UC Santa Cruz
Physics 115
Spring 2018

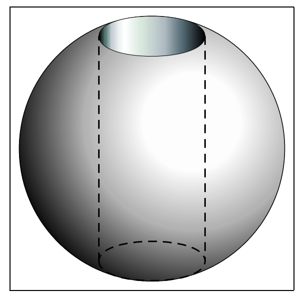
Final Exam

Due online Thursday, June 14 at 10:30 PM

Because this is the final exam for the course, I need to see your own work.
No collaboration with other students is allowed.
You may use your notes, books, and online references to answer the 2 problems.

1. **Solid Body Calculations**

A sphere of radius R and uniform density ρ has a cylindrical hole of radius r drilled through its exact center. For the purposes of this problem, we can assume $\rho = 1\,\mathrm{kg/m^3}$, $R = 0.5\,\mathrm{m}$, and $r = 0.3\,\mathrm{m}$.
Use Monte Carlo integration to calculate the inertial properties of this object.



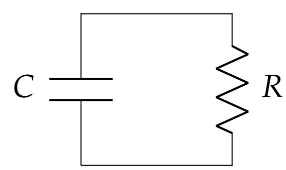
(a) Calculate the total mass of the object.

(b) Calculate the moment of inertia about the shared axis of symmetry (call it $\hat{z}$).

(c) Calculate the moment of inertia about the "$\hat{x}$ axis" (any axis orthogonal to $\hat{z}$).

2. **RC Circuit Behavior**

The "RC circuit" is composed of a resistor R and capacitor C in a series circuit driven by a voltage or current source.



The circuit behavior is given by the equation

$$R\frac{dq}{dt} + \frac{1}{C}q = U(t),$$

where q is the capacitor charge, $dq/dt = I$ is the circuit current, and $U(t)$ is the applied voltage.

We would like to know how the system responds to a square input voltage pulse:

$$U(t) = \begin{cases} 0, & t < t_1 \\ U_0 = 6\,\mathrm{V}, & t_1 \le t \le t_2 \\ 0, & t > t_2 \end{cases}$$

In this problem, we will assume $R = 100\,\Omega$, $C = 4.7 \times 10^{-5}$ A s V^{-1} *.

(a) Assuming $q(0) = 0$, solve the differential equation numerically and plot $q(t)$ and $I(t)$ over the interval $0 < t < t_{\text{final}}$, with $t_1 = 0$, $t_2 = 2RC$, and $t_{\text{final}} = 4RC$.

(b) Compare your result for the current to the analytic solution

$$\bar{I}(t) = \frac{U_0}{R}\left(e^{-(t-t_1)/RC}\,\Theta(t-t_1) - e^{-(t-t_2)/RC}\,\Theta(t-t_2)\right),$$

where $\Theta(x)$ is the Heaviside step function.

*This unit is almost exactly the same as the Farad, but not exactly!