Name: ______________________________
SID: ______________________________

MATH 135: SET THEORY
MIDTERM # 2

There are **five** questions. Each question is worth 20 points. Write your answers in the space provided. If you need more space, you may continue your answers on the blank pages at the end of the exam. **Please submit all of the pages, including the extra blank pages even if you have not written on them.**

For questions 1a and 1b, express your solution in the formal language of set theory having the binary relation symbol $\in$, the constant symbol $\varnothing$, the unary function symbol $\{\cdot\}$, the unary function symbol $\mathcal{P}$, the unary predicate f `is a function`, the unary function symbol $\operatorname{ran}(\cdot)$, the unary function symbol $\operatorname{dom}(\cdot)$, the binary function symbol $\cap$, and the binary function symbol $\cup$.

1a. State precisely the Axiom of Infinity.

Name: ______________________________
SID: ______________________________

1b. Give a formal definition of the expression X `is finite` .

Name: ______________________________

SID: ______________________________

2. Let A be a set such that for every $x \in A$, x is a transitive set. **Show** that $\bigcup A$ is a transitive set.

Name: ____
SID: ____

3. Show that if a, b, and c are natural numbers and $a < b$, then $a + c < b + c$.

Name: ______________________________
SID: ______________________________

4. Let X be a set and suppose that $\omega \preceq X$. **Show** that X is Dedekind-infinite (that is, there is some one-to-one function $f : X \hookrightarrow X$ which is *not* onto).

Name: ______________________________

SID: ______________________________

5. Prove that

$$2^{\aleph_0} = 3^{\aleph_0} \ .$$

Name: ____
SID: ____

For which problem should this work be credited?

Name: ______________________________

SID: ______________________________

For which problem should this work be credited?