MATH 53 WORKSHEET 12/02

(1) Calculate $\int\int_S \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S}$ where $\mathbf{F} = \langle x+z, x+y, x^3 \rangle$ and S is the portion of the surface $z = 25 - x^2 - y^2$ strictly above the plane $z = 9$, oriented upwards.

(2) True or false

(a) Let $\mathbf{F}$ and $\mathbf{G}$ be smooth vector fields in $\mathbb{R}^3$. If C is a path from point (a_1, b_1, c_1) to (a_2, b_2, c_2), then $\int_C (\mathbf{F} \times \mathbf{G}) \cdot d\mathbf{r}$ is independent of the path C.

(b) If f is a smooth function on $\mathbb{R}^3$, then $\nabla \times (f \nabla f) = \mathbf{0}$.

(c) If $\mathbf{F}$ is a smooth conservative vector field on $\mathbb{R}^3$, then its flux through an smooth closed surface is zero.

(3) Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle xy, yz, xz \rangle$, and C, oriented counterclockwise, is the curve of the intersection of the surfaces $x^2 + y^2 - z^2 = 9$ and $z = 4$. Sketch C and the surfaces.

(4) Calculate $\int\int_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = \langle z \arctan y^2, z^3 \ln(x^2 + 1), z \rangle$, and S is the part of the paraboloid $z = 2 - x^2 - y^2$ that lies strictly above the plane $z = 1$ oriented downward.