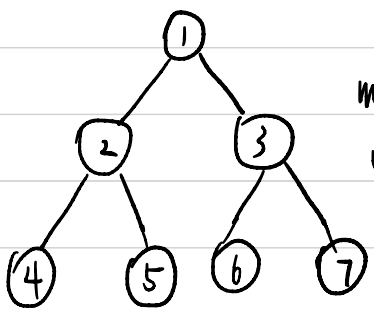
EL504 Homework 3

1.

(a) True

(b) True



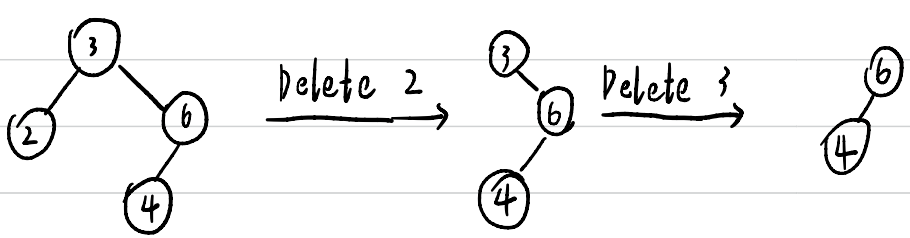
start with the root node the minimum height of the tree is 0 and the maximum height of the tree is 1. So the value of $h=1$. Put the value of h in the given formula $2^{h+1}-1$.

For the tree above, $2^{2+1}-1 = 2^3-1 = 8-1 = 7$

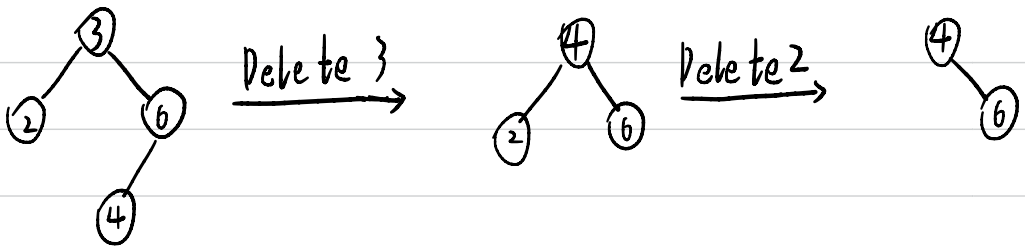
So the statement is correct.

(c) False

For example, First delete 2 then delete 3



First delete 3 then delete 2



So the statement is false

(d) False

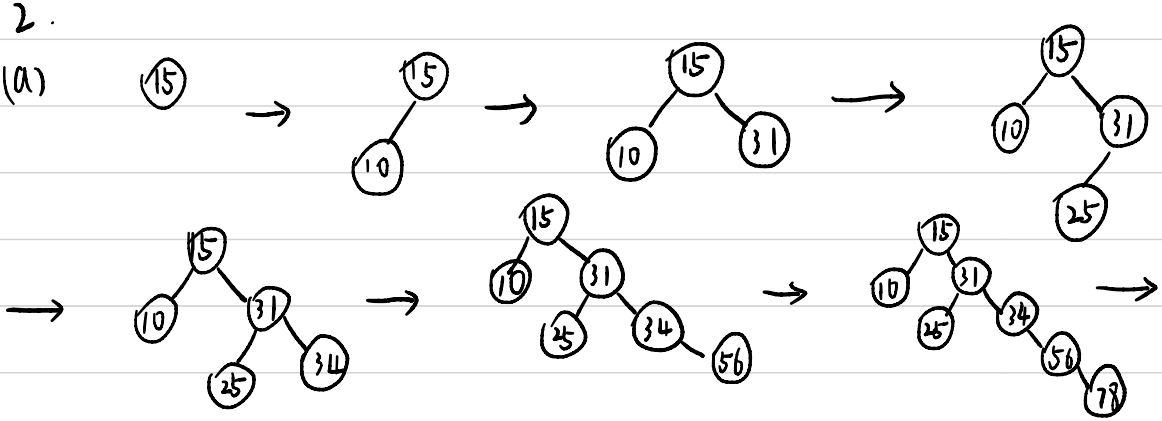
Inserting one number in binary min-heap will take O(logn) time.

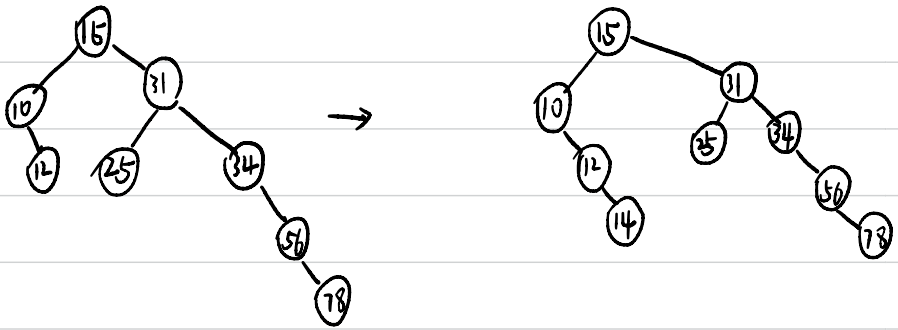
(e) True

Because it is the min-heap property.

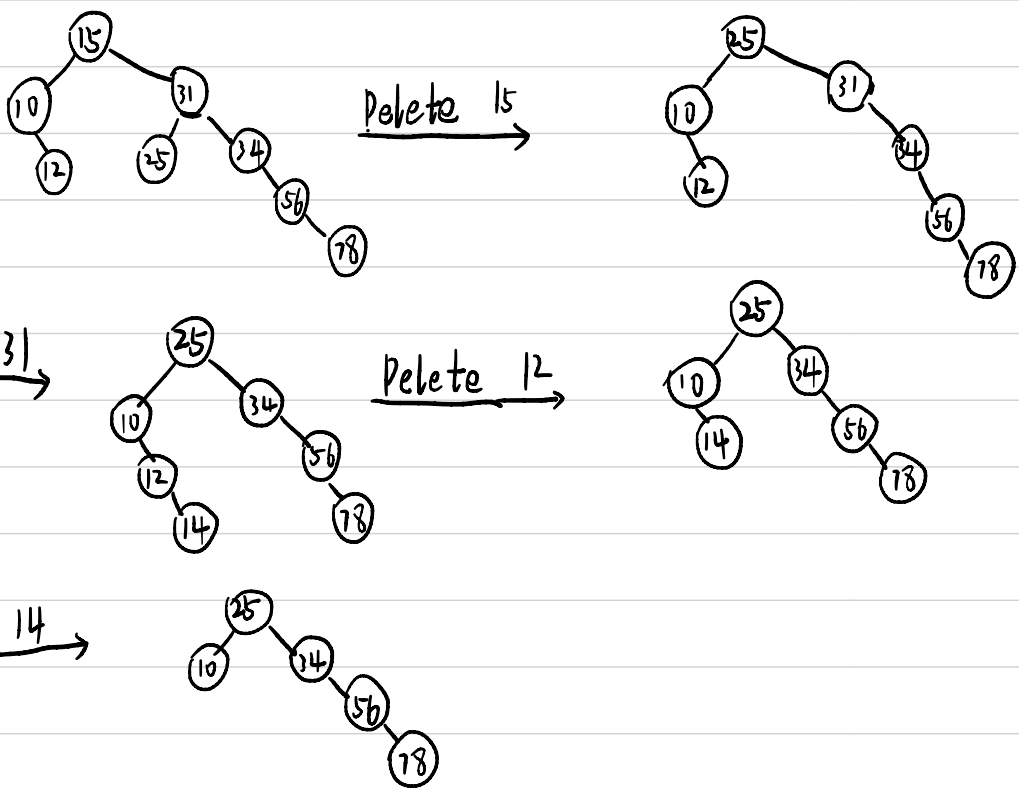
2.

(a)




15
10
31
12
25
34
56
78
15
10
31
25
34
12
56
14
78

(b)


15
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Delete 15
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Delete 31
25
10
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12
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Delete 12
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Delete 14
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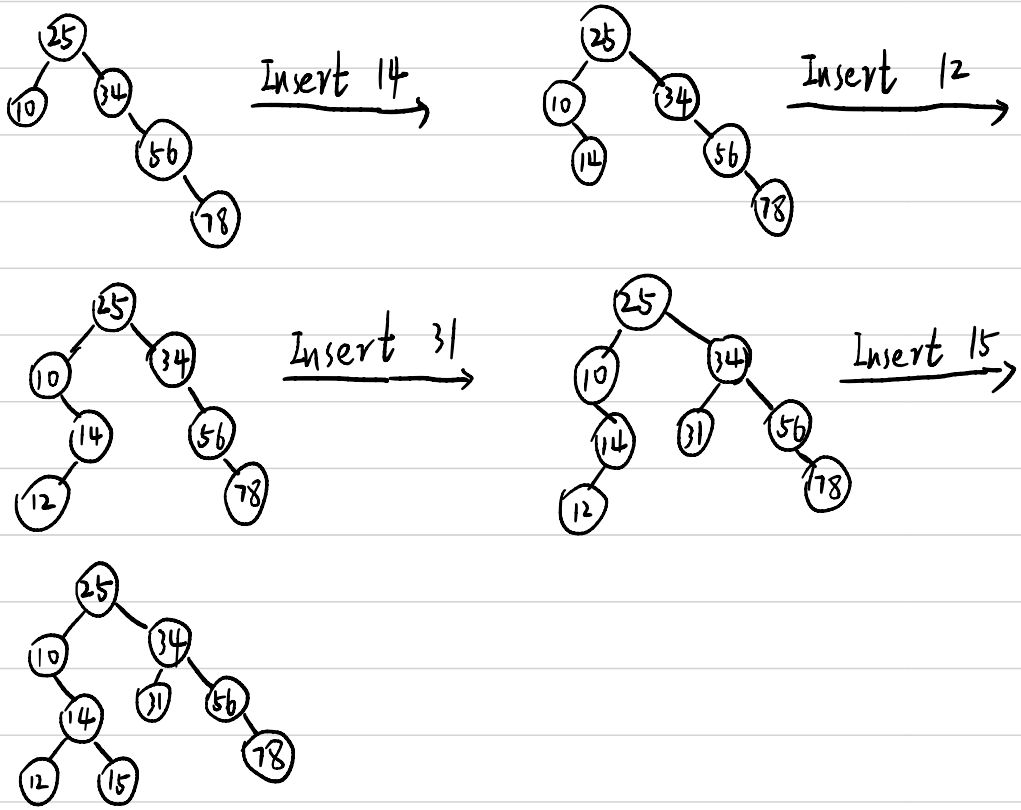
34
56
78

(c)



3.

(a)

6.1-3: If the largest element in the subtree were somewhere orther than the root, it has a parent that is in the subtree. So, it is larger than it's parent, so, the heap property is violated at the

parent of the maximum element in the subtree.

6.1-4: In any of the leaves, that is, elements with index $\lfloor n/2 \rfloor + k$, where $k \geq 1$. that is, in the second half of the heap array.

6.1-6: No. Since the parent of 7 is 6. This violates the max-heap property.

6.3-3: We know that the leaves of a heap are the nodes indexed by

$$\lfloor n/2 \rfloor + 1, \lfloor n+2 \rfloor + 2, \cdots, n$$

Note that those elements corresponds to the second half of the array (plus the middle element if n is odd). Thus, the number of leaves in any heap of size n is $\lceil n/2 \rceil$. Let's prove by induction. Let n_h denote the number of nodes at height h. The upper bound holds for the base since $n_0 = \lceil n/2^0 \rceil$ is exactly the number of leaves in a

heap of size n.
Now assume it holds for $h-1$. We have prove that it also holds for h. Note that if n_{h-1} is even each node at height h has exactly two children, which also implies $n_h = [n_{h-1}/2] + 1 = [n_{h-1}/2]$. Thus, we have

$$n_h = \left[\frac{n_{h-1}}{2}\right] \leq \left[\frac{1}{2} \cdot \left[\frac{n}{2^{(h-1)}+1}\right]\right]$$

$$= \left[\frac{1}{2} \cdot \left[\frac{n}{2^h}\right]\right]$$

$$= \left[\frac{n}{2^{h+1}}\right]$$

which implies that it holds for h.

12.3-2:
Number of nodes examined while searching also includes the node which is searched for, which isn't the case when we inserted it.

12.3-3

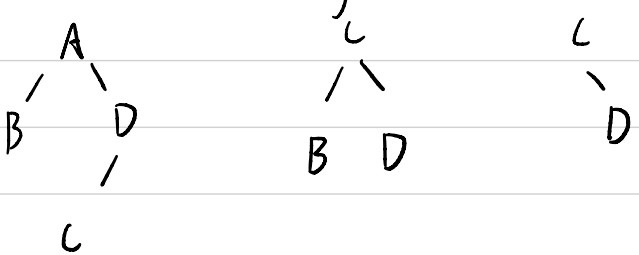
· The worst-case is that the tree formed has height n because we were inserted them in already sorted order. This will result in a runtime of $\Theta(n^2)$

· The best-case is that the tree formed is approximately balanced. This will mean that the height doesn't exceed $O(\lg n)$. Note that it can't have a smaller height, because a complete binary tree of height h only has $\Theta(2^h)$ elements. This will result in a runtime of $O(n \lg n)$.

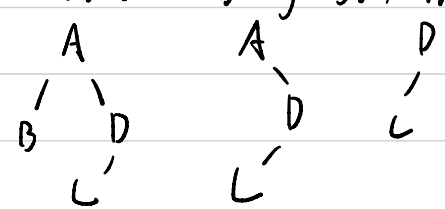
12.3-4

No, giving the following counterexample

· Delete A first, then delete B



· Delete B first, then delete A



4.

(a) False, it takes $n \log n$ time.

(b) False, it's a complete binary tree.

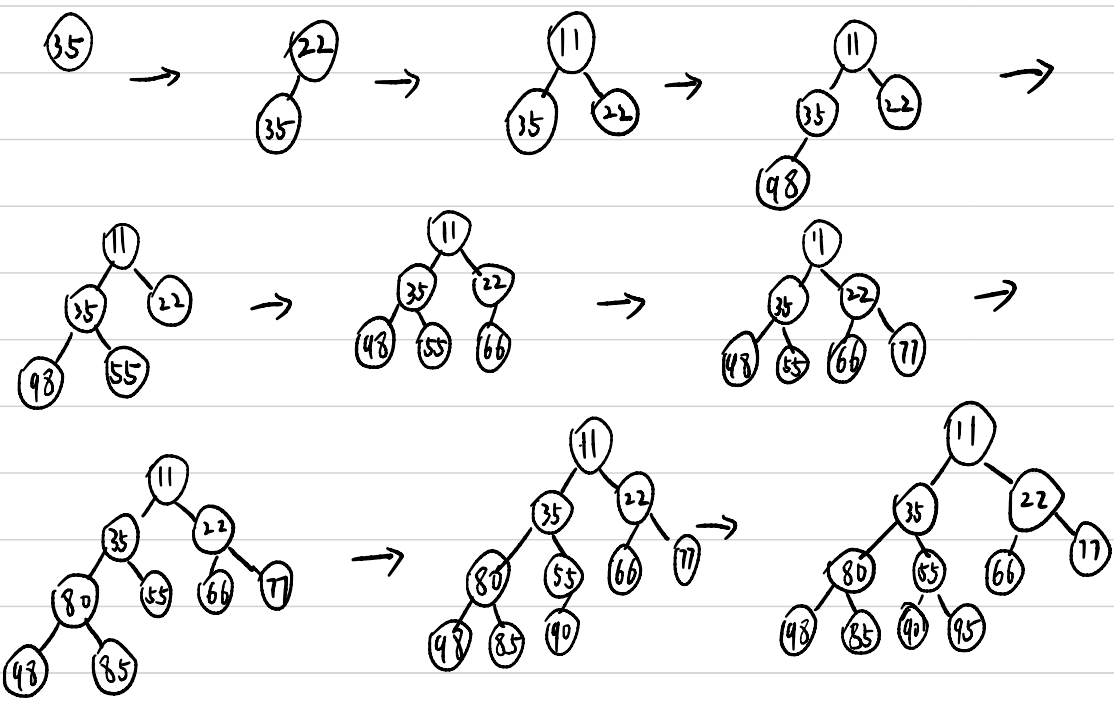
(c) true

(d) true

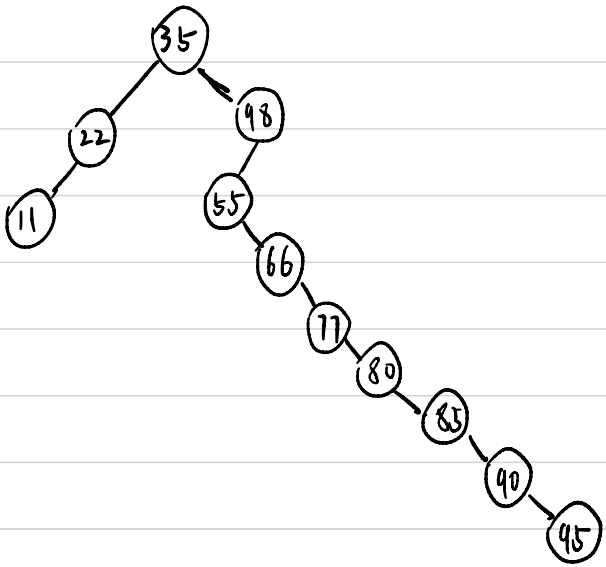
(e) true

5.

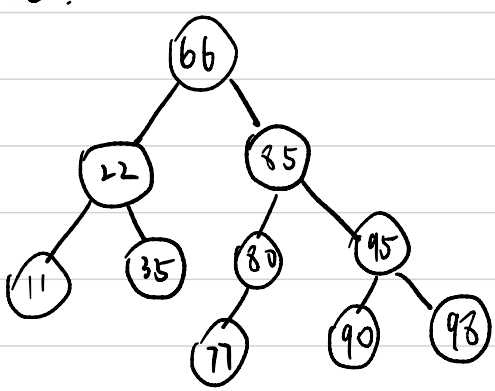
(a)



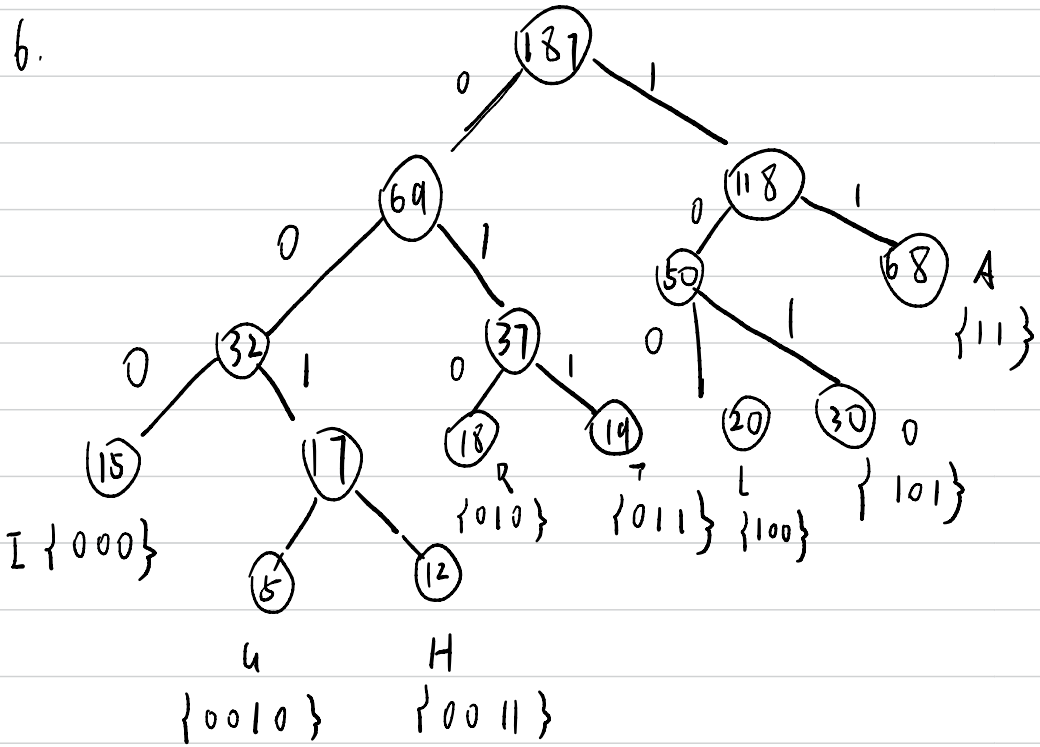
(b) BST tree:



(c) AVL tree:



6.



(a) Minimum external path length

$20\times3 + 5\times4 + 30\times3 + 18\times3 + 15\times3 + 19\times3 + 12\times4 + 68\times2$

$= 510$

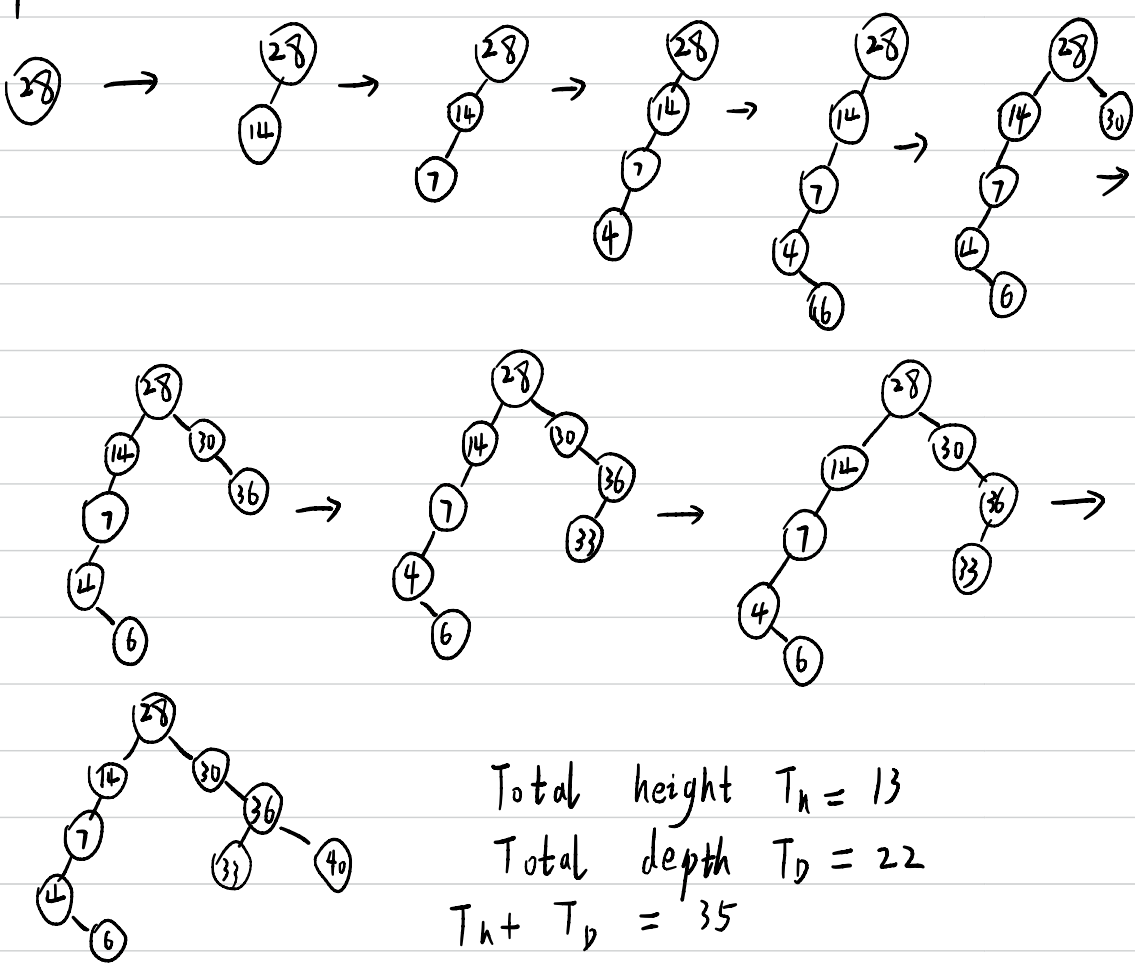
(b) Number of bit for each letter is given in image like for

A - 2bits, L - 4bits, O - 3bits, R - 3bits, I - 3bits, T

- 3 bits H - 4 bits

Compute the average number of bit per symbol = 510 / 187 = 2.72, yes it is less than 3.

7.

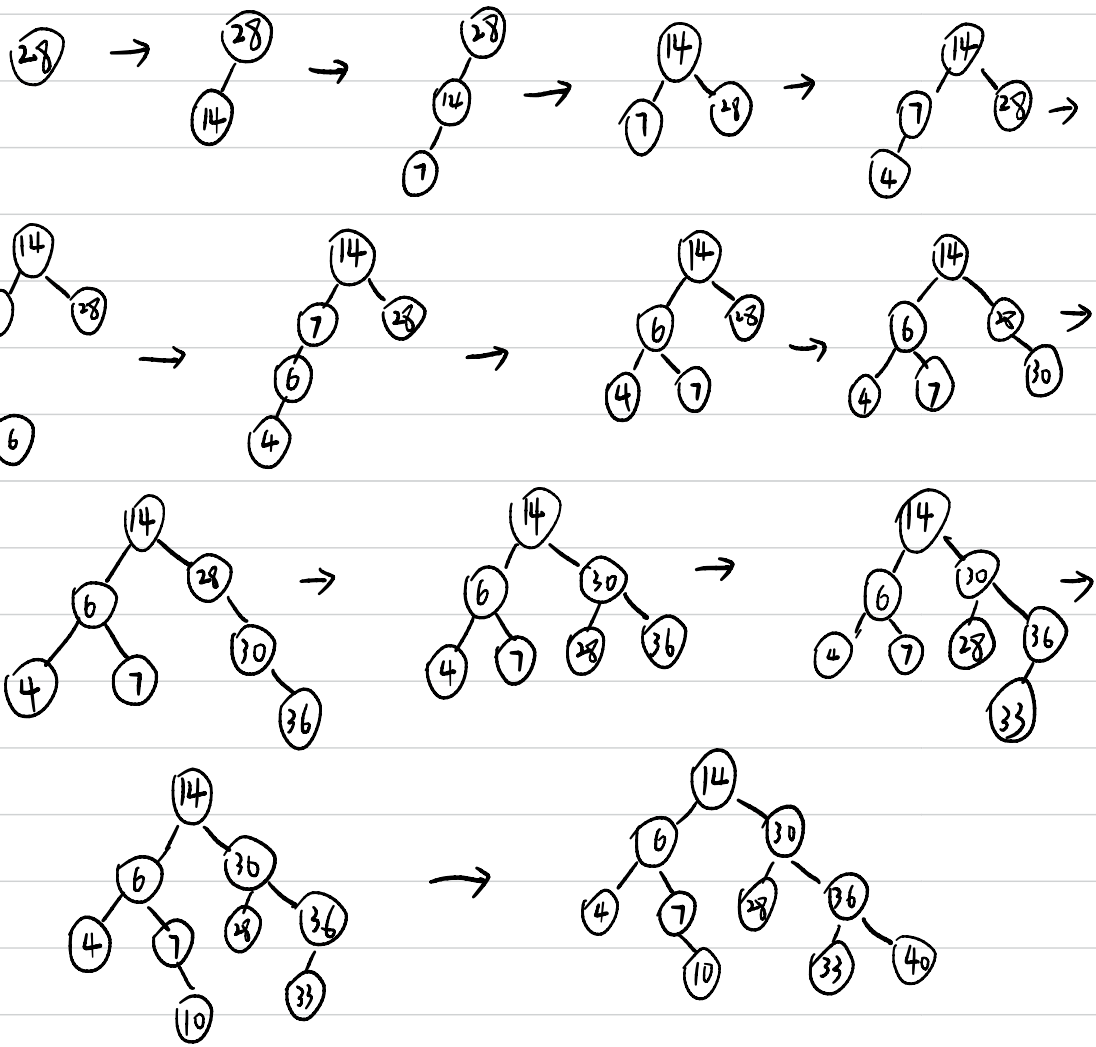


Total height $T_h = 13$

Total depth $T_D = 22$

$T_h + T_D = 35$

(c)



Total height = 9 = T_H

Total depth = 19 = T_D

$T_H + T_D = 28$