

MATH 53 PRACTICE EXAM 2

Please note that this practice test is designed to give you examples of the types of questions that will appear on the actual exam. Your performance on these sample questions should not be taken as an absolute prediction of your performance on the actual exam. When you take the actual exam, the questions you see will cover some of the same content included in the practice exam, as well as some content not tested on the practice exam.

- (1) Find the maximum and minimum values of the function $g(x,y) = x$ in the region given by $2x^2 + 6xy + 9y^2 \leq 9$.

(2) Find the limit or show it does not exist:

(a)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}.$$

(b)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{3xy}{x^2 + y^2}.$$

- (3) Draw and label some level curves of $f(x, y) = \frac{-4x}{x^2+y^2+1}$. Find the absolute extrema of this function and where they occur.

- (4) Find and classify all critical points for the function $f(x, y) = y \sin x$.

- (5) (a) Find $\partial z/\partial x$ and $\partial z/\partial y$ at $(0, 0, 0)$ if $x^3 + z^2 + z \cos y = 0$.
(b) Find $\partial w/\partial r$ when $r = 1, s = -1$ if $w = (x + y + z)^2, x = r - s, y = \cos(r + s), z = \sin(r + s)$.

- (6) Show that if $w = f(u, v)$ satisfies the equation $f_{uu} + f_{vv} = 0$ and if $u = (x^2 - y^2)/2$ and $v = xy$, then w satisfies the equation $w_{xx} + w_{yy} = 0$.

- (7) Find the equation of tangent plane to $f(x, y) = x^2 \cos(\pi y)$ at the point $(2, -1)$ and the equation of a normal line that passes through $(2, -1)$. Find the length of the path a particle would take as it moves along the surface of $f(x, y) = x^2 \cos(\pi y)$ from the point $(2, -1, 4)$ to $(2, 1, 4)$ on the trace in the plane $x = 2$.