MATH 53 WORKSHEET 11/22

(1) An electric charge at the origin generates an electric field given by $\mathbf{E}(r, \theta, \phi) = \frac{c\mathbf{r}}{|\mathbf{r}|^3}$, where c is a constant and $\mathbf{r} = \langle x, y, z \rangle$. Show that if S is the surface of a sphere centered at the origin then $\int\int_S \mathbf{E} \cdot d\mathbf{S}$ does not depend on S the radius of the sphere.

(a) Solve this by parameterizing the sphere by using spherical coordinates.

(b) Sketch the vector field $\mathbf{E}$. Show that $\mathbf{E} \cdot \mathbf{n} = \frac{c}{|\mathbf{r}|^2}$ is a constant. Solve the integral using this fact.

(2) Compute $\int\int_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \langle\ -y, x, z^2\rangle$ and S is the part of the cone $z = \sqrt{x^2 + y^2}$ between $z = 0$ and $z = 1$, oriented downward.

(3) True or false (some review of older material)

(a) If $\int_C f\ ds = 0$, then C is a closed curve.

(b) If (x, y) is a local minimum of a function g, then g is differentiable at (x, y) and $\nabla g(x, y) = 0$.

(c) If $f(x, y) = g(x)h(y)$, then $\int\int_E f(x, y)dA = (\int\int_E g(x)dA)(\int\int_E h(y)dA)$ over a region E.