UC Santa Cruz
Physics 115
Spring 2018

Exercise Set #5
due online Wednesday, May 16 at 10:10 AM

1. **Solving a Differential Equation**
   We are given the following differential equation and initial condition:

$$\begin{cases} \frac{dx}{dt} = -tx^2 \\ x(0) = 2 \end{cases}$$

   Find the value of x at $t = -0.2$, correct to two decimal places, using

   (a) one step of the Taylor series method of order 2, and

   (b) one step of the Runge-Kutta method of order 2.

   You can do these with or without the computer. Remember, just write one step!

2. **Projectile Motion with Air Resistance**
   The parabolic motion of a projectile is spoiled by air resistance (drag force). Given a drag coefficient k, we can write the time change of projectile momentum as

$$\frac{dp}{dt} = mg - kv^2$$

   For a spherical projectile, take values of $m = 10^{-2}\,\text{kg}$ and $k = 10^{-4}\,\text{kg/s}^2$.

   (a) Use a 4th-order Runge-Kutta program with adaptive step sizes to find the velocity $v = p/m$ as a function of time for $t < 10\,\text{s}$, assuming the projectile is released from rest. Try to adapt the step sizes so that you obtain 4 significant digits of accuracy.

   (b) Compare your numerical result to the analytic result for v expected with $k = 0$.

3. **Nonlinear Oscillations**

   In this problem, we will study anharmonic oscillations in potentials of the form $V(x) = \frac{1}{p}kx^p$. (See also Landau Chapter 8.8.)

   (a) Use the 4th-order Runge-Kutta technique to plot the oscillations $x(t)$ of systems with $p = 2, 4, 8$. Confirm that energy is conserved during the oscillations.

(b) Plot the period of oscillations as a function of initial amplitude for $p = 2, 4, 8$. Because the motion may be asymmetric, you must record the time for at least three cycles in order to find the period.

4. **Oscillations with External Driving Force**

Now we will add an external driving force F_{ext} to the harmonic oscillator system with $p = 2$.

$$F_{\text{ext}}(t) = F_0 \sin \omega t$$

(See also Landau Chapter 8.10.)

(a) Solve for and plot the oscillation of this system for a very large F_0 value.

(b) Now reduce F_0 until it is approximately equal to the system restoring force $F_k(x) = -dV(x)/dx$, and verify that you see a beat frequency of $(\omega - \omega_0)/2\pi$, where ω_0 is the natural frequency of the system.

(c) Plot the maximum amplitude as a function of the driving frequency ω by scanning the interval $\omega_0/10 \leq \omega \leq 10\omega_0$. What features do you observe?

(d) Repeat the previous scan over ω with a new, non-linear system having $p = 4$. (You may have to retune F_0.) How does the response of this system differ from the linear system having $p = 2$?