UC Santa Cruz
Physics 115
Spring 2018

Exercise Set #6
due online Wednesday, May 23 at 10:10 AM

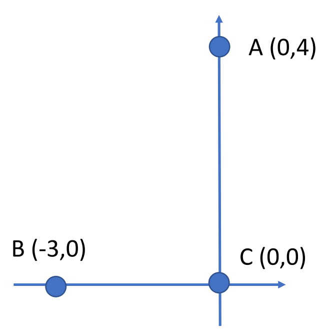
1. **Quantized Energy Levels**

   For this exercise we will follow the discussion of the energy levels in Chapter 9.2.2 and 9.3.

   (a) Start by assuming the finite square well with depth 83 MeV and radius 2 fm, as in Eqn. 9.5. Use the mass of the proton, as in Eqn. 9.8. Use the Numerov method to solve for the allowed energies in the well.
   (b) Plot the allowed wave functions on the same figure as the potential, as in Figure 9.1. (You will have to scale one of them to make them both fit.) Draw a horizontal line to represent each of the allowed energies.
   (c) Check to see how the bound-state energies change when the radius of the well changes. Start by increasing the radius by a factor of 2.

2. **Pythagorean (or Euler) 3-body Problem**

   The Pythagorean version of the Euler 3-body problem has 3 masses at the corners of a 3-4-5 right triangle.



   The force on each mass m_i is the sum of gravitational forces from the other masses.

$$\vec{F}_i = -G \sum_{j \neq i} m_i m_j \frac{\vec{r}_i - \vec{r}_j}{|\vec{r}_i - \vec{r}_j|^3}$$

   With units arranged so that $G = 1$, the masses have values $m_A = 3$, $m_B = 4$, and $m_C = 5$, and they are at rest at $t = 0$ as shown in the figure.

(a) Find the motion of the system over the interval $t = 0$ to $t = 10$. (If you have time, try to use the animation functions.)

(b) A new, stable solution for the equal-mass 3-body problem was discovered in 2000 by Prof. R. Montgomery (UCSC) and A. Chenciner (Annals of Mathematics **152**: 881-901). Find the motion of the 3-body system with all 3 masses set to $m = 1$, $G = 1$, and initial conditions given in Figure 1 of the paper. This system is called the "figure-eight" orbit.

(c) Check the stability of the "figure-eight" orbit by changing the initial conditions slightly and checking the orbit again.

3. **Vibrating String**

Even though the equation of a vibrating string (with length L, linear mass density $\mu(x)$, and tension T) is a partial differential equation

$$\frac{\partial^2 u(x,t)}{\partial t^2} = \frac{T}{\mu(x)} \frac{\partial^2 u(x,t)}{\partial x^2}$$

the use of a solution $u(x,t) = y(x)\tau(t)$ allows us to separate the equation into a spatial side and a temporal side:

$$\frac{1}{y(x)} \frac{T}{\mu(x)} \frac{d^2 y}{dx^2} = \frac{1}{\tau(t)} \frac{d^2 \tau}{dt^2}$$

The separation constant is taken to be $-\omega^2$.

(a) Show that if $\mu(x)$ is a constant μ_0, then the spatial solution is

$$y(x) = \alpha \sin \omega \sqrt{\frac{\mu_0}{T}} x + \beta \cos \omega \sqrt{\frac{\mu_0}{T}} x$$

(b) Set the boundary conditions $y(x) = 0$ at the ends of the string, and show that the allowed values of ω are

$$\omega = \frac{n\pi}{L} \sqrt{\frac{T}{\mu_0}}$$

(c) Now use the shooting method with the boundary conditions to find the lowest frequency of the string and plot the eigenfunction (shape). Take $L = 1\,\mathrm{m}$, $m = 0.954\,\mathrm{g}$, and $T = 1000\,\mathrm{N}$. Assume a constant linear mass density μ_0 for this part.

(d) Repeat the shooting method with a non-uniform $\mu(x) = 0.954\,\mathrm{g/m} + \left(x - \frac{L}{2}\right) 0.8\,\mathrm{g/m^2}$. Plot this eigenfunction (shape) and compare with the previous part.