Name: ____

SID: ____

MATH 135: SET THEORY
MIDTERM # 1

There are **six** questions. The first question has **five** parts. Write your answers in the space provided. If you need more space, you may continue your answers on the blank pages at the end of the exam. **Please submit all of the pages, including the extra blank pages even if you have not written on them.**

1a. (5 pts) Express the Union Axiom using the signature of set theory, that is, with only the binary relation symbol $\in$ as a nonlogical symbol. [**N.B. :** We mean the Union Axiom in our official formulation, not the preliminary version concerning binary unions.]

Name or initials: ______________________________

1b. (5 pts) Express the Pair Set Axiom using only the signature of set theory.

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1c. (5 pts) Give a precise formal definition of the relation $y = \mathcal{P}x$, that is, of "y `is the power set of` x", using only the the binary relation symbol $\in$ as a nonlogical symbol. In particular, if your definition should *not* use the symbol $\subseteq$.

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1d. (5 pts) Give a precise formal definition of the relation $z = \langle x, y \rangle$ using only the binary relation symbol $\in$ as a nonlogical symbol. In particular, your definition should *not* use the relation $c = \{a, b\}$.

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1e. (5 pts) Give a precise formal definition of the condition "f `is a function`" using the binary relation symbol $\in$ and the ordered pair function symbol $\langle \cdot, \cdot \rangle$ as primitives.

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2. (15 pts) **Prove:** $(\forall S)(\forall R)\ (R \circ S)^{-1} = S^{-1} \circ R^{-1}$.

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3. (15 pts) **Prove or disprove:** If B is a transitive set, then $(\forall a)[a \in B \to \mathcal{P}a \in \mathcal{P}B]$.

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4. (15 pts) **Prove:** that if $X \subseteq \omega$ and $X \neq \varnothing$, then X has a least element. That is, there is some $a \in X$ such that for all $b \in X$ one has $a = b$ or $a \in b$.

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5. (15 pts) **Prove:** that there is no set of all functions. That is, show that there is no set $\mathbb{F}$ such that

$$(\forall t)[\ t \in \mathbb{F} \longleftrightarrow t \texttt{ is a function }]\ .$$

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6. (16 pts) **Prove:** $(\forall \ell \in \omega)(\forall m \in \omega)(\forall n \in \omega)[\ \ell \in m \rightarrow (\ell + n) \in (m + n)\ ]$

Name or initials: ______________________________

6.

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For which problem should this work be credited?

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