Deep Q-Learning hyper-parameters optimization for portfolio management

CS 229 - Project

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Introduction

- The proposed framework is to utlize machine learning methods to tune portfolio optimization parameters.
- Bayesian Optimization (Gaussian Process) and Deep Q-Learning are considered for the purpose of this research.
- The tax-managed equity portfolio optimization will be studied as the underlying system.

The Tax-Managed portfolio optimization

- The tax-managed portfolio optimization aims to track the return of a benchmark while minimizing the tax impact.

$$\underset{w^p}{\text{minimize}} \quad TE^2 + \sum_{i=1}^{M} (w_i{}^0 - w_i) TR_i (1 - \frac{c_i}{p_i})$$

S.t:

$$LB \leq \mathbf{w_A} \leq UB \quad \text{Assets bound constraints}$$

$$LB^f \leq \mathbf{w'_A B_f} \leq UB^f \quad \text{Factor f exposure constraints}$$

$$\sum_{k=1}^{m} w_k = w_i \quad \text{sum of (m) lots weights to asset i weight}$$

$$\mathbf{w_p} \geq 0 \quad \text{long-only constraints}$$

$$\sum q_i \leq N \quad \text{max positions (N) constraint}$$

The Tax-Managed portfolio optimization - Cont'd

- The mixed integer quadratic optimization goal is to minimize tax cost in addition to the square of tracking error :

$$TE^2 = \mathbf{w}_\mathbf{A}' \mathbf{B} \mathbf{\Omega}_F \mathbf{B}' \mathbf{w}_\mathbf{A} + \mathbf{w}_\mathbf{A}' \mathbf{D} \mathbf{w}_\mathbf{A}$$

- The continuous Decision variables:
  - $w_j{}^p$ Portfolio weights (at asset level)
  - w_i lot i weight
- The Binary Decision variables:
  - $q_i = 1$ if $w_j{}^p > 0$
- Other Variables/Constants:
  - $w_i{}^0$ initial lot i weight
  - w_i final lot i weight
  - TR_i Tax Rate (long term or short term)
  - c_i cost basis (asset price at purchase)
  - p_i current price (current asset price)

The Tax-Managed portfolio optimization - Cont'd

- Other Variables/Constants (cont'd):
  - $\mathbf{w_A}$ Active weights (vector of $w_j{}^p - w_j{}^b$)
  - $\mathbf{\Omega}_F$ Factors Covariance Matrix
  - $\mathbf{B}$ Factors Loadings/Exposures Matrix
- the factors covariance matrix and exposures can be estimated using Principle components analysis of historical stocks returns
- we also can use the quandl covariance matrix. In this case we won't need to use factor analysis altogether.

Bayesian Optimization

Deep Q-Learning

References