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MATH 135: SET THEORY
FINAL EXAM

There are **eight** questions. Question 1 is worth thirty points. The other questions are worth ten points each. Write your answers in the space provided. If you need more space, you may continue your answers on the blank pages at the end of the exam. **Please submit all of the pages, including the extra blank pages even if you have not written on them.**

For each part of Question 1, express your solution in the specified formal language.

1a. State precisely the Power Set Axiom in the language $\mathcal{L}(\in)$ having only the binary relation symbol $\in$.

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1b. Give a formal definition of the expression X `is a transitive set` in the language of set theory $\mathcal{L}(\in)$ having only the binary relation symbol $\in$.

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1c. Give a formal definition of the expression α `is an ordinal` (or as we wrote this in class $\alpha \in \mathbb{ON}$) in the language $\mathcal{L}(\in)$ having only the binary relation symbol $\in$.

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1d. Give a formal definition of the expression $Y = \mathrm{ran}(R)$ in the language of set theory $\mathcal{L}(\in)$ having only the binary relation symbol $\in$.

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1e. State precisely the Regularity Axiom Scheme in the language of set theory $\mathcal{L}(\in)$ having only the binary relation symbol $\in$.

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1f. State precisely the Empty Set Axiom in the language of set theory $\mathcal{L}(\in)$ having only the binary relation symbol $\in$.

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2. Let X be any set and $R \subseteq X \times X$ any relation on X. By the recursion theorem on ω there is a unique function $f : \omega \to \mathcal{P}(X \times X)$ satisfying $f(0) = I_X \cup R \cup R^{-1}$ and $f(n^+) = f(n) \circ f(n)$ for all $n \in \omega$. Let $E := \bigcup \operatorname{ran}(f)$. **Show** that for any equivalence relation E' on X with $R \subseteq E'$, we have $E \subseteq E'$.

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3. Show that for every set X there is some set K so that for every $x \in X$ one has $x \prec K$.

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4. Show that if X is a finite set and $\leq$ is a total ordering on X, then $\leq$ is a well-ordering.

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5. **Show** that there is an **infinite** set N and an **onto** function $f : \omega \to N$ for which $f(0) = \varnothing$ and for every $n \in \omega$ one has $f(n^+) = \{f(n)\}$.

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6. Let $(X, \leq)$ be a well-ordered set and $f : X \to X$ a function satisfying for all x and y from X, $x < y \to f(x) < f(y)$. **Show**: for all $x \in X$ one has $x \leq f(x)$.

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7. Recall that for a set X, the product is the set

$$\prod X := \{f \in \mathcal{P}(X \times \bigcup X) \ : \ (\forall x)[x \in X \to f(x) \in x\} \ .$$

Show that if X is a set of nonempty disjoint sets, then $\bigcup X \prec \prod X$. (Note: There are two parts to this. You need to establish that $\bigcup X \preceq \prod X$ and that $\bigcup X \not\approx \prod X$.)

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8. Show that for every set X there is an inductive set I with $X \subseteq I$.

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