Name: ______________________________
SID: ______________________________

MATH 135: SET THEORY
MIDTERM # 2

There are **six** questions. The first question has **five** parts. Write your answers in the space provided. If you need more space, you may continue your answers on the blank pages at the end of the exam. Please submit all of the pages, including the extra blank pages even if you have not written on them. **Mark your name, or at least your initials, on** EVERY **page.**

1a. (5pts) Express the **Axiom of Infinity** in the formal language having nonlogical symbols for $\in$, $\cup$ (the binary union as a binary function symbol), $\{\cdot\}$ (the singleton set operator as a unary function symbol), $\emptyset$ (as a constant symbol).

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1b. (5 pts) Express the **Power Set Axiom** using only the signature of set theory.

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1c. (5 pts) Give a precise formal definition of the relation $\mathrm{card}(M) = \mathrm{card}(K)^{\mathrm{card}(L)}$. You may use as primitives the relation f `is a function`, a binary function symbols for the ordered pair set $\langle \cdot, \cdot \rangle$ and $\times$, and unary function symbols ran, dom, and $\mathcal{P}$. [**Hint:** It may be easier to first define the relations $X \approx Y$ and $Z = {}^{Y}X$, and then to use these to define $\mathrm{card}(M) = \mathrm{card}(K)^{\mathrm{card}(L)}$.]

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1d. (5 pts) Give a precise formal definition of the relation $x = \mathbb{R}$. You may use as primitives a constant symbol for $\mathbb{Q}$, the binary relation symbol $<$ for the order relation on $\mathbb{Q}$, a constant symbol $\emptyset$ for the empty set, the power set $\mathcal{P}$ as a function symbol, and binary relation symbols $\subseteq$ for the subset relation and $\neq$ for inequality.

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1e. (5 pts) Give a precise formal definition of the condition $x = \mathbb{Z}$. You may use the following primitives: $\in$, ω, E `is an equivalence relation`, X/E (as a binary function symbol), $+$ (as a binary function symbol on ω), $\langle \cdot, \cdot \rangle$, and $\times$.

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2. (15 pts) **Prove:** If $A \subseteq \mathbb{R}$, $A \neq \emptyset$, and A is bounded from below: that is

$$(\exists b \in \mathbb{R})(\forall a \in A)\ b \leq a\ ,$$

then A has a greatest lower bound, that is, there us some

$$(\exists c \in \mathbb{R})([(\forall a \in A)\ c \leq a]\ \&\ (\forall d \in \mathbb{R})[((\forall a \in A)\ d \leq a) \rightarrow d \leq c])\ .$$

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3. (15 pts) **Prove (without using the Axiom of Choice):** If X is a finite set and $f : X \twoheadrightarrow Y$ is onto, then Y is finite.

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4. (15 pts) **Prove (without using the Axiom of Choice):** If $X \subseteq \omega$ is infinite, then $X \approx \omega$.

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5. (15 pts) **Prove:** For every nonempty set K, there does not exist a set $\mathbb{K}$ having the property that for all sets x, $x \in \mathbb{K}$ if and only if $x \approx K$.

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6. (15 pts) **Prove:** The Axiom of Choice implies that if X is infinite then there is a function $f : X \hookrightarrow X$ which is one-to-one, but is not onto. [Hint: It will be easier to use a result we proved in class about the consequences of the Axiom of Choice than to attempt to apply the Axiom of Choice directly.]

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For which problem should this work be credited?

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