

Probability practice

Part A

[1] 0.7142857

Given from the problem:

- The fraction of Random Clickers (RC) is $P(\text{RC}) = 0.3$, so the fraction of Truthful Clickers (RC) is $P(\text{TC}) = 1 - P(\text{RC}) = 0.7$
- The probability that a Random Clicker answers “Yes” is $P(\text{Yes}|\text{RC}) = 0.5$, and the probability that they answer “No” is $P(\text{No}|\text{RC}) = 0.5$
- The overall fraction of people who answered “Yes” is 65%, so $P(\text{Yes}) = 0.65$

Find the probability that a Truthful Clicker answered “Yes”: $P(\text{Yes}|\text{TC})$

- Total Probability: $P(\text{Yes}) = P(\text{Yes}|\text{RC}) * P(\text{RC}) + P(\text{Yes}|\text{TC}) * P(\text{TC})$
- $0.65 = 0.5 * 0.3 + P(\text{Yes}|\text{TC}) * 0.7$
- $P(\text{Yes}|\text{TC}) = 0.7142857$

The fraction of people who are truthful clickers that answered yes is **0.7142857**, or approximately 71.43%.

Part B

[1] 0.1988824

Given from the problem:

- $P(\text{Positive}|\text{Disease}) = 0.993$
- $P(\text{Negative}|\text{No Disease}) = 0.9999$, so $P(\text{Positive}|\text{No Disease}) = 1 - 0.9999 = 0.0001$
- $P(\text{Disease}) = 0.000025$, so $P(\text{No Disease}) = 1 - 0.000025 = 0.999975$

Find probability that someone has the disease given that they tested positive: $P(\text{Disease}|\text{Positive})$

- Bayes Theorem: $P(\text{Disease}|\text{Positive}) = (P(\text{Positive}|\text{Disease}) * P(\text{Disease})) / P(\text{Positive})$
- Total Probability: $P(\text{Positive}) = P(\text{Positive}|\text{Disease}) * P(\text{Disease}) + P(\text{Positive}|\text{No Disease}) * P(\text{No Disease})$
- $P(\text{Positive}) = (0.993 * 0.000025) + (0.0001 * 0.999975)$
- $p_{\text{positive}} = (0.993 * 0.000025) + (0.0001 * 0.999975) = 0.0001248225$
- $P(\text{Disease}|\text{Positive}) = (0.993 * 0.000025) / p_{\text{positive}}$
- $(0.993 * 0.000025) / p_{\text{positive}} = 0.1988824$

The probability that someone has the disease given that they test positive is **0.1988824**, or approximately 19.89%.