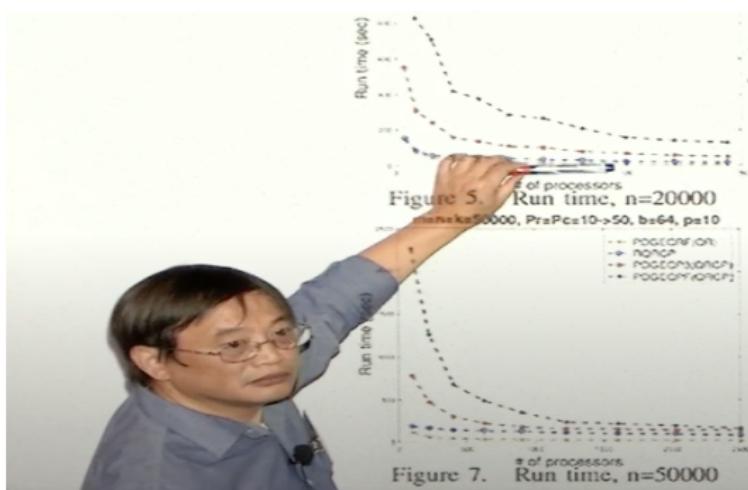


Self Introduction: Prof. Ming Gu, UC Berkeley

- ▶ Email: mgu@berkeley.edu
- ▶ Office hours: MWF 1:10-2:00PM
in 861 Evans or by appointment



Self Introduction

- ▶ **research:** data analysis, scientific computing

Self Introduction

- ▶ **research:** data analysis, scientific computing
- ▶ **teaching:** numerical analysis (over 10 years)

Self Introduction

- ▶ **research:** data analysis, scientific computing
- ▶ **teaching:** numerical analysis (over 10 years)
- ▶ **student paper:** Gautam was 128A student

DOI: 10.1007/s00211-012-0480-x · Corpus ID: 12099422

A modified Brent's method for finding zeros of functions

Gautam Williams, Ming Gu · Published 2013 · Mathematics, Computer Science · Numerische Mathematik

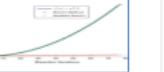
Brent's method, also known as zeroin, has been the most popular method for finding zeros of functions since it was developed in 1970. It is very robust and converges to a zero, for the occasional difficult functions encountered in practice, it typically takes 250(350) iterations to converge, where 250(350) is the number of steps required for the bisection method to find the zero to approximately the same accuracy. While it has long been known that in theory Brent's method could require... [CONTINUE](#)

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ABSTRACT FIGURES, TABLES, AND TOPICS 2 CITATIONS 8 REFERENCES

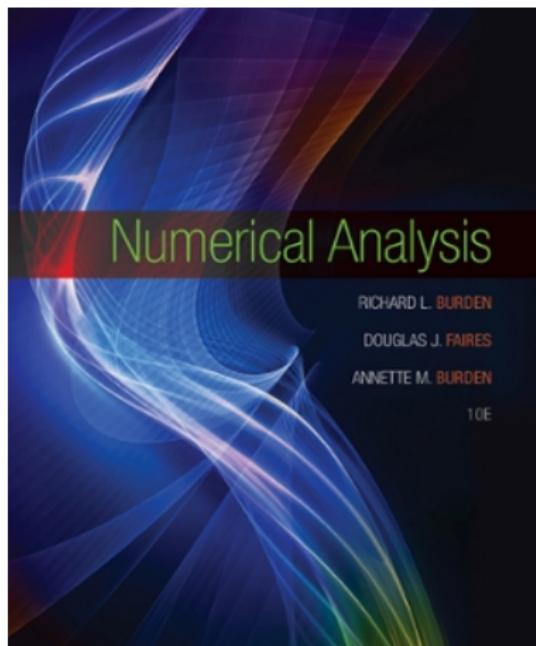
Figures, Tables, and Topics from this paper.

Figures and Tables

Figure 1  Table 1  Table 1  Figure 2 

Text Book

- Burden and Faires, **Numerical Analysis.**
10th edition, required.



Matlab

← → C ⓘ www.mathworks.com

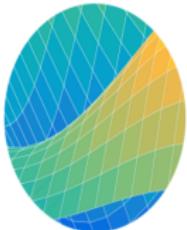
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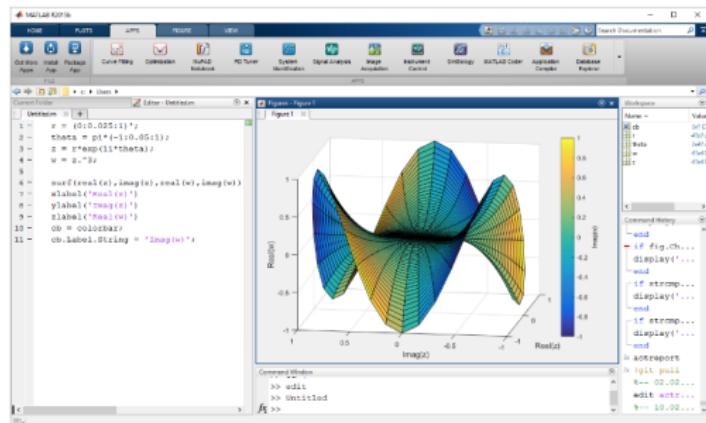
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by K Sigmon - 1989 - Cited by 24 - Related articles
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MATLAB Primer (Book by Kermit Sigmon)
Originally published: 1993
Author: Kermit Sigmon

MATLAB Primer, Eighth Edition (Book by Timothy A. Davis)
Originally published: December 29, 2004
Author: Timothy A. Davis



► Matlab **free download** for UC Berkeley students

<https://www.mathworks.com/academia/tah-portal/berkeley-731130.html>

Math 98: Introduction to MATLAB Programming (1 unit, Pass/No Pass)

- ▶ runs 3 weeks, starting 8/24
- ▶ two sections (tentative)
 - ▶ Tu/Th 9:30-12:30PM (back to back)
- ▶ Instructor: Michael Heinz
(michael_heinz@berkeley.edu)

Class Work

- ▶ 13 weekly home work sets,
up to 1.5 points each, maximum 15 points.
 - ▶ homework due on Wednesdays (first one due on Sep. 6)
submit on gradscope (Entry Code:3J4V26)

Class Work

- ▶ 13 weekly home work sets,
up to 1.5 points each, maximum 15 points.
 - ▶ homework due on Wednesdays (first one due on Sep. 6)
submit on gradsphere (Entry Code:3J4V26)
- ▶ 6 quizzes;
up to 3 points each, maximum 15 points.
 - ▶ quiz schedule
(Sept. 6, Sept. 20, Oct. 4, Nov. 1, Nov. 15, Nov. 29)
 - ▶ first quiz covers material in the first homework set
 - ▶ every other quiz covers material in the two most recent homework sets

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(Sept. 6, Sept. 20, Oct. 4, Nov. 1, Nov. 15, Nov. 29)
 - ▶ first quiz covers material in the first homework set
 - ▶ every other quiz covers material in the two most recent homework sets
- ▶ Midterm exam (Oct. 18), 20 points.
- ▶ Programming Assignment (Due Nov. 22), 20 points.
- ▶ Final exam (TBA), 30 points.
 - ▶ Final worth 50 points if midterm missing

Grade Scale

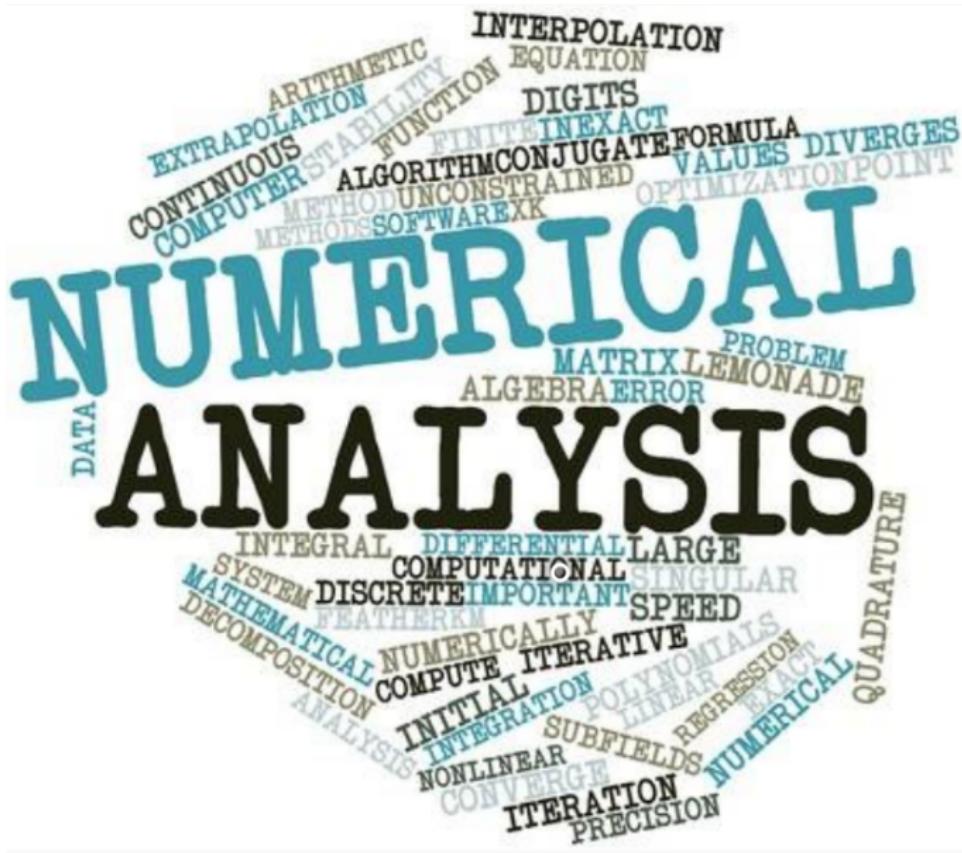
- ▶ **A-** to **A+**: at least 85 points;
- ▶ **B-** to **B+**: between 70 and 85 points;
- ▶ **C-** to **C+**: between 60 and 70 points;
- ▶ **D**: between 55 and 60 points;
- ▶ **F**: less than 55 points.

No grade curve: your grade **will** be your own effort

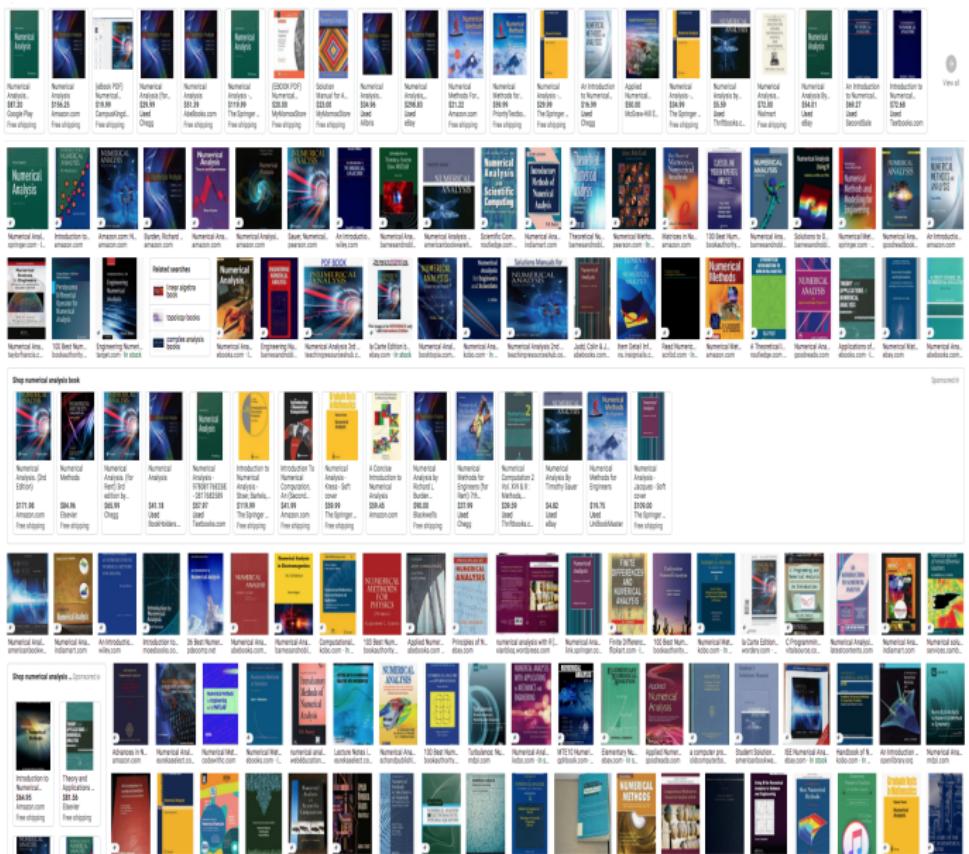
Most people get *A* level or *B* level grades.

More refined grade scales will be discussed later on based on class performance.

Many Topics in Numerical Analysis



Numerical Analysis, books for every need



Numerical Analysis = Calculus on a computer

First 6 Chapters of Text Book

- ▶ **Chapter 1:** Calculus Review, Computer arithmetic
- ▶ **Chapter 2:** Solution to non-linear equation $f(x) = 0$.
- ▶ **Chapter 3:** Function approximations.
- ▶ **Chapter 4:** Numerical differentiation and integration.
- ▶ **Chapter 5:** Numerical Initial value ODEs.
(ordinary differential equations)
- ▶ **Chapter 6:** Solution to linear equation $Ax = b$.

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(ordinary differential equations)
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Prerequisite: Calculus/simple `matlab` programming skill

YBC 7289, Babylonian clay tablet, about 3800 years old



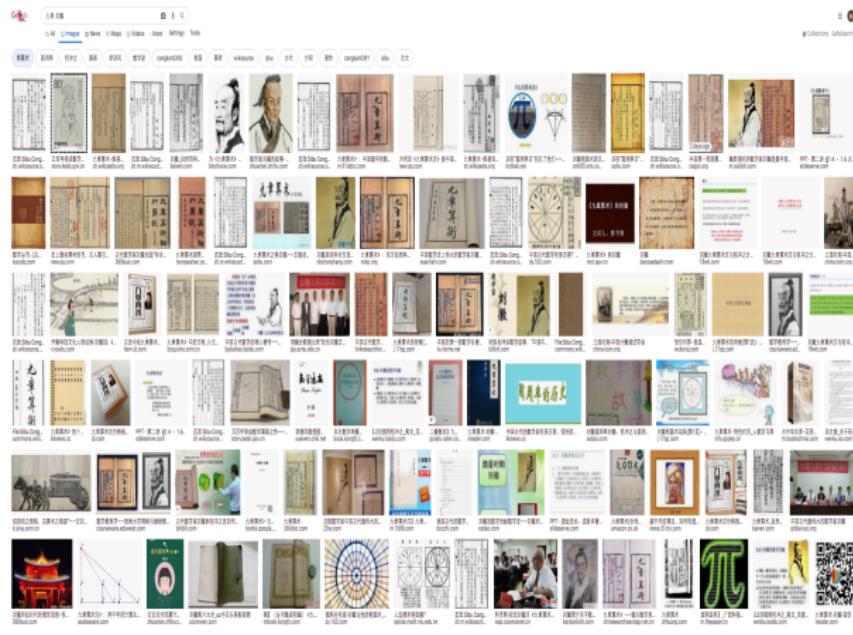
$$\sqrt{2} = \underline{1.4142135\cdots} \approx 1 + \frac{24}{60} + \frac{51}{60^2} + \frac{10}{60^3} = \underline{1.4142129\cdots}$$

Rhind Papyrus, 3600 years old

The image shows the Rhind Mathematical Papyrus, a document from ancient Egypt that contains mathematical problems and their solutions. The text is written in hieratic script and is organized into several columns. Some of the text is enclosed in rectangular boxes. There are also some geometric drawings, such as a triangle and a circle, integrated into the text.

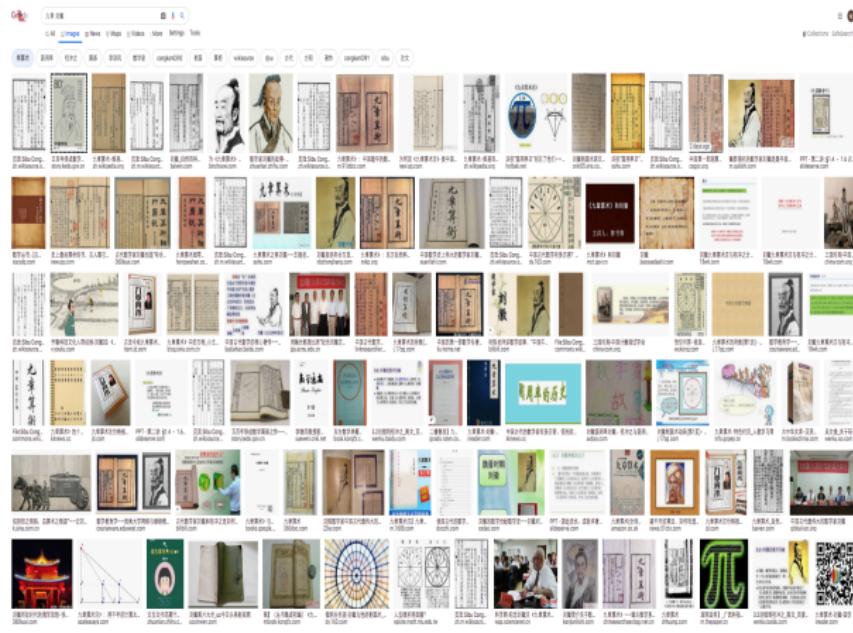
Solving linear equations in one unknown

Liu Hui (editor): Nine Chapters of Arithmetic Art (> 2000 years old)



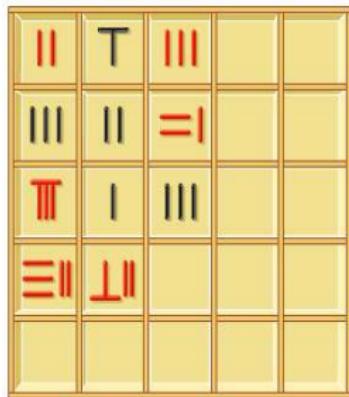
Solving linear system of equations in multiple unknowns,

Liu Hui (editor): Nine Chapters of Arithmetic Art (> 2000 years old)



Solving linear system of equations in multiple unknowns,
(Gaussian elimination without pivoting, Ch. 6)

Counting sticks for linear equations



Columns were used for equations, from right to left:

$$\begin{aligned}3x + 21y - 3z &= 0 \\-6x - 2y - z &= 62 \\2x - 3y + 8z &= 32\end{aligned}$$

- Negative
- Positive

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Counting sticks for linear equations

	T				
		=			

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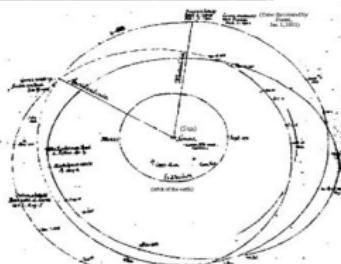


Japanese mathematicians solve linear equations with sticks

Gauss (200 years): Gaussian elimination, Gaussian quadrature



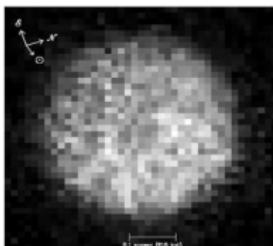
(Left) Carl Friedrich Gauss, considered one of the three greatest mathematicians of all time (along with Archimedes and Sir Isaac Newton).
(Right) Gauss at 24, when he computed the orbit of Ceres.



Sketch of the orbits of Ceres and Pallas (nachstall Gauß, Handb. 4). Courtesy of Universitätsbibliothek Göttingen.

(Left) Gauss' sketch of the orbit of Ceres.

(Right) Image of Ceres from the Hubble telescope.



https://www.youtube.com/watch?v=Y4B6fqNlfAc&ab_channel=TomoNewsUS

Fibonacci's Problem in 1224, with Emperor Frederick II

Solve

$$f(x) = x^3 + 2x^2 + 10x - 20 = 0.$$

Fibonacci's Solution

$$x = 1 + 22 \left(\frac{1}{60} \right) + 7 \left(\frac{1}{60} \right)^2 + 42 \left(\frac{1}{60} \right)^3 + 33 \left(\frac{1}{60} \right)^4 + 4 \left(\frac{1}{60} \right)^5 + 40 \left(\frac{1}{60} \right)^6.$$

There can only be ONE real solution since

$$f'(x) = 3x^2 + 4x + 10 = 3(x + 2/3)^2 + \frac{26}{3} > 0,$$

$f(x)$ is a monotonically increasing function.

Fibonacci's Problem in 1224, with Emperor Frederick II

Solve

$$f(x) = x^3 + 2x^2 + 10x - 20 = 0.$$

Fibonacci's Solution

$$\begin{aligned}x_{\text{Fibon}} &= 1 + 22 \left(\frac{1}{60} \right) + 7 \left(\frac{1}{60} \right)^2 + 42 \left(\frac{1}{60} \right)^3 + 33 \left(\frac{1}{60} \right)^4 + 4 \left(\frac{1}{60} \right)^5 + \underline{\underline{40}} \left(\frac{1}{60} \right)^6 \\&\approx 1.3688081078\underline{5322}, \quad f(x_{\text{Fibon}}) \approx 6.7185 \times 10^{-10}\end{aligned}$$

A better base-60 approximation

$$\begin{aligned}x_{\text{Comp}} &= 1 + 22 \left(\frac{1}{60} \right) + 7 \left(\frac{1}{60} \right)^2 + 42 \left(\frac{1}{60} \right)^3 + 33 \left(\frac{1}{60} \right)^4 + 4 \left(\frac{1}{60} \right)^5 + \underline{\underline{39}} \left(\frac{1}{60} \right)^6 \\&\approx 1.3688081078\underline{3179}, \quad f(x_{\text{Comp}}) \approx 2.1976 \times 10^{-10}\end{aligned}$$

Going too far

$$\begin{aligned}x_{\text{Far}} &= 1 + 22 \left(\frac{1}{60} \right) + 7 \left(\frac{1}{60} \right)^2 + 42 \left(\frac{1}{60} \right)^3 + 33 \left(\frac{1}{60} \right)^4 + 4 \left(\frac{1}{60} \right)^5 + \underline{\underline{38}} \left(\frac{1}{60} \right)^6 \\&\approx 1.3688081078\underline{1036}, \quad f(x_{\text{Far}}) \approx -2.3239 \times 10^{-10}\end{aligned}$$

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What is the range for the true root?

Fibonacci's Cubic Root is correct to 11 digits

```
>> format long e;
>> h = [1 2 10 -20];
>> r = roots(h)

r =
-1.684404053910685e+00 + 3.431331350197691e+00i
-1.684404053910685e+00 - 3.431331350197691e+00i
1.368808107821373e+00

>> Fibonacci = (((((40/60+4)/60+33)/60+42)/60+7)/60+22)/60+1

Fibonacci =
1.368808107853224e+00

>> r(3)-Fibonacci

ans =
-3.185118835347112e-11

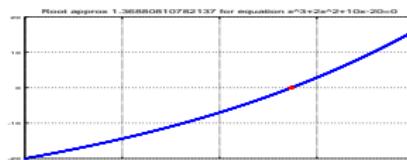
>> Better = (((((31/60+38)/60+4)/60+33)/60+42)/60+7)/60+22)/60+1

Better =
1.368808107821430e+00

>> r(3)-Better

ans =
-5.795364188543317e-14
```

One real root, two complex roots



Roots of a Random Quintic Polynomial: No closed form formula

```
>> format short g
>> hrand = randn(1,6)

hrand =
    -1.3499      3.0349      0.7254     -0.063055      0.71474     -0.20497

>> rrand = roots(hrand)

rrand =
    2.4872
   -0.70735
    0.105 + 0.56831i
    0.105 - 0.56831i
    0.2584
```

Numerical Methods can find roots for any polynomial

Simple Numerical Integration (I)

- ▶ Some integrals are easy to integrate by hand

$$I_1 = \int_0^1 \sqrt{1+x} dx = \frac{2}{3} \left(2^{\frac{3}{2}} - 1 \right).$$

- ▶ But others are harder

$$I_2 = \int_0^1 \sqrt{1+x^{1.1}} dx = ?$$

Numerical Methods attempt to numerically evaluate ANY integral

Numerical Integration for $\int_{-1}^1 \sqrt{1 + |x|^\alpha} dx$, $\alpha = 1, 2$: (6 correct digits)

```
>> format long g
>> I2 = quad(@(x) sqrt(1+x.^2), -1 ,1 )
I2 =
2.29558701441275

>> I1 = quad(@(x) sqrt(1+x), -1 ,1 )
I1 =
1.88561089016424

>> I1 - (4/3)*sqrt(2)
ans =
-7.19299988971578e-06
```

Numerical Integration for $\int_{-1}^1 \sqrt{1 + |x|^\alpha} dx$, $\alpha = 1, 1.1$: (12 correct digits)

```
%
>> I1 = quad(@(x) sqrt(1+x.^^(1.0)), -1, 1, 1e-12)
I1 = 1.88561808316413
>>
>> I1 - 4/3*sqrt(2)
ans = -8.88178419700125e-16
>>
>> I2 = quad(@(x) sqrt(1+abs(x).^(1.1)), -1, 1, 1e-12)
I2 = 2.41785196129704
%
```

Simple Numerical Integration: can't go overboard in accuracy

```
>> [I1,fcnt1] = quad(@(x) sqrt(1+x), -1 ,1, 1e-4);disp([I1-(4/3)*sqrt(2),fcnt1])
-4.5143e-04    2.1000e+01

>> [I1,fcnt1] = quad(@(x) sqrt(1+x), -1 ,1, 1e-8);disp([I1-(4/3)*sqrt(2),fcnt1])
-4.0728e-08    1.1300e+02

>> [I1,fcnt1] = quad(@(x) sqrt(1+x), -1 ,1, 1e-12);disp([I1-(4/3)*sqrt(2),fcnt1])
-3.5500e-12    6.9700e+02

>> [I1,fcnt1] = quad(@(x) sqrt(1+x), -1 ,1, 1e-16);disp([I1-(4/3)*sqrt(2),fcnt1])
-4.4409e-16    4.3930e+03

>> [I1,fcnt1] = quad(@(x) sqrt(1+x), -1 ,1, 1e-20);disp([I1-(4/3)*sqrt(2),fcnt1])
Warning: Maximum function count exceeded; singularity likely.
> In quad at 106
-5.6018e-06    1.0017e+04
```

Trajectory with ODE

Problem to solve: find function $y(t)$ so that

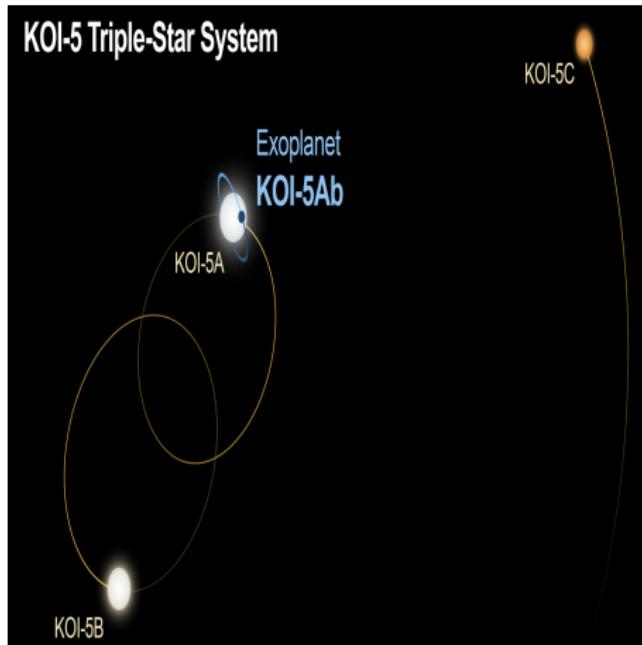
$$y'(t) = f(y(t), t), \quad y(t_0) = y_0$$

$y(t)$ could be trajectory of a flying bullet, y_0 is initial position.

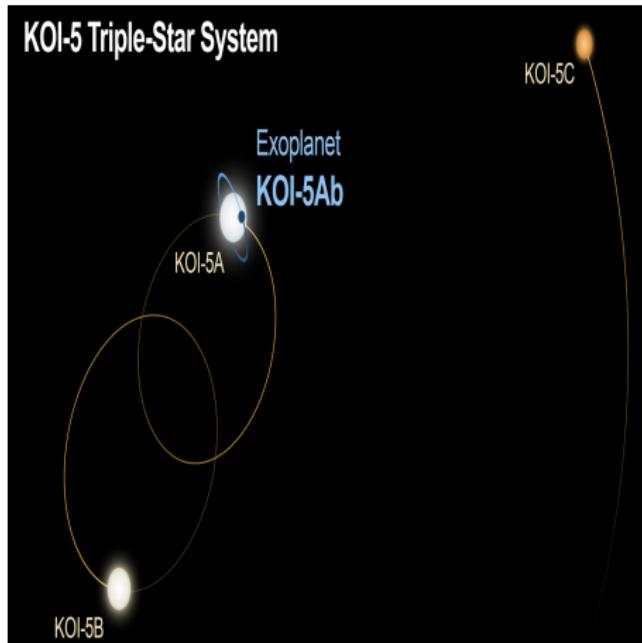
trajectory follows ODE, aiming = initial conditions



Planet orbit with 3 Suns



Planet orbit with 3 Suns



Another planet with 3 suns

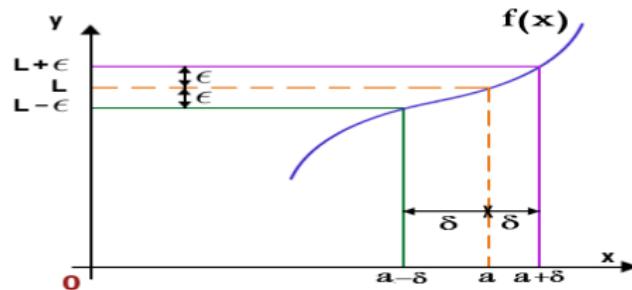
<https://www.nasa.gov/feature/goddard/2016/newly-discovered-planet-has-3-suns>

Teaching Philosophy



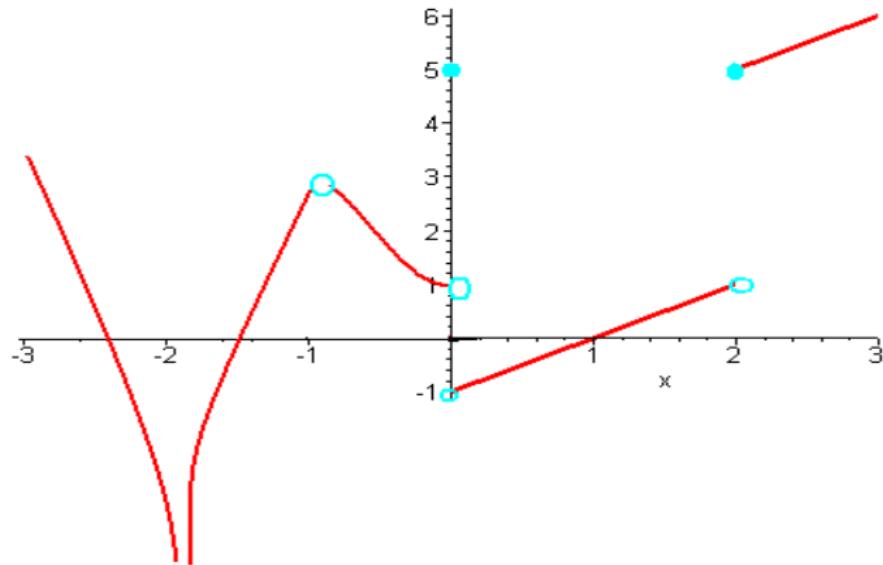
*"I expect you all to be independent, innovative,
critical thinkers who will do exactly as I say!"*

§1.1 Calculus Review: Limit/Continuity

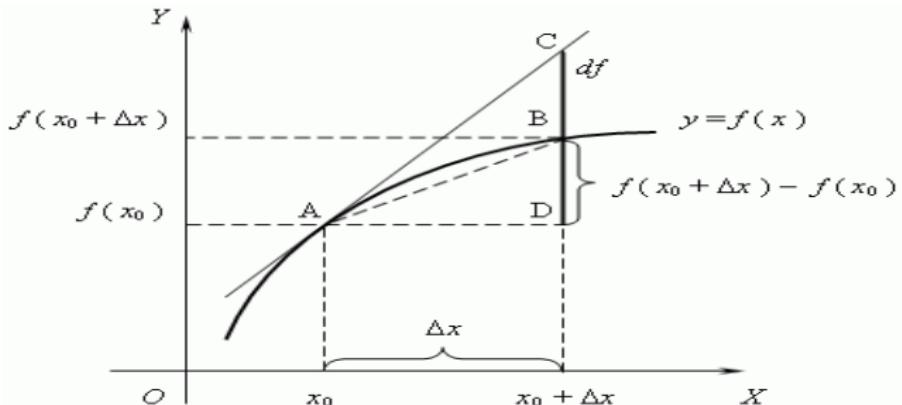


$$\lim_{x \rightarrow a} f(x) = f(a) = L$$

Continuity vs. Discontinuity

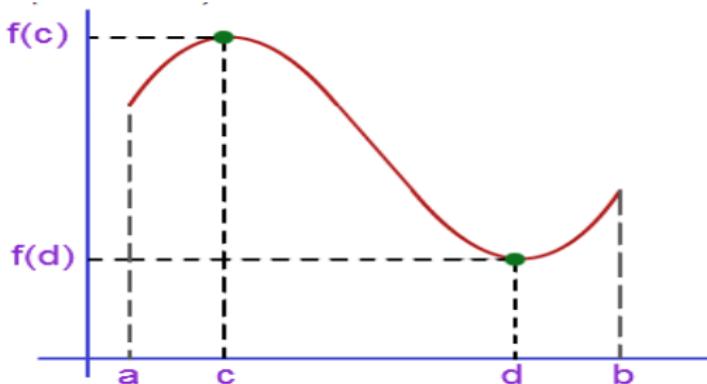


Def: Differentiability



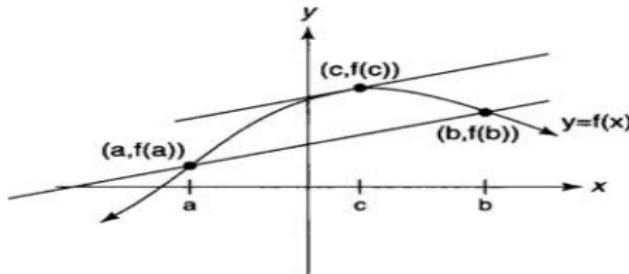
$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}.$$

Extreme Value Theorem



- ▶ Maximum $f(c)$ and minimum $f(d)$ attainable in $[a, b]$ if $f(x)$ continuous.
- ▶ For very high dimensions, this means data analysis, artificial intelligence, etc

Mean Value Theorem

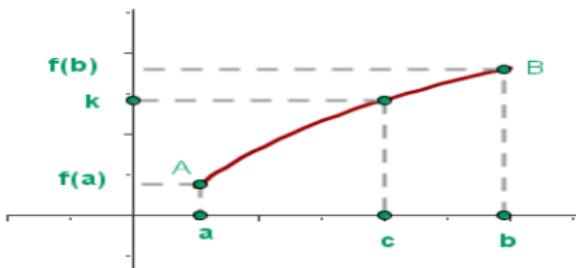


- If $f(x)$ continuous, then c exists in $[a, b]$ so

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

- Basis of much of theoretical analysis.

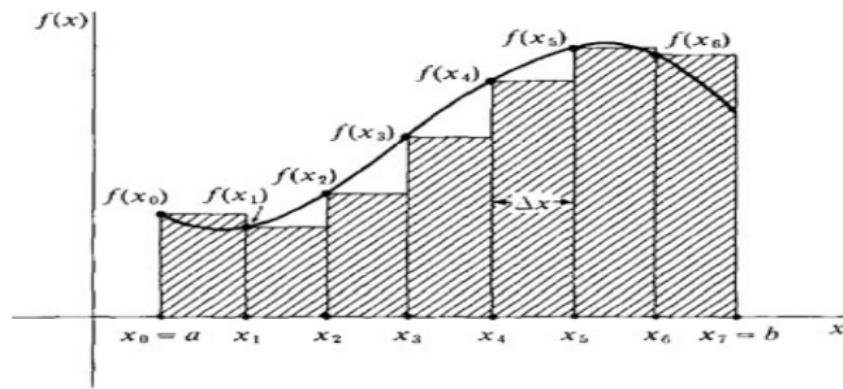
Intermediate Value Theorem



- ▶ If $f(x)$ continuous, then c exists in $[a, b]$ so $f(c) = k$ for any k between $f(a)$ and $f(b)$.
- ▶ Basis of methods for solving $f(x) = 0$.

Calculus says c exists; we will find c to within a TOLERANCE.

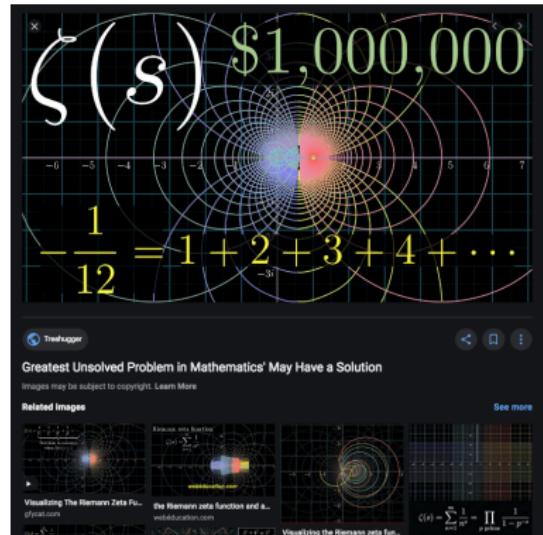
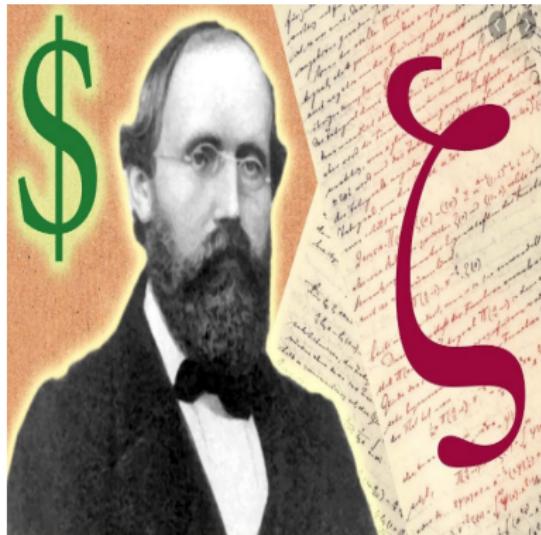
Riemann Sum



$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{k=1}^n f(x_k).$$

Calculus shows limit exists; we find it to within a TOLERANCE.

Riemann, 1826 – 1866



Machine Precision

- ▶ Computer numbers (floating point numbers) are a **finite** subset of rational numbers.
- ▶ There is a smallest positive computer number ϵ so that

$$1 + \epsilon > 1$$

§1.2 Machine Precision

```
>> eps
ans =
2.220446049250313e-16

>> x = 1 + eps; disp([eps, (x-1)/eps])
2.220446049250313e-16    1.000000000000000e+00

>> delta = 0.75*eps
delta =
1.665334536937735e-16

>> x = 1 + delta; disp([delta, (x - 1)/delta])
1.665334536937735e-16    1.333333333333333e+00

>> Delta = 0.5*eps
Delta =
1.110223024625157e-16

>> x=1+Delta;disp([Delta, (x - 1)/Delta])
1.110223024625157e-16          0
```

Overflow

```
>> x=2^1023
x=2^1023

x =
8.9885e+307

>> x = 2*x
x = 2*x

x =
Inf

>> y = 2^(-1023)
y = 2^(-1023)

y =
1.1125e-308

>> y = y/(2^51)
y = y/(2^51)

y =
4.9407e-324

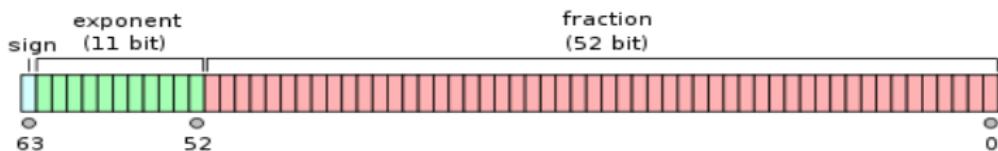
>> y = y / 2
y = y / 2

y =
0
```

Apple Memory Chip: Huge but finite capacity



IEEE 754 Double Precision Format



Every floating point number fits within 64 bits.

Round-off Errors and Floating Point Arithmetic

- ▶ **Binary Machine Numbers:** any double precision non-zero *floating point number* has form

$$x = (-1)^s 2^{c-1023} (1 + f), \quad \text{with 64 bits.}$$

- ▶ s = SIGN BIT: 0 for $x > 0$ and 1 for $x < 0$.
- ▶ c = CHARACTERISTIC, with 11 bits:

$$c = c_1 \cdot 2^{10} + c_2 \cdot 2^9 + c_3 \cdot 2^8 + c_4 \cdot 2^7 + c_5 \cdot 2^6 + c_6 \cdot 2^5 + c_7 \cdot 2^4 + c_8 \cdot 2^3 + c_9 \cdot 2^2 + c_{10} \cdot 2^1 + c_{11} \cdot 2^0,$$

with each $c_j = 0$ or 1.

- ▶ f = MANTISSA with 52 bits

$$f = f_1 \cdot \left(\frac{1}{2}\right) + \cdots + f_{52} \cdot \left(\frac{1}{2}\right)^{52} = \sum_{j=1}^{52} f_j \cdot \left(\frac{1}{2}\right)^j, \quad \text{each } f_j = 0 \text{ or } 1.$$

- ▶ **Floating Point:** Binary point always comes after 1, independent of c .

- ▶ Special cases for special numbers

Round-off Errors and Floating Point Arithmetic

- ## ► Binary Machine Numbers: Example binary string

- $s = 0$, $c = (10000000011)_2 = 1024 + 2 + 1 = 1027$, and

$$f = 1 \cdot \left(\frac{1}{2}\right)^1 + 1 \cdot \left(\frac{1}{2}\right)^3 + 1 \cdot \left(\frac{1}{2}\right)^4 + 1 \cdot \left(\frac{1}{2}\right)^5 + 1 \cdot \left(\frac{1}{2}\right)^8 + 1 \cdot \left(\frac{1}{2}\right)^{12}.$$

$$(-1)^s 2^{c-1023} (1+f) = (-1)^0 \cdot 2^{1027-1023} \left(1 + \left(\frac{1}{2} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{256} + \frac{1}{4096} \right) \right)$$

$$= 27.56640625.$$

binary representation: ineffective for humans, magical for machines

Street Numbers in Binary? (City of machines)



Round-off Errors and Floating Point Arithmetic

- ▶ **k -digit Decimal Machine Numbers:**

$$x = \pm 0.d_1 d_2 \cdots d_k \times 10^n, \quad \text{where} \quad 1 \leq d_1 \leq 9, \quad 0 \leq d_i \leq 9, i \geq 2.$$

- ▶ Any positive real number

$$\begin{aligned} y &= 0.d_1 d_2 \cdots d_k d_{k+1} d_{k+2} \cdots \times 10^n, \\ &\approx 0.d_1 d_2 \cdots d_k \times 10^n \stackrel{\text{def}}{=} fl(y) \quad (\textbf{chopping}) \\ &\approx 0.\delta_1 \delta_2 \cdots \delta_k \times 10^n \stackrel{\text{def}}{=} fl(y) \quad (\textbf{rounding}), \end{aligned}$$

where

$$\textbf{rounding} = \textbf{chopping on } y + 5 \times 10^{n-(k+1)}.$$

- ▶ If $d_{k+1} < 5$: **rounding = chopping**.
- ▶ If $d_{k+1} \geq 5$: cut off d_{k+1} and below, then add 1 to d_k .

Round-off Errors and Floating Point Arithmetic

- ▶ 5-digit Decimal Machine Numbers for π :

$$\pi = 0.314159265 \dots \times 10^1$$

$$\approx 0.31415 \times 10^1 = 3.1415 \quad (\text{chopping})$$

$$\approx (0.31415 + 0.00001) \times 10^1 = 3.1416 \quad (\text{rounding}).$$

Absolute error vs. relative error

Suppose that p^* is an approximation to $p \neq 0$.

- ▶ **absolute error** = $|p - p^*|$,
- ▶ **relative error** = $\frac{|p - p^*|}{|p|}$.

Example

- ▶ **absolute errors:**

$$|\pi - 3.1415| \approx 9 \times 10^{-5}, \quad |\pi - 3.1416| \approx 7 \times 10^{-6}.$$

- ▶ **relative errors:**

$$\frac{|\pi - 3.1415|}{\pi} \approx 3 \times 10^{-5}, \quad \frac{|\pi - 3.1416|}{\pi} \approx 2 \times 10^{-6}.$$

Cool \$200,000 wager by LeSean McCoy, 2017



Cool \$200,000 wager by LeSean McCoy, 2017



David Payne Purdum @DavidPurdum

LeSean McCoy bet \$200,000 on Warriors to win Finals. It's largest bet Planet Hollywood took on Finals. Would pay \$62,500.
<http://ift.tt/2mzqH2t>
8:00 AM - Jun 4, 2017

Cool \$200,000 wager by LeSean McCoy, 2017



- ▶ Wager: Warriors to win NBA Finals
- ▶ McCoy made \$6M in 2017. $\frac{\text{wager}}{\text{salary}} \approx 3\%$
- ▶ If lost, wager would be a **huge** absolute error, but **small** relative error, to his salary. He won \$62,500