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STAT 155

1. a)

$$\begin{array}{ccccccc} & & & \text{min} & & & \\ & \left(\begin{matrix} 1 & 5 & 4 & 0 \\ 4 & 0 & -1 & 3 \\ 2 & 4 & 3 & -1 \\ 1 & 1 & -2 & 2 \end{matrix}\right) & & & & & \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \\ & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} & & & & & \end{array}$$

col 2 is dominated by col 3.
row 4 is dominated by row 1. max
col 1 is dominated by col 4.
row 3 is dominated by row 0.

the payoff matrix is now:

$$\begin{array}{c|cc} & y & 1-y \\ \hline x & 4 & 0 \\ 1-x & -1 & 3 \end{array}$$

$4x-(1-x)=0+3(1-x)$ $\quad$ $4y=-1y+3(1-y)$

$4x-1+x=3-3x$ $\quad$ $4y=-y+3-3y$

$8x=4$ $\quad$ $8y=3$

$x=\frac{1}{2}$ $\quad$ $y=\frac{3}{8}$

optimal solutions for p1 and p2:

$\left[\frac{1}{2}, \frac{1}{2}\right]$ $\quad$ $\left[\frac{3}{8}, \frac{5}{8}\right]$

there are no pure strategies because the value of the game is $\frac{3}{2}$ and HW1Q4 says all 2×2 pure NE must be (pure, pure) and there are no cells in M that equal $3/2$

b) $\begin{pmatrix} 9 & 6 & 7 \\ \cancel{3} & \cancel{0} & \cancel{1} \\ 4 & 16 & 12 \end{pmatrix} \Rightarrow \begin{pmatrix} 9 & 6 & 7 \\ 4 & 16 & 12 \end{pmatrix}$

row 1 is dominated by row 0.

$\frac{1}{3}$ of col 1 + $2/3$ of col 2 is col 3

so the matrix $\begin{array}{c|cc} & y & 1-y \\ \hline x & 9 & 6 \\ 1-x & 4 & 16 \end{array}$

$9x + 4 - 4x = 6x + 16 - 16x$

$5x + 4 = -10x + 16$

$15x = 12$

$x = \frac{4}{5}$

$9y + 6 - 6y = 4y + 16 - 16y$

$3y + 6 = -12y + 16$

$15y = 10$

$y = \frac{2}{3}$

an optimal strategy for p1 is $(\frac{4}{5}, \frac{1}{5})$ and for p2 is $(\frac{2}{3}, \frac{1}{3})$ which gives the game a value of 8.

By HW3Q2, all optimal strategies y for p2 satisfy $\begin{pmatrix} 1 & 0 & 1/3 \\ 0 & 1 & 2/3 \end{pmatrix} y = \left(\frac{2}{3}, \frac{1}{3}\right)^T$

Solving the inhomogeneous system of linear equations gives $\{\frac{2}{3}-t, \frac{1}{3}-2t, 3t\}$ where $t \in \mathbb{R}$. to Finish we must restrict so all probs are nonnegative and sum to 1.

$\frac{2}{3} - t \geq 0 \wedge \frac{1}{3} - 2t \geq 0 \wedge 3t \geq 0$

$\Rightarrow t \geq 0$

and

$\frac{2}{3} - t + \frac{1}{3} - 2t + 3t = 1 \Rightarrow t \leq \frac{1}{6}$

Because there are no pure strategies because $\{a_{ij} \neq v : a_{ij} \in A\}$ for all cells

pI optimal: $\left(\frac{4}{5}, \frac{1}{5}\right)$

pII optimal: $\left\{\left(\frac{2}{3} - t, \frac{1}{3} - 2t, 3t : 0 \leq t \leq \frac{1}{6}\right)\right\}$

5.

a)

i)
$$\begin{pmatrix} (3,5) & (9,19) \\ (7,8) & (1,10) \end{pmatrix}$$

$$4 + 2 + 8 - 14 = 0$$ potential game

for a 2x2 game there is only 1 potential function

ii)
$$\begin{pmatrix} (7,6) & (9,5) & (12,9) \\ (1,4) & (3,4) & (3,3) \\ (6,6) & (2,2) & (6,9) \end{pmatrix}$$

$$1 + 3 + 6 - 3 = 7 \neq 0$$

the game is not a potential game because there is a non zero cycle (shown in red)

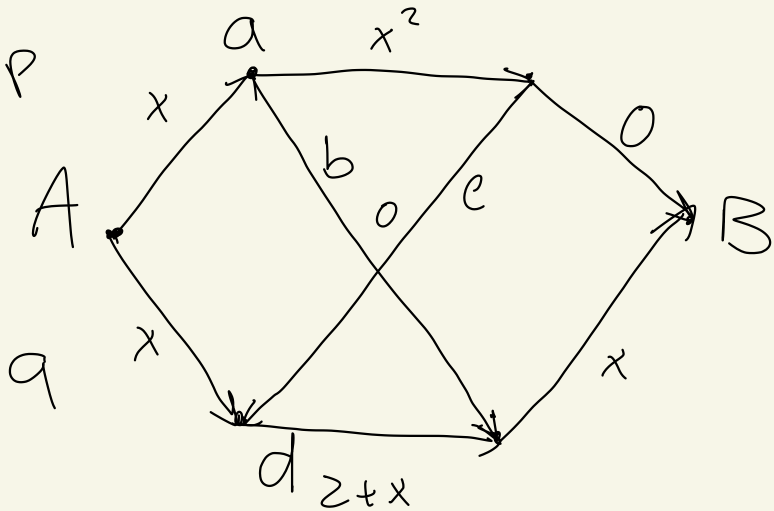
b) this game can be thought of as a congestion game where

$\{$primes from 1 to 1000$\}$ are the facilities

each player i picks a set $D(k_i)$ for some k_i between 1 and 1000.

and $c(k_i)$ is the negative of the utility function that counts the number of overlapping prime factors

congestion games are guaranteed to have a pure Nash Equilibrium



$$a+b+c+(a+c)^2$$

$$a+b+c+b+c+d$$

$$a+b+c+(a+c)^2+2+c+d+b+c+d$$

$$d+2+c+d+b+c+d$$

3.

a)

$$\begin{array}{ccc} & A & B \\ & y & 1-y \\ \begin{array}{c} A \quad x \\ B \quad 1-x \end{array} & \left(\begin{array}{cc} (1,2) & (3,2) \\ (2,4) & (2,0) \end{array}\right) \end{array}$$

pure NE: (B,A) (A,B)

mixed NE:

$$\cancel{2x} + 4 - 4x = \cancel{2x} \qquad y + 3 - 3y = \cancel{2y} + 2 - \cancel{2y}$$

$$x = 1 \qquad 1 = 2y$$

$$y = \frac{1}{2}$$

this method also finds all possible {mixed, pure} strategies as well

there is a mixed NE for strategies $p1 = (1,0)$ and $p2 = \left(\frac{1}{2}, \frac{1}{2}\right)$

b) first find all NE

$$\begin{array}{c|ccc} & A & B & C \\ A & (1,1) & \underline{(6,7)} & (1,3) \\ B & \underline{(7,6)} & (0,0) & (1,3) \\ C & (3,1) & (3,1) & \underline{(2,2)} \end{array}$$

Pure NE: $(A,B), (B,A), (C,C)$
(symmetric)

finding fully mixed strategies:
using equalization with $p+q+r=1$

$$\cancel{p} + 6q + 1 - \cancel{p} - \cancel{q} = 7p + 1 - p - \cancel{q}$$

$$6q + 1 = 6p + 1$$

$p = q$ and $p = r = q$ by symmetry

there is a fully mixed strategy of
$\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

finding mixed strategies weighted on only two actions?

$$\begin{array}{cccc} & A & B & C \\ A & (1,1) & \underline{(6,7)} & (1,3) \\ B & \underline{(7,6)} & (0,0) & (1,3) \\ C & (3,1) & (3,1) & \underline{(2,2)} \end{array}$$

for $(x, 1-x, 0)$

$x + 6 - 6x = 7x$

$\boxed{x = \frac{1}{2}}$ $\boxed{\left(\frac{1}{2}, \frac{1}{2}, 0\right)}$

the payoffs for pII

$x^T A$ where $x = \left(\frac{1}{2}, \frac{1}{2}, 0\right)$

$= (3.5 \quad 3.5 \quad 3)$

so y must put all it's weight on A and B

$y + 6 - 6y = 7y$

$\boxed{y = \frac{1}{2}}$ $\boxed{\left(\frac{1}{2}, \frac{1}{2}, 0\right)}$

for $(x, 0, 1-x)$

$7x + 1 - x = 3x + 2 - 2x$

$6x + 1 = x + 2$

$\boxed{x = \frac{1}{5}}$ $\boxed{\left(\frac{1}{5}, 0, \frac{4}{5}\right)}$

the payoffs for pII

$x^T A$ where $x = \left(\frac{1}{5}, 0, \frac{4}{5}\right)$

$= (1, 2.2, 2.2)$

y must put all weight on B and C

$6y + 1 - y = 3y + 2 - 2y$

$5y + 1 = y + 2$

$\boxed{y = \frac{1}{4}}$ $\boxed{\left(0, \frac{1}{4}, \frac{3}{4}\right)}$

all strategies are $(1,0,0)$, $(0,1,0)$, $(0,0,1)$,
$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$, $\left(\frac{1}{2},\frac{1}{2},0\right)$,
$\left(\frac{1}{5},0,\frac{4}{5}\right)$, $\left(0,\frac{1}{4},\frac{3}{4}\right)$

and are symmetric

X is ESS if for any pure strategy z

(a) $z^T A x \le x^T A x$

(b) If $z^T A x = x^T A x$, then $z^T A z < x^T A z$

$$\begin{bmatrix}1\\0\\0\end{bmatrix} A \begin{bmatrix}1&0&0\end{bmatrix} \le \begin{bmatrix}1\\0\\0\end{bmatrix} A \begin{bmatrix}1&0&0\end{bmatrix} \quad 1 \le 1$$

$$\begin{bmatrix}0\\1\\0\end{bmatrix} A \begin{bmatrix}1&0&0\end{bmatrix} \le \begin{bmatrix}1\\0\\0\end{bmatrix} A \begin{bmatrix}1&0&0\end{bmatrix} \quad \cancel{7 \le 1}$$

not ESS

$$\begin{bmatrix}0\\0\\1\end{bmatrix} A \begin{bmatrix}1&0&0\end{bmatrix} \le \begin{bmatrix}1\\0\\0\end{bmatrix} A \begin{bmatrix}1&0&0\end{bmatrix} \quad \cancel{3 \le 1}$$

$$\begin{bmatrix}1\\0\\0\end{bmatrix} A \begin{bmatrix}0&1&0\end{bmatrix} \le \begin{bmatrix}0\\1\\0\end{bmatrix} A \begin{bmatrix}0&1&0\end{bmatrix} \quad \cancel{6 \le 0}$$

not ESS

$$\begin{bmatrix}0\\1\\0\end{bmatrix} A \begin{bmatrix}0&1&0\end{bmatrix} \le \begin{bmatrix}0\\1\\0\end{bmatrix} A \begin{bmatrix}0&1&0\end{bmatrix} \quad 0 \le 0$$

$$\begin{bmatrix}0\\0\\1\end{bmatrix} A \begin{bmatrix}0&1&0\end{bmatrix} \le \begin{bmatrix}0\\1\\0\end{bmatrix} A \begin{bmatrix}0&1&0\end{bmatrix} \quad \cancel{3 \le 0}$$

$$\begin{bmatrix}1\\0\\0\end{bmatrix} A \begin{bmatrix}0 & 0 & 1\end{bmatrix} \le \begin{bmatrix}0\\0\\1\end{bmatrix} A \begin{bmatrix}0 & 0 & 1\end{bmatrix} \quad 1 \le 2$$

$$\begin{bmatrix}0\\1\\0\end{bmatrix} A \begin{bmatrix}0 & 0 & 1\end{bmatrix} \le \begin{bmatrix}0\\0\\1\end{bmatrix} A \begin{bmatrix}0 & 0 & 1\end{bmatrix} \quad 1 \le 2$$

$$\begin{bmatrix}0\\0\\1\end{bmatrix} A \begin{bmatrix}0 & 0 & 1\end{bmatrix} \le \begin{bmatrix}0\\0\\1\end{bmatrix} A \begin{bmatrix}0 & 0 & 1\end{bmatrix} \quad 2 \le 2$$

since $z^T A x = x^T A x$, check $z^T A z < x^T A z$

$\cancel{2 < 2}$ not ESS

$$\begin{bmatrix}1\\0\\0\end{bmatrix} A \begin{bmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\end{bmatrix} \le \begin{bmatrix}\frac{1}{3}\\ \vdots\end{bmatrix} A \begin{bmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\end{bmatrix} \quad \frac{8}{3} \le \frac{8}{3}$$

$$\begin{bmatrix}0\\1\\0\end{bmatrix} A \begin{bmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\end{bmatrix} \le \begin{bmatrix}\frac{1}{3}\\ \frac{1}{3}\\ \frac{1}{3}\end{bmatrix} A \begin{bmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\end{bmatrix} \quad \frac{8}{3} \le \frac{8}{3}$$

$$\begin{bmatrix}0\\0\\1\end{bmatrix} A \begin{bmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\end{bmatrix} \le \begin{bmatrix}\frac{1}{3}\\ \frac{1}{3}\\ \frac{1}{3}\end{bmatrix} A \begin{bmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\end{bmatrix} \quad \frac{8}{3} \le \frac{8}{3}$$

since $z^T A x = x^T A x$, check $z^T A z < x^T A z$

$1 < \frac{11}{3}$, $0 < 3$, $\cancel{2 < \frac{4}{3}}$ not ESS

$\begin{bmatrix}1\\0\\0\end{bmatrix} A \begin{bmatrix}\frac{1}{2} & \frac{1}{2} & 0\end{bmatrix} \le \begin{bmatrix}\frac{1}{2}\\\frac{1}{2}\\0\end{bmatrix} A \begin{bmatrix}\frac{1}{2} & \frac{1}{2} & 0\end{bmatrix}$ $\quad \frac{7}{2} \le \frac{7}{2}$

$\begin{bmatrix}0\\1\\0\end{bmatrix} A \begin{bmatrix}\frac{1}{2} & \frac{1}{2} & 0\end{bmatrix} \le \begin{bmatrix}\frac{1}{2}\\\frac{1}{2}\end{bmatrix} A \begin{bmatrix}\frac{1}{2} & \frac{1}{2} & 0\end{bmatrix}$ $\quad \frac{7}{2} \le \frac{7}{2}$

$\begin{bmatrix}0\\0\\1\end{bmatrix} A \begin{bmatrix}\frac{1}{2} & \frac{1}{2} & 0\end{bmatrix} \le \begin{bmatrix}\frac{1}{2}\\\frac{1}{2}\end{bmatrix} A \begin{bmatrix}\frac{1}{2} & \frac{1}{2} & 0\end{bmatrix}$ $\quad 3 \le \frac{7}{2}$

since $z^T A x = x^T A x$, check $z^T A z < x^T A z$

$1 < 4, \quad 0 < 3$ $\quad \boxed{\left(\frac{1}{2}, \frac{1}{2}, 0\right) \text{ is ESS}}$

$\begin{bmatrix}1\\0\\0\end{bmatrix} A \begin{bmatrix}\frac{1}{5} & 0 & \frac{4}{5}\end{bmatrix} \le \begin{bmatrix}\frac{1}{5}\\0\\\frac{4}{5}\end{bmatrix} A \begin{bmatrix}\frac{1}{5} & 0 & \frac{4}{5}\end{bmatrix}$ $\quad 1 < 1.96$

$\begin{bmatrix}0\\1\\0\end{bmatrix} A \begin{bmatrix}\frac{1}{5} & 0 & \frac{4}{5}\end{bmatrix} \le \begin{bmatrix}\frac{1}{5}\\0\\\frac{4}{5}\end{bmatrix} A \begin{bmatrix}\frac{1}{5} & 0 & \frac{4}{5}\end{bmatrix}$ $\quad \cancel{2.2 < 1.96}$

$\begin{bmatrix}0\\0\\1\end{bmatrix} A \begin{bmatrix}\frac{1}{5} & 0 & \frac{4}{5}\end{bmatrix} \le \begin{bmatrix}\frac{1}{5}\\0\\\frac{4}{5}\end{bmatrix} A \begin{bmatrix}\frac{1}{5} & 0 & \frac{4}{5}\end{bmatrix}$ $\quad \cancel{2.2 < 1.96}$

not ESS

$\begin{bmatrix}1\\0\\0\end{bmatrix} A \begin{bmatrix}0 & \frac{1}{4} & \frac{3}{4}\end{bmatrix} \le \begin{bmatrix}0\\\frac{1}{4}\\\frac{3}{4}\end{bmatrix} A \begin{bmatrix}0 & \frac{1}{4} & \frac{3}{4}\end{bmatrix}$ $\quad 2.25 < 1.875$

$\begin{bmatrix}0\\1\\0\end{bmatrix} A \begin{bmatrix}0 & \frac{1}{4} & \frac{3}{4}\end{bmatrix} \le \begin{bmatrix}0\\\frac{1}{4}\\\frac{3}{4}\end{bmatrix} A \begin{bmatrix}0 & \frac{1}{4} & \frac{3}{4}\end{bmatrix}$ $\quad 0.75 < 1.875$

$\begin{bmatrix}0\\0\\1\end{bmatrix} A \begin{bmatrix}0 & \frac{1}{4} & \frac{3}{4}\end{bmatrix} \le \begin{bmatrix}0\\\frac{1}{4}\\\frac{3}{4}\end{bmatrix} A \begin{bmatrix}0 & \frac{1}{4} & \frac{3}{4}\end{bmatrix}$ $\quad 2.25 < 1.875$

not ESS

$\boxed{\text{All ESS: } \left(\frac{1}{2}, \frac{1}{2}, 0\right)}$

c)

x is ESS under a small invasion of population $\left(\frac{2}{3}\ \frac{1}{3}\ 0\right)$

(a) $z^T A x \leq x^T A x$

(b) If $z^T A x = x^T A x$, then $z^T A z < x^T A z$

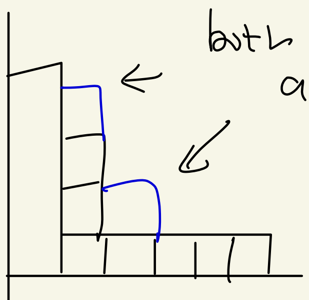
$\begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ 0 \end{bmatrix} A \left[\frac{1}{2}\ \frac{1}{2}\ 0\right] \leq x^T A x \Rightarrow 3.5 \leq 3.5$

check (b) $z^T A z < x^T A z = \frac{10}{3} < \frac{11}{3}$

$\boxed{x \text{ is stable}}$

2.

b) i) for any state that corresponds to a non increasing function, the only valid moves are on values of a_i that are greater than 1 and less than $n-1$ and $a_{i-1} > a_i$. This is so half of the sides will be touched. A valid move can only add to a column up to the column height before it.



both are valid because $a_{i-1} > a_i$ and $a_i > 1$

using induction, assume a valid move was made. the state will maintain its non increasing property because no move can bring any column higher than the one before it.

ii) from (i) we know that all columns valid moves can only bring to the height of the column directly before it. The very first column is of height n by construction so all valid moves will not create a column taller than n. Thus as more moves are played, all columns that have at least one block in them will eventually fill their column up to n.

therefore, the number of turns it takes to reach a terminal state is $mn - \text{sum}(\{a_i\}_{i \in \mathbb{Z}_{\geq 0}})$ which defines the excess area.

iii) for the n-staircase, the only legal moves reduce the excess are by 1 because the valid move criteria ($a_{i-1} > a_i \wedge a_i > 1$) a_i is defined to be one less then a_{i-1} and no move can surpass the height of colum a_{i-1}.

let $P' = \{\{a_i\}_{i \in \mathbb{Z}_{\geq 0}} : E(a) \% 2 = 0\}$

let $N' = \{\{a_i\}_{i \in \mathbb{Z}_{\geq 0}} : E(a) \% 2 \neq 0\}$

suppose $a \in P'$, this would be

the specific case of the n-staircase where $E(a)$ is even but the only legal move is 1 which brings to ~~a~~ a state in N.

suppose $a \in N'$, if $E(a)$ is odd then there exist a move by placing a 1×1 block that makes it even.

Thus $P = P'$ ~~and~~ $N = N'$ when

the n-staircase is in P which is losing

4.

III: purify

I \ II	purity	pollute
purity	(1,1,1)	(1,0,1)
pollute	(0,1,1)	(3,3,3)

III: pollute

I \ II	purity	pollute
purity	(1,1,0)	(3,3,3)
pollute	(3,3,3)	(3,3,3)

Pure NE are (pur, pur, pol), (pol, pur, pur)
(pur, pol, pur)

to find {pure, mixed, mixed} strategies
lock p3 to their pure strategies and use
symmetry.

$$\begin{pmatrix} (1,1,1) & (1,0,1) \\ (0,1,1) & (3,3,3) \end{pmatrix}$$

pIII purity

$\cancel{x} + 1 - \cancel{x} = 0 + 3 - 3x$ $\quad y + 1 - y = 0 + 3 - 3y$

$3x = 2$ $\quad y = \frac{2}{3}$

$x = \frac{2}{3}$

pIII pollute

$$\begin{pmatrix} (1,1,0) & (3,3,3) \\ (3,3,3) & (3,3,3) \end{pmatrix}$$

$x + \cancel{3-3x} = 3x + \cancel{3-3x}$

$x = 0$

no (pure, mixed, mixed) strategies

with pure = pollute.

(pure mixed, mixed) NE:

$$(\text{pur}, (\tfrac{2}{3}, \tfrac{1}{3}), (\tfrac{2}{3}, \tfrac{1}{3})), ((\tfrac{2}{3}, \tfrac{1}{3}), (\tfrac{2}{3}, \tfrac{1}{3}), \text{pur})$$

$$((\tfrac{2}{3}, \tfrac{1}{3}), \text{pur}, (\tfrac{2}{3}, \tfrac{1}{3}))$$

for fully mixed strategies:

$$C_i(\text{pur}, x_{-i}) = C_i(\text{pol}, x_{-i})$$

1 1 1 3   0 3 3 3

$$p_1 p_2 + p_1(1-p_2) + p_2(1-p_1) + \cancel{3(1-p_1)(1-p_2)}$$

$$= 0 + 3p_1(1-p_2) + 3p_1(1-p_2) + \cancel{3(1-p_1)(1-p_2)}$$

$$\cancel{p_1 p_2} + p_1 - \cancel{p_1 p_2} + p_2 - p_1 p_2$$

$$p_1 + p_2 - p_1 p_2$$

$$= 3p_1 - 3p_1 p_2 + 3p_2 - 3p_1 p_2$$

$$3p_1 + 3p_2 - 6p_1p_2 = p_1 + p_2 - p_1p_2$$

$$(0)\ 2p_1 + 2p_2 - 5p_1p_2 = 0$$

$$(1)\ 2p_2 + 2p_3 - 5p_2p_3 = 0$$

$$(2)\ 2p_1 + 2p_3 - 5p_1p_3 = 0$$

$$(0) - (2) = 2p_2 - 2p_1 - 5p_1p_2 + 5p_1p_3$$

$$2p_2 - 5p_1p_2 - 2p_1 + 5p_1p_3$$

$$p_2(2 - 5p_1) - p_1(2 - 5p_1)$$

$$0 = (p_2 - p_1)(2 - 5p_1)$$

$p_1 = p_2$ and $p_1 = \frac{2}{5}$

plugging in $p_1 = \frac{2}{5}$ $p_2 = 0$ and $p_3 = 0$ so throw it out

$$4p_1 = 5p_1^2$$

$$p = \frac{4}{5}$$

Fully mixed NE:

$$\left(\left(\frac{4}{5}, \frac{1}{5}\right), \left(\frac{4}{5}, \frac{1}{5}\right), \left(\frac{4}{5}, \frac{1}{5}\right)\right)$$

p1 = they/them

b) if p1 is told to pollute, they know the other two were told to purify so there is no incentive to deviate

if p1 is told to purify, they are either one of the 2 people who were told to purify or the 3 people that were told to purify.

let

$q(p) = P(\text{everyone was told to purify} \mid p1 \text{ was told to purify})$

If they follow directions, p1's expected payoff is $1 \cdot q(p) + 1(1-q(p)) = 1$

if they do not follow directions, and pollute instead, their payoff is

$$0 \cdot q(p) + 3(1-q(p)) = 3-3q(p)$$

this strategy is a correlated equilibrium iff $1 \geq 3 - 3q(p)$

cost (minimize)

$$q(p) = \frac{p}{p + \frac{2}{3}(1-p)}$$

$$\frac{1}{3} \leq \frac{\frac{2}{3}(1-p)}{p + \frac{2}{3}(1-p)} \qquad \frac{2}{3} \leq \frac{p}{p + \frac{2}{3}(1-p)}$$

$$2p + \frac{4}{3}(1-p) \leq 3p$$

$$1 - p \leq \frac{3}{4}p$$

$$1 \leq \frac{7}{4}p$$

$$\boxed{p = \frac{4}{7}}$$

c) finding all correlated equilibria is finding $z = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that

$$(1-0)a + (1-3)b \geq 0$$
$$(0-1)c + (3-1)d \geq 0$$
$$(1-0)a + (1-3)c \geq 0$$
$$(0-1)b + (3-1)d \geq 0$$

I \ II	purity	pollute
purity	(1,1,1)	(1,0,1)
pollute	(0,1,1)	(3,3,3)

$$\begin{array}{l} a - 2b \geq 0 \\ 2d - c \geq 0 \\ a - 2c \geq 0 \\ 2d - b \geq 0 \\ a+b+c+d = 1 \end{array} \;=\; \boxed{\begin{array}{l} a \geq 2b \\ 2d \geq c \\ a \geq 2c \\ 2d \geq b \\ a+b+c+d = 1 \end{array}}$$

these equations

7. a) $\begin{pmatrix} (2,1) & (1,1) & (4,4) & (3,3) \\ (3,3) & (6,2) & (6,6) & (2,2) \\ (2,2) & (8,8) & (4,3) & (1,1) \\ (3,3) & (0,0) & (3,3) & (5,1) \end{pmatrix}$

$A = \begin{pmatrix} 2 & 1 & 4 & 3 \\ 3 & 6 & 6 & 2 \\ 2 & 8 & 4 & 1 \\ 3 & 0 & 3 & 5 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 1 & 4 & 3 \\ 3 & 2 & 6 & 2 \\ 2 & 8 & 3 & 1 \\ 3 & 0 & 3 & 1 \end{pmatrix}$

$A - B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ $A + B = \begin{pmatrix} 3 & 2 & 8 & 6 \\ 6 & 8 & 12 & 4 \\ 4 & 16 & 7 & 2 \\ 6 & 0 & 6 & 6 \end{pmatrix}$

We can find a safety strategy from A-B by recognizing that p1 will either choose 1 or 4 and the game can be reduced to

$$\begin{array}{c} x \\ 1-x \end{array}\begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix} \quad x = 4-4x$$

$$x = \frac{4}{5}$$

without loss of generality as long as 4/5 is divided by 2 for the two possible choices for y

$$\boxed{\left(\frac{1}{10}, \frac{2}{5}, \frac{2}{5}, \frac{1}{10}\right) \text{ for p1 and because A-B is symmetric } \left(\frac{1}{10}, \frac{2}{5}, \frac{2}{5}, \frac{1}{10}\right)}$$

is also a safety strat for p2.

to find disagreement point, $(x_*^T A y_*, x_*^T B y_*)$

disagreement point $= (4.61, 3.76)$

for A+B, the matrix is maximized

at (e_3, e_2) where the max is $\sigma = 16$

and the vale of A-B is $2/5$

$$\frac{16 + \frac{2}{5}}{2}, \frac{16 - \frac{2}{5}}{2}$$

$\left(\frac{83}{10}, \frac{78}{10}\right)$ is the final payoff

the solution for the game is to have

p1 play e_3 and p2 to play e_2

and p1 transfers $8.2 - 2/5 = 7.8$ to

pII

b)

i) the solution to this game must be on the Pareto boundary and $\boxed{(3,0)}$ is the only point on the boundary

ii) the solution is the projection of the disagreement point onto the Pareto boundary. Because the feasible set has an edge with slope of -1 when can find the intersection of $y=x$ and Pareto boundary.

$2 = -8 + b$

$b = 10$

$x = -x + 10$

$y = -x + 10 \quad y = x$

$x = 5$ and $y = 5$

the solution is $\boxed{(5,5)}$

8.