Game Theory (STAT 155) Worksheet #6 03/01/2021

1. Find some pure Nash equilibrium in the following congestion game:

   - $k = 3$ players,
   - $m = 3$ facilities $\{1 \dots m\}$,
   - For player i , there is a set S_i of strategies that are subsets of facilities, $s \subseteq \{1 \dots m\}$, $S_1 = S_2 = S_3 = \big\{\{1,2\},\{3,2\},\{1,3\}\big\}$.
   - For facility j , there is a cost vector $c_j \in \mathbb{R}^k$, where $c_j(n)$ is the cost of facility j when it is used by n players.

$$c_1 = (1,2,3), \quad c_2 = (3,2,1), \quad c_3 = (2,2,2).$$

2. Show that the following games are not potential:

$$\begin{pmatrix} (5,3) & (1,5) \\ (0,4) & (3,3) \end{pmatrix}, \quad \begin{pmatrix} (2,1) & (3,1) & (2,3) \\ (2,1) & (4,2) & (2,2) \\ (3,2) & (4,0) & (1,3) \end{pmatrix}$$

3. Consider the following general-sum game of n players. For each i from 1 to n the i-th player picks a natural number k_i from 1 to 1000. Denote the set of natural divisors of k as $D(k)$. The utility functions are the following:

$$u_i(k_i, k_{-i}) = \sum_{d \in D(k_i)} \#\{j : k_j \neq 0 \bmod d\}.$$

   Show that this game has a pure Nash equilibrium.