Business Economics Homework

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2021-11-30

1 Question 1

We are given that there are $J+1$ goods; I interpret this as meaning there are $J+1$ goods on the market, *not including* the outside option. We are provided with the assumtions that: $J = 8$, $p_j = 10 - j/2$, $\xi_j = j/5$, and $x_j = 10 - j/10$, where p_j is the price of good j, ξ_j and x_j are some characteristics of good j that are common across individuals. Each person purchases at most one good, and the utility of the outside option is normalized to zero.

Assuming that individual indirect utility is given by:

$$u_{ij} = \beta x_j - \alpha p_j + \xi_j + \varepsilon_{ij}$$

and that $\varepsilon_{ij} \sim \text{GeneralizedExtremeValue}(0, 1, 0)$, I simulate a market with $N = 10000$ individuals and $J+1$ products. The dataframe of simulated data is described below:

	variable	mean	min	median	max	nmissing	eltype
	Symbol	Float64	Real	Float64	Real	Int64	DataType
1	i	5000.5	1	5000.5	10000	0	Int64
2	j	4.0	0	4.0	8	0	Int64
3	epsilon	0.57267	-2.48973	0.360029	12.0708	0	Float64
4	u	2.97267	-2.33889	2.93982	16.8708	0	Float64
5	chosen1	0.111111	0	0.0	1	0	Bool
6	p	8.0	6.0	8.0	10.0	0	Float64
7	xi	0.8	0.0	0.8	1.6	0	Float64
8	x	9.6	9.2	9.6	10.0	0	Float64
9	u_maximum	6.1698	3.14087	5.97472	16.8708	0	Float64
10	u_given_purchase	0.685534	-0.0	0.0	16.8708	0	Float64
11	alpha	1.0	1.0	1.0	1.0	0	Float64

Given that $\alpha = \beta = 1$, we have the following purchase decisions. The share of the individual choices can be interpreted in two equivalent ways: the share of the market captured by each product, or the probability of purchasing a given product *j*. Choice probability is calculated both directly (`prob_mean`) and by simulated market shares. Note that in this first case (Case 1), the market is covered, and also note that `cum_chosen` is the cumulative number of purchases.

	j	prob_mean	market_share1	cum_chosen1
	Int64	Float64	Float64?	Int64?
1	0	0.37	0.41	41
2	1	0.68	0.71	112
3	2	1.23	1.2	232
4	3	2.25	2.41	473
5	4	4.1	3.88	861
6	5	7.46	7.64	1625
7	6	13.6	13.37	2962
8	7	24.78	24.69	5431
9	8	45.16	45.69	10000

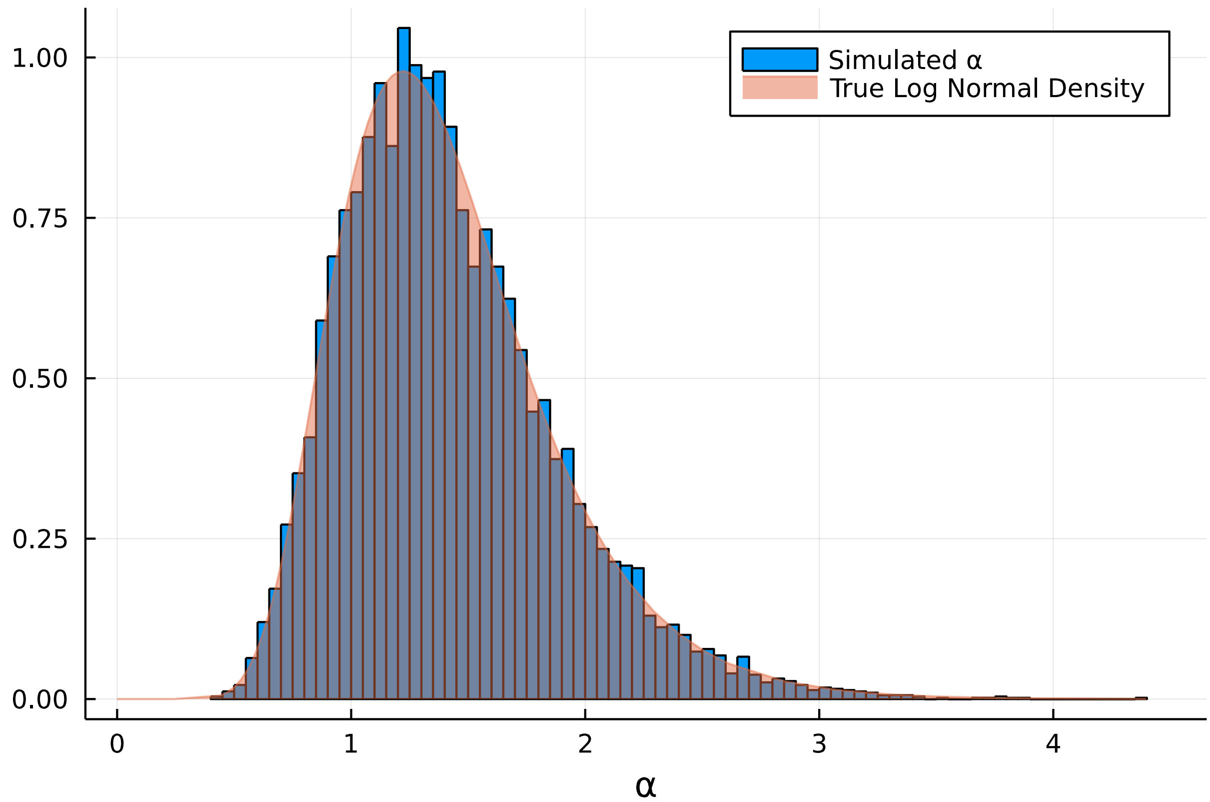
Let $P_i(j)$ indicate that individual i chooses good j. Since we assume that the distribution of ε_{ij} conditional on i is the same as its unconditional distribution:

$$P_i(0) = P(0) = 0.41\%$$
$$P(6) = 13.37\%$$

and so on, as in the table above.

2 Question 2

Now we assume that $\alpha_i \sim LogNormal(\mu = 0.3, \sigma = \sqrt{0.1})$. I again use 10000 individuals to simulate this market; I use more than the suggested 100 draws to more closely approximate the true choice probabilities / market shares.

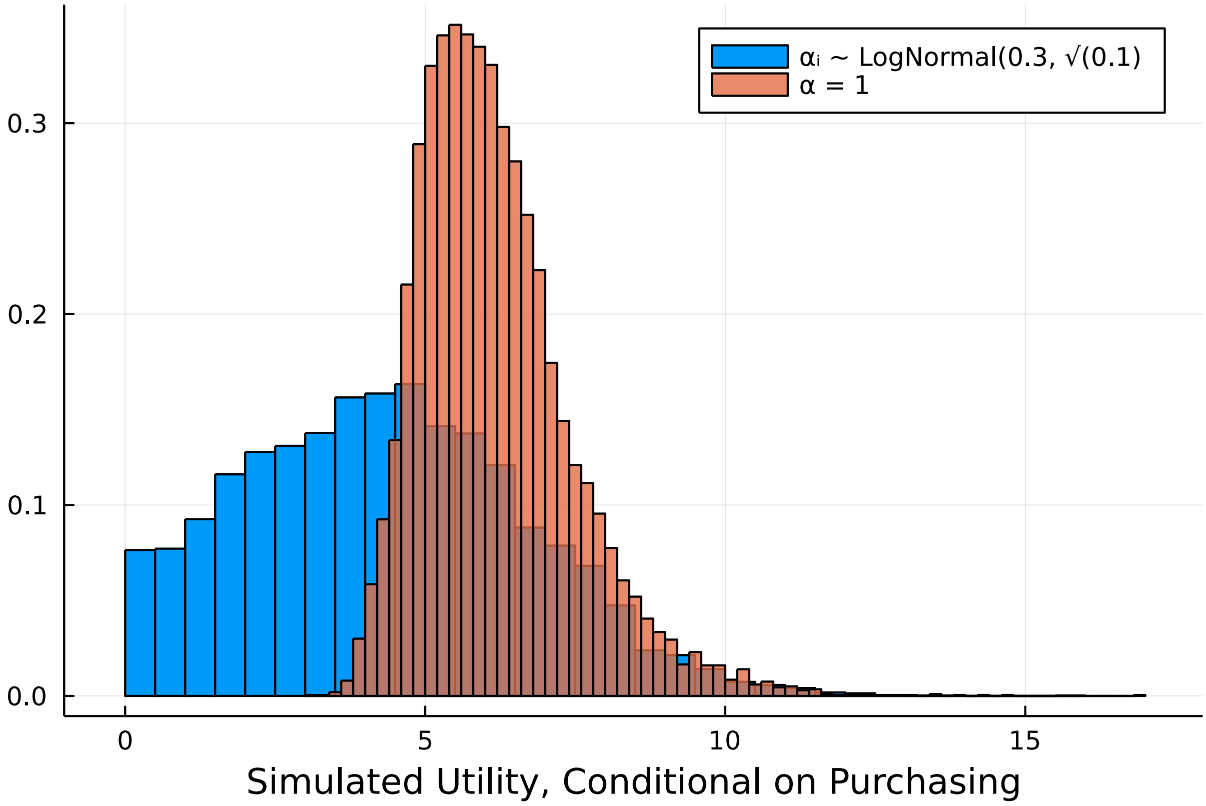


In this case (Case 2), the market is no longer covered: some 1314 people (13.14%) choose not to purchase any good.

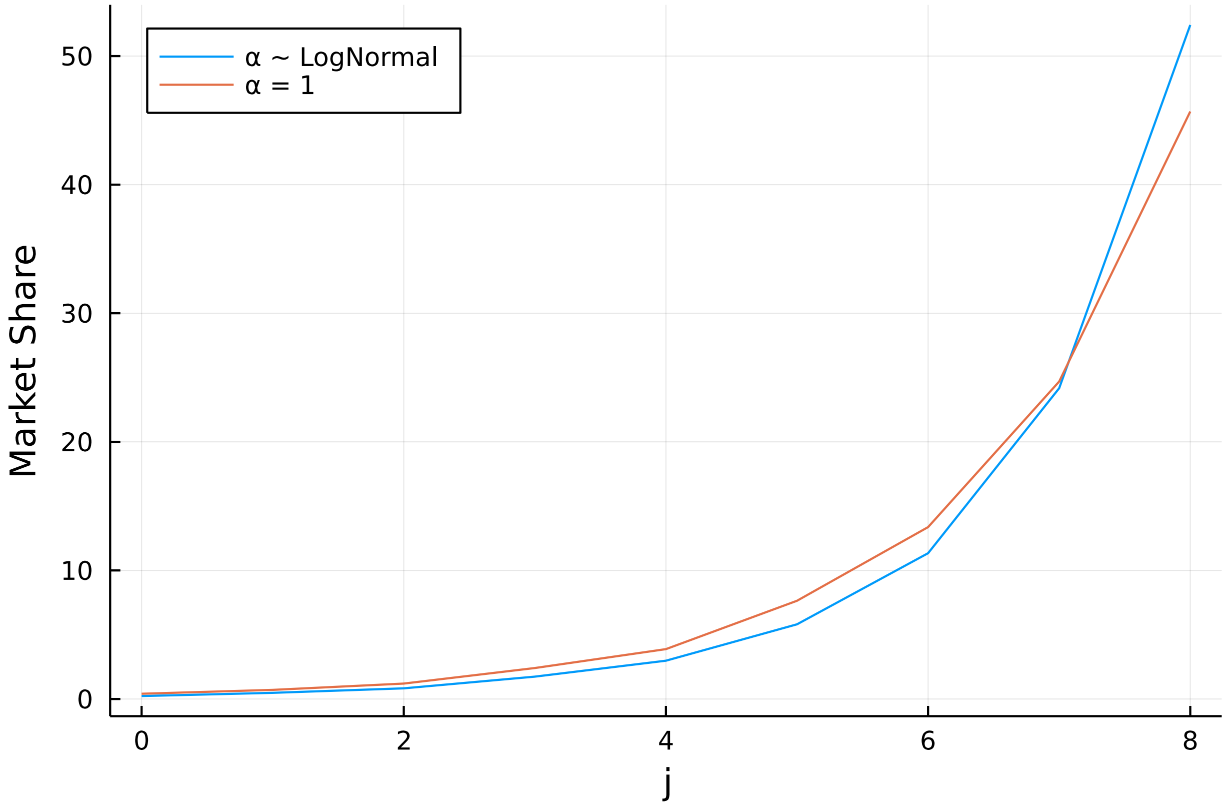
The following table shows the market share of each good j, which can be interpreted as the probability that an individual purchases good j conditional on having chosen to purchase any good. Column names ending in 1 refer to Case 1 when $\alpha = 1$, the others to Case 2 with $\alpha_i \sim LogNormal(\mu, \sigma)$.

	j	market_share1	cum_chosen1	market_share2	cum_chosen2
	Int64	Float64	Int64	Float64	Int64?
1	0	0.41	41	0.23	20
2	1	0.71	112	0.48	62
3	2	1.2	232	0.83	134
4	3	2.41	473	1.74	285
5	4	3.88	861	2.98	544
6	5	7.64	1625	5.81	1049
7	6	13.37	2962	11.34	2034
8	7	24.69	5431	24.17	4133
9	8	45.69	10000	52.42	8686

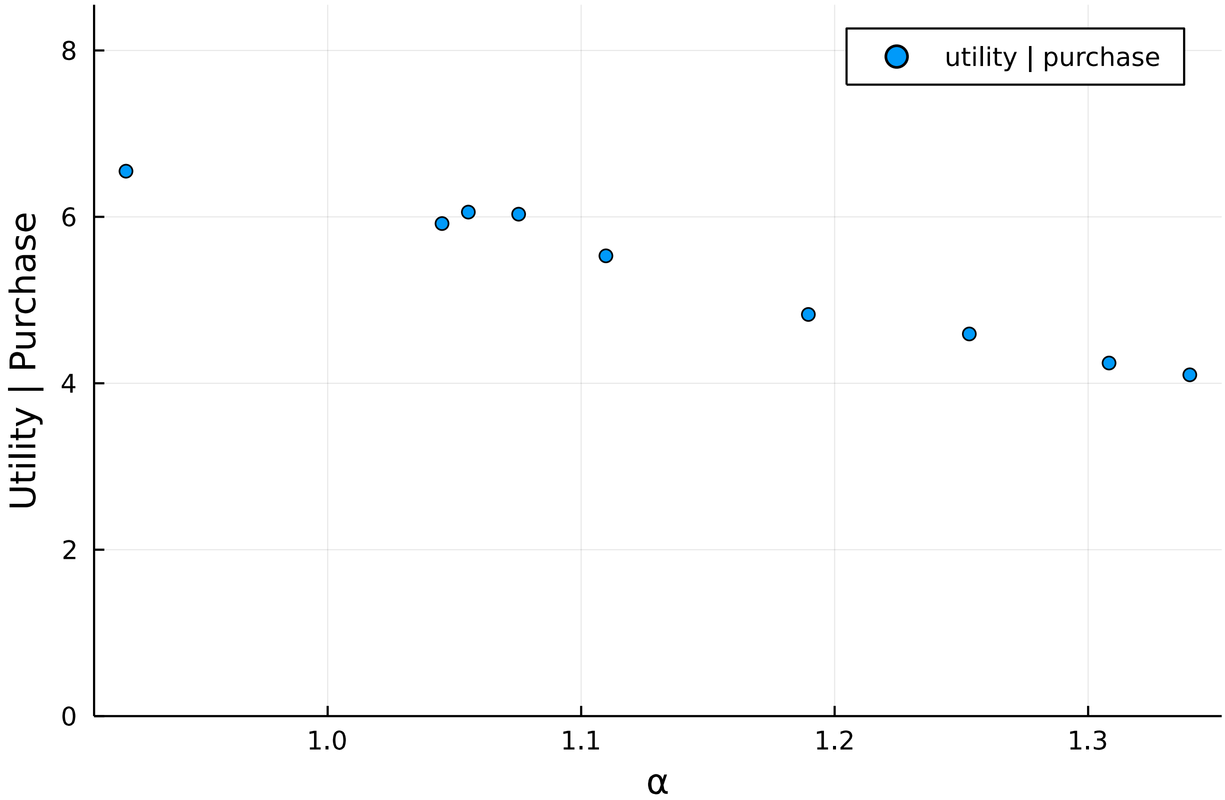
The conditional distributions of utility are normalized and plotted below. More people purchase under Case 1 with the constant α, and among those who purchase, mean utility is much higher and variance of utility much lower under Case 1 than Case 2.



Market shares under Case 1 and Case 2 are shown below. Market share is increasing in j; this is explained by the fact that price is decreasing faster than is x_j, and additionally ξ_j is increasing in j, so utility is increasing in j. However, we have the random error term ϵ that is uncorrelated across observations, and it influences each person's decision. For a given (i, j), ϵ may be very high and therefore, through its effect on utility, influence the person to choose a good which he otherwise would not have chosen.



We may also be interested in how utility varies with the value of α. Since α is is the marginal utility of income, we would expect that a *higher* α would be associated with a *lower* utility; a person with a greater marginal utility of income would have fewer profitable options, and would be more sensitive to prices. The following graph suggests that this may be the case.



3 Question 3

In this question, we assume that there is a good, the utility of which is independent of j; its utility depends only on the marginal utility of income α and on the random shock ε_{ij}. In particular, we are given that for this good k, $x_k = 3$, $\xi_k = 8$ and $p_k = 10$. The new good is denoted by $k = 9$.

With the addition of the new good, I refer to the new scenarios as Case 3 (mixed logit with new good) and Case 4 (logit with new good).

Under the given conditions, we find:

	j	market_share3	cum_chosen3	market_share4	cum_chosen4
	Int64	Float64	Int64	Float64?	Int64?
1	0	0.22	19	0.41	41
2	1	0.48	61	0.7	111
3	2	0.81	131	1.18	229
4	3	1.7	279	2.39	468
5	4	2.9	531	3.8	848
6	5	5.78	1033	7.57	1605
7	6	11.26	2011	13.23	2928
8	7	24.05	4100	24.38	5366
9	8	52.21	8635	45.23	9889
10	9	0.59	8686	1.11	10000

Under the mixed logit (Case 3), the new good has a market share of 0.59%, with only 51.0 people choosing to purchase it. Those who purchase have a mean α equal to 0.985. The exact same number of people (8686) choose to purchase as in Case 2, but now 51.0 people choose good $j = 9$.

Under the standard logit from Case 4, the new good has a market share of 1.11%, with only 111.0 people choosing to purchase it.

4 Question 4

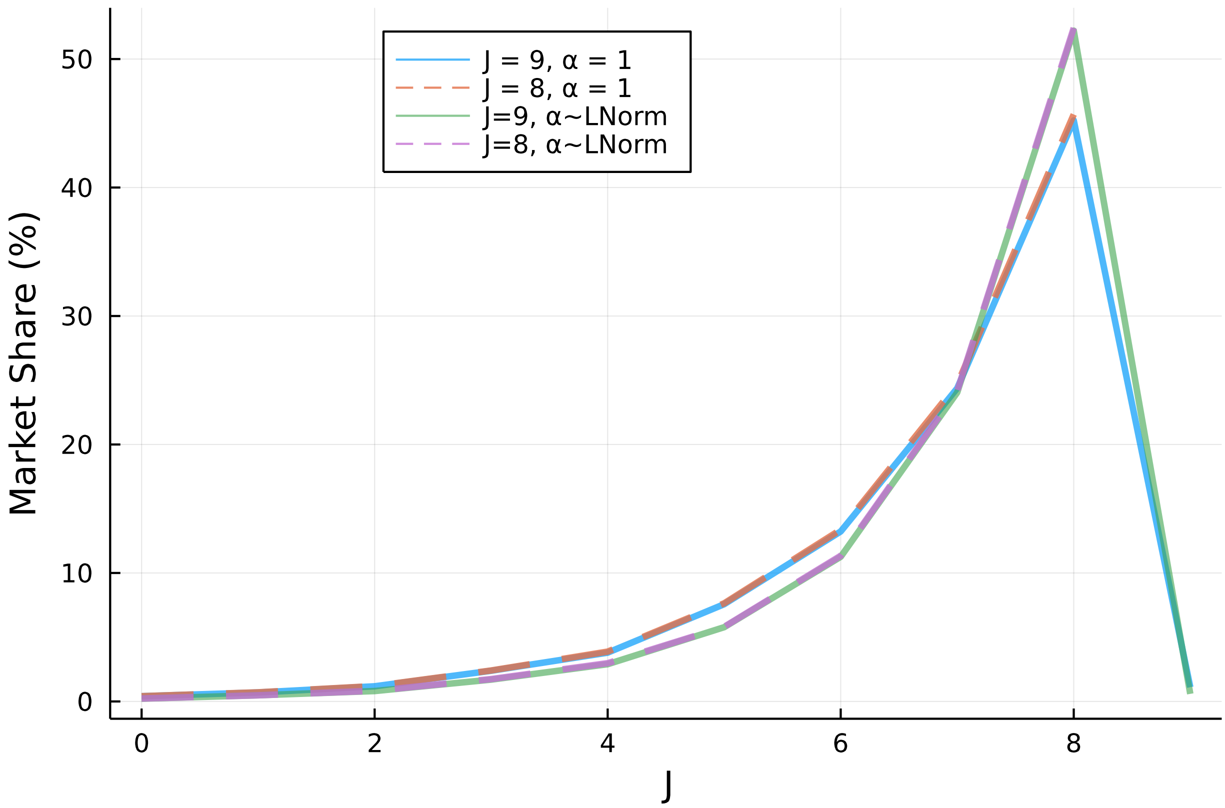
The welfare gains brought by this new good are negligible. Under Case 2, the welfare gain is 0.197%, and under Case 1 it is 0.19%.

The total consumer surplus gain from the addition of the new good under the mixed logit is 57.75, or 0.0066 per consumer, on average.

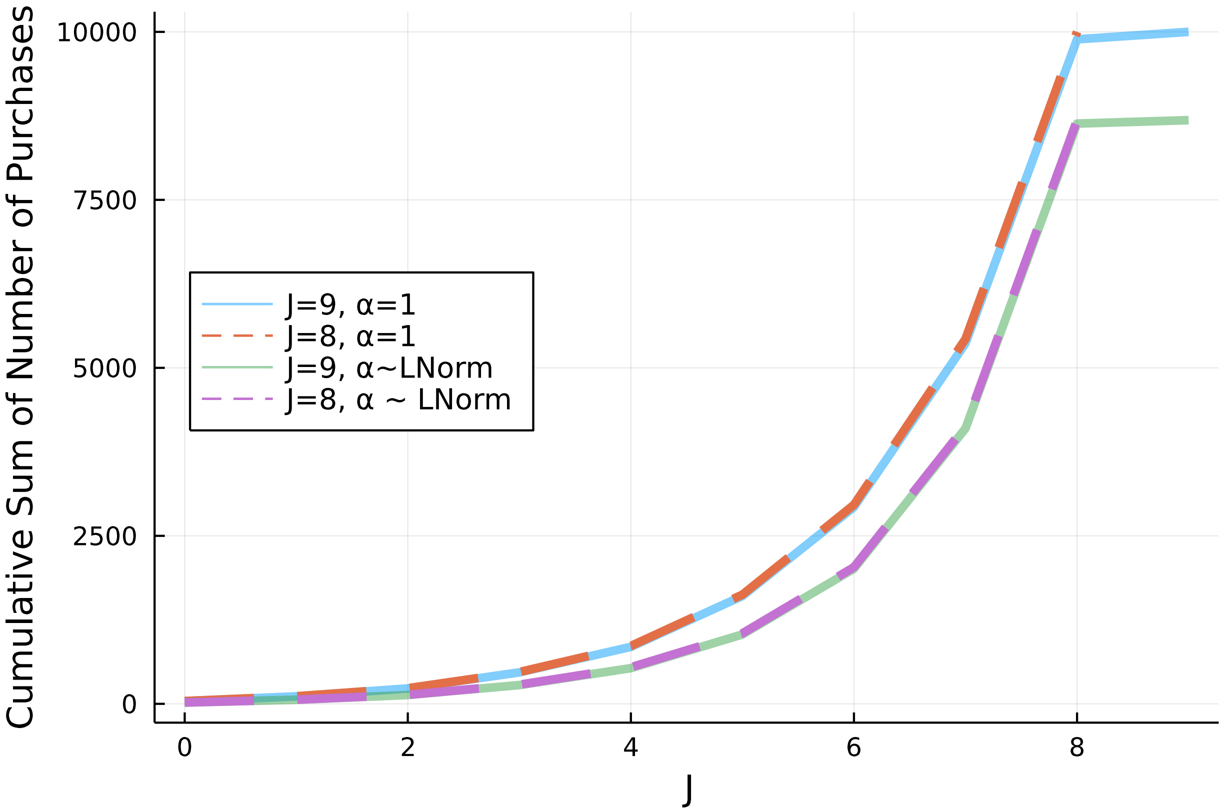
Under the original model with $\alpha = 1$ the welfare gain is 117.11, or 0.0117 per consumer, on average.

The market share of the new good is so low that only very few consumers prefer it. With only 1% market share and consumers choosing it on very narrow margins, the addition of the new good leads only to a limited increase in consumer surplus.

Let's look at how market shares and number of purchases change in these different scenarios:



The market shares are almost identical before and after the addition of the new good. In both Case 3 and 4, the market share of good 9 is negligible. We can obtain a better understanding of what is happening in these markets by also looking at the cumulative number of purchases:



We can see that with the uncertainty in α and the fact that it has a mean higher than 1, the cumulative number of purchases is always lower for any j.