

بسم

۱۰۰ فرمول مثلثات

$$۱) \sin^2 \alpha + \cos^2 \alpha = 1$$

$$۲) \sin \alpha = \sqrt{1 - \cos^2 \alpha}$$

$$۳) \cos \alpha = \sqrt{1 - \sin^2 \alpha}$$

$$۴) \tan \alpha \cdot \cot \alpha = 1$$

$$۵) \tan \alpha = \frac{1}{\cot \alpha}$$

$$۶) \cot \alpha = \frac{1}{\tan \alpha}$$

$$۷) \tan \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$۸) \cot \alpha = \frac{\cos \alpha}{\sin \alpha}$$

$$۹) \sec \alpha = \frac{1}{\cos \alpha}$$

$$۱۰) \csc \alpha = \frac{1}{\sin \alpha}$$

$$11) \quad 1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha}$$

$$12) \quad \cos^2 \alpha = \frac{1}{1 + \tan^2 \alpha}$$

$$13) \quad \cos^2 \alpha = \frac{\cot^2 \alpha}{1 + \cot^2 \alpha}$$

$$14) \quad 1 + \cot^2 \alpha = \frac{1}{\sin^2 \alpha}$$

$$15) \quad \sin^2 \alpha = \frac{1}{1 + \cot^2 \alpha}$$

$$16) \quad \sin^2 \alpha = \frac{\tan^2 \alpha}{1 + \tan^2 \alpha}$$

$$17) \quad \sin^3 \alpha + \cos^3 \alpha = (\sin \alpha + \cos \alpha) (1 - \sin \alpha \cos \alpha)$$

$$18) \quad \sin^3 \alpha - \cos^3 \alpha = (\sin \alpha - \cos \alpha) (1 + \sin \alpha \cos \alpha)$$

$$19) \quad \sin^4 \alpha + \cos^4 \alpha = 1 - 2 \sin^2 \alpha \cdot \cos^2 \alpha$$

$$20) \quad \sin^4 \alpha + \cos^4 \alpha = 1 - \frac{1}{2} \sin^2 2\alpha$$

$$21) \quad \sin^6 \alpha + \cos^6 \alpha = 1 - 3 \sin^2 \alpha \cdot \cos^2 \alpha$$

$$22) \quad \sin^6 \alpha + \cos^6 \alpha = 1 - \frac{3}{4} \sin^2 2\alpha$$

$$\textbf{۲۳)} \quad \sin(\alpha + \beta) = \sin \alpha . \cos \beta + \cos \alpha . \sin \beta$$

$$\textbf{۲۴)} \quad \sin(\alpha - \beta) = \sin \alpha . \cos \beta - \cos \alpha . \sin \beta$$

$$\textbf{۲۵)} \quad \cos(\alpha + \beta) = \cos \alpha . \cos \beta - \sin \alpha . \sin \beta$$

$$\textbf{۲۶)} \quad \cos(\alpha - \beta) = \cos \alpha . \cos \beta + \sin \alpha . \sin \beta$$

$$\textbf{۲۷)} \quad \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha . \tan \beta}$$

$$\textbf{۲۸)} \quad \tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha . \tan \beta}$$

$$\textbf{۲۹)} \quad \tan\left(\frac{\pi}{4} + \alpha\right) = \frac{1 + \tan \alpha}{1 - \tan \alpha}$$

$$\textbf{۳۰)} \quad \tan\left(\frac{\pi}{4} - \alpha\right) = \frac{1 - \tan \alpha}{1 + \tan \alpha}$$

$$\textbf{۳۱)} \quad \cot(\alpha + \beta) = \frac{\cot \alpha . \cot \beta - 1}{\cot \alpha + \cot \beta}$$

$$\textbf{۳۲)} \quad \cot(\alpha - \beta) = \frac{\cot \alpha . \cot \beta + 1}{\cot \alpha - \cot \beta}$$

$$۳۳) \sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$۳۴) \sin 2\alpha = \frac{2 \tan \alpha}{1 + \tan^2 \alpha}$$

$$۳۵) \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$۳۶) \cos 2\alpha = \cos^4 \alpha - \sin^4 \alpha$$

$$۳۷) \cos 2\alpha = 2\cos^2 \alpha - 1$$

$$۳۸) \cos 2\alpha = 1 - 2\sin^2 \alpha$$

$$۳۹) \cos 2\alpha = \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha}$$

$$۴۰) \tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$۴۱) \cot 2\alpha = \frac{\cot^2 \alpha - 1}{2 \cot \alpha}$$

$$۴۲) \sin^2 \alpha = \frac{1 - \cos 2\alpha}{2}$$

$$۴۳) \cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$$

$$۴۴) \tan^2 \alpha = \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha}$$

$$۴۵) \tan \alpha = \frac{1 - \cos 2\alpha}{\sin 2\alpha}$$

$$۴۶) \tan \alpha = \frac{\sin 2\alpha}{1 + \cos 2\alpha}$$

$$\textcircled{27}) \sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$$

$$\textcircled{28}) \cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$$

$$\textcircled{29}) \cos 4\alpha = 1 - 8 \sin^2 \alpha \cdot \cos^2 \alpha$$

$$\textcircled{30}) \cos 4\alpha = 8 \sin^4 \alpha - 8 \sin^2 \alpha + 1$$

$$\textcircled{31}) \tan 3\alpha = \frac{3 \tan \alpha - \tan^3 \alpha}{1 - 3 \tan^2 \alpha}$$

$$\textcircled{32}) \cot 3\alpha = \frac{3 \cot \alpha - \cot^3 \alpha}{1 - 3 \cot^2 \alpha}$$

$$\textcircled{33}) \cot \alpha - \tan \alpha = 2 \cot 2\alpha$$

$$\textcircled{34}) \tan \alpha + \cot \alpha = \frac{1}{\sin \alpha \cdot \cos \alpha}$$

$$\textcircled{35}) \tan \alpha + \cot \alpha = \frac{2}{\sin 2\alpha}$$

$$\textcircled{36}) \sin \alpha \cdot \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\textcircled{37}) \cos \alpha \cdot \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

$$\textcircled{38}) \sin \alpha \cdot \sin \beta = \frac{-1}{2} [\cos(\alpha + \beta) - \cos(\alpha - \beta)]$$

$$\textcircled{39}) \sin \alpha \cdot \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$٩٠) \sin \alpha + \cos \alpha = \sqrt{2} \sin \left(\frac{\pi}{4} + \alpha \right)$$

$$٩١) \sin \alpha + \cos \alpha = \sqrt{2} \cos \left(\frac{\pi}{4} - \alpha \right)$$

$$٩٢) \cos \alpha - \sin \alpha = \sqrt{2} \sin \left(\frac{\pi}{4} - \alpha \right)$$

$$٩٣) \cos \alpha - \sin \alpha = \sqrt{2} \cos \left(\frac{\pi}{4} + \alpha \right)$$

$$٩٤) (\sin \alpha + \cos \alpha)^2 = 1 + 2 \sin \alpha \cos \alpha$$

$$٩٥) (\sin \alpha + \cos \alpha)^2 = 1 + \sin 2\alpha$$

$$٩٦) (\sin \alpha - \cos \alpha)^2 = 1 - 2 \sin \alpha \cos \alpha$$

$$٩٧) (\sin \alpha + \cos \alpha)^2 = 1 - \sin 2\alpha$$

$$٩٨) \sin^2 \alpha - \sin^2 \beta = \sin(\alpha + \beta) \cdot \sin(\alpha - \beta)$$

$$٩٩) \cos^2 \alpha - \cos^2 \beta = -\sin(\alpha + \beta) \cdot \sin(\alpha - \beta)$$

$$١٠٠) \cos^2 \alpha - \sin^2 \beta = \cos(\alpha + \beta) \cdot \cos(\alpha - \beta)$$

$$\text{११)} \quad \sin p + \sin q = 2 \sin \frac{p+q}{2} \cdot \cos \frac{p-q}{2}$$

$$\text{१२)} \quad \sin p - \sin q = 2 \sin \frac{p-q}{2} \cdot \cos \frac{p+q}{2}$$

$$\text{१३)} \quad \cos p + \cos q = 2 \cos \frac{p+q}{2} \cdot \cos \frac{p-q}{2}$$

$$\text{१४)} \quad \cos p - \cos q = -2 \sin \frac{p-q}{2} \cdot \sin \frac{p+q}{2}$$

$$\text{१५)} \quad \tan p + \tan q = \frac{\sin(p+q)}{\cos p \cdot \cos q}$$

$$\text{१६)} \quad \tan p - \tan q = \frac{\sin(p-q)}{\cos p \cdot \cos q}$$

$$\text{१७)} \quad \cot p + \cot q = \frac{\sin(p+q)}{\sin p \cdot \sin q}$$

$$\text{१८)} \quad \cot p - \cot q = \frac{\sin(p-q)}{\sin p \cdot \sin q}$$

$$79) \sin^{-1}(-x) = -\sin^{-1} x$$

$$80) \cos^{-1}(-x) = \pi - \cos^{-1} x$$

$$81) \tan^{-1}(-x) = -\tan^{-1} x$$

$$82) \cot^{-1}(-x) = \pi - \cot^{-1} x$$

$$83) \sin(\sin^{-1} x) = x$$

$$84) \cos(\cos^{-1} x) = x$$

$$85) \tan(\tan^{-1} x) = x$$

$$86) \cot(\cot^{-1} x) = x$$

$$87) \sin(\cos^{-1} x) = \cos(\sin^{-1} x) = \sqrt{1-x^2}$$

$$88) \tan(\cot^{-1} x) = \cot(\tan^{-1} x) = \frac{1}{x}$$

$$89) \sin(\tan^{-1} x) = \frac{x}{\sqrt{1+x^2}}$$

$$90) \cos(\tan^{-1} x) = \frac{1}{\sqrt{x^2+1}}$$

$$91) \tan^{-1} x = \cot^{-1} \left(\frac{1}{x} \right)$$

$$92) \cot^{-1} x = \tan^{-1} \left(\frac{1}{x} \right)$$

$$93) \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$94) \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$$

$$95) \tan^{-1} x + \tan^{-1} \left(\frac{1}{x} \right) = \frac{\pi}{2}$$

$$96) \cot^{-1} x + \cot^{-1} \left(\frac{1}{x} \right) = \frac{\pi}{2}$$

$$97) \tan(\tan^{-1} x + \tan^{-1} y) = \frac{x+y}{1-xy}$$

$$98) \tan(\tan^{-1} x - \tan^{-1} y) = \frac{x-y}{1+xy}$$

$$99) \sin(2 \sin^{-1} x) = 2x \sqrt{1-x^2}$$

$$100) \sin(2 \cos^{-1} x) = 2x \sqrt{1-x^2}$$