Mech 568 - Assignment 01 - Finite Difference

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Problem 1

1.a Problem Statement

The 1-D wave equation for an acousting wave using the finite difference method is defined As:

$$\frac{\partial^2 P}{\partial t^2} = c^2 \frac{\partial^2 P}{\partial x^2} \tag{1}$$

Assumptions and conditions:

- simulating the first 0.05s of the wave with 0.001s time increments.
- Length of the pipe: 100 ft
- Number of spatial points 10 (one point each 10 ft)
- $c = 1125 \frac{ft}{s}$ is the speed of sound in water.
- The initial pressure is the steady-state value of 850psi
- Since the the wave equation is a 2nd order differential with respect to time we need to initial conditions which is the pressure of 850psi at the time step 0 and -1
- In order to make the equations simpler:

$$r = \frac{c^2 \Delta t^2}{\Delta x^2} \tag{2}$$

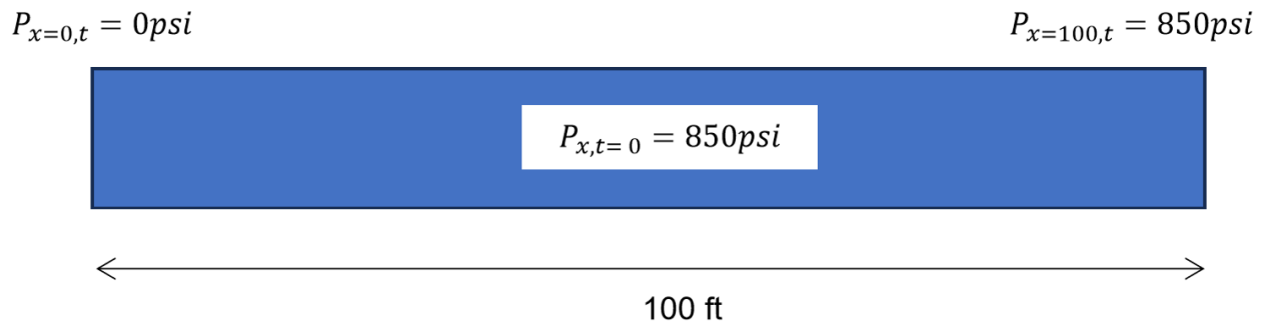

Figure 1: Schematic of the boundary conditions of the 1-D pipe under an acoustic wave

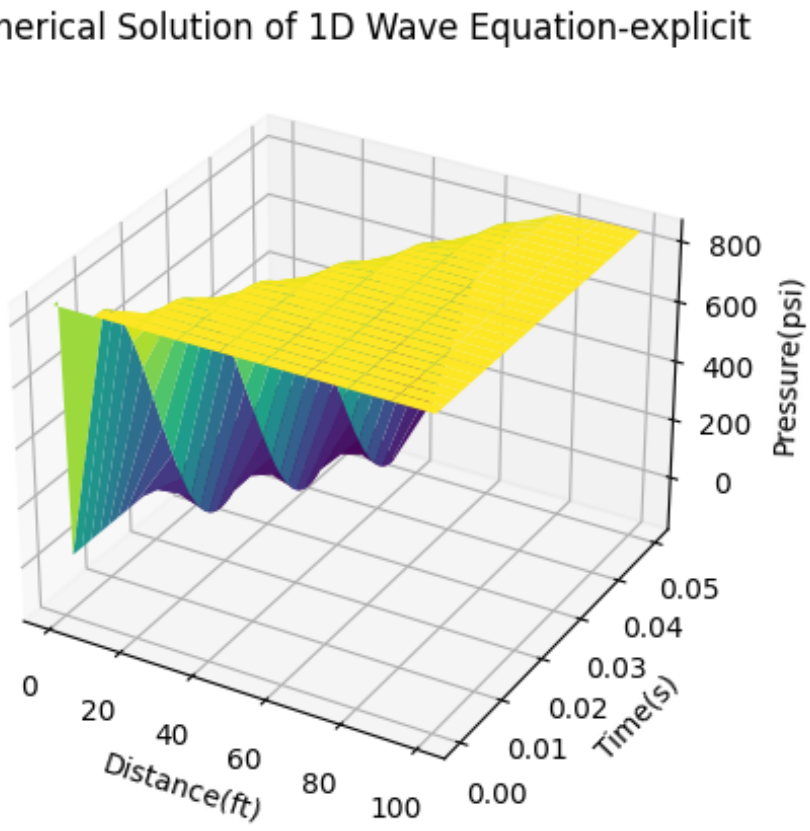

Figure 2: The plot of the solution for 1-D wave equation using explicit method

1.b Explicit Method

The equation after discretizizng the equation using the explicit method:

$$P_{k+1,i} = rP_{k,i-1} + [2-2r]P_{k,i} + rP_{k,i+1} - P_{k-1,i} \tag{3}$$

Starting at the first time step at each time step, for any point this equation is solved.

1.c Implicit Method

The equation after discretizizng the equation using the Implicit method:

$$P_{k,i} - \frac{1}{2}P_{k-1,i} = -\frac{r}{2}P_{k+1,i-1} + \frac{(2r+1)}{2}P_{k+1,i} - \frac{r}{2}P_{k+1,i+1} \tag{4}$$

The $P_{k,i}$ and $P_{k-1,i}$ are know. The left side of the equation are the unknowns that make a matrix for all the points of the domain.

1.d Discussion

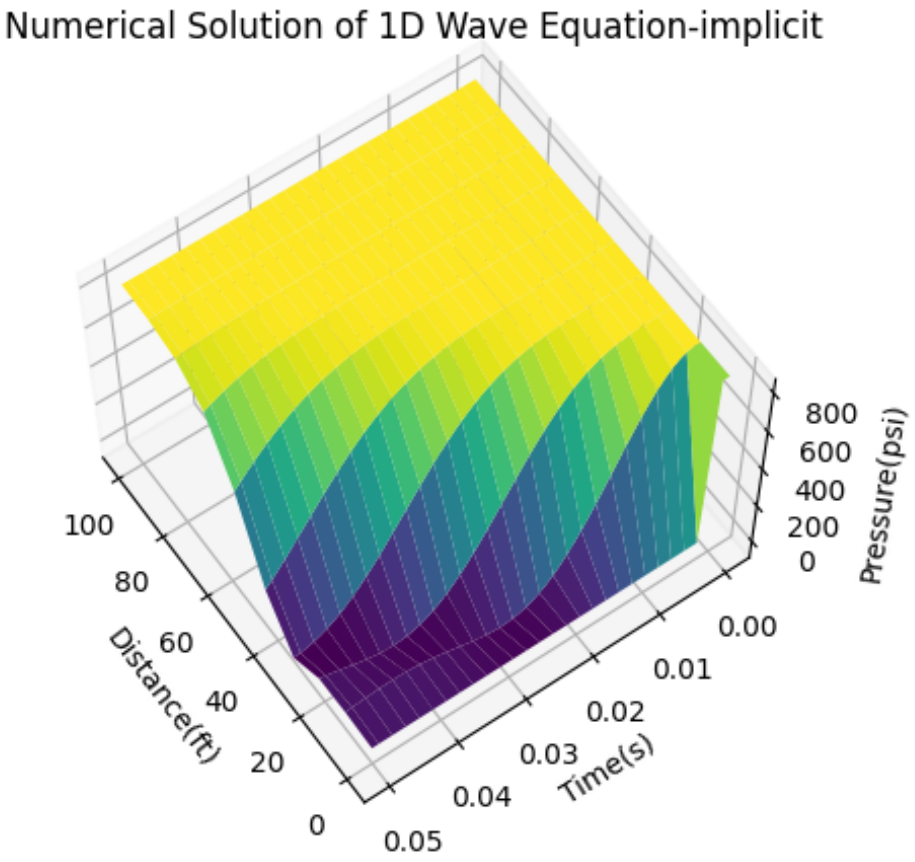

Figure 3: The plot of the solution for 1-D wave equation using implicit method

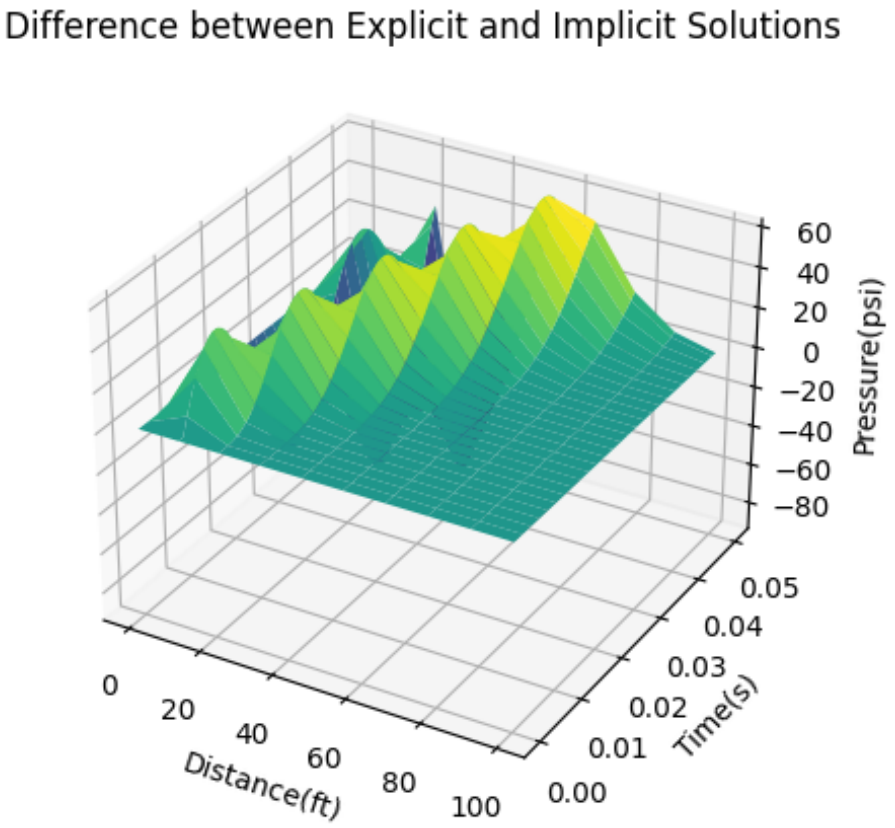

Figure 4: The plot of the difference of the implicit and explicit method