Business Economics Homework

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2021-11-21

1 Question 1

Assuming that individual indirect utility is given by:

$$u_{ij} = \beta x_j - \alpha p_j + \xi_j + \varepsilon_{ij}$$

and that $\varepsilon_{ij} \sim \text{GeneralizedExtremeValue}(0, 1, 0)$, I simulate a market with $N = 10000$ individuals and $J + 1$ products.

Given that $\alpha = \beta = 1$, we have the following purchase decisions. Choice probability is calculated both directly and by simulated market shares. Note that in this first case (Case 1), the market is covered:

j Int64	prob_mean Float64	market_share1 Float64?	cum_chosen1 Int64?
0	0.37	0.41	41
1	0.68	0.71	112
2	1.23	1.2	232
3	2.25	2.41	473
4	4.1	3.88	861
5	7.46	7.64	1625
6	13.6	13.37	2962
7	24.78	24.69	5431
8	45.16	45.69	10000

The share of the individual choices can be interpreted in two equivalent ways: the share of the market captured by each product, or the probability of purchasing a given product j.

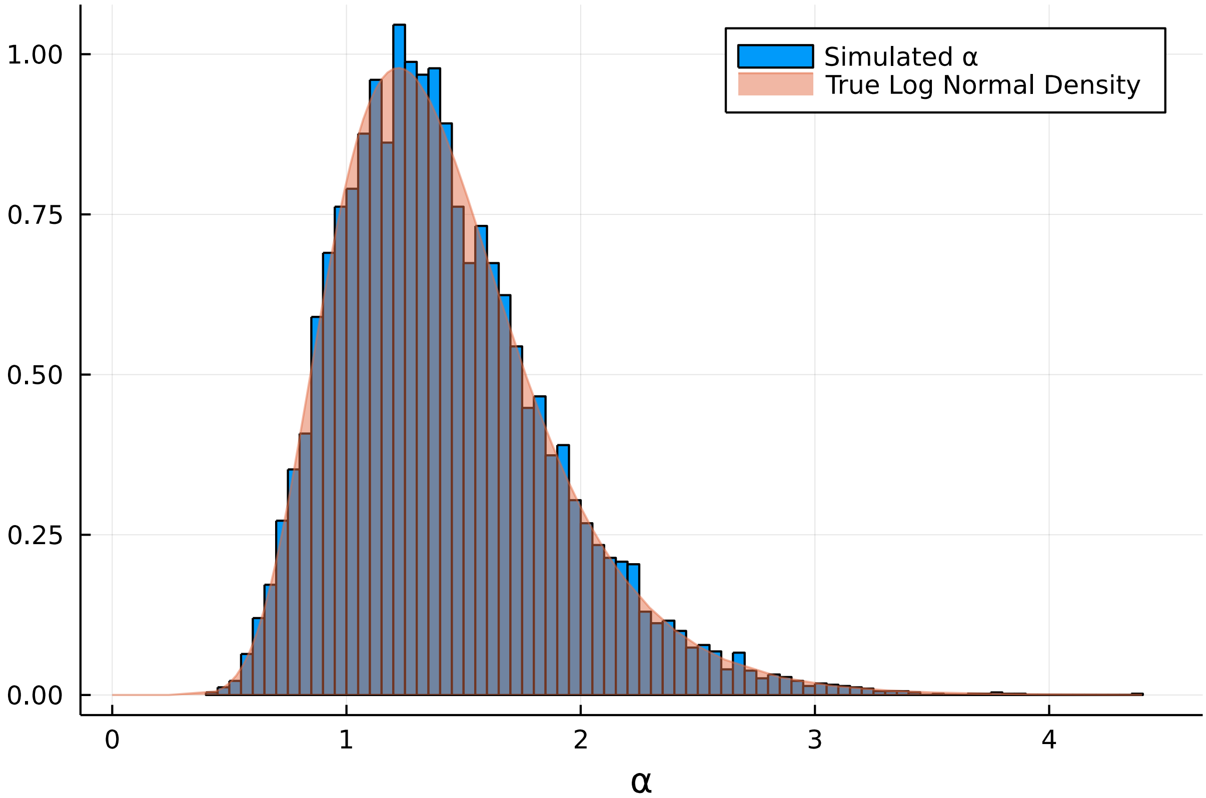
Let $P_i(j)$ indicate that individual i chooses good j. Since we assume that the distribution of ε_{ij} conditional on i is the same as its unconditional distribution:

$$P_i(0) = P(0) = 0.41\%$$
$$P(6) = 13.37\%$$

and so on, as in the table above.

2 Question 2

Now we assume that $\alpha_i \sim LogNormal(\mu = 0.3, \sigma = \sqrt{0.1})$. I again use 10000 individuals to simulate this market; I use more than the suggested 100 draws to more closely approximate the true choice probabilities / market shares.

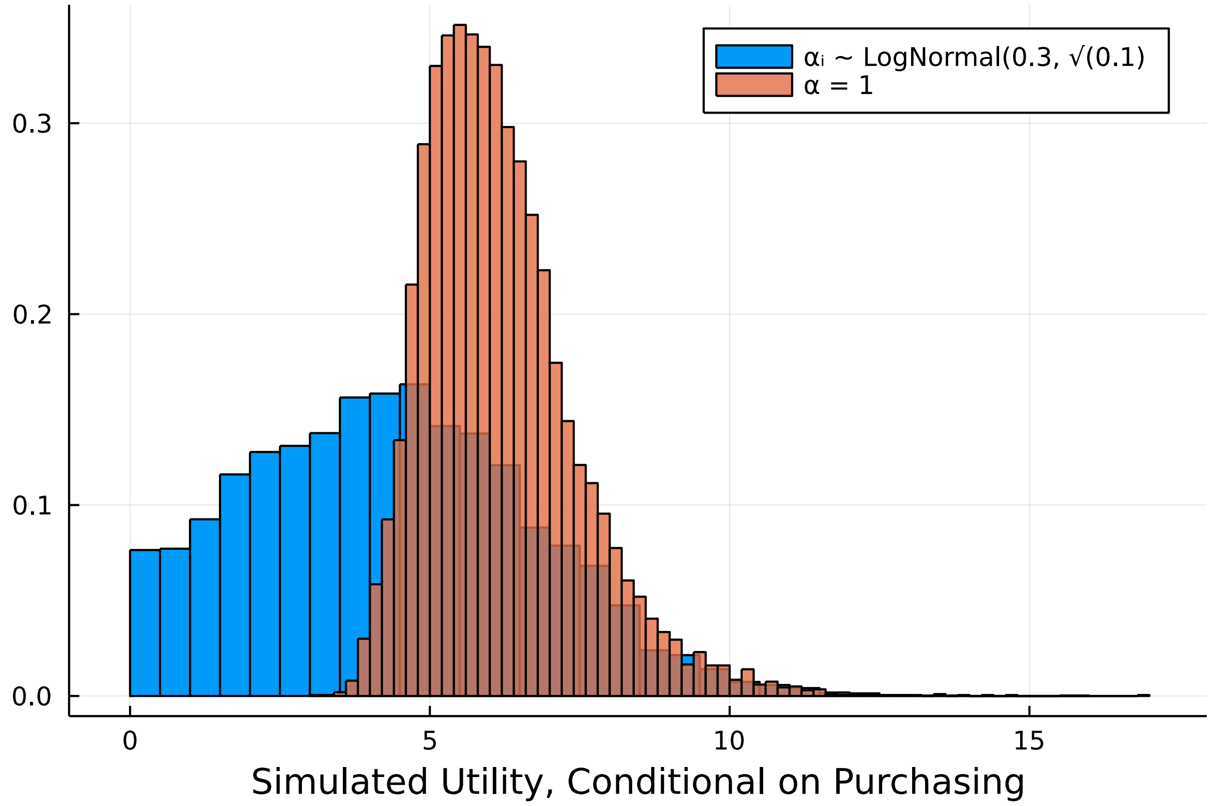


In this case (Case 2), the market is no longer covered: some 1314 people (13.14%) choose not to purchase any good.

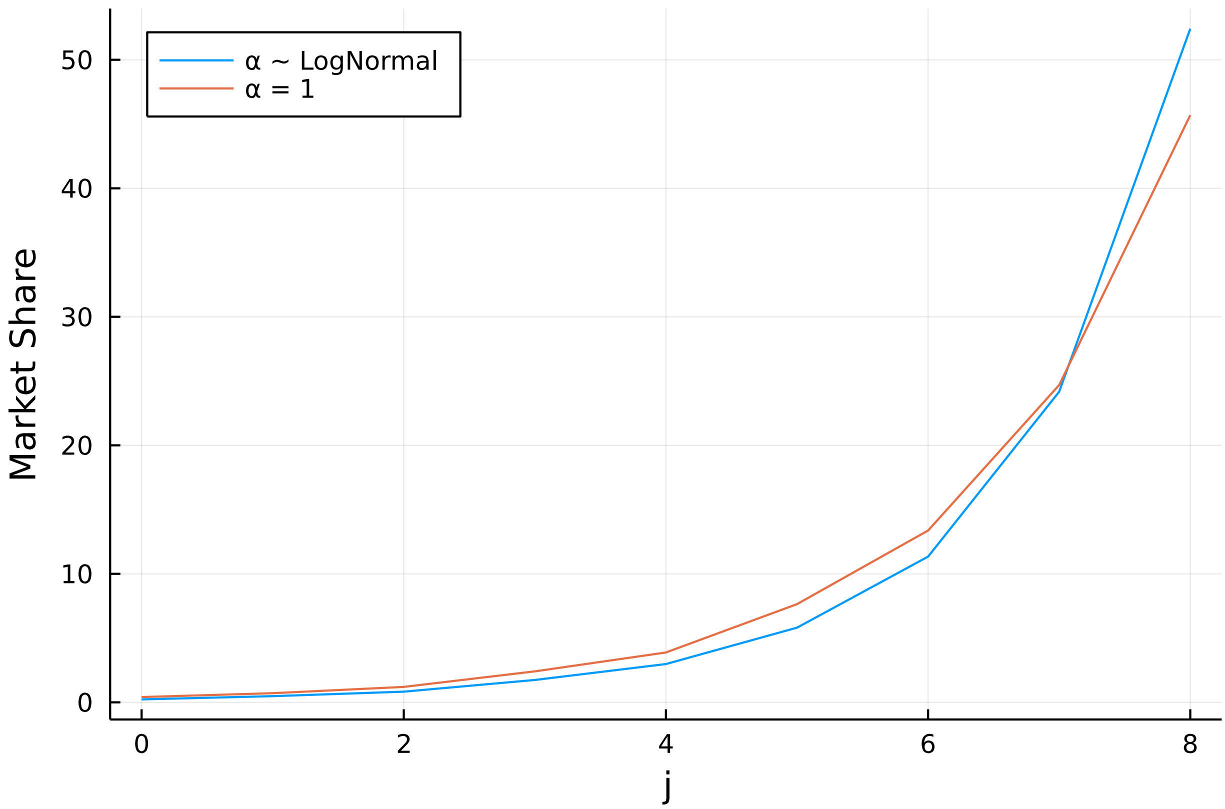
The following table shows the market share of each good j, which can be interpreted as the probability that an individual purchases good j conditional on having chosen to purchase any good. Column names ending in 1 refer to Case 1 when $\alpha = 1$, the others to Case 2 with $\alpha_i \sim LogNormal(\mu, \sigma)$.

j Int64	market_share1 Float64	cum_chosen1 Int64	market_share2 Float64	cum_chosen2 Int64?
0	0.41	41	0.23	20
1	0.71	112	0.48	62
2	1.2	232	0.83	134
3	2.41	473	1.74	285
4	3.88	861	2.98	544
5	7.64	1625	5.81	1049
6	13.37	2962	11.34	2034
7	24.69	5431	24.17	4133
8	45.69	10000	52.42	8686

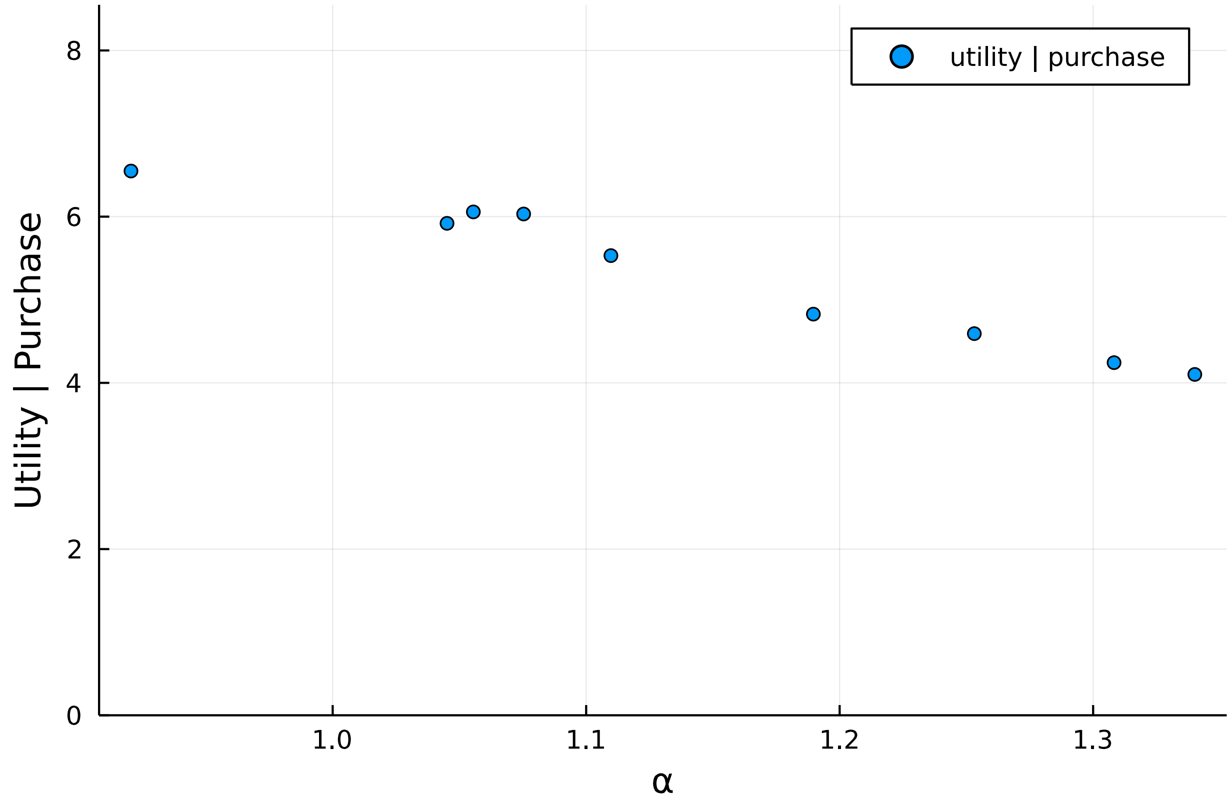
The conditional distributions of utility are normalized and plotted below:



Market shares under Case 1 and Case 2 are shown below:



We may also be interested in how utility varies with the value of α. Since α is is the marginal utility of income, we would expect that a *higher* α would be associated with a *lower* utility; a person with a greater marginal utility of income would have fewer profitable options, and would be more sensitive to prices. The following graph suggests that this may be the case.



3 Question 3

In this question, we assume that there is a good, the utility of which is independent of j; its utility depends only on the marginal utility of income α and on the random shock ε_{ij}. With the addition of the new good, I refer to these as Case 3 (mixed logit with new good) and Case 4 (logit with new good).

Under the given conditions, we find:

j Int64	market_share3 Float64	cum_chosen3 Int64	market_share4 Float64?	cum_chosen4 Int64?
0	0.22	19	0.41	41
1	0.48	61	0.7	111
2	0.81	131	1.18	229
3	1.7	279	2.39	468
4	2.9	531	3.8	848
5	5.78	1033	7.57	1605
6	11.26	2011	13.23	2928
7	24.05	4100	24.38	5366
8	52.21	8635	45.23	9889
9	0.59	8686	1.11	10000

Under the mixed logit (Case 3), the new good has a market share of 0.59%, with only 51.0

people choosing to purchase it. Those who purchase have a mean α equal to 0.985. The exact same number of people (8686) choose to purchase as in Case 2, but now 51.0 people now choose good $j = 9$.

Under the standard logit from Case 4, the new good has a market share of 1.11%, with only 111.0 people choosing to purchase it.

4 Question 4

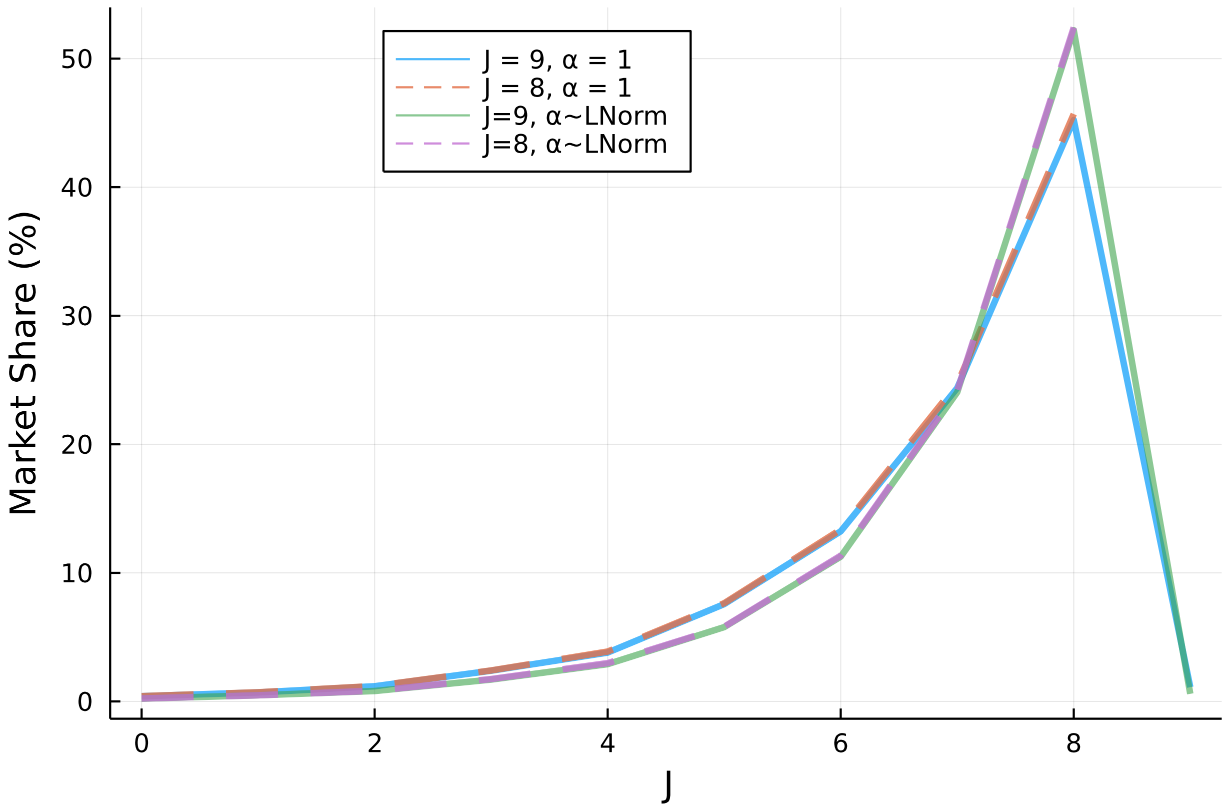
The welfare gains brought by this new good are negligible. Under Case 2, the welfare gain is 0.197%, and under Case 1 it is 0.19%.

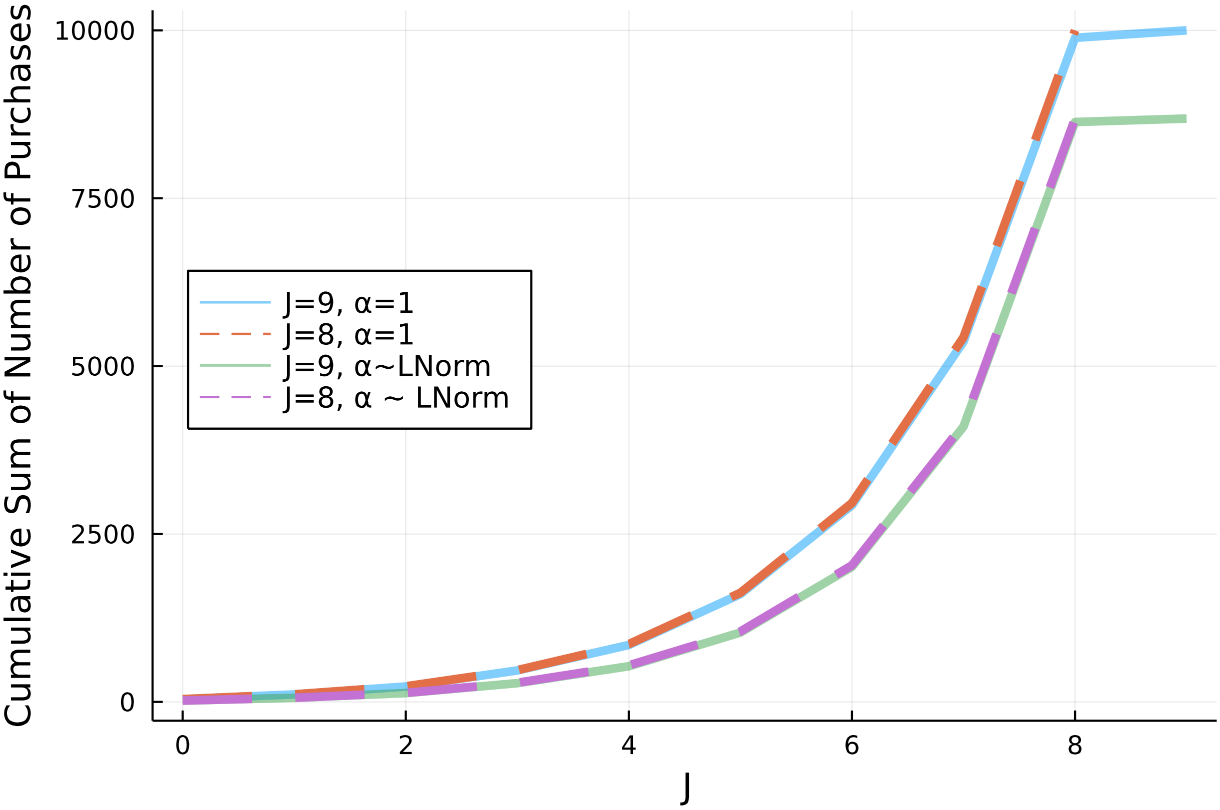
The total consumer surplus gain from the addition of the new good under the mixed logit is 57.75, or 0.0066 per consumer, on average.

Under the original model with $\alpha = 1$ the welfare gain is 117.11, or 0.0117 per consumer, on average.

The market share of the new good is so low that only very few consumers prefer it. With only 1% market share and consumers choosing it on very narrow margins, the addition of the new good leads only to a limited increase in consumer surplus.

Let's look at how market shares and number of purchases change in these different scenarios:




Cumulative Sum of Number of Purchases
10000
7500
5000
2500
0
0
2
4
6
8
J
J=9, α=1
J=8, α=1
J=9, α~LNorm
J=8, α ~ LNorm