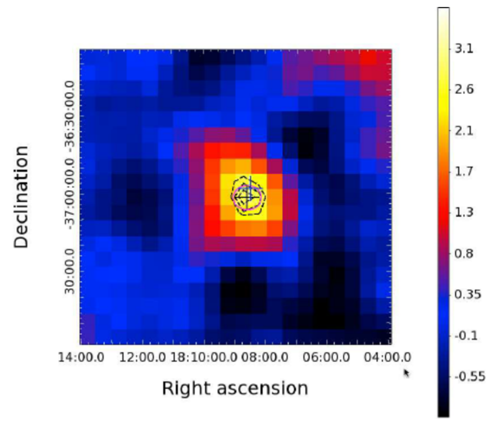
UC Santa Cruz
Physics 115
Spring 2018

Exercise Set #4
due online Wednesday, May 9 at 10:10 AM

1. **Pulsar analysis with Point Spread Function**

The Large Area Telescope (LAT) is one of two instruments currently in orbit aboard the Fermi satellite. It is able to map the entire sky in the energy range from less than 20 MeV to more than 300 GeV. However, it functions more like a particle detector than an optical telescope. As such, while optical telescopes have an angular resolution, the LAT has a point spread function (PSF). The PSF describes the region of sky over which a point source appears to “smear out” when viewed by the LAT.



In this problem, you will produce a rough simulation of PSR B1509-58, a soft gamma ray pulsar, as seen by the Fermi LAT.

(a) The spectrum of PSR B1509-58 can be described by a power law with exponential cutoff:

$$F(E) = k * E^{-\alpha} \exp\left(-\frac{E}{E_c}\right),$$

where $\alpha = 1.87$ is the spectral index, $k = 1 \times 10^{-4}\,\mathrm{s}^{-1}\mathrm{cm}^{-2}$ is a normalization factor, and $E_c = 81\,\mathrm{MeV}$ is the energy cutoff. Normalize this distribution, and use it to generate $100,000$ photon energies from $E = 20\,\mathrm{MeV}$ to $E = 500\,\mathrm{MeV}$. Plot the distribution in a log-log histogram.

Hint: You will need to re-normalize the equation in Part (a) for use as a probability distribution. As written, it has an area of about 4.14575×10^{-6} in the interval from 20 MeV to 500 MeV.

(b) We will approximate the Fermi PSF as a 2-D normal distribution in x and y (the actual function is much more complex). The 1-D normal distribution is given by:

$$F(x \mid \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right),$$

where σ is the standard deviation, and μ is the center of the peak. For Fermi, the value of σ is energy dependent:

$$\sigma = \left(\frac{E}{100\,MeV}\right)^{-0.8}$$

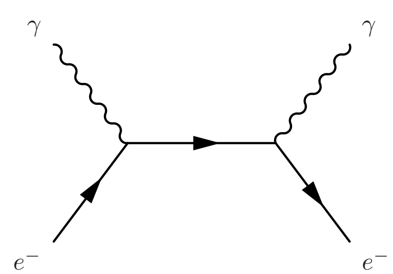
PSR B1509-58 is located at equatorial coordinates (right ascension [RA], declination [DEC]) = (48.04, -30.92). Draw $100,000$ photons of energy $100\,\mathrm{MeV}$, and make a 2D histogram of their positions, with RA on the x axis and DEC on the y axis.

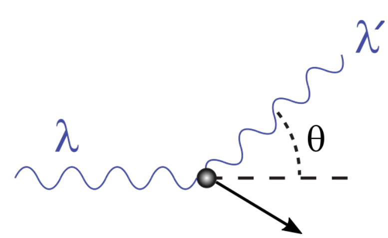
(c) Simulate a set of $100,000$ photons from PSR B1509-58 in a Monte Carlo simulation. Use the equation from Part (a) to assign each photon an energy. Then use the equations in Part (b) to assign each photon a (RA, DEC) position appropriate for that energy.

Make a 2-D histogram of your data, with RA on the X axis and DEC on the Y axis. You may need to explore some plotting options for indicating the number of particles (the 3rd dimension) in each bin.

(d) How might one experimentally determine the LAT's PSF? Due to the unique pulsating nature of pulsars, their position can be measured to extreme accuracy in other wavelengths. "Photon gating" can be used to select photons highly likely to originate from the pulsar. Put together, this can be studied to arrive at the PSF. To increase statistics, LAT scientists "cut out" multiple pulsars, and analyzed the data as a single generic source in a technique known as "stacking." If you wanted to "cut out" PSR B1509-58 in such a way that you kept 95% of the data, what radius around the pulsar would you use?

2. **Compton Scattering**





Compton scattering is the inelastic scattering of a photon by a charged particle, usually an electron. The scattering angle of the electron (ϕ) depends on on the initial energy of the photon (E) and its scattered energy (E').

$$\cos\phi = \frac{E^2 - E'^2 + K^2(1 + 2E_0/K)}{2EK\sqrt{1 + 2E_0/K}},$$

where $K = 2.5\,\mathrm{keV}$ and $E_0 = 511\,\mathrm{keV}$ is the electron rest mass.

Suppose that X-rays are incident on a target and undergo Compton scattering from the atomic electrons.

(a) Calculate the scattered energy E' if $E = 100\,\text{keV}$ and $\phi = 73^\circ$. For each iteration step i, print out E'_i and $\Delta \equiv \left|E'_i - E'_{i-1}\right|$.

(b) Repeat the calculation for scattering from a nuclear proton in hydrogen, with $E_0 = 938\,\text{MeV}$.

3. **Quantum Monte Carlo**

The hydrogen atom (1-electron atom) is one of the simplest quantum mechanical systems, and the ground state energy of hydrogen can be estimated via the variational method, wherein we calculate the local energy of a trial state function and search for a minimum value:

$$\langle E_L \rangle = \frac{1}{\psi_T} H \psi_T$$

This minimum value approximates the true energy given by $\langle H \rangle$.

The 3-dimensional version of Schrodinger's equation leads to the following Hamiltonian for this system:

$$H = -\frac{1}{2}\frac{\partial^2}{\partial \rho^2} - \frac{1}{\rho} + \frac{\ell(\ell+1)}{2\rho^2}$$

A suitable trial function for the hydrogen atom ground state might be

$$u_T(\rho) = \alpha \rho e^{-\alpha \rho}$$

(a) With this trial function, show that the local energy operator E_L can be written as

$$E_L(\rho) = -\frac{1}{\rho} - \frac{\alpha}{2}\left(\alpha - \frac{2}{\rho}\right)$$

and that

$$\langle H \rangle = \int_0^\infty P(\rho) E_L(\rho) d\rho,$$

where $P(\rho)$ is the probability density.

(b) Use Monte Carlo integration to evaluate $\langle H \rangle$ as a function of α, thereby finding the ground state energy. (*Hint*: What should the weighting function be?) The energy will be in terms of the constant $e^2/4\pi\epsilon_0 a_0 = 27.21\,\text{eV}$.