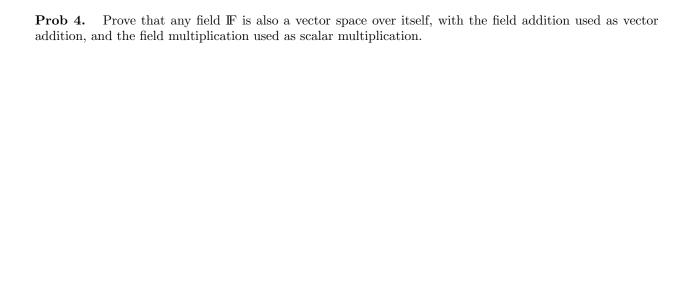
## Math 110, Spring 2023. Homework 1, due January 28.

**Prob 1.** Suppose  $a \in \mathbb{F}$  (field),  $v, w \in V$  (vector space over  $\mathbb{F}$ ), and av = aw. Prove that a = 0 or v = w.

**Prob 2.** Is  $\mathbb{R}^{\mathbb{Z}}$  a vector space over  $\mathbb{Z}$ ? Over  $\mathbb{Q}$ ? Over  $\mathbb{R}$ ? Over  $\mathbb{C}$ ? Explain.

**Prob 3.** Suppose that  $\{0,1,x\}$  is a field with exactly three elements. What do the addition and multiplication tables *have to be* in that case? Based on the addition and multiplication tables you get, check this is indeed a field. What is the natural way to think of this field (and of x)?



**Prob 5.** For which values of a is the set of all real-valued twice differential functions f on the interval  $(0, \infty)$  such that f''(2) - af(0) = a (equipped with the usual addition of functions and multiplication by real scalars) a vector space over  $\mathbb{R}$ ?