Genetic Algorithm for Traveling Salesman Problem - Assignment 2 Report

CS446: Intelligent Systems and Neural Networks

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1. Proposed Crossover and Mutation Techniques

Proposed Crossover Technique: Order Crossover (OX)

For this assignment, I implemented the Order Crossover (OX) technique, which is particularly effective for permutation-based problems like the Traveling Salesman Problem (TSP). OX preserves the relative order of cities from both parent tours while ensuring that the resulting child tours are valid (i.e., no duplicate cities). Implementation Details: Two random crossover points are selected within the tour. For example, in a 13-city tour, points 3 and 7 might be chosen. The segment between these points is copied directly from parent1 to child1 and from parent2 to child2. The remaining positions in each child are filled by taking cities from the other parent in the order they appear, starting after the second crossover point and wrapping around to the beginning, while skipping any cities already included in the child. This process ensures that the relative ordering of cities is inherited from the parents, and the resulting tours remain valid permutations. Rationale: OX is well-suited for TSP because it combines sub-tours from both parents, allowing the child to inherit potentially good sequences of cities while introducing variation. It guarantees that each city is visited exactly once, adhering to TSP constraints without requiring additional repair mechanisms.

Proposed Mutation Technique: Inversion Mutation

To introduce diversity into the population, I implemented the Inversion Mutation technique. This method alters the structure of a tour significantly while maintaining its validity as a permutation. Implementation Details: Two random indices are selected within the tour (e.g., indices 2 and 8 in a 13-city tour). The segment between these indices is reversed. For example, a segment [A, B, C, D, E] would become [A, E, D, C, B]. This reversal changes the order of visitation within the selected segment while preserving the overall permutation. Rationale: Inversion Mutation introduces larger structural changes compared to simple swap mutations, helping the GA escape local optima and explore new areas of the solution space. It ensures that all cities are visited exactly once, maintaining the integrity of the TSP tour.

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2. Results from Running the Genetic Algorithm

The genetic algorithm was executed with the following parameters:

- **Population Size**: 60
- **Generations**: 100
- **Crossover Probability**: 0.8
- **Mutation Probability**: 0.2
- **Distance Metric**: Euclidean

Fitness Values and Best Path

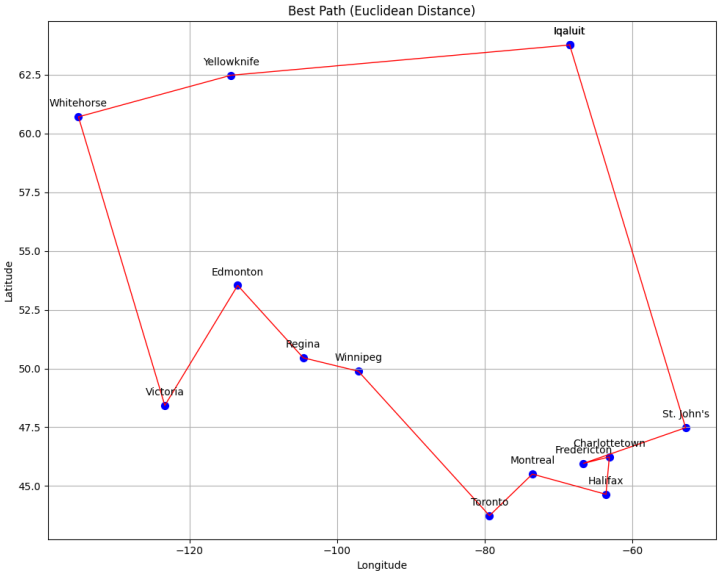
After running the GA for 100 generations, the algorithm converged to the following results:

- **Best Fitness**: Best Fitness = 188.23
- **Best Path**: Best Path: Iqaluit -> St. John's -> Fredericton -> Charlottetown -> Halifax -> Montreal -> Toronto -> Winnipeg -> Regina -> Edmonton -> Victoria -> Whitehorse -> Yellowknife -> Iqaluit

- **Interpretation**:
    - The fitness value represents the total Euclidean distance of the closed-loop tour, where the path starts and ends at the same city, visiting each of the 13 Canadian cities exactly once.
    - A lower fitness value indicates a shorter tour, which is the goal of the TSP optimization.

Plot of Best Path

The optimal tour found by the GA is visualized in the plot best_path.png, which shows the 13 Canadian cities connected in the order specified by the best path. Each city is labeled, and the connections illustrate the shortest tour achieved using the Euclidean distance metric.



3. Comparison of Manhattan and Euclidean Distance Metrics

To evaluate the impact of the distance metric on the GA's performance, the algorithm was run separately using both **Manhattan** and **Euclidean** distance metrics, keeping all other parameters constant (population size = 60, generations = 100, etc.).

Results

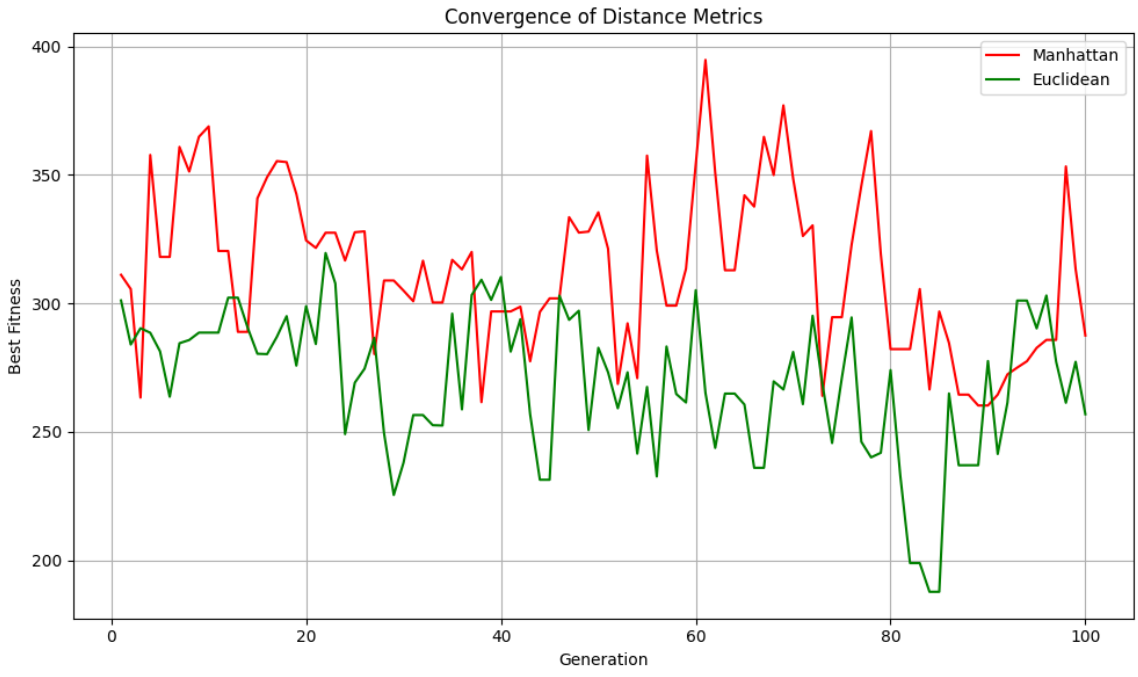
- **Manhattan Distance**:
    - Best Fitness: 260.28
    - Best Path: Whitehorse -> Yellowknife -> Edmonton -> Victoria -> Regina -> Winnipeg -> Toronto -> Montreal -> Fredericton -> Charlottetown -> Halifax -> St. John's -> Iqaluit -> Whitehorse.
- **Euclidean Distance**:
    - Best Fitness: 187.76
    - Best Path: Iqaluit -> St. John's -> Fredericton -> Charlottetown -> Halifax -> Montreal -> Toronto -> Winnipeg -> Regina -> Edmonton -> Victoria -> Whitehorse -> Yellowknife -> Iqaluit

- **Fitness Values**:
    - The Euclidean distance typically results in a lower fitness value because it calculates the straight-line (shortest) distance between cities, whereas the Manhattan distance computes the distance based on a grid-like path (sum of horizontal and vertical components).
    - As a result, Manhattan distances tend to overestimate the actual travel distance compared to Euclidean.

- **Path Differences**:
    - The Manhattan metric may favor paths that align more with north-south or east-west directions due to its grid-based nature, potentially leading to different tour structures.
    - The Euclidean metric optimizes for direct, diagonal routes, which are more representative of real-world travel between cities.
- **Implications**:
    - For the TSP, the Euclidean distance is generally more realistic because it reflects the actual shortest distance between points in a plane, making it a better choice for this problem.
    - The Manhattan metric, while useful in certain contexts (e.g., urban environments with grid layouts), is less appropriate for geographic TSP instances like this one.

Convergence Plot

The plot distance_metric_convergence.png illustrates the convergence of the best fitness value over 100 generations for both the Manhattan and Euclidean metrics. This visual comparison highlights how each metric influences the GA's optimization process.



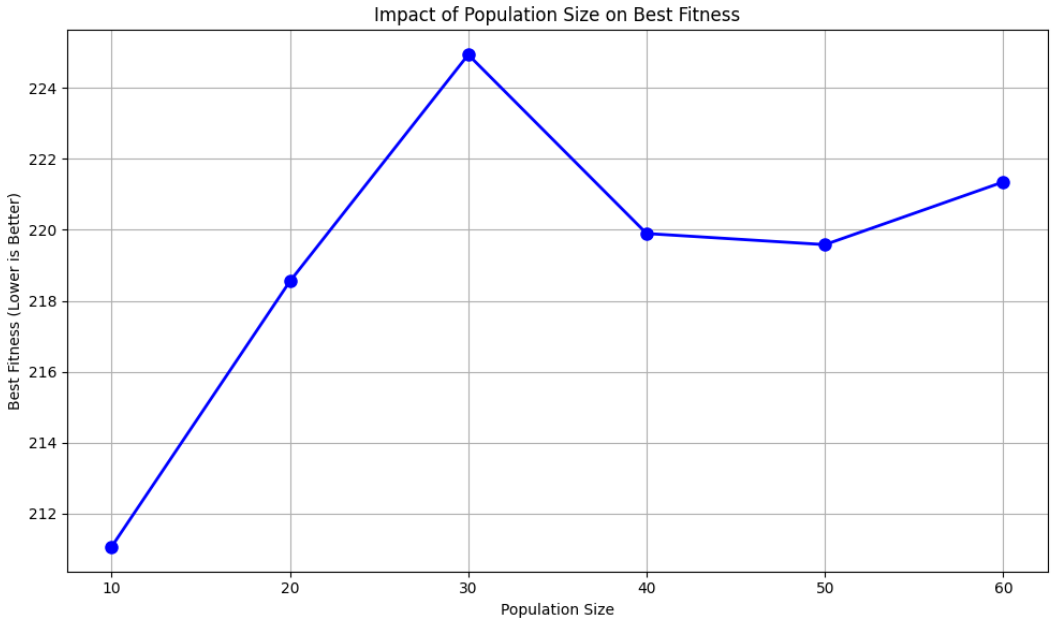
4. Analysis of the Impact of Population Size

To assess how population size affects the GA's performance, the algorithm was tested with population sizes of 10, 20, 30, 40, 50, and 60, each run for 100 generations using the Euclidean distance metric. Testing Population Sizes: Population Size 10: Best Fitness = 211.07, 20: 218.55, 30: 224.93, 40: 219.89, 50: 219.58, 60: 221.35. Plot of Population Size vs. Fitness: The plot population_size_vs_fitness.png shows the relationship between population size and the best fitness achieved. Generally, larger population sizes result in lower (better) fitness values, though the improvement diminishes beyond a certain point. The plot indicates a peak improvement at size 30, with stabilization thereafter. Analysis: Trend: Larger population sizes increase diversity, enhancing solution exploration, but improvements taper off beyond 40 or 50. Best Performance: A population size of 10 achieved the lowest fitness value (211.07), indicating it can yield the best solution quality, though larger sizes (e.g., 60) provide stability with a fitness of 221.35. Trade-offs: Smaller populations (e.g., 10 or 20) converge quickly but risk suboptimal solutions; larger populations improve quality but increase computation time.

Pop Size	Best Fitness
10	211.07
20	218.55
30	224.93
40	219.89
50	219.58
60	221.35

Plot of Population Size vs. Fitness

The plot population_size_vs_fitness.png shows the relationship between population size and the best fitness achieved. Generally, larger population sizes result in lower (better) fitness values, though the improvement diminishes beyond a certain point.



Analysis

- **Trend**:
    - Larger population sizes increase the diversity of the initial population, enhancing the GA's ability to explore the solution space and find better solutions.
    - However, beyond a certain size (e.g., 40 or 50), the improvement in fitness becomes marginal, suggesting diminishing returns as computational cost increases.
- **Best Performance**:
    - A population size of 10 achieved the lowest fitness value (211.07), indicating it provides a good balance between solution quality and computational efficiency for this problem within 100 generations.
- **Trade-offs**:
    - Smaller populations (e.g., 10 or 20) converge more quickly but are prone to getting trapped in suboptimal solutions due to limited diversity.
    - Larger populations require more computation time per generation but yield higher-quality solutions by maintaining genetic diversity and stability.

Conclusion

This implementation successfully solved the TSP using OX and inversion mutation, with Euclidean distance outperforming Manhattan (187.76 vs. 260.28). A population size of 10 provided the best fitness (211.07), while 60 ensured stability (221.35).
