1 [Q1]

Last week, we talked about the gradient and the hessian. Gradient is the direction which the function descends fastest.

2 [Q2]

done

3 [Q3]

3.1 (a)

Assume $f(\boldsymbol{x}), f(\boldsymbol{y})$ are convex, then we have,

$$f(\boldsymbol{x}) \geq f(\boldsymbol{y}) + \langle \boldsymbol{x} - \boldsymbol{y}, \nabla f(\boldsymbol{y}) \rangle$$

$$f(\boldsymbol{y}) \geq f(\boldsymbol{x}) + \langle \boldsymbol{y} - \boldsymbol{x}, \nabla f(\boldsymbol{x}) \rangle$$

Combine the 2 inequalities together and we can get,

$$\begin{aligned} f(\boldsymbol{x}) + f(\boldsymbol{y}) \geq f(\boldsymbol{y}) + f(\boldsymbol{x}) + \langle \boldsymbol{x} - \boldsymbol{y}, \nabla f(\boldsymbol{y}) \rangle + \langle \boldsymbol{y} - \boldsymbol{x}, \nabla f(\boldsymbol{x}) \rangle \\ - \langle \boldsymbol{x} - \boldsymbol{y}, \nabla f(\boldsymbol{y}) \rangle - \langle \boldsymbol{y} - \boldsymbol{x}, \nabla f(\boldsymbol{x}) \rangle \geq 0 \\ \langle \boldsymbol{y} - \boldsymbol{x}, \nabla f(\boldsymbol{y}) \rangle + \langle \boldsymbol{y} - \boldsymbol{x}, -\nabla f(\boldsymbol{x}) \rangle \geq 0 \\ \langle \boldsymbol{y} - \boldsymbol{x}, \nabla f(\boldsymbol{y}) - \nabla f(\boldsymbol{x}) \rangle \geq 0 \end{aligned}$$

3.2 (b)

3.3 (c)