Mason McBride
3035925411
mason7@berkeley.edu
STAT 155

1. to show the market sharing game is a potential game we must show

$$\Phi(j_1, c_{-i}) - \Phi(j_2, c_{-i}) = u_i(j_1, c_{-i}) - u_i(j_2, c_{-i})$$

with $\Phi(j_i, c_{-i}) = \sum_{j \in c} \sum_{l=1}^{n_j(c)} \frac{v_j}{l}$

$$u_i(j_1, c_{-i}) - u_i(j_2, c_{-i}) = \frac{v_{j_1}}{n_{j_1}(c)} - \frac{v_{j_2}}{n_{j_2}(c)+1}$$

and

$$\Phi(j_1, c_{-i}) - \Phi(j_2, c_{-i}) = \frac{v_{j_1}}{n_{j_1}(c)} - \frac{v_{j_2}}{n_{j_2}(c)+1}$$

so the market sharing game is a potential game.

2. From section, it is sufficient to
a non-zero cycle in the payoff mat
to show it is not a potential gam

for $\begin{pmatrix} (4,4) & (2,3) \\ (5,3) & (1,5) \end{pmatrix}$

$$+1+2+1+2 = \boxed{6 \neq 0}$$

for $\begin{pmatrix} (6,1) & (9,4) & (1,1) \\ (2,3) & (3,1) & (4,2) \\ (6,2) & (4,0) & (1,1) \end{pmatrix}$

$$0-2+5+3 = \boxed{6 \neq 0}$$

3.

$$\begin{array}{ccc} & A & B \\ A & (4,4) & (2,5) \\ B & (5,2) & (3,3) \end{array}$$

Pure Strategies:

(A,B) (B,A)

$$4x + (1-x)\cdot 2 = 5x + (1-x)\cdot 3$$
$$4x + 2 - 2x = 5x + 3 - 3x$$
$$2x + 2 = 2x + 3$$

$2 \neq 3$ no mixed strategy

There are no symmetric NE for this payoff matrix

for

$$\begin{array}{ccc} & A & B \\ A & (4,4) & (3,2) \\ B & (2,3) & (5,5) \end{array}$$

NE: (A,A) (B,B)

$$4x + (1-x)\cdot 3 = 2x + (1-x)\cdot 5$$
$$4x + 3 - 3x = 2x + 5 - 5x$$
$$4x = 2$$
$$x = \tfrac{1}{2}$$

$\left(\tfrac{1}{2}, \tfrac{1}{2}\right)$

Three NE are symmetric (A, A), (B, B), and $(\frac{1}{2}, \frac{1}{2})$

x is ESS if for any pure strategy z

(a) $z^T A x \leq x^T A x$

(b) If $z^T A x = x^T A x$, then $z^T A z < x^T A z$

$$\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} = 3.5$$

$$\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} = 3.5$$

Both are equal so (b)

$z^T A z = 5 \not< x^T A z = 4$

$z A z = 4 \not< x^T A z = 3$

$\boxed{\text{not an ESS}}$

for (A,A):

$$\begin{bmatrix}0\\1\end{bmatrix} A [1, 0] = 2 < x^T A x = 4$$

(A,A) is ESS

for (B,B):

$$\begin{bmatrix}1\\0\end{bmatrix} A [0 \; 1] = 3 < \begin{bmatrix}0\\1\end{bmatrix} A [0 \; 1] = 5$$

(B,B) is ESS

4. x is ESS if for any pure strategy z

(a) $z^TAx \leq x^TAx$

(b) If $z^TAx = x^TAx$, then $z^TAz < x^TAz$

define strategy i to be any pure strategy i in a symmetric game.

define $j \in J$ where j is a pure strategy where $j \neq i$.

for (a) $j^TAi \leq i^TAi$ $\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \begin{bmatrix} & & \\ & a_{ii} & \\ & & \end{bmatrix} [0\, 0 \ldots 1 \ldots 0]$

j^TA picks out the jth row of payoff Matrix A because there are 0's for every index but the jth which turns all values of those rows to 0

by similar logic, j^TAi picks the ith column from the jth row of A so $j^TAi = a_{ji}$

The same is true for $i^T A i = a_{ii}$

Because $a_{ji} < a_{ii}$ by construction, $j^T A i < i^T A i$ and pure strategy i is an ESS

5. d1 has two options: ABC or ADC

all others are dominated because they will overlap with {ABC, ADC}

d2 also has two options: BAD and BCD

I \ II	BAD	BCD
ABC	(16+7, 16+10)	(13+23, 23+21)
ADC	(15+21, 15+13)	(10+14, 7+14)

	BAD	BCD
ABC	(23, 26)	(36, 44)
ADC	(36, 28)	(24, 21)

Pure NE: (ADC, BAD) where d1 takes roads AD, DC and d2 takes roads BC, CD

(ABC, BCD) where d1 takes roads AB, BC and d2 takes roads BC, CD

6 $\begin{pmatrix} (0,0) & (6,2) \\ (2,6) & (0,0) \end{pmatrix}$ $\quad \begin{pmatrix} (0,1) & (6,a) \\ (a,6) & (0,1) \end{pmatrix}$

$6\cdot(1-x) = 2x$

$6-6x = 2x$

$\boxed{x = \frac{3}{4}}$

$6\cdot(1-x) = ax$

$\cancel{6-6x} = ax$

$\cancel{2-2x} = 3x$

$\boxed{x = \frac{2}{5}}$

X is ESS if for any pure strategy z

(a) $z^T A x \leq x^T A x$

(b) If $z^T A x = x^T A x$, then $z^T A z < x^T A z$

$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} A \left[\frac{3}{4}, \frac{1}{4}, 0\right] = \frac{1}{5}$ $\quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} A \left[\frac{3}{4}\ \frac{1}{4}\ 0\right] = \frac{1}{5}$

$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} A \left[\frac{3}{4}\ \frac{1}{4}\ 0\right] = \frac{1}{5}$ $\quad x^T A x = \frac{1}{5}$

~~because~~ of equality, $z^T A z < x^T A z$ must be checked

$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} Ax = 0 < x^T A z = 0.5$

$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} Ax = 0 < 4.5$

$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} Ax = 0 < 0.75$

$\boxed{\text{so } \left[\frac{3}{4}, \frac{1}{4}, 0\right] \text{ is ESS}}$

for $\left(0, \frac{2}{5}, \frac{3}{3}\right)$

$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} Ax = 1.8 \qquad x^T A x = 3.6$

$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} Ax = 3.6$

$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} Ax = 3.6$

to check cases where $z^T A x = x^T A x$,

$z_1^T A z_1 = 0 < x^T A z_1 = 5.4$

$z_2^T A z_1 = 0 < x^T A z_2 = 2.4$

$\boxed{\text{so } \left(0, \frac{2}{5}, \frac{3}{5}\right) \text{ is an ESS}}$

Use the stability criteria of an arbitrarily small population of half B and C's with $z = (0, \frac{1}{2}, \frac{1}{2})$

(a) $z^T A x \leq x^T A x$

(b) If $z^T A x = x^T A x$, then $z^T A z < x^T A z$

$$\begin{bmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} A \left[\frac{3}{4}, \frac{1}{4}, 0 \right] = 1.5 = x^T A x = 1.5$$

for (b)

$$\cancel{z^T A z = 3.75 \nless x^T A z = 2.625}$$

$$\boxed{\text{Thus AB is not stable against strategy } z \text{ and pop. BC will overtake pop. AB}}$$