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A general approach for optimal allocation of FACTS devices using equivalent impedance models of VSCs

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SUMMARY

Flexible AC transmission system (FACTS) devices such as static synchronous series compensators, static synchronous compensators, and unified power flow controllers which use voltage source convertors (VSCs) can noticeably improve different characteristics of power systems. Therefore, finding their optimal allocation is a vital issue. To find the optimized placement of different FACTS devices, it is necessary to calculate the effects of such devices in the whole system. Sensitivity analysis has been recently proposed as a fast and reliable method to find the effects of voltage source convertors on system variables. In this paper, a novel approach is introduced to optimize the allocation of different FACTS devices based on the equivalent impedance model obtained from sensitivity analysis. This method requires neither the exact modeling of voltage source convertors nor more than one load flow, so it is much faster than conventional methods. Due to its simplicity and fastness, it can cover all possible locations, continuous sizing, type, and number of compensators. Therefore, its performance and accuracy are considerably high. The proposed method based on a novel objective function, which includes voltage profile, transmission capability, and stability, is applied to a typical 6-bus and the 30-bus IEEE test systems. Furthermore, genetic algorithm is implemented to solve the optimization problem. The test results illustrate the effectiveness of the introduced method. Copyright © 2014 John Wiley & Sons, Ltd.

KEY WORDS: static synchronous series compensators (SSSC); static synchronous compensators (STATCOM); unified power flow controllers (UPFC); optimized placement; genetic algorithm (GA); transmission capability; transient stability; sensitivity analysis; flexible AC transmission systems (FACTS)

1. INTRODUCTION

Flexible AC transmission systems (FACTS) broadly use voltage source converters (VSCs). They can inject active and reactive power to the power system and control the active and reactive power flow. VSCs are also used in FACTS devices in order to improve power transfer capability and increase the efficiency of power systems [1–3]. FACTS controllers which use VSCs can be utilized in series or parallel with transmission lines. In this regard, they are categorized into different groups. First, static synchronous series compensators (SSSC) are used in series, and they can improve voltage profiles (VPs) and compensate the reactive power of the system by reduction of the equivalent impedances of transmission lines [4,5]. Second, static synchronous compensators (STATCOM) are installed and operated in parallel with lines which supply reactive power and improve VPs [6–8]. Third, unified power flow controllers (UPFC) include two different sections: one operates in series like SSSCs, and another operates in parallel with lines like STATCOMs. However, these two sections are electrically interconnected which can interchange power with together [9–11]. All of the three discussed groups of FACTS installed in series, parallel and combinatorial series and parallel contain

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VSCs and can be operated under different control conditions and diverse topologies of power system. Thus, their performance and effects on the power system should be investigated. Moreover, it is important to obtain the effects of FACTS devices (using VSCs) on the specified system in large scales. Sensitivity analysis is one of the most useful methods for determining the effect of variations of one variable on the total system. In references [12–15], there are several approaches of sensitivity analysis related to FACTS devices (e.g. voltage source model based and equivalent impedance model based). Sensitivity analysis appropriately makes direct relations between the control variables and the observed variables such as power flow and VP in power systems including FACTS devices [13,14]. The relations of sensitivity analysis based on each equivalent impedance or voltage source modeling lead to obtain an optimal location for FACTS devices in order to remove overloads and congestion in many applications. Furthermore, it can help power system improve transient stabilities after using VSCs.

In reference [14], sensitivity analysis has been used to find proper locations of FACTS devices. In this reference, both direct and indirect approaches to analyze the effects of the FACTS equipments on load flow equations, active power flow, and the voltages of different buses have been used. In reference [15], an economical dispatching approach has been proposed based on the nodal price calculations wherein there are also some FACTS devices in the power system. Also, the optimization subject achieves both the suitable stability of the system and the decreasing of the cost in this reference. The applied method in [15] has been proposed by use of sensitivity analysis on the results of the transient stability simulations in time domain and the collections of the linear limitations related to the desired inputs of active power suppliers and FACTS devices. There are two general methods to implement voltage source convertors in system equations and analyze the effects of the equipments on a specified system. The first method is based on the modeling of the VSC with an injecting voltage source [13,14]. In the direct method, transmission active power variations in different lines of the system and the proportion of different point voltages to variations of the injecting voltages of the VSCs can be calculated based on analytical equations. Furthermore, the second method indirectly implements VSCs into system analysis as equivalent impedances [14,16]. In the second approach, transmission active power variations and different points' voltages in proportion to lines' impedance variations (equivalent to VSC impedance) can be considered. In both methods, by first-order linear extension with no need to complicated and numerous calculations, the response of the system to variations of voltage source converters can be estimated acceptably. The indirect method has the advantage of being independent from exact modeling of the equipments in the system. This would be very useful if the existence of FACTS devices is analyzed in various locations [14].

One of the important aspects of FACTS devices including VSCs is how to find their proper location in large-scale power systems in order to achieve the desired situations with minimum change in cost and undesired variations in the system [14,15] and [17,18]. The transmission capability, load flow equations, different point voltages [14,19], voltage stabilities [20], transient stabilities and market optimization [21,22], consumer load variations, and variable amount of renewable energy generation [23] are the most important factors which are considered in FACTS controller allocation.

In this paper, the proper allocation of FACTS devices is proposed by equivalent impedance modeling of VSCs (used in SSSCs, STATCOMs, and UPFCs) and using sensitivity analysis. This optimization allocation is based on VP, maximum transmittable active power of lines (LMTAP), static and transient stability (STS), and cost function. One of the most important advantages of equivalent impedance modeling of VSCs is its simplicity, fast calculations, and the ability to solve an optimization problem by only one load flow calculation in initial state. Another advantage of this method is analysis of locating of FACTS devices in all possible places and all continuous amounts of sizing. Furthermore, the selectivity between different types of FACTS devices and their numbers is accurately reachable with the proposed methodology. This approach can evaluate the power system conditions with all of three types and all amounts of compensations. The genetic algorithm (GA) is used to solve the introduced optimization problem. This optimization method will be applied on a typical 6-bus test system to clarify its performance.

2. EQUIVALENT IMPEDANCES

There are three different FACTS devices which will be considered in this paper (SSSC, STATCOM, and UPFC). Therefore, it is needed to obtain and analyze their formulations. To do so, equivalent impedance modeling will be presented. By these modeling, a fast and proper method is possible for locating FACTS devices in the power system and finding their numbers, types, and input values with no need to load flow in every step.

2.1. Equivalent impedance of SSSC

Figure 1 shows an SSSC which is connected in series to transmission lines with a transformer. The reactive power exchanged between the system and SSSC can be modeled by equivalent impedance. However, SSSC does not have active power exchange with the network because it is not connected to the energy source in normal conditions. The equivalent voltage-sourced and equivalent impedance schematics of an SSSC have been shown in Figures 1-b and 1-c, respectively.

The relations between the voltage, current, and the equivalent impedance of SSSC can be achieved by:

$$\vec{I}_{ij} = \frac{\vec{V}_i - \vec{V}_j + \vec{V}\ e_{ij}}{jXt_{ij} + Z_L} \tag{1}$$

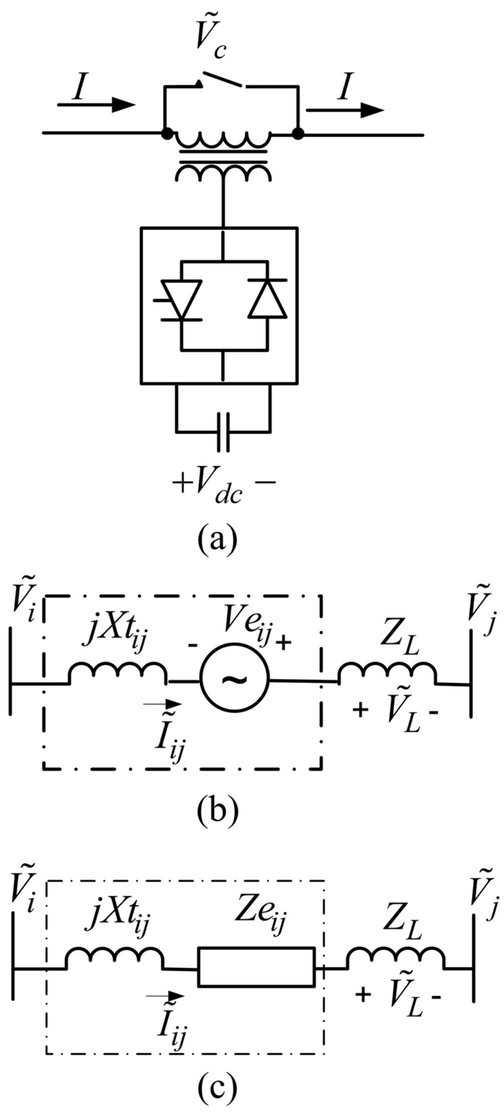

Figure 1. SSSC. (a) Schematic. (b) Equivalent voltage-sourced view. (c) Equivalent impedance view.

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$$Ze_{ij} = \frac{\vec{V}\,e_{ij}}{\vec{I}_{ij}} \tag{2}$$

Due to lack of active power exchange between SSSC and the network, the voltage of SSSC and its current is orthogonal. The SSSC voltage vectors and current vector diagram are demonstrated in Figure 2. Usually, the voltage of the VSC lags its current; therefore, it shows capacitor equivalent impedance. Furthermore, a VSC can be inductive as a series inductance with the line if its voltage leads its current [24].

2.2. *Equivalent impedance of STATCOM*

Figure 3 shows a STATCOM connected in parallel to bus (i). The reactive power exchange between the system and STATCOM can be modeled by equivalent impedance (in parallel). Like SSSC, STATCOM does not have active power exchange with the network, too. Figures 3-b and 3-c show the equivalent voltage-sourced and equivalent impedance schematic of a STATCOM connected to bus (i), respectively.

The relation of calculating equivalent impedance of the STATCOM can be obtained by:

$$\vec{I}_i = \frac{\vec{V}\,e_i - \vec{V}_i}{jXt_i} \tag{3}$$

$$Ze_i = -\frac{\vec{V}\,e_i}{\vec{I}_i} \tag{4}$$

2.3. *Equivalent impedance of UPFC*

Due to constraints of the SSSC and STATCOM in terms of the controllability, it is better to apply UPFC. They can exchange the active power with the network independent of its reactive power exchange. So, its controllability and degrees of freedom are much higher than other two types of mentioned compensators (SSSC and STATCOM). Figure 4-a shows a UPFC. The reactive power exchange between the system and each of voltage source converters. Also, the active power flow is possible through a DC capacitor.

To model its equivalent impedance, it is better to consider it as two impedances, one connected in parallel, another one connected in series. Figures 4-b and 4-c show the equivalent voltage-sourced and equivalent impedance schematic of a UPFC, respectively.

To calculate the series equivalent impedance (Ze_{ij}) of UPFC, Equations (1) and (2) should be followed again. To obtain the parallel equivalent impedance ($Z'e_i$), Equations (5) and (6) can be used:

$$r_i = -\frac{\vec{V}_i^{\,2}}{\vec{I}\,2_{ij}.real\left(Ze_{ij}\right)}, \quad x_i = -\frac{\vec{V}_i^{\,2}}{Q_i} \tag{5}$$

$$Z'e_i = \frac{r_i.jx_i}{r_i + jx_i} - jXt_i \tag{6}$$

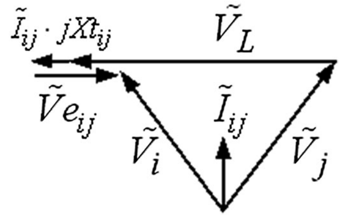
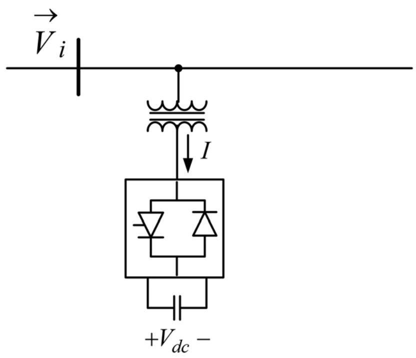
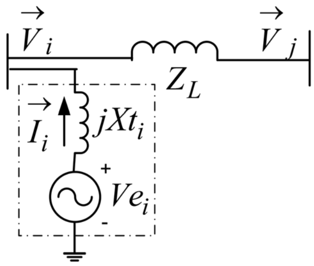


Figure 2. Vector diagram of SSSC when voltage lags from the current.





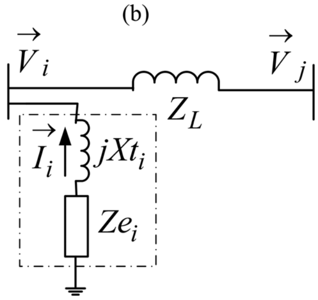


Figure 3. STATCOM. (a) Schematic. (b) Equivalent voltage-sourced view. (c) Equivalent impedance view.

3. ESTIMATION OF THE ADMITTANCE VARIATION MATRICES

To find the voltage matrix of the compensated state, both initial voltage matrix and voltage variations one should be considered:

$$\vec{V}_{new} = \vec{V}_{init} + \Delta\vec{V} \tag{7}$$

The initial voltage matrix is obtained by base case load flow (without any compensator). However, the voltage variation matrix needs an iterative process to be calculated. At first, the primary equivalent impedances of compensators (SSSC, STATCOM, and UPFC) should be achieved. Therefore, the admittance variation matrix (ΔY) is initially available by Equations (8) to (10):

$$SSSC:\begin{cases}\begin{bmatrix}\Delta Y_{ii} & \Delta Y_{ij}\\ \Delta Y_{ji} & \Delta Y_{jj}\end{bmatrix}=\begin{bmatrix}\dfrac{1}{jXt_{ij}+Ze_{ij}+Z_L}-\dfrac{1}{Z_L} & -\dfrac{1}{jXt_{ij}+Ze_{ij}+Z_L}+\dfrac{1}{Z_L}\\ -\dfrac{1}{jXt_{ij}+Ze_{ij}+Z_L}+\dfrac{1}{Z_L} & \dfrac{1}{jXt_{ij}+Ze_{ij}+Z_L}-\dfrac{1}{Z_L}\end{bmatrix}\\ \Delta Y_{pq}=0 \quad for \quad p,q\neq i \;\; or \;\; j\end{cases} \tag{8}$$

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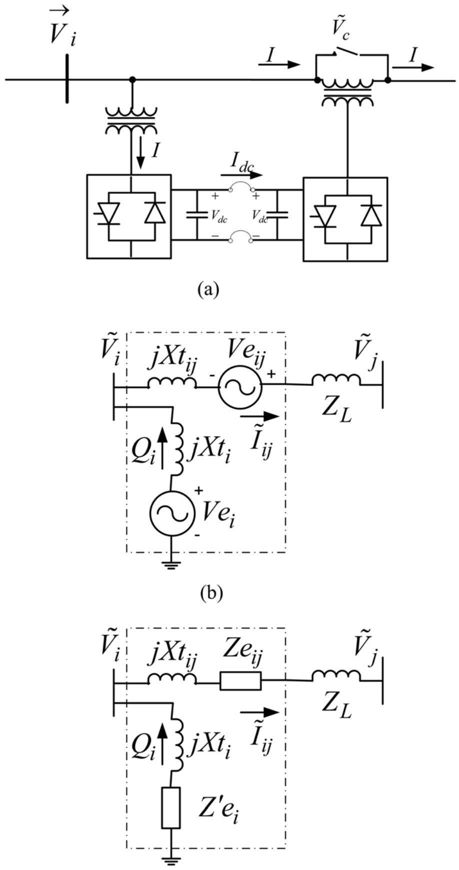

Figure 4. UPFC. (a) Schematic. (b) Equivalent voltage-sourced view. (c) Equivalent impedance view.

$$STATCOM: \begin{cases} \begin{bmatrix} \Delta Y_{ii} & \Delta Y_{ij} \\ \Delta Y_{ji} & \Delta Y_{jj} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{Ze_i + jXt_i} & 0 \\ 0 & 0 \end{bmatrix} \\ \Delta Y_{pq} = 0 \quad for \quad p, q \neq i \;\; or \;\; j \end{cases} \tag{9}$$

$$UPFC: \begin{cases} \begin{bmatrix} \Delta Y_{ii} & \Delta Y_{ij} \\ \Delta Y_{ji} & \Delta Y_{jj} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{jXt_i + Z'e_i} + \dfrac{1}{Ze_{ij} + jXt_{ij} + Z_L} - \dfrac{1}{Z_L} & -\dfrac{1}{Ze_i + jXt_{ij} + Z_L} + \dfrac{1}{Z_L} \\ -\dfrac{1}{Ze_i + jXt_{ij} + Z_L} + \dfrac{1}{Z_L} & +\dfrac{1}{Ze_i + jXt_{ij} + Z_L} - \dfrac{1}{Z_L} \end{bmatrix} \\ \Delta Y_{pq} = 0 \quad for \quad p, q \neq i \;\; or \;\; j \end{cases} \tag{10}$$

The current matrix can be calculated for the next step as indicated below:

$$\vec{I}_{new} = Y_{new}\vec{V}_{new} = (Y_{init} + \Delta Y)\left(\vec{V}_{init} + \Delta \vec{V}\right) \tag{11}$$

Expanding of Equation (11) leads to:

$$\vec{I}_{new} = Y_{init}.\vec{V}_{init} + Y_{init}.\Delta \vec{V} + \Delta Y.\vec{V}_{init} + \Delta \mathrm{Y}.\Delta \vec{V} \tag{12}$$

In a large power system, if the inserted voltage by FACTS device is a small perturbation, also remote to the generation and load locations, the effect of voltage insertion would be small. Hence, it is possible to ignore the change of current matrix, still resulting in good approximation [14,17]. According to this explanation, the compensated and initial current matrices ($\vec{I}_{new}$ and $\vec{I}_{init}$) can be assumed equal. Furthermore, the multiplying of the voltage and admittance variations matrices is negligible, too.

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$$\begin{cases} \vec{I}_{new} = \vec{I}_{init} + \vec{I}_{injected} \simeq \vec{I}_{init} = Y_{init}\vec{V}_{init} \\ \Delta \mathrm{Y}.\Delta \vec{V} \simeq 0 \end{cases} \tag{13}$$

Thus, by neglecting the second-order item $\Delta \mathrm{Y}.\Delta \vec{V}$ and assumption of being unchanged of the current matrix, the following equation can be derived:

$$\Delta \vec{V} = -Y_{init}{}^{-1}.\Delta \mathrm{Y}.\vec{V}_{init} \tag{14}$$

The assumption of being unchanged of the current matrix through voltage insertion of FACTS devices has been examined in [14,17] by comparison of the mentioned method and accurate computations. The comparison results of [14,17] confirm the validation of the mentioned method.

This equation should be solved with an iterative process in order to reach a constant amount for $\Delta \vec{V}$. The flowchart of the iterative algorithm to calculate the voltage variations by the presented equations has been illustrated in Figure 5.

First, load flow results have been taken for the initial state of the system. Then, the algorithm uses Equations (1) to (6) in order to find the admittance variation matrix (ΔY). Afterward, it calculates the voltage variations ($\Delta \vec{V}$) by Equation (14). Thus, the values of ΔY and $\Delta \vec{V}$ are obtained in each iteration by network bus voltages calculated in the previous iteration. If voltage variations are variable (not constant over iterations), they should be continued to calculate. In the end of the each iteration, they are compared with voltage variations of the previous iterations. If voltage variations of buses are approximately constant, the algorithm will stop, and the desired results will be achieved.

The advantage of Equation (14) is that it determines a relation between admittance variations and voltage variations of the compensators. Also, calculations can be only processed by the inverse matrix of the admittance in the initial state, and there is no need to obtain the admittance inverse matrix for

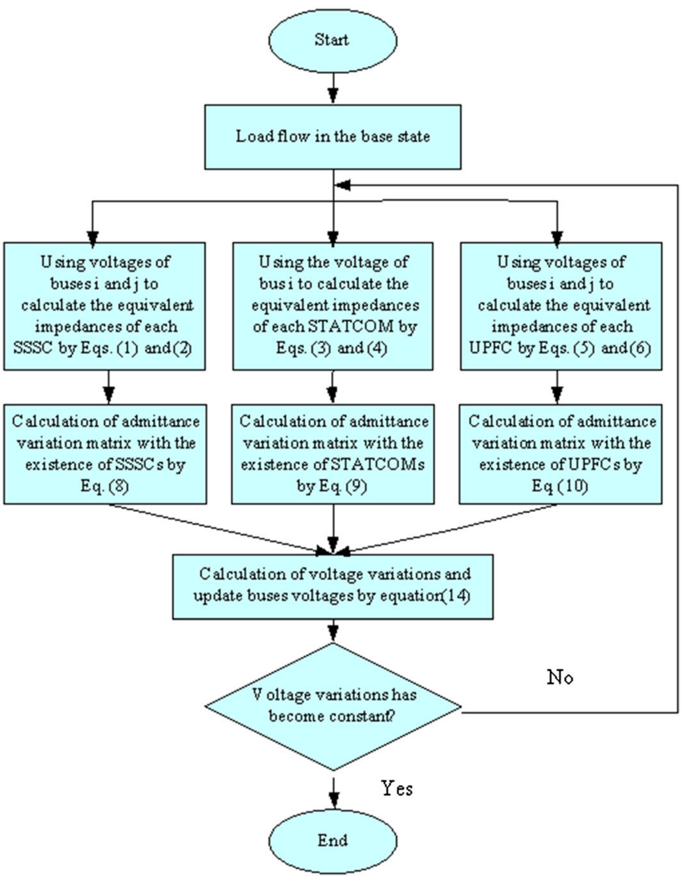

Figure 5. Flow chart of the iterative process to calculate the compensated voltage matrix.

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each possible allocation of compensators (SSSC, STATCOM, and UPFC). This method shows its performance greatly where there is a large amount of candidate locations for several compensators.

4. OBJECTIVE FUNCTION

Different criteria have been implied for allocation of FACTS devices in recent researches. One of the most important objects of using FACTS controllers is improvement of the VPs (VP) in power systems' different buses. Another parameter which affects the reliability and the performance of power systems is transmission line capability. Furthermore, the phase difference between voltages of connected buses (δ) is a prominent parameter which affects the transient stability of the system. If δ decreases, the stability margin of the transmission line will be increased [12,24]. In Equation (15), all the above mentioned parameters of a power system are presented:

$$\begin{cases} VP_i = |V_i - 1| & i = 1:N \\ P_{t,Max} = \dfrac{|V_s||V_r|}{X_t} & t = 1:M \\ \delta_k = |\varphi_s - \varphi_r| & k = 1:M \end{cases} \tag{15}$$

In the above equation, VP_i is the difference between the voltage magnitude of bus (i) and unit voltage. Also, $P_{ij,Max}$ is the maximum amount of transmittable active power between two connected buses (i) and (j). Moreover, δ_k is the difference between the voltage angles of two connected buses (i) and (j) in the two sides of line k which are called phase angles (PAs). In this paper, VP, LMTAP, and STS are applied to obtain the objective function for allocation and sizing of FACTS devices.

$$\mathrm{OF} = \sum_{i=1}^{N}\omega_i\left(\frac{VP_i^{comp} - VP_i^{init}}{\overline{VP_{init}}}\right) + \sum_{j=1}^{M}\left[\omega_j^{'}\left(\frac{p_{j,\max}^{init} - p_{j,\max}^{comp}}{\overline{p_{\max}^{init}}}\right) + \omega_j^{''}\left(\frac{\delta_j^{comp} - \delta_j^{init}}{\overline{\delta^{init}}}\right)\right] \tag{16}$$

In Equation (16), OF indicates the objective function and includes the sum of the VP term of all buses, LMTAP, and PA between two connected buses with their own weight coefficients ($\omega_i, \omega_i^{'}$, and $\omega_i^{''}$, respectively). These coefficients determine the relative importance between VP_i, $P_{ij,Max}$, and δ_j and make a reasonable relation between them. As indicated in Equation (16), compensated values of VP_i and δ_j should be less than their initial values in order to minimize the OF. However, compensated values of $P_{ij,Max}$ should be more than their initial values.

It should be mentioned that the independent variable of the proposed objective function and optimization problem is the inserted voltage of FACTS devices as the VSCs. At the end of solving the optimization problem, it is desired to find the values of the inserted voltage by different FACTS devices based on an optimum solution. Hence, in the studies of the optimum solution and in order to reach the best scenario, the most important parameter of decision making is the recommended values of VSCs.

5. OPTIMIZATION PROCESS

In this paper, to optimize the proposed OF, the GA is used. The flowchart related to the optimization process is shown in Figure 6. In order to obtain the optimum solution and the best value of the OF, selecting the proper parameters such as population size, generation size, and other GA parameters is necessary [25]. In this paper, the GA toolbox of MATLAB is used to implement the optimization problem solving. In order to solve the optimization problem, the steps shown in the optimization flow chart are performed.

6. TEST RESULTS

The proposed algorithm is applied to a typical 6-bus and 30-bus IEEE test systems. In this paper, the following assumptions are considered. Network designers' budget can be allocated to provide three

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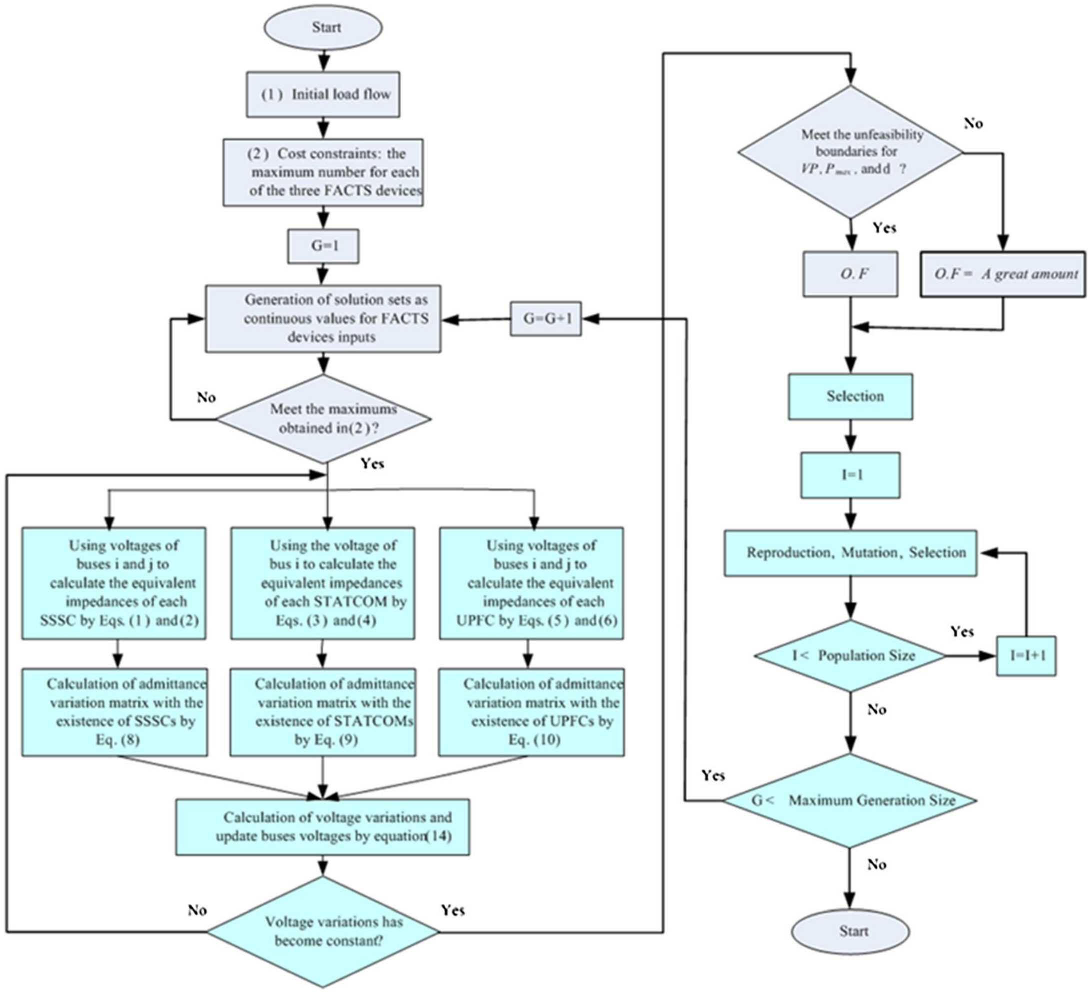

Figure 6. Flowchart of the proposed algorithm.

SSSCs, three STATCOMs, or two UPFCs in three scenarios. Due to the high cost of UPFCs in relation to that amount for SSSCs and STATCOMs, the cost of providing 2 UPFCs in the third scenario is approximately equivalent to the spent cost for purchasing three SSSCs or three STATCOMs in the first or second scenarios.

First, the load flow of the networks in initial state was taken by DIgSILENT. Consequently, in the proposed method based on initial load flow computation and sensitivity analysis and equivalent impedance model, the effect of FACTS devices is determined. The power flow results of the 6-bus test system can be found in Table I The initial values of voltage profile, transmission capacity, and phase angle of the 6-bus system presented in Table I are used in the next steps of optimization process.

The information of the 6-bus test system shown in Figure 7 is available in Tables II and III. After solving the optimization problem using GA, the results of compensations with three devices are shown in Tables IV–VI. In order to solve the optimization problem and find the best solution for the 6-bus test system, population and generation sizes, elite value, and crossover fraction of GA have been selected to be 30 and 100, 0.2, and 0.8, respectively. Furthermore, the migration direction is assumed forward, and the stopping criterion is set as generations that are equal to 100 in this case study. It should be mentioned that, by using the convergence diagram, reproducibility of responses, etc., it is possible to confirm the global optimality of the obtained results.

As demonstrated in these tables, compensated amounts of VP and δ have decreased in order to reach better conditions. Furthermore, the amount of P_{max} has increased according to the results of three compensations. To select the best result of the three compensations with SSSCs, STATCOMs, and UPFCs,

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Table I. Initial values of voltage profile, transmission capacity, and phase angle of six-bus test system.

Bus no.	Vinit (p.u.)	VPinit (p.u.)	Sending bus no.	Receiving bus no.	$P_{\max}^{init}$	δ_{init} (degree)
1	1.0000	0.0000	1	4	1.5831	10.7850
2	0.8699	0.1301	1	5	4.6987	10.9474
3	0.9999	0.0001	2	3	4.3491	9.3203
4	0.95	0.0501	2	4	2.0658	25.6717
5	0.9398	0.0602	3	5	4.6985	5.3809
6	1.0500	0.0500	4	6	3.3247	11.8411

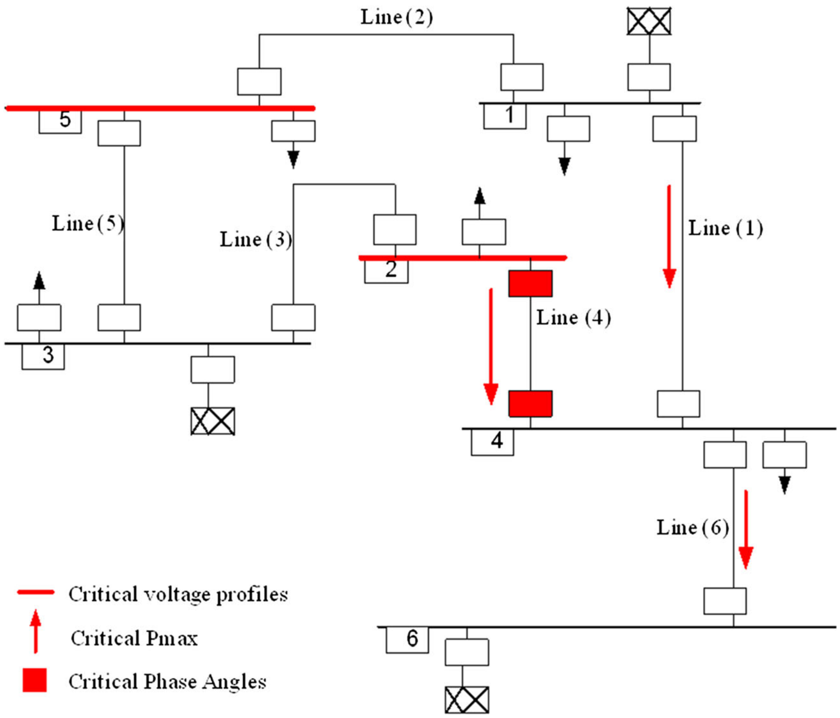

Figure 7. Single line diagram of the 6-bus test system.

Table II. Six-bus system's loads and generations data.

Load (MVAr)	Load (MW)	Generation	Bus type
0	80	150	PV
40	240	0	PQ
0	40	360	PV
24	160	0	PQ
40	240	0	PQ
0	0	600	Slack

Table III. Six-bus system topology data.

Reactance (p.u.)	Resistance (p.u.)	Length (mile)	To bus	From bus
0.6	0.15	60	4	1
0.2	0.05	20	5	1
0.2	0.05	20	3	2
0.4	0.1	40	4	2
0.2	0.05	20	5	3
0.3	0.075	30	6	4

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Table IV. Results of optimal compensation of six-bus test system with SSSCs.

Bus no.	VP_{init}	VP_{comp}	Line no.	$P_{\max}^{init}$	$P_{\max}^{comp}$	δ_{init} (degree)	δ_{comp} (degree)
1	0.000	0.061	1	1.583	1.563	10.785	12.885
2	0.131	0.094	2	4.699	5.712	10.947	6.835
3	0.001	0.001	3	4.349	4.788	9.320	5.413
4	0.051	0.063	4	2.066	2.070	25.672	24.169
5	0.063	0.045	5	4.699	4.931	5.381	7.963
6	0.049	0.023	6	3.325	3.025	11.841	12.614

Table V. Results of optimal compensation of six-bus test system with STATCOMs.

Bus no.	VP_{init}	VP_{comp}	Line no.	$P_{\max}^{init}$	$P_{\max}^{comp}$	δ_{init} (degree)	δ_{comp} (degree)
1	0.000	0.027	1	1.583	1.7253	10.785	10.383
2	0.131	0.091	2	4.699	4.6577	10.947	11.245
3	0.001	0.029	3	4.349	4.6777	9.320	8.855
4	0.051	0.027	4	2.066	2.3350	25.672	23.521
5	0.063	0.076	5	4.699	4.7551	5.381	6.963
6	0.049	0.040	6	3.325	3.4383	11.841	10.876

Table VI. Results of optimal compensation of six-bus test system with UPFCs.

Bus no.	VP_{init}	VP_{comp}	Line no.	$P_{\max}^{init}$	$P_{\max}^{comp}$	δ_{init} (degree)	δ_{comp} (degree)
1	0.000	0.008	1	1.583	1.5960	10.785	6.332
2	0.131	0.055	2	4.699	4.8021	10.947	12.319
3	0.001	0.099	3	4.349	5.2006	9.320	10.901
4	0.051	0.071	4	2.066	2.2458	25.672	23.010
5	0.063	0.058	5	4.699	5.2394	5.381	6.453
6	0.049	0.001	6	3.325	3.1669	11.841	7.205

the proposed method selects UPFC optimization due to its lower OF value according to Table VII ($OF_{UPFC} = -1.124$) in comparison with OF values of SSSC ($OF_{SSSC} = -0.889$) and STATCOM ($OF_{STATCOM} = -0.524$). Considering the mentioned assumption of this paper, the cost of providing and installation of two UPFCs is equal to the cost of three SSSCs or three STATCOM. Therefore, the proposed method prefers to choose and utilize two UPFCs due to their lower OF. The locations and input values of UPFC compensators used in the network are demonstrated in Table VIII. According to Table VIII, the optimal compensation with SSSC uses three SSSCs in lines (1) to (3) with

Table VII. Objective functions of three compensations of six-bus test system with SSSCs, STATCOMs, and UPFCs.

OF of SSSC compensation (OF_{SSSC})	−0.889
OF of STATCOM compensation ($OF_{STATCOM}$)	−0.524
OF of UPFC compensation (OF_{UPFC})	−1.124

Table VIII. Optimal locations and input values of three compensations of six-bus test system with SSSCs, STATCOMs, and UPFCs.

Line no.	1	2	3	4	5	6
Input voltage values of SSSCs	0.08	0.06	0.24			
Bus no.	1	2	3	4	5	6
Input voltage values of STATCOMs	1.04				0.94	0.92
Line and bus no.	1	2	3	4	5	6
Input shunt voltage values of UPFCs		1.02		0.97		
Input series voltage values of UPFCs		0.05		−0.05		

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Table IX. Thirty-bus IEEE test system loading information.

Bus no.	Bus type	Generation (MW)	Active load (MW)	Reactive load (MVAr)	Bus no.	Bus type	Generation (MW)	Active load (MW)	Reactive load (MVAr)
1	Slack	23.54	0	0	16	PQ	0	3.5	1.8
2	PV	60.97	21.7	12.7	17	PQ	0	9	5.8
3	PQ	0	2.4	1.2	18	PQ	0	3.2	0.9
4	PQ	0	7.6	1.6	19	PQ	0	9.5	3.4
5	PQ	0	0	0	20	PQ	0	2.2	0.7
6	PQ	0	0	0	21	PQ	0	17.5	11.2
7	PQ	0	22.8	10.9	22	PV	21.59	0	0
8	PQ	0	30	30	23	PV	19.2	3.2	1.6
9	PQ	0	0	0	24	PQ	0	8.7	6.7
10	PQ	0	5.8	2	25	PQ	0	0	0
11	PQ	0	0	0	26	PQ	0	3.5	2.3
12	PQ	0	11.2	7.5	27	PV	26.91	0	0
13	PV	37	0	0	28	PQ	0	0	0
14	PQ	0	6.2	1.6	29	PQ	0	2.4	0.9
15	PQ	0	8.2	2.5	30	PQ	0	10.6	1.9

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Table X. Thirty-bus IEEE test system topology information.

Line no.	Sending bus	Receiving bus	Resistance (Ohm)	Reactance (Ohm)	Line no.	Sending bus	Receiving bus	Resistance (Ohm)	Reactance (Ohm)
1	1	2	0.02	0.06	22	15	18	0.11	0.22
2	1	3	0.05	0.19	23	18	19	0.06	0.13
3	2	4	0.06	0.17	24	19	20	0.03	0.07
4	3	4	0.01	0.04	25	10	20	0.09	0.21
5	2	5	0.05	0.2	26	10	17	0.03	0.08
6	2	6	0.06	0.18	27	10	21	0.03	0.07
7	4	6	0.01	0.04	28	10	22	0.07	0.15
8	5	7	0.05	0.12	29	21	22	0.01	0.02
9	6	7	0.03	0.08	30	15	23	0.1	0.2
10	6	8	0.01	0.04	31	22	24	0.12	0.18
11	6	9	0	0.21	32	23	24	0.13	0.27
12	6	10	0	0.56	33	24	25	0.19	0.33
13	9	11	0	0.21	34	25	26	0.25	0.38
14	9	10	0	0.11	35	25	27	0.11	0.21
15	4	12	0	0.26	36	28	27	0	0.4
16	12	13	0	0.14	37	27	29	0.22	0.42
17	12	14	0.12	0.26	38	27	30	0.32	0.6
18	12	15	0.07	0.13	39	29	30	0.24	0.45
19	12	16	0.09	0.2	40	8	28	0.06	0.2
20	14	15	0.22	0.2	41	6	28	0.02	0.06
21	16	17	0.08	0.19					

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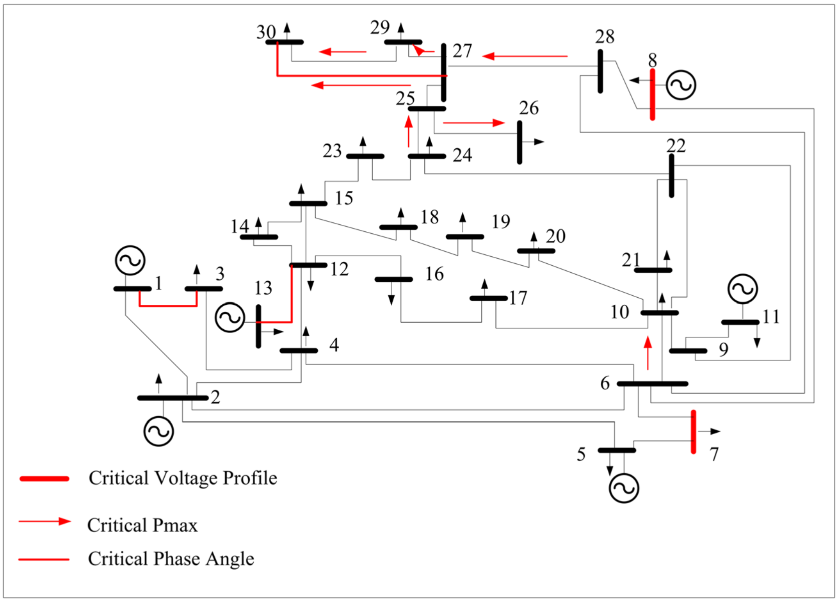

Figure 8. Single line diagram of the 30-bus IEEE test system.

input voltages of '0.08', '0.06', and '0.24' p.u., respectively. Also, optimal allocation of STATCOMs utilizes three STATCOMs located in buses (1), (5), and (6) with input voltage values of '1.04', '0.94', and '0.92' p.u., respectively. Furthermore, the best allocation result is obtained by using 2 UPFCs connected to buses (1) and (2) in lines (2) and (4) with input shunt voltage values of '1.02' and '0.97' p.u., and input series voltage values of '0.05' and '−0.05' p.u.

To clear the effectiveness of the proposed method in order to apply on the large-scale power system, it is applied on the 30-bus IEEE test system. The information of this test system has been presented in Tables IX and X. The single line diagram of this test system is shown in Figure 8. The critical lines and buses of this test system have been demonstrated by different colors. In this case study, the optimization problem is solved by GA with 300 population size and 400 generations. The selected crossover, mutation, and other parameters are similar to ones considered in the 6-bus test system optimization problem. The optimum solution can be obtained by 400 iterations because the generation size is considered as the stop criterion. Since using the proposed method significantly simplifies and decreases the equations and calculations, selecting the great iterations or population sizes can be feasible based on the required process time.

The optimization results of compensation using SSSC, STATCOM, and UPFC have been expressed in Tables XI–XIV. It should be noted the values of ω, ω', and ω'' are assumed to be 0.9, 0.02, and 0.08, respectively.

Table XI. Thirty-bus IEEE test system optimization results of by SSSC compensation.

Line no.	Reference voltage of SSSCs VSC (p.u.)
9	0.6
10	0.3
38	0.4

Table XII. Thirty-bus IEEE test system optimization results by STATCOM compensation.

Bus no.	Reference voltage of STATCOM's VSC (p.u.)
6	0.98
7	0.98
8	0.97

Table XIII. Thirty-bus IEEE test system optimization results by UPFC compensation.

Line no.	Reference voltage of UPFC's series VSC (p.u.)	Reference voltages of UPFC's parallel VSC (p.u.)
Line 9 on bus 6	0.5	0.98
Line 10 on bus 8	0.3	0.97

Table XIV. Thirty-bus IEEE test system optimization results.

Optimization results	Value (p.u.)
OF of SSSC compensation (OF_{SSSC})	−0.1264
OF of STATCOM compensation ($OF_{STATCOM}$)	−0.1587
OF of UPFC compensation (OF_{UPFC})	−0.1118

As shown in the optimization results, the best approach is using three STATCOMs on buses 6, 7, and 8 for this 30-bus system due to its lower OF. The second option is using three SSSCs in lines 9, 10, and 38. The third choice can be using two UPFCs connected to buses 6 and 8 and series in lines 9 and 10, respectively.

7. CONCLUSION

The optimal allocation of different FACTS devices in power systems improves many essential characteristics of systems. So far, many attempts have been taken into account to find the best solution of optimal allocation of one specific FACTS device. However, time-consuming load flows decrease the performance and effectiveness of ordinary methods to apply specially in large-scale systems. In this paper, a novel approach is proposed to obtain the best allocation of different FACTS devices (SSSC, STATCOM, and UPFC) using equivalent impedance models of such devices. This method is very time efficient and also applicable to find the best result in a noticeably large search area. Therefore, all the desired unknown parameters of the optimum situation can be easily found. Applying the proposed method on typical 6-bus and 30-bus IEEE test systems shows the desirable performance of the introduced method.

8. NOMENCLATURE

VP	Voltage profile index
$LMTAP$	Maximum active power transmission capability of transmission lines
STS	Static and transient stability
PA	Phase angle index
$\vec{I}_{ij}$	Current passing through connecting line of buses i and j
$\vec{I}_i$	Current of the parallel branch connected to bus i
$\vec{V}_i$	Voltage of bus i
Ve_i	Equivalent voltage of the parallel part of FACTS controller connected to bus i
$\vec{V}\,e_{ij}$	Equivalent voltage of series part of FACTS controller located in line connecting buses i and j
ϕ_i	Phase of the voltage of bus i
Z_L	Line impedance
X_j	Reactance of j-th line
X_{ij}	Reactance of line connecting buses i and j
Xt	FACTS controller transformer reactance

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Ze_i	Equivalent impedance of the parallel part of FACTS controller connected to bus i
Ze_{ij}	Equivalent impedance of the series part of FACTS controller located in line connecting buses i and j
Q_i	Injected reactive power from UPFC to bus i
r_i	Resistance parameter of the parallel part of UPFC
x_i	Reactance parameter of the parallel part of UPFC
$\overline{V}_{init}, \overline{V}_{new}$	Voltage matrix of initial (without any compensating device) and compensated states
$\overline{\Delta V}$	Voltage variation matrix
$\overline{\Delta Y}$	Admittance variation matrix
Δy_{ij}	i-th row and j-th column of ΔY
$\underline{Y}_{init}, \underline{Y}_{new}$	Admittance matrix of initial and compensated states
$\vec{I}_{init}, \vec{I}_{new}$	Current matrix of initial and compensated states
$\vec{I}_{injected}$	Injected current matrix of FACTS controllers
$p_{ij,\max}$	Maximum transmittable active power between buses i and j
$p_{\max}^{init}, p_{j,\max}^{comp}$	Initial and compensated amounts of $p_{j,\max}$
$\overline{p}_{\max}^{init}$	Average value of the initial maximum transmission capability of all lines
$\delta_j^{init}, \delta_j^{comp}$	Initial and compensated amounts of the phase angle of j-th line
$\overline{\delta}^{init}$	Average value of the initial phase angle of all lines
$VP_i^{init}, VP_i^{comp}, VP_{init}$	Initial and compensated amounts of VP_i
$\overline{VP}_{init}$	Average value of the initial voltage profile of all buses
ω_i	Weight coefficient regarding the voltage profile of bus i
ω_j'	Weight coefficient regarding the maximum transmission capability of line j
ω_j''	Weight coefficient regarding the phase angle of line j
N	Number of network buses
M	Number of network lines

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