MATH 53 PRACTICE EXAM 3

Please note that this practice test is designed to give you examples of the types of questions that will appear on the actual exam. Your performance on these sample questions should not be taken as an absolute prediction of your performance on the actual exam. When you take the actual exam, the questions you see will cover some of the same content included in the practice exam, as well as some content not tested on the practice exam.

(1) Rewrite the following integral as an equivalent iterated integral in the five other orders:

$$\int_0^1 \int_0^{1-y^2} \int_0^{1-y} dz dx dy.$$

(2) True or False. Give a short explanation for each. You may cite any definitions or theorems that have been covered so far.

(a) The region $A = \{(x, y) : \; |x^2 + y^2 - 2| < 1\}$ is a simply connected region in the xy-plane.

(b) If $\mathbf{F}(x, y, z)$ is a vector field whose divergence is zero everywhere, and $f(x, y, z)$ is a scalar function, then $\text{div}(f\mathbf{F})$ is zero.

(c) Let $\mathbf{F}(x, y, z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ have continuous first partial derivatives in a open connected region R, and let C be a piecewise smooth curve in R. If $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for every closed path C in R, then $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path in R.

(3) Find the mass of the wire in the shape of a helix

$$\mathbf{r}(t) = \frac{1}{\sqrt{2}}(\cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t\mathbf{k}), \quad 0 \le t \le 6\pi$$

where the density of the wire is $\rho(x, y, z) = 1 + z$.

(4) Consider the solid P bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$. Suppose the density at each point is proportional to the distance between the point and the z-axis. Find the moment of inertia about the axis of symmetry.

(5) Let R be the region bounded by the lines $x - 2y = 0$, $x - 2y = -4$, $x + y = 4$, and $x + y = 1$. Evaluate $\int_R \int 3xy \, dA$ using the change of coordinates $x = \frac{1}{3}(2u + v)$ and $y = \frac{1}{3}(u - v)$.

(6) Let C_1 be the boundary of the the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, oriented in the counterclockwise direction, and let C_2 be the boundary of the circle $x^2 + y^2 = 1$, oriented in the clockwise direction. Let R be the region inside the ellipse and outside the circle. Use Green's Theorem to evaluate the line integral

$$\int_C 2xydx + (x^2 + 2x)dy,$$

where $C = C_1 + C_2$ is the boundary of R.

(7) Find the work done by $\mathbf{F} = (e^x \cos y + yz)\mathbf{i} + (xz - e^x \sin y)\mathbf{j} + (xy + z)\mathbf{k}$ on an object moving along a curve from the north pole to the south pole on the surface of a sphere of radius 2 centered at $(1, \frac{\pi}{2}, 0)$. Assume that north is in the positive z direction.