Game Theory Homework #4, 02/17/2021

All the problems are worth 4 points each and will be graded on a 0/1/2/3/4 scale. Due on Wednesday 02/24/2021 before 11:59 pm to be uploaded via Gradescope.

1. Two smart students form a study group in some math class where homework is handed in jointly by each group. In the last homework of the semester, each of the two students can choose to either work ('W') or party ('P'). If at least one of them solves the homework that week (chooses 'W'), then they will both receive 10 points. But solving the homework incurs a substantial effort, worth -7 points for a student doing it alone, and an effort worth -2 points for each student, if both students work together. Partying involves no effort, and if both students party, they both receive 0 points. Assume that the students do not communicate prior to deciding whether they will work or party. Write this situation as a matrix game and determine all Nash equilibria.

2. Consider the two player general sum game with payoff matrix

$$\begin{pmatrix} (1,1) & (2,0) \\ (2,0) & (-1,5) \end{pmatrix}.$$

Find safety strategies (for both players), and all mixed and pure Nash Equilibria, and the respective expected payoffs for both players in each case.

3. Two cheetahs and three antelopes: Two cheetahs each chase one of three antelopes. If they catch the same one, they have to share. The antelopes are Large, Small, and Tiny, and their values to the cheetahs are ℓ, s and t. Write the 3×3 matrix for this game. Assume that $t < s < \ell < 2s$. Find a condition for a symmetric fully mixed equilibrium to exist and find the latter as well as all the pure equilibria in that case.

4. Volunteering dilemma: There are n players in a game show. Each player is put in a separate room. If some of the players volunteer to help the others, then each volunteer will receive 1000 and each of the remaining players will receive 1500. If no player volunteers, then they all get zero. Show that for this game the set of symmetric (mixed) Nash equilibria contains exactly one element. Let p_n denote the probability that player 1 volunteers in this equilibrium. Find p_2 and show that

$$\lim_{n\to\infty} np_n = \log(3).$$