MATH 135: INTRODUCTION TO THE THEORY OF SETS
AUTUMN 2011
MIDTERM II

There are six questions. This exam will last eighty minutes. Write your answers in your own blue (or green) book. With the exception of clarifications provided by the instructor, you may not refer to any external source in the course of completing this examination.

1. (20 points) Define the following terms. When referring to another term introduced in this course, completely define that term as well unless it is included in the following list: $x \subseteq y$, $\varnothing$, subset, $\mathscr{P}x$, $\{x,y\}$, $\{x\}$, $\langle x,y\rangle$, $x \cup y$, $\bigcup x$, $x \cap y$, $\bigcap x$, $x \times y$, relation, function, $\mathrm{dom}(f)$, $\mathrm{ran}(f)$, one-to-one, onto, $f \circ g$, transitive set, transitive relation

(1) A set I is *inductive* if and only if ____________.
(2) A set n is a *natural number* if and only if ____________.
(3) A set x is a *finite* if and only if ____________.
(4) A set Y is *countable* if and only if ____________.

2. (10 points) State the following axioms in the formal language. You may write the axiom in mathematical English also to explain the formal sentence. As in question **1**, if you refer to a term introduced in this course, you must define that term unless it was listed above.

(1) Zorn's Lemma
(2) Axiom of Infinity

3. (15 points) Prove that if X is a finite set and $Y \subseteq X$ is a subset of X, then Y is finite.

4. (15 points) Show that if $A \subseteq \omega$ and $\bigcup A = A$, then $A = \varnothing$ or $A = \omega$.

5. (15 points) Let $E : \mathbb{Q} \to \mathbb{R}$ be the natural embedding given by $q \mapsto \{s \in \mathbb{Q} \ : \ s < q\} \in \mathbb{R}$. Give a complete proof (*ie* without referring to any lemmata proven in class or your textbook) that if $x < y$ are real numbers, then there is some $q \in \mathbb{Q}$ with $x < E(q) < y$.

6. (15 points) We say that a set A is *almost disjoint* if for any two distinct x and y from A the set $x \cap y$ is finite. Prove that there is a set $A \subseteq \mathscr{P}(\omega)$ which is almost disjoint and has $|A| = 2^{\aleph_0}$.

Date: 1 November 2011.