Final Project: Modified `zero-in` for root-finding

We would like to find a root of the equation

$$f(x) = 0, \quad \text{for} \quad x \in \mathbf{R}$$

given an initial interval $[a, \ b]$ with

$$f(a) \cdot f(b) < 0.$$

with a combination of two methods

- **bisection** method, for its reliability
- **inverse quadratic interpolation** (IQI) method, for its higher order of convergence.

inverse quadratic interpolation (IQI) method

Given three pairs of points $(x_0,\ f_0)$, $(x_1,\ f_1)$, $(x_2,\ f_2)$, IQI defines a quadratic polynomial in f that goes through these points,

$$x(f) = \frac{(f - f_1)(f - f_2)}{(f_0 - f_1)(f_0 - f_2)}\, x_0 + \frac{(f - f_0)(f - f_2)}{(f_1 - f_0)(f_1 - f_2)}\, x_1 + \frac{(f - f_0)(f - f_1)}{(f_2 - f_0)(f_2 - f_1)}\, x_2$$

This leads to an estimate for the root $x_3 \overset{def}{=} x(0)$:

$$x_3 = \frac{f_1\, f_2}{(f_0 - f_1)(f_0 - f_2)}\, x_0 + \frac{f_0\, f_2}{(f_1 - f_0)(f_1 - f_2)}\, x_1 + \frac{f_0\, f_1}{(f_2 - f_0)(f_2 - f_1)}\, x_2$$

Modified `zero-in` for root-finding in a sketch

For given initial interval $[a,\ b]$ with

$$f(a) \cdot f(b) < 0.$$

We would like to find a root of the equation $\quad f(x) = 0, \quad$ for $\quad x \in \mathbf{R}$

Modified `zero-in` for root-finding in a sketch

For given initial interval $[a,\ b]$ with

$$f(a) \cdot f(b) < 0.$$

We would like to find a root of the equation $\quad f(x) = 0, \quad$ for $\quad x \in \mathbf{R}$

1. **set** $x_0,\ x_1,\ x_2 = a,\ b,\ c \stackrel{def}{=} \frac{a+b}{2}$
2. **let** $x_3 = \mathtt{IQI}\left(x_0,\ x_1,\ x_2\right)$
   - **if** $x_3 \notin [a,\ b]$
     - **do** `bisection` steps on $[a,\ b]$
     - **set** new interval $[a^{\text{new}},\ b^{\text{new}}] \subset [a, b]$ with
       $$f(a^{\text{new}}) \cdot f(b^{\text{new}}) < 0, \qquad \textbf{repeat}\ \text{step (1)}$$
   - **else if** $|f\left(x_3\right)|$ has not DECREASED by a factor of 2 within 4 consecutive **IQI** iterations,
     - **do** `bisection` steps on $[a,\ b]$
     - **set** new interval $[a^{\text{new}},\ b^{\text{new}}] \subset [a, b]$ with
       $$f(a^{\text{new}}) \cdot f(b^{\text{new}}) < 0, \qquad \textbf{repeat}\ \text{step (1)}$$
   - **repeat IQI** in step (2)
3. **stop** when iteration converged