Math 135: Introduction to Set Theory
Autumn 2000
Midterm Exam II

15 November 2000
2:10pm - 3:00pm

1. (30 points) Complete the following definitions. If your definition involves a term introduced in this course after the first midterm, you must define that term as well.

a. The set $\mathbb{Z}$ of *integers* is defined to be _______.

b. If K is a set of cardinality κ and L is a set of cardinality λ, then the set X has cardinality κ^λ if and only if _______.

c. If $(P, <)$ is a partially ordered set and $\mathcal{C} \subseteq P$, then $\mathcal{C}$ is a *chain* if and only if _______.

2. (10 points)

- State the (transfinite) recursion theorem.
- State the principle of cardinal comparability.

3. (20 points) Suppose that $f : \omega \to \mathbb{R}$ is a function satisfying

1. f is increasing: $(\forall m < n \in \omega)\ f(n) < f(m)$ and
2. f is bounded: $(\exists B \in \mathbb{R})(\forall n \in \omega)\ f(n) < B$.

Show that f has a limit:

$$(\exists \ell \in \mathbb{R})(\forall \epsilon > 0)(\exists N \in \omega)(\forall n > N)\ 0 < \ell - f(n) < \epsilon$$

4. (20 points) For a set X define $\mathrm{Fin}(X) := \{A \in \mathcal{P}X : A \text{ is finite }\}$. Show that if X is infinite, then $X \approx \mathrm{Fin}(X)$.

5. (20 points) Let $(X, <)$ be a well-ordered set. Define a new ordering on X by $a <^* b \Leftrightarrow b < a$. Show that if $(X, <^*)$ is also well-ordered, then X is finite.