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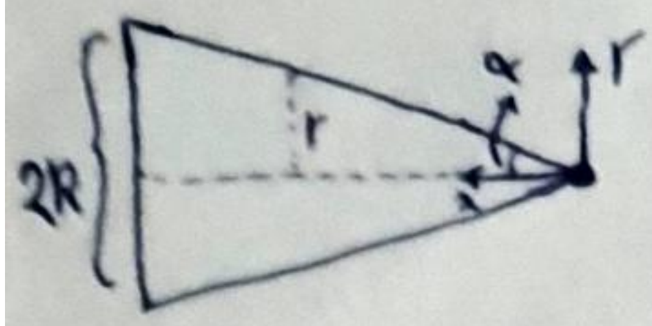
حالت اول:



$$\tan\alpha = \frac{R}{L} \rightarrow \frac{r}{x} = \tan\alpha \Rightarrow$$

$$r = \frac{R}{L}x \Rightarrow P.dx = 2\pi\frac{R}{L}x\,dx$$

سطح جانبی

سطح مقطع عمودی: $\pi r^2 = \pi\left(\frac{R^2}{L^2}\right)x^2$

$$\rightarrow \frac{\partial}{\partial x}\left(kA_x\frac{\partial T}{\partial x}\right)dx - h_2 P dx (T - T_\infty) = 0$$

$$\rightarrow \frac{\partial}{\partial x}\left(k\pi\left(\frac{R}{L}\right)^2 x^2\frac{\partial\theta}{\partial x}\right)dx - h_2 2x\pi\frac{R}{L}dx = 0$$

$$\rightarrow \frac{\partial}{\partial x}\left(x^2\frac{\partial\theta}{\partial x}\right) - \frac{2h_2L}{kR}x\theta = 0 \Rightarrow m^2 = \frac{2h_2L}{kR}$$

$$\frac{\partial}{\partial x}\left(x^2\frac{\partial\theta}{\partial x}\right) - m^2x\theta = 0 \Rightarrow \begin{cases}\alpha = 2 & \beta = 1\\ \beta - \alpha + 2 = 1 \neq 0\end{cases}$$

$$\Rightarrow \theta(x) = C_1 x^{-\frac{1}{2}} I_{-1}\left(2x^{\frac{1}{2}}m\right) + C_2 x^{-\frac{1}{2}} K_{-1}\left(2x^{\frac{1}{2}}m\right)$$

$$BC\begin{cases} x = 0 \quad \frac{\partial\theta}{\partial x} = 0 \text{ or } \theta = \text{finite} \Rightarrow \boxed{C_2 = 0} \\ x = L \quad \theta = \theta_0 \Rightarrow \theta_0 = C_1 L^{-\frac{1}{2}} I_{-1}\left(2L^{\frac{1}{2}}m\right)\end{cases}$$

$$\Rightarrow C_1 = \frac{\theta_0}{L^{-\frac{1}{2}} I_{-1}\left(2L^{\frac{1}{2}}m\right)} \Rightarrow \theta(x) = \frac{\theta_0}{L^{-\frac{1}{2}} I_{-1}\left(2L^{\frac{1}{2}}m\right)} x^{-\frac{1}{2}} I_{-1}\left(2x^{\frac{1}{2}}m\right)$$

حل به روش تقریبی approximate:

$$\int_{x/2}^{x} \frac{1}{x}\frac{\partial}{\partial x}\left(x^2\frac{\partial\theta}{\partial x}\right) - m^2 x^{-1}\theta = 0 \quad \Rightarrow \int_0^L \frac{\partial}{\partial x}\left(x^2\frac{\partial\theta}{\partial x}\right) - m^2 x\theta \Big]^{A^2} dx = 0$$

به روش HA-Ritz: $\Leftarrow$

$$\theta = A_0 + A_1 x + A_2 x^2$$

$$BC\begin{cases} x=0 & \frac{\partial\theta}{\partial x}=0 \Rightarrow \boxed{A_1=0} \\ x=L & \theta=\theta_0 \Rightarrow \theta_0 = A_0 + A_2 L^2 \Rightarrow \boxed{A_0=\theta_0 - A_2 L^2} \end{cases}$$

$$\int_0^L \left[\frac{\partial}{\partial x}\left(x^2 2A_2 x\right) - m^2 x\left[A_0 + A_2 x^2\right]\right] dx = 0$$

$$\int_0^L \left(6A_2 x^2 - m^2 A_0 x - m^2 A_2 x^3\right)^{x^2} dx = 0$$

$$2A_2 L^3 - \frac{m^2 A_0 L^2}{2} - \frac{m^2 A_2 L^4}{4} = 0$$

$$2A_2 L^3 - \frac{m^2 L^2}{2}\left(\theta_0 - A_2 L^2\right) - \frac{m^2 A_2 L^4}{4} = 0$$

$$A_2\left(2L^3 + \frac{m^2 L^4}{2} - \frac{m^2 L^4}{4}\right) = \frac{m^2 L^2}{2}\theta_0$$

$$\Rightarrow \boxed{A_2 = \frac{\frac{m^2 L^2}{2}\theta_0}{\left(2L^3 + \frac{m^2 L^4}{2} - \frac{m^2 L^4}{4}\right)}}$$

☑ برای عددگذاری و بدست آوردن مقادیر از متلب استفاده شد که فایل آن هم قرار دادم.

$$\theta(x) = A_0 + A_2 x^2 \qquad ARD = \frac{|\theta_{exact} - \theta_{approximate}|}{\theta_{exact}}$$

$\theta_0=30$, $k=94$, $h_2=10$ $(L=0.1)$: exact حل برای θ محاسبات

$R=0.1 \quad L=0.1 \Rightarrow m^2=0.2128 \Rightarrow m=0.4613$

$x=0.001 \Rightarrow \theta_{exact}(0)=29.6151$

$x=0.02 \Rightarrow \theta_{exact}(0.02)=29.7344$

$x=0.04 \Rightarrow \theta_{exact}(0.04)=29.8117$

$x=0.06 \Rightarrow \theta_{exact}(0.06)=29.8663$

$x=0.08 \Rightarrow \theta_{exact}(0.08)=29.9359$

$x=0.1 \Rightarrow \theta_{exact}(0.1)=30$

$(L=0.1)$ approximate حل برای θ محاسبات

$x=0.001 \Rightarrow \theta=29.8409 \Rightarrow ARD=0.0076$

$x=0.02 \Rightarrow \theta=29.8472 \Rightarrow ARD=0.0038$

$x=0.04 \Rightarrow \theta=29.8663 \Rightarrow ARD=0.0018$

$x=0.06 \Rightarrow \theta=29.8981 \Rightarrow ARD=0.0011$

$x=0.08 \Rightarrow \theta=29.9427 \Rightarrow ARD=0.0002$

$x=0.1 \Rightarrow \theta=30 \Rightarrow ARD=0$

$m^2=1.0638$

$L=0.5$: طول برای exact حل برای θ محاسبات

$x=0.01 \Rightarrow \theta(0.01)=23.3746$

$x=0.2 \Rightarrow \theta(0.2)=25.8096$

$x=0.3 \Rightarrow \theta(0.3)=27.1602$

$x=0.35 \Rightarrow \theta(0.35)=27.8496$

$x=0.4 \Rightarrow \theta(0.4)=28.5560$, $x=0.5 \Rightarrow \theta=30$

$\theta = \underbrace{26.2594}_{A_0} + \underbrace{14.9626}_{A_2} x^2$ ← $L=0.5$ برای approxima حل

$x=0.01 \Rightarrow \theta = 26.2608 \Rightarrow ARD = 0.1235$

$x=0.2 \Rightarrow \theta = 26.8579 \Rightarrow ARD = 0.0406$

$x=0.3 \Rightarrow \theta = 27.6060 \Rightarrow ARD = 0.0164$

$x=0.35 \Rightarrow \theta = 28.0923 \Rightarrow ARD = 0.0087$

$x=0.4 \Rightarrow \theta = 28.6534 \Rightarrow ARD = 0.0034$

$\Rightarrow ARD = 0$

$x=0.5 \Rightarrow \theta = 30$

$m^2 = 2.1277 \quad m = 1.4586$: ($L=1$) Jobcis. exact حل

$x=0.01 \Rightarrow \theta_{enatc}(0.01) = 12.0535$

$x=0.1 \Rightarrow \theta_{enact}(0.1) = 13.2405$

$x=0.2 \Rightarrow \theta_{enact}(0.2) = 14.6498$

$x=0.5 \Rightarrow \theta_{enact}(0.5) = 19.4980$

$x=0.8 \Rightarrow \theta_{enact}(0.8) = 25.3996$

$x=1 \Rightarrow \theta_{exact}(1) = 30$

$A_0 = 17.3950 \quad A_2 = 12.6050 \leftarrow L=1$ طول برای approximate حل

$\Rightarrow \theta = 17.3950 + 12.6050 x^2$

$\theta = 17.3950 + 12.6050\, x^2$ ← $L=1$ جواب کلی. approximate

$x=0.01 \Rightarrow \theta = 17.3962 \Rightarrow ARD = 0.4432$

$x=0.1 \Rightarrow \theta = 17.5210 \Rightarrow ARD = 0.3233$

$x=0.2 \Rightarrow \theta = 17.8992 \Rightarrow ARD = 0.2218$

$x=0.5 \Rightarrow \theta = 20.5462 \Rightarrow ARD = 0.0538$

$x=0.8 \Rightarrow \theta = 25.4622 \Rightarrow ARD = 0.0025$

$x=1 \Rightarrow \theta = 30 \Rightarrow ARD = 0$

$m^2 = 10.6383 \Rightarrow m = 3.2616$ ← $L=5$ جواب کلی. exact حل

$x=0.01 \Rightarrow \theta_{exact}(0.01) = 0.0010$

$x=1 \Rightarrow \theta_{exact}(1) = 0.0305$

$x=2 \Rightarrow \theta_{exact}(2) = 0.2755$

$x=3 \Rightarrow \theta_{exact}(3) = 1.6297$

$x=4 \Rightarrow \theta_{exact}(4) = 7.5790$

$x=5 \Rightarrow \theta_{exact}(5) = 30$

$A_0 = -22.155 \quad A_2 = 2.0862$ ← ($L=5$) جواب کلی. approximate حل

$\theta = -22.1558 + 2.0862\, x^2$

$\theta = -22.1558 + 2.0862 x^2$ ← $L=5$ جواب تقریبی approximate

$x = 0.01 \Rightarrow \theta = -22.1556 \Rightarrow ARD = 2.1119 \times 10^{4}$

$x = 1 \Rightarrow \theta = -20.0695 \Rightarrow ARD = 0.0660 \times 10^{4}$

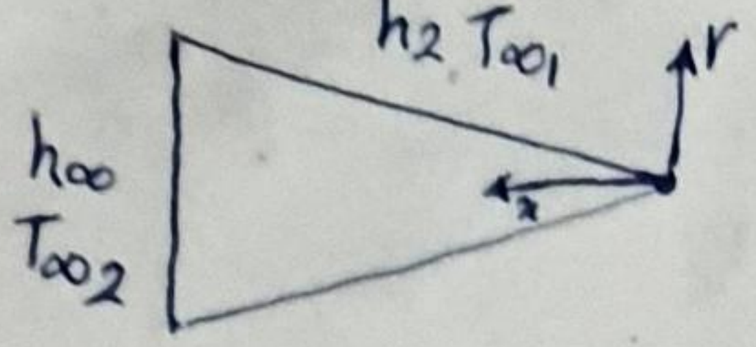
$x = 2 \Rightarrow \theta = -13.8108 \Rightarrow ARD = 0.0051 \times 10^{4}$

$x = 3 \Rightarrow \theta = -3.3797 \Rightarrow ARD = 0.0003 \times 10^{4}$

$x = 4 \Rightarrow \theta = 11.2239 \Rightarrow ARD = 0.00002$

$x = 5 \Rightarrow \theta = 30 \Rightarrow ARD = 0$

حالت دوم یا سؤال دوم:



مانند حالت قبل مساحت ها و معادلات می شود

و فقط تفاوت در شرط مرزی ها است.

$$\frac{\partial}{\partial x}\left(x^2 \frac{\partial \theta}{\partial x}\right) - m^2 x \theta = 0 \qquad m^2 = \frac{2 h_2 L}{KR} \Rightarrow \begin{cases} \alpha = 2 & \beta = 1 \\ \beta - \alpha + 2 = 1 \neq 0 \end{cases} \Rightarrow$$

$$\theta = c_1 x^{-\frac{1}{2}} I_{-1}\left(2 x^{\frac{1}{2}} m\right) + c_2 x^{-\frac{1}{2}} k_{-1}\left(2 x^{\frac{1}{2}} m\right)$$

$$Bc \begin{cases} x = 0 & \frac{\partial \theta}{\partial x} = 0 \quad \theta = finite \Rightarrow \boxed{C_2 = 0} \\ x = L \Rightarrow & -k \frac{\partial \theta}{\partial x} = h\left(\theta + T_{\infty 1} - T_{\infty 2}\right) \end{cases}$$

$\theta = c_1 x^{-\frac{1}{2}} I_{-1}\left(2 x^{\frac{1}{2}} m\right) \Rightarrow$ از طریق متلب مشتق $\Rightarrow$ را حساب می کنیم

$$\frac{\partial \theta}{\partial x} = c_1 \left[\frac{-I_1\left(2 m x^{\frac{1}{2}}\right)}{2 x^{\frac{3}{2}}} - \left(\frac{I_1'\left(2 m x^{\frac{1}{2}}\right)}{2x} - \frac{m I_0\left(2 m x^{\frac{1}{2}}\right)}{x^{\frac{1}{2}}} \right) x \frac{1}{x^{\frac{1}{2}}} \right]$$

$\Rightarrow -k \frac{\partial \theta}{\partial x} = h\left(\theta + T_{\infty 1} - T_{\infty 2}\right) \Rightarrow \boxed{C_1 = ?}$ با حل این معادله داخل فایل متلب بدست می آید.

$\Rightarrow \theta = c_1 I_1\left(2 x^{\frac{1}{2}} m\right) x^{-\frac{1}{2}}$

$$\boxed{C_1 = \frac{T_{\infty 1} - T_{\infty 2}}{-\left(\frac{k}{h} \frac{\partial \theta}{\partial x} + \theta\right)}}$$

حالت دوم (حل به روش approximate):

$$\theta = A_0 + A_1 x + A_2 x^2 \qquad \int_\forall \frac{\partial}{\partial x}\left(x^2 \frac{\partial \theta}{\partial x}\right) - m^2 \theta x = 0$$

$$BC\begin{cases} x=0 \quad \frac{\partial \theta}{\partial x} = 0 \Rightarrow \boxed{A_1 = 0} \Rightarrow \theta = A_0 + A_2 x^2 \\ x=L \quad -k\frac{\partial \theta}{\partial x} = h_\infty(\theta + T_{\infty 1} - T_{\infty 2}) \end{cases}$$

$$\Rightarrow \frac{\partial \theta}{\partial x} = 2A_2 x \Rightarrow -2kA_2 L = h_\infty(A_0 + A_2 L^2 + T_{\infty 1} - T_{\infty 2})$$

$$\Rightarrow \boxed{A_0 = \frac{-2kA_2 L}{h_\infty} - A_2 L^2 - T_{\infty 1} + T_{\infty 2}}$$

$$\int_\forall \frac{\partial}{\partial x}\left(x^2 \frac{\partial \theta}{\partial x}\right) - m^2 x\theta\, dx = 0 \Rightarrow \int_0^L \frac{\partial}{\partial x}\left(x^2 \frac{\partial \theta}{\partial x}\right) - m^2 x\theta\, dx = 0$$

$$\Rightarrow \int_0^L 6A_2 x^2 - m^2 x(A_0 + A_2 x^2) = \int_0^L 6A_2 x^2 - m^2 A_0 x - m^2 A_2 x^3$$

$$\Rightarrow 2A_2 L^3 - \frac{m^2 A_0 L^2}{2} - \frac{m^2 A_2 L^4}{4} = 0 \Rightarrow$$

$$2A_2 L^3 - \frac{m^2 L^2}{2}\left(\frac{-2kA_2 L}{h_\infty} - A_2 L^2 - T_{\infty 1} + T_{\infty 2}\right) - \frac{m^2 A_2 L^4}{4} = 0$$

$\Rightarrow A_2 = \surd$ از طریق متلب مقدار $\Rightarrow \boxed{A_0 = \surd} \Rightarrow \boxed{\theta = A_0 + A_2 x^2}$

محاسبات برای $L=0.1$

$k=94,\ R=0.1,\ h_2=70\quad h_\infty=150$

$T_{\infty 1}=20\quad T_{\infty 2}=100$

برای حل exact داریم که:

$m^2=0.2128 \rightarrow m=0.4613$

$x=0.01 \Rightarrow \theta_{exact}(0.01)=74.3031$

$x=0.02 \Rightarrow \theta_{exact}(0.02)=74.3822$

$x=0.03 \Rightarrow \theta_{exact}(0.03)=74.4613$

$x=0.06 \Rightarrow \theta_{exact}(0.06)=74.6989$

$x=0.07 \Rightarrow \theta_{exact}(0.07)=74.7782$

$x=0.1 \Rightarrow \theta_{exact}(0.1)=75.0165$

برای طول $L=0.1$ محاسبات به روش approximate:

$A_0=74.6145$

$A_2=39.7944 \Rightarrow \theta=74.6145+39.7944x^2$

$x=0.01 \Rightarrow \theta_{app}=74.6185 \Rightarrow ARD=0.0042$

$x=0.02 \Rightarrow \theta_{app}=74.6304 \Rightarrow ARD=0.0033$

$x=0.03 \Rightarrow \theta_{app}=74.6503 \Rightarrow ARD=0.0025$

$x=0.06 \Rightarrow \theta_{app}=74.7578 \Rightarrow ARD=0.0008$

$x=0.07 \Rightarrow \theta_{app}=74.8095 \Rightarrow ARD=0.0004$

$x=0.1 \Rightarrow \theta_{app}=75.0124 \Rightarrow ARD=0.0001$

$k=94$, $R=0.1$ $h_2=10$ $h_\infty=150$

محاسبات برای $\underline{L=0.5}$

$T_{\infty_1}=20$ $T_{\infty 2}=100$

محاسبات برای حالت exact :

$m^2=1.0638 \Rightarrow m=1.0314$

C_1 به $= 45.9737$

$x=0.01 \Rightarrow \theta_{exact}(0.01)=47.6709$

$x=0.2 \Rightarrow \theta_{exact}(0.2)=52.6448$

$x=0.3 \Rightarrow \theta_{exact}(0.3)=55.3983$

$x=0.35 \Rightarrow \theta_{exact}(0.35)=56.8112$

$x=0.4 \Rightarrow \theta_{exact}(0.4)=58.2486$

$x=0.5 \Rightarrow \theta_{exact}(0.5)=61.1983$

محاسبات برای حل approximate برای طول $\underline{L=0.5}$:

$A_0=53.3502$

$A_2=30.399$

$\Rightarrow \theta=53.3502+30.399x^2$

$x=0.01 \Rightarrow \theta_{app}=53.3533 \Rightarrow ARD=0.1192$

$x=0.2 \Rightarrow \theta_{app}=54.5662 \Rightarrow ARD=0.0365$

$x=0.3 \Rightarrow \theta_{app}=56.0861 \Rightarrow ARD=0.0124$

$x=0.35 \Rightarrow \theta_{app}=57.0741 \Rightarrow ARD=0.0046$

$x=0.4 \Rightarrow \theta_{app}=58.2141 \Rightarrow ARD=0.0006$

$x=0.5 \Rightarrow \theta_{app}=60.9500 \Rightarrow ARD=0.0041$

$k=94,\ R=0.1\quad h_2=10\quad h_\infty=150$ محاسبات برای حل $\underline{L=1}$

$T_{\infty 1}=20\quad T_{\infty 2}=100$

محاسبات برای حل enact برای $L=1$:

$m^2=2.1277 \Rightarrow m=1.4586$

$G_{1,\bar{x}\bar{x}}=14.4316 \Rightarrow \theta(n)=c_1 x^{-\frac{1}{2}} I_1(2mx^{\frac{1}{2}})$

$x=0.01 \Rightarrow \theta_{exact}(0.01)=21.2754$

$x=0.1 \Rightarrow \theta_{exact}(0.1)=23.3709$

$x=0.2 \Rightarrow \theta_{exact}(0.2)=25.8587$

$x=0.5 \Rightarrow \theta_{exact}(0.5)=34.4188$

$x=0.8 \Rightarrow \theta_{exact}(0.8)=44.8344$

$x=1 \Rightarrow \theta_{exact}(1)=52.9552$

محاسبات برای حل $\underline{L=1}$ برای حل approximate:

$A_0=30.3853$

$A_2=22.0183$

$\Rightarrow \theta=30.3853+22.0183x^2$

$x=0.01 \Rightarrow \theta_{app}=30.3875 \Rightarrow ARD=0.4283$

$x=0.1 \Rightarrow \theta_{app}=31.2661 \Rightarrow ARD=0.3096$

$x=0.2 \Rightarrow \theta_{app}=31.2661 \Rightarrow ARD=0.2091$

$x=0.5 \Rightarrow \theta_{app}=35.8899 \Rightarrow ARD=0.0427$

$x=0.8 \Rightarrow \theta_{app}=44.4771 \Rightarrow ARD=0.0080$

$x=1 \Rightarrow \theta_{app}=52.4037 \Rightarrow ARD=0.0104$

$k=94$, $R=0.1$, $h_2=10$, $h_\infty=150$ محاسبات برای Job $L=5$

$T_{\infty 1}=20$ $T_{\infty 2}=100$

محاسبات Job برای exact $L=5$:

$m^2=10.6383 \Rightarrow m=3.2616$

C_1 ,$\theta_{\infty}=4.4655\times10^{-4} \Rightarrow \theta(x)=C_1 x^{-\frac{1}{2}} I_{-1}(2mx^{\frac{1}{2}})$

$x=0.01 \Rightarrow \theta_{exact}(0.01)=0.0015$

$x=1 \Rightarrow \theta_{exact}(1)=0.0446$

$x=2 \Rightarrow \theta_{exact}(2)=0.4032$

$x=3 \Rightarrow \theta_{exact}(3)=2.3853$

$x=4 \Rightarrow \theta_{exact}(4)=11.0932$

$x=5 \Rightarrow \theta_{exact}(5)=43.9121$

محاسبات Job برای approximate $L=5$:

$A_0=-41.1495$ $A_2=3.8747$

$$\theta(x)=-41.1495+3.8747x^2$$

$x=0.01 \Rightarrow \theta_{app}=-41.1491 \Rightarrow ARD=2.6802\times10^{4}$

$x=1 \Rightarrow \theta_{app}=-37.2748 \Rightarrow ARD=0.0837\times10^{4}$

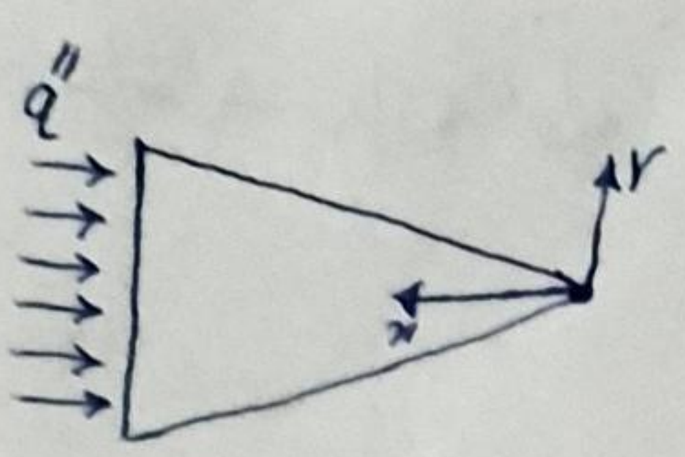
$x=2 \Rightarrow \theta_{app}=-25.6506 \Rightarrow ARD=0.0065\times10^{4}$

$x=3 \Rightarrow \theta_{app}=-6.2770 \Rightarrow ARD=0.0004\times10^{4}$

$x=4 \Rightarrow \theta_{app}=20.8460 \Rightarrow ARD=0.0001\times10^{4}$

$x=5 \Rightarrow \theta_{app}=55.7184 \Rightarrow ARD=0.00006$

حالت سوم:



مانند حالت های قبل مساحت هاو در نتیجه معادلات می شود

و تفاوت تنها در شرط مرزی ها است.

$$\frac{\partial}{\partial x}\left(x^2\frac{\partial\theta}{\partial x}\right)-m^2\theta x=0 \qquad m^2=\frac{2h_2L}{KR}$$

حل روش exact:

$$Bc\begin{cases} x=0 \rightarrow \dfrac{\partial\theta}{\partial x}=0 \\ x=L \Rightarrow K\dfrac{\partial\theta}{\partial x}=+q'' \end{cases}$$

$$\frac{\partial}{\partial x}\left(x^2\frac{\partial\theta}{\partial x}\right)-m^2\theta x=0 \Rightarrow \begin{cases} \alpha=2 \quad \beta=1 \\ \beta-\alpha+2=1\neq 0 \end{cases} \Rightarrow$$

$$\theta(x)=C_1x^{-\frac{1}{2}}I_{-1}\left(2x^{\frac{1}{2}}m\right)+C_2x^{-\frac{1}{2}}K_{-1}\left(2x^{\frac{1}{2}}m\right)$$

at Bc: $x=0 \Rightarrow \dfrac{\partial\theta}{\partial x}=0$ or $\theta=$finite $\Rightarrow$ $\boxed{C_2=0}$

$\Rightarrow$ at Bc: $x=L$ $K\dfrac{\partial\theta}{\partial x}=+q'' \Rightarrow \theta(x)=C_1x^{-\frac{1}{2}}I_{-1}\left(2x^{\frac{1}{2}}m\right)$

$$\frac{\partial\theta}{\partial x}=C_1\left[\frac{-I_1\left(2mx^{\frac{1}{2}}\right)}{2x^{\frac{3}{2}}}-\left(\frac{I_1\left(2mx^{\frac{1}{2}}\right)}{2x}-\frac{mI_0\left(2mx^{\frac{1}{2}}\right)}{x^{\frac{1}{2}}}\right)x\frac{1}{x^{\frac{1}{2}}}\right]=$$

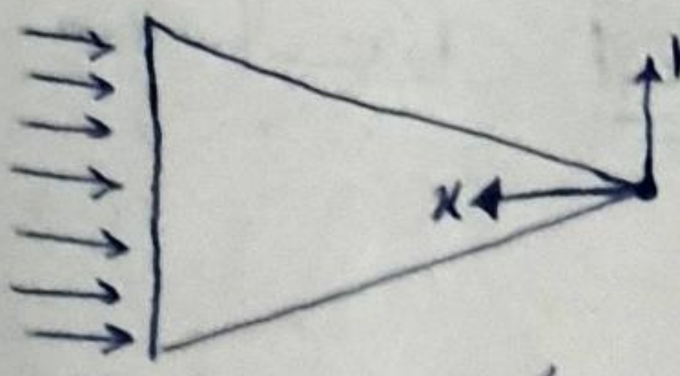
$\Rightarrow x=L \Rightarrow K\left.\dfrac{\partial\theta}{\partial x}\right|_{x=L}=q'' \Rightarrow \left.\dfrac{\partial\theta}{\partial x}\right|_{x=L}=\dfrac{q''}{K}$ *

از طریق متلب محاسبه شد مشتق θ.

$\rightarrow$ * از طریق حل معادله $\Rightarrow$ $\boxed{C_1=\surd}$ حل این معادله در متلب انجام می شود.

$$\theta(x)=C_1I_{-1}\left(2mx^{\frac{1}{2}}\right)x^{-\frac{1}{2}}$$

حالت سوم: حل به روش approximate



معادله‌ی حاکم: $\frac{\partial}{\partial x}\left(x^2\frac{\partial\theta}{\partial x}\right)-m^2\theta x=0$ $\quad m^2=\frac{2h_2L}{kR}$

$\theta(x)=A_0+A_1x+A_2x^2$ $\Rightarrow$ $BC\begin{cases} x=0 & \frac{\partial\theta}{\partial x}=0 \text{ or } \theta=\text{finite} \\ x=L & k\frac{\partial\theta}{\partial x}=q'' \end{cases}$

at BC: $x=0$ $\frac{\partial\theta}{\partial x}=0$ or $\theta=$finite $\Rightarrow$ $\boxed{A_1=0}$

at BC: $x=L$ $k\frac{\partial\theta}{\partial x}=q''$ $\Rightarrow$ $k(2A_2x)\Big|_{x=L}=q''$

$\Rightarrow 2kA_2L=q''$ $\Rightarrow$ $\boxed{A_2=\frac{q''}{2kL}}$

$\int_x \frac{\partial}{\partial x}\left(x^2\frac{\partial\theta}{\partial x}\right)-m^2\theta x=0 \Rightarrow \int_0^L \frac{\partial}{\partial x}\left(x^2\frac{\partial\theta}{\partial x}\right)-m^2\theta x)dx=0$

$\Rightarrow \int_0^L 6A_2x^2-m^2[A_0+A_2x^2]x)dx=0$

$\Rightarrow 2A_2x^3-\frac{m^2A_0x^2}{2}-\frac{m^2A_2x^4}{4}\Big|=0$ $\Rightarrow$ $\Big|_{x=L}$

$2A_2L^3-\frac{m^2A_2L^4}{4}=\frac{m^2A_0L^2}{2}$ $\Rightarrow$ $\boxed{A_0=\frac{4A_2L^3-\frac{m^2A_2L^4}{2}}{m^2L^2}}$

$\Rightarrow \theta(x)=A_0+A_2x^2$

محاسبات برای $L=0.1$

$K=94,\ R=0.1,\ h_2=10$

$q''=200$

برای حل exact داریم:

$m^2=0.2128 \Rightarrow m=0.4613$

C_1 محاسبه شده $=43.0528 \Rightarrow \theta(x)=C_1 x^{-\frac{1}{2}} I_1(2mx^{\frac{1}{2}})$

$x=0.01 \Rightarrow \theta_{exact}(0.01)=19.8799$

$x=0.02 \Rightarrow \theta_{exact}(0.02)=19.9011$

$x=0.03 \Rightarrow \theta_{exact}(0.03)=19.9222$

$x=0.06 \Rightarrow \theta_{exact}(0.06)=19.9858$

$x=0.07 \Rightarrow \theta_{exact}(0.06)=20.0070$

$x=0.1 \Rightarrow \theta_{exact}(0.1)=20.0708$

برای طول $L=0.1$ محاسبات به روش approximate:

$A_0=19.9468$

$A_1=10.6383 \Rightarrow \theta(x)=A_0+A_2x^2$

$x=0.01 \Rightarrow \theta_{app}=19.9479 \Rightarrow ARD=0.0034$

$x=0.02 \Rightarrow \theta_{appro}=19.9511 \Rightarrow ARD=0.0025$

$x=0.03 \Rightarrow \theta_{appro}=19.9584 \Rightarrow ARD=0.0017$

$x=0.06 \Rightarrow \theta_{appro}=19.9851 \Rightarrow ARD=0.0000$

$x=0.07 \Rightarrow \theta_{appro}=19.9989 \Rightarrow ARD=0.0004$

$x=0.1 \Rightarrow \theta_{appro}=20.0532 \Rightarrow ARD=0.0009$

محاسبات برای طول $L=0.5$

$K=94,\ R=0.1,\ h_2=10$

$q''=200$

محاسبات برای روش exact : $L=0.5$ برای طول

$m^2=1.0638 \Rightarrow m=1.0314$

$\theta(x)=C_1 I_1(2mx^{\frac{1}{2}})x^{-\frac{1}{2}} \Rightarrow$ محاسبه شده C_1 برابر $= 3.2603$

$x=0.01 \Rightarrow \theta_{exact}(0.01)=3.3806$

$x=0.2 \Rightarrow \theta_{exact}(0.2)=3.7333$

$x=0.3 \Rightarrow \theta_{exact}(0.3)=3.9286$

$x=0.35 \Rightarrow \theta_{exact}(0.35)=4.0288$

$x=0.4 \Rightarrow \theta_{exact}(0.4)=4.1307$

$x=0.5 \Rightarrow \theta_{exact}(0.5)=4.3399$

محاسبات به روش approximate برای طول $L=0.5$:

$A_0=3.7340$

$A_2=2.1277 \Rightarrow \theta(x)=A_0+A_2x^2$

$x=0.01 \Rightarrow \theta_{appro}=3.7343 \Rightarrow ARD=0.1046$

$x=0.2 \Rightarrow \theta_{appro}=3.8191 \Rightarrow ARD=0.0230$

$x=0.3 \Rightarrow \theta_{appro}=3.9255 \Rightarrow ARD=0.0008$

$x=0.35 \Rightarrow \theta_{appro}=3.9947 \Rightarrow ARD=0.00085$

$x=0.4 \Rightarrow \theta_{appro}=4.0745 \Rightarrow ARD=0.0136$

$x=0.5 \Rightarrow \theta_{appro}=4.2660 \Rightarrow ARD=0.0170$

محاسبات برای طول $L=1$:

$K=94,\ R=0.1,\ h_2=10$

$q''=200$

محاسبات به روش exact برای طول $L=1$:

$m^2=2.1277 \Rightarrow m=1.4586$

$\Rightarrow \theta(x)=c_1 x^{-1/2} I_1(2m x^{1/2})$

مقدار c_1 محاسبه شده $=0.7115$

$x=0.01 \Rightarrow \theta_{exact}(0.01)=1.0489$

$x=0.1 \Rightarrow \theta_{exact}(0.1)=1.1522$

$x=0.2 \Rightarrow \theta_{exact}(0.2)=1.2749$

$x=0.5 \Rightarrow \theta_{exact}(0.5)=1.6969$

$x=0.8 \Rightarrow \theta_{exact}(0.8)=2.2104$

$x=1 \Rightarrow \theta_{exact}(1)=2.6107$

محاسبات به روش approximate برای طول $L=1$:

$A_0=1.4681$

$A_2=1.0638$

$\Rightarrow \theta(x)=A_0+A_2 x^2$

$x=0.01 \Rightarrow \theta_{appr_0}=1.4682 \Rightarrow ARD=0.3997$

$x=0.1 \Rightarrow \theta_{appr_0}=1.4787 \Rightarrow ARD=0.2834$

$x=0.2 \Rightarrow \theta_{appr_0}=1.5106 \Rightarrow ARD=0.1849$

$x=0.5 \Rightarrow \theta_{appr_0}=1.7340 \Rightarrow ARD=0.0219$

$x=0.8 \Rightarrow \theta_{appr_0}=2.1489 \Rightarrow ARD=0.0278$

$x=1 \Rightarrow \theta_{appr_0}=2.5319 \Rightarrow ARD=0.0302$

محاسبات برای طول $L=5$

$k=94$, $R=0.1$, $h_2=10$

$q''=200$

محاسبات به روش exact برای طول $L=5$:

$m^2=10.6368 \Rightarrow m=3.2616$

$\theta(x)=C_1 x^{-\frac{1}{2}} I_1(2m x^{\frac{1}{2}})$ $\Rightarrow$ مقدار محاسبه شده $C_1 = 1.6499 \times 10^{-5}$

$x=0.01 \Rightarrow \theta_{exact}(0.01) = 0.0001$

$x=1 \Rightarrow \theta_{exact}(1) = 0.0016$

$x=2 \Rightarrow \theta_{exact}(2) = 0.0149$

$x=3 \Rightarrow \theta_{exact}(3) = 0.0881$

$x=4 \Rightarrow \theta_{exact}(4) = 0.4099$

$x=5 \Rightarrow \theta_{exact}(5) = 1.6224$

محاسبات به روش approximate برای طول $L=5$:

$A_0 = -2.2596$

$A_2 = 0.2128$ $\Rightarrow \theta(x) = A_0 + A_2 x^2$

$x=0.01 \Rightarrow \theta_{appro} = -2.2596$ $\Rightarrow ARD = 3.9833 \times 10^{4}$

$x=1 \Rightarrow \theta_{appro} = -2.0468$ $\Rightarrow ARD = 0.1243 \times 10^{4}$

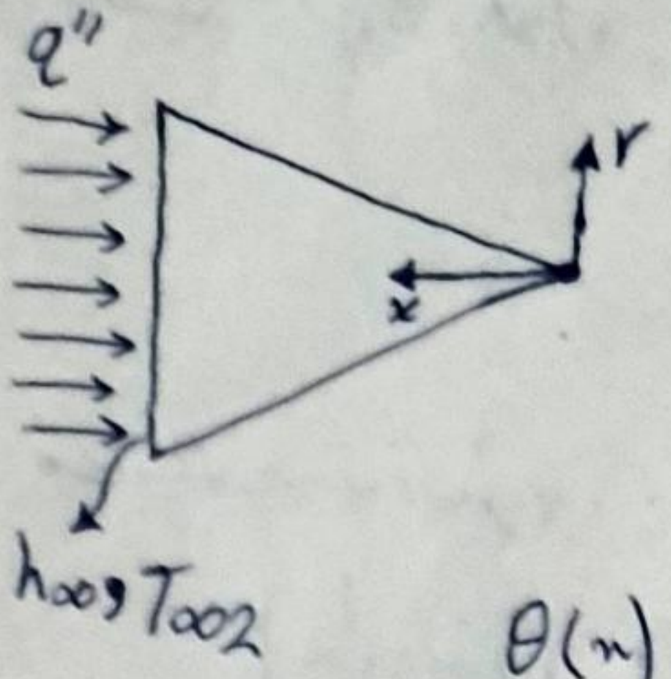
$x=2 \Rightarrow \theta_{appro} = -1.4085$ $\Rightarrow ARD = 0.0096 \times 10^{4}$

$x=3 \Rightarrow \theta_{appro} = -0.3447$ $\Rightarrow ARD = 0.0005 \times 10^{4}$

$x=4 \Rightarrow \theta_{appro} = 1.1447$ $\Rightarrow ARD = 0.0002 \times 10^{4}$

$x=5 \Rightarrow \theta_{appro} = 3.0596$ $\Rightarrow ARD = 0.0001 \times 10^{4}$

حالت چهارم: حل به روش approximate:



معادله‌ی حاکم: $\frac{\partial}{\partial x}\left(x^2 \frac{\partial \theta}{\partial x}\right) - m^2 \theta x = 0 \qquad m^2 = \frac{2h_2 L}{kR}$

$$\theta(x) = A_0 + A_1 x + A_2 x^2 \Rightarrow$$

$$BC \begin{cases} x=0 & \frac{\partial \theta}{\partial x} = 0 \quad \text{or } \theta = \text{finite} \rightarrow \boxed{A_1 = 0} \\ x=L & k\frac{\partial \theta}{\partial x} = q'' - h_\infty(\theta + T_{\infty 1} - T_{\infty 2}) \end{cases}$$

$$\rightarrow k(2A_2 x)\Big|_{x=L} = q'' - h_\infty\left(\left[A_0 + A_2 x^2\right] + T_{\infty 1} - T_{\infty 2}\right)\Big|_{x=L}$$

$$\Rightarrow \frac{2kA_2 L}{h_\infty} = \frac{q''}{h_\infty} - \left(A_0 + A_2 L^2 + T_{\infty 1} - T_{\infty 2}\right)$$

$$\Rightarrow \boxed{A_0 = \frac{q''}{h_\infty} - \frac{2kA_2 L}{h_\infty} - A_2 L^2 - T_{\infty 1} + T_{\infty 2}} \quad (1)$$

$$\int_x \left(\frac{\partial}{\partial x}\left(x^2 \frac{\partial \theta}{\partial x}\right) - m^2 x\theta\right) dx = 0 \Rightarrow \int_0^L \left(\frac{\partial}{\partial x}\left(x^2 \frac{\partial \theta}{\partial x}\right) - m^2 x\theta\right) dx = 0$$

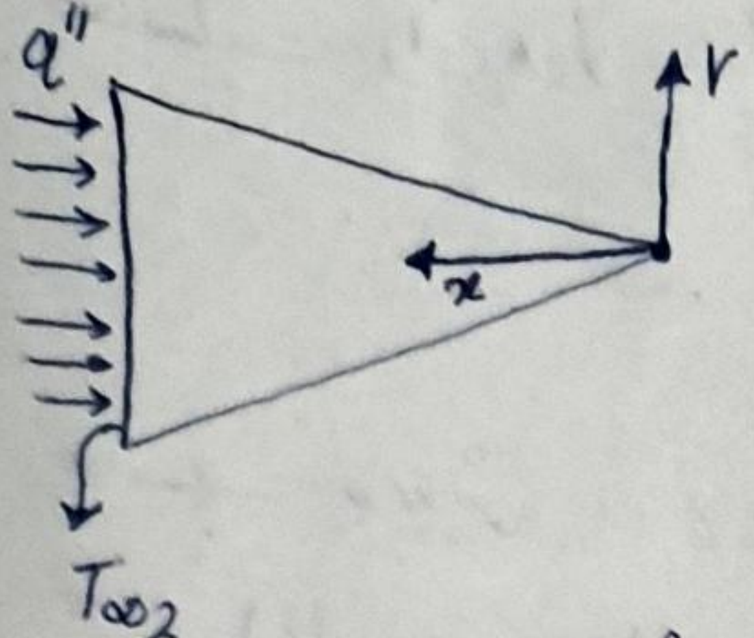
$$\int_0^L \left(6A_2 x^2 - m^2\left[A_0 + A_2 x^2\right] x\right) dx = 0$$

$$2A_2 x^3 - \frac{m^2 A_0}{2} x^2 - \frac{m^2 A_2 x^4}{4}\Big|_0^L = 0$$

$$\boxed{2A_2 L^3 - \frac{m^2 A_2 L^4}{4} = \frac{m^2 A_0 L^2}{2}} \quad (2)$$

از حل دو معادله و دو مجهول (1) و (2) مقادیر A_0 و A_2 بدست می‌آیند که حل این معادلات فایل متلب است.

حالت وچهارم: این حالت هم مانند حالت های قبل مساحت ها ودرنتیجه معادله ی حاکم یکسان است وفقط در شرط مرزی ها تفاوت است.



$$\frac{\partial}{\partial x}\left(x^2\frac{\partial\theta}{\partial x}\right)-m^2\theta x=0 \qquad m^2=\frac{2h_2L}{KR}$$

$$\Rightarrow \frac{\partial}{\partial x}\left(x^2\frac{\partial\theta}{\partial x}\right)-m^2\theta x=0 \Rightarrow \begin{cases}\alpha=2 & \beta=1\\ \beta-\alpha+2=1\neq 0\end{cases} \Rightarrow$$

$$\theta(x)=c_1x^{-\frac{1}{2}}I_{-1}\left(2mx^{\frac{1}{2}}\right)+c_2x^{-\frac{1}{2}}K_{-1}\left(2mx^{\frac{1}{2}}\right)$$

$$BC\begin{cases} x=0 & \frac{\partial\theta}{\partial x}=0 \text{ or } \theta=\text{finite} \Rightarrow \boxed{C_2=0}\\ x=L & -K\frac{\partial\theta}{\partial x}+q''-h_\infty(\theta+T_{\infty 1}-T_{\infty 2})=0\end{cases}$$

$$\Rightarrow x=L \Rightarrow K\frac{\partial\theta}{\partial x}\Big|_{x=L}=q''-h_\infty(\theta+T_{\infty 1}-T_{\infty 2})$$

$$\frac{\partial\theta}{\partial x}=c_1\left[-\frac{I_1\left(2mx^{\frac{1}{2}}\right)}{2x^{\frac{3}{2}}}-\left(\frac{I_1\left(2mx^{\frac{1}{2}}\right)}{2x}-\frac{mI_0\left(2mx^{\frac{1}{2}}\right)}{x^{\frac{1}{2}}}\right)\times\frac{1}{x^{\frac{1}{2}}}\right]$$

سپس از طریق حل معادله مقدار c_1 را بدست می آوریم در مطلب.

$$\theta(x)=c_1x^{-\frac{1}{2}}I_{-1}\left(2mx^{\frac{1}{2}}\right)$$

$K=94,\ R=0.1,\ h_2=10,\ h_\infty=150$ محاسبات برای طول $\underline{L=0.1}$

$T_{\infty 1}=20 \quad T_{\infty 2}=100 \quad q''=200$

$m^2=0.2128 \Rightarrow m=0.4613$: $\underline{L=0.1}$ برای حل exact طول

مقدار محاسبه شده $C_1=163.5959 \Rightarrow \theta(x)=C_1 x^{-\frac{1}{2}} I_{-1}(2mx^{\frac{1}{2}})$

$x=0.01 \Rightarrow \theta_{exact}(0.01)=75.5415$

$x=0.02 \Rightarrow \theta_{exact}(0.02)=75.6219$

$x=0.03 \Rightarrow \theta_{enact}(0.03)=75.7023$

$x=0.06 \Rightarrow \theta_{enact}(0.06)=75.9439$

$x=0.07 \Rightarrow \theta_{enact}(0.07)=76.0245$

$x=0.1 \Rightarrow \theta_{exact}(0.1)=76.2668$

$A_0=75.8581$: $\underline{L=0.1}$ برای طول approximate روش حل

$A_2=40.4576 \rightarrow \theta(x)=A_0+A_2x^2$

$x=0.01 \rightarrow \theta_{appro}=75.8621 \Rightarrow ARD=0.0042$

$x=0.02 \rightarrow \theta_{appro}=75.8742 \Rightarrow ARD=0.0033$

$x=0.03 \rightarrow \theta_{appro}=75.8945 \Rightarrow ARD=0.0025$

$x=0.06 \Rightarrow \theta_{appro}=76.0037 \Rightarrow ARD=0.0008$

$x=0.07 \Rightarrow \theta_{appro}=76.0563 \Rightarrow ARD=0.0004$

$x=0.1 \Rightarrow \theta_{appro}=76.2626 \Rightarrow ARD=0.0001$

محاسبات برای طول $L=0.5$:

$k=94, R=0.1, h_2=10, h_\infty=150$

$T_{\infty_1}=20, T_{\infty_2}=100, q''=200$

حل به روش enact برای طول $L=0.5$:

$m^2=1.0638 \Rightarrow m=1.0314$

$\Rightarrow \theta(n)=C_1 I_{-1}(2mn^{1/2}) n^{-1/2}$

مقدار محاسبه شده $C_1=46.7399$

$n=0.01 \Rightarrow \theta_{enact}(0.01)=48.4654$

$n=0.02 \Rightarrow \theta_{enact}(0.2)=53.5222$

$n=0.3 \Rightarrow \theta_{enact}(0.3)=56.3216$

$n=0.35 \Rightarrow \theta_{enact}(0.35)=57.7581$

$n=0.4 \Rightarrow \theta_{enact}(0.4)=59.2194$

$n=0.5 \Rightarrow \theta_{enact}(0.5)=62.2183$

حل به روش approximate برای طول $L=0.5$:

$A_0=54.2394$

$A_2=30.9056 \Rightarrow \theta(n)=A_0+A_2 n^2$

$n=0.01 \Rightarrow \theta_{appro}=54.2425 \Rightarrow ARD=0.1192$

$n=0.2 \Rightarrow \theta_{appro}=55.4756 \Rightarrow ARD=0.0365$

$n=0.3 \Rightarrow \theta_{appro}=57.0209 \Rightarrow ARD=0.0124$

$n=0.35 \Rightarrow \theta_{appro}=58.0253 \Rightarrow ARD=0.0046$

$n=0.4 \Rightarrow \theta_{appro}=59.1843 \Rightarrow ARD=0.0006$

$n=0.5 \Rightarrow \theta_{appro}=61.9658 \Rightarrow ARD=0.0041$

محاسبات برای طول $L=1$:

$k=94,\ R=0.1,\ h_2=10,\ h_\infty=150$

$T_{\infty 1}=20,\ T_{\infty 2}=100,\ q''=200$

محاسبات به روش Exact برای طول $L=1$:

$m^2=2.1277 \Rightarrow m=1.4586$

مقدار محاسبه شده $c_1=14.6721 \Rightarrow \theta(x)=c_1 I_1(2mx^{\frac{1}{2}})x^{-\frac{1}{2}}$

$x=0.01 \Longrightarrow \theta_{exact}(0.01)=21.6300$

$x=0.1 \Longrightarrow \theta_{exact}(0.1)=23.7604$

$x=0.2 \Longrightarrow \theta_{exact}(0.2)=26.2897$

$x=0.5 \Longrightarrow \theta_{exact}(0.5)=34.9225$

$x=0.8 \Longrightarrow \theta_{exact}(0.8)=45.5817$

$x=1 \Longrightarrow \theta_{exact}(1)=53.8378$

محاسبات به روش Approximate برای طول $L=1$:

$A_0=30.8917$

$A_2=22.3853 \Rightarrow \theta(x)=A_0+A_2x^2$

$x=0.01 \Longrightarrow \theta_{appro}=30.8940 \Longrightarrow ARD=0.4283$

$x=0.1 \Longrightarrow \theta_{appro}=31.1156 \Longrightarrow ARD=0.3096$

$x=0.2 \Longrightarrow \theta_{appro}=31.7872 \Longrightarrow ARD=0.2091$

$x=0.5 \Longrightarrow \theta_{appro}=36.4881 \Longrightarrow ARD=0.0427$

$x=0.8 \Longrightarrow \theta_{appro}=45.2183 \Longrightarrow ARD=0.0080$

$x=1 \Longrightarrow \theta_{appro}=53.2771 \Longrightarrow ARD=0.0104$

محاسبات برای طول $L=5$:

$K=94,\ R=0.1,\ h_2=10,\ h_\infty=150$

$T_{\infty_1}=20,\ T_{\infty_2}=100,\ q''=200$

محاسبات برای روش enact طول $L=5$:

$m^2=10.6383, m=3.26$

مقدار محاسبه شده $C_1=4.539941\times10^{-4} \Rightarrow \theta(n)=C_1 n^{-\frac{1}{2}} I_1(2mn^{\frac{1}{2}})$

$n=0.01 \Rightarrow \theta_{enact}(0.01)=0.0016$

$n=1 \Rightarrow \theta_{enact}(1)=0.0453$

$n=2 \Rightarrow \theta_{enact}(2)=0.4100$

$n=3 \Rightarrow \theta_{enact}(3)=2.4251$

$n=4 \Rightarrow \theta_{enact}(4)=11.2781$

$n=5 \Rightarrow \theta_{enact}(5)=44.6440$

محاسبات به روش approximate طول $L=5$:

$A_0=-41.8353$

$A_2=3.9393 \Rightarrow \theta(n)=A_0+A_2n^2$

$n=0.01 \Rightarrow \theta_{appro}=-41.8349 \Rightarrow ARD=2.6802\times10^{-4}$

$n=1 \Rightarrow \theta_{appro}=-37.8960 \Rightarrow ARD=0.0837\times10^{-4}$

$n=2 \Rightarrow \theta_{appro}=-26.0781 \Rightarrow ARD=0.0065\times10^{-4}$

$n=3 \Rightarrow \theta_{appro}=-6.3817 \Rightarrow ARD=0.0004\times10^{-4}$

$n=4 \Rightarrow \theta_{appro}=+21.1934 \Rightarrow ARD=0.0001\times10^{-4}$

$n=5 \Rightarrow \theta_{appro}=56.6471 \Rightarrow ARD=0.0000$