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Decline Curve Analysis in Unconventional Resource Plays Using Logistic Growth Models

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Decline Curve Analysis in Unconventional Resource Plays Using Logistic Growth Models

by

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Dedication

To my parents for their support and encouragement

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Abstract

Decline Curve Analysis in Unconventional Resource Plays Using Logistic Growth Models

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Current models used to forecast production in unconventional oil and gas formations are often not producing valid results. When traditional decline curve analysis models are used in shale formations, Arps b-values greater than 1 are commonly obtained, and these values yield infinite cumulative production, which is non-physical.. Additional methods have been developed to prevent the unrealistic values produced, like truncating hyperbolic declines with exponential declines when a minimum production rate is reached. Truncating a hyperbolic decline with an exponential decline solves some of the problems associated with decline curve analysis, but it is not an ideal solution. The exponential decline rate used is arbitrary, and the value picked greatly effects the results of the forecast.

A new empirical model has been developed and used as an alternative to traditional decline curve analysis with the Arps equation. The new model is based on the concept of logistic growth models. Logistic growth models were originally developed in the 1830s by Belgian mathematician, Pierre Verhulst, to model population growth. The new logistic model for production forecasting in ultra-tight reservoirs uses the concept of a carrying capacity. The carrying capacity provides the maximum recoverable oil or gas from a single well, and it causes all forecasts produced with this model to be within a reasonable range of known volumetrically available oil. Additionally the carrying capacity causes the production rate forecast to eventually terminate as the cumulative production approaches the carrying capacity.

The new model provides a more realistic method for forecasting reserves in unconventional formations than the traditional Arps model. The typical problems encountered when using conventional decline curve analysis are not present when using the logistic model.

Predictions of the future are always difficult and often subject to factors such as operating conditions, which can never be predicted. The logistic growth model is well established, robust, and flexible. It provides a method to forecast reserves, which has been shown to accurately trend to existing production data and provide a realistic forecast based on known hydrocarbon volumes.

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Chapter 1: Decline Curve Analysis

1.1 Introduction

A practicing reservoir engineer performs many tasks, however, among the most important is the estimation of petroleum reserves and prediction of production rate decline. This can be done through various methods, including numerical reservoir simulation, analytic modeling, or empirical mathematical models. The most commonly used is the empirical mathematical models, typically referred to as decline curve analysis (DCA).

This work consists of two parts. The first part is a look at (i) the history of reserves estimates and decline curve analysis, (ii) improvements made to the traditional DCA techniques, and (iii) some of the new methods invented to use DCA in extremely low permeability oil and gas reservoirs. The second part introduces a new model to be used for DCA in low permeability reservoirs based on logistic growth models. This part includes an explanation of logistic growth models, development of the form of the model used here, and examples of the new model being used on specific field examples.

1.2 History of Reserves Estimates

Hydrocarbons, be they oil or gas, are finite and accumulated in limited quantities within the earth (Hubbert, 1956). It would then follow that the production rate of oil or gas from any individual well would in fact vanish. It was from the basic understanding that the earliest estimations of ultimate recovery began. In the later part of the 19th century until the early 20th century, reserves were estimated very crudely based on assumed geologic properties and observed reservoir boundaries (Lombardi, 1915; Requa, 1918). The basic method of estimating reserves came to be known as the saturation method. To forecast reserves with the saturation method, five parameters must be known: the thickness of the sands, the areal extent of the reservoir, the oil saturation, the percentage of rock occupied by oil (oil saturated porosity) and the amount of oil that could be recovered. The thickness of the sand could be determined prior to logging technology by assembling cross sections over numerous wells, and estimating the target formation from drill cuttings. The areal extent could be determined in conventional oil accumulations by drilling a sufficient number of wells to determine the boundaries of the productive reservoir. The oil saturation, porosity and recoverable percentage were all determined from rock cuttings. The amount of oil that remained after all the naturally flowing oil had been removed from the sample roughly estimated the recovery factor of the reservoir (Lombardi, 1915). There was a tremendous amount of uncertainty associated with all of these measurements, but ultimately the calculation used to determine recoverable oil from the saturation method is still used today. Today it is known as a volumetric calculation of original oil in place, and the methods for determining thickness, areal extent, saturation, porosity, and recovery factor have all improved thanks in particular to well logging and a greater understanding of the physics of fluid flow in porous media, but essentially it is the same calculation.

The second method used to estimate reserves began to evolve in the early 20th century. Initially it was referred to as the production decline method, but ultimately became known as decline curve analysis (DCA). Reservoir engineers had noted that the production of oil declined over time, but it was not until 1908 in a report by Arnold and Anderson (1908) that any attempt to quantify and calculate this decline was reported. The initial report simply mentioned the decline in production as a percentage of the previous production rate over a given time interval. By the mid 1910s, engineers and geoscientists began making efforts to form a kind of production decline methodology based on graphic representations of the declining production rate. This method grew rapidly in popularity because of the increased demand for hydrocarbons brought about by the war in Europe at the time (Arps, 1944). Lewis and Beal (1918), Requa (1918), Pack (1917), Arnold (1915) and numerous other petroleum engineers began to employ the early DCA methods to estimate future production and quantify reserves. The original method was to plot the percent decrease from the initial production rate versus time shown in the following figures.

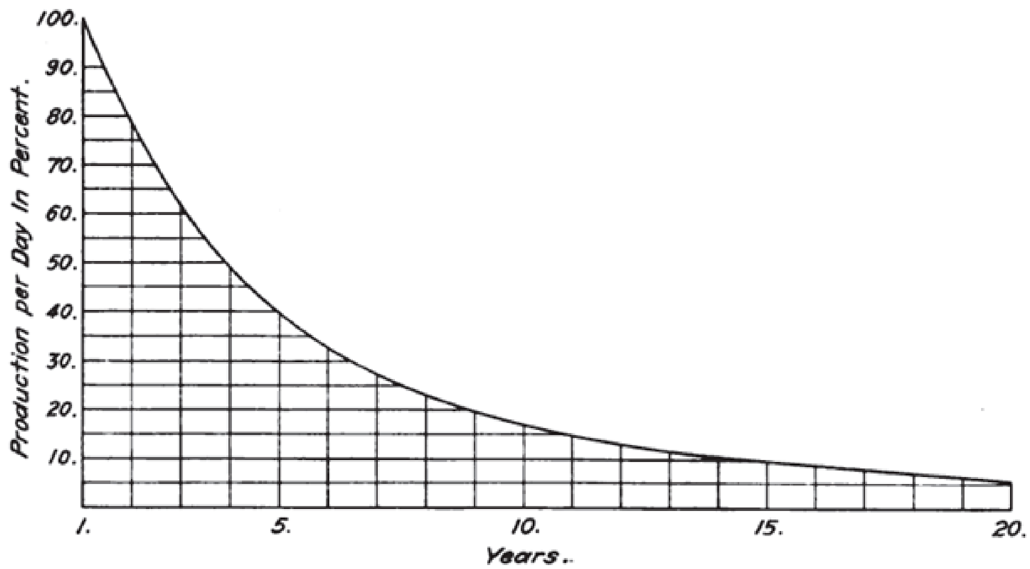

Figure 1 - Early attempt at decline curve analysis (Lombardi, 1915)

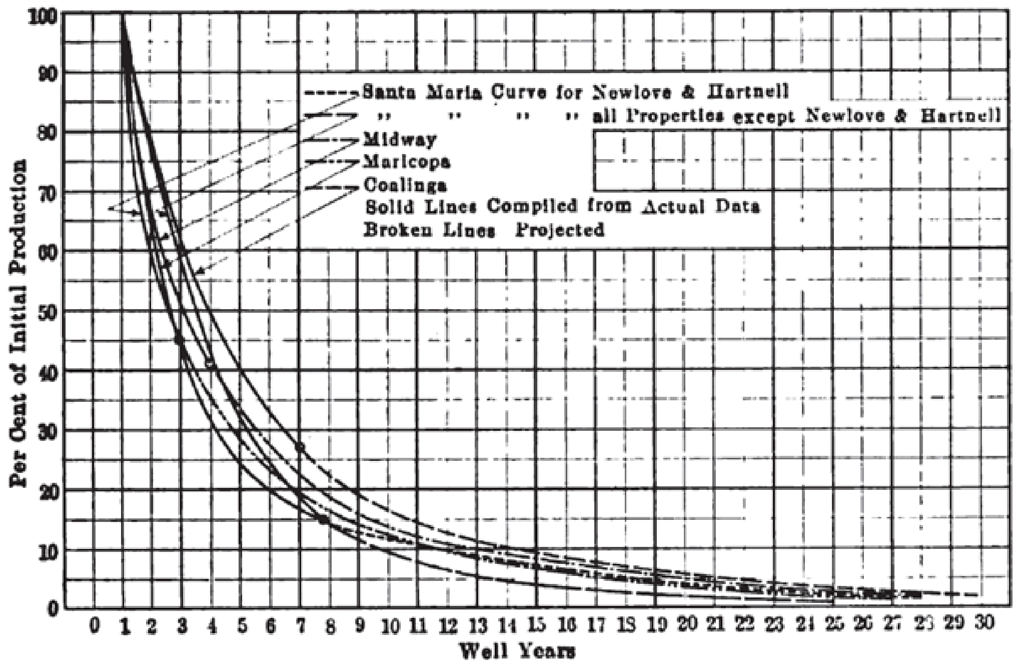

Figure 2 - Early attempt at decline curve analysis (Requa, 1915)

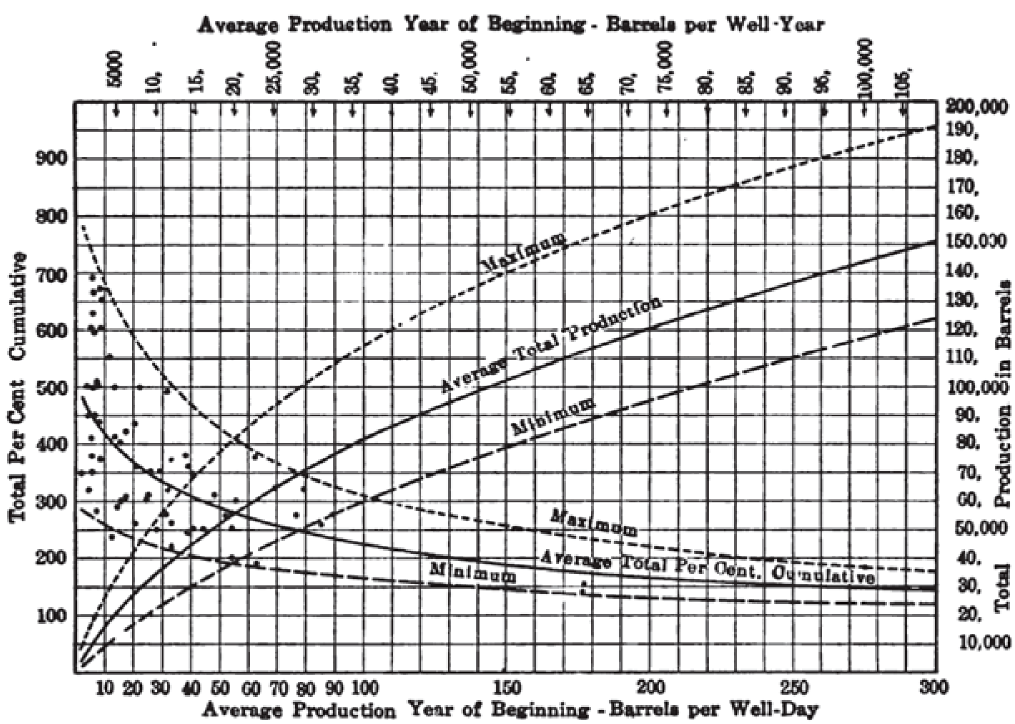

Figure 3 - Early attempt at decline curve analysis (Lewis and Beal, 1918)

Figure 1 by Lombardi shows a decline curve that was constructed for a large field in California. There were seven years of data for the field, and the forecast of decline into time was based on declines seen in similar and adjacent fields (Lombardi, 1915). The Y-axis is the current production rate at a given time divided by the initial peak production rate, to give a percent decline. **Figure 2** shows a decline curve done by Requa in 1915. It shows the decline percentages for various fields in California. The declines show a constant percentage decrease each year for the known production, and that is extrapolated into the future for the unknown production (Requa, 1915). **Figure 3** from Lewis and Beal shows a more advanced method that incorporates the uncertainty involved in forecasting with the production decline method. To account for the scatter of the data and the inability to predict the future, a probabilistic estimate was established offering a range of potential outcomes. The figure shows forecasts of both rate versus time and cumulative production versus time, with both cases offering a minimum, maximum and average estimate (Lewis and Beal, 1918).

The percent decline in production was relatively constant for entire fields, and could thus be used to estimate production rate in the area based only on initial rate. The rate of decrease for a single field was not necessarily the same for other fields. A paper by Johnson and Bollens (1927) laid the ground work for traditional DCA. Johnson and Bollens observed that the current production rate divided by the change in production rate over a given time period was constant. They then provided a method for calculating future production based on the observation.

$$y_n = y_{n-1}\left(\frac{r_n}{r_n+1}\right) \tag{1.1}$$

where

y	=	Production rate, volume/time
r	=	Ratio of production rate over change in production rate
n	=	Time interval

Equation 1.1 shows the method that Johnson and Bollens used to calculate the production rate out to some point in time. It was from **eq. 1.1** that the form of DCA used today was born.

1.3 DEVELOPMENT OF DECLINE CURVE ANALYSIS

Equation 1.1 proved to work very well for conventional oil wells and laid the ground work for Arps 1944 paper. Arps applied some simple mathematical formulation to Johnson and Bollens observation to come up with the by far most widely used method for estimating reserves. (Lee, 2010).

Arps observed that when the ratio of production rate over change in production rate was constant, it plotted as a straight line on semi log paper, and was an exponential decline. Arps rewrote the Johnson and Bollens equation in differential form yielding:

$$-a = \frac{q}{\left(\frac{dq}{dt}\right)} \tag{1.2}$$

where

a	=	Exponential decline constant, time
q	=	Production rate, volume/time
t	=	Time

The equation can then be rearranged and put into the form:

$$\frac{dq}{dt} = -\frac{q}{a}$$

(1.3)

Then by separating and integrating from the initial rate q_i to the rate at any later time q, the equation yields:

$$\int_{qi}^{q} \frac{1}{q} dq = -\frac{1}{a}\int_{0}^{t} dt \;\; => \;\; \ln\left(\frac{q}{qi}\right) = -\frac{t}{a}$$

(1.4)

where

q_i = Initial production rate, volume/time

Finally, by exponentiation of both sides of the equation and rearrangement, the Arps equation for exponential decline takes on the form that it is commonly seen today:

$$q(t) = q_i \exp\left(-\frac{t}{a}\right)$$

(1.5)

Equation 1.5 is a frequently encountered equation referred to as exponential growth or decay. According to Goldstein *et al.*, the basic idea is that "at every instant, the rate of increase of the quantity is proportional to the quantity at that instant" (Goldstein *et al.*, 1977). In this case, it is the rate of decrease seen in the figure below.

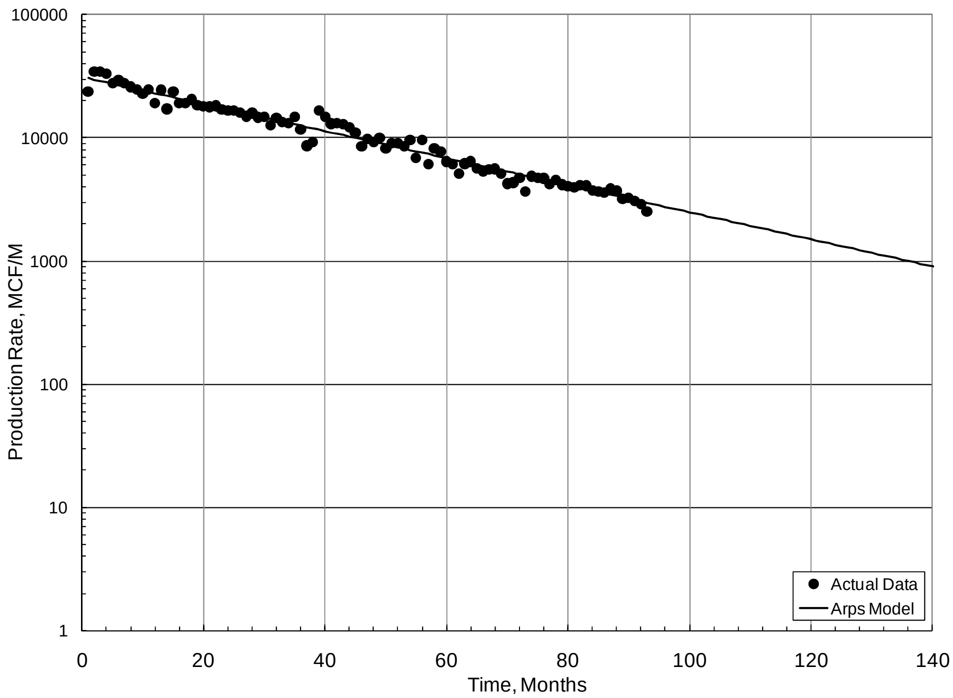

Figure 4 - Example of actual production data for a gas well being fit with the Arps exponential model

Figure 4 is an example of the Arps equation for exponential decline being used to forecast production for a gas well. In the example shown, the model (the line) fits the data (the points) very well. If it is assumed that the gas well continues to decline in the same manner, than the model can easily be used to forecast the total production for the well. When represented on a semi-log plot, the exponential model plots as a straight line.

Arps noted that some wells did not plot as a straight line on semi-log paper, but instead the decline exponent changed over time at a constant rate. These wells were hyperbolic in nature, and Arps applied another mathematical formulation to represent this behavior. His observation took on the form:

$$\frac{da}{dt} = -b = \frac{d\left(\frac{q}{dq/dt}\right)}{dt}$$

(1.6)

where

b = Hyperbolic decline exponent, unitless

Multiplying through by dt and integrating, the equation takes on the form:

$$-b\int dt = \int d\left(\frac{q}{dq/dt}\right) \Rightarrow -bt-c = \frac{q}{dq/dt}$$

(1.7)

where

c = Constant of integration

c can be solved keeping in mind the definition of a from the exponential decline equation, and knowing the initial condition that $a = a_i$ at $t = 0$, the c can be found to be equal to a.

$$\frac{q}{\left(\frac{dq}{dt}\right)} = -a \Rightarrow bt+c = a \Rightarrow c = a_i$$

(1.8)

where

a_i = Initial exponential decline constant at t=0, time

Substituting **eq. 1.8** into **eq. 1.6** and rearranging changes the equation to the form:

$$\frac{dq}{dt} = \frac{q}{-bt-a_i}$$

(1.9)

Then once again separating and integrating, the equation takes on the form:

$$\int_{q_i}^{q} \frac{1}{q} = \int_{0}^{t} \frac{-1}{bt+a_i} dt \;\; => \;\; \ln\left(\frac{q}{q_i}\right) = -\frac{1}{b}\ln\left(1+\frac{bt}{a_i}\right)$$

(1.10)

Finally, by exponentiation of both sides and solving for q, the equation takes on the present form, noting that what Arps defined as a is equal to $1/D_i$ today (Blasingame and Rushing, 2005).

$$q(t) = \frac{q_i}{\left(1+\frac{bt}{a}\right)^{1/b}} \tag{1.11}$$

Or in the form it is seen in today:

$$q(t) = \frac{q_i}{\left(1+bD_it\right)^{1/b}}$$

(1.12)

where

D_i = Initial decline constant, 1/time

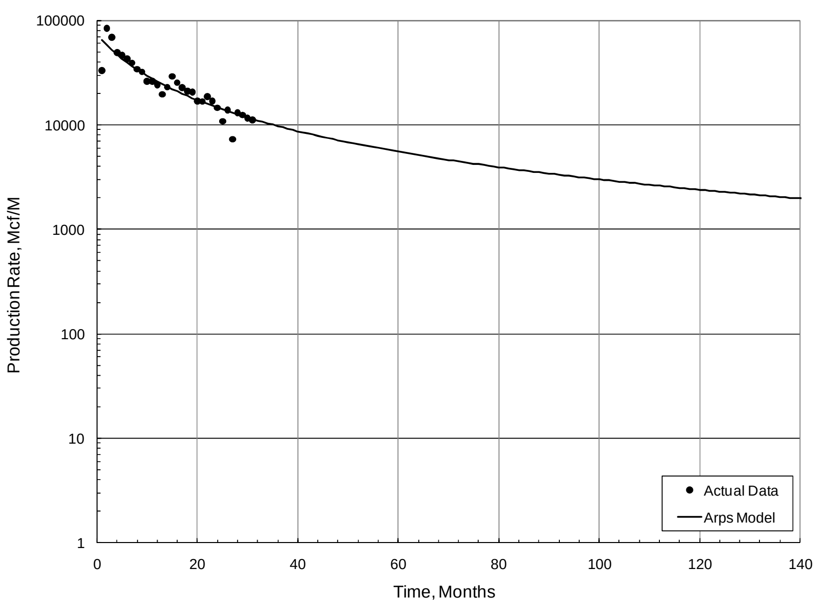

Figure 5 - Example of production data for a gas well fit with the Arps hyperbolic model

In **figure 5** the Arps model is fit to monthly production data for a gas well. The Arps hyperbolic equation fits the data very well, and provides a reliable method for forecasting the future production of the well, if it is assumed that the mathematical trend which governed production in the past will continue to govern in the future (Arps, 1944).

The empirical observation by Arps was perhaps one of the most important advances in the field of reservoir engineering, particularly with respect to reserves estimates. The Arps equation is still the most commonly used method for forecasting reserves today.

1.4 Improvements to Decline Curve Analysis

The Arps equation was discovered by empirical means, however, recent work shows that there is physical meaning to the parameters. For certain situations, both the exponential and hyperbolic equations can be derived theoretically. It has also been shown by various authors that the parameters in the Arps equation relate directly to actual reservoir parameters. Fetkovich (1986) has done extensive work relating the Arps equation to known physical parameters. Fetkovich relates a rigorous analytic solution of the diffusion equation for boundary dominated flow to the exponential form of the Arps equation, and he relates rate and material balance equations to the hyperbolic form of the Arps equation. In later work he goes on to identify specific hyperbolic decline exponent values to be expected from specific reservoir production mechanisms.

Walsh and Lake (2004) show that if the reservoir is treated as a homogenous tank, the exponential form of the Arps equation can be derived. The decay constant can then be related directly to reservoir properties. Additionally they show that for a multi-layered reservoir with no cross-flow, the permeability heterogeneity can be equated to the hyperbolic decline exponent.

Fetkovich asserted that if DCA was used purely empirically without any attempt to relate it to physics and known petroleum engineering concepts, it would be done so with error. For the exponential decline equation, he showed that it could be coupled with a rigorous analytic solution to the pressure diffusion equation for boundary dominated flow. When coupled with the diffusion equation, the Arps equation can be related to physical parameters. For instance, the q_i term in the Arps equation is commonly used as the peak production rate. Fetkovich stated that q_i is not the peak, but the point at which the well first sees the boundary. The point where boundary dominated flow begins can be observed on the backpressure curve when the pressure stabilizes (Fetkovich, 1996). The solution to the diffusivity equation for boundary dominated flow takes on the dimensionless form:

$$q_D = \frac{141.3q(t)B}{kh(p_i\text{-}p_{wf})} \tag{1.13}$$

where

q_D	=	Dimensionless production rate
μ	=	Viscosity, cp
B	=	Formation volume factor, res bbl/STB
k	=	Permeability, md
h	=	Formation thickness, ft
P_i	=	Initial reservoir pressure, psia
P_{wf}	=	Flowing bottomhole pressure, psia

With dimensionless time:

$$t_D = \frac{0.00634\,kt}{\mu\phi c_t r_w^2} \tag{1.14}$$

where

c_t = Total system compressibility, 1/psi

ϕ = Porosity, fraction

r_w = Wellbore diameter, ft

More detailed explanations of this solution can be found in work published by Moore, Schilthuis, and Hurst (1933) or Hurst (1934). The solution for rate can also be written in terms of the productivity index where:

$$J_0 = \frac{q_i}{\left(p_i - p_{wf}\right)} \tag{1.15}$$

where

J_0 = Productivity index, bbl/(day-psia)

Fetkovich (1996) states that the rate of a well with a constant pressure at the aquifer boundary will take on the form of:

$$q(t) = \frac{J_0\left(p_i - p_{wf}\right)}{\exp\left(\frac{\left(q_i\right)_{max} t}{N_{pi}}\right)} \tag{1.16}$$

where

$(q_i)_{max}$ = Maximum production rate, bbl/day

N_{pi} = Cumulative oil production to shut-in reservoir pressure of 0, bbl

This equation is similar to the Arps exponential equation, which takes on the form:

$$q(t) = \frac{q_i}{\exp(D_i t)}$$

(1.17)

q_i can be found from the equation for productivity index and the decline exponent D_i is equal to:

$$D_i = \frac{(q_i)_{max}}{N_{pi}}$$

(1.18)

Where N_{pi} and $(q_i)_{max}$ both are directly calculated from reservoir parameters with the following equations:

$$N_{pi} = \frac{\pi\left(r_e^2 - r_w^2\right)\phi c_t h p_i}{5.615B}$$

(1.19)

$$(q_i)_{max} = \frac{khp_i}{141.3\mu B\left[\ln\left(\frac{r_e}{r_w}\right) - \frac{1}{2}\right]}$$

(1.20)

where

r_e = Radius of entire reservoir, ft

Fetkovich was able to show that for a specific case, the Arps exponential equation can be derived theoretically, and the parameters related to reservoir parameters.

Fetkovich (1973) showed similar work for the hyperbolic equation by combining an empirical rate equation with material balance. He proposed the following equation:

$$q_o = J_{oi}\left(\frac{\overline{p}_R}{\overline{p}_{Ri}}\right)\left(\overline{p}_R^2 - p_{wf}^2\right)^n$$

(1.21)

where

q_o	=	Oil production rate, bbl/day
J_{oi}	=	Initial oil productivity index, bbl/(day-psi^{2n})
$\bar{p}_R$	=	Average reservoir pressure, psia
$\bar{p}_{Ri}$	=	Average initial reservoir pressure, psia
p_{wf}	=	Flowing bottomhole pressure, psia
n	=	Exponent of back-pressure curve

The back-pressure exponent, n, can be used to determine the hyperbolic decline exponent b. Fetkovich assumes that the well will be operated at optimal conditions and the flowing bottomhole pressure, p_{wf}, will be zero.

$$q_o = J_{oi}\left(\frac{\bar{p}_R}{\bar{p}_{Ri}}\right)\left(\bar{p}_R^{2n}\right) \tag{1.22}$$

The material balance equation used for this derivation is given in terms of average reservoir pressure as:

$$\bar{p}_R = -\left(\frac{\bar{p}_{Ri}}{N_{pi}}\right)N_p + \bar{p}_{Ri} \tag{1.23}$$

where

N_p	=	Cumulative oil production, bbl
N_{pi}	=	Cumulative oil production to shut-in reservoir pressure of 0, bbl

By combining the material balance equation with the empirical rate equation, a new rate equation is formed. For a more detailed derivation please reference the appendix of Fetkovich (1973). The combined form of the equation then takes on the form:

$$q_o(t) = \frac{q_{oi}}{\left[2n\left(\frac{q_{oi}}{N_{pi}}\right)t+1\right]^{\frac{2n+1}{2n}}} \tag{1.24}$$

where

q_{oi} = Initial oil production rate, bbl/day

The rate equation can be related to the Arps equation, which takes on the form:

$$q(t) = \frac{q_i}{\left(1+bD_i t\right)^{1/b}} \tag{1.25}$$

where D_i and q_{oi} have the same value as the exponential equation, and the b parameter can be related to the slope of the backpressure curve, n, by the following means:

$$b = \frac{2n}{2n+1} \tag{1.26}$$

Both the exponential and hyperbolic decline equations, despite their empirical origins, have been derived for specific situations. Fetkovich went on to relate the specific hyperbolic exponent, b, with specific reservoir drive mechanisms Certain production declines will not yield unique solutions to the Arps equation. In the event of non-unique solutions, the knowledge of drive mechanism can be used to get the following appropriate b values:

- b = undeterminable – Any well still in transient flow, when no additional engineering or geologic information is needed.
- b = 0 – Single phase liquid, high pressure gas, solution gas drive with very poor gas relative permeability, poor waterflood wells.

- b = 0.3 – Typical solution-gas-drive wells.
- b = 0.4 – 0.5 – Typical gas wells.
- b = 0.5 – Gravity drainage wells or water-drive in oil reservoirs.

Fetkovich also pointed out that b values greater than 1 should never be observed, and never will be if the Arps equation is used appropriately.

Walsh and Lake (2004) showed that by treating the reservoir as a tank with closed system boundaries, both forms of the Arps equation could be approximated. The first model is a homogenous reservoir with single phase oil production, and the second model is a multi-layer reservoir with no cross flow between layers. Several assumptions must be made to complete the derivations. The assumptions are:

- Two phases present in the reservoir, water and oil.
- Water phase must be immobile.
- The total saturation is 1, i.e., So+Sw = 1

Additionally, the following equations for oil rate at standard conditions and productivity index are used:

$$q_{osc} = \frac{J\left(\bar{p}-p_{wf}\right)}{\bar{B}_o} \tag{1.27}$$

$$J = \frac{0.00708hk}{\mu_o\left[\frac{1}{2}\ln\left(\frac{4A}{r_w^2 C_A}\right)+5.75+s\right]} \tag{1.28}$$

where

q_{osc}	=	Production rate at standard conditions, rb/day
$\bar{p}$	=	Average reservoir pressure, psia
$\bar{B}_o$	=	Average formation volume factor, res bbl/STB
μ_o	=	Oil viscosity, cp

A	=	Area, acres
C_A	=	Shape factor
s	=	Skin factor

From here, the material balance equations can be used to derive the exponential decline equation. Keeping in mind that the water phase is immobile, the material-balance rate equations for both oil and water are as follows:

$$\frac{d\left[V_p\left(\frac{\bar{S}_w}{\bar{B}_w}\right)\right]}{dt} = 0$$

(1.29)

$$\frac{d\left[V_p\left(\frac{\bar{S}_o}{\bar{B}_o}\right)\right]}{dt} = -q_{osc}$$

(1.30)

where

V_p	=	Pore Volume, bbl
$\bar{S}_w$	=	Average water saturation
$\bar{B}_w$	=	Average water formation factor, res bbl/STB
$\bar{S}_o$	=	Average oil saturation
$\bar{B}_o$	=	Average oil formation factor, res bbl/STB

The oil rate is negative for mass being removed from the reservoir. These material balance equations are stating that the rate of fluid removal from the pore spaces is zero for water, and q_{osc} at standard conditions for oil. By expanding both equations and combining them the equation takes on the form:

$$\frac{V_p}{\bar{B}_o}\left[\bar{S}_o\bar{B}_o\frac{\partial\left(\frac{1}{\bar{B}_o}\right)}{\partial t}+\bar{S}_w\bar{B}_w\frac{\partial\left(\frac{1}{\bar{B}_w}\right)}{\partial t}+\frac{1}{V_p}\frac{\partial V_p}{\partial p}\right]\frac{d\bar{p}}{dt}=-q_{osc} \tag{1.31}$$

The total system compressibility can be defined as:

$$c_t=\bar{S}_oc_o+\bar{S}_wc_w+c_f=\bar{S}_o\bar{B}_o\frac{\partial\left(\frac{1}{\bar{B}_o}\right)}{\partial t}+\bar{S}_w\bar{B}_w\frac{\partial\left(\frac{1}{\bar{B}_w}\right)}{\partial t}+\frac{1}{V_p}\frac{\partial V_p}{\partial p} \tag{1.32}$$

where

c_o	=	Oil compressibility, 1/psi
c_w	=	Water compressibility, 1/psi
c_f	=	Formation compressibility, 1/psi
c_t	=	Total system compressibility, 1/psi

Substituting the total system compressibility into the combined material balance rate equations reduces to:

$$\frac{V_pc_t}{\bar{B}_o}\frac{d\bar{p}}{dt}=-q_{osc} \tag{1.33}$$

Combining **eq. 1.33** with the productivity index, the equation further reduces to:

$$\frac{V_pc_t}{\bar{B}_o}\frac{d\bar{p}}{dt}=\frac{J\left(\bar{p}-p_{wf}\right)}{\bar{B}_o} \tag{1.34}$$

The formation volume factors cancel, and the equation can be separated and integrated with the limits:

$$\int_{p_i}^{\bar{p}} \frac{d\bar{p}}{J\left(\bar{p}-p_{wf}\right)} = -\frac{1}{V_p c_t}\int_0^t dt \;\Rightarrow\; \log\left(\frac{\bar{p}-p_{wf}}{p_i-p_{wf}}\right) = -\frac{Jt}{V_p c_t} \tag{1.35}$$

The solution can then be exponentiated, and the resulting solution looks like this:

$$\frac{J\left(\bar{p}-p_{wf}\right)}{J\left(p_i-p_{wf}\right)} = \exp\left(\frac{Jt}{V_p c_t}\right) \tag{1.36}$$

This is now in the form of the exponential decline equation, where the productivity index utilizing the initial pressure corresponds with the initial production rate, and the decline constant relates to the following reservoir parameters:

$$D_i = \frac{J}{V_p c_t} \tag{1.37}$$

Walsh and Lake (2004) also state that the hyperbolic decline equation can be used to forecast production from a reservoir with multiple layers and no cross-flow between layers. If the individual layers are homogenous, and the permeability between layers is distributed log-normally, then the decline will be hyperbolic. If the independent layers are declining exponentially at different rates, then the sum of the rates will not be exponential, but hyperbolic. **Figure 6** shows an example of a well with multiple layers and no crossflow declining hyperbolically, and being fit well with the Arps hyperbolic decline equation. With this analogy for hyperbolic decline, there are no physical reservoir parameters correlated with the hyperbolic decline exponent, b, however the decline exponent can be correlated with the Dysktra-Parsons coefficient. The Dykstra-Parsons coefficient can be used to represent the spread of a log-normal distribution with maximum spread having a value of 1.

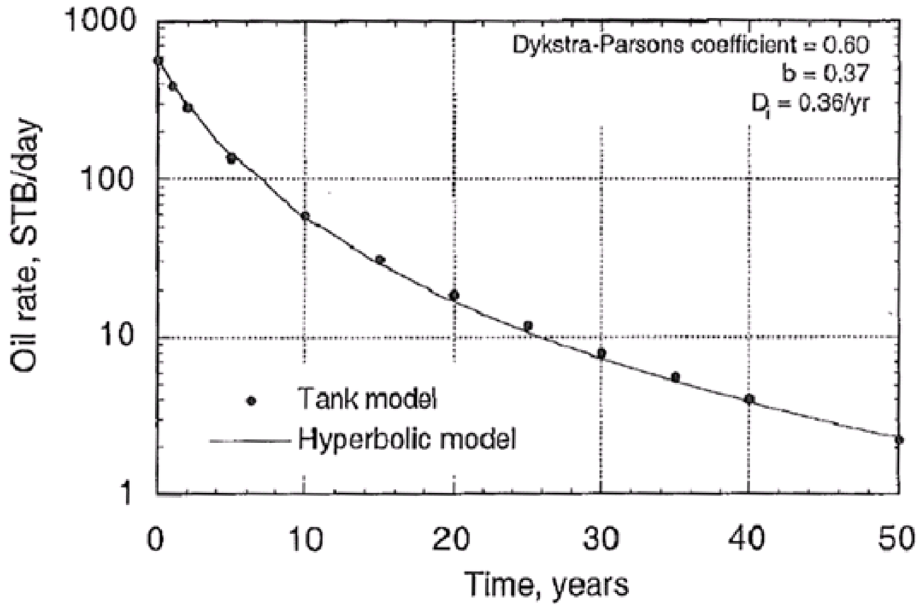

Figure 6 - Example fit of Arps equation to multi-layer reservoir tank model with no crossflow (Walsh and Lake, 2004)

Figure 7 shows the correlation between the decline exponent, b, and the coefficient. The relationship between the two terms is non-linear. If the permeabilities of the various layers are known, they can be used to approximate the b values. The figure also shows that the initial decline percent can be correlated with the Dykstra-Parsons coefficient as well.

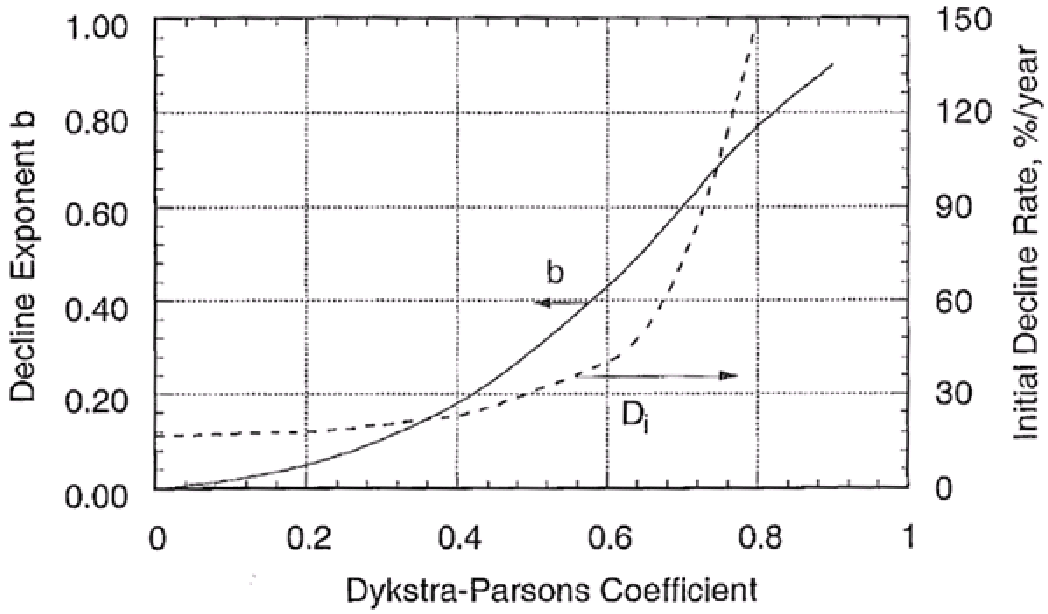

Figure 7 - Correlation between the hyperbolic exponent and the Dysktra-Parsons coefficient (Walsh and Lake, 2004)

Considerable work has been done to show that the Arps equation is more than just an empirical equation, and that it can be used by practicing reservoir engineers as more than just a statistical fit to the production data.

1.5 Development of Unconventional Resources

The Arps equation has proven to be very useful for forecasting reserves in what have now come to be referred to as conventional oil and gas formations. The Arps equation, however, encounters problems when being used in formations considered to be "unconventional," particularly with regards to over estimation of reserves.

The exact definition of "unconventional" resources is difficult to pinpoint. In very broad terms an unconventional resource refers to any oil or gas well that requires advanced drilling or completion techniques to produce at an economic rate. This can include low permeability or tight formations, heavy oil, coalbed methane, shale oil, or gas hydrates. In the case of this study, *unconventional* reservoirs will be used to refer to extremely low permeability reservoirs. According to the National Petroleum Council report on unconventional gas, the U.S. government defined a tight gas reservoir as any reservoir with permeability below 0.1md (Holditch *et al.*, 2007). The specific reservoirs looked at in this thesis, the Barnett Shale in the Fort Worth Basin and the Bakken Shale in the Williston Basin, have average permeabilities less than 0.1 md. According to Holditch *et al.*, this definition is limited in significance. Another way to define unconventional gas would be "natural gas that cannot be produced at economic flow rates nor in economic volumes of natural gas unless the well is stimulated by a large hydraulic fracture treatment, a horizontal wellbore, or by using multilateral wellbores or some other technique to expose more of the reservoir to the wellbore." While this definition is more complete, it is still somewhat misleading as many of the wells being drilled today have switched to horizontal well technology, not because it is required for the well to be economic, but because it provides for better economics than a vertical well.

According to Davis (1986) natural resources are distributed log normally in nature. This idea is further interpreted in the work of Masters (1979) who pointed out that oil and gas reservoirs could be represented by a triangle. Masters' triangle implied that at the top of the triangle, are the highest permeability easy to recover resources, while the tighter, deeper and more difficult to recover resources make up the larger portion of the triangle, seen in **figure 8**.

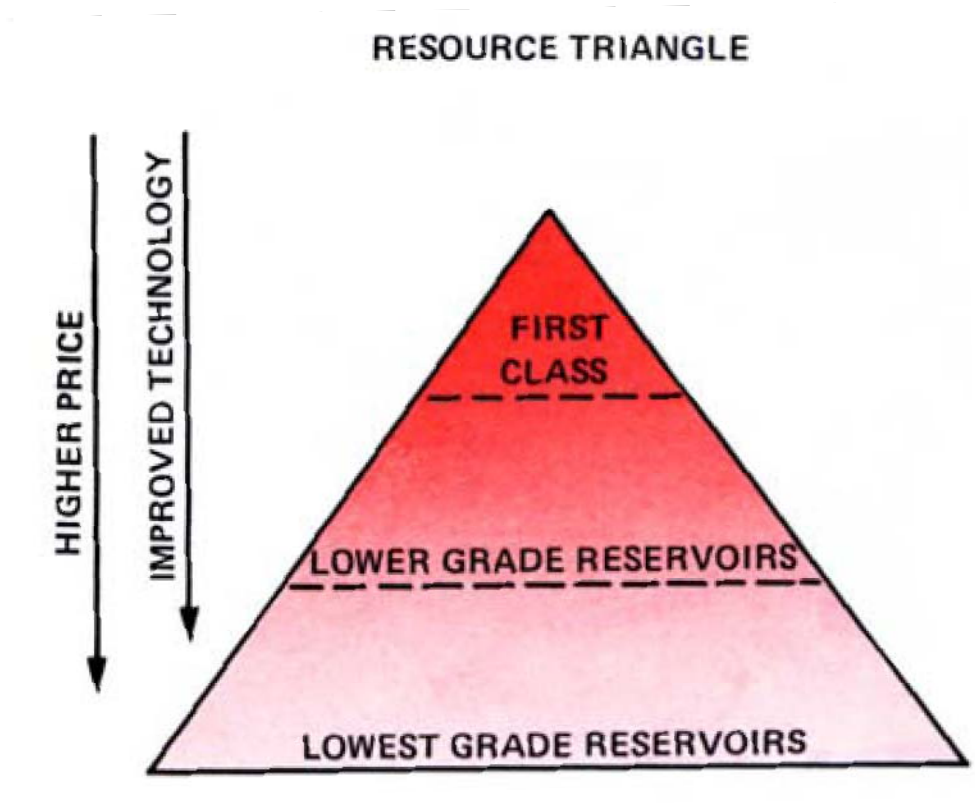

Figure 8 - Hydrocarbon reservoir resource triangle (Masters, 1979)

Masters' triangle differs from a triangular distribution seen in statistics that refers to the shape of the probability density function of the variable approximating a triangular shape (Jensen, *et al.*, 1996). As the cost and technology required to recover the gas from lower grade reservoirs increase, so does the volume of gas contained in these reservoirs. Holditch equated Masters' triangle to a log normal distribution in statistics (Davis, 1986).

The development of low grade reservoirs has increased drastically over the last decade. The causes for the increased development are numerous, but there are three main factors that will be discussed here. The first factor is the increased demand for energy from both the developed and developing nations. The second factor is the decrease of oil and gas supply from conventional reservoirs. The final factor contributing to the boost in development is the improvement in technology, which Masters clearly identified as one of the needs to develop these reservoirs in 1979.

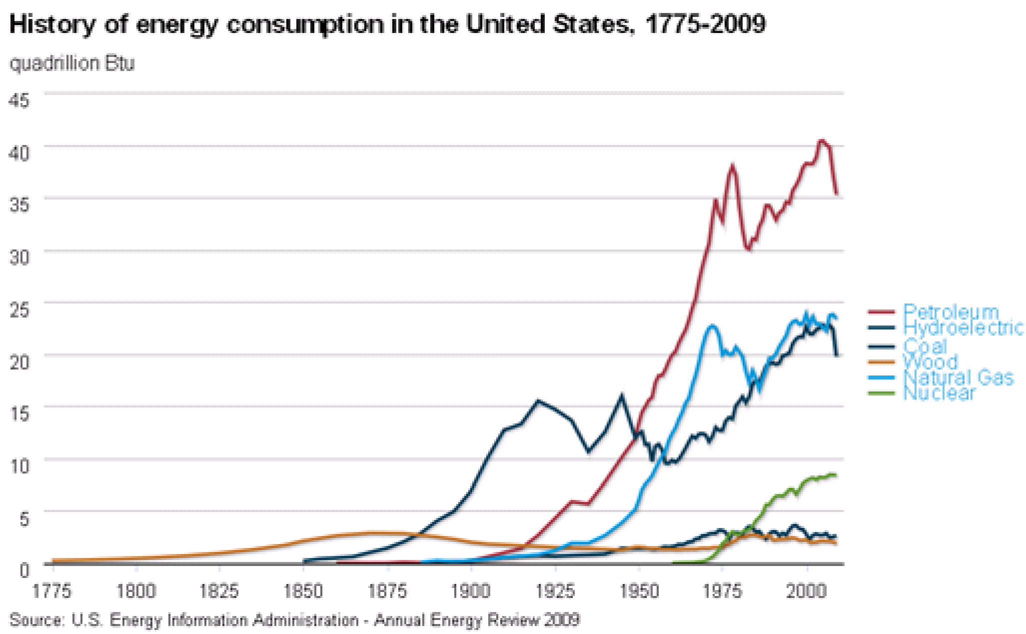

Figure 9 - History of energy consumption in the U.S. by energy source (Annual Energy Review 2009)

The need for energy around the world has increased exponentially in the last 200 years. **Figure 9** shows the continual increase in energy used by the U.S. which will continue to increase the demand for oil and gas development. The need for development of unconventional resources extends beyond just the United States. The U.S. has been on the forefront of energy usage, and is still the largest per capita consumer of energy in the world. Rapidly developing nations particularly China and India which combine to make up more than one third of the world's population are putting an even larger strain on the need for oil and gas. An extensive report published by the National Petroleum Council (NPC) found that the bulk of energy consumption around the world comes from developed nations referred to as the Organization for Economic Co-operation and Development (OECD), and they use roughly half the energy while making up only about one sixth of the world's population. In 2004 the non-OECD countries consumed only 44% of the world's energy, however, **figure 10** shows the NPC forecast that by the year 2030 non-OECD nations will be consuming 60% of the world's energy.

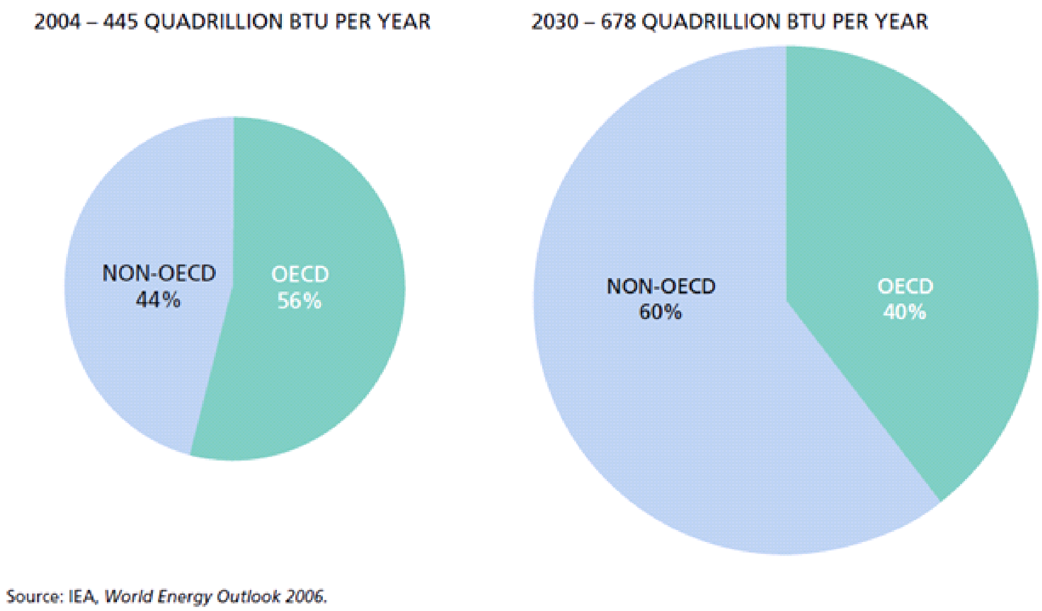

Figure 10 – Projected world energy demand growth in the next 20 years (World Energy Outlook, 2006)

Energy consumption will continue to grow as time moves forward, and new sources of energy will need to be discovered. One of the most readily available sources will be unconventional gas and oil reservoirs.

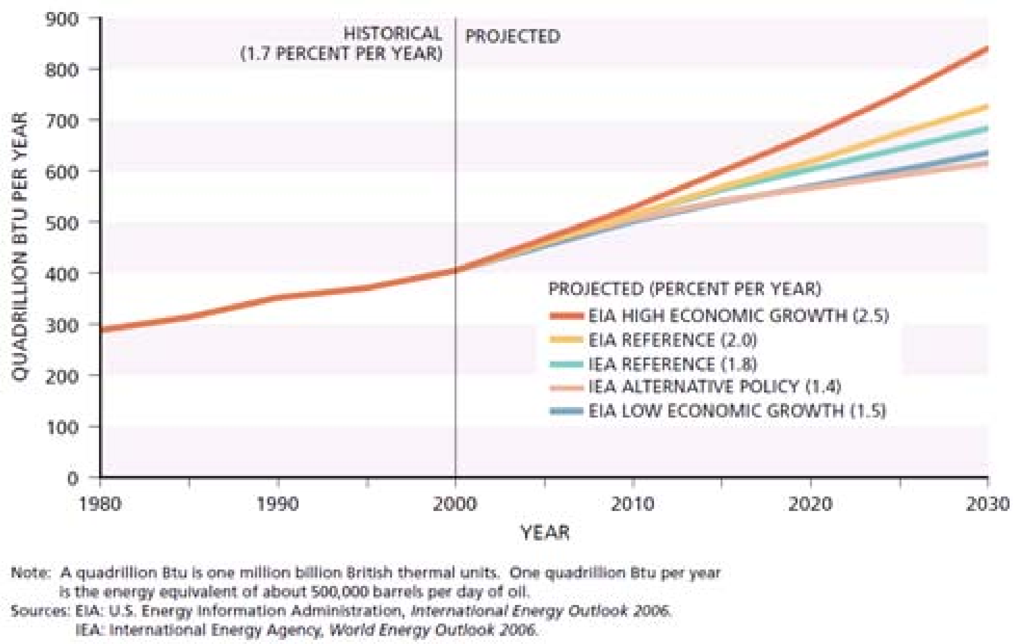

Figure 11 - Various forecasts for increase in world energy demand (World Energy Outlook, 2006)

The second factor contributing to the increased development of unconventional resources is the declining production from conventional reservoirs around the world. The concept of declining global oil production was first introduced by Hubbert (1956). Hubbert determined using his model, that oil production for any given producing province will increase until half of the supply has been produced, then production will subsequently decrease until the supply has been exhausted. Hubbert predicted with his model that U.S. onshore production would peak in 1971. This prediction proved to be very accurate as seen in **figure 12**.

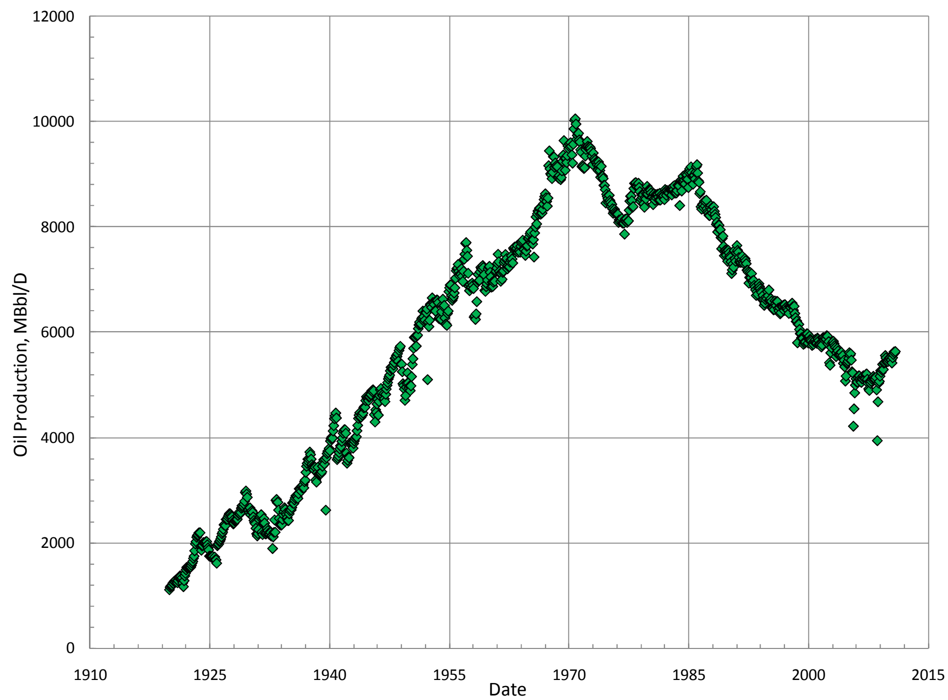

Figure 12 - Monthly US oil production over the last century

A similar trend can be seen in Norwegian oil production, which is done primarily offshore in the North Sea. **Figure 13** shows the Hubbert model accurately fitting the annual oil production rate of the North Sea. It can be assumed that the oil production for the world will follow a similar trend.

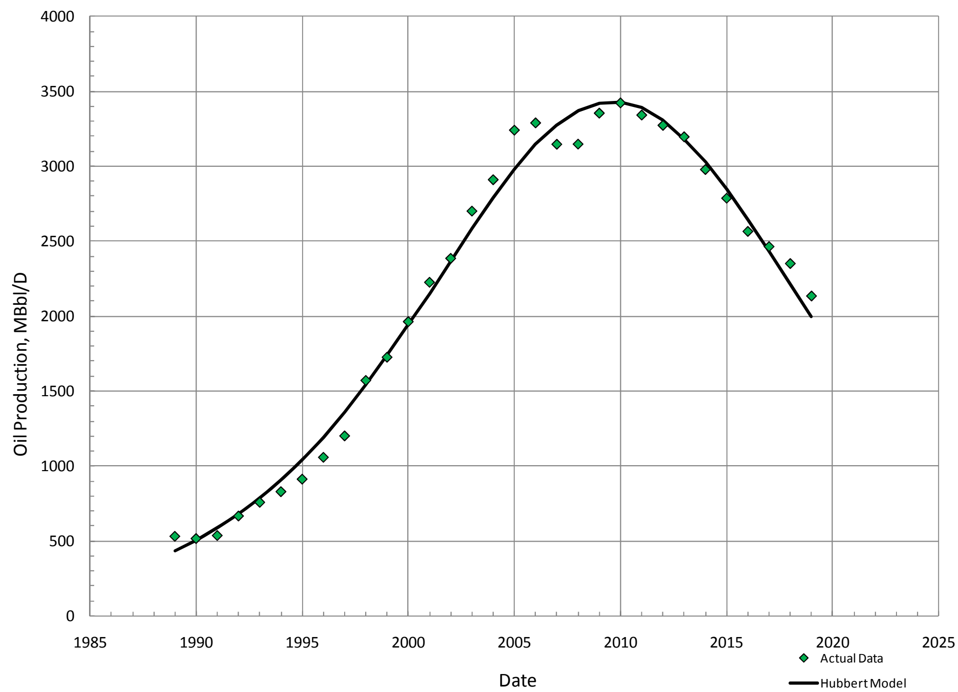

Figure 13 - Yearly Norwegian oil production from the North Sea fit with Hubbert's model

It is unclear when the world will reach its peak. Various sources predict different times for peak production to occur. The best way to mitigate this inevitable decline in production is by development of the more difficult or unconventional oil and gas resources. Thanks to advances in technology, this has begun to occur.

The third factor contributing to the growth of oil and gas production from unconventional reservoirs is improved technology. The technological improvements have come primarily from two distinct areas of field development: drilling and completions, specifically horizontal well drilling, and hydraulic fracturing. The improvement in horizontal drilling technology has proven to be invaluable to the development of tight oil and gas. Horizontal drilling allows for more of the reservoir to be accessed with a single well. The cost of drilling a horizontal well is greater than the cost of drilling a vertical well, but drilling horizontal wells reduces the total number of wells necessary to exhaust a field. Drilling fewer total wells improves the economics of the entire field. A good example is the Barnett Shale. Vertical wells were the primary method of drilling in the area until 2002 when Devon drilled the first horizontal wells. The industry rapidly adopted the technology, and by 2004 every single operator in the area had switched to horizontal well technology (Martineau, 2007).

The other technological improvements have come from hydraulic fracturing. The improvements across the field of fracturing are numerous. Improvements in fracturing fluid have helped to increase effectiveness of fracture treatments. Friction reduced water, referred to as slick water, has proven to be the best method for propagating fractures in the Barnett Shale (Palisch *et al.*, 2010), while in the Bakken shale, extreme depths and temperatures, have required advances in the cross-linked gels used to fracture the tight oil formation. The type of proppant used in the Bakken formation has helped as well. At high depths, there is much higher closure pressure on the proppant, and typical silica based proppants prove to be inadequate. Lightweight high strength ceramic proppants have been used with great success in the Bakken (Rankin *et al.*., 2010).

Perhaps the most significant technological development in hydraulic fracturing has been increased efficiency and effectiveness in multi-stage fracturing. Multi-stage technology allows multiple fracture stages to propagate on a single horizontal well, greatly increasing the volume of reservoir accessed, and the volume of oil or gas recovered.

The rate of development of unconventional resources has increased drastically over the past decade. There is another factor that is quite possibly the most important in the onset of the development of these resources, and that is the price of oil and gas. The price of oil steadily increased over the past decade, and with it so too did the development of unconventional fields. As the price of oil increases, fields that previously were uneconomical become more desirable to produce. The cause for the increase in price was caused largely by the first two factors discussed above. An increase in the demand for oil, with a corresponding fear of decrease in supply caused the price to climb quickly over time. The increased price allowed for more advanced technology, which comes with a higher price tag, to be used economically in an unconventional field. There are many factors that have contributed to the increased production from unconventional fields in the U.S., and these factors have now led fossil fuel producing nations around the world to begin looking more and more towards low permeability formations.

1.6 Decline Curve Analysis in Unconventional Formations

The Arps model for decline curve analysis has proven to work very well in conventional oil and gas wells. It has not proven to work well when used in extremely low permeability oil and gas wells. The complex nature of the reservoirs and the drilling technique causes the reservoir decline performance to behave differently from conventional reservoirs. These complexities in the reservoir have caused the Arps equation to be inadequate for reserves estimates. The biggest issue with the Arps equation in unconventional formation comes from the hyperbolic decline exponent b. When fitting the decline equation to unconventional wells, very typically a b value greater than 1 is experienced. When this occurs, the fit to the data still appears very reasonable, however, mathematically this causes the production rate to continue on indefinitely, and causes an over estimation of reserves. Additionally, the physical meaning that has been rigorously applied to the model loses all value, and the forecasts become non-physical. This is because of the complexity of the reservoir, and the amount of time the well has produced before the decline curve analysis is performed.

Numerous authors have discussed the issues that occur when b values are greater than 1. Lee states, "we find that "best fits" require values of "b" to be greater than 1, beyond the limit that Arps specified. It turns out that values of b equal to or greater than 1 can cause the reserves derived using Arps decline equation to have physically unreasonable properties." (Lee, and Sidle, 2010). The problem is largely mathematical in nature. If the Arps hyperbolic rate equation is integrated to yield a cumulative form of the equation, it can be shown that as time approaches infinity, the total cumulative production also approaches infinity. This obviously is not a possible solution, since it is known that the amount of hydrocarbons in the ground is finite. The Arps hyperbolic rate equation can be integrated to yield the cumulative form:

$$Q(t) = \frac{q_i^b}{D_i(b-1)}\left(q(t)^{(1-b)} - q_i^{(1-b)}\right)$$

(1.38)

If b lies between 0 and 1, and the limit is taken as time approaches infinity, a finite number is obtained:

$$\frac{\lim}{t \to \infty} Q = \frac{q_i}{D_i(1-b)}$$

(1.39)

However, if the b value exceeds 1, then the limit becomes:

$$\frac{\lim}{t \to \infty} Q = \frac{q_i^b}{D_i(b-1)}\left(\frac{1}{q(t)^{(b-1)}} - \frac{1}{q_i^{(b-1)}}\right) \to \infty$$

(1.40)

This is clearly an unphysical interpretation of the Arps equation. Despite this flaw, the Arps equation is still frequently used in industry. When used for economic purposes, the production is not forecasted infinitely into the future, but instead truncated at an uneconomic production rate. The cut off rate solves the problem of infinite reserves, but it is still an incorrect way to use the model. The results of b values greater than 1 never terminating can be displayed graphically, and are best presented on a plot of rate versus cumulative production.

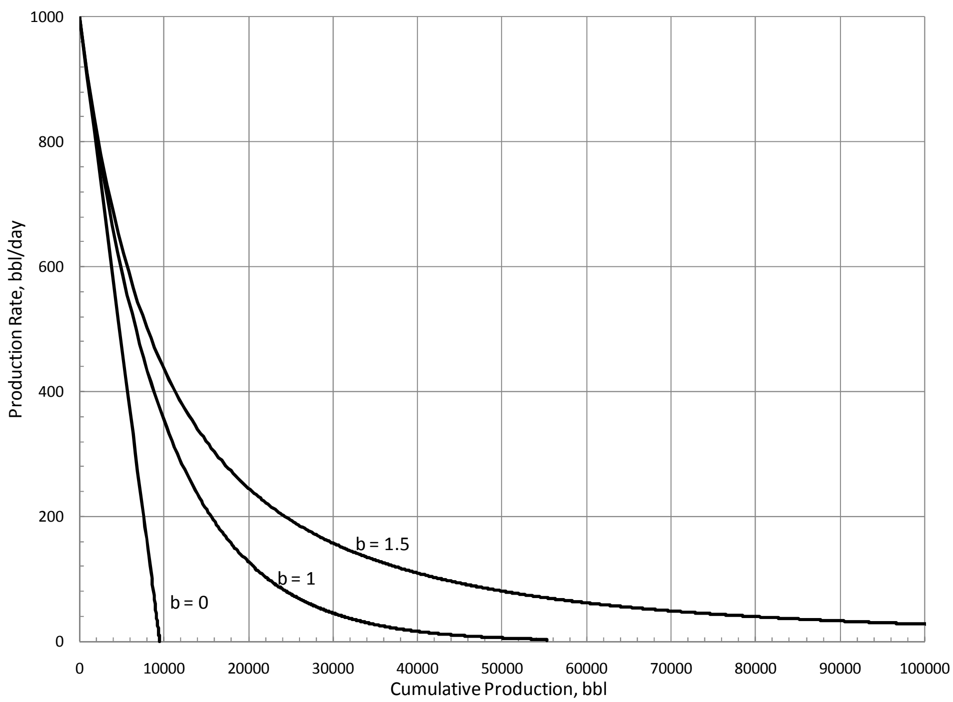

Figure 14 - Rate vs. cumulative plot of Arps Hyperbolic equation with various b values

Figure 14 shows an example of the Arps hyperbolic model being plotted as rate versus cumulative production. Three curves are shown, with b values equal to 0, 1 and 1.5. The initial rate, q_i, and decline constant, D_i, are both arbitrary in this example. In **figure 14**, the curves for b equal to 0 and 1 both eventually reach a production rate of 0. The curve for the b value of 1.5, however, never reaches 0. Hyperbolic exponents greater than 1 will commonly be obtained when curve fitting in unconventional formations.

High permeability conventional reservoirs will produce in two flow regimes: transient flow and boundary dominated flow. In transient flow, the pressure response is constantly moving outward, and the reservoir appears to be infinite. Once this pressure response reaches the boundary of the reservoir, the flow regime switches to boundary dominated flow. The rate at which the pressure response moves towards the boundary is directly related to the permeability of the formation. In conventional reservoirs where the permeability is much higher than unconventional reservoirs, the boundary is seen within a few days, and the well produces much of its life in boundary dominated flow. In unconventional reservoirs, the permeability is so low, that it is unclear when, if ever in the producing life of the well the boundary is seen. The extended transient flow regime is the cause for b values greater than 1 to be observed. It can be recalled from Fetkovich's work that when coupling the Arps equation with the solution to the pressure diffusivity equation for boundary dominated flow, that the q_i used should be production rate when the boundary is first seen on the backpressure plot. In unconventional reservoirs that point is not seen for many years if it is seen at all. In order for the Arps equation to be used without neglecting the physics that govern fluid flow several assumptions must be met. These assumptions are constant bottomhole pressure, boundary dominated flow, unchanging drainage area and a constant skin factor (Fetkovich, 1996; Lee and Sidle, 2010). Several of these assumptions are clearly violated in unconventional wells, particularly the boundary dominated flow and the unchanging drainage area.

Performing decline curve analysis in unconventional reservoirs with hyperbolic decline exponent values greater than 1 has both mathematical and physical shortcomings. The necessity to forecast reserves for operational and financial purposes requires the estimate to be done before adequate information is available, and before the boundary has been seen in low permeability formations. Using the Arps hyperbolic equation without regard for its shortcomings is a risky practice. The flaws in using b values greater than 1 have been recognized for many years, and efforts have been made to correct them.

1.7 New Methods of Decline Curve Analysis

Numerous efforts have been made to correct the shortcomings of using the Arps equation in unconventional reservoirs. Four methods will be reviewed here. The first method looked at will be the truncation of the hyperbolic equation with an exponential decline at a certain point in time, first proposed by Maley (1985). The second method is the use of multiple transient hyperbolic exponents suggested by Spivey *et al.*. (2001) and further expanded by Kupchenko *et al.* (2008). The third method looked at will be a model proposed by Ilk *et al.* (2008) which is more commonly known as the power law model. The final method looked at will be the stretched exponential model proposed by Valko (2009).

1.7.1 Exponential Truncation of Hyperbolic Equation

The most commonly used method to deal with the problem of b values greater than 1 was also one of the first proposed. Aware of the problem with unrealistic reserves estimates, Maley (1985) suggested that at some point in time the hyperbolic decline should switch to an exponential decline. Switching to an exponential decline will cause the production rate to go to 0. The instantaneous decline rate in the hyperbolic equation continuously decreases with time. To implement his method, Maley suggested that at some pre-determined decline rate, the model could switch to an exponential decline. This practice requires the use of two separate models. The initial data can be fit with hyperbolic a curve, and at the point in time when the set instantaneous decline rate is reached, a new exponential curve is used for the second part of production. There is no physical basis for this change in behavior; it simply prevents the issues of production continuing indefinitely. Maley stated that when the well switches to exponential decline it would be so far out the future that because of present-value discounting, there will be no monetary value to the oil produced at that time. He says, “reserves produced beyond 20 years in the future have almost no economic worth due to low producing rates and present value discounting.” (Maley, 1985). Maley’s method is arbitrary and does not reflect actual reservoir performance. Additionally, the value of the final exponential decline chosen is subjective, and will alter the reserve estimate.

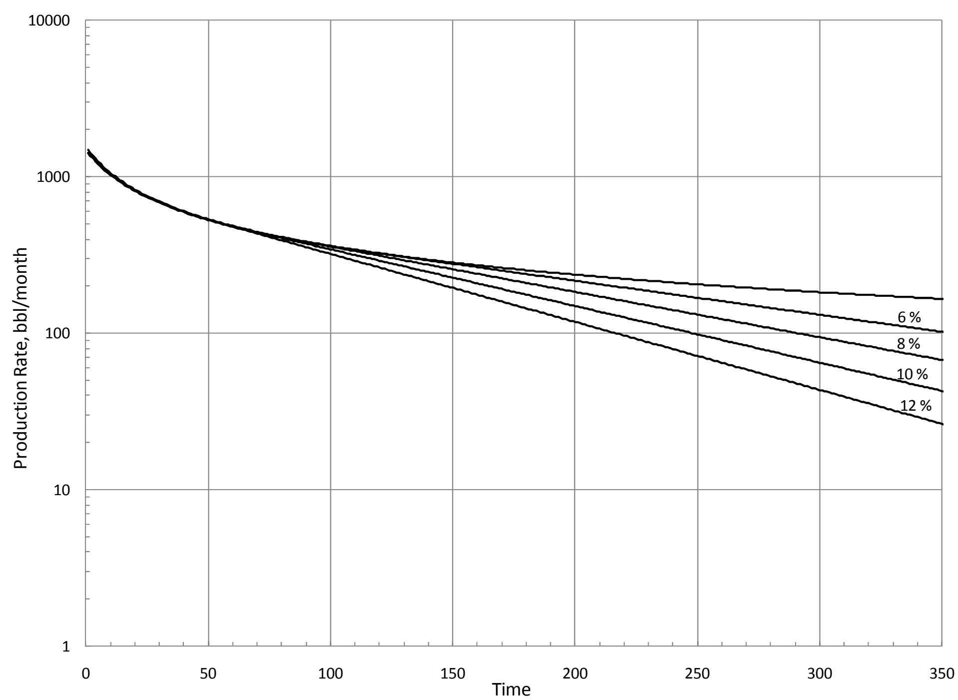

Figure 15 - Rate vs. time of various fixed exponential decline rates

Figure 15 shows the production rate versus time for varying exponential decline rates ranging from 6% to 12%. Depending on which decline rate is used, the production rate after 30 years changes drastically. In this case the final producing rate at 30 years is anywhere from 24 bbl/month with a 12% final exponential decline rate to 97 bbl/month with a 6% final exponential decline. This shows that if a final decline of 6% is chosen the model will predict the well producing 300% more oil per month after 30 years than if a 12% final decline is chosen.

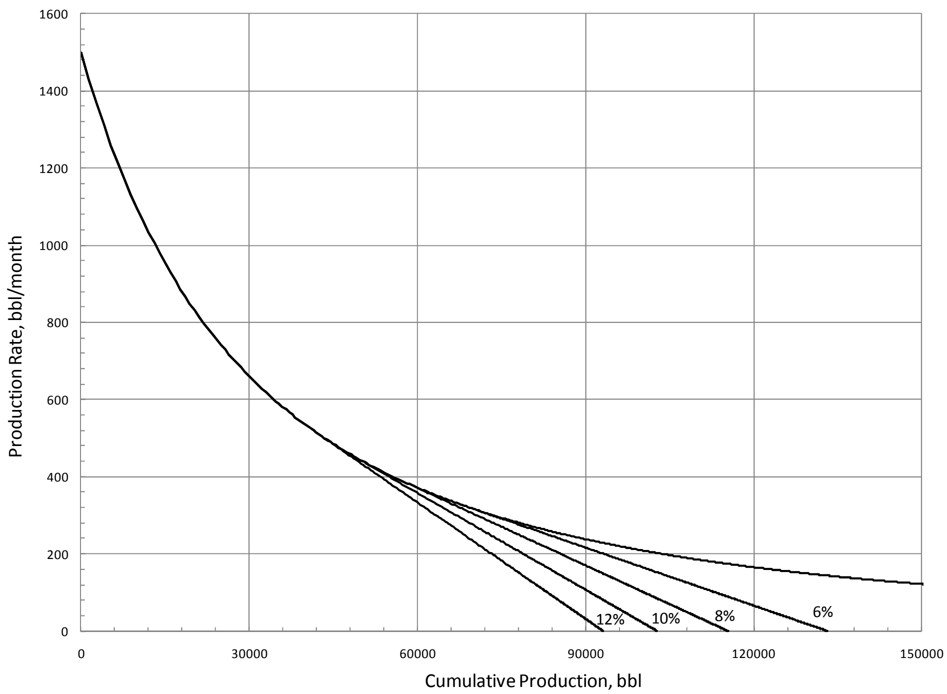

Figure 16 - Rate vs. cumulative for various fixed exponential decline rates

Figure 16 shows a rate versus cumulative plot of various fixed exponential models. This illustrates how the final decline rate can affect the reserves estimates. Again the declines range from 6 to 12% annual decline in production. In the case of a 12% final decline the well would be expected to recover 93,000 bbls of oil, however, when a decline of 6% is used the recovery is expected to be 133,000 bbls. This causes a roughly 50% higher estimate. Additionally, both **figure 15** and **figure 16** show the Arps model fit with a b value of 1.5 which is not truncated with an exponential decline.

1.7.2 Multiple Transient Hyperbolic Exponents

The second method uses multiple hyperbolic exponents to forecast production, and is not entirely dissimilar to the method proposed by Maley. With the multiple hyperbolic exponents method, various hyperbolic decline exponents are used to represent different periods of flow in the reservoir. Spivey *et al.*. were the first ones to suggest using multiple b values. The work has further been refined by Kupchenko. Spivey *et al.*. showed that the b value will change with time, and hence gave it the name transient hyperbolic exponent. In the early stages of production of a tight gas well, the dominant flow regime is referred to as linear flow. The flow regime is characterized by linear flow in the fractures with corresponding transient flow in the matrix (Ozkan *et al.*, 1987). When this flow regime occurs, it should result in a b value of 2 (Spivey *et al.*, 2001). A good example of the trend is in the Bakken Shale.

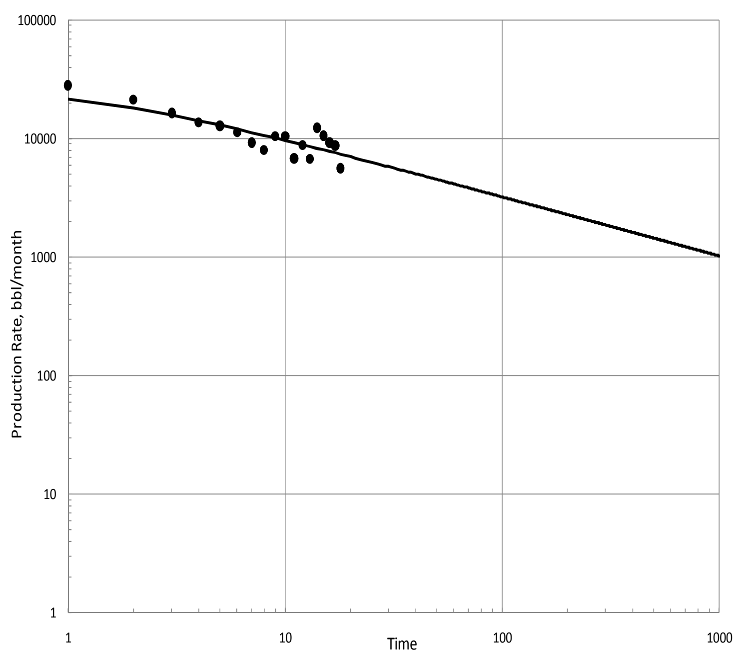

Figure 17 - Rate vs. Time plot of Bakken Shale well exhibiting linear flow

Figure 17 shows a typical Bakken shale well that has been producing for roughly 2 years. The Arps hyperbolic model fits the data with a b value of 2, which according to Spivey would be indicative of a linear flow regime. The linear flow regime is finite in duration, and once the fracture system has been depleted of its stored oil or gas, the flow regime will change along with the b value. For shale gas purposes, Kupchenko elaborated on the point and has shown through numerical simulation and the use of a transient hyperbolic exponent that various b values should fit various regimes. Following the linear flow regime, the reservoir should experience a boundary dominated flow where a b value of 0.25 should match the data (Kupchenko *et al.*, 2008).

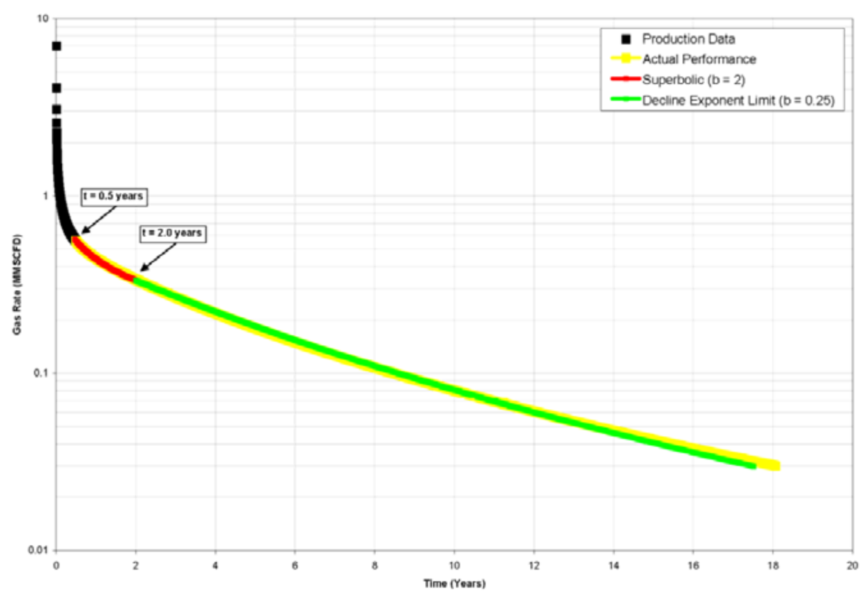

Figure 18 - Rate vs. time plot of simulated production data showing the fit of multiple hyperbolic exponents (Kupchenko *et al.*., 2008)

Figure 18 shows the rate versus cumulative production for a numerical simulation of a fractured horizontal well in a tight gas reservoir. The first 2 years of production can be fit with a b value of 2 while the reservoir is in linear flow. After 2 years when the reservoir transitions into boundary dominated flow, the duration of the data can be fit with a b value of 0.25. If enough is known about the performance of wells in an area, multiple hyperbolic exponents could yield more realistic results than Maley's method. Unfortunately, because of the limited amount of data available, it is unclear as to when a well will transition into the second flow regime, and thus makes the point at which the forecaster chooses to transition little more than a guess. Additionally, as with the fixed exponential method, this requires using multiple discontinuous models.

1.7.3 Power Law Model

An entirely new third model was introduced for decline curve analysis by Ilk *et al.*. in 2008. This model is based on an exponential decline; however, unlike the Arps exponential decline equation where the decay is constant, the power law model considers the decay to be a power law function. The power law loss-ratio method, or power law model for short takes on the form:

$$q(t) = \hat{q}_i \exp\left(-D_\infty t - \hat{D}_i t^n\right) \tag{1.41}$$

where

$\hat{q}_i$	=	Rate intercept or q(t=0), Mscf/D
$\hat{D}_i$	=	Decline constant defined by D_1/n, 1/D
D_1	=	Decline constant after 1 time unit, 1/D
D_∞	=	Decline constant at infinite time, 1/D
n	=	Time exponent

The power law model behaves differently from the exponential model with several distinct advantages. At early times the t^n term will match the transient flow regime. At late times the D_∞ will govern the decline behavior, eventually causing the rate to terminate (Ilk *et al*.., 2008). The behavior has distinct advantage over the previous methods in that a single continuous function can be used to forecast production. One concern with the power law model is that the final decline rate D_∞ is arbitrary, and with insufficient data it is unclear what its value should be.

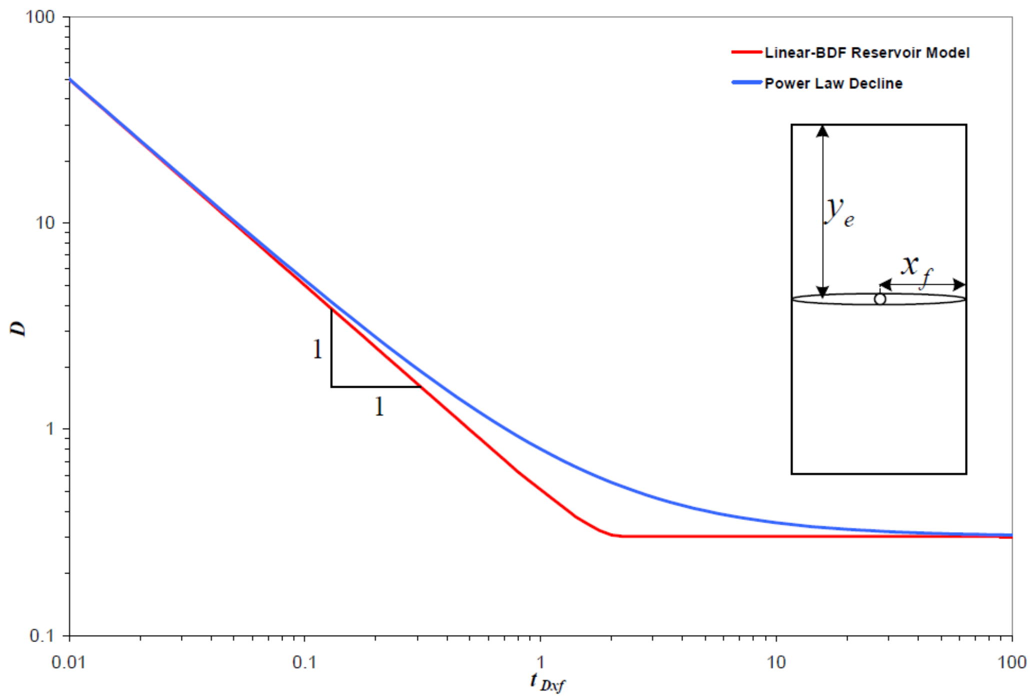

Figure 19 - Decline behavior vs. dimensionless time for analytic linear flow model (Mattar and Moghadam, 2009)

Figure 19 shows the decline behavior versus dimensionless time for an analytic flow model which transitions from linear flow to boundary dominated flow. The decline behavior is then matched with a power law model, showing that the dominant behaviors can be matched. The analytic model shows a more gradual transition between regimes than the power law model, but the power law trends the general behavior.

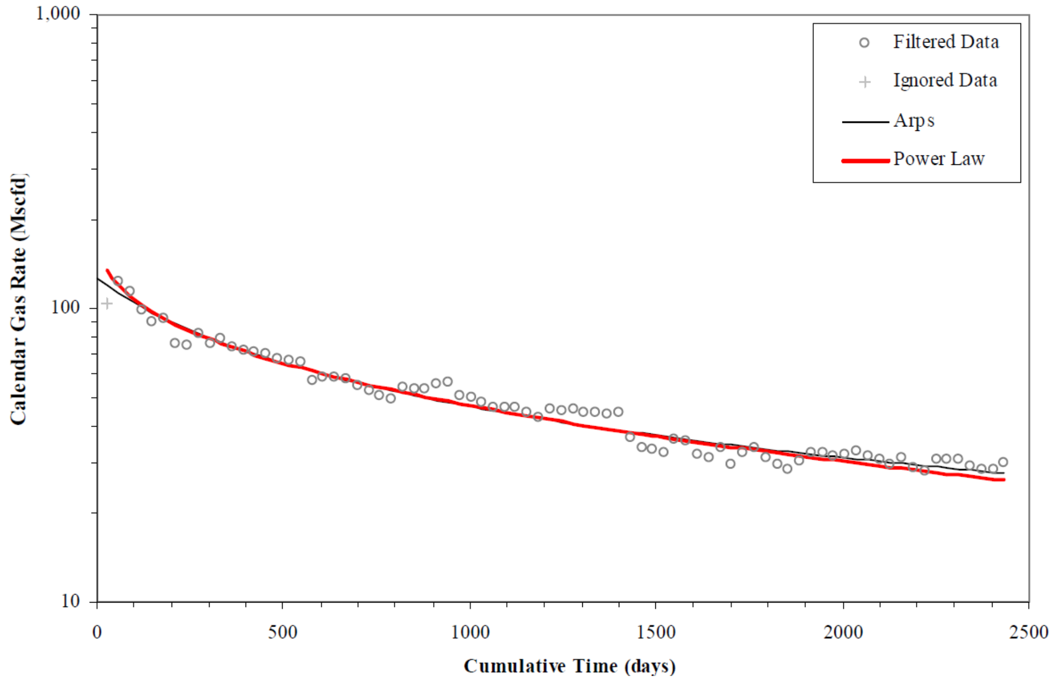

Figure 20 - Rate vs. time plot showing power law model match with actual data (McNeil *et al.*, 2009)

Figure 20 shows the rate versus time data for a tight gas well fit with both the power law and Arps model. Both the models trend the data very well. The Arps model, however, is fit with a b value of 1.4 which will cause it to ultimately result in unreliable results (Mcneil *et al.*, 2009). The power law model on the other hand will eventually terminate, and yield a finite total production.

Practical application of the power law model is not as simple as an empirical fit with the Arps equation. The power law model has 4 unknown parameters, and thus 4 degrees of freedom resulting in non-unique solutions. To get an optimal fit, a more complicated procedure than the other methods is required. The procedure involves several steps including calculating initial decline parameters, and then curve fitting different portions of the data in order to determine particular parameters. This method requires more personal attention from the reserve estimator, and does not lend the power law model to large scale field evaluations which can be very quickly performed with the Arps equation.

1.7.4 Stretched Exponential Model

The final method for decline curve analysis in tight gas wells is the stretched exponential model first proposed by Valko in 2009. The stretched exponential model is also empirical, however, unlike the others it does have a basis in physics and is governed by a defining differential equation (Valko, 2009). The concept of using stretched exponential models in the petroleum industry was proposed by Laherrere and Sornette in 1998. The stretched exponential model is related to fat tail distributions. Fat tail distributions can experience any number of standard deviations away from the mean and still have a likelihood of occurrence. The behavior is unlike normal distributions in which the large majority of instances occur within three standard deviations of the mean. Fat tail distributions have also shown to be closely related to fractals, and they are also said to show power law decay (Bahat *et al.*, 2006). The power law decay phenomenon is very similar to the power law model proposed by Ilk *et al.* discussed above. When Laherrere and Sornette first applied the fat tail distribution to the petroleum industry, they showed that it could be used to describe the size of Gulf of Mexico oil fields. Valko re-interpreted the stretched exponential idea and applied it to production declines in tight gas wells. The defining differential equation that was devised takes on the following form:

$$\frac{dq}{dt} = -n\left(\frac{t}{\tau}\right)^{n}\frac{q}{t} \tag{1.42}$$

where

n	=	Exponential parameter
q_0	=	Peak production rate, Mscf/month
τ	=	Characteristic time parameter, month

The equation can then be separated and integrated to take on a rate form:

$$q(t) = q_o \exp\left[-\left(\frac{t}{\tau}\right)^n\right] \tag{1.43}$$

Equation 1.43 looks very similar to the power law model, and can be equated to a particular instance where D_∞ is equal to zero and τ is equal to $1/\hat{D}_i$. The stretched exponential model differs from the power law model in that it does not rely on a single interpretation of the parameters, but instead uses two-parameter gamma functions (Valko, 2009). This is because of the fact that not a single set of τ and n parameters exists but instead there is a sum of multiple exponential declines which follows the fat tail distribution (Valko, 2010).

Application of the stretched exponential model for decline curve analysis is different from the other models, and more complicated to apply. The stretched exponential model is used for large scale evaluation of entire fields, and does not emphasize the individual well analysis. It can be used for individual well forecasts, but as seen in **figure 21**, it has at this point been applied to entire fields.

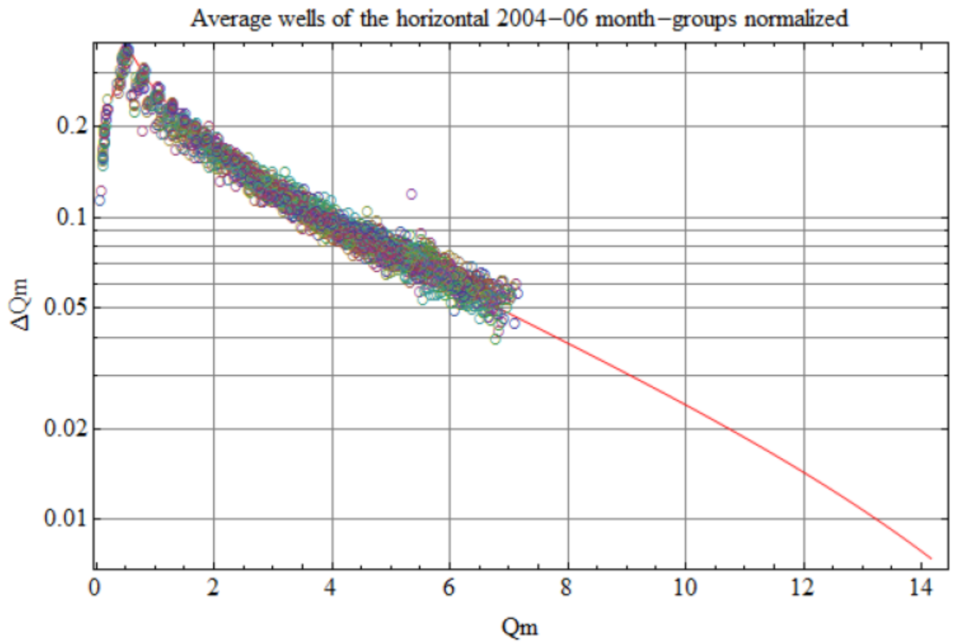

Figure 21 - Normalized production rate vs. cumulative using the stretched exponential model (Valko, 2010)

Figure 21 shows normalized production rate versus cumulative production for all wells completed between January 2004 and December 2006 in the Barnett Shale. When production rate is normalized by dividing the current month's production rate by the peak months production rate, all of the data follows a similar trend. To forecast production for individual wells, the equation is linearized by plotting recovery potential versus cumulative production on a Cartesian grid as depicted in **figure 22**.

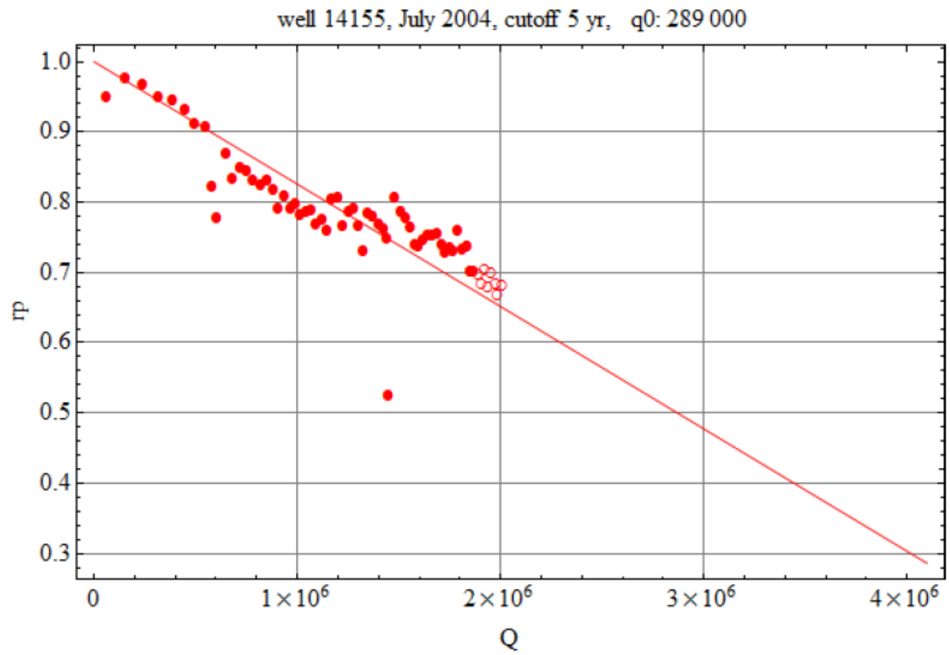

Figure 22 - Recovery potential vs. cumulative production in Barnett Shale (Valko, 2009)

Figure 22 shows recovery potential versus cumulative production for a horizontal well in the Barnett shale. The recovery potential as defined by Valko is:

$$rp = 1 - \frac{Q}{EUR} = \frac{1}{\Gamma\left[\frac{1}{n}\right]} \Gamma\left[\frac{1}{n}, -\ln\left(\frac{q}{q_0}\right)\right] \tag{1.44}$$

where

rp	=	Recovery potential, dimensionless
EUR	=	Estimated ultimate recovery, Mscf
Q	=	Cumulative production, Mscf
q	=	Production rate, Mscf/month
q_0	=	Peak production rate, Mscf/month
n	=	Exponential parameter

The recovery potential calculation is used to transform the decline to a straight line, which allows for easy extrapolation to ultimate recovery. Upon obtaining the gamma function solutions to the τ and n parameters, the EUR can be determined, and the rp calculated.

Chapter 2: Logistic Growth Models

2.1 Origins of Logistic Growth Models

Logistic growth models are a family of mathematical models used to forecast growth for numerous industries. Originally developed for the biological modeling of population growth, they have been adapted to numerous disciplines extending far beyond the biological sciences. Logistic models originally were used to model population growth, but have also been used to model the growth of yeast, the regeneration of organs, and the market penetration of new products (Tsoularis and Wallace, 2001). They have even been used to model the oil production of entire regions (Hubbert, 1956).

Logistic growth models, first developed by Verhulst in 1838, are based on the philosophical ideas regarding population proposed by Malthus in 1803. At this time the population of the world was growing rapidly. Malthus believed that growth could not continue to occur at this rate, and that there was a limit the natural resources of any given area could hold. Once the population was too large for the natural resources, the growth rate would slow down and the size of the population would stabilize (Patzek and Croft, 2010). The size limit was referred to as the carrying capacity. Verhulst developed a mathematical model to represent this trend in population growth. By applying a multiplicative factor to the exponential growth equation, he was able to restrict the population size to the carrying capacity. The most basic equation for exponential growth is:

$$\frac{dN}{dt} = rN \tag{1.45}$$

where

N = Population

r = Constant

t = Time

This equation can be separated and integrated to obtain the cumulative population growth:

$$N(t) = N_0 e^{rt} \tag{1.46}$$

where

N_0 = Initial population

When the equation for exponential growth is plotted versus time the population will grow indefinitely, which, according to Malthus was unrealistic.

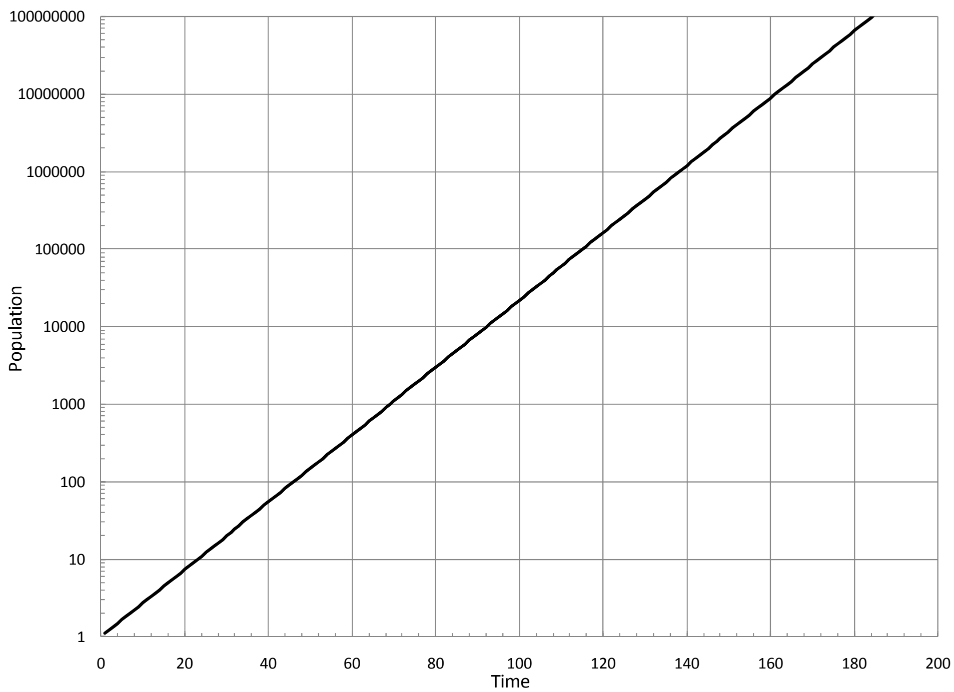

Figure 23 - Population growth vs. time with exponential model

Figure 23 shows the population growth versus time for an exponential model. If a model of this nature were used to forecast the growth of anything, it would continue to grow infinitely throughout time.

To limit population growth predictions, Verhulst applied a multiplicative factor, and the exponential growth equation took on the new form:

$$\frac{dN}{dt} = \left(1-\frac{N}{K}\right)rN \tag{1.47}$$

where

K = Carrying capacity

K is the carrying capacity, or the maximum size to which a population can grow before being limited by natural resources. This equation can be integrated to obtain the cumulative form:

$$N(t) = \frac{KN_0}{\left(K-N_0\right)e^{-rt}+N_0} \tag{1.48}$$

Figure 24 shows the logistic growth model developed by Verhulst plotted versus a typical exponential growth. The carrying capacity for the logistic model was arbitrarily set at 10,000 to illustrate behavior. The population grows exponentially until it begins to approach the carrying capacity, at which point the behavior changes and the population size levels off at the carrying capacity K.

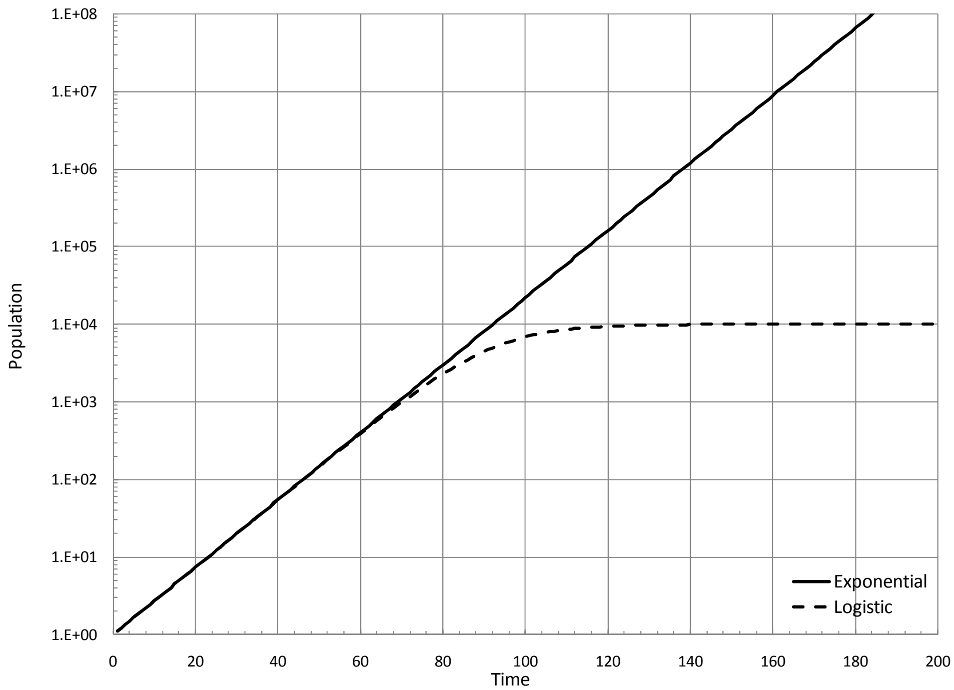

Figure 24 - Population growth vs. time with both the exponential and logistic growth model

Figure 25 shows the population growth rate versus time of both the logistic and exponential models. The exponential growth rate continues to grow indefinitely, while the logistic models growth rate increases until the population has reached half of the carrying capacity. At this midpoint, the rate of population growth decreases exponentially until the carrying capacity is reached. After initially being proposed by Verhulst, the logistic model has been developed and expanded upon over time, until it has reached the widely used form it is seen in today.

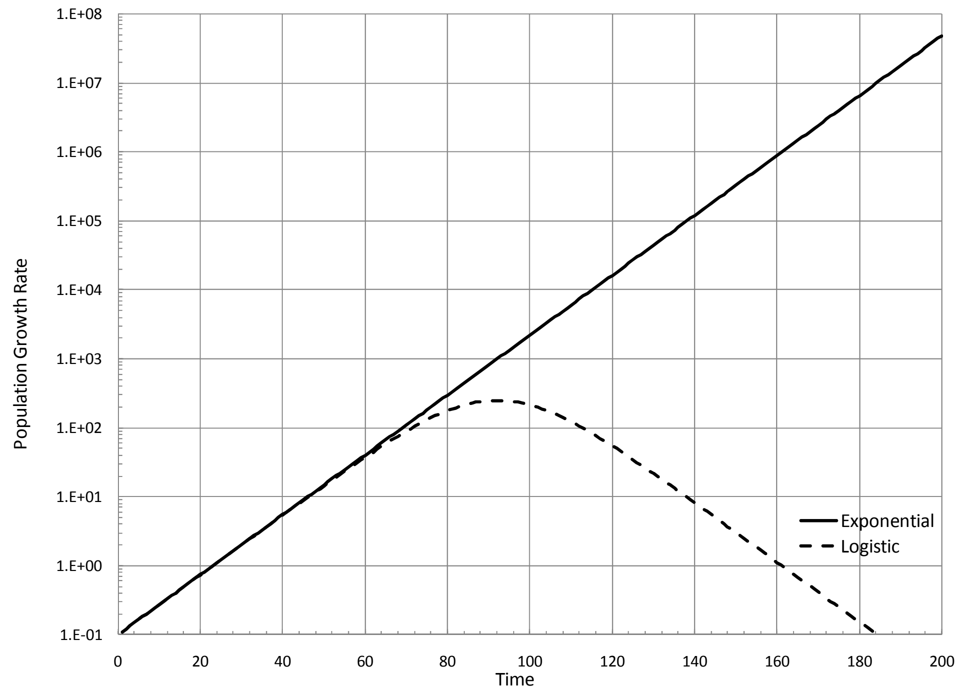

Figure 25 - Population growth rate vs. time with both the exponential and logistic growth model.

2.2 Development of Logistic Growth Models

The Verhulst logistic growth equation eventually came to be known as the Verhulst-Pearl equation after Pearl (1920) who used the logistic model to estimate population growth of the United States (Tsoularis and Wallace, 2001). The Verhulst-Pearl equation was somewhat inflexible in predicting growth behavior. For this reason, the logistic growth model has been adapted and changed to fit more complicated behaviors that exhibit the sigmoidal rate of growth but which are not necessarily exponential. A more generalized form of the logistic model was proposed by Richards in 1959 and used to empirically fit the growth of plants. This equation takes on the form:

$$\frac{dN}{dt} = rN\left[1-\left(\frac{N}{K}\right)^{\beta}\right]$$

(1.49)

where

β = Exponential constant

The Verhulst-Pearl equation is a specific form of the Richards model where β=1. The Richards equation can be integrated to obtain a cumulative growth equation of the form:

$$N(t) = \frac{N_0 K}{\left[N_0^{\beta}+\left(K^{\beta}-N_0^{\beta}\right)e^{-\beta rt}\right]^{1/\beta}}$$

(1.50)

Further improvements were made to the logistic model by Blumberg (1968), who observed that certain things, particularly the growth of organs occur not exponentially, but hyperbolically. By introducing an additional exponent to the Richards form of the equation, the Blumberg hyper-logistic growth model takes on the form:

$$\frac{dN}{dt} = rN^{\alpha}\left(1-\frac{N}{K}\right)^{\gamma}$$

(1.51)

where

α = Exponential constant

γ = Exponential constant

The Blumberg equation both increases and decreases hyperbolically; again the Verhulst-Pearl equation is a specific form of the Blumberg equation where the exponential constants are equal to one. An equation of the Blumberg type will be used to model oil and gas production from unconventional reservoirs, because it is well known that the production decline is hyperbolic. An even more generalized form of the logistic model has been proposed by Tsoularis and Wallace(2001), which incorporates both the Richards and the Blumberg equations. The generalized form of the logistic model takes on the form:

$$\frac{dN}{dt} = rN^{\alpha}\left[1-\left(\frac{N}{K}\right)^{\beta}\right]^{\gamma} \tag{1.52}$$

The even more generalized form of the logistic growth model incorporates the behaviors of the Verhulst-Pearl, Richards and Blumberg equations. The general form is equivalent to the Verhulst-Pearl equation when all of the exponential constants are equal to one. It is equivalent to the Richards equation when α and γ are equal to one. It is also equivalent to the Blumberg equation when β is equal to one.

2.3 Use of Logistic Models in Petroleum Industry

Logistic models are very flexible and robust empirical mathematical models that have been used to model numerous physical trends. The model being proposed in this thesis is not the first time logistic models have been used in the petroleum industry. Hubbert (1956) proposed what has now come to be referred to as the Hubbert model. Hubbert observed that all finite natural resources will follow similar behavior to that of population growth. The production of fossil fuels from an entire field or producing region will follow the same sigmoidal growth as the logistic growth model.

Initially, the production will be zero, and as the first wells are drilled, the total rate of production will continue to increase until half of the existing hydrocarbons have been produced, at which point the production rate of the field will begin to decline. Eventually, the field production will terminate when all of the economically producible hydrocarbons have been depleted. As was discussed earlier, the Hubbert model has been shown to fit production profiles for various areas very well. It very accurately predicted the decline of U.S. and North Sea oil production. There are however, numerous factors which affect production of oil from a field, particularly the price of oil. When oil prices are high, wells are drilled quickly to capitalize on the high price. When prices drop down, the development slows, and production rates are curtailed.

Because of the various changes in conditions which affect production, it has been proposed that field production can be modeled more accurately with multi-Hubbert cycle analysis, instead of a single Hubbert curve (Patzek and Croft, 2010). Patzek and Croft showed that various forecasts for the emissions of CO_2 from coal burning power plants predicted unreasonably high values. This was caused by the assumption that the production and burning of coal would continue to increase, almost indefinitely in some cases, over the next 30 years. Coal is a fossil fuel like oil and gas, and thus finite in quantity. The same physical constraints which restrict oil and gas restrict coal. Based on multi-Hubbert cycle analysis of global coal production, coal appears to be approaching its peak. Therefore, increased CO_2 emission from coal is unlikely. As coal production declines, so too will the emissions generated from its burning.

Multi-Hubbert cycle analysis can also be used to match field production in the Barnett Shale. **Figures 26** and **27** show examples of multi-Hubbert cycle analysis of the production rate and cumulative production of gas versus time in the Barnett Shale.

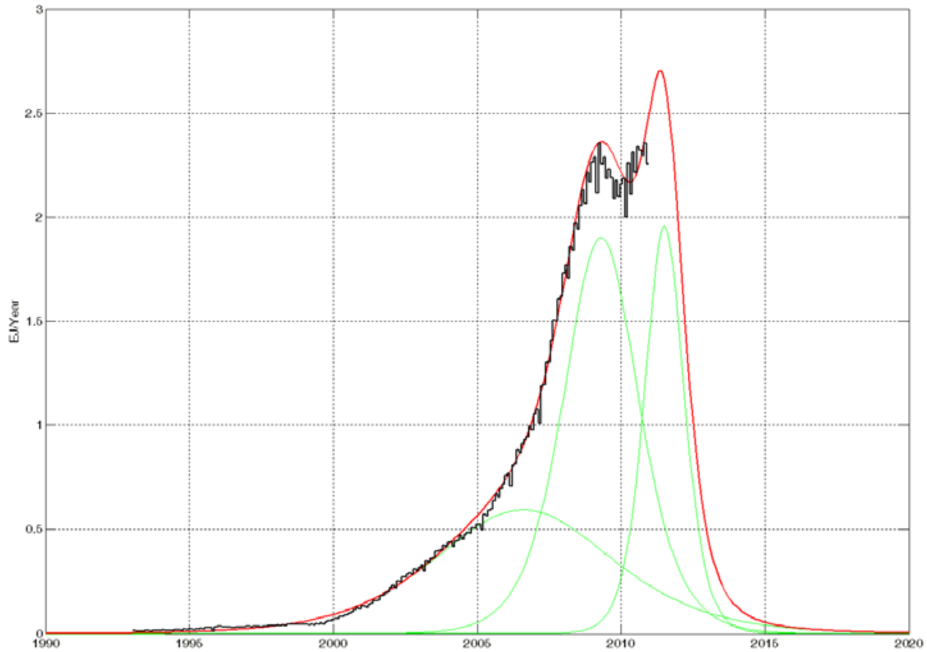

Figure 26 - Production rate vs. time mutli-Hubbert cycle analysis of production from the Barnett Shale

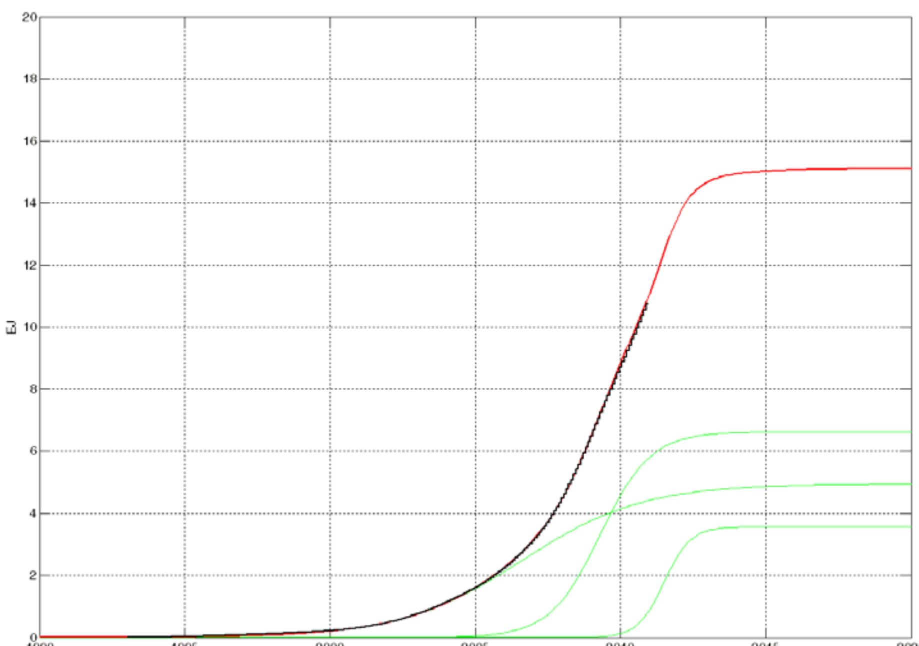

Figure 27 - Production cum vs. time mutli-Hubbert cycle analysis of Barnett Shale

Figures 26 and **27** depict three cycles of production in this field. The black line represents the actual production from the field and the red line represents the multi-Hubbert cycle analysis. The first cycle is the original production, and corresponding increases in drilling activity which occurred during the first decade of production in the field. The second cycle begins in 2004 with the introduction of horizontal drilling and a corresponding increase in the price of gas. The cycle is marked by a steep increase in production rate followed by a steep decrease when the price of gas plummeted. The third cycle began early in 2010 when gas prices began to rise again, and the economic prospects of the Barnett Shale once again improved.

The Hubbert model has been used by Juvkam-Wold and Dessler (2009) to forecast world oil production. By plotting the ratio of annual oil production over cumulative production versus cumulative production, the Hubbert model appears as a straight line which follows the actual production data very well. This straight line can easily be extrapolated until the rate reaches zero, and the total production can be determined.

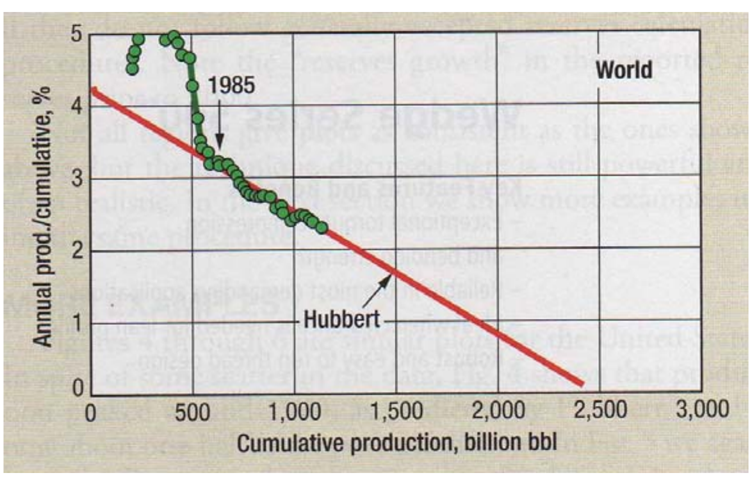

Figure 28 –World oil production forecast using the Hubbert model (Juvkam-Wold and Dessler, 2009)

Figure 28 shows the annual world oil production over the cumulative production plotted versus cumulative production. As can be seen in the figure, the actual production data has fallen into a straight line behavior, and the Hubbert model coincides very well with the actual production data. The model can then be extrapolated until the production reaches zero, and the x-intercept will be cumulative oil production for the world. Based on this forecast, the ultimate total oil production for the world will be close to 2.5 trillion barrels of oil.

2.4 NEW MODEL FOR DECLINE CURVE ANALYSIS IN UNCONVENTIONAL WELLS

A new model has been developed to forecast production and estimate reserves in extremely low permeability formations. The new model is based on the logistic growth model family of curves. The model was originally discovered in Tsoularis and Wallace (2001), and is of the Blumberg type. The Blumberg family of growth curves are hyperbolic, as is the decline of unconventional tight oil and gas wells. As the model appeared in the paper it took on the form:

$$N(t) = \frac{K(t+a)^n}{b+(t+a)^n} \tag{1.53}$$

where

N	=	Population
K	=	Carrying capacity
a	=	Constant
b	=	Constant
n	=	Exponential parameter
t	=	Time

The equation first appeared in Spencer and Coulombe (1966) and it predicted the hepatic regeneration, or the rate of regrowth of rat livers. The rats' livers were reduced to one third their original size, and the rate at which the livers regenerated was observed to follow a hyperbolic trend (Spencer and Coulombe, 1966). Spencer and Coulombe observed that various models could be used to trend the growth. The model they used took on the form:

$$L = \frac{L_0 t}{K+t}$$

(1.54)

where

L	=	Liver size
L_0	=	Original liver size before reduction (carrying capacity)
K	=	Time at which liver has grown to half original size
t	=	time

The form that it appeared, **eq. 1.53,** in the Tsoularis paper, updated the equation to contain both the a parameter, and the exponential n parameter.

After empirically fitting the model to oil and gas well production data using least-squares regression with the Excel Solver add-in, the a term consistently went to zero. For this reason, the a-term seen in **eq. 1.53** has been removed, and the nomenclature has been changed to fit current oil and gas production nomenclature. The logistic growth model for use in forecasting oil and gas reserves in ultra-tight formations takes on the form:

$$Q(t) = \frac{Kt^n}{a+t^n}$$

(1.55)

where

Q = Cumulative production

K = Carrying capacity

n = Hyperbolic exponent

a = Constant

The carrying capacity, K, is the physically recoverable oil or gas from the reservoir and production mechanism. The carrying capacity does not take into account economic constraints that might be used to forecast EUR. The a constant is the time raised to the power n at which half the oil or gas has been recovered. The n parameter is the hyperbolic decline exponent.

The logistic growth model for use in forecasting oil and gas reserves can be converted to a rate form by taking the derivative with respect to time. The rate form of the equation is:

$$q(t)=\frac{dQ}{dt}=\frac{Knbt^{n-1}}{\left(a+t^{n}\right)^{2}} \tag{1.56}$$

where

q = Production rate

The equation in this form does not appear to resemble the generalized form of the logistic growth model proposed by Tsoularis and Wallace. It can, however, be shown through some algebraic manipulation that this equation is a Blumberg type logistic model, or the generalized form with β equal to one. If the cumulative form of the equation is substituted into the rate form, the equation becomes:

$$q(t) = \frac{anQt^{-1}}{a+t^{n}} \tag{1.57}$$

The cumulative form of the equation can be solved for t^n to yield:

$$t^n = \frac{aQ}{K\text{-}Q} \tag{1.58}$$

Equation 1.58 can then be inserted into **eq. 1.57** to bring about the form:

$$q(t) = anQ\left(\frac{aQ}{K\text{-}Q}\right)^{-1/n}\left(a+\frac{aQ}{K\text{-}Q}\right)^{-1} \tag{1.59}$$

The a terms cancel out, and the Q term can be brought outside the parenthesis and combined to give:

$$q(t) = nQ^{1-1/n}\left(\frac{a}{K\text{-}Q}\right)^{-1/n}\left(1+\frac{Q}{K\text{-}Q}\right)^{-1} \tag{1.60}$$

The last term of **eq. 1.60** can further be reduced to:

$$q(t) = nQ^{1-1/n}\left(\frac{a}{K\text{-}Q}\right)^{-1/n}\left(\frac{K\text{-}Q}{K}\right) \tag{1.61}$$

Finally, by multiplying the equation by $K^{1+1/n}$ over $K^{1+1/n}$ **eq. 1.61** can be rearranged to take on the general form of the logistic growth model.

$$q(t) = n\left(\frac{K}{a}\right)^{1/n} Q^{1-1/n}\left(1\text{-}\frac{Q}{K}\right)^{1+1/n} \tag{1.62}$$

Equation 1.62 is a specific form of the generalized logistic growth model proposed by Tsoularis and Wallace where r is equal to $n(K/a)^{1/n}$, α is equal to 1-1/n, β is equal to 1 and γ is equal to 1+1/n. The growth model will be shown to fit production data for extremely low permeability oil and gas reservoirs very well. **Equation 1.55** is a new method to forecast reserves that provides finite estimates.

2.5 Results and Applications of New Model

The logistic growth model for production forecasting in unconventional reservoirs is a simple and robust empirical model. The usage of the model is relatively easy and can be done in at least three ways. In this research, it has been applied to both Bakken Shale and Barnett Shale oil and gas wells using three different methods. The model has two or three unknown parameters, depending on how much is known about each individual well. To determine the parameters the methods employed are least-squares regression coupled with Excel's solver add-in, the non-linear regression function in MATLAB and finally using a linearization of the equation to determine the parameters.

There are two to three unknown parameters in the logistic equation that must be determined to obtain a good fit of the data. The parameters are K, the carrying capacity, a, the time to the power n at which half of the oil or gas has been produced, and n, the hyperbolic decline exponent. K is the term which may or may not be known a priori. The carrying capacity is the total amount of oil or gas that can be recovered from the well from primary depletion, not taking into account economic or time related cutoffs. K is the unique aspect of the logistic model that causes it to behave more realistically than the Arps model. K is also the parameter that allows known physical reservoir volumes to be accounted for. If the volumetrically available oil or gas from primary depletion is known ahead of time, the carrying capacity can be fixed before the other two parameters are determined. With good petrophysical data, the EUR from volumetrics can be determined, and the carrying capacity fixed to this value. There is uncertainty in volumetric estimates. Based on the saturation method explained previously, if the drainage area, formation thickness, water saturation, porosity and recovery factor are known, the volumetric oil and gas can be calculated.

In extremely tight reservoirs, there is uncertainty related to all of the parameters. Wells are drilled with long laterals, ranging from 5,000 to 10,000 ft depending on the formation or the goals of the operator. The long laterals are then completed with multi-stage fracture treatments where large quantities of sand and fracturing fluid are pumped into the formation. Each stage of the fracture treatment is estimated to propagate perpendicular to the wellbore outward into the formation. The length of each fracture stage is uncertain, as is the effectively propped length of the fracture. A new technique referred to as microseismic monitoring is being used during the fracturing process to attempt to estimate how much of the reservoir is accessed by each fracture stage. According to Mayerhofer *et al.*, one theory of the drainage area of ultra-tight reservoirs is the concept of stimulated reservoir volume. SRV theorizes that the drainage radius does not extend beyond the volume of reservoir rock that has physically been accessed by the fracture treatment (Mayerhofer *et al.*, 2008). If the concept holds true, it is essential to have an accurate understanding of exactly how far each fracture stage propagates into the formation to determine drainage radius.

Figure 29 shows the drainage radius versus time calculated from a well test equation which determines the rate at which pressure responses propagate away from the wellbore in vertical wells. As permeability decreases, the time it takes the response to propagate outwards increases. If the reservoir permeability were on the order of 1 microdarcy, it would take roughly 2500 days or more than 6 years for the drainage radius to reach just 500 ft. If the reservoir permeability were 100 nanodarcies, the drainage radius would not have extended 500 feet beyond the wellbore even after 20 years.

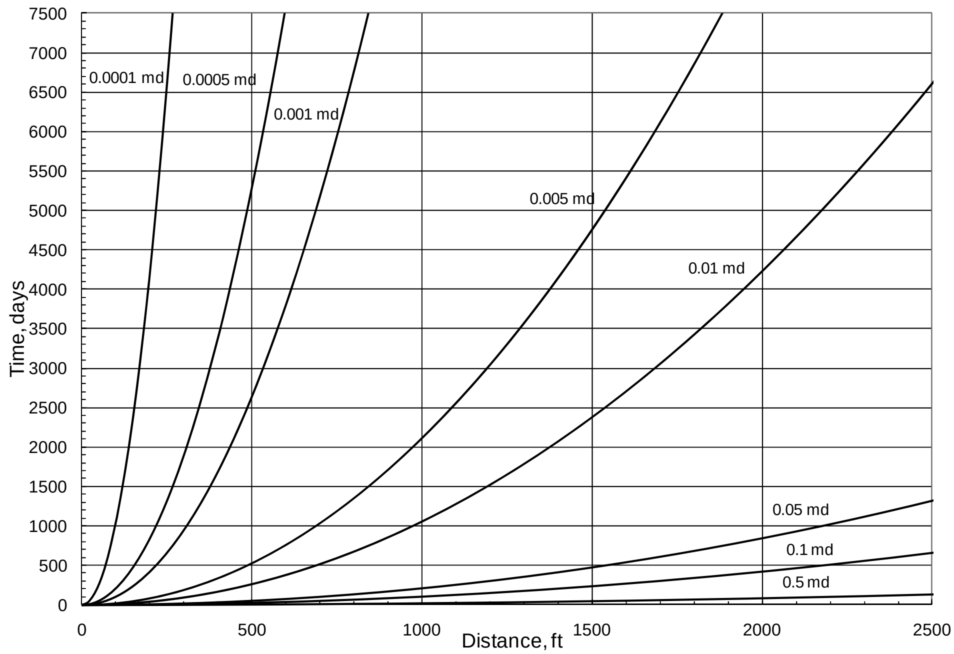

Figure 29 - Drainage radius vs. time for low permeability formations

With known matrix permeabilities in the micro to nanodarcy range common in tight formations, it is reasonable to assume that the concept of stimulated reservoir volume provides the appropriate drainage radius.

There are similar uncertainties associated with all of the necessary parameters to determine the volumetric oil and gas available in resource plays. In gas shales, there is not only gas stored in the matrix, but also gas adsorbed into the organic matter which is produced as the reservoir pressure is depleted (Lee, 2010). The volume of adsorbed gas is difficult to estimate. The water saturation and porosity obtained from log values in shales is unreliable, however core data can provide more realistic estimates. The recovery factor of shales is also highly unknown. Initially, recovery factor in gas shale was expected to be somewhere in the 7-10% range (Holditch *et el*., 2007). The believed recovery factor has increased recently with better than expected recoveries in wells, and is estimated to be around 30%. In oil shales, the recovery from primary depletion is expected to be very low, but typically in the 5-7% range (Clark, 2009). A volumetric estimate of EUR in ultra tight reservoirs is subject to high uncertainty, but with good data an estimate can be obtained, and this estimate can be used to constrain the carrying capacity with the logistic growth model to reasonable values. If, however, the carrying capacity is not known, the K parameter can be used as a fitting parameter in one of the three methods of fitting the equation listed below.

2.5.1 Methods for Using the Logistic Growth Model

When it has been determined whether or not the carrying capacity is known before producing the well, the logistic model can be fit empirically using a least-squares regression and Excel's solver add-in. Fitting is done by making initial guesses as to the values of the parameters, then using the solver to minimize the sum of the differences between the model estimates and the actual data. When matching to the logistic model, the cumulative form of the equation is used as opposed to the rate form. The integral form of the model (the cumulative) serves to eliminate some of the noise seen in the rate data, which is commonly caused by operational issues. All of the matches performed in this work were done with the cumulative form. All of the production data used in this study was obtained from DrillingInfo.com, which gives all volumes of oil or gas in monthly volumes. Outliers were removed from the data by inspection. **Figures 30** and **31** are examples of the logistic model being fit to Barnett Shale gas wells in which the carrying capacity was not known. **Figures 32** and **33** are examples of the logistic model being fit to Bakken Shale oil wells.

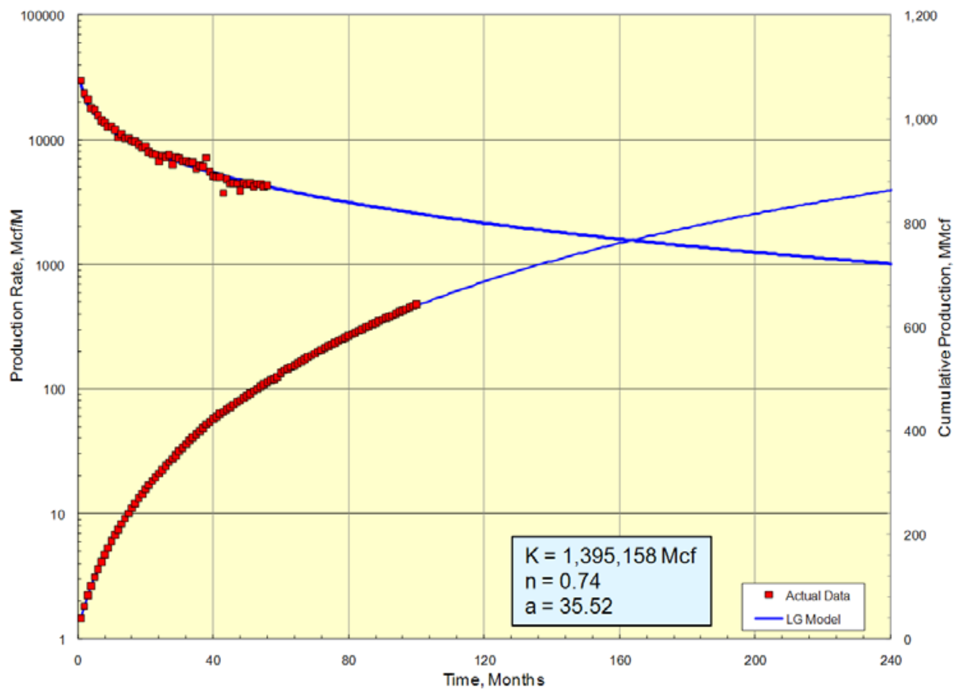

Figure 30 - Rate and cumulative vs. time of the logistic model on Barnett Shale data

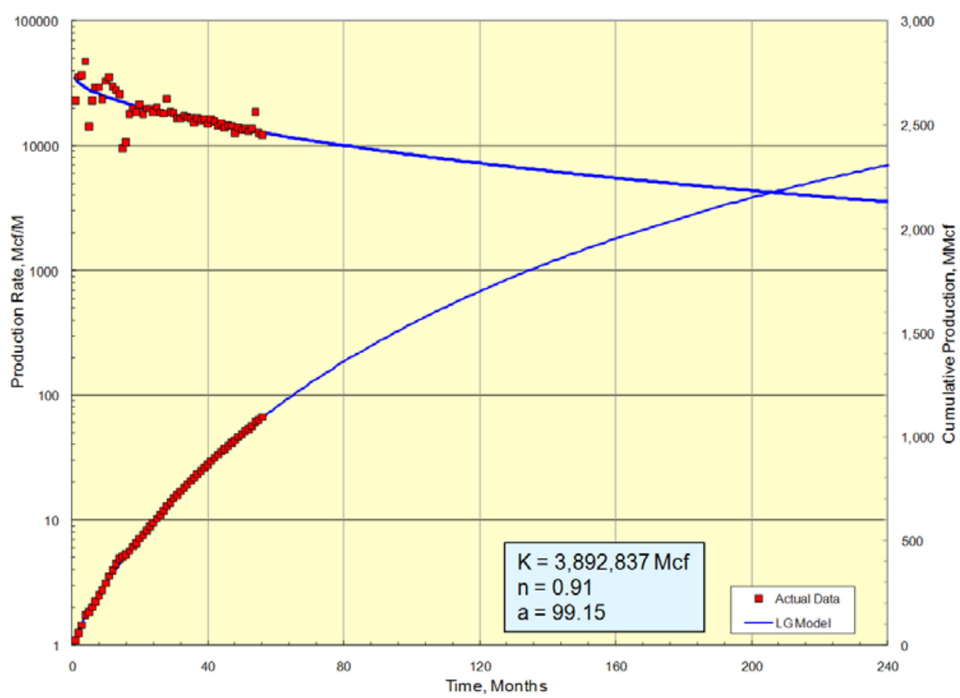

Figure 31 - Rate and cumulative vs. time of the logistic model on Barnett Shale data

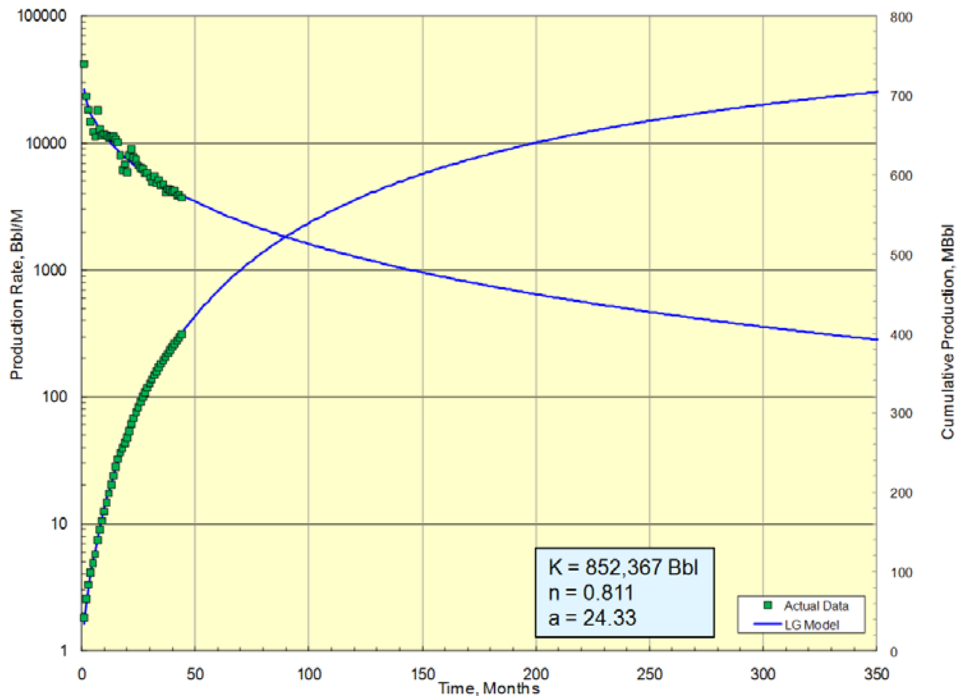

Figure 32 - Rate and cumulative vs. time of the logistic model on Bakken Shale data

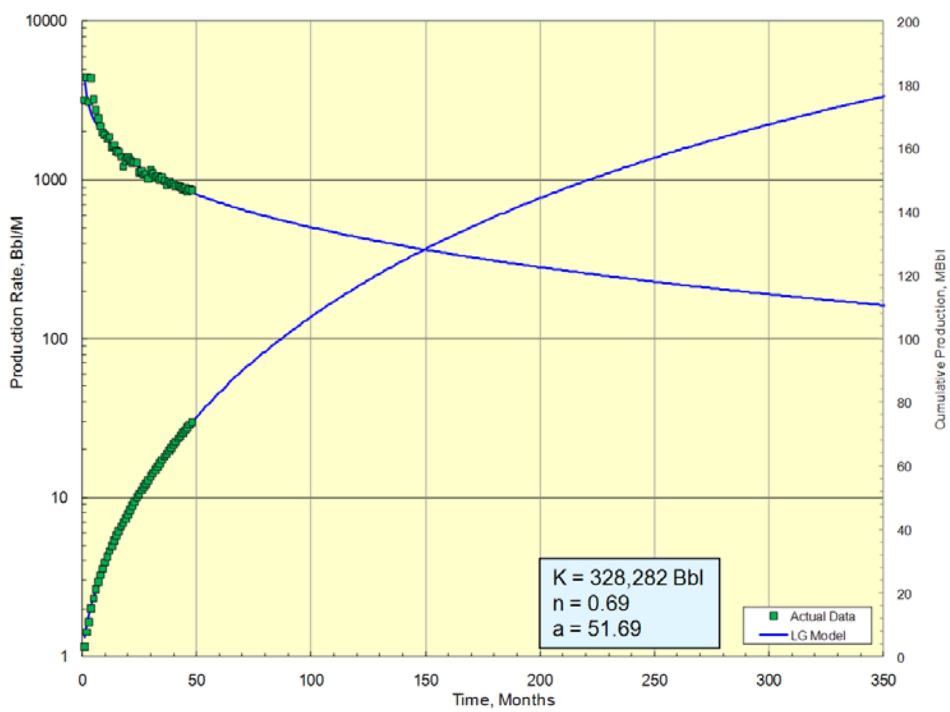

Figure 33 - Rate cumulative vs. time of the logistic model on Bakken Shale data

Figures 30 through **33** show that using the logistic model and least-squares regression, a very accurate fit of the data can be obtained. The smoothing nature of the cumulative production versus the rate can be observed in **figures 31** and **32**. In the two figures the trend of the rate data is inconsistent; however, the trend in cumulative data is more consistent. The smoothing nature causes fitting to the cumulative data to yield a better estimate than fitting to the rate data. The variability of the production rate is also clearly identified in both the Barnett shale and Bakken Shale wells. The two examples for both reservoirs vary drastically in EUR estimate, however, in all cases the logistic model trends the production data very well.

The second method for using the logistic growth model to fit production data and obtain an estimate of reserves is to use the built in nonlinear regression function in MATLAB. The function 'nlinfit' optimizes the parameters of the logistic growth model to obtain the best fit to the actual production data. The production data has to first be parsed and read into MATLAB with a script file designed to read the data in its original Excel format. The only outliers removed from the data set are the months in which oil or gas production was 0. Once the data has been formatted, it can be run with another script file which automatically fits the logistic model to each well. The MATLAB file does not offer the user room for personal interpretation; however, it is very convenient for fitting the model to large sets of data. The nonlinear regression function does not always yield realistic results, so the script was edited to exclude wells which yield unrealistic parameters. **Figures 34** and **35** show examples of the logistic model fit to Barnett Shale data with the nonlinear regression function. Both **figures 34** and **35** show the cumulative gas production versus time in months for Barnett Shale wells. The circles represent the actual data points and the straight line represents the logistic model.

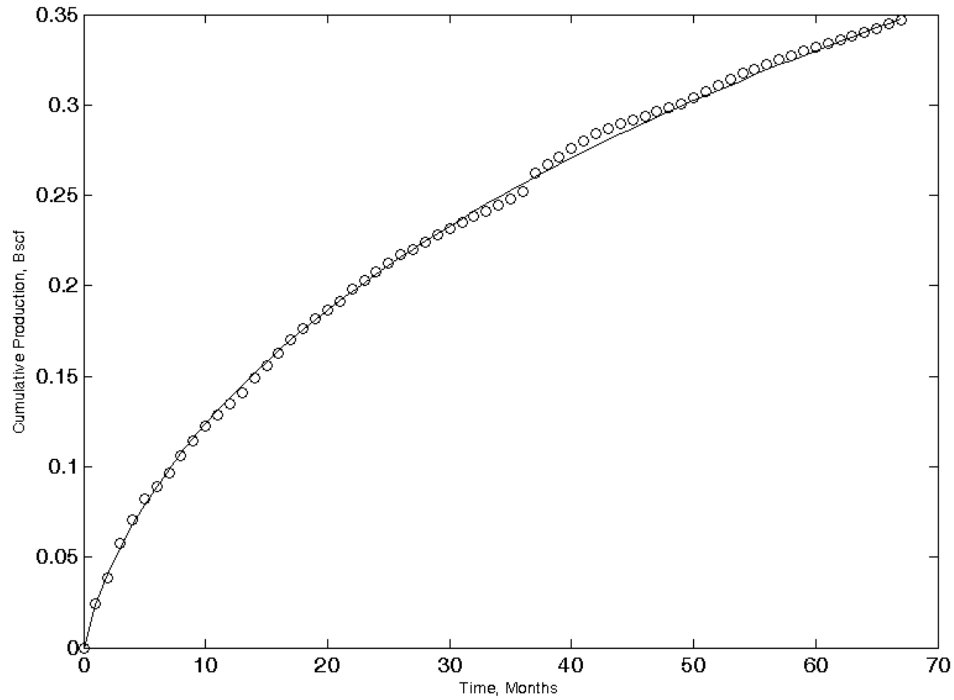

Figure 34 - Cumulative vs. time of logistic model fit with nonlinear regression function

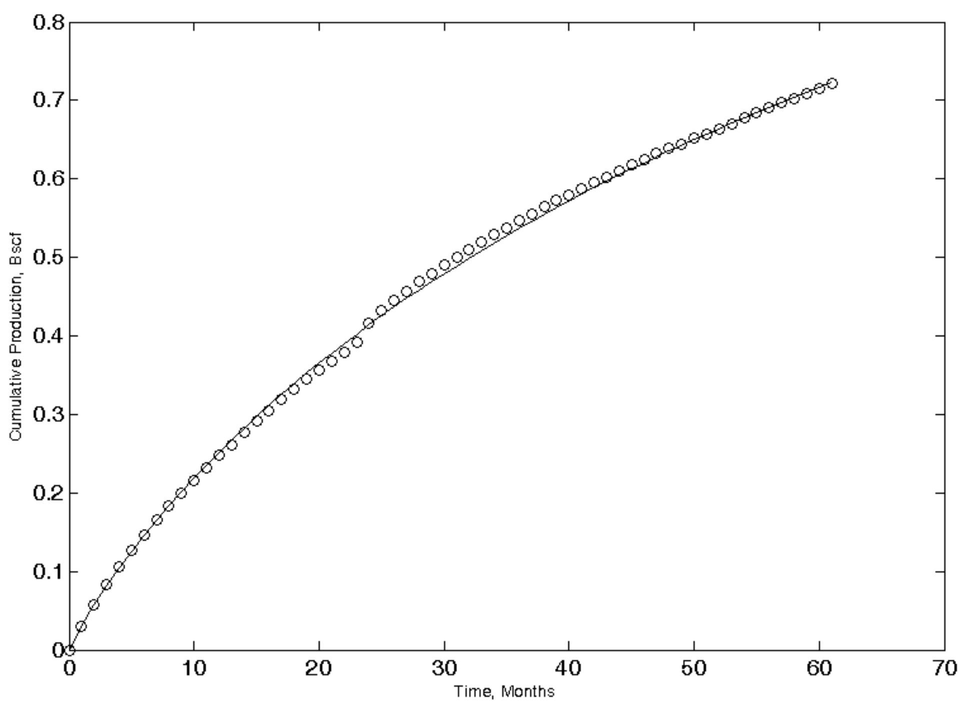

Figure 35 - Cumulative vs. time of logistic model fit with nonlinear regression function

The third method to determine the parameters of the logistic growth model is to linearize the equation. To linearize the equation, the first step is to take the reciprocal of the cumulative form.

$$\frac{1}{Q} = \frac{a+t^n}{Kt^n}$$

(1.63)

After the reciprocal has been taken, the numerator can be separated and the equation takes on the new form:

$$\frac{1}{Q} = \frac{a}{Kt^n}+\frac{1}{K}$$

(1.64)

After this, the equation can be multiplied by K, and 1 can be subtracted at which point the model becomes:

$$\frac{K}{Q}-1 = at^{-n}$$

(1.65)

Finally, the log of both sides is taken, and the equation can now be plotted as a straight line to determine parameters.

$$\log\left(\frac{K}{Q}-1\right) = \log a - n \log t$$

(1.66)

To obtain the parameters with linearization, the carrying capacity must be known before hand, and the logarithm of (K/Q-1) can be plotted versus the logarithm of time. The points will plot as a straight line, and the value of n and a can be determined. The negative of the slope of the line will be the n value and 10 taken to the power of the y-intercept will yield the a-parameter. **Figure 36** shows an example of the linearization of the logistic model.

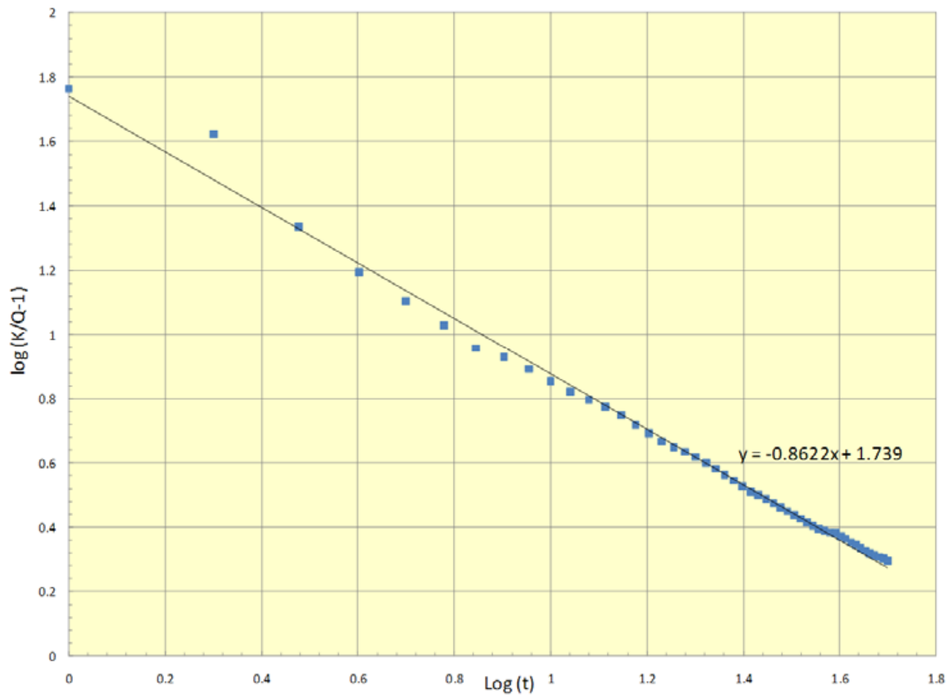

Figure 36 – A linear fit of the logistic growth model to a Barnett Shale well

The carrying capacity was set at 1.5 Bcf. From the linear fit of the data, the slope is 0.8622 which is the n value. The y-intercept of the line is 1.739. By taking 10 to the 1.739 the a value is found to be 54.83. **Figure 37** shows the logistic model being fit to the production data using the parameters obtained from the linear fit.

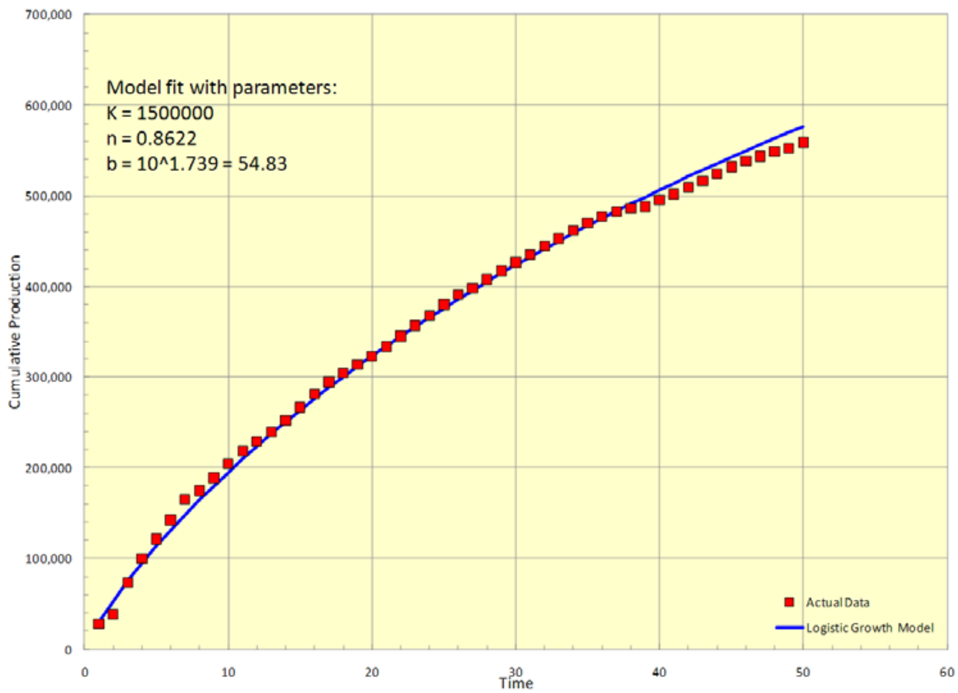

Figure 37 - Cumulative production vs. time fit using linearized fitting

The linear method provides a good alternative for estimating the parameters of the logistic growth model. **Figures 38** and **39** show another example of parameters obtained by linearization. In the example the carrying capacity was set at 3 billion cubic feet. The n value obtained from the method was 0.9944 and the a value obtained was 95.

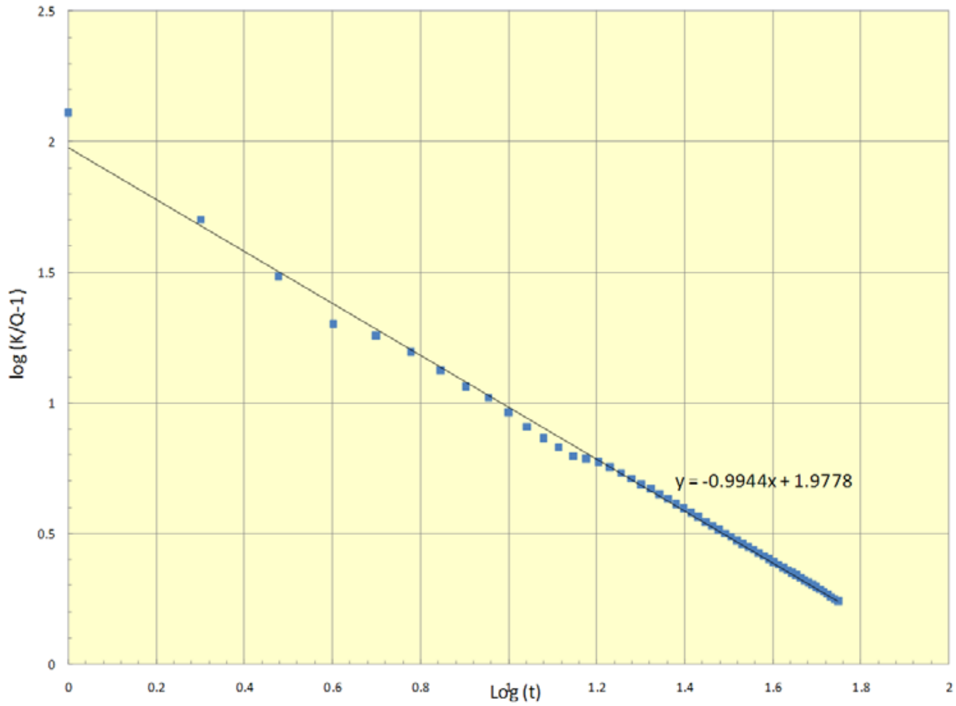

Figure 38 – A linear fit of the logistic growth model to a Barnett Shale well

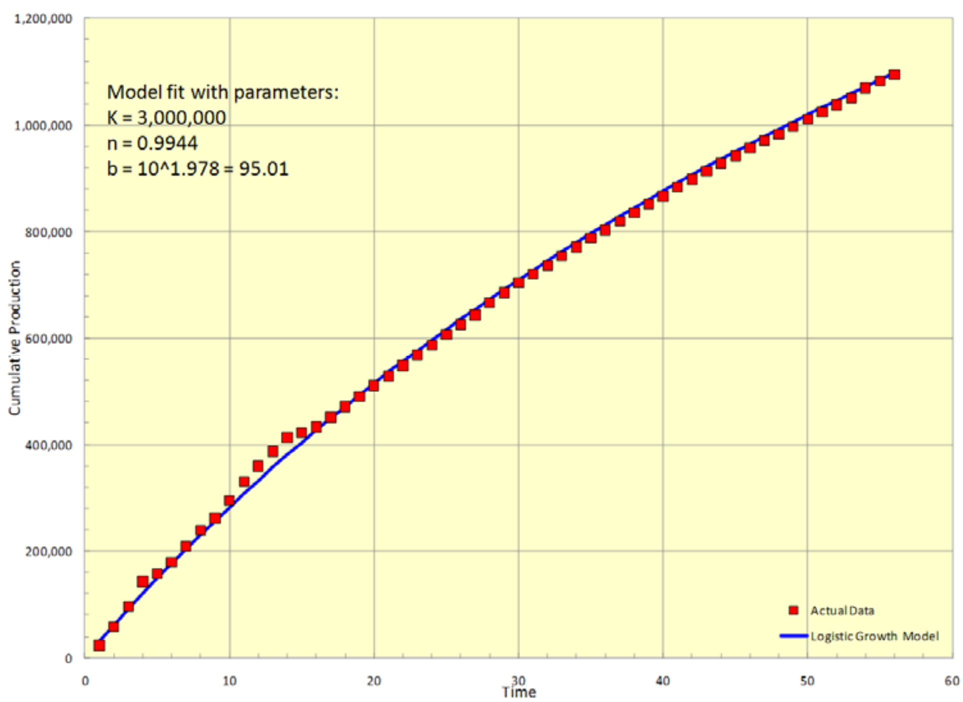

Figure 39 - Cumulative production vs. time fit using linearization

2.5.2 Discussion of Parameters

The logistic growth model being used to forecast production in extremely tight formations is hyperbolic. The n and a parameters in the model affect how the model behaves. To illustrate how the a and n parameters affect the performance of the logistic model, dimensionless cumulative production and production rate terms were developed. The dimensionless cumulative is the cumulative production over the carrying capacity.

$$Q_D = \frac{Q}{K} \tag{1.67}$$

where

Q_D = Dimensionless cumulative production

The dimensionless production is also the fraction of total recovery. When the cumulative production reaches the carrying capacity, the dimensionless cumulative will be equal to one. The dimensionless production rate is the current production rate over the peak production rate, which is also the initial production rate.

$$q_D = \frac{q}{q_0} \tag{1.68}$$

where

q_D = Dimensionless production rate

Using the dimensionless variables, type curves exhibiting the behavior of the model can be made. **Figure 40** shows the dimensionless rate versus dimensionless cumulative for varying values of n.

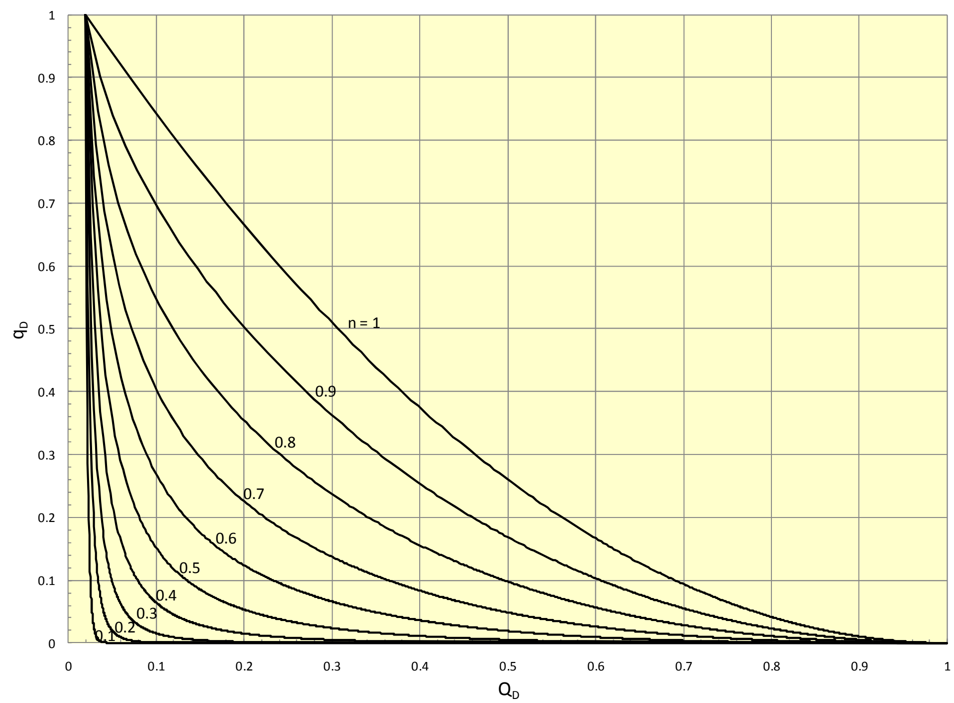

Figure 40 - Dimensionless rate vs. dimensionless cum type curve for varying n values

The values of n in **figure 40** vary between 0 and 1, and the values of a and K are arbitrary. The n determines the amount of curvature in the model. The wells with higher n values will have a more gradual production decline, while the wells with lower n values will decline quickly before stabilizing at a lower rate. If the n parameter exceeds 1, the decline curve will have an inflection point. It will be seen later, that for certain wells an n value greater than 1 will yield the best fit to the data.

Varying a is similar to the initial decline parameter in the Arps equation. The a value is the time to the power n at which half of the oil or gas is produced. This can be seen from **eq. 1.55** by taking the limit as t^n approaches the a value.

$$\lim_{t^n \to a}\left[\frac{Kt^n}{a+t^n}\right] = \frac{K}{2} \tag{1.69}$$

The time it takes for half the oil or gas to be produced is going to determine how quickly the well declines initially, and then when it stabilizes. The a should not to be confused with half the time that it will take for the oil to be produced. With low values of a, the well will decline very steeply initially, and then stabilize for long periods of time at lower rates. With higher values of a, the well will produce at a more steady decline. This behavior can be seen in **figures 41** and **42.**

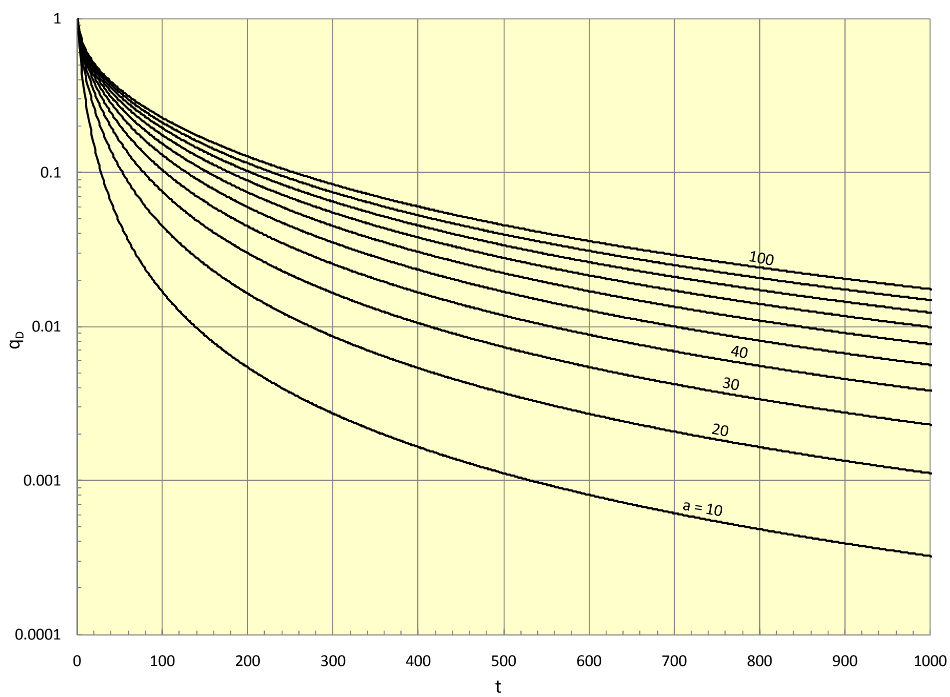

Figure 41 - Dimensionless rate vs. time type curve for varying a values

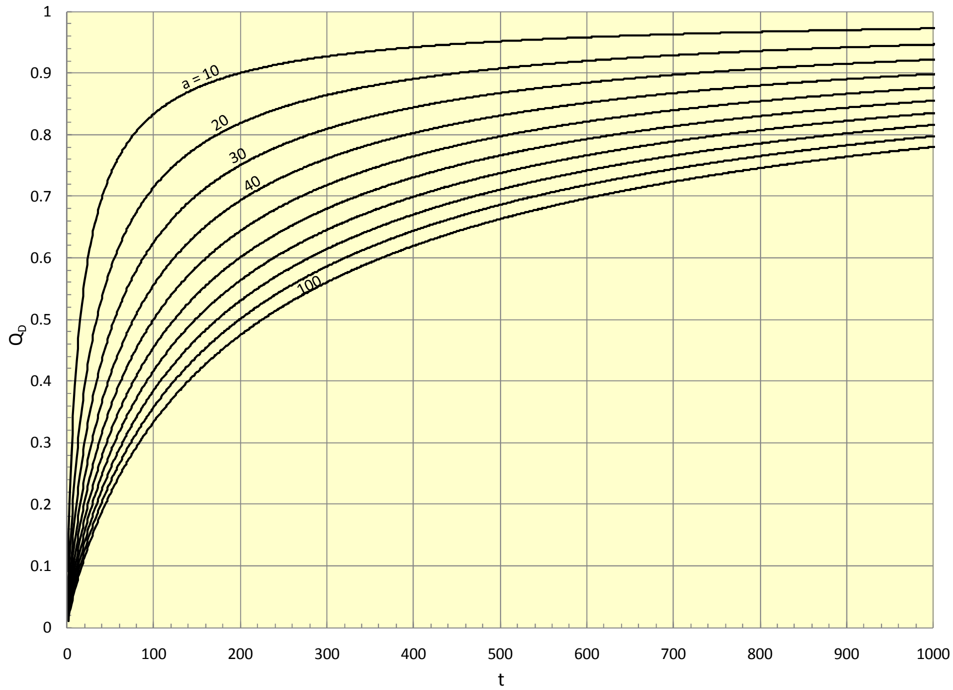

Figure 42 - Dimensionless rate vs. time type curve for varying a values

Figures 41 and **42** show the dimensionless rate versus time and dimensionless cumulative versus time for varying values of a from 10 to 100. The carrying capacity, K, and the exponent n are both arbitrary. **Figure 41** shows that at higher values of a, the well produces at a steadier rate for longer periods of time, and that as the a value decreases, the rate of production decreases more drastically. The dimensionless cumulative, however, in **figure 42** shows that the lower a value will result in a quicker recovery of the oil or gas in the reservoir.

2.5.3 Comparison Between Arps, Power Law and Logistic Growth Model

Microsoft Excel was used to fit the logistic growth model, the Arps model and the Power Law model to production data for 50 randomly chosen Barnett Shale wells in order to compare the predictive capabilities of the models. In all cases, the solver function was used with least squares optimization in order to obtain unbiased predictions with the models. The 50 random wells used in the study were all horizontal wells completed between January 2002 and December 2006. The models were fit to the first 36 months of production data, and the cumulative total at the end of the actual production data was compared with the predicted value of the three models in order to determine which model was closest to the actual total production. **Figure 43** shows an example of the results for one well in this analysis. The figure shows the rate versus time and cumulative versus time comparisons of the actual data to the three models.

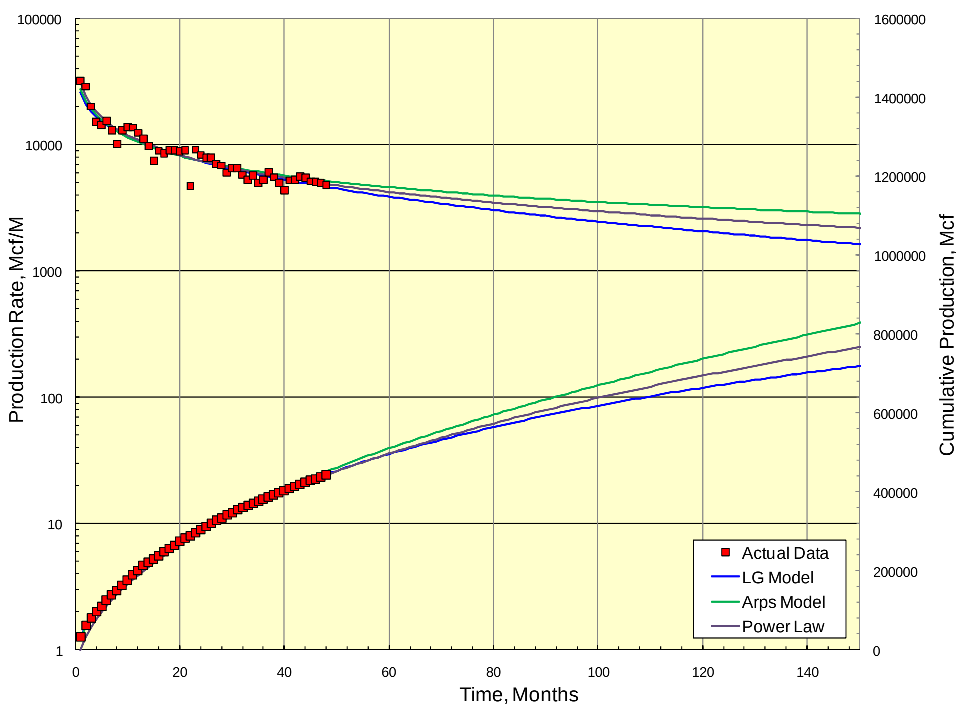

Figure 43 - Example rate and cum vs. time of model comparisons in Barnett Shale

The results of the analysis have been compiled in **Table 1**. The method of fitting the first 36 months of production is not the best practice for forecasting reserves. The decline curve should be fit to, and extrapolated out from the best trend in the data, often times from the start of the latest recompletion. Wells are often recompleted several months or several years after being drilled, and the recompletion results in a change in behavior. For this analysis, however, the first 36 months of the producing life were used to obtain the estimate, regardless of operational changes that may have occurred. An unbiased optimization of the parameters for the Arps and logistic models is a preferred way to forecast reserves. The recommended best practice for using the power law model is a more involved process than the one used in this research. Therefore, the results of the Power Law model could be in error. They are being included to show how the model behaves when its parameters are determined from an unbiased optimization.

Table 1 - Results of comparison of logistic, Arps and Power Law models

	Actual Data	LGM	Arps	Power Law
Total production (all wells)	718,702	664,484	695,261	642,190
Average cum difference		54,217	23,441	76,512
Absolute average cum difference		72,551	74,793	94,076
Absolute average % error		9.62%	9.98%	13.39%
Average 30 year EUR estimate		1,116,240	1,380,302	975,626
Total 30 year EUR estimate		55,812,018	69,015,113	48,781,305

All of the categories in **table 1**, with the exception of the percent error are in units of thousand cubic feet. The total production for all wells is the sum of the total production for the 50 wells at the end of their current production. The total production category shows the Arps model being closer to the actual value than the other two models, however, all of them predict less total recovery than the actual value. The under prediction is caused by recompletions which occurred at later times in the wells. Recompletions do not necessarily cause the total EUR for the well to increase, but they do reduce the total time it takes to recover the gas from the well. **Figures 44** and **45** show an example of one of the wells used in the analysis in which multiple recompletions caused the model estimates to fall greatly below the actual cumulative value for the well.

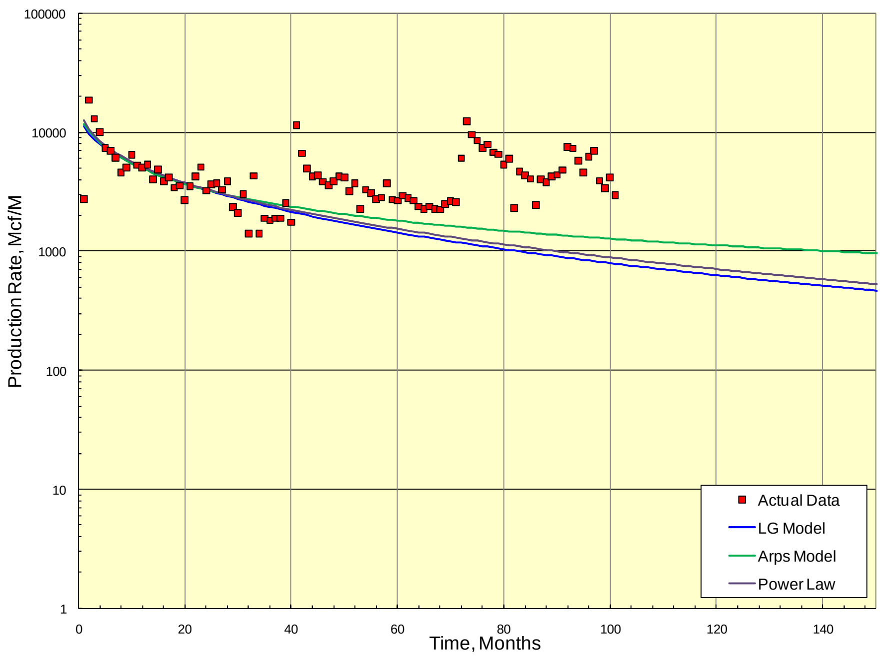

Figure 44 - Rate vs. time of a well with multiple recompletions

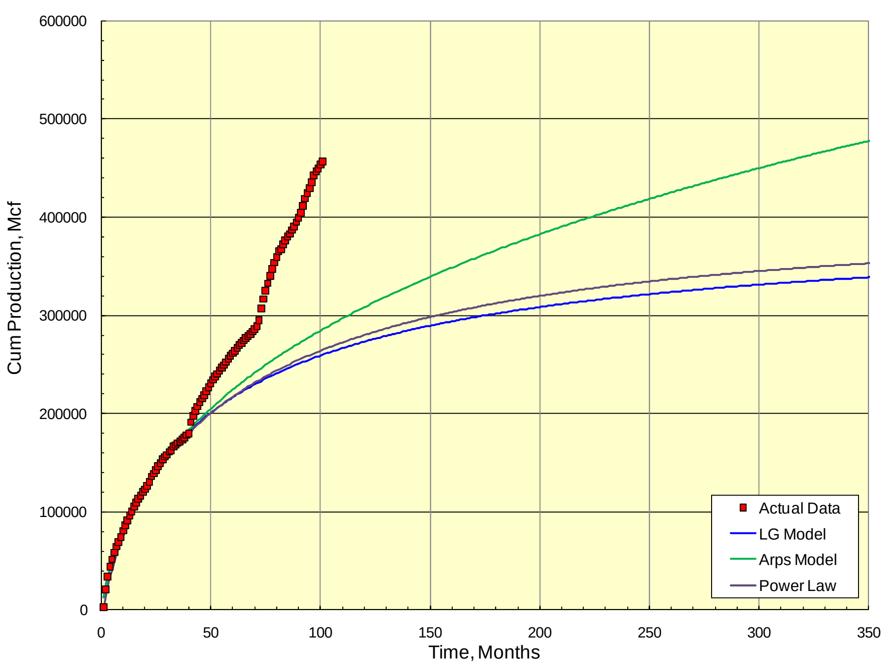

Figure 45 - Cumulative vs. time of a well with multiple recompletions

Figures 44 and **45** show that in both the rate versus time and the cumulative versus time plots that all three models fit the trend in the first 36 months of production very well. Approximately 41 months into production, however, the trend increases drastically. The increase is particularly evident in **figure 45,** the cumulative versus time plot, where the actual cumulative production takes a drastic upward spike, and quickly departs from the models. The changes in operating conditions, which no empiricle model can ever account for are the cause of the actual production data outperforming any of the model predictions.

The next category is the average difference between the real and predicted cumulative production at the end of the current producing life of the well. The Arps model prediction is closer than the other two models, however, this is not necessarily a good measure of the accuracy of the model. Use of the average, without squaring or taking the absolute value of the difference can serve to mask inaccuracies in the models. On certain wells, the model will over predict, while on other wells it will under predict. These two phenomena can cancel each other out. A better measure of the accuracy of the model is the next category, average of the absolute value of the difference between actual production and forecasted production. In this case the logistic model outperformed the other two models having an average difference of 72 MMcf between the actual production and the forecasted production. The average percent error for the logistic model was lower than the Arps model, but the difference between the two is small, differing by less than 1%.

The next two categories involve the forecasting tendencies of the models. The average 30 year EUR estimate and the total 30 year EUR estimate. These categories are telling of some of the differences between the models. The average EUR estimates were 1.38 Bcf for the Arps model, 1.12 Bcf for the logistic growth model, and 0.98 Bcf for the power law model. The Arps prediction was often the highest of the three and the Power Law was often the lowest, with the logistic model somewhere in between. The total EUR for all 50 wells follows the same trend with the logistic model's estimate being in between the Arps and Power Law models. The Arps model which is the commonly used industry model, predicts the total of the wells to be approximately 24% higher than the logistic model, and more than 40% higher than the Power Law model. Accurately estimating the recovery of an oil or gas well is extremely difficult, and impossible to determine until the well has actually completed producing. It is clear from the analysis that the Arps model is always the most optimistic method for forecasting reserves in unconventional formations.

2.5.4 Comparison of Arps and Logistic Growth Model for 1,000 Wells

The logistic model and Arps model were fit to 1,000 randomly selected horizontal wells in the Barnett Shale using an automated script file. The results were then analyzed by looking at both the comparison between the two models, and the statistical distribution of the parameters obtained.

Wells that obtained erroneous results from either the logistic model or Arps model were discarded from the data set before final analysis was performed. Wells were discarded if the Arps equation parameters returned a b value greater than 3, or a D_i value greater than 1. Wells were discarded for the logistic model if they returned a carrying capacity greater than 10 Bcf, an n value greater than 2, or an a value greater than 200. With either model, the well was discarded if any parameters of either model returned a value less than 0, or an imaginary number. The discarding of wells acted as a filter to remove wells with erratic production behavior. Of the 1,000 wells looked at, 421 of them were discarded leaving only 579 with usable data.

The results of the 1,000 well comparisons were similar to the results of the previous comparison. **Figures 46- 48** show the cumulative versus time fits for the actual data, the logistic model and the Arps model respectively.

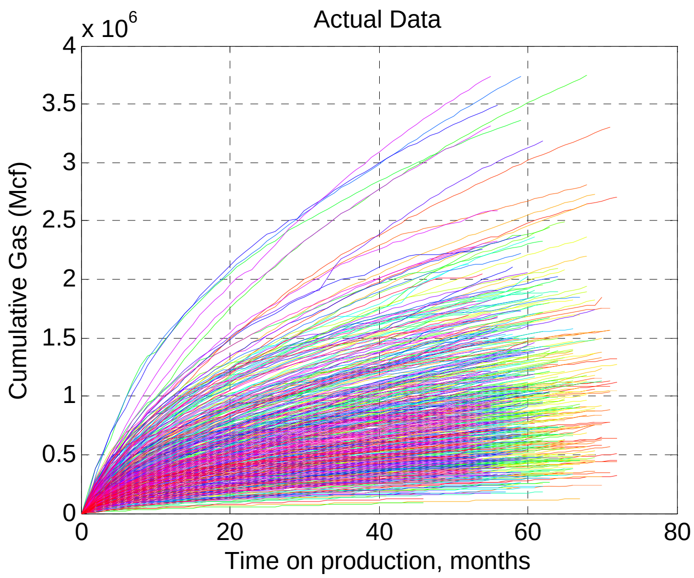

Figure 46 - Cumulative production vs. time for 1000 actual wells

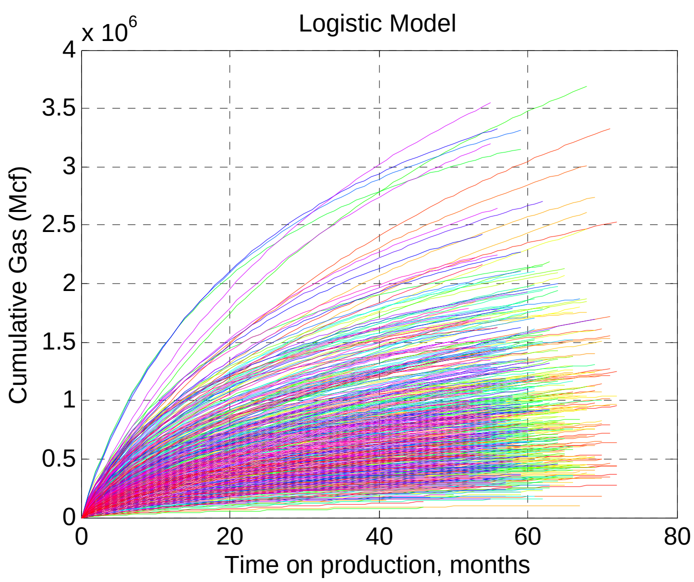

Figure 47 - Cum vs. time curves obtained from logistic growth model

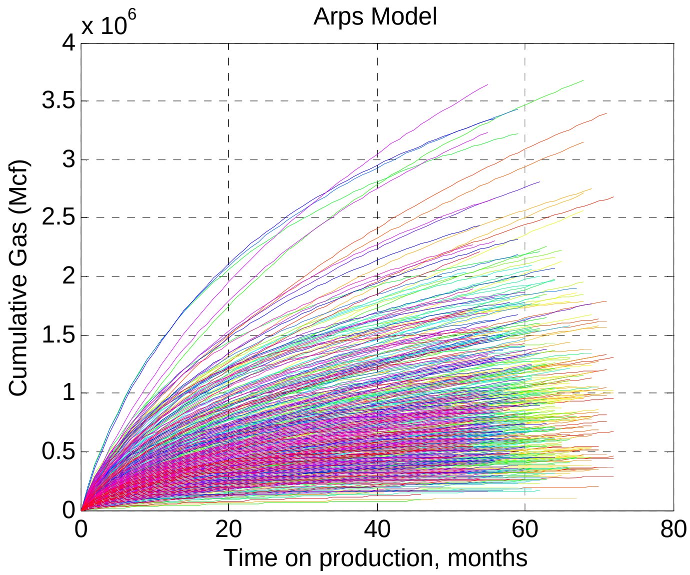

Figure 48 - Cum vs. time curves obtained from Arps' model

The trends seen in all 3 figures appear similar with no distinct outliers or erroneous estimates being apparent. The tabulated results from the analysis can be seen in **Table 2**.

Table 2 - Results of MATLAB comparison of logistic and Arps' models

	Actual	LGM	Arps
Total production	555,494,337	531,423,708	542,264,055
Absolute average cum difference		58,665	53,517
Absolute average % error		6.36%	5.84%
Average EUR 30 year estimate		1,438,363	1,827,160
Total EUR 30 year estimate		834,250,517	1,059,752,631

All of the values in the table, aside from the percent error, are in thousand cubic feet. The results from both the logistic model and Arps model are very close to the actual production values. The main difference between the models comes from their predictive capabilities. The predictions from the Arps model are more optimistic than those of the logistic model, as was the case seen in the previous analysis. The average EUR predicted from the logistic model was approximately 1.4 Bcf, while the average EUR predicted from the Arps modal was over 1.8 Bcf. The results from the Arps model were more than 25% higher than the results from the logistic model. The Arps model predicts that the entire group of wells will produce over 1 Tcf, while the logistic model only predicts 0.8 Tcf.

In addition to comparing the results of the logistic and Arps models, statistical analysis was done on the data obtained. The distributions of the forecasts obtained from the models, and the values of the parameters obtained were reviewed. **Figures 49** and **50** show the distributions of the EUR forecasts obtained from the logistic growth model and the Arps model respectively. The distributions of both models are log-normally distributed, similarly to the distribution of natural resources. The high productivity wells do not occur frequently. The Arps equation provides a minimum forecast of approximately 0.15 Bcf and a maximum of over 8 Bcf, while the logistic model has a range from 0.13 Bcf to just under 6 Bcf. The Arps model again shows the best wells in the set having more favorable expected recoveries than the logistic model.

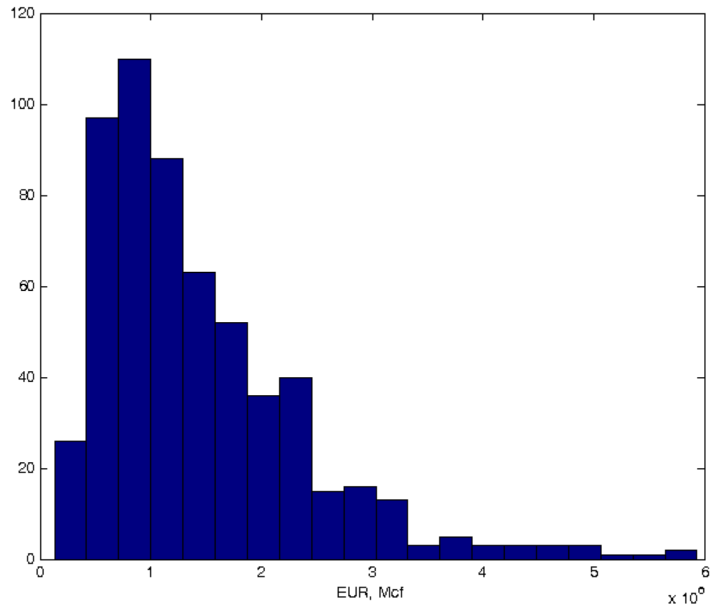

Figure 49 - Distribution of EUR forecasts obtained from logistic growth model in the Barnett Shale

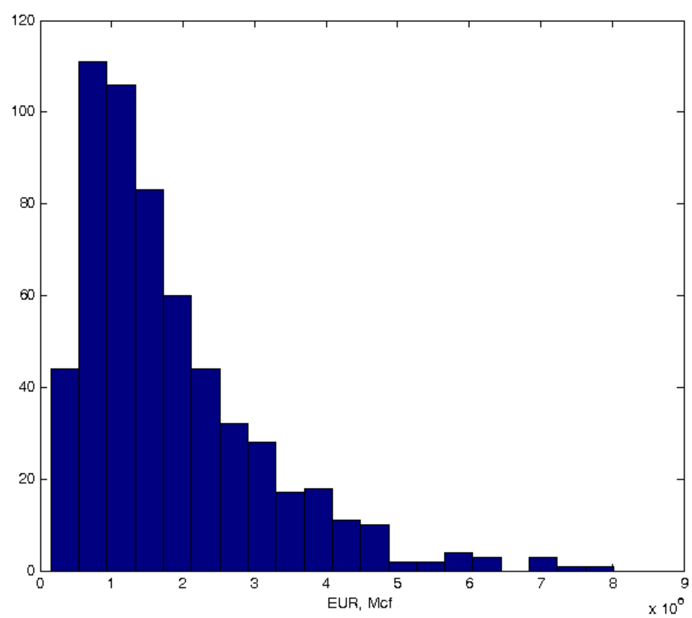

Figure 50 - Distribution of EUR forecasts obtained from Arps' model in the Barnett Shale

The analysis of the resulting parameters gives an idea for what kind of values should be obtained when forecasting with the logistic growth model in unconventional gas reservoirs. The parameters obtained for the Arps equation will be discussed briefly, to demonstrate that most b values obtained in the Barnett shale exceed 1. **Figure 51** shows the distribution of D_i values obtained for the Arps equation after matching the model to 579 wells in the Barnett shale.

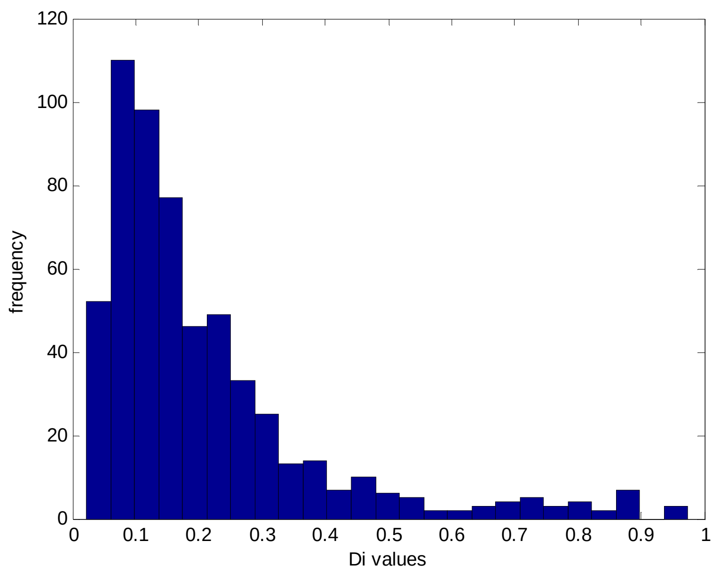

Figure 51 - Distribution of D_i parameters for the Arps model in the Barnett Shale

The D_i values, much like the EUR values follow a log-normal distribution. **Figure 52** shows the distribution of the b values obtained for the wells in the Barnett Shale, and **figure 53** shows the cumulative distribution function (CDF) of the b values.

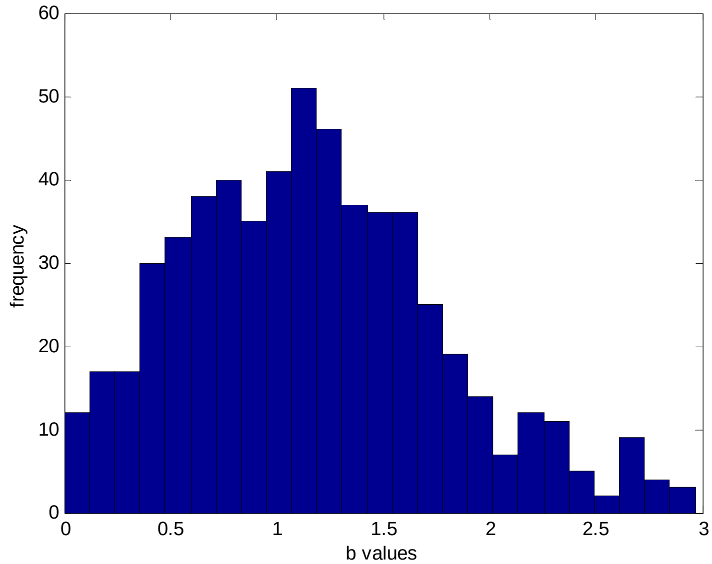

Figure 52 - Distribution of b parameter for Arps' model in the Barnett Shale

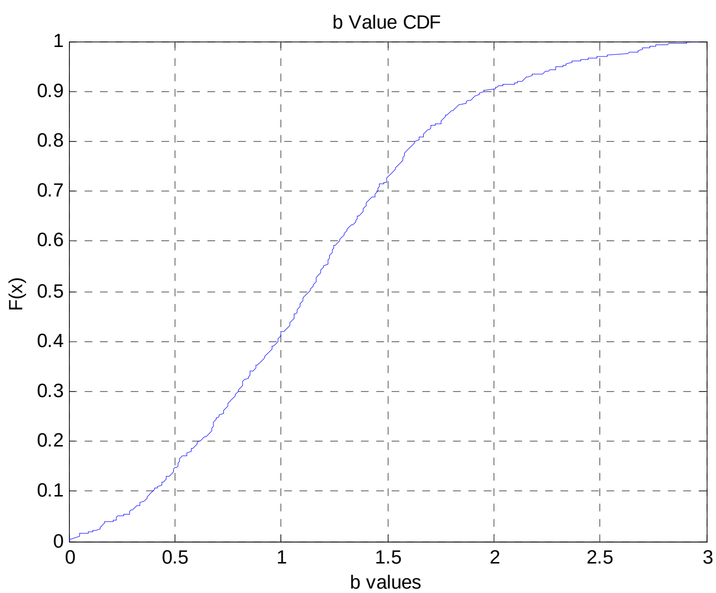

Figure 53 - CDF of b values for Arps' model in the Barnett Shae

Unlike the EURs or D_i parameter for the Arps model, the b values follow a normal distribution. The majority of the values obtained, however, are well above the cut off with a mean b value of 1.17 and a standard deviation of 0.61. The CDF shows that approximately 60% of the b values obtained are above 1.

Figure 54 shows the distribution of the carrying capacities, or K values. The K value distribution is log-normal, and is similar to the EUR distribution. The carrying capacity mean is higher than the EUR mean because the EUR determination is truncated after 30 years. The carrying capacity reflects the total recovery the well would achieve if allowed to produce without time constraints until depletion.

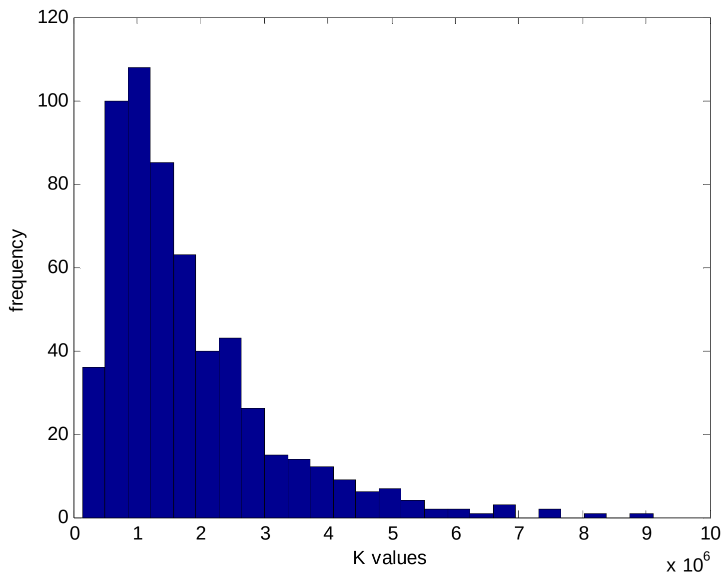

Figure 54 - Distribution of K values for the logistic growth model in the Barnett Shale

The a parameter acts much like the D_i parameter in the Arps model, and like the D_i parameter, the distribution is log-normal, seen in **figure 55**.

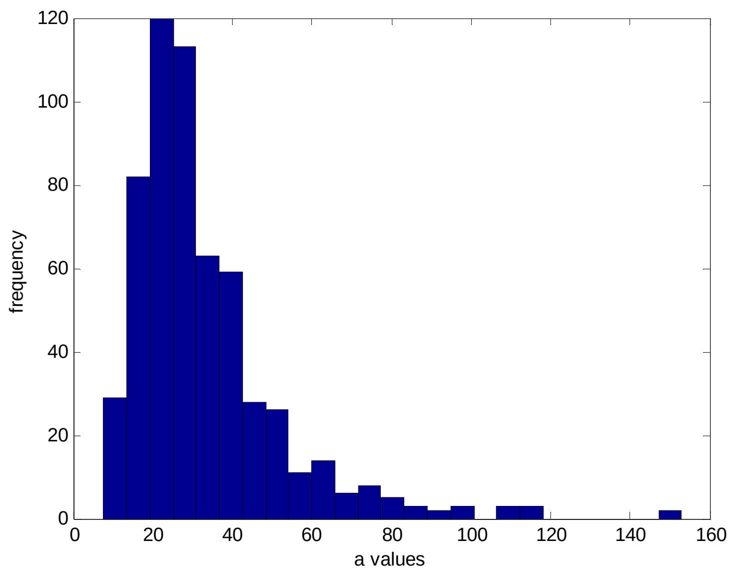

Figure 55 - Distribution of a parameter for the logistic growth model in the Barnett Shale

Figure 56 shows the distribution of the n parameter for the logistic growth model. The n parameter is normally distributed among the almost 600 wells analyzed in the Barnett Shale.

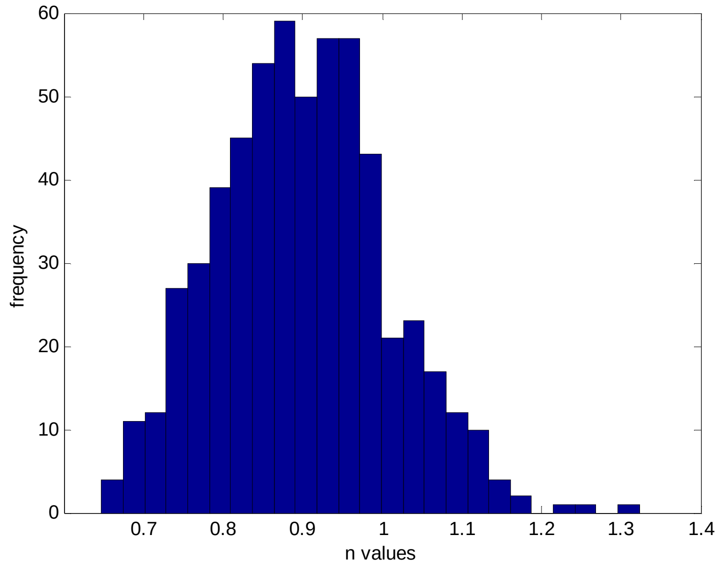

Figure 56 - Distribution of n parameter for the logistic growth model in the Barnett Shale

The tabulated results of the analysis can be seen in **Table 3**. The average carrying capacity of Barnett Shale wells is approximately 1.8 Bcf, with a standard deviation of 1.3 Bcf. The high standard deviation shows that the average well will recover around 1.8 Bcf, but the well has potential to be considerably higher or lower.

Table 3 - Results of statistical analysis of logistic growth model parameters

	mean	st. dev	min	median	max
K	1,782,382	1,309,691	139,360	1,386,216	9,102,227
n	0.90	0.11	0.65	0.90	1.32
a	33.07	19.37	7.66	27.63	152.98

The average value for the a parameter is around 33. In this particular case the time unit is in months. The a value implies that in the Barnett Shale the average well is expected to recover half its total gas in roughly 33 months. The producing life of these wells is expected to be on the order of 25 to 30 years, or around 400 months. If half the gas is produced in the first 33 months, it means that in less than one tenth of the life of the well, most of its gas will be produced. The trend has been observed in the Barnett, as many wells will begin producing with very high rates that decline very quickly, and then stabilize at low rates that decline much slower. The average n value obtained in the study was 0.90 with a standard deviation of 0.11. The range for the n values was very small with the minimum value obtained being 0.65 and the maximum being 1.32.

2.5.5 Determining Infill Drilling Potential with the Logistic Growth Model

The carrying capacity in the logistic growth model allows for it to be used in creative new ways. One unique aspect is a method developed for determining infill drilling potential. To do this, several assumptions must be made, specifically that the volumetrically recoverable oil or gas from a section is known ahead of time, that additional wells drilled in the section will behave identical to the original well in the early time in the section and that given enough time, a single well will be sufficient to drain the entire section.

With the assumptions in mind, assume that there is a section of land known to have a carrying capacity of 3 Bcf. A single well can be drilled in the center of the section, and it will eventually recover all of the gas in the section, given a long enough period of time. Another potential option for effectively draining the section would be to drill two evenly spaced well. In this case, each well will produce exactly half of the gas in the section, so the carrying capacity will be 1.5 Bcf/well. A third scenario would be to drill three evenly spaced wells in the section, and in this case the carrying capacity would be 1 Bcf/well for each of the 3 wells. An example of the three scenarios is in **figure 57**.

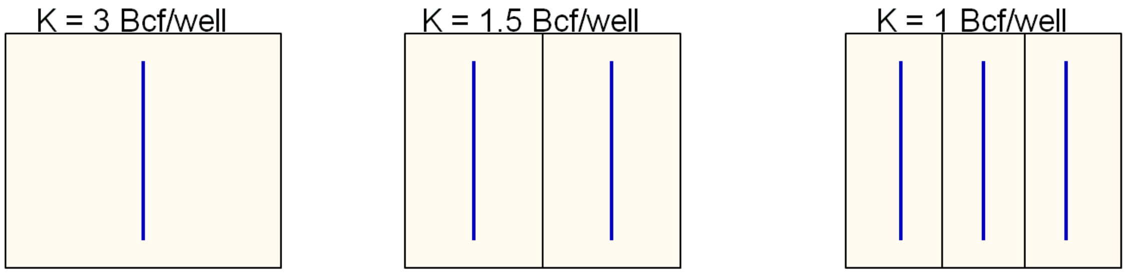

Figure 57 - Different scenarios for infill drilling

Because of the flexible nature of the logistic growth model, “good fits” to the data can be obtained with different carrying capacities. In other words, the solutions obtained are non-unique

If a single well has already been drilled in the section, and as assumed the additional wells will have the same production behavior in the early time as the original well, the carrying capacity can be varied, and good fits can be obtained to the data. It could then be assumed that the production declines observed for the various carrying capacities, would be the correct declines if a single well, two wells, or three wells were drilled in the section. **Figure 58** shows the production data for the first well being fit with the logistic model using the various K values.

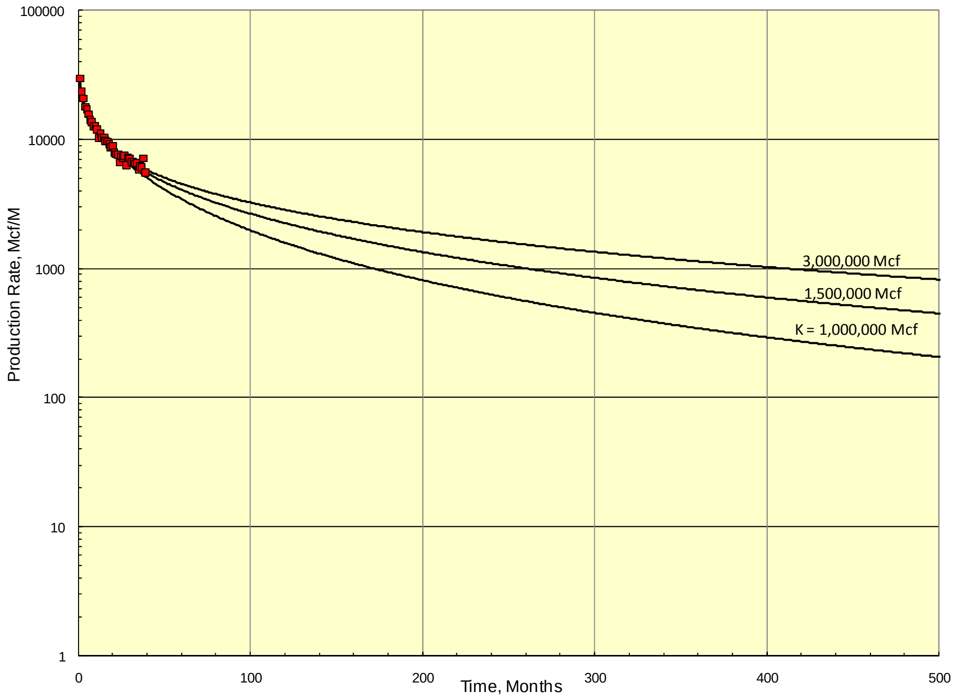

Figure 58 - Logistic model fit to production data using various carrying capacities

The lower the carrying capacity, the more quickly the well will decline. Economic indicators can be calculated for the different scenarios to determine the optimal number of wells to drill in the section. A very basic net present value (NPV) calculation was done in this example to determine the best scenario. The values used for determining the NPV can be seen in **table 4**, and the results of the NPV calculation can be seen in **table 5**.

Table 4 - Parameters used to determine net present value

Well Cost	Discount Rate	Gas Price	Operating Cost
$2,000,000	10%	4.00 $/Mcf	0.50 $/Mcf

Table 5 - Results of net present value calculation for various scenarios

NPV		
Single Well	Two Wells	Three Wells
$118,023	($109,367)	($613,174)

The well cost was assumed to be 2 million dollar per well. A discount rate of 10% was used, and the cost of gas was estimated at a constant 4.00 $/Mcf. For the case of two wells, or three wells, the NPV of a single well was calculated, and then doubled or tripled respectively to obtain the NPV. The results show that the only economic way to develop the section would be to drill a single well. Because of the marginal economics, and the high cost of drilling the wells, drilling additional wells in this scenario would result in a negative net present value, and an ultimate loss of money on development.

2.6 Conclusions

The research done reviews the history of decline curve analysis, and proposes a new model for forecasting reserves in extremely low permeability reservoirs The new model contains several distinct advantages over the existing Arps model, as well as over other the new models discussed. The logistic growth model for production forecasting uses the carrying capacity to help constrain forecasts to known physical volumes of available oil and gas in the reservoirs. Additionally, the carrying capacity causes the forecasted production rate to always terminate at infinite time.

Attempting to predict events that will occur in the future is at best an extremely uncertain practice. The actual amount of oil or gas that will be recovered from a well will never actually be known until the day that the well is plugged and abandoned. The logistic growth model provides an alternative model to forecast reserves that has been shown to accurately fit existing production data, and which provides reasonable predictions of how they will decline in the future.

Nomenclature

A	=	Area, acres
a	=	Constant
a	=	Exponential decline constant, time
a_i	=	Initial exponential decline constant at t=0, time
b	=	Constant
B	=	Formation volume factor, res bbl/STB
b	=	Hyperbolic decline exponent, unitless
$\bar{B}_o$	=	Average oil formation factor, res bbl/STB
$\bar{B}_w$	=	Average water formation factor, res bbl/STB
c	=	Constant of integration
C_A	=	Shape factor
c_f	=	Formation compressibility, 1/psi
c_o	=	Oil compressibility, 1/psi
c_t	=	Total system compressibility, 1/psi
c_w	=	Water compressibility, 1/psi
D_1	=	Decline constant after 1 time unit, 1/D
D_i	=	Initial decline constant, 1/time
$\hat{D}_i$	=	Decline constant defined by D_1/n, 1/D
D_∞	=	Decline constant at infinite time, 1/D
EUR	=	Estimated ultimate recovery, Mscf
h	=	Formation thickness, ft
J_0	=	Productivity index, bbl/(day-psia)
J_{oi}	=	Initial oil productivity index, bbl/(day-psi^{2n})

K	=	Carrying capacity
k	=	Permeability, md
K	=	Time at which liver has grown to half original size
L	=	Liver size
L_0	=	Original liver size before reduction (carrying capacity)
n	=	Time interval
n	=	Exponent of back-pressure curve
n	=	Exponential parameter
N	=	Population
n	=	Time exponent
N_0	=	Initial population
N_p	=	Cumulative oil production, bbl
N_{pi}	=	Cumulative oil production to shut-in reservoir pressure of 0, bbl
$\overline{p}$	=	Average reservoir pressure, psia
P_i	=	Initial reservoir pressure, psia
$\overline{p}_R$	=	Average reservoir pressure, psia
$\overline{p}_{Ri}$	=	Average initial reservoir pressure, psia
P_{wf}	=	Flowing bottomhole pressure, psia
Q	=	Cumulative production
q	=	Production rate
$\hat{q}_i$	=	Rate intercept or q(t=0), Mscf/D
$(q_i)_{max}$	=	Maximum production rate, bbl/day
q_0	=	Peak production rate, Mscf/month
Q_D	=	Dimensionless cumulative production
q_D	=	Dimensionless production rate

q_i	=	Initial production rate, volume/time
q_o	=	Oil production rate, bbl/day
q_{oi}	=	Initial oil production rate, bbl/day
q_{osc}	=	Production rate at standard conditions, rb/day
r	=	Ratio of production rate over change in production rate
r	=	Constant
r_e	=	Radius of entire reservoir, ft
rp	=	Recovery potential, dimensionless
r_w	=	Wellbore diameter, ft
s	=	Skin factor
$\bar{S}_o$	=	Average oil saturation
$\bar{S}_w$	=	Average water saturation
t	=	time
V_p	=	Pore Volume, bbl
y	=	Production rate, volume/time
α	=	Exponential constant
β	=	Exponential constant
γ	=	Exponential constant
τ	=	Characteristic time parameter, month
ϕ	=	Porosity, fraction
μ	=	Viscosity, cp
μ_o	=	Oil viscosity, cp

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