1 [Q1]

Last week, we talked about Lagrange.

2 [Q2]

done

3 [Q3]

3.1 (a)

$$\|x\|_\infty = \max\{|x|\} = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases} \tag{1}$$

Hence, the gradient of $\|x\|_\infty$ is,

$$\partial\|x\|_\infty = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases} \tag{2}$$

And we can write it as,

$$\partial\|x\|_\infty = \left\{u : \|u\|_1 = 1, u^T x = \|x\|_\infty\right\} \tag{3}$$

3.2 (b)

4 [Q7]

4.1 (a)

According to KKT condtion (K1), we have,

$$(x-2)(x-4) \leq 0 \tag{4}$$

the feasible set is $\{x : 2 \leq x \leq 4\}$

4.2 (b)

The Lagrangian for this problem is,

$$L(x, \lambda) = x^2 + 1 + \lambda(x-2)(x-4)$$

According to KKT conditions (K4), we have,

$$\frac{\partial L}{\partial x} = 2x + 2\lambda x - 6\lambda$$

Set it to 0, we can get x as follows,

$$x = \frac{3\lambda}{\lambda + 1}$$

Lagrange dual program is

$$\underset{\lambda}{\text{maximize}} \frac{-\lambda^3 + 8\lambda^2 + 10\lambda + 1}{(\lambda + 1)^2}, \quad \text{subject to } \lambda \geq 0$$

Apprently, this will be maximize when $\lambda = 2$, which, when substituted into the dual problem yields,

$$d(\lambda^\star) = 5$$

Hence, the optimal $x^\star$ is **2**, accordingly, the value of the objective function at the minimizer is **5**.

4.3 (c)

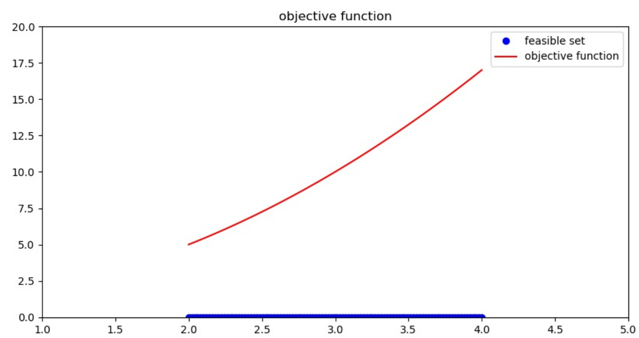

Figure 1: Plot of Objective Function

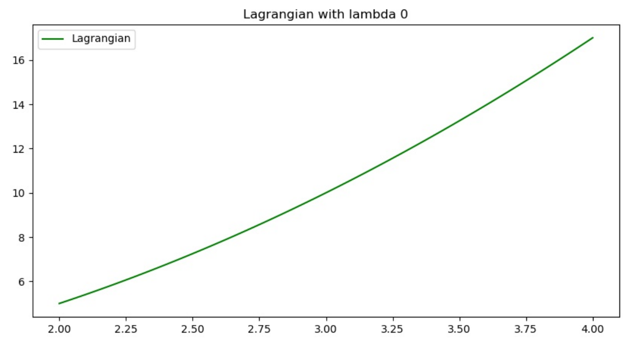

Figure 2: Plot of Lagrangian with $\lambda = 0$

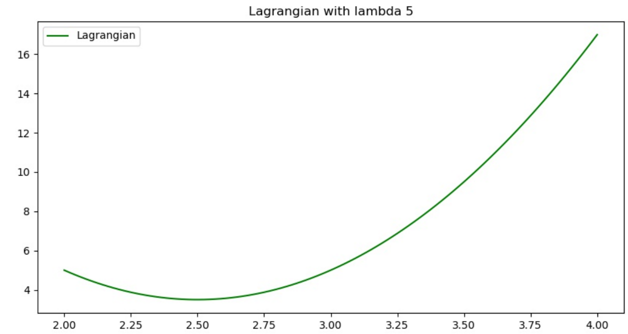

Figure 3: Plot of Lagrangian with $\lambda = 5$

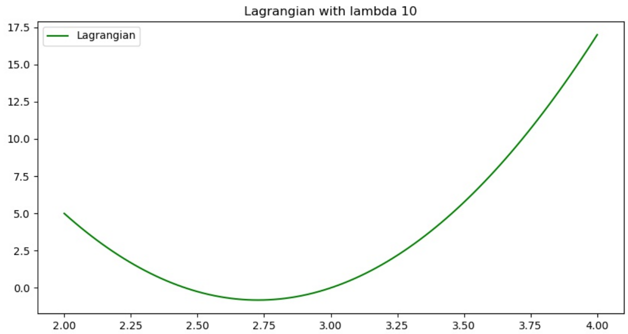

Figure 4: Plot of Lagrangian with $\lambda = 10$

4.4 (d)

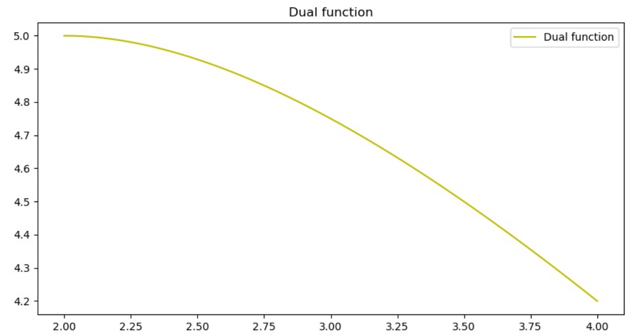

Figure 5: Plot of Dual function

4.5 (e)

The dual problem is

$$\underset{\lambda}{\text{maximize}} \; \frac{-\lambda^3 + 8\lambda^2 + 10\lambda + 1}{(\lambda+1)^2}, \quad \text{subject to } \lambda \geq 0$$

And the maximizer $\lambda^\star = 2$, accordingly $d(\lambda^\star) = 5$.

Strong duality holds because the value of primal problem $p^\star$ is **5** and the value of dual problem $d^\star$ is **5**.