Math 110, Spring 2023.

Homework 1, due January 28.

Prob 1. Suppose $a \in \mathbb{F}$ (field), v, $w \in V$ (vector space over $\mathbb{F}$), and $av = aw$. Prove that $a = 0$ or $v = w$.

Prob 2. Is $\mathbb{R}^{\mathbb{Z}}$ a vector space over $\mathbb{Z}$? Over $\mathbb{Q}$? Over $\mathbb{R}$? Over $\mathbb{C}$? Explain.

Prob 3. Suppose that $\{0, 1, x\}$ is a field with exactly three elements. What do the addition and multiplication tables *have to be* in that case? Based on the addition and multiplication tables you get, check this is indeed a field. What is the natural way to think of this field (and of x)?

Prob 4. Prove that any field $\mathbb{F}$ is also a vector space over itself, with the field addition used as vector addition, and the field multiplication used as scalar multiplication.

Prob 5. For which values of a is the set of all real-valued twice differential functions f on the interval $(0, \infty)$ such that $f''(2) - af(0) = a$ (equipped with the usual addition of functions and multiplication by real scalars) a vector space over $\mathbb{R}$?