poly(`volatile.acidity:residual.sugar`, 5)1  0.453 0.650374
## poly(`volatile.acidity:residual.sugar`, 5)2 -1.426 0.153968
## poly(`volatile.acidity:residual.sugar`, 5)3  0.351 0.725540
## poly(`volatile.acidity:residual.sugar`, 5)4  0.507 0.612458
## poly(`volatile.acidity:residual.sugar`, 5)5 -0.539 0.589691
## poly(`volatile.acidity:total.sulfur.dioxide`, 2)1 -3.311 0.000951 ***
## poly(`volatile.acidity:total.sulfur.dioxide`, 2)2  2.978 0.002944 **
## `volatile.acidity:ln.sulphates`           -0.583 0.559714
## poly(`citric.acid:ln.chlorides`, 5)1      0.840 0.400846
## poly(`citric.acid:ln.chlorides`, 5)2      -1.073 0.283369
## poly(`citric.acid:ln.chlorides`, 5)3      -0.447 0.654805
## poly(`citric.acid:ln.chlorides`, 5)4      1.967 0.049402 *
## poly(`citric.acid:ln.chlorides`, 5)5      -1.867 0.062135 .
## `citric.acid:density`                   -2.005 0.045151 *
## poly(`citric.acid:ln.sulphates`, 4)1     -1.470 0.141661
## poly(`citric.acid:ln.sulphates`, 4)2     0.681 0.495688
## poly(`citric.acid:ln.sulphates`, 4)3     2.449 0.014452 *
## poly(`citric.acid:ln.sulphates`, 4)4     2.783 0.005448 **
## `residual.sugar:ln.chlorides`           1.646 0.100022
## `residual.sugar:pH`                    1.190 0.234392
## poly(`ln.chlorides:density`, 5)1       -2.790 0.005334 **
## poly(`ln.chlorides:density`, 5)2       -2.774 0.005612 **
## poly(`ln.chlorides:density`, 5)3       -2.840 0.004572 **
## poly(`ln.chlorides:density`, 5)4       -3.136 0.001744 **
## poly(`ln.chlorides:density`, 5)5       -3.581 0.000353 ***
## poly(`ln.chlorides:alcohol`, 5)1        -1.624 0.104628
## poly(`ln.chlorides:alcohol`, 5)2        0.730 0.465578
## poly(`ln.chlorides:alcohol`, 5)3        -0.890 0.373513
## poly(`ln.chlorides:alcohol`, 5)4        0.741 0.458862
## poly(`ln.chlorides:alcohol`, 5)5        -1.469 0.142069
## poly(`free.sulfur.dioxide:ln.sulphates`, 2)1 -2.893 0.003871 **
## poly(`free.sulfur.dioxide:ln.sulphates`, 2)2 -0.909 0.363538
## poly(`density:pH`, 5)1                 -0.210 0.834012
## poly(`density:pH`, 5)2                 0.210 0.834021
## poly(`density:pH`, 5)3                 -0.209 0.834272
## poly(`density:pH`, 5)4                 0.221 0.825470
## poly(`density:pH`, 5)5                 -0.066 0.947685
## `pH:ln.sulphates`                     4.486 7.79e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6251 on 1535 degrees of freedom
## Multiple R-squared:  0.4245, Adjusted R-squared:  0.4008
## F-statistic: 17.97 on 63 and 1535 DF,  p-value: < 2.2e-16
##
## Call:
## lm(formula = quality ~ citric.acid + density + poly(ln.chlorides,
##   5) + `fixed.acidity:volatile.acidity` + poly(`fixed.acidity:citric.acid`,
##   2) + poly(`fixed.acidity:alcohol`, 3) + poly(`volatile.acidity:total.sulfur.dioxide`,
##   2) + poly(`citric.acid:ln.chlorides`, 4) + `citric.acid:density` +
##   poly(`citric.acid:ln.sulphates`, 4) + poly(`ln.chlorides:density`,
##   5) + `free.sulfur.dioxide:ln.sulphates` + `pH:ln.sulphates`,

```

```

##      data = wine.interactions)
##
## Residuals:
##      Min      1Q   Median      3Q     Max
## -2.66862 -0.37253 -0.02931  0.42185  1.86442
##
## Coefficients:
##                               Estimate Std. Error
## (Intercept)                7.138e+03  3.106e+03
## citric.acid               -6.064e+01  6.188e+01
## density                   -7.155e+03  3.116e+03
## poly(ln.chlorides, 5)1      4.578e+04  2.002e+04
## poly(ln.chlorides, 5)2      1.123e+04  4.657e+03
## poly(ln.chlorides, 5)3      3.362e+03  1.461e+03
## poly(ln.chlorides, 5)4      1.575e+03  5.273e+02
## poly(ln.chlorides, 5)5      3.379e+02  1.286e+02
## `fixed.acidity:volatile.acidity` -1.135e-01  1.672e-02
## poly(`fixed.acidity:citric.acid`, 2)1 -4.546e+01  7.063e+00
## poly(`fixed.acidity:citric.acid`, 2)2      9.862e+00  1.982e+00
## poly(`fixed.acidity:alcohol`, 3)1      2.277e+01  1.885e+00
## poly(`fixed.acidity:alcohol`, 3)2      -4.484e+00  8.154e-01
## poly(`fixed.acidity:alcohol`, 3)3      -2.392e+00  6.927e-01
## poly(`volatile.acidity:total.sulfur.dioxide`, 2)1 -2.506e+00  1.015e+00
## poly(`volatile.acidity:total.sulfur.dioxide`, 2)2      1.834e+00  6.836e-01
## poly(`citric.acid:ln.chlorides`, 4)1      3.007e-01  7.025e+00
## poly(`citric.acid:ln.chlorides`, 4)2      -3.093e-01  1.416e+00
## poly(`citric.acid:ln.chlorides`, 4)3      -2.196e-01  7.508e-01
## poly(`citric.acid:ln.chlorides`, 4)4      8.881e-01  6.764e-01
## `citric.acid:density`       6.490e+01  6.215e+01
## poly(`citric.acid:ln.sulphates`, 4)1      -3.971e+00  1.800e+00
## poly(`citric.acid:ln.sulphates`, 4)2      -9.845e-01  1.937e+00
## poly(`citric.acid:ln.sulphates`, 4)3      3.571e+00  1.689e+00
## poly(`citric.acid:ln.sulphates`, 4)4      2.381e+00  8.963e-01
## poly(`ln.chlorides:density`, 5)1      -4.565e+04  1.996e+04
## poly(`ln.chlorides:density`, 5)2      -1.116e+04  4.627e+03
## poly(`ln.chlorides:density`, 5)3      -3.317e+03  1.442e+03
## poly(`ln.chlorides:density`, 5)4      -1.557e+03  5.204e+02
## poly(`ln.chlorides:density`, 5)5      -3.298e+02  1.264e+02
## `free.sulfur.dioxide:ln.sulphates` -9.546e-03  4.012e-03
## `pH:ln.sulphates`            4.426e-01  5.067e-02
##
## (Intercept)                2.298  0.02167 *
## citric.acid               -0.980  0.32723
## density                   -2.297  0.02177 *
## poly(ln.chlorides, 5)1      2.287  0.02235 *
## poly(ln.chlorides, 5)2      2.412  0.01599 *
## poly(ln.chlorides, 5)3      2.301  0.02150 *
## poly(ln.chlorides, 5)4      2.987  0.00286 **
## poly(ln.chlorides, 5)5      2.626  0.00871 **
## `fixed.acidity:volatile.acidity` -6.784 1.65e-11 ***
## poly(`fixed.acidity:citric.acid`, 2)1 -6.436 1.62e-10 ***
## poly(`fixed.acidity:citric.acid`, 2)2      4.975 7.23e-07 ***
## poly(`fixed.acidity:alcohol`, 3)1      12.075 < 2e-16 ***
## poly(`fixed.acidity:alcohol`, 3)2      -5.499 4.45e-08 ***

```

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## poly(`fixed.acidity:alcohol`, 3)3           -3.453  0.00057 ***
## poly(`volatile.acidity:total.sulfur.dioxide`, 2)1 -2.469  0.01364 *
## poly(`volatile.acidity:total.sulfur.dioxide`, 2)2  2.683  0.00736 **
## poly(`citric.acid:ln.chlorides`, 4)1          0.043  0.96586
## poly(`citric.acid:ln.chlorides`, 4)2          -0.218  0.82716
## poly(`citric.acid:ln.chlorides`, 4)3          -0.293  0.76993
## poly(`citric.acid:ln.chlorides`, 4)4          1.313  0.18938
## `citric.acid:density`                      1.044  0.29652
## poly(`citric.acid:ln.sulphates`, 4)1         -2.207  0.02749 *
## poly(`citric.acid:ln.sulphates`, 4)2         -0.508  0.61141
## poly(`citric.acid:ln.sulphates`, 4)3          2.114  0.03467 *
## poly(`citric.acid:ln.sulphates`, 4)4          2.657  0.00797 **
## poly(`ln.chlorides:density`, 5)1            -2.287  0.02233 *
## poly(`ln.chlorides:density`, 5)2            -2.412  0.01597 *
## poly(`ln.chlorides:density`, 5)3            -2.301  0.02153 *
## poly(`ln.chlorides:density`, 5)4            -2.992  0.00282 **
## poly(`ln.chlorides:density`, 5)5            -2.608  0.00919 **
## `free.sulfur.dioxide:ln.sulphates`        -2.379  0.01748 *
## `pH:ln.sulphates`                         8.735 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6302 on 1567 degrees of freedom
## Multiple R-squared:  0.4028, Adjusted R-squared:  0.391
## F-statistic: 34.09 on 31 and 1567 DF,  p-value: < 2.2e-16
##
## Call:
## lm(formula = quality ~ density + poly(ln.chlorides, 5) + `fixed.acidity:volatile.acidity` +
##      poly(`fixed.acidity:citric.acid`, 2) + poly(`fixed.acidity:alcohol`,
##      3) + poly(`volatile.acidity:total.sulfur.dioxide`, 2) + poly(`citric.acid:ln.sulphates`,
##      4) + poly(`ln.chlorides:density`, 5) + `free.sulfur.dioxide:ln.sulphates` +
##      `pH:ln.sulphates`, data = wine.interactions)
##
## Residuals:
##       Min     1Q   Median     3Q    Max 
## -2.7709 -0.3944 -0.0321  0.4232  1.9161 
##
## Coefficients:
##             Estimate Std. Error
## (Intercept) 6.105e+03 3.042e+03
## density      -6.118e+03 3.052e+03
## poly(ln.chlorides, 5)1 3.902e+04 1.962e+04
## poly(ln.chlorides, 5)2 9.545e+03 4.566e+03
## poly(ln.chlorides, 5)3 2.968e+03 1.426e+03
## poly(ln.chlorides, 5)4 1.303e+03 5.161e+02
## poly(ln.chlorides, 5)5 3.033e+02 1.253e+02
## `fixed.acidity:volatile.acidity` -1.234e-01 1.672e-02
## poly(`fixed.acidity:citric.acid`, 2)1 -1.068e+01 2.220e+00
## poly(`fixed.acidity:citric.acid`, 2)2 5.219e+00 1.020e+00
## poly(`fixed.acidity:alcohol`, 3)1 1.651e+01 1.455e+00
## poly(`fixed.acidity:alcohol`, 3)2 -4.351e+00 7.758e-01
## poly(`fixed.acidity:alcohol`, 3)3 -1.662e+00 6.845e-01
## poly(`volatile.acidity:total.sulfur.dioxide`, 2)1 -1.939e+00 1.005e+00

```

```

## poly(`volatile.acidity:total.sulfur.dioxide`, 2)2 1.658e+00 6.825e-01
## poly(`citric.acid:ln.sulphates`, 4)1 -6.018e+00 1.711e+00
## poly(`citric.acid:ln.sulphates`, 4)2 -1.031e+00 1.905e+00
## poly(`citric.acid:ln.sulphates`, 4)3 3.312e+00 1.656e+00
## poly(`citric.acid:ln.sulphates`, 4)4 3.096e+00 8.516e-01
## poly(`ln.chlorides:density`, 5)1 -3.891e+04 1.957e+04
## poly(`ln.chlorides:density`, 5)2 -9.485e+03 4.537e+03
## poly(`ln.chlorides:density`, 5)3 -2.929e+03 1.407e+03
## poly(`ln.chlorides:density`, 5)4 -1.286e+03 5.094e+02
## poly(`ln.chlorides:density`, 5)5 -2.968e+02 1.232e+02
## `free.sulfur.dioxide:ln.sulphates` -7.915e-03 4.010e-03
## `pH:ln.sulphates` 4.874e-01 4.929e-02
##
## t value Pr(>|t|)
## (Intercept) 2.007 0.044973 *
## density -2.004 0.045213 *
## poly(ln.chlorides, 5)1 1.988 0.046937 *
## poly(ln.chlorides, 5)2 2.090 0.036757 *
## poly(ln.chlorides, 5)3 2.081 0.037574 *
## poly(ln.chlorides, 5)4 2.524 0.011693 *
## poly(ln.chlorides, 5)5 2.421 0.015607 *
## `fixed.acidity:volatile.acidity` -7.382 2.52e-13 ***
## poly(`fixed.acidity:citric.acid`, 2)1 -4.812 1.64e-06 ***
## poly(`fixed.acidity:citric.acid`, 2)2 5.116 3.50e-07 ***
## poly(`fixed.acidity:alcohol`, 3)1 11.348 < 2e-16 ***
## poly(`fixed.acidity:alcohol`, 3)2 -5.609 2.40e-08 ***
## poly(`fixed.acidity:alcohol`, 3)3 -2.428 0.015287 *
## poly(`volatile.acidity:total.sulfur.dioxide`, 2)1 -1.929 0.053852 .
## poly(`volatile.acidity:total.sulfur.dioxide`, 2)2 2.429 0.015241 *
## poly(`citric.acid:ln.sulphates`, 4)1 -3.517 0.000449 ***
## poly(`citric.acid:ln.sulphates`, 4)2 -0.541 0.588331
## poly(`citric.acid:ln.sulphates`, 4)3 2.000 0.045652 *
## poly(`citric.acid:ln.sulphates`, 4)4 3.636 0.000286 ***
## poly(`ln.chlorides:density`, 5)1 -1.989 0.046913 *
## poly(`ln.chlorides:density`, 5)2 -2.091 0.036717 *
## poly(`ln.chlorides:density`, 5)3 -2.081 0.037561 *
## poly(`ln.chlorides:density`, 5)4 -2.525 0.011673 *
## poly(`ln.chlorides:density`, 5)5 -2.409 0.016122 *
## `free.sulfur.dioxide:ln.sulphates` -1.974 0.048590 *
## `pH:ln.sulphates` 9.888 < 2e-16 ***
##
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6362 on 1573 degrees of freedom
## Multiple R-squared: 0.3892, Adjusted R-squared: 0.3795
## F-statistic: 40.09 on 25 and 1573 DF, p-value: < 2.2e-16
##
## Confusion Matrix and Statistics
##
##          Reference
## Prediction 3 4 5 6 7 8
##      3 0 0 0 0 0 0
##      4 1 1 3 0 0 0
##      5 8 32 474 180 7 0
##      6 1 20 202 431 147 11

```

```

##      7   0   0   2   27  45   7
##      8   0   0   0   0   0   0
##
## Overall Statistics
##
##          Accuracy : 0.5947
## 95% CI : (0.5702, 0.6189)
##  No Information Rate : 0.4259
## P-Value [Acc > NIR] : < 2.2e-16
##
##          Kappa : 0.3293
## Mcnemar's Test P-Value : NA
##
## Statistics by Class:
##
##          Class: 3  Class: 4  Class: 5  Class: 6  Class: 7
## Sensitivity      0.000000 0.0188679  0.6960  0.6755  0.22613
## Specificity      1.000000 0.9974127  0.7527  0.6035  0.97429
## Pos Pred Value    NaN 0.2000000  0.6762  0.5308  0.55556
## Neg Pred Value    0.993746 0.9673777  0.7695  0.7370  0.89855
## Prevalence        0.006254 0.0331457  0.4259  0.3990  0.12445
## Detection Rate    0.000000 0.0006254  0.2964  0.2695  0.02814
## Detection Prevalence 0.000000 0.0031270  0.4384  0.5078  0.05066
## Balanced Accuracy  0.500000 0.5081403  0.7244  0.6395  0.60021
##
##          Class: 8
## Sensitivity      0.000000
## Specificity      1.000000
## Pos Pred Value    NaN
## Neg Pred Value    0.98874
## Prevalence        0.01126
## Detection Rate    0.000000
## Detection Prevalence 0.000000
## Balanced Accuracy  0.500000

## Confusion Matrix and Statistics
##
##          Reference
## Prediction  3   4   5   6   7   8
##      3   0   0   0   0   0   0
##      4   1   1   3   0   0   0
##      5   8  32 474 190   7   0
##      6   1  20 202 423 148  10
##      7   0   0   2  25  44   8
##      8   0   0   0   0   0   0
##
## Overall Statistics
##
##          Accuracy : 0.5891
## 95% CI : (0.5645, 0.6134)
##  No Information Rate : 0.4259
## P-Value [Acc > NIR] : < 2.2e-16
##
##          Kappa : 0.3195
## Mcnemar's Test P-Value : NA

```

```

## Statistics by Class:
##          Class: 3  Class: 4  Class: 5  Class: 6  Class: 7
## Sensitivity 0.000000 0.0188679  0.6960  0.6630  0.22111
## Specificity 1.000000 0.9974127  0.7418  0.6035  0.97500
## Pos Pred Value      NaN 0.2000000  0.6667  0.5261  0.55696
## Neg Pred Value   0.993746 0.9673777  0.7669  0.7296  0.89803
## Prevalence     0.006254 0.0331457  0.4259  0.3990  0.12445
## Detection Rate 0.000000 0.0006254  0.2964  0.2645  0.02752
## Detection Prevalence 0.000000 0.0031270  0.4447  0.5028  0.04941
## Balanced Accuracy 0.500000 0.5081403  0.7189  0.6333  0.59805
##          Class: 8
## Sensitivity    0.00000
## Specificity    1.00000
## Pos Pred Value      NaN
## Neg Pred Value   0.98874
## Prevalence     0.01126
## Detection Rate 0.00000
## Detection Prevalence 0.00000
## Balanced Accuracy 0.50000

## Confusion Matrix and Statistics
##
##          Reference
## Prediction 3 4 5 6 7 8
##       3 0 0 0 0 0 0
##       4 1 2 2 0 0 0
##       5 8 38 497 190 7 0
##       6 1 13 181 414 137 10
##       7 0 0 1 34 55 8
##       8 0 0 0 0 0 0
##
## Overall Statistics
##
##          Accuracy : 0.6054
##          95% CI : (0.5809, 0.6294)
##          No Information Rate : 0.4259
##          P-Value [Acc > NIR] : < 2.2e-16
##
##          Kappa : 0.3494
## Mcnemar's Test P-Value : NA
##
## Statistics by Class:
##          Class: 3  Class: 4  Class: 5  Class: 6  Class: 7  Class: 8
## Sensitivity 0.000000 0.037736  0.7298  0.6489  0.27638  0.00000
## Specificity 1.000000 0.998060  0.7353  0.6441  0.96929  1.00000
## Pos Pred Value      NaN 0.400000  0.6716  0.5476  0.56122      NaN
## Neg Pred Value   0.993746 0.968005  0.7858  0.7343  0.90406  0.98874
## Prevalence     0.006254 0.033146  0.4259  0.3990  0.12445  0.01126
## Detection Rate 0.000000 0.001251  0.3108  0.2589  0.03440  0.00000
## Detection Prevalence 0.000000 0.003127  0.4628  0.4728  0.06129  0.00000
## Balanced Accuracy 0.500000 0.517898  0.7326  0.6465  0.62283  0.50000

```

```

## Confusion Matrix and Statistics
##
##             Reference
## Prediction 3 4 5 6 7 8
##          3 0 0 0 0 0 0
##          4 1 2 0 0 0 0
##          5 8 37 489 191 6 0
##          6 1 14 190 416 140 9
##          7 0 0 2 31 53 9
##          8 0 0 0 0 0 0
##
## Overall Statistics
##
##                 Accuracy : 0.6004
##                 95% CI : (0.5759, 0.6245)
##      No Information Rate : 0.4259
##      P-Value [Acc > NIR] : < 2.2e-16
##
##                 Kappa : 0.3402
## Mcnemar's Test P-Value : NA
##
## Statistics by Class:
##
##                  Class: 3 Class: 4 Class: 5 Class: 6 Class: 7 Class: 8
## Sensitivity          0.000000 0.037736 0.7181 0.6520 0.26633 0.00000
## Specificity          1.000000 0.999353 0.7364 0.6316 0.97000 1.00000
## Pos Pred Value       NaN 0.666667 0.6689 0.5403 0.55789 NaN
## Neg Pred Value       0.993746 0.968045 0.7788 0.7322 0.90293 0.98874
## Prevalence           0.006254 0.033146 0.4259 0.3990 0.12445 0.01126
## Detection Rate       0.000000 0.001251 0.3058 0.2602 0.03315 0.00000
## Detection Prevalence 0.000000 0.001876 0.4572 0.4816 0.05941 0.00000
## Balanced Accuracy    0.500000 0.518545 0.7272 0.6418 0.61817 0.50000

## Confusion Matrix and Statistics
##
##             Reference
## Prediction 3 4 5 6 7 8
##          3 0 0 0 0 0 0
##          4 1 2 2 0 0 0
##          5 8 35 475 182 8 0
##          6 1 16 202 431 138 10
##          7 0 0 2 25 53 8
##          8 0 0 0 0 0 0
##
## Overall Statistics
##
##                 Accuracy : 0.601
##                 95% CI : (0.5765, 0.6251)
##      No Information Rate : 0.4259
##      P-Value [Acc > NIR] : < 2.2e-16
##
##                 Kappa : 0.3409
## Mcnemar's Test P-Value : NA
##

```

```

## Statistics by Class:
##
##          Class: 3 Class: 4 Class: 5 Class: 6 Class: 7 Class: 8
## Sensitivity      0.000000 0.037736  0.6975  0.6755  0.26633  0.00000
## Specificity     1.000000 0.998060  0.7462  0.6181  0.97500  1.00000
## Pos Pred Value    NaN 0.400000  0.6709  0.5401  0.60227  NaN
## Neg Pred Value   0.993746 0.968005  0.7688  0.7416  0.90338  0.98874
## Prevalence       0.006254 0.033146  0.4259  0.3990  0.12445  0.01126
## Detection Rate   0.000000 0.001251  0.2971  0.2695  0.03315  0.00000
## Detection Prevalence 0.000000 0.003127  0.4428  0.4991  0.05503  0.00000
## Balanced Accuracy 0.500000 0.517898  0.7218  0.6468  0.62067  0.50000

## Confusion Matrix and Statistics
##
##          Reference
## Prediction 3 4 5 6 7 8
##           3 1 0 0 0 0 0
##           4 0 4 5 0 0 0
##           5 8 34 487 174 5 0
##           6 1 15 187 433 129 11
##           7 0 0 2 31 65 7
##           8 0 0 0 0 0 0
##
## Overall Statistics
##
##          Accuracy : 0.6191
##          95% CI : (0.5948, 0.643)
##          No Information Rate : 0.4259
##          P-Value [Acc > NIR] : < 2.2e-16
##
##          Kappa : 0.375
## Mcnemar's Test P-Value : NA
##
## Statistics by Class:
##
##          Class: 3 Class: 4 Class: 5 Class: 6 Class: 7
## Sensitivity      0.1000000 0.075472  0.7151  0.6787  0.32663
## Specificity     1.0000000 0.996766  0.7593  0.6431  0.97143
## Pos Pred Value   1.0000000 0.444444  0.6879  0.5580  0.61905
## Neg Pred Value   0.9943680 0.969182  0.7823  0.7509  0.91031
## Prevalence       0.0062539 0.033146  0.4259  0.3990  0.12445
## Detection Rate   0.0006254 0.002502  0.3046  0.2708  0.04065
## Detection Prevalence 0.0006254 0.005629  0.4428  0.4853  0.06567
## Balanced Accuracy 0.5500000 0.536119  0.7372  0.6609  0.64903
##
##          Class: 8
## Sensitivity      0.00000
## Specificity      1.00000
## Pos Pred Value    NaN
## Neg Pred Value   0.98874
## Prevalence       0.01126
## Detection Rate   0.00000
## Detection Prevalence 0.00000
## Balanced Accuracy 0.50000

## Confusion Matrix and Statistics

```

```

##  

##          Reference  

## Prediction 3 4 5 6 7 8  

##            3 0 0 0 0 0 0  

##            4 1 4 4 0 0 0  

##            5 9 36 480 170 8 0  

##            6 0 13 196 435 133 9  

##            7 0 0 1 33 58 9  

##            8 0 0 0 0 0 0 0  

##  

## Overall Statistics  

##  

##          Accuracy : 0.611  

## 95% CI : (0.5866, 0.635)  

## No Information Rate : 0.4259  

## P-Value [Acc > NIR] : < 2.2e-16  

##  

##          Kappa : 0.3608  

## Mcnemar's Test P-Value : NA  

##  

## Statistics by Class:  

##  

##          Class: 3 Class: 4 Class: 5 Class: 6 Class: 7 Class: 8  

## Sensitivity      0.000000 0.075472 0.7048 0.6818 0.29146 0.00000  

## Specificity      1.000000 0.996766 0.7571 0.6348 0.96929 1.00000  

## Pos Pred Value   NaN 0.444444 0.6828 0.5534 0.57426 NaN  

## Neg Pred Value   0.993746 0.969182 0.7757 0.7503 0.90587 0.98874  

## Prevalence        0.006254 0.033146 0.4259 0.3990 0.12445 0.01126  

## Detection Rate   0.000000 0.002502 0.3002 0.2720 0.03627 0.00000  

## Detection Prevalence 0.000000 0.005629 0.4396 0.4916 0.06316 0.00000  

## Balanced Accuracy 0.500000 0.536119 0.7310 0.6583 0.63037 0.50000  

##  

## Confusion Matrix and Statistics  

##  

##          Reference  

## Prediction 3 4 5 6 7 8  

##            3 2 0 0 0 0 0  

##            4 6 14 10 0 0 0  

##            5 2 37 515 143 2 0  

##            6 0 2 155 464 83 4  

##            7 0 0 1 31 114 11  

##            8 0 0 0 0 0 3  

##  

## Overall Statistics  

##  

##          Accuracy : 0.6954  

## 95% CI : (0.6722, 0.7179)  

## No Information Rate : 0.4259  

## P-Value [Acc > NIR] : < 2.2e-16  

##  

##          Kappa : 0.5121  

## Mcnemar's Test P-Value : NA  

##  

## Statistics by Class:

```

```

##                                     Class: 3 Class: 4 Class: 5 Class: 6 Class: 7 Class: 8
## Sensitivity                  0.200000 0.264151   0.7562   0.7273   0.57286 0.166667
## Specificity                 1.000000 0.989651   0.7996   0.7461   0.96929 1.000000
## Pos Pred Value              1.000000 0.466667   0.7368   0.6554   0.72611 1.000000
## Neg Pred Value              0.994991 0.975143   0.8156   0.8047   0.94105 0.990602
## Prevalence                   0.006254 0.033146   0.4259   0.3990   0.12445 0.011257
## Detection Rate               0.001251 0.008755   0.3221   0.2902   0.07129 0.001876
## Detection Prevalence        0.001251 0.018762   0.4371   0.4428   0.09819 0.001876
## Balanced Accuracy            0.600000 0.626901   0.7779   0.7367   0.77108 0.583333

## Confusion Matrix and Statistics
##
##          Reference
## Prediction 3 4 5 6 7 8
##           3 0 0 0 0 0 0
##           4 1 2 2 0 0 0
##           5 9 39 486 184 6 0
##           6 0 12 193 425 127 9
##           7 0 0 0 29 66 9
##           8 0 0 0 0 0 0
##
## Overall Statistics
##
##          Accuracy : 0.6123
##          95% CI : (0.5879, 0.6362)
##          No Information Rate : 0.4259
##          P-Value [Acc > NIR] : < 2.2e-16
##
##          Kappa : 0.3621
##          Mcnemar's Test P-Value : NA
##
## Statistics by Class:
##
##                                     Class: 3 Class: 4 Class: 5 Class: 6 Class: 7 Class: 8
## Sensitivity                  0.000000 0.037736   0.7137   0.6661   0.33166 0.00000
## Specificity                 1.000000 0.998060   0.7407   0.6452   0.97286 1.00000
## Pos Pred Value              NaN 0.400000   0.6713   0.5548   0.63462     NaN
## Neg Pred Value              0.993746 0.968005   0.7771   0.7443   0.91104 0.98874
## Prevalence                   0.006254 0.033146   0.4259   0.3990   0.12445 0.01126
## Detection Rate               0.000000 0.001251   0.3039   0.2658   0.04128 0.00000
## Detection Prevalence        0.000000 0.003127   0.4528   0.4790   0.06504 0.00000
## Balanced Accuracy            0.500000 0.517898   0.7272   0.6557   0.65226 0.50000

## Confusion Matrix and Statistics
##
##          Reference
## Prediction 3 4 5 6 7 8
##           3 0 0 0 0 0 0
##           4 1 2 2 0 0 0
##           5 9 39 486 184 6 0
##           6 0 12 193 425 127 9
##           7 0 0 0 29 66 9
##           8 0 0 0 0 0 0
##

```

```

## Overall Statistics
##
##          Accuracy : 0.6123
##          95% CI : (0.5879, 0.6362)
##          No Information Rate : 0.4259
##          P-Value [Acc > NIR] : < 2.2e-16
##
##          Kappa : 0.3621
##  Mcnemar's Test P-Value : NA
##
## Statistics by Class:
##
##          Class: 3 Class: 4 Class: 5 Class: 6 Class: 7 Class: 8
## Sensitivity      0.000000 0.037736  0.7137  0.6661  0.33166  0.00000
## Specificity      1.000000 0.998060  0.7407  0.6452  0.97286  1.00000
## Pos Pred Value    NaN 0.400000  0.6713  0.5548  0.63462  NaN
## Neg Pred Value    0.993746 0.968005  0.7771  0.7443  0.91104  0.98874
## Prevalence        0.006254 0.033146  0.4259  0.3990  0.12445  0.01126
## Detection Rate    0.000000 0.001251  0.3039  0.2658  0.04128  0.00000
## Detection Prevalence 0.000000 0.003127  0.4528  0.4790  0.06504  0.00000
## Balanced Accuracy  0.500000 0.517898  0.7272  0.6557  0.65226  0.50000

## Confusion Matrix and Statistics
##
##          Reference
## Prediction 3 4 5 6 7 8
##       3 0 0 0 0 0 0
##       4 1 3 3 0 0 0
##       5 8 35 481 165 7 0
##       6 1 15 195 444 132 9
##       7 0 0 2 29 60 9
##       8 0 0 0 0 0 0
##
## Overall Statistics
##
##          Accuracy : 0.6179
##          95% CI : (0.5936, 0.6418)
##          No Information Rate : 0.4259
##          P-Value [Acc > NIR] : < 2.2e-16
##
##          Kappa : 0.3716
##  Mcnemar's Test P-Value : NA
##
## Statistics by Class:
##
##          Class: 3 Class: 4 Class: 5 Class: 6 Class: 7 Class: 8
## Sensitivity      0.000000 0.056604  0.7063  0.6959  0.30151  0.00000
## Specificity      1.000000 0.997413  0.7658  0.6337  0.97143  1.00000
## Pos Pred Value    NaN 0.428571  0.6911  0.5578  0.60000  NaN
## Neg Pred Value    0.993746 0.968593  0.7785  0.7584  0.90727  0.98874
## Prevalence        0.006254 0.033146  0.4259  0.3990  0.12445  0.01126
## Detection Rate    0.000000 0.001876  0.3008  0.2777  0.03752  0.00000
## Detection Prevalence 0.000000 0.004378  0.4353  0.4978  0.06254  0.00000
## Balanced Accuracy  0.500000 0.527008  0.7361  0.6648  0.63647  0.50000

```

```

## Confusion Matrix and Statistics
##
##             Reference
## Prediction 3 4 5 6 7 8
##          3 0 0 0 0 0 0
##          4 1 3 1 0 0 0
##          5 8 35 468 177 9 0
##          6 1 15 211 434 133 10
##          7 0 0 1 27 57 8
##          8 0 0 0 0 0 0
##
## Overall Statistics
##
##                 Accuracy : 0.6016
##                 95% CI : (0.5771, 0.6257)
## No Information Rate : 0.4259
## P-Value [Acc > NIR] : < 2.2e-16
##
##                 Kappa : 0.343
## Mcnemar's Test P-Value : NA
##
## Statistics by Class:
##
##                Class: 3 Class: 4 Class: 5 Class: 6 Class: 7 Class: 8
## Sensitivity      0.000000 0.056604 0.6872 0.6803 0.28643 0.00000
## Specificity     1.000000 0.998706 0.7505 0.6150 0.97429 1.00000
## Pos Pred Value   NaN 0.600000 0.6714 0.5398 0.61290  NaN
## Neg Pred Value   0.993746 0.968632 0.7639 0.7434 0.90571 0.98874
## Prevalence       0.006254 0.033146 0.4259 0.3990 0.12445 0.01126
## Detection Rate   0.000000 0.001876 0.2927 0.2714 0.03565 0.00000
## Detection Prevalence 0.000000 0.003127 0.4359 0.5028 0.05816 0.00000
## Balanced Accuracy 0.500000 0.527655 0.7189 0.6476 0.63036 0.50000

```

Multivariate Analysis

Talk about some of the relationships you observed in this part of the investigation. Were there features that strengthened each other in terms of looking at your feature(s) of interest?

Were there any interesting or surprising interactions between features?

OPTIONAL: Did you create any models with your dataset? Discuss the strengths and limitations of your model.

Final Plots and Summary

Tip: You've done a lot of exploration and have built up an understanding of the structure of and relationships between the variables in your dataset. Here, you will select three plots from all of your previous exploration to present here as a summary of some of your most interesting findings.

Make sure that you have refined your selected plots for good titling, axis labels (with units), and good aesthetic choices (e.g. color, transparency). After each plot, make sure you justify why you chose each plot by describing what it shows.

Plot One

Description One

Plot Two

Description Two

Plot Three

Description Three

Reflection

Tip: Here's the final step! Reflect on the exploration you performed and the insights you found. What were some of the struggles that you went through? What went well? What was surprising? Make sure you include an insight into future work that could be done with the dataset.

When scoring new data points for quality using the regression functions determined above, we may find correlated residuals $e_i = y_i - \hat{y}_i$ in the new data points x_i . New data points may be taken from different growing regions or from different growing times. For example, differences in soil, climate, etc., may result in correlated residuals when using prior regression functions. As a result, it may be appropriate to determine new regression functions or consider adding features for geographic region, soil chemistry, or climate data.