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Electric Power Systems Research 78 (2008) 605–617
ELECTRIC POWER SYSTEMS RESEARCH
www.elsevier.com/locate/epsr

A new approach for three-phase loads compensation based on the instantaneous reactive power theory

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Received 6 July 2006; received in revised form 8 March 2007; accepted 9 May 2007
Available online 27 June 2007

Abstract

The original instantaneous reactive power theory or p–q theory has been systematically used in the control of active power filters (APFs). When the APF is connected in parallel to a non-linear and unbalanced load, the p–q theory application has allowed a compensation strategy named constant power to be obtained. This means that, after the APF connection, the instantaneous power supplied by the source is constant and it has the same value as the average power consumed by the load. Nevertheless, the use of other compensation strategies is possible: unity power factor or sinusoidal and balanced supply currents, among others. This paper shows that any compensation strategy may be developed into the p–q theory frame. Besides, the paper presents a p–q theory reformulation without using mapping matrices, which makes easier the obtention of the compensation currents. Finally, an exhaustive analysis of practical cases has been carried out at simulation and experimental level through a laboratory prototype which has allowed the proposed approach to be verified.

Keywords: Active power filters; Harmonics; Instantaneous reactive power; Power quality; Three-phase systems

1. Introduction

The integration of power electronics in the commercial and industrial processes has originated a considerable increase of non-linear currents. The problems associated with these distorted currents have made of harmonics compensation a main task. There are different equipments to achieve the non-linear currents compensation. One of the most used currently is the active power filter (APF) [1–5]. An APF is a power electronics converter which supplies the non-useful current required by the load, making possible that the source only supplies the useful power required by the load. The active power filter control is based on the instantaneous reactive power theory or other formulations later proposed [6–15].

Akagi et al. [6] introduced the instantaneous reactive power or p–q theory in the control of the APFs. With this formulation, its authors got to design an APF control circuit in an easy way. In this theory approach, the more ambitious objective is achieved when the instantaneous power supplied by the source is constant and with the same value as the average power consumed by the load. This has been named constant power compensation. If the source voltage is balanced and sinusoidal, the constant power compensation allows a balanced and sinusoidal source current to be obtained. If the source voltage is non-sinusoidal and/or unbalanced, the current will not be balanced and sinusoidal after compensation. This point has been one of the aspects more criticized of the p–q theory. Nevertheless, certainly, the p–q theory has been the most used till nowadays in the APFs control.

This paper demonstrates that, with a reformulation of the p–q theory, all the compensation objectives can be obtained: unity power factor compensation or balanced and sinusoidal source current strategy [7]. And this is true for any source voltage condition.

The p–q theory has been formulated from its origin referring the voltage and current variables in the $0\alpha\beta$ reference system obtained after applying the Clark transformation [8]. So, the compensation currents are obtained by means of successive application of different mapping matrices. This makes truly hard the mathematical analysis when other control strategies are applied.

In this paper, a p–q theory reformulation has been carried out. It avoids the mapping matrices, and it makes use of the vectorial

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doi:10.1016/j.epsr.2007.05.003

representation, which simplifies the obtention of compensation currents.

Finally, the different compensation strategies have been applied to a practical case, which has allowed the proposed approach to be verified.

The paper is organized as follows. In Section 2, the p–q theory is summarized and a new formulation, without using mapping matrices, is presented. Section 3 presents the way of obtaining the compensation currents in the p–q theory reference frame according to the strategy denominated constant power. In Section 4, the procedure to obtain the compensation currents with the compensation objective named unity power factor is established. In Section 5, a similar development with the compensation objective of obtaining balanced and sinusoidal source currents is presented. The two last control strategies have not been developed up now in the p–q original theory frame. Finally, Section 6 presents the simulation results corresponding to the three compensation strategies applied to a practical case and Section 7 shows the results obtained by applying the three strategies to an experimental laboratory prototype.

2. Original p–q theory

The instantaneous reactive power theory was formulated at the beginning of the eighties [6], and this is the formulation with the largest diffusion along these years. For that reason, it is the most used as control strategy in the APF. The p–q theory was developed for three-phase three-wire systems with balanced and sinusoidal source voltages. The compensation objective assumed was the obtention of a constant instantaneous source power.

The theory is based on a translation from the phase reference system (123) to the $0\alpha\beta$ system, Fig. 1.

The transformation matrix associated is as follows:

$$\begin{bmatrix} e_0 \\ e_\alpha \\ e_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{1}$$

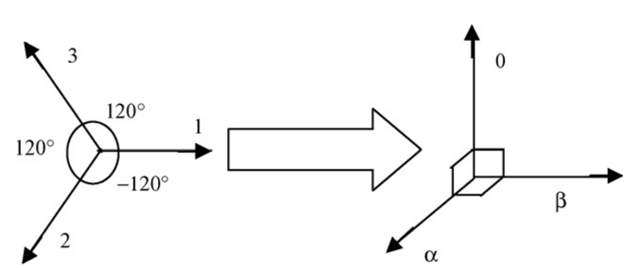


Fig. 1. $0\alpha\beta$ referente system.

$$\begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \tag{2}$$

From Eqs. (1) and (2), it can be deduced that

$$i_N = i_1 + i_2 + i_3 = i_0\sqrt{3} \tag{3}$$

The different power terms are defined as follows:

$$\begin{bmatrix} p_0 \\ p_{\alpha\beta} \\ q_{\alpha\beta} \end{bmatrix} = \begin{bmatrix} e_0 & 0 & 0 \\ 0 & e_\alpha & e_\beta \\ 0 & -e_\beta & e_\alpha \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = [T] \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} \tag{4}$$

where p_0 is the zero sequence (real) instantaneous power, $p_{\alpha\beta}$ the $\alpha\beta$ real instantaneous power and $q_{\alpha\beta}$ is the imaginary instantaneous power.

Considering the $[T]$ inverse matrix, the calculation of the current components from the different power terms is possible. The expression is shown in the next equation:

$$\begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = \frac{1}{e_0 e_{\alpha\beta}^2} \begin{bmatrix} e_{\alpha\beta}^2 & 0 & 0 \\ 0 & e_0 e_\alpha & -e_0 e_\beta \\ 0 & e_0 e_\beta & e_0 e_\alpha \end{bmatrix} \begin{bmatrix} p_0 \\ p_{\alpha\beta} \\ q_{\alpha\beta} \end{bmatrix} \tag{5}$$

where $e_{\alpha\beta}^2 = e_\alpha^2 + e_\beta^2$

Although original p–q theory is developed from the mapping matrices (4) and (5), an alternative develop is possible. This new model does not use any mapping matrices. Therefore, it makes easier the theory treatment and its application to the active filters control. In fact, the vectorial frame allows to obtain, in a simple way, any compensation strategy, while the mapping matrices have been always limited to the constant power strategy. Actually, the vectorial frame is a conceptual approach. So, three new voltage vector will be defined, $\vec{e}_{0\alpha\beta} : \vec{e}_0, \vec{e}_{\alpha\beta}$ and $\vec{e}_{-\beta\alpha}$, Fig. 2.

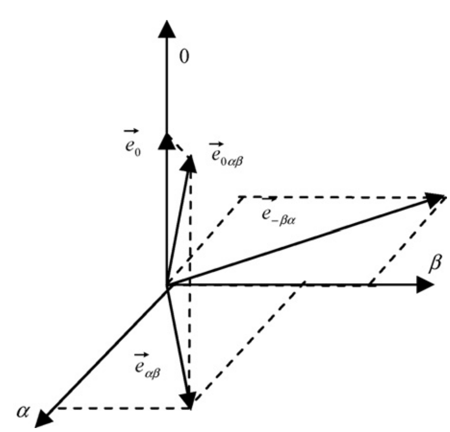


Fig. 2. New voltage vectors in $0\alpha\beta$ coordinates system.

The voltage and current space vectors are defined as follows:

$$\vec{e}_{\alpha\beta}=\begin{bmatrix}0\\ e_\alpha\\ e_\beta\end{bmatrix};\qquad \vec{e}_{-\beta\alpha}=\begin{bmatrix}0\\ -e_\beta\\ e_\alpha\end{bmatrix};\qquad \vec{v}=\vec{e}_0=\begin{bmatrix}e_0\\ 0\\ 0\end{bmatrix};$$

$$\vec{i}=\begin{bmatrix}i_0\\ i_\alpha\\ i_\beta\end{bmatrix}\tag{6}$$

where $\vec{e}_{-\beta\alpha}$ is the orthogonal voltage vector and $\vec{e}_0$ is the zero sequence voltage vector. It is always verified that

$$\vec{e}_{0\alpha\beta}=[\,e_0\quad e_\alpha\quad e_\beta\,]^t=\vec{e}_{\alpha\beta}+\vec{e}_0\tag{7}$$

and

$$\vec{e}_{-\beta\alpha}\cdot\vec{e}_{0\alpha\beta}=0\tag{8}$$

In the $0\alpha\beta$ reference frame, the current vector $\vec{i}$ may be expressed as the sum of its projections over the vectors $\vec{e}_0$, $\vec{e}_{\alpha\beta}$ and $\vec{e}_{-\beta\alpha}$, that is

$$\begin{aligned}\vec{i}&=\frac{p_{\alpha\beta}(t)}{\vec{e}_{\alpha\beta}\cdot\vec{e}_{\alpha\beta}}\vec{e}_{\alpha\beta}+\frac{q_{\alpha\beta}(t)}{\vec{e}_{-\beta\alpha}\cdot\vec{e}_{-\beta\alpha}}\vec{e}_{-\beta\alpha}+\frac{p_0(t)}{\vec{e}_0\cdot\vec{e}_0}\vec{e}_0\\ &=\frac{p_{\alpha\beta}(t)}{\vec{e}_{\alpha\beta}\cdot\vec{e}_{\alpha\beta}}\vec{e}_{\alpha\beta}+\frac{q_{\alpha\beta}(t)}{\vec{e}_{\alpha\beta}\cdot\vec{e}_{\alpha\beta}}\vec{e}_{-\beta\alpha}+\frac{p_0(t)}{\vec{e}_0\cdot\vec{e}_0}\vec{e}_0\end{aligned}\tag{9}$$

where $p_{\alpha\beta}=\vec{e}_{\alpha\beta}\cdot\vec{i}$ is the instantaneous real power in α–β components, $q_{\alpha\beta}=\vec{e}_{-\beta\alpha}\cdot\vec{i}$ the instantaneous imaginary power and $p_0(t)$ is the zero sequence instantaneous real power. They are identical to those power terms defined in Eq. (4). The fact that the orthogonal voltage vector norm and the voltage vector without zero-sequence component norm are the same has been considered in Eq. (9). Since now the compensation currents will be obtained from several strategies within the model presented in the Eq. (9), without using mapping matrices.

3. Constant power compensation

The strategy assumed by the original or p–q theory since its origin has been the obtention of a constant instantaneous power in the source side with the only restriction of getting a null average instantaneous power exchanged by the compensator, P_c. Along the paper, lower case represents instantaneous values, upper case average values, subindex "L" load requirement, subindex "C" compensator supply and subindex "S" source supply.

In fact, total power required by the load can be expressed as follows:

$$p_L(t)=p_{L\alpha\beta}(t)+p_{L0}(t)=P_{L\alpha\beta}+\tilde{p}_{L\alpha\beta}(t)+P_{L0}+\tilde{p}_{L0}(t)\tag{10}$$

where uppercase are referred to the average values and the terms with the character ∼ over it are referred to the power oscillatory component.

To calculate the compensator current, and according to Fig. 3, it is verified that

$$p_C(t)=p_L(t)-p_S(t)=p_L(t)-P_L\tag{11}$$

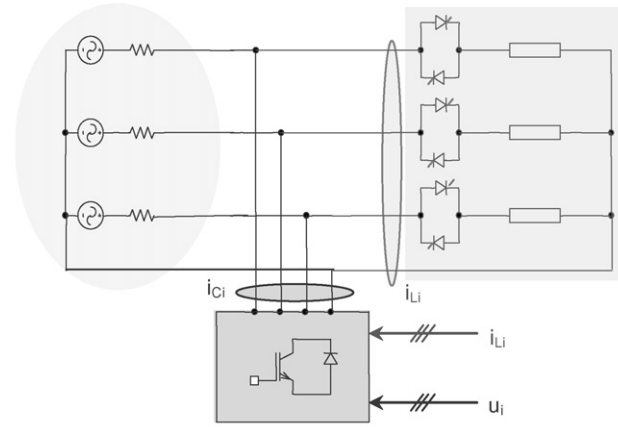


Fig. 3. Power system compensated by an active power line conditioner.

where P_L is the total active power incoming to the load. $p_S(t)=P_L$ after compensation. Taking into account Eq. (10), and the independence of $0\alpha\beta$ coordinates, the Eq. (11) can be expressed in the next way:

$$p_{C\alpha\beta}(t)=p_{L\alpha\beta}(t)-P_{L\alpha\beta}=\tilde{p}_{L\alpha\beta}(t)\tag{12}$$

$$p_{C0}(t)=p_{L0}(t)-P_{L0}=\tilde{p}_{L0}(t)\tag{13}$$

These equations, effectively, means that the average value $\langle p_C(t)\rangle=P_C=0$, where $\tilde{p}_L(t)$ represents the ac or oscillatory part of $p_L(t)$.

On the other hand, the instantaneous imaginary power exchanged by the compensator must be the same as the instantaneous imaginary power required by the load:

$$q_C(t)=q_{L\alpha\beta}(t)\tag{14}$$

Therefore, from Eq. (5), the compensation current is

$$\begin{bmatrix}i_{C0}\\ i_{C\alpha}\\ i_{C\beta}\end{bmatrix}=\frac{1}{e_0e_{\alpha\beta}^2}\begin{bmatrix}e_{\alpha\beta}^2 & 0 & 0\\ 0 & e_0e_\alpha & -e_0e_\beta\\ 0 & e_0e_\beta & e_0e_\alpha\end{bmatrix}\begin{bmatrix}\tilde{p}_{L0}(t)\\ \tilde{p}_{L\alpha\beta}(t)\\ q_{L\alpha\beta}(t)\end{bmatrix}\tag{15}$$

This strategy achieves a constant instantaneous power supplied by the source. Nevertheless, it does not eliminate the neutral current.

After compensation, the source supplies a constant instantaneous power with a value identical to the load active power with a neutral current not null.

The results presented in Eq. (15) have been got by mean of the mapping matrices introduced by Akagi et al. [6].

In the new model of the p–q theory, without the use of mapping matrices, the constant power compensation strategy can be developed. Besides, the elimination of the neutral current is possible [9]. The corresponding procedure is presented now.

After compensation, the source current $\vec{i}_S(t)$ is the next:

$$\vec{i}_S(t)=K\vec{e}_{\alpha\beta}\tag{16}$$

Taking into account that $\vec{e}_0\cdot\vec{e}_{\alpha\beta}=0$, from Eq. (6) to (7), the source total instantaneous real power must be as follows:

$$p_S(t)=\vec{e}_{0\alpha\beta}\cdot\vec{i}_S=K\vec{e}_{0\alpha\beta}\cdot\vec{e}_{\alpha\beta}=K\vec{e}_{\alpha\beta}\cdot\vec{e}_{\alpha\beta}=P_L\tag{17}$$

Imposing that $p_S(t) = P_L$, the proportionality factor can be obtained and it gets the next value:

$$K = \frac{P_L}{e_{\alpha\beta}^2} \quad (18)$$

Therefore, from Eqs. (9) and (18), the compensation currents may be expressed like a vector as follows:

$$\begin{aligned}\vec{i}_C(t) &= \vec{i}_L(t) - i_S(t)\\ &= \frac{p_{\alpha\beta}(t)}{e_{\alpha\beta}^2}\vec{e}_{\alpha\beta} - \frac{P_L}{e_{\alpha\beta}^2}\vec{e}_{\alpha\beta} + \frac{p_{L0}(t)}{e_0^2}\vec{e}_0 + \frac{q_{L\alpha\beta}}{e_{\alpha\beta}^2}\vec{e}_{-\beta\alpha}\\ &= \frac{\tilde{p}_{\alpha\beta}(t) - P_{L0}}{e_{\alpha\beta}^2}\vec{e}_{\alpha\beta} + \frac{p_{L0}(t)}{e_0^2}\vec{e}_0 + \frac{q_{L\alpha\beta}}{e_{\alpha\beta}^2}\vec{e}_{-\beta\alpha} \quad (19)\end{aligned}$$

where $\tilde{p}_{\alpha\beta}(t)$ represents the oscillatory part of the instantaneous real power in the α–β plane.

From Eq. (19), such component of the source current is as follows:

$$i_{c0} = \frac{p_{L0}(t)}{e_0^2}e_0 \quad (20)$$

$$\begin{aligned}i_{c\alpha} &= \frac{\tilde{p}_{L\alpha\beta}(t) - P_{L0}}{e_{\alpha\beta}^2}e_\alpha - \frac{q_{L\alpha\beta}}{e_{\alpha\beta}^2}e_\beta,\\ i_{c\beta} &= \frac{\tilde{p}_{L\alpha\beta}(t) - P_{L0}}{e_{\alpha\beta}^2}e_\beta + \frac{q_{L\alpha\beta}}{e_{\alpha\beta}^2}e_\alpha \quad (21)\end{aligned}$$

4. Unity power factor compensation

The unity power factor compensation is a strategy widely used along the time. The target is to obtain source currents with the same distortion and symmetry condition as source voltages. It means that the source current is collinear to the supply voltage. In this situation, the source supplies the load active power, but the instantaneous real power is not constant after compensation [7].

When the source voltages are sinusoidal and balanced, constant power and unity power factor compensations results are the same. In fact, for sinusoidal and balanced source voltages, if compensation currents cancel reactive, distortion and unbalanced current components, source currents will be sinusoidal and balanced in the same way as voltage. And for that reason, instantaneous power supplied by the source will be constant. However, when the supply voltage is unbalanced and non-sinusoidal the situation won't be the same.

Since now, a control strategy will be obtained. It is based on the p–q theory. This new strategy allows unity power factor in the source side to be obtained after compensation:

$$\vec{i}_s = G_e\vec{e}_{0\alpha\beta} \quad (22)$$

Taking into account Eqs. (11) and (22) and to satisfy that the active power exchanged by the compensator is null:

$$p_C(t) = p_L(t) - p_S(t) = p_L(t) - G_e e^2(t) \quad (23)$$

Thus, knowing that $\langle p_C(t)\rangle = P_C = 0$ the proportionality constant is as follows:

$$G_e = \frac{P_L}{E^2} \quad (24)$$

since $\langle p_L(t)\rangle = P_L$ and where E^2 is the square root of voltage vector RMS value:

$$E^2 = \frac{1}{T}\int_T e_{0\alpha\beta}^2\,\mathrm{d}t \quad (25)$$

Finally, after compensation:

$$\begin{bmatrix} i_{S0}\\ i_{S\alpha}\\ i_{S\beta}\end{bmatrix} = \frac{P_L}{E^2}\begin{bmatrix} e_0\\ e_\alpha\\ e_\beta\end{bmatrix} \quad (26)$$

From Eq. (11) and considering the reference axes, compensation power can be expressed as follows:

$$p_{C0}(t) = p_{L0}(t) - p_{S0}(t) = p_{L0}(t) - \frac{P_L}{E^2}e_0^2 \quad (27)$$

$$p_{C\alpha\beta}(t) = p_{L\alpha\beta}(t) - p_{S\alpha\beta}(t) = p_{L\alpha\beta}(t) - \frac{P_L}{E^2}e_{\alpha\beta}^2 \quad (28)$$

Besides, instantaneous imaginary power verifies:

$$q_C(t) = q_{L\alpha\beta}(t) \quad (29)$$

So,

$$\begin{aligned}\vec{i}_C = \vec{i}_L - \vec{i}_S &= \frac{p_{L\alpha\beta}(t)}{e_{\alpha\beta}^2}\vec{e}_{\alpha\beta} - \frac{P_{L\alpha\beta}}{E_{\alpha\beta}^2}\vec{e}_{\alpha\beta} + \frac{p_0(t)}{e_0^2}\vec{e}_0 - \frac{P_L}{E_0^2}\vec{e}_0\\ &\quad + \frac{q_{L\alpha\beta}(t)}{e_{\alpha\beta}^2}\vec{e}_{-\beta\alpha} \quad (30)\end{aligned}$$

In this way, the explicit expressions of the compensation currents in the 0–α–β coordinates are as follows:

$$\begin{aligned}i_{C0} &= \left(\frac{p_0(t)}{e_0^2} - \frac{P_L}{E_0^2}\right)e_0,\\ i_{C\alpha} &= \left(\frac{p_{L\alpha\beta}(t)}{e_{\alpha\beta}^2} - \frac{P_{L\alpha\beta}}{E_{\alpha\beta}^2}\right)e_\alpha - \frac{q_{L\alpha\beta}(t)}{e_{\alpha\beta}^2}e_\beta,\\ i_{C\beta} &= \left(\frac{p_{L\alpha\beta}(t)}{e_{\alpha\beta}^2} - \frac{P_{L\alpha\beta}}{E_{\alpha\beta}^2}\right)e_\beta + \frac{q_{L\alpha\beta}(t)}{e_{\alpha\beta}^2}e_\alpha \quad (31)\end{aligned}$$

This control algorithm does not permit to eliminate the neutral current, since:

$$i_{S0} = i_{L0} - i_{C0} = \frac{p_0(t)}{e_0^2}e_0 - \frac{p_0(t)}{e_0^2}e_0 + \frac{P_L}{E_0^2}e_0 = \frac{P_L}{E_0^2}e_0 \quad (32)$$

This expression will be different of zero if zero-sequence voltage component exists.

5. Sinusoidal and balanced source current compensation

For last years, both compensation objectives presented in this paper have not been enough to solve the new problems associated to the voltage and current distortion. It is necessary to define a new compensation objective. It is the sinusoidal source current compensation strategy. The aim is to obtain a sinusoidal and balanced source current, in phase with voltage [11–13], when the voltage is sinusoidal and balanced or in phase with voltage positive sequence fundamental component in any other case. It must be fulfilled in any supply voltage conditions and load kind.

If voltages are balanced and sinusoidal, currents proposed will be proportional to the voltages (as in previous strategy). The proportionality constant must have the value established in the Eqs. (24) and (25), too.

If voltages are balanced non-sinusoidal, the reference source current must be proportional to its fundamental component. The proportionality constant is fixed to satisfy the restriction of having null active power exchanged by the compensator. The ideal current expression is as follows [12,13]:

$$\begin{bmatrix} i_{S0} \\ i_{S\alpha} \\ i_{S\beta} \end{bmatrix} = \frac{P_L}{E_1^2} \begin{bmatrix} 0 \\ e_{\alpha 1} \\ e_{\beta 1} \end{bmatrix} \tag{33}$$

$e_{\alpha 1}$ and $e_{\beta 1}$ represent the fundamental components of the α, β voltage, respectively, and E_1^2 represents the square root of the voltage $\vec{e}_{\alpha\beta 1}$ fundamental component RMS value.

To analyze the case of unbalanced non-sinusoidal voltage, it is necessary to consider that the vector $\vec{e}_{\alpha\beta}$ from Fig. 2 may be divided in the positive sequence fundamental component $\vec{e}^{\,+}_{\alpha\beta 1}$, negative sequence fundamental component $\vec{e}^{\,-}_{\alpha\beta 1}$, and harmonic components $\vec{e}_{\alpha\beta N1}$. Current vector can be expressed, in this way, as follows:

$$\begin{aligned} \vec{i}_L &= \frac{p_{L\alpha\beta}(t)}{e_{\alpha\beta}^2}\vec{e}_{\alpha\beta} + \frac{P_{L0}}{e_0^2}\vec{e}_0 + \frac{q_{L\alpha\beta}(t)}{e_{\alpha\beta}^2}\vec{e}_{-\beta\alpha} \\ &= \frac{p_{L\alpha\beta}(t)}{e_{\alpha\beta}^2}\vec{e}^{\,+}_{\alpha\beta 1} + \frac{p_{L\alpha\beta}(t)}{e_{\alpha\beta}^2}\vec{e}^{\,-}_{\alpha\beta 1} + \frac{P_{L\alpha\beta}(t)}{e_{\alpha\beta}^2}\sum_{\forall n \geq 2}\vec{e}_{\alpha\beta n} \\ &\quad + \frac{P_{L0}}{e_0^2}\vec{e}_0 + \frac{q_{L\alpha\beta}(t)}{e_{\alpha\beta}^2}\vec{e}_{-\beta\alpha} \end{aligned} \tag{34}$$

So, in the case of unbalanced non-sinusoidal voltage, the source current expression becomes as shown in the following equation:

$$\begin{bmatrix} i_{S0} \\ i_{S\alpha} \\ i_{S\beta} \end{bmatrix} = \frac{P_L}{E_1^{+2}} \begin{bmatrix} 0 \\ e^{+}_{\alpha 1} \\ e^{+}_{\beta 1} \end{bmatrix} \tag{35}$$

E_1^{+2} represents the square root of the voltage positive sequence fundamental component RMS value.

In (35), i_{Si} is proportional to the voltage positive sequence fundamental component. The proportionality constant value is calculated imposing the restriction of null active power exchanged by the compensator.

Thus, the compensation current equations to obtain balanced and sinusoidal supply currents are calculated, from Eq. (35), as follows:

$$\begin{aligned} \vec{i}_C = \vec{i}_L - \vec{i}_S &= \left(\frac{p_{L\alpha\beta}(t)}{e_{\alpha\beta}^2} - \frac{P_L}{E^{+2}_{\alpha\beta 1}}\right)\vec{e}^{\,+}_{\alpha\beta 1} + \frac{p_{L\alpha\beta}(t)}{e_{\alpha\beta}^2}\vec{e}^{\,-}_{\alpha\beta 1} \\ &\quad + \frac{P_{L\alpha\beta}(t)}{e_{\alpha\beta}^2}\sum_{\forall n\geq 2}\vec{e}_{\alpha\beta n} + \frac{P_{L0}}{e_0^2}\vec{e}_0 + \frac{q_{L\alpha\beta}(t)}{e_{\alpha\beta}^2}\vec{e}_{-\beta\alpha} \end{aligned} \tag{36}$$

In this case, it is necessary to impose restrictions not only to the instantaneous real powers $p_0(t)$ and $p_{\alpha\beta}(t)$, but besides to the instantaneous imaginary power $q_{\alpha\beta}(t)$. This new restriction is necessary to get sinusoidal and balanced source currents in the case of non-sinusoidal or unbalanced voltages.

In fact:

$$\begin{aligned} q_{C\alpha\beta} = \vec{e}_{-\beta\alpha}\cdot\vec{i}_C &= -\frac{P_L}{E^{+2}_{\alpha\beta 1}}\vec{e}_{-\beta\alpha}\cdot e^{+}_{\alpha\beta 1} + \frac{q_{L\alpha\beta}(t)}{e_{\alpha\beta}^2}e_{\alpha\beta}^2 \\ &= -\frac{P_L}{E^{+2}_{\alpha\beta 1}}\vec{e}_{-\beta\alpha}\cdot e^{+}_{\alpha\beta 1} + q_{L\alpha\beta}(t) \end{aligned} \tag{37}$$

since in general $\vec{e}_{-\beta\alpha}\cdot e^{+}_{\alpha\beta 1} \neq 0$

In mapping matrices format, the control strategy may be expressed as

$$\begin{bmatrix} i_{C0} \\ i_{C\alpha} \\ i_{C\beta} \end{bmatrix} = \frac{1}{e_0 e_{\alpha\beta}^2}\begin{bmatrix} e_{\alpha\beta}^2 & 0 & 0 \\ 0 & e_0 e_\alpha & -e_0 e_\beta \\ 0 & e_0 e_\beta & e_0 e_\alpha \end{bmatrix} \times \begin{bmatrix} p_{L0}(t) \\ p_{L\alpha\beta}(t) - \dfrac{P_L}{E_1^{+2}}(e_\alpha e^{+}_{\alpha 1} + e_\beta e^{+}_{\beta 1}) \\ q_{L\alpha\beta}(t) - \dfrac{P_L}{E_1^{+2}}(e_\alpha e^{+}_{\beta 1} - e_\beta e^{+}_{\alpha 1}) \end{bmatrix} \tag{38}$$

Moreover, this strategy achieves to eliminate the neutral current with a null active power exchanged by the compensator.

6. Simulation results

In this paper, the different control strategies proposed within instantaneous reactive power theory have been applied to a three-phase four-wire system similar to the presented in Fig. 3. It is a three-phase four-wire system composed of an unbalanced and non-sinusoidal voltage source that supplies a non-linear unbalanced load. The compensator injects the non-useful current required by the load.

The three strategies presented in previous sections have been applied to the simulation platform shown in Fig. 4. This figure presents a Matlab-Simulink implementation diagram. On the one hand, the source is composed of three single-phase sources and a serial resistor in star configuration. On the other hand, the load is composed of two back to back SCRs and a serial resistor,

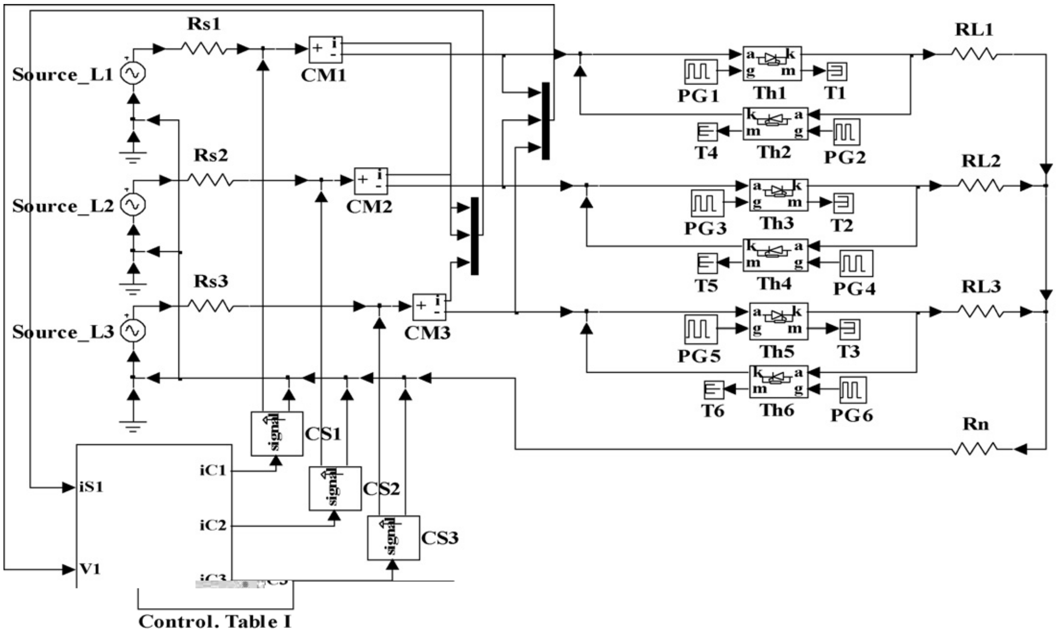


Fig. 4. Matlab-Simulink implementation diagram.

whose value is different in each phase, in star connection. Thus, a non-linear load is obtained. The main objective is the control strategies analysis to calculate the reference current. So, for the purposes of the simulation analysis, a relatively simple model has been considered for the compensator. Therefore, the converter is modeled by means of three controlled current sources. The compensation current is obtained through a set of calculation blocks, and they are directly injected in the system by the compensator. The block identified as "control block" is a mask whose content implements the different specified control algorithm. All the algorithms used in this paper are summarized in Fig. 5, where u_R is the voltage used to calculate the reference current by each compensation strategy.

The method used to calculate the voltage fundamental component, necessary in the last implemented strategy, is based on the use of low band filters, multipliers and adders, as shown in Fig. 6.

In fact, any voltage or current signal, in $\alpha\beta$ coordinates, may be expressed in the next way:

$$e_\alpha = E_{\alpha 1}\cos(\omega t + \varphi_1) + E_{\alpha 2}\cos(\omega t + \varphi_2) + \cdots \tag{39}$$

The product of the voltage component $e_\alpha(t)$ and the waveform $\sin\omega t$ has a constant term: $1/2E_{\alpha 1}\cos\varphi_1$ being $E_{\alpha 1}$ the voltage fundamental component amplitude and φ_1 is the voltage fundamental component phase.

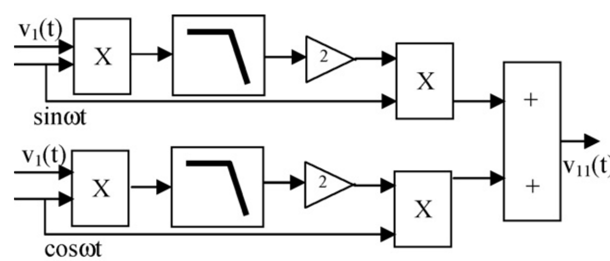


Fig. 6. Method to get voltage fundamental component.

On the other hand, the product composed by the same voltage component $e_\alpha(t)$ and the waveform $\cos\omega t$ has a constant term: $1/2E_{\alpha 1}\sin\varphi_1$. The product of both constant terms per 2 and per $\sin\omega t$ and $\cos\omega t$, respectively, compose the phase 1 voltage fundamental component waveform, as shown in Fig. 6. The angular speed used to generate the sine and the cosine functions corresponds to the nominal pulsation of the voltage waveform fundamental component. It is because the objective is to extract the voltage fundamental component waveform, whose frequency is the nominal one.

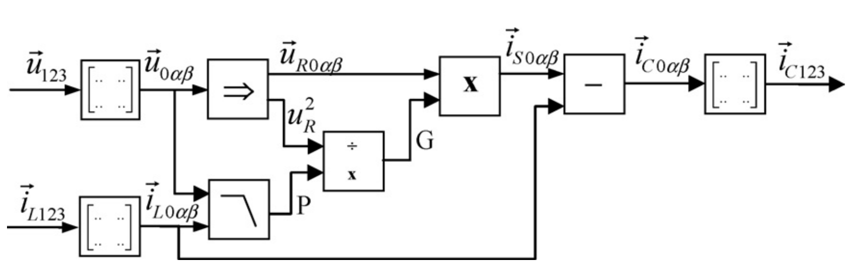


Fig. 5. Control block algorithms diagram.

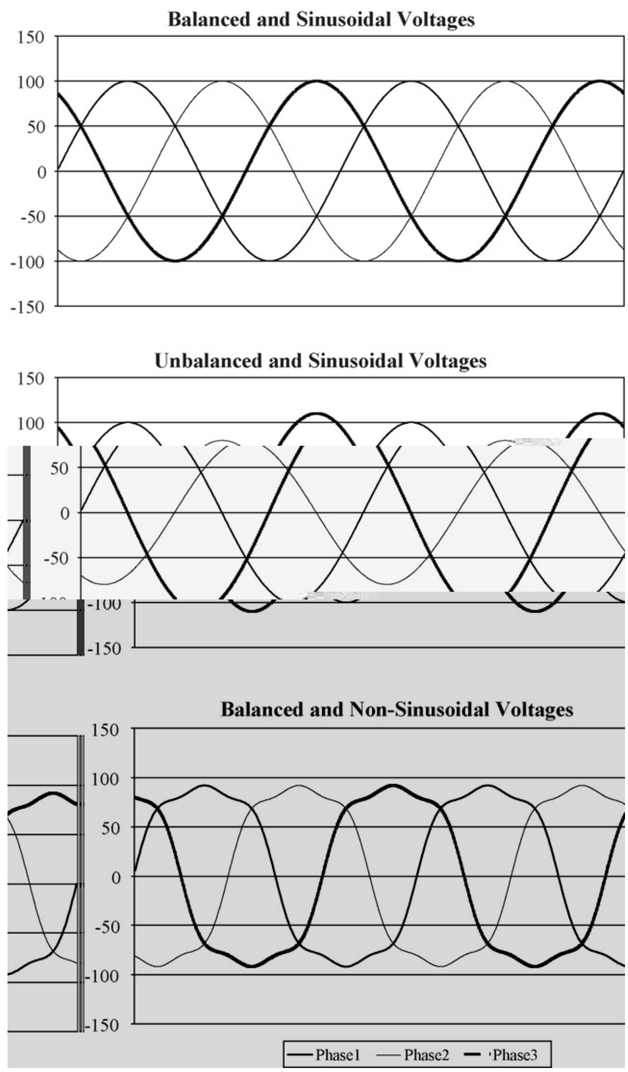


Fig. 7. Supply voltages (*V*), used in simulation assays.

The results got from the different simulation strategies are presented in Figs. 7–9 which show two periods (0.04 s) of the waveforms. Fig. 7 presents the voltages supply correspondent to each case. The first graph shows a balanced and sinusoidal voltage system, the second one an unbalanced and sinusoidal voltage system and the last one a balanced and non-sinusoidal voltage system.

The results obtained from each strategy application in case 1, balanced and sinusoidal supply voltage, are identical: balanced and sinusoidal source current. Fig. 8 presents the current waveforms which are obtained applying an unbalanced and sinusoidal voltage system, i.e., the case 2. And, finally, Fig. 9 presents the case 3, balanced and non-sinusoidal supply voltage.

The first graph in Figs. 8 and 9 present the three-phase source current before compensation, i.e., the three-phase current required by the load. It is a strongly non-linear and unbalanced current even if the voltage supply system is balanced and sinusoidal. Of course, the load currents present a different distortion and unbalanced behavior for each supply condition corresponding to each case.

6.1. Case 1: balanced and sinusoidal voltage supply

When the supply voltage is balanced and sinusoidal, the results obtained applying the three control strategy (constant power, unity power factor and sinusoidal source current) are the same.

6.2. Case 2: unbalanced and sinusoidal voltage supply

In this case, load current shown in Fig. 8 first graph is unbalanced due to the load unbalance effect and to the supply voltage asymmetry, simultaneously.

The second graph presents the three-phase source current after compensation when the strategy named constant power is applied. This graph shows a source current distortion considerably reduced. This strategy does not get sinusoidal source current although it obtains constant instantaneous power supplied by the source after compensation. The third graph presents the source current after compensation according to the unity power factor strategy. In this case, the source current waveform is collinear to the voltage and the strategy is not able to compensate the part of the load current due to the supply voltage zero sequence-phase component. So, this strategy does not eliminate the neutral current and it does not get constant instantaneous power supplied by the source after compensation. However, the power factor is the unity. The three-phase current waveform shown in last graph is the only one which is balanced and sinusoidal. It corresponds to the sinusoidal source current strategy that imposes getting sinusoidal and balanced source currents in phase with the voltage positive sequence phase and carrying the active power required by the load.

However, the power supplied by the source is not constant and the power factor is not exactly the unity.

6.3. Case 3: balanced and non-sinusoidal voltage supply

In this case, the load current shown in Fig. 9 is strongly distorted due to the simultaneous action of the load nor linearity and the applied voltage distortion.

As in the two earlier cases, in Fig. 9 second to fourth graphs the three-phase source currents correspond to the different compensation strategies are presented. The constant power one presents a low distortion although the waveforms are not sinusoidal. The third graph presents the results got applying the strategy named unity power factor, where the source current reproduces the voltages distortion. The power factor is the unity. At last, the fourth graph presents the results obtained using the sinusoidal source current strategy. The source current is now balanced and sinusoidal. The load unbalanced and distortion have been compensated.

In summary, a strongly non-linear and unbalanced load has been considered to be studied in three different cases of supply voltage. In case 1, the load unbalanced effect is completely compensated by all the control strategies. Nevertheless, if voltage is unbalanced and/or non-sinusoidal, each strategy achieve the target imposed in its design although they do not achieve any of the other two, e.g., constant power strategy obtain constant

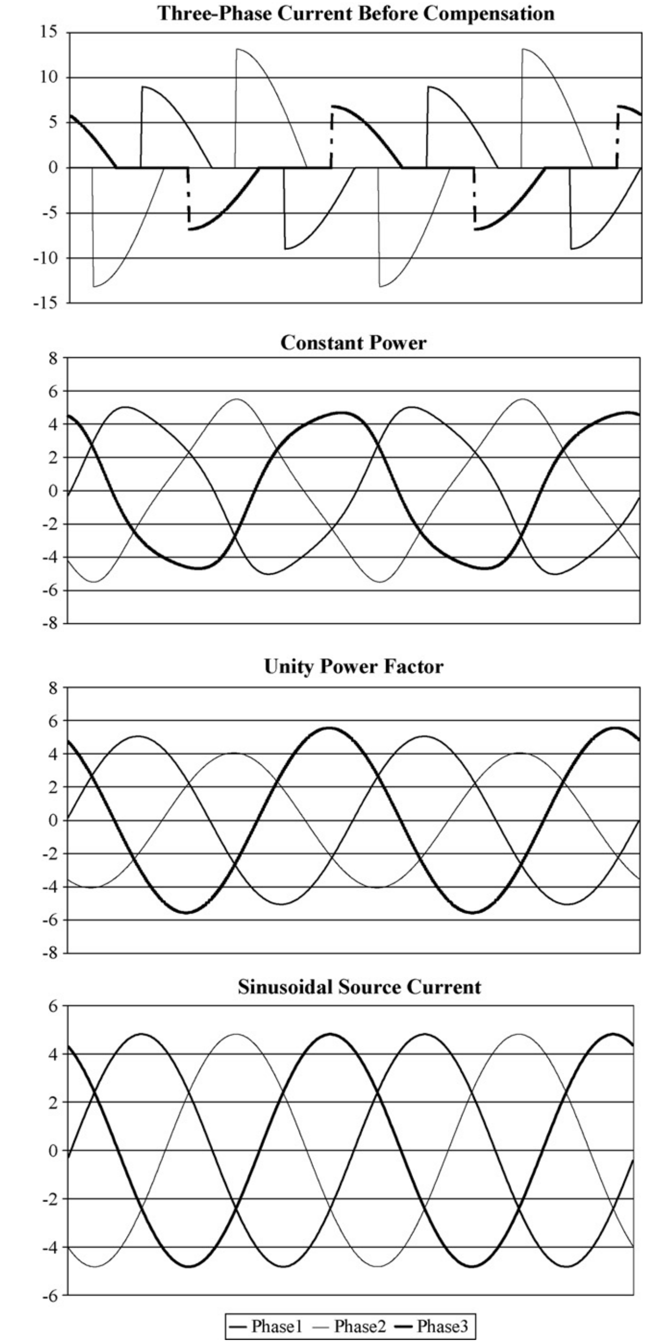


Fig. 8. Three-phase source current (*A*) before/after compensation with unbalanced and sinusoidal source voltages obtained in simulation assays.

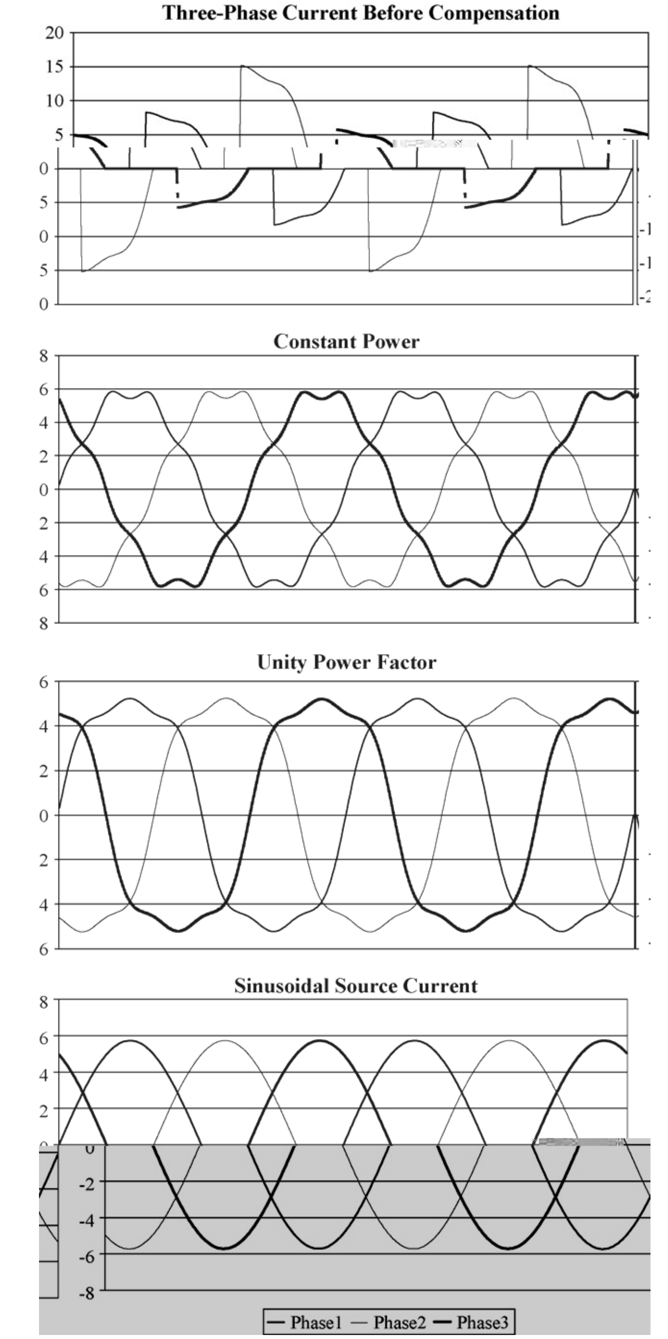


Fig. 9. Three-phase source current (*A*) before/after compensation with balanced and non-sinusoidal source voltages obtained in simulation assays.

power supplied by the source, the power factor is not the unity and the source current is not balanced and sinusoidal. And so on, applying the other two compensation strategies.

7. Experimental results

In this work, the APF control has been implemented by means of a specific digital signal processor (DSP) board developed by dSPACE. It includes a real-time processor and the necessary In/Out interfaces that allow the control operation to be carried out. In particular, DS1103 peer to peer connection (PPC) controller board is equipped with a Power PC processor for fast floating point calculation at 400 MHz. This hardware allows to program via Simulink. In this way, all the control circuit components are configured graphically within the Simulink environment. The RTI translates the Simulink model to C language, it generates the real-time executable program, and it downloads it in the controller board.

To check the approaches proposed in this paper, they were applied to the three-phase four-wire unbalanced ac-regulator compensation. Fig. 10 shows a general scheme of compensated system.

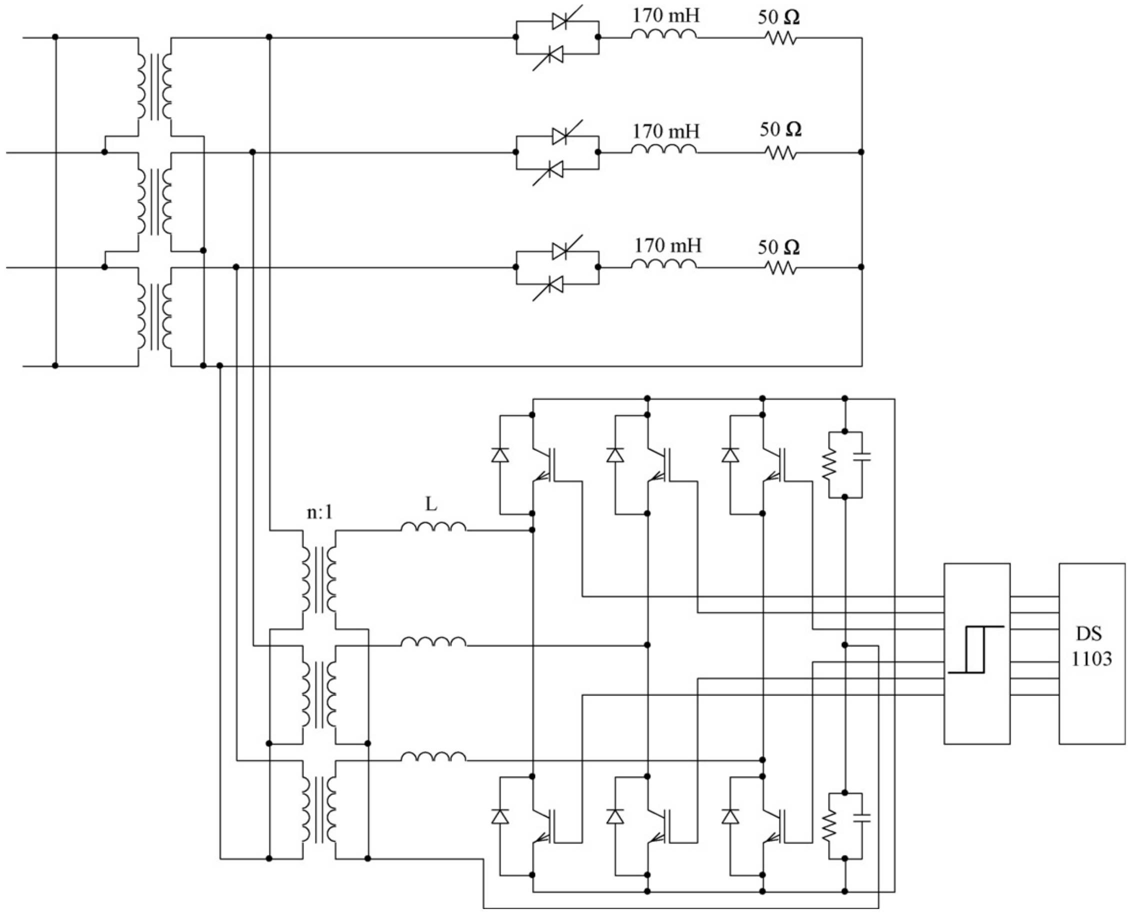


Fig. 10. A general scheme of the compensated system.

The voltage source is constituted by three autotransformer. They allow the voltage to be unbalanced before applying it to the system. The load is composed of three regulators with a serial inductive load in each phase connected in star. The compensator is constituted by a Semikron power inverter model SKM50GB123, with a three-phase IGBT bridge and two capacitors of 2200 μF in dc side. In each phase, the connection transformers voltage ratio is 230/460 V and the inverter output inductance is Ls = 17 mH. The use of the coupling transformers allows working with a low dc voltage. The control block inputs are nine: the load voltages and currents, and the compensation currents. Besides, it is necessary another input to control the capacitor dc voltage. Additional inputs are used to check the APF compensation performance. The voltage and current sensors used are AD/DC LEM LA-35 NP and AC/DC LEM LV 25-600 models, respectively.

The software running in the real-time processor carries out the control. It calculates the reference source current as it was presented in Sections 2–4. The difference between the real compensation current and the calculated before is the input to the PWM module. Its output are the power circuit IGBTs trigger signals. In fact, in an experimental develop the current control used is very important. In our experimental system, a hysteresis band PWM control has been implemented. The converter IGBTs commutation introduces unavoidable high order harmonics. To

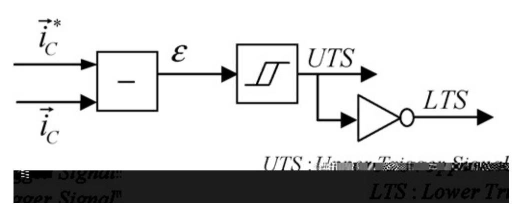


Fig. 11. Control system used in the experimental set-up.

reduce its influence, the PWM control is carried out by mean of a specific circuit instead of by software, Fig. 11.

The RMS voltage values and the load parameters values are shown in Table 1.

The voltage applied is the net voltage unbalanced using an autotransformer. The three phases root mean square (RMS) values are indicated in Table 1 first row. These values point out a great level of unbalanced applied to the system.

Table 1
System parameters

	Phase 1	Phase 2	Phase 3
RMS voltage	203.84	147.81	221.92
Resistor (Ω)	52.2	52.2	52.2
Inductance (mH)	150	150	150
SCRs angle	90	90	90

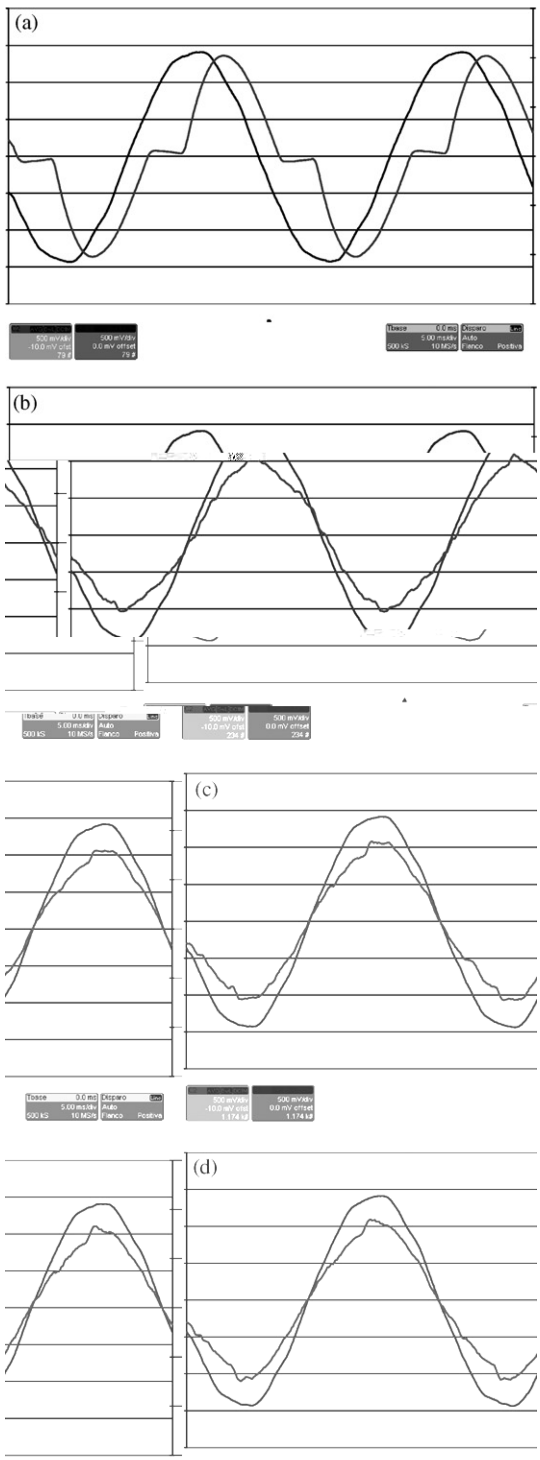


Fig. 12. Phase 1 voltage (*V*) and current (*A*) obtained in the laboratory prototype. (a) Before compensation, (b) after compensation applying constant power strategy, (c) after compensation applying unity power factor strategy and (d) after compensation applying sinusoidal source current compensation.

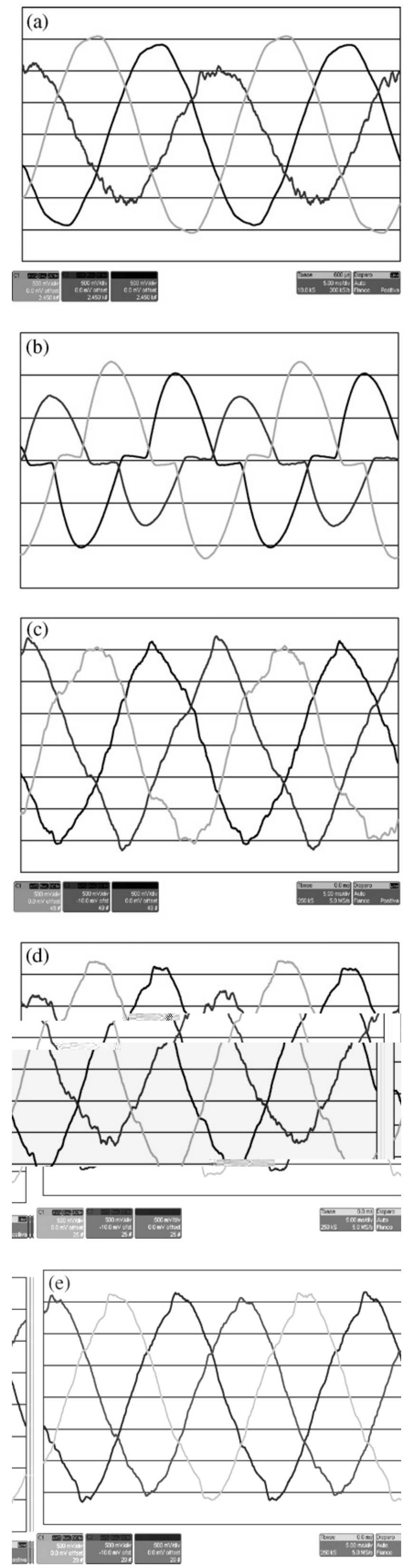


Fig. 13. Three-phase waveforms obtained in the laboratory prototype. (a) Voltage applied, (b) source current before compensation, (c) source current after compensation applying constant power strategy, (d) source current after compensation applying unity power factor strategy and (e) source current after compensation applying sinusoidal source current compensation.

Besides, the feedback method used to control self-supporting dc bus is very important. It supposes, in general, a modification of compensation currents. Summarizing, the procedure used consists of controlling the capacitor voltage to a reference value. To achieve it, a PI control may be chosen for the error between the reference value and the capacitor voltage value at the end of each period. The result is the value of the compensator lost power. This value modifies the power terms which appear in the compensation currents expressions corresponding to each strategy. The modification is in the next way: referring to the first strategy, Eqs. (20) and (21), the power lost is subtracted to the $\tilde{p}_{L\alpha\beta}$ term in the expressions of $i_{c\alpha}$ and $i_{c\beta}$; referring to the second strategy, Eq. (31), the power lost is added to the $P_{L\alpha\beta}$ term in the expressions of $i_{c\alpha}$ and $i_{c\beta}$; referring to the third strategy, Eq. (38), the power lost is added to the load power P_L.

Fig. 12 presents the phase one source voltage and current before and after compensation. Fig. 12(a) graph shows the situation before compensation, i.e., phase one voltage and load current. Fig. 12 graphs (b), (c) and (d) show the results got by all the three control strategies: constant power, unity power factor and sinusoidal source current, respectively. The phase difference between voltage and current is eliminated by applying the three strategies. In fact, the first graph shows a phase difference between both waveforms. It is not in the other three graphs.

Fig. 13 presents the three-phase waveforms corresponding to: voltage applied (graph 13 (a)), load current or source current before compensation (graph 13 (b)), and source current after compensation using each control strategy (graphs 13 (c–e)): constant power, unity power factor and sinusoidal source current, respectively. Fig. 13(a) shows the voltage unbalanced. Fig. 13(b) presents an unbalanced and distorted three-phase load current due to the load non-linearity and to the source voltage unbalance.

The source current distortion after compensation using any of the three control strategy is much lower than the presented by the load current. The unity power factor strategy does not get balanced source current and it does not eliminate the neutral current.

The sinusoidal source current strategy obtains balanced and sinusoidal source current after compensation, Fig. 13(e). Even so, a ripple in the waveforms is unavoidable due to the threshold band imposed by the PWM control. The source current corresponding to the constant power strategy presents a considerably distortion and the corresponding to the unity power factor strategy shows the same unbalance as the voltage applied to the system.

The voltage RMS and total distortion demand (TDD) values and the source current RMS and TDD values before and after compensation are presented in Table 2. The TDD is calculated as follows:

$$\mathrm{TDD} = \sqrt{\frac{h_2^2 + h_3^2 + \cdots + h_n^2}{\mathrm{RMS}^2}} \tag{40}$$

where h_i represent the order i harmonic RMS value and RMS represents the waveform RMS value.

Besides, Table 2 presents the three-phase RMS value corresponding to voltage and currents. It is defined as follows [16]:

$$V_e = \sqrt{\frac{V_{10}^2 + V_{20}^2 + V_{30}^2}{3}}, \qquad I_e = \sqrt{\frac{I_1^2 + I_2^2 + I_3^2 + I_4^2}{3}} \tag{41}$$

where V_{i0} represents the phase voltage corresponding to the phase i and I_i represents the phase i current RMS value. Phase 4 corresponds to the neutral current.

This table shows as the three control strategies decrease the current distortion considerably. Except the values corresponding to the phase two, the distortion values are very similar applying the three control strategies. However, that distortion represents high order harmonics in the case of sinusoidal current compensation and low and high order harmonics in the other two, Fig. 13. It is corroborated by the values corresponding to the phase two. The values achieved applying sinusoidal current is lower than the achieved in the other two phases. In an ideal case, they would be null, but in this case the PWM block has a great influence on the results, and therefore the compensation current does not track its reference exactly. On the other hand, the source current after compensation applying the control strategy derived from constant power compensation presents a distortion appreciatively bigger than the corresponding to the sinusoidal source current strategy as can be corroborated by Fig. 13. The unity power factor compensation, does not obtain balanced source currents.

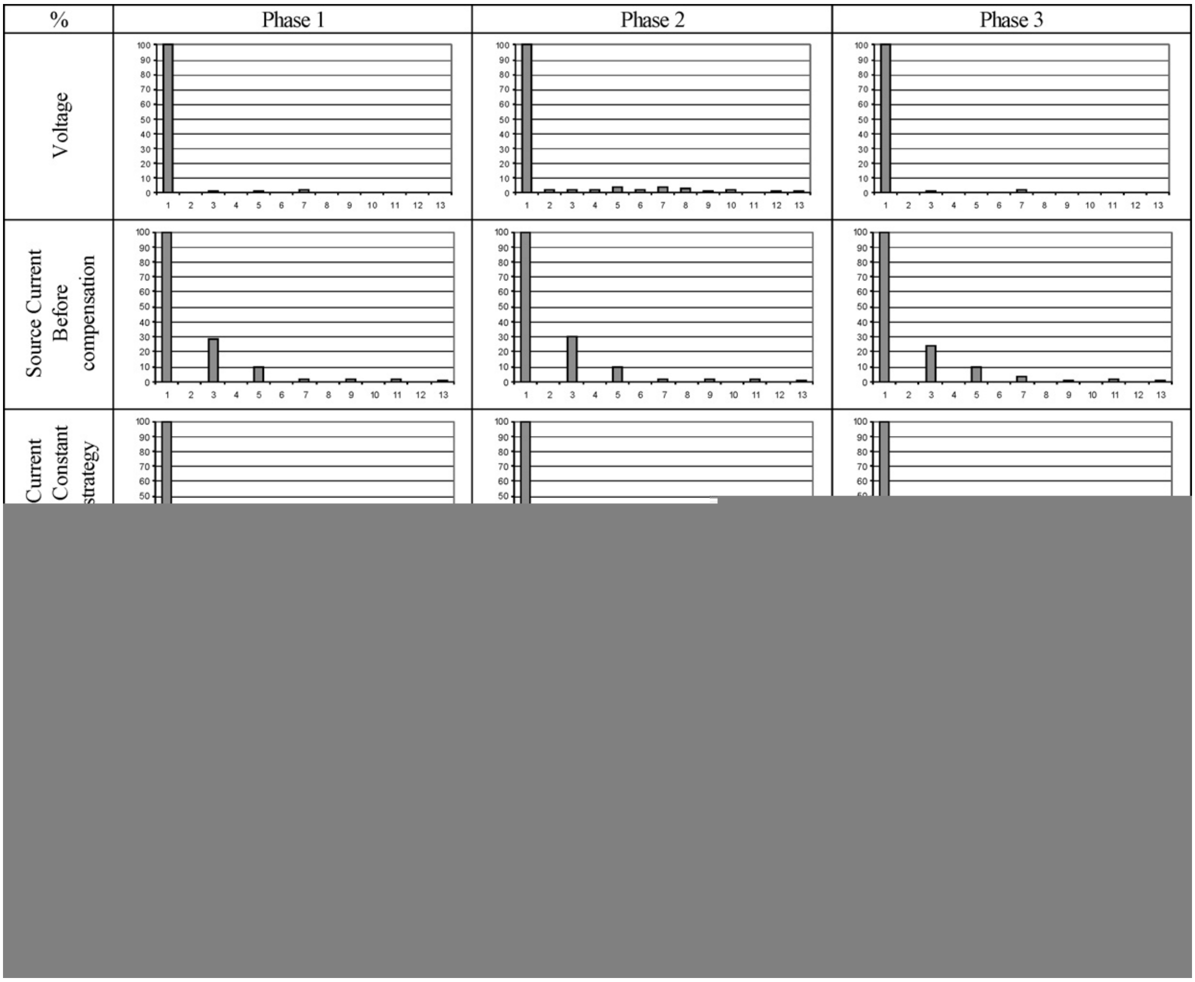
Respect to the three-phase RMS values, they are very similar in the case of applying constant power and unity power factor compensation strategies and a little bigger in the case of sinusoidal source current. It is necessary to obtain balanced and sinusoidal source current if voltage is not balanced and sinusoidal.

The column "Power factor" in Table 2 shows that the three strategies improve the power factor value after compensation.

Table 2
Experimental results, voltage values (V) and currents values (A)

	RMS						TDD		
	Phase 1	Phase 2	Phase 3	Neutral	V_e or I_e	Power factor	Phase 1	Phase 2	Phase 3
Voltage	203.84	147.81	221.92	69.04	202.16		0.02	0.10	0.02
Source current									
Before compensation	1.25	0.91	1.43	1.02	1.53	0.752	0.28	0.31	0.25
Constant power	1.04	1.00	1.07	0.19	1.26	0.986	0.07	0.16	0.13
Unity power factor	1.08	0.79	1.17	0.39	1.25	0.995	0.07	0.11	0.06
Sinusoidal source current	1.06	1.03	1.08	0.19	1.29	0.990	0.08	0.06	0.08

Table 3
Source voltage and current harmonic content (% over the fundamental component) before and after compensation



Besides, the corresponding to the unity power factor strategy is the highest of them.

Table 3 presents the main harmonic order corresponding to the source current three phases before and after compensation.

8. Conclusions

This paper has developed an exhaustive analysis of the p–q instantaneous reactive power theory based on a new formulation without using mapping matrices. The calculation of the reference compensation currents is based on the current vector projections over three new voltage vectors defined in $0\alpha\beta$ coordinates system. The three main compensation objectives (constant power, unity power factor and sinusoidal source current) have been studied to generate three control strategies based on the p–q instantaneous reactive power theory, one corresponding to each compensation objective. These control strategies have been implemented in a simulation platform to study the results obtained in three cases of voltage applied: sinusoidal and balanced, sinusoidal but unbalanced and balanced but non-sinusoidal. Thus, the analysis shows that, with few modifications in the control strategy developed by the authors of the original theory, any compensation objective imposed to the system may be achieved. So, the original p–q theory can be used as the base of active power filters control algorithm, to get any compensation objective and with any voltage condition in the PCC. The strategies have been assayed by simulations in the Matlab-Simulink environment and tested in an experimental laboratory prototype.

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