# CSC 406 Lecture Notes (9/22/22)

## In 2D:

2 non-colinear ($\neq$ direction) vectors $\vec{u}, \vec{v}$ define a basis of the plane

$B = (\vec{u}, \vec{v})$

Any vector $\vec{w}$ can be expressed as $\vec{w} = \alpha \vec{u} + \beta \vec{v}$ $(\alpha, \beta)$ are unique

We write $(\vec{w})_B = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ coordinates of $\vec{w}$ in basis $B$

## In 3D:

i.e. there is no $\alpha, \beta$, such that $\vec{w} = \alpha\vec{u} + \beta\vec{v}$

3 non-coplanar vectors $\vec{u}, \vec{v}, \vec{w}$ define a basis of 3D space

$B = (\vec{u}, \vec{v}, \vec{w})$

Any vector $\vec{t}$ can be expressed as $\vec{t} = \alpha\vec{u} + \beta\vec{v} + \gamma\vec{w}$

$\alpha, \beta, \gamma$ are unique

$$(\vec{t})_B = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

## Change of Coordinates for Vectors:

2 Bases: $B = (\vec{u}, \vec{v})$ $B' = (\vec{u}', \vec{v}')$

We know that $(\vec{u}')_B = \begin{pmatrix} a \\ b \end{pmatrix}$ $(\vec{v}')_B = \begin{pmatrix} c \\ d \end{pmatrix}$

Consider $\vec{w}$ such that $(\vec{w})_{B'} = \begin{pmatrix} x' \\ y' \end{pmatrix}$

$(\vec{w})_B = \begin{pmatrix} ? \\ ? \end{pmatrix}$

$$(\vec{w})_{B'} = \begin{pmatrix} x' \\ y' \end{pmatrix} \Longleftrightarrow \vec{w} = x'\vec{u}' + y'\vec{v}'$$

$$= x' \cdot (a\vec{u} + b\vec{v}) + y' \cdot (c\vec{u} + d\vec{v})$$

$$= (x'a + y'c)\vec{u} + (x'b + y'd)\vec{v}$$

$$\boxed{(\vec{w})_B = \begin{pmatrix} ax' + cy' \\ bx' + dy' \end{pmatrix}}$$

$$(\vec{w})_B = \begin{pmatrix} ax' + cy' \\ bx' + dy' \end{pmatrix}$$

This can be represented by matrix multiplication:

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \end{pmatrix}$$

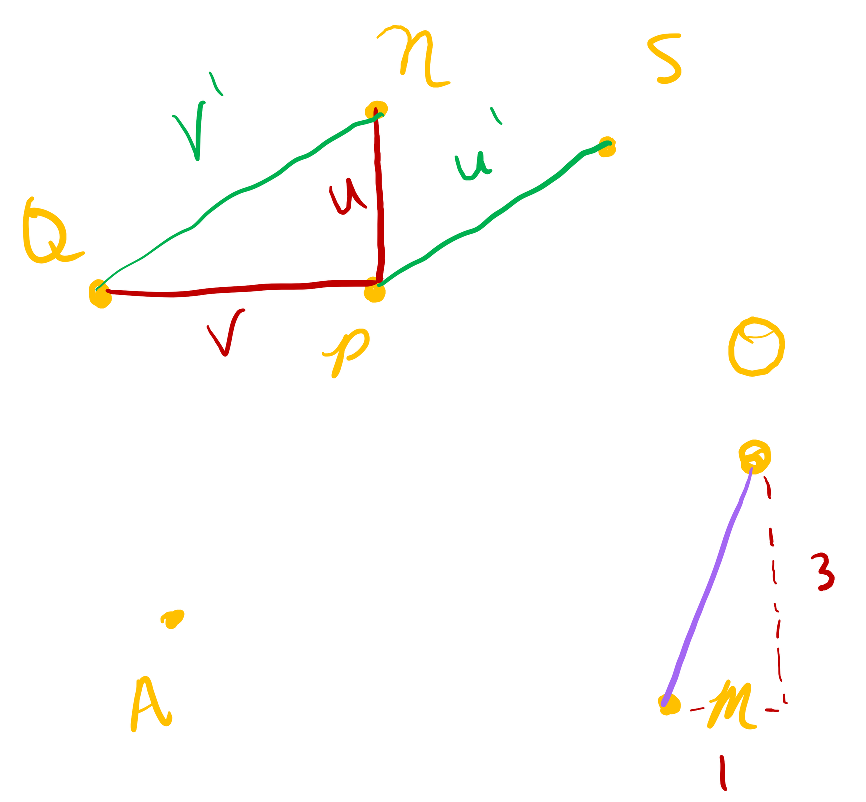
* $m \times n \cdot n \times p \rightarrow m \times p$ matrix

## Coordinates for points (reference frames)

$\mathcal{E}$ is a planar space of points. $E$ associated vector space

$B = (\vec{u}, \vec{v})$ a basis of $E$. A point $O$ of $\mathcal{E}$ together with $B$ define a reference frame for $\mathcal{E}$. $O$ is the origin of $\mathcal{R}$.

Coordinates of a point $M$ relative to $R$: $(M)_R = (\vec{OM})_B$




$\vec{u} = \vec{np}$  $\vec{v} = \vec{PQ}$  $B = (\vec{u}, \vec{v})$

$\vec{u}' = \vec{sp}$  $\vec{v}' = \vec{nQ}$  $B = (\vec{u}', \vec{v}')$

$R = (O, B)$  $R' = (A, B')$ , $R'' = (O, B')$

$$(M)_R = (\vec{OM})_B = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

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... Big Skip in Notes

## Rules for Transformations:

- Follow Transformations in order but transforms relative to new frame
  ↓
  multiply on right side

  transforms relative to old frame
  ↓
  multiply on left side

Ex. Blue = old reference

Red = new reference

List of Translations:

$T_1$

$R_2$

$T_3$

$R_4$

$R_5$

$T_6$

$R_7$

→ $R_4 \circ R_2 \circ T_1 \cdot \text{Orig.} \cdot T_3 \cdot R_5 \cdot T_6 \cdot R_7$

This method will work when you want to string a bunch of matrix multiplication transformations into one expression.