Game Theory (STAT 155) Midterm 03/10/2020 8:00 AM Pacific time- 03/11/2020 8:00 AM Pacific time. To be uploaded via gradescope.

Read the following carefully.

- No collaboration is allowed. You are supposed to only consult the textbook and any other material posted on bcourses and nothing else. Also no electronic devices should be used to perform computations.
- Questions may not appear in order of difficulty.
- You are allowed to type up your solutions. If uploading handwritten solutions, please write as neatly as you can. We will not grade what is not legible.
- Present your solution in a clear and coherent manner. We will deduct points for significant logical gaps.
- Unless explicitly indicated otherwise, you must justify your answer to receive full credit. You can cite any theorem from lecture or homework without reproving it - when doing so, you must write down clearly the relevant parts of the theorem statement.
- Any announcements during the exam will be made on bcourses. For clarifications, you should mail the instructor or the GSI. Responses will be provided only if deemed appropriate.

1. Find the values of the zero-sum games given by the following payoff matrices, and determine all the optimal strategies for both players in the games:

a) [5 points]

$$\begin{pmatrix} 19 & 1 & 4 & 1 & 3 \\ 22 & 3 & 8 & 4 & 2 \\ 14 & 6 & 7 & 6 & 7 \\ 15 & 2 & 1 & 2 & 1 \\ 23 & 1 & 9 & 1 & 5 \end{pmatrix}$$

b) [5 points]

$$\begin{pmatrix} 2 & 5 & 12 & 3 & 13 \\ 8 & 4 & 6 & 7 & 9 \end{pmatrix}$$

2. Answer the following questions:

a) [4 points] Consider the following game. From any state $x \in \mathbb{Z}_{\geq 1} = \{1, 2, \ldots\}$, a legal move is to move to $x + 2^k$, for any non-negative integer k such that 2^k divides x. The first player to move to a state larger than 2021 is the winner. If the game starts in the state 2, which player has a winning strategy?

b) [2 points] Consider the subtraction game with one pile where players may remove $1, 2, 3, 4$ or 6 chips. Under normal play classify all the starting states i.e., elements of $\{0, 1, 2, \ldots, \}$, into N or P positions and determine which player wins if the game starts with a pile of 1173 chips.

c) [4 points] In a subtraction game there are 3 piles where the i^{th} pile has n_i chips. Players must choose one pile, and if their choice is the i^{th} pile they must remove a number, in the set $\{1, 2, \ldots, k_i - 1, 3k_i + 1\}$, of chips from the chosen pile. If $(k_1, k_2, k_3) = (4, 6, 12)$, then is $(n_1, n_2, n_3) = (100, 25, 32)$ a P or an N?

3. Answer the following questions:

a) [3 points] Find all pure Nash Equilibria in the following two-player general-sum game with the strategies labelled $(1, 2, \ldots, 4)$ and $(1, 2, \ldots, 6)$ respectively.

$$\begin{pmatrix} (2,1) & (4,3) & (7,2) & (7,4) & (0,5) & (3,2) \\ (4,0) & (5,4) & (1,6) & (0,4) & (0,3) & (5,1) \\ (1,3) & (5,3) & (3,2) & (4,1) & (1,0) & (4,3) \\ (4,3) & (2,5) & (4,0) & (1,0) & (1,5) & (2,1) \end{pmatrix}.$$

b) Consider the following two-player game:

$$\begin{pmatrix} (4,4) & (8,9) \\ (9,8) & (3,3) \end{pmatrix}$$

i.) [3 points] Compute all pairs of safety strategies for both players and show that they are not in equilibrium.

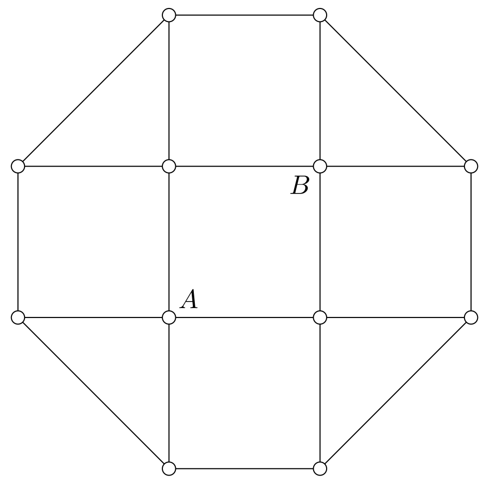
ii.) [4 points] Find all fully mixed Nash equilibrium, and show that they result in the same player payoffs as the safety strategies.

4. Answer the following questions:

a) A zebra has four possible locations to cross the Zembezi river, call them a, b, c, d, arranged from north to south. A crocodile can wait (undetected) at one of these locations. If the Zebra and the Crocodile choose the same location, the payoff

to the crocodile (since it will catch the zebra) is 3. The payoff to the crocodile is 1 if they choose adjacent locations (since the chance is slightly less), and 0 in the remaining cases, when the locations chosen are distinct yet non-adjacent.
i.) [2 points] Write the payoff matrix for this zero-sum game.
ii.) [2 points] Reduce this game to a 2×2 game.
iii.) [2 points] Find the value of the game (to the crocodile) and at least one optimal strategy for each player.

b) [4 points] The picture below describes a road network, where nodes represent sites and edges represent roads.



Player I chooses a path from A to B, and Player II chooses a site other than A and B to place a toll. If Player I's path drives through Player II's chosen site, she pays a \$1 toll to Player II. Else, there is no payment exchanged. Find the value of the game and some pair of optimal strategies.

5. Three firms (players I, II, and III) put three items (one each) on the market and advertise them either on morning or evening TV. A firm advertises exactly once per day. If more than one firm advertises at the same time, the profits of the firms advertising simultaneously are zero. If exactly one firm advertises in the morning, its profit is \$250. If exactly one firm advertises in the evening its profit is \$550. Firms must make their advertising decisions simultaneously.
a) [2 points] Write down the payoffs for various players in form of two matrices, one for each of the pure strategies for III. Explain your answers.
b) [8 points] Find all the symmetric mixed Nash equilibria.