MATH 135: SET THEORY
SECOND MIDTERM

Name: ____ **SID:** ___________________________

Instructions

(1) **Very important:** Write your name and SID on **both sides** of each page. This exam will be scanned and then graded from a computer screen. It is very likely that individual pages may appear in the file out of order and will thus not be credited correctly if the name is missing.
(2) Write your answers in the space provided. If you find that you need additional space, you may use an additional blank piece of paper provided to you. Write your name, SID and problem number on each side of each additional page. On the page where the problem is set, write "Additional work to be found on attached sheets."
(3) When you are finished, stack your papers in order, and fasten them with the paper clip. Do **not** use a staple.
(4) Each question is worth ten points.
(5) The extra credit at the end of the exam is worth an additional three points.

Date: 27 October 2014.

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For the first two questions, in addition to the primitive notions of $x \in y$ and $x = y$, you may take the following terms as primitive:

`function`, `relation`, `ordered pair`, $z = \langle x, y\rangle$, $z = \{x, y\}$, $z = \{x\}$, $y = \bigcup x$, $y = \mathcal{P}x$, $x \subseteq y$, `subset`, $x = \varnothing$, $x \notin y$, $z = x \cup y$, $z = x \cap y$, `transitive relation`, `equivalence relation`, `symmetric relation`, $y = R^{-1}$, $y = R \circ S$, $y = f(x)$, $y = f[[x]]$, $y = \operatorname{dom} R$, $y = \operatorname{ran} R$, $y = \operatorname{fld} R$, `equivalence class`, $y = X/E$, $y = [x]_E$, `one-to-one`, `onto`, $z = x \times y$.

1. For each of the following questions give a *formal definition* of the following terms. You may write your definition using other defined terms, but these must also be defined.

a. A set I is *inductive* if and only if

b. A set x is a *transitive* if and only if

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2. With the following questions you are asked to state precisely and formally some of the axioms of Zermelo-Frankel set theory which we have introduced. As with the definitions, you may use defined terms but those terms must be defined formally.

a. The Axiom of Infinity

b. The Union Axiom

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3. Prove that if $A \subseteq \omega$ is nonempty and bounded (meaning that there is some $n \in \omega$ for which $(\forall a \in A)[a \leq n]$), then A has a greatest element. That is, there is some $m \in A$ so that $(\forall a \in A)[a \leq m]$.

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4. Recall that for sets X and Y,

$$^{X}Y = \{f \subseteq X \times Y \mid f : X \to Y \text{ is a function from } X \text{ to } Y\} \ .$$

Prove that for all natural numbers m and n, $^{n}m \approx m^n$ where $m^n \in \omega$ is the natural number m raised to the n^{th} power as defined by the theorem on recursion from repeated multiplication.

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5. Suppose that Y is a finite set, $f : X \to Y$ is a function from X to Y, and for all $y \in Y$ the set $f^{-1}\{y\}$ is finite. Prove that X is finite.

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6. Assume that A is a set, G is a function, and $f_1 : \omega \to A$ and $f_2 : \omega \to A$ are two function from ω to A. Further assume that for each $n \in \omega$, $f_1 \upharpoonright n \in \operatorname{dom} G$, $f_2 \upharpoonright n \in \operatorname{dom} G$ and

$$f_1(n) = G(f_1 \upharpoonright n) \text{ and } f_2(n) = G(f_2 \upharpoonright n) \ .$$

Show that $f_1 = f_2$.

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Extra credit: Construct a one-to-one and onto function $f : (0,1) \to \mathbb{R}$ which takes rational numbers to rational numbers and irrational numbers to irrational numbers. (You should prove that your function has the required properties.)

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For which question should the material on this page be credited? ________________________

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