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Integration of Discrete-Event Dynamics and Machining Dynamics for Machine Tool: Modeling, Analysis and Algorithms

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Outline

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2. Problem Description
3. A Machine Learning-based Cost Function
4. Mathematical Models for Integrated Optimization Problems
5. Computational Experiments
6. Conclusion and Future Work

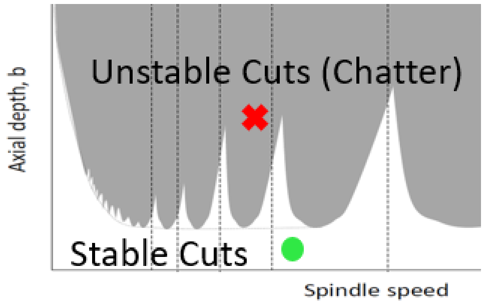
Introduction

Background

Daily operations for machine tool in the context of machine shop

Machining Dynamics:
A single part

Stability Lobe Diagram (SLD)



Tool Wear and Tool Life

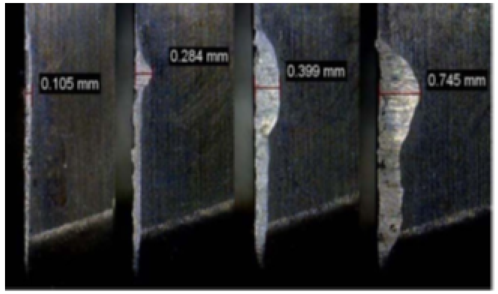
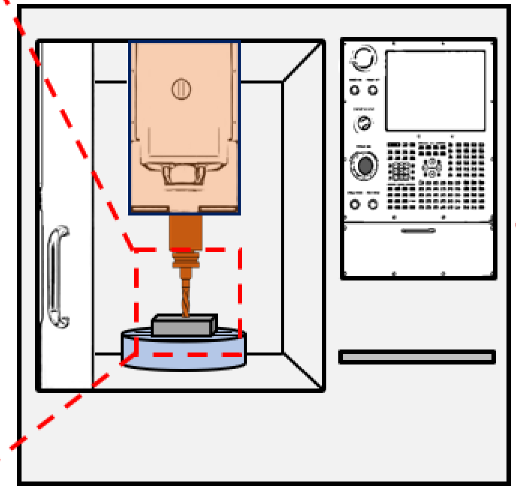


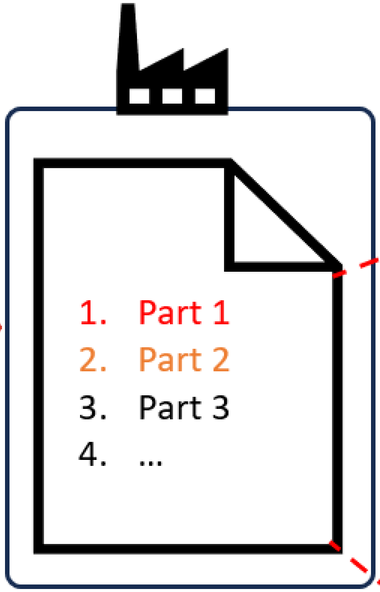
Image source: Karandikar et al., 2021

Disconnected!



Machine Tool

Discreet Event Dynamics:
A series of parts of customer orders



Machine Shop

Customer Orders Constraints
- Order-part relationship
- Quantity
- Due date
- Priority/Rush order
- …

Production Constraints
- Cost
- Makespan/shift
- Capacity
- …

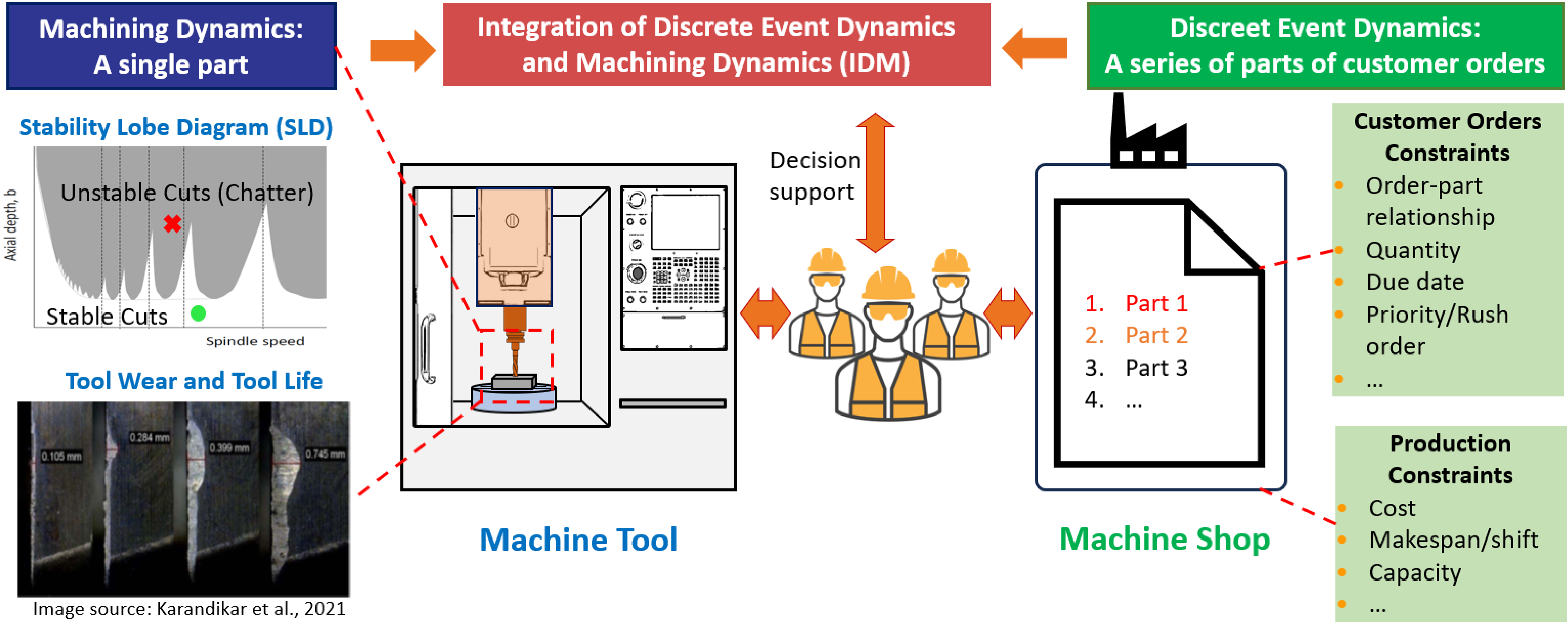
Research Gap

- **Machining Dynamics**
  - Physical modeling of machining:
    - C. Tyler and T. Schmitz, 2013; C. Tyler and T. Schmitz, 2014
  - Stable/unstable (chatter) parameters
    - Y. Altintas et al., 2016; J. Karandikar et al., 2020
  - Tool wear and tool life
    - J. Karandikar et al., 2015; J. Karandikar et al., 2021
  - **Gap**: The state-of-the-art machining dynamics studies focus on only one part. No consideration of multiple parts from multiple orders under practical machine shop context for saving cost

- **Discrete event dynamics**
  - Production planning and scheduling under various order/machine environment
    - Order scheduling on parallel machines
      - Z. Shi et al., 2015; Z. Shi et al., 2021
    - Job scheduling on batch machines
      - Z. Shi et al., 2018; M. Qin et al., 2019
  - Buffer allocation for machines
    - H. Ma et al., 2020
  - **Gap**: The discrete-event dynamics research for manufacturing has not considered the physical constraints and practical operating of machine tools, including the stability and tool life.

Our Idea: IDM

We address the integration of discrete-event dynamics and machining dynamics (IDM) to achieve cost savings for machining



Problem Description

Integration of Discrete Event Dynamics and Machining Dynamics (IDM) for Machine Tool

Problem 0:

The problem of integration of discrete-event dynamics and machining dynamics for machine tool, denoted by **IDM.**

- **Objective:** minimize the total production cost
- **Constraints:**
  - **Physical constraints of machining dynamics:** stability and tool life
  - **Production constrains of discrete event dynamics:** order-part relationship, quantity, makespan, and due date.

Single part

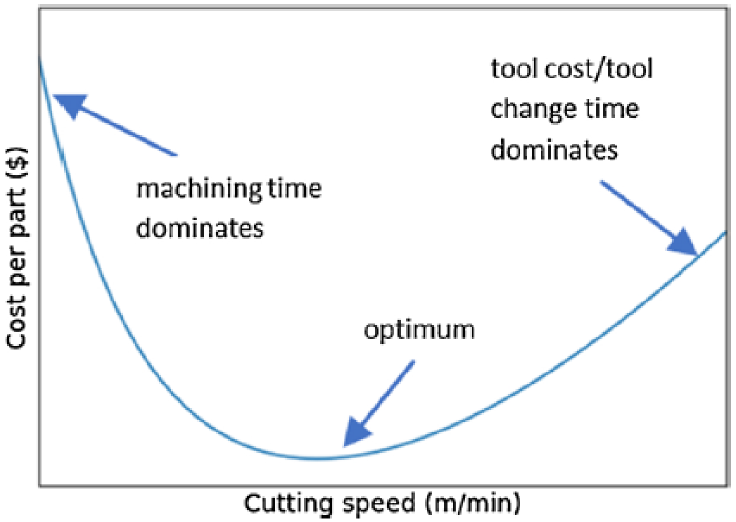


Image source: Karandikar et al., 2021

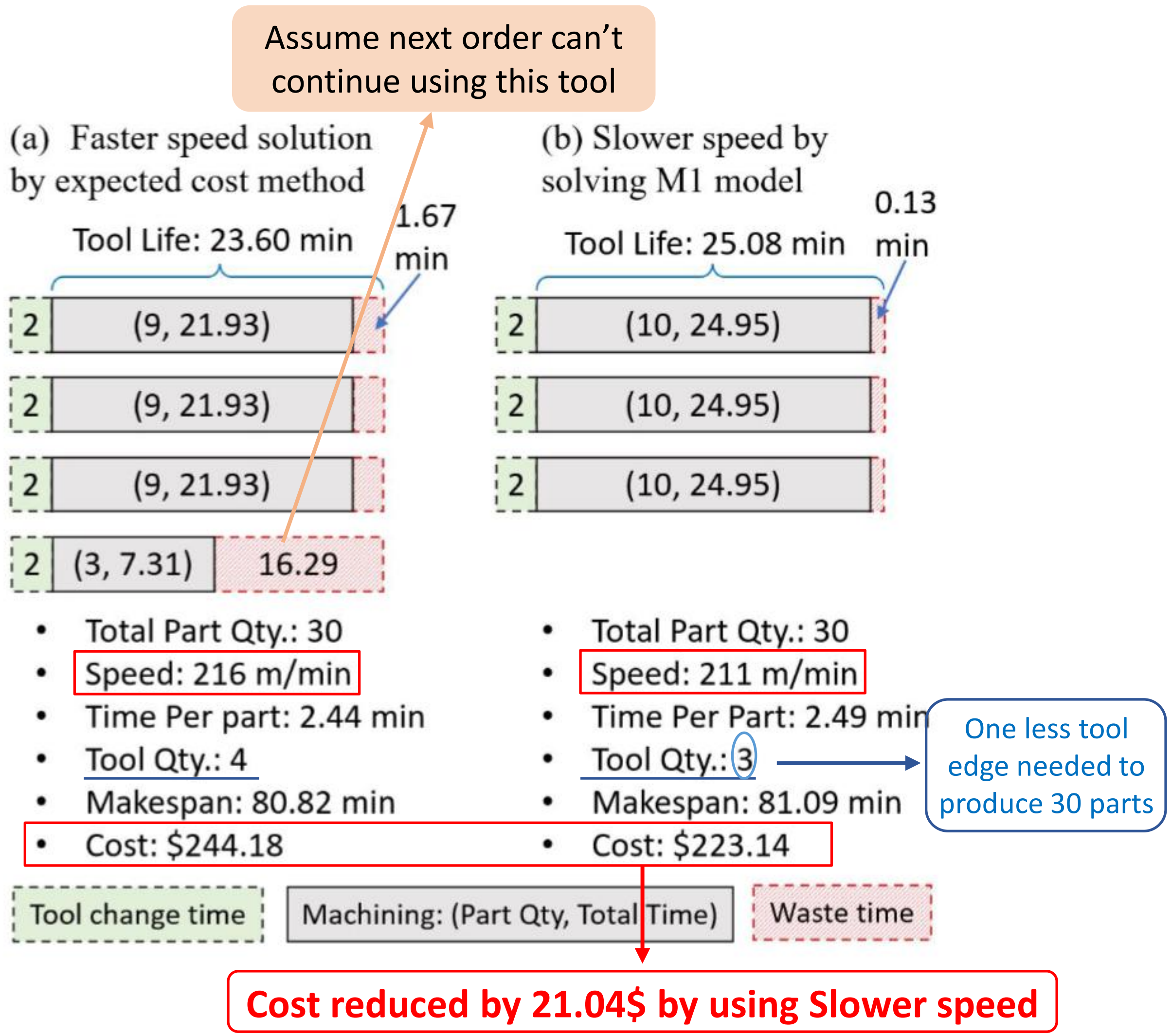
Our basic idea:

We choose different cutting speeds for different orders/parts

Multiple parts from multiple orders ?

Illustrative Example

- Illustrative Example IDM: choosing slower speed leads to smaller cost to process multiple parts of an order.
  - (a): The faster speed is determined by the expected cost method in Jaydeep's paper [1].
  - (b): The slower speed is calculated by our developed model with only cost consideration.



1. Karandikar, J., Schmitz, T. and Smith, S., 2021. Physics-guided logistic classification for tool life modeling and process parameter optimization in machining. *Journal of Manufacturing Systems*, *59*, pp.522-534.

Problem Settings

- **Discrete-Event Dynamics: Order and Part Setting**
  - Define a set of n orders as $\mathcal{J} = \{1, 2, \dots, n\}$
  - Each of order consists of a specific quantity of parts l_j, $j \in \mathcal{J}$.
  - Assume all parts of an order are the same type, but parts for different orders can be different.
  - In the order j, the volume to be removed for one part is defined as v_j.

- **Machining Dynamics: Spindle Speed and Tool Life Setting**
  - The stable spindle speed Ω is selected from SLD obtained by conducting impact test
  - The machining time in the machining process can be calculated by:

$$t = \frac{v}{MRR} = \frac{v}{abf_tN_t\Omega}$$

where MRR is the material removal rate, a and b are the radial and axial depth of cut, f_t is the feed per tooth, N_t is the number of tooth.

  - Tool life prediction is calculated based on the Taylor's tool life equation

$$T = \left(\frac{C}{\pi D\Omega}\right)^{\frac{1}{N}}$$

where V is the cutting speed, T is the tool life, and N and C are constants.

Problem Settings: Production and Cost Setting

- Only single machine tool with sufficient tool edges of same type for processing all n orders.
- Tool edge will be changed if the rest of tool life is not enough to produce one part.
- No preemption and interruption.
- Same cutting speed for one order and different speeds are allowed for different orders.
- Overall cost for producing multiple parts in multiple orders:

$$C_0 = \sum_{j \in \mathcal{J}} [\underbrace{r_m p_j}_{\text{Machine tool operating cost for processing order } j} + \underbrace{(r_m t_{ch} + C_{te}) q_j}_{\text{Cost induced by changing tool edges for processing order } j}]$$

where we denote $r_m, p_j, t_{ch}, C_{te}, q_j$ as the machine tool operating cost in \$ per unit time, total machining time for order j, time require for changing a tool edge, cost per tool edge, and number of tool edge we needed for order j.

A Machine Learning-based Cost Function

Machine Learning Model for Tool Life Prediction

Physics-guided Machine Learning Model for Tool Life Prediction [1]

- Physics: Taylor tool life equation:

$$VT^N = C$$

where V is the cutting speed and $V = \pi D \Omega$, D is the diameter of the tool, and T is the tool life. N and C are constants that depend on the tool-workpiece combination

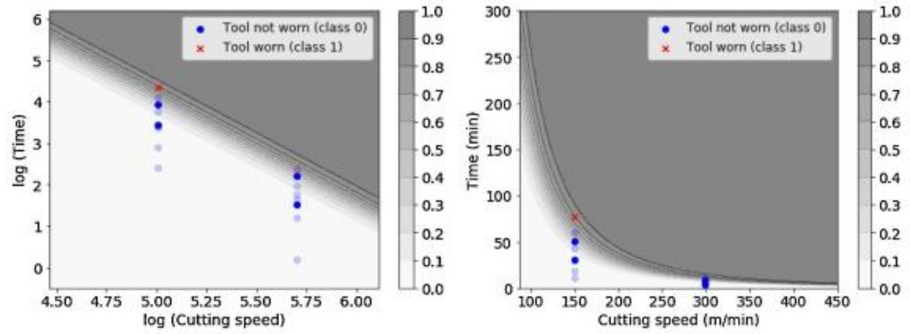
- Logistic regression model: logistic decision boundary for tool life prediction

$$Log(T) = -\frac{\theta_1}{\theta_2}\log(V) - \frac{\theta_0}{\theta_2} \quad \Rightarrow$$

$$T = \left(\frac{C}{V}\right)^{\frac{1}{N}} \quad \Rightarrow \quad \boxed{T = \left(\frac{C}{\pi D \Omega}\right)^{\frac{1}{N}}}$$

where N= $\frac{\theta_2}{\theta_1}$ and $C = e^{-n\frac{\theta_0}{\theta_2}}$,

$N = 0.388$ and $C = 735.2$ m/min



Probability of tool worn obtained by trained logistic model in machining dynamics research. Image source: [1]

Calculation of Learning-based Cost Function

- Discrete variables to indicate speed – order assignment relationship

$$x_{ij} = \begin{cases} 1, & \text{if speed } i \text{ is chosen for processing order } j \\ 0, & \text{otherwise} \end{cases}$$

The **cost function** for processing all orders is reorganized as :

Two parts of cost:

1. Machine Tool operating cost for order j at speed i
2. Total cost for changing tool edges for order j at speed i

This term based on cost function for parts of orders shows **machining dynamics**.

$$f_{cost}(x) = \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}} \overbrace{(\underbrace{r_m h_{ij}}_{1} + \underbrace{C_{te} q_{ij}}_{2})}\, x_{ij}$$

This discrete variable can choose different speed for different order indicates **discrete-event dynamics.**

This cost function integrates both machining dynamics, the term $r_m h_{ij} + C_{te} q_{ij}$, and the discrete-event dynamics, the term x_{ij}.

Calculation of Learning-based Cost Function

Data Matrices for modeling preparation:

- $\mathbf{\Omega}$: includes all candidate speeds for different order j. for a given Ω_{ij}, it refers to the specific spindle speed for processing order j with spindle speed i, $i \in \mathcal{I}$, and $\mathcal{I} = \{1, 2, \dots, m\}$ is the index set of available stable spindle speeds.
- $\mathbf{T}$ and $\mathbf{P}$: are matrices of tool life and machining time. The total machining time p_{ij} for processing order j with Ω_{ij} is $p_{ij} = \frac{v_j l_j}{a b f_t N_t \Omega_{ij}}$, where $v_j l_j$ is the total removed volume of order j, the term on denominator indicates the material removal rate..
- $\mathbf{Q}$: is a matrix of number of tool edge needed for each order with different spindle speeds. It can be defined as $q_{ij} = \left\lceil \frac{l_j}{\lfloor T_{ij}/t_{ij} \rfloor} \right\rceil$, where t_{ij} is the machining time for a single part of order j with Ω_{ij} and $t_{ij} = \frac{v_j}{a b f_t N_t \Omega_{ij}}$.
- $\mathbf{H}$: is the total processing time occupied by order j with speed i. Within $\mathbf{H}$, h_{ij} is equal to the total machining time plus the time of changing tool edges in order j , $h_{ij} = p_{ij} + t_{ch} q_{ij}$.

Mathematical Models for Integrated Optimization Problems

P0 - Model with Only Cost Consideration

- **Integer Programming (IP) model** is proposed to find the optimal spindle speed which minimize the cost function:

P Problem

$$\min f_{cost}(x) \qquad (M1)$$

$$\text{s.t.} \sum_{i \in \mathcal{I}} x_{ij} = 1, \qquad \forall j \in \mathcal{J}$$

$$x_{ij} \in \{0, 1\}, \qquad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}.$$

- **linear programing (LP) model**: Since the constraint matrix of this formulation is **totally unimodular** here, we can solve this **IP** model as a **LP** model.

$$\min f_{cost}(x) \qquad (M1 - LP)$$

$$\text{s.t.} \sum_{i \in \mathcal{I}} x_{ij} = 1, \qquad \forall j \in \mathcal{J}$$

$$x_{ij} \in [0, 1], \qquad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}.$$

If the constraints matrix of **M1** is totally unimodular, the optimal solutions of its **LP-relaxation** are **integral**.

P0 - Model with Only Cost Consideration

- **Decomposition-based Greedy Algorithm ($\boldsymbol{DG}$):** The objective function and constraints of formulation $M1$ can be decomposed, since the assignment of spindle speed for each order is independent. Thus, for each order j, we can select the optimal speed from m candidates to minimize the cost $r_m h_{ij} + C_{te} q_{ij}$ in objective function.

Algorithm 1: The DG algorithm

Input : Data matrices $\mathbf{\Omega}$; order set $\mathcal{J}$ and associated l_j and v_j; the cost parameters r_m, t_{ch} and C_{te}.

Initialize $f_{cost}(x) = 0$; $x_{ij} = 0, \forall i \in \mathcal{I}, j \in \mathcal{J}$;
Calculate matrices $\mathbf{T}$, $\mathbf{Q}$, $\mathbf{P}$ and $\mathbf{H}$ with Eqs. (6) - (12);
for $j \in \mathcal{J}$ **do** `/* Decompose as n subproblems */`
    for $i \in \mathcal{I}$ **do** `/* Find best speed for j */`
        Compute $Obj_{ij} = r_m h_{ij} + C_{te} q_{ij}$;
    end
    Find the best speed i^* of job j by $i^* := \arg\min_{i \in \mathcal{I}} Obj_{ij}$;
    Update $x_{i^* j} = 1$;
    $f_{cost}(x) \leftarrow f_{cost}(x) + Obj_{i^* j}$
end
Output Optimal cost $f_{cost}(x)$, speed $x_{ij}, \forall i \in \mathcal{I}, j \in \mathcal{J}$

P1 - Model with Cost and Makespan Consideration

Makespan: length of time elapses from start of work to the end.

- The constraints of makespan refer to the hard constraint that all the orders must be completed within a given time period.

- Parameter c: a given constant of upper bound for makespan

$$
\begin{aligned}
&\min f_{cost}(x) && && (M2) \\
&\text{s.t.} \sum_{i \in \mathcal{I}} x_{ij} = 1\,, && \forall j \in \mathcal{J} \\
&\boxed{\sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}} h_{ij} x_{ij} \le c,} \\
&x_{ij} \in \{0, 1\}, && \forall i \in \mathcal{I}, \forall j \in \mathcal{J}.
\end{aligned}
$$

Complexity of P1 problem is still **OPEN!**

P2 - Model with Cost and Due Date Consideration

Due date: d_j the date that the order must be done.

- The sequences of order processing need to be considered to determine the cost.
- Three categories of model based on the choices of decision variables are proposed to determine the order sequence and further determine the spindle speed and minimize the cost function.

1. Modeling with Completion Time Variables $(M3-1)$

$$
\begin{aligned}
&\min f_{cost}(x) \\
&\text{s.t.} \sum_{i \in \mathcal{I}} x_{ij} = 1, && \forall j \in \mathcal{J} \\
&C_j \geq \sum_{i \in \mathcal{I}} h_{ij} x_{ij}, && \forall j \in \mathcal{J} \\
&C_j + \sum_{i \in \mathcal{I}} h_{ij} x_{ij} \leq C_k + M(1 - y_{jk}), && \forall j < k \in \mathcal{J} \\
&C_k + \sum_{i \in \mathcal{I}} h_{ij} x_{ij} \leq C_j + M y_{jk}, && \forall j < k \in \mathcal{J} \\
&C_j \in [0, d_j] && \forall j \in \mathcal{J} \\
&x_{ij}, y_{jk} \in \{0, 1\}, && \forall i \in \mathcal{I}, \forall j \in \mathcal{J}.
\end{aligned}
$$

Completion time of order j

Constraints to satisfy due date requirement with **completion time variables**

$y_{jk} = 1$, if order j is processed before order k. Otherwise, $y_{jk} = 0$

P2 - Model with Cost and Due Date Consideration

2. Modeling with Linear Ordering Variables (M3-2)

$$\min f_{cost}(x) \qquad (M3-2)$$

$$\text{s.t.} \sum_{i \in \mathcal{I}} x_{ij} = 1, \qquad \forall j \in \mathcal{J},$$

$$y_{jk} + y_{kj} = 1, \qquad \forall j,k \in \mathcal{J}, j < k,$$

$$y_{jk} + y_{kl} + y_{lj} \leq 2, \qquad \forall j,k,l \in \mathcal{J}, j \neq k \neq l,$$

$$\sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}} h_{ij} x_{ij} y_{jk} + \sum_{i \in \mathcal{I}} h_{ik} x_{ik} \leq d_k \qquad \forall k \in \mathcal{J},$$

$$x_{ij}, y_{jk} \in \{0,1\}, \qquad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}.$$

Quadratic constraints!

$y_{jk} = 1$, if order j is processed before order k. Otherwise, $y_{jk} = 0$

Constraints to satisfy due date requirement with **linear ordering variables**

Linearization Technique

Reformulation with Linearization Technique

- **Proved to keep all optimal solutions**

$$\min f_{cost}(x) \qquad (\boldsymbol{M3-2-L})$$

$$\text{s.t.} \sum_{i \in \mathcal{I}} x_{ij} = 1, \qquad \forall j \in \mathcal{J},$$

$$y_{jk} + y_{kj} = 1, \qquad \forall j,k \in \mathcal{J}, j < k,$$

$$y_{jk} + y_{kl} + y_{lj} \leq 2, \qquad \forall j,k,l \in \mathcal{J}, j \neq k \neq l,$$

$$\sum_{j \in \mathcal{J}} \alpha_{jk} + \sum_{i \in \mathcal{I}} h_{ik} x_{ik} \leq d_k, \qquad \forall k \in \mathcal{J},$$

$$\sum_{i \in \mathcal{I}} h_{ij} x_{ij} - M(1 - y_{jk}) \leq \alpha_{jk} \leq M y_{jk}, \qquad \forall j,k \in \mathcal{J},$$

$$\alpha_{jk} \geq 0, \qquad \forall j,k \in \mathcal{J},$$

$$x_{ij}, y_{jk} \in \{0,1\}, \qquad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}.$$

where $\alpha_{jk} = (\sum_{i \in \mathcal{I}} h_{ij} x_{ij}) y_{jk}$.

P2 - Model with Cost and Due Date Consideration

3. Modeling with Positional Variables (M3-3)

$z_{jk} = 1$, if order j is assigned on position k. Otherwise, $z_{jk} = 0$

Completion time of order at position k

$$\min f_{cost}(x) \qquad (M3-3)$$

$$\text{s.t.} \sum_{i \in \mathcal{I}} x_{ij} = 1, \qquad \forall j \in \mathcal{J},$$

$$\text{s.t.} \sum_{k \in \mathcal{J}} z_{jk} = 1, \qquad \forall j \in \mathcal{J},$$

$$\text{s.t.} \sum_{j \in \mathcal{J}} z_{jk} = 1, \qquad \forall j \in \mathcal{J},$$

$$0 \le C_k \le \sum_{j \in \mathcal{J}} d_j z_{jk}, \qquad \forall k \in \mathcal{J},$$

$$C_1 \ge \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}} h_{ij} x_{ij}\, z_{j1},$$

$$C_k \ge C_{k-1} + \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}} h_{ij} x_{ij}\, z_{jk}, \qquad \forall k \in \mathcal{J}/\{1\}$$

$$x_{ij}, z_{jk} \in \{0,1\}, \qquad \forall i \in \mathcal{I}, j, k \in \mathcal{J}.$$

Constraints to satisfy due date requirement with **positional variables**

Quadratic constraints!

Linearization Technique

Reformulation with Linearization Technique

- **Proved to keep all optimal solutions**

$$\min f_{cost}(x) \qquad (\boldsymbol{M3-3-L})$$

$$\text{s.t.} \sum_{i \in \mathcal{I}} x_{ij} = 1, \qquad \forall j \in \mathcal{J},$$

$$\text{s.t.} \sum_{k \in \mathcal{J}} z_{jk} = 1, \qquad \forall j \in \mathcal{J},$$

$$\text{s.t.} \sum_{j \in \mathcal{J}} z_{jk} = 1, \qquad \forall j \in \mathcal{J},$$

$$0 \le C_k \le \sum_{j \in \mathcal{J}} d_j z_{jk}, \qquad \forall k \in \mathcal{J},$$

$$C_1 \ge \sum_{j \in \mathcal{J}} \beta_{j1},$$

$$C_k \ge C_{k-1} + \sum_{j \in \mathcal{J}} \beta_{jk}, \qquad \forall k \in \frac{\mathcal{J}}{\{1\}}$$

$$\sum_{i \in \mathcal{I}} h_{ij} x_{ij} - M(1 - z_{jk}) \le \beta_{jk} \le M z_{jk}, \qquad \forall j, k \in \mathcal{J},$$

$$\beta_{jk} \ge 0, \qquad \forall j, k \in \mathcal{J},$$

$$x_{ij}, z_{jk} \in \{0,1\}, \qquad \forall i \in \mathcal{I}, j, k \in \mathcal{J}.$$

where $\beta_{jk} = (\sum_{i \in \mathcal{I}} h_{ij} x_{ij}) z_{jk}$.

Computational Experiments

Data Generation

Computational Environment

- All methods are coded in Python.
- The optimization solver: CPLEX 20.1.0.
- Time limit for CPLEX: 600 seconds.
- The experiments are deployed on a mobile workstation with Intel® Xeon® W-10885M CPU @ 2.40GHz, 128 GB memory, 64-bit Windows 10 Pro operating system for workstations.

Machine tool:

- Single-insert Endmill Tool.
- Diameter: D = 19.05 mm.
- Number of tooth: N_t = 1.
- Radial depth of cut: a= 4.7 mm.
- Axial depth of cut: b = 3 mm.
- Feed per tooth: f_t = 0.06 mm.

Constant in Taylor tool life equation:

- N = 0.388.
- C = 735.2.

Order and part:

- The number of order n is from a set $\{2, 4, 6, 8, 10,12,14,16,18,20\}$ with a single part type for each order.
- l_j is sampled from uniform distribution $U[10,200]$ randomly.
- v_j is sampled from uniform distribution $U[5000,15000]$ randomly.
- Makespan limitation c is the integer number by rounding the average number of the makespan of solution by solving problem IDM-C and the minimum makespan when adopting the minimum h_{ij} for each order.

Due date:

- Due date d_j: randomly sampling from uniform distribution. $U[0.4\bar{C}_{max}, 0.5\bar{C}_{max}]$, then choose the feasible due date by using earliest due date rule.

Production and cost:

- Candidate cutting speed Ω: is evenly spaced in $[220, 420]$m/min with the interval 2 m/min.
- Operation cost: $r_m = 2$ \$/min.
- Time to change a tool edge: $t_{ch} = 2$ min.
- Cost per tool edge: $C_{te} = 12.5$ \$.

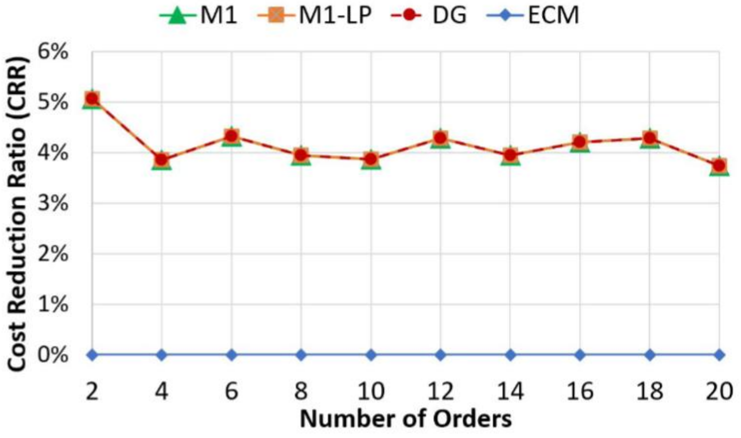
Results for Integrated Optimization P0 - Cost

- Methods we compare: ECM (Expected Cost Method), M1, M1-LP, DG

Table 1. Numerical results of solving problem IDM-C by the expected cost method (ECM) [6], solving models M1 and M1-LP, and the exact algorithm DG.

Order #	ECM		M1			M1-LP			DG		
	Obj: Cost	Time (s)	Obj: Cost	CRR (%)	Time (s)	Obj: Cost	CRR (%)	Time (s)	Obj: Cost	CRR (%)	Time (s)
2	2082.55	2.81	**1977.23**	5.06	0.01	**1977.23**	5.06	0.01	**1977.23**	5.06	**0.000**
4	3301.91	5.58	**3174.74**	3.85	0.01	**3174.74**	3.85	0.02	**3174.74**	3.85	**0.000**
6	5418.85	8.28	**5184.44**	4.33	0.01	**5184.44**	4.33	0.03	**5184.44**	4.33	**0.000**
8	6670.49	11.01	**6406.96**	3.95	0.01	**6406.96**	3.95	0.03	**6406.96**	3.95	**0.003**
10	9688.88	13.87	**9314.33**	3.87	0.01	**9314.33**	3.87	0.03	**9314.33**	3.87	**0.003**
12	11872.06	16.66	**11363.36**	4.28	0.01	**11363.36**	4.28	0.05	**11363.36**	4.28	**0.003**
14	13597.67	19.27	**13061.32**	3.94	0.02	**13061.32**	3.94	0.05	**13061.32**	3.94	**0.003**
16	15417.88	21.77	**14768.81**	4.21	0.02	**14768.81**	4.21	0.06	**14768.81**	4.21	**0.005**
18	16849.63	24.57	**16127.64**	4.28	0.02	**16127.64**	4.28	0.06	**16127.64**	4.28	**0.003**
20	20654.23	27.29	**19883.29**	3.73	0.02	**19883.29**	3.73	0.08	**19883.29**	3.73	**0.003**

DG is the fastest method within 0.005



The comparison of cost reduction ratio (CRR) for problem IDM-C.

Conclusion:

- M1, M1-LP, and DG all find the optimal solutions.
- DG algorithm performs the fastest among three methods, within 0.005s.
- The overall cost is reduced by 3% - 5% on average by selecting different spindle speed for different orders.

Results for Integrated Optimization P1 – Cost & Makespan

- Methods we compare: ECM and M2

M2 can achieve both smaller cost & shorter makespan

Table 2. Numerical results of solving problem IDM-CM by expected cost method (ECM) [6] and solving model M2.

Order #	Makespan Limit: c	ECM			M2					
		Obj: Cost	Time (s)	Makespan	Obj: Cost	Num	Gap (%)	LP Relaxation	Time (s)	Makespan
2	633	2082.55	2.81	763.15	**2052.42**	**10**	0.00	2034.58	0.00	624.96
4	1027	3301.91	5.58	1209.71	**3245.44**	**10**	0.00	3233.46	0.05	1021.47
6	1678	5418.85	8.28	2000.68	**5281.53**	**10**	0.00	5271.44	0.08	1672.02
8	2070	6670.49	11.01	2449.62	**6532.52**	**10**	0.00	6526.56	0.08	2067.51
10	3007	9688.88	13.87	3580.07	**9512.87**	**10**	0.00	9502.75	0.11	3003.93
12	3669	11872.06	16.66	4362.28	**11571.73**	**10**	0.00	11566.20	0.08	3667.11
14	4198	13597.67	19.27	5013.83	**13364.26**	**10**	0.00	13354.87	0.05	4195.26
16	4775	15417.88	21.77	5687.06	**15047.90**	**10**	0.00	15042.45	0.08	4772.70
18	5202	16849.63	24.57	6221.69	**16426.20**	**10**	0.00	16421.71	0.05	5199.98
20	6413	20654.23	27.29	7656.49	**20286.36**	**10**	0.00	20281.15	0.08	6411.31

Makespan limit c: the average makespan limit within which all parts must be finished

Conclusion:

- Practical to achieve solutions that lead to both smaller cost and shorter makespan
- The LP relaxation results show M2 is tight, which indicate the good computational efficiency of M2.

Results for Integrated Optimization P2 – Cost & Due Date

- Methods we compare: M3-1, M3-2-L, M-3-3-L

Solution quality, the smaller the better

Time efficiency, the smaller the better

Table 3. Numerical results of problem IDM-CD by solving models M3-1, M3-2-L and M3-3-L.

Order #	M3-1				M3-2-L				M3-3-L			
	Obj: Cost	Num	Gap (%)	Time (s)	Obj: Cost	Num	Gap (%)	Time (s)	Obj: Cost	Num	Gap (%)	Time (s)
2	**1920.68**	**10**	0.00	0.02	**1920.68**	**10**	0.00	0.02	**1920.68**	**10**	0.00	0.03
4	**4463.55**	**10**	0.00	0.08	**4463.55**	**10**	0.00	0.05	**4463.55**	**10**	0.00	0.07
6	**5412.14**	**10**	0.00	0.25	**5412.14**	**10**	0.00	0.07	**5412.14**	**10**	0.00	0.14
8	**7293.43**	**10**	0.00	0.27	**7293.43**	**10**	0.00	0.10	**7293.43**	**10**	0.00	1.83
10	**11129.40**	**10**	0.00	1.02	**11129.40**	**10**	0.00	0.39	**11129.40**	**10**	0.01	25.43
12	**12700.74**	**10**	0.01	14.17	**12700.74**	**10**	0.00	2.66	12700.74	5	3.14	508.27
14	12615.67	8	0.44	163.11	**12615.67**	**10**	0.01	10.80	12615.67	1	6.05	601.27
16	13232.52	4	2.52	485.71	**13232.52**	**10**	0.01	45.09	13232.52	1	6.06	602.53
18	18001.38	0	6.47	600.30	**18001.38**	**10**	0.01	184.39	18001.38	0	8.06	602.33
20	21394.91	0	8.76	600.60	**21394.91**	8	0.76	380.35	21394.91	0	8.76	602.19

M3-2-L is the best one that give the best solution within the smallest time

Conclusion:

- M3-2-L can solve the optimization problem faster than the other two methods.
- M3-2-L has the best solution quality among three methods because the maximum gap is only 0.76%.
- Among all 100 instances, M3-2-L can solve 98 to optimality.

Conclusion and Future Work

Conclusion and Future Work

- **Conclusions**
  - A new optimization problem of integrated discrete-event dynamics and machining dynamics for minimizing production cost of machine tool.
  - A new learning-based cost function for the new integrated optimization problem.
  - Multiple effective mathematical optimization models and algorithms considering practical production scenarios including cost, makespan, and due date.
  - Numerical results show the potential of proposed models for cost saving in practice.

- **Future work**
  - Consider other machining parameters as decision variables, such as axial depth of cut and feed rate.
  - Investigate the stochastic tool life prediction.
  - Consider the computational complexities of the problems for makespan and due date constraints.
  - Consider more practical production constraints, like different candidate speeds for orders, various part types of an order and multiple machine tools in machine shop