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Math128A: Numerical Analysis
Sample Midterm

This is a closed book exam. You are allowed to cite any results, up to Section 4.5 but excluding those in the exercises, from the textbook. Results from anywhere else will be treated the same as your answers, which need to be justified. Completely correct answers given without justification will receive little credit. Partial solutions will get partial credit.

Problem	Maximum Score	Your Score
1	18	
2	16	
3	16	
4	16	
5	16	
6	18	
Total	100	

1. Determine the absolute error, the relative error, and the number of significant digits in the approximation

$$\pi \approx 3.14$$

2. Let $x_0 < x_1 < x_2$. Find a second degree polynomial $P(x)$ such that

$$P(x_0) = f_0, \quad P(x_1) = f_1, \quad \text{and} \quad P'(x_2) = f_2'.$$

3. Show that the cubic equation

$$2x^3 - 6x + 1 = 0 \qquad (1)$$

has a real root in the interval $[0, \quad 1/2]$. Perform one step of Bisection method with this interval.

4. Reformulate the above equation (1) as

$$x = \frac{2x^3 + 1}{6}.$$

Define the fixed point iteration (FPI) based on this equation, and show that FPI convergences for any initial guess in $[0, \quad 1/2]$.

5. Construct the natural cubic spline that approximates

$$f(x) = \frac{\sin x}{x}$$

at the nodes $-1, 0, 1$. Note that at $x = 0$, we define $f(0) = 1$.

6. Suppose that

$$L = \lim_{h \to 0} f(h) \quad \text{and} \quad L - f(h) = c_6 h^6 + c_9 h^9 + \cdots.$$

Find a combination of $f(h)$ and $f(h/2)$ with an $O(h^9)$ error estimate of L.