MATH 53 WORKSHEET 11/30

(1) Consider the vector field

$$\mathbf{F}(x, y, z) = \frac{1}{(x^2 + y^2 + z^2)^{3/2}} \langle x, y, z \rangle.$$

Does $\mathbf{F} = \nabla \times \mathbf{G}$ for some vector field $\mathbf{G}$? Hint: Review question 3 on the worksheet on 11/28.

(2) True or false

(a) If a smooth vector field $\mathbf{F}$ on $\mathbb{R}^3$ is curl free and divergence free, then $\mathbf{F}$ has a potential function f and satisfies $\Delta f = 0$.

(b) There exists a vector field $\mathbf{F}$ such that $\nabla \times \mathbf{F} = \langle 1, 1, 1 \rangle$.

(c) There exists a vector field $\mathbf{F}$ such that $\nabla \times \mathbf{F} = \langle x, y, z \rangle$.

(d) Let $\mathbf{F}$ be a smooth vector field on $\mathbb{R}^3$ and suppose that S_1 and S_2 are oriented surfaces with boundary curve C, and C is in the direction that is compatible with the orientations of S_1 and S_2. Then

$$\int\int_{S_1} \mathbf{F} \cdot \mathbf{n} \, dS = \int\int_{S_2} \mathbf{F} \cdot \mathbf{n} \, dS.$$

(3) Find $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where $F = \langle \cos x + y + z, x + z, x + y \rangle$ and C is the intersection of the surfaces $x^2 + \frac{y^2}{2} + \frac{z^2}{3} = 1$ and $z = x^2 + 2y^2$, oriented counterclockwise if viewed from above.

(4) Find $\int\int_S \mathbf{F} \cdot \mathbf{n}\, dS$, where

$$\mathbf{F} = \langle 2x, 2z - 2x, 2x - 2z \rangle$$

and

$$S = \{(x, y, z) \mid z = e^y(1 - x^2 - y^2)(1 - y^5)\cos x,\ \ x^2 + y^2 \leq 1\}$$

with upward pointing normal. Hint: Find a $\mathbf{G}$ such that $\mathbf{F} = \nabla \times \mathbf{G}$. Let $\mathbf{G} = \langle G_1, G_2, 0 \rangle$ for some scalar functions G_1 and G_2.