MATH 53 WORKSHEET 11/28

(1) (a) Consider a simple solid region D with smooth boundary ∂D. Let f and g be smooth functions. Show that

$$\int\int\int_D f\Delta g\; dV = \int\int_{\partial D} f\nabla g \cdot d\mathbf{S} - \int\int\int_D \nabla f \cdot \nabla g\; dV.$$

Note here that Δ denotes the Laplacian operator on a function.

(b) Find a function f such that

$$\int\int_{\partial D} \nabla g \cdot d\mathbf{S} = \int\int\int_D \Delta g\; dV$$

(2) Find the flux of the vector field

$$\mathbf{F}(x, y, z) = \langle y^3 z, x^3 z, 1 + e^{x^2+y^2} \rangle$$

through the surface given by $x^2 + y^2 + z \leq 1$ above the xy-plane, oriented upwards.

(3) Consider the vector field

$$\mathbf{F}(x, y, z) = \frac{1}{(x^2 + y^2 + z^2)^{3/2}} \langle x, y, z \rangle.$$

(a) What is the domain of $\mathbf{F}$?

(b) Consider a closed bounded solid region Q in the domain of $\mathbf{F}$ with smooth boundary surface ∂Q. Show that

$$\int\int_{\partial Q} \mathbf{F} \cdot d\mathbf{S} = 0.$$

(c) Consider S, a sphere of radius R centered at the origin. Show that

$$\int\int_{S} \mathbf{F} \cdot d\mathbf{S} = 4\pi.$$