ECE 6270: Convex Optimization: Theory, Algorithms, and Applications
Spring 2021 Syllabus

Summary

This course will cover the fundamentals of convex optimization. We will talk about mathematical fundamentals, modeling (i.e., how to set up optimization problems in different applications), and algorithms.

Prerequisites

Students should be familiar with linear algebra (e.g., solving systems of equations, least squares, matrix factorizations including SVD), basic probability (e.g., you should be comfortable with multivariate probability densities), and have good MATLAB or Python programming skills.

Instructors

Mark Davenport (mdav@gatech.edu)
Magnus Egerstedt (magnus.egerstedt@gatech.edu)
Office hours: Fridays, 1-2pm

Teaching Assistant

Nauman Ahad
Email: nauman.ahad@gatech.edu
Office hours: Mondays, 11-noon

Lecture

Lectures are Mondays and Wednesdays from 9:30-10:45am and will be held either online or in Klaus 1456.

Instruction Modality

In Spring 2021, ECE 6270 will be taught in a hybrid mode. What this will mean in this course is that, while we do have a room on campus reserved and available (Klaus 1456), some lectures will be delivered remotely. Moreover, all lectures will be recorded and available to watch at whatever time is most convenient for you. If you are uncomfortable with (or unable to) attend lectures in person, it will be possible to complete this course fully online.

The course will be fully online (meaning that our discussions and office hours will be conducted online) for at least the first week of class, and possibly through the end of January. We then hope to transition to holding at least some of the lectures on campus during our regularly scheduled class time for those who wish to attend. These in-person meetings will continue to be recorded for those who cannot or do not want to attend. (Note that, depending on the demand, we may have to create a rotating schedule for in-person attendance depending on final enrollment and room capacity.)

Grading

- **Homeworks (40%):** There will be ≈ 10 homework assignments. See further details below.
- **Midterm exam (30%):** There will be a midterm exam to be completed remotely. This is tentatively scheduled for March 8.
- **Final exam (30%):** The final exam will be completed remotely, and is scheduled for April 30 from 8-10:50am.

Your final grade will be assigned as a letter grade according to the scale:

A: 90-100% B: 80-89% C: 70-79% D: 60-69% F: $\leq$ 59%

We may exercise the option to "curve" exam scores as necessary (by adjusting the grades higher, but not lower) if we determine that an exam was more difficult than intended.

Homework

Homework will be assigned weekly (approximately). **Homework will be turned in via canvas. Unless you have made prior arrangements with me, late submissions will get zero credit.** Each homework assignment will be graded out of 100 points. Over the course of the semester, **the maximum number of homework points that you can earn is $(N-1)\cdot 100$, where N is the number of assignments** – this serves a similar role to allowing you to drop one homework assignment, but should encourage you to still submit a partially completed one (and avoid panic if there is occasionally a problem that you do not finish in time.)

The homework assignments will be hard; many of them will require significant amounts of time and effort to complete. But this is really where most of the learning takes place. You will get out of the assignments what you put into them. Students who complete all of the assignments in full will be rewarded with a deep understanding of the role that linear algebra and optimization play in data science, machine learning, robotics, and controls (among other things). Effectively, homework is worth much more than 30% of your grade. In teaching many courses over the years, the instructors **have never seen a case where a student does not put effort into the homework assignments but does well on the exams**.

Students are *strongly* encouraged to discuss homework problems with one another. However, **each student must write up and turn in their own solutions written in their own words. Cases where solutions appear to be identical or nearly identical will be immediately referred to the Office of Student Integrity.**

Unauthorized use of any previous semester course materials, such as tests, quizzes, and homework, is prohibited in this course. Furthermore, redistributing materials from this semester is also prohibited. For any questions involving these or any other Academic Honor Code issues, please consult me or www.honor.gatech.edu.

Online resources

The course webpage is at: mdav.ece.gatech.edu/ece-6270-spring2021. This page will provide general course information, copies of the lecture notes, homework assignments, relevant papers, and other resources. Homework solutions and some additional resources will be posted in canvas as necessary.

We plan to make exclusive use of Piazza to make announcements and answer questions. This site can be accessed via: piazza.com/gatech/spring2021/ece6270/home. Piazza is a great platform for you to work with your fellow students to discuss problems, form study/project groups, etc. Please direct any questions you might have to Piazza. Unless your questions are personal in nature, please do not make private posts – if you have a question you are probably not the only one, and other students may benefit from seeing the discussion.

Text

There is no required text. Course notes will be posted as they become available at the course website. These notes will be based on material sourced from several different texts. The one we will follow most (but certainly not all) the time is

- Boyd and Vanderberghe: *Convex Optimization*
  Available at `http://amzn.to/2RBbH30`, but also available as a free pdf at `http://web.stanford.edu/~boyd/cvxbook/`

Here are other books which are very useful:

- Bertsekas, Nedic, and Ozdaglar: *Convex Analysis and Optimization*
  `http://amzn.to/2C6cxek`

- Nocedal and Wright: *Numerical Optimization*
  `http://amzn.to/2VEpmp0`

- Ben-Tal and Nemirovski: *Lectures on Modern Convex Optimization*
  http://amzn.to/2RDoKRx
- Luenberger: *Optimization by Vector Space Methods*
  http://amzn.to/2GZs0Cx

I may also provide some additional resources (e.g., papers and excerpts from other books) on the course website and/or using canvas as appropriate.

We will also use the CVX MATLAB package, which makes it easy to prototype many of the optimization programs we will see this semester. It is available (for free) at http://cvxr.com/cvx/.

Course Objectives

Upon successful completion of this course, students should:

1. Be able to recognize and differentiate between common classes of optimization problems.
2. Have an understanding of how duality can be exploited to develop alternative approaches to solving an optimization problem.
3. Be able to implement and analyze the convergence properties of common iterative optimization algorithms.
4. Be able to translate practical engineering problems into optimization problems (modeling).

Course Expectations and Guidelines

COVID-19 considerations

All Georgia Tech faculty and students are required to wear face coverings while inside campus facilities/buildings. This includes classrooms and offices. If/when we transition to on-campus meetings, you must wear an appropriate face covering. This, however, is not a substitute for other social distancing measures.

As noted above, you may complete this course fully online if you have any discomfort with attending in-person activities.

Academic integrity

Georgia Tech aims to cultivate a community based on trust, academic integrity, and honor. Students are expected to act according to the highest ethical standards. For information on Georgia Tech's Academic Honor Code, please visit www.catalog.gatech.edu/policies/honor-code. Any student suspected of cheating or plagiarizing on a quiz, exam, or assignment will be reported to the Office of Student Integrity, who will investigate the incident and identify the appropriate penalty for violations.

Redistributing materials from this course and/or using external sites for assistance (e.g., contributing to test banks, CourseHero, Chegg, or similar sites) is prohibited.

Collaboration and group work

Students are *strongly* encouraged to discuss homework problems with one another. However, **each student must write up and turn in their own solutions written in their own words**. **Cases where solutions appear to be identical or nearly identical will be immediately referred to the Office of Student Integrity.**

Absences, late assignments, and missed exams

Active participation in the class discussions is expected. Please attend class (either in person or online) unless you have a compelling reason not to do so. However, you will not be penalized for any excused absences (e.g., due to illnesses, religious observances, career fairs, job interviews, etc.) We would like to be able to discuss the homework assignments in class the day after they are due, and thus **we cannot accept late homeworks** in the absence of prior approval. In the event that an excused absence prevents you from submitting an assignment, your homework grade will be calculated on a pro-rated basis. **Exams will be completed remotely, but during specified time frames. If you expect to miss an exam, please contact me as soon as you realize this so we can make alternative arrangements.** We may consider options to take the exam at an alternate time or instead may adjust the grading allocation to place more emphasis on other exams, depending on the circumstances.

Accommodations for students with disabilities

If you are a student with learning needs that require special accommodation, contact the Office of Disability Services at (404)894-2563 or disabilityservices.gatech.edu, as soon as possible, to make an appointment to discuss your special needs and to obtain an accommodations letter. Please also e-mail us as soon as possible in order to set up a time to discuss your learning needs.

Student-Faculty expectations agreement

At Georgia Tech we believe that it is important to strive for an atmosphere of mutual respect, acknowledgement, and responsibility between faculty members and the student body. In the end, simple respect for knowledge, hard work, and cordial interactions will help build the environment we seek. Therefore, we encourage you to remain committed to the ideals of Georgia Tech while in this class. See www.catalog.gatech.edu/rules/22 for an articulation of some basic expectation that you can have of us and that we have of you.

Digital etiquette

Much of this course will occur online. Active participation in the Piazza forum and in remote discussions will be critical to replicating the in-person experience we would all prefer to be having. Please come to our discussion sessions prepared with questions to ask. In general, we would prefer if everyone leaves their video on. It is much easier to have a conversation when we can see each other, and we often rely on this feedback to know when we say something that doesn't actually make sense. Ultimately, however, we do understand that internet connections do not always allow for this, and we will not penalize you if you must disable your video.

Outline

The outline below should be treated as an approximation; it is subject to (hopefully small) changes.

1. Introduction to optimization, examples of convex optimization problems
2. Convexity
   (a) convex sets
   (b) convex functions
   (c) convexity and gradients
3. Unconstrained minimization
   (a) gradient descent
   (b) line search methods for 1D problems
   (c) convergence analysis
   (d) accelerated first order methods (Heavy ball, Nesterov)
   (e) incremental and stochastic gradients
   (f) Newton's method, convergence analysis
   (g) Quasi-Newton methods
   (h) trust region methods
   (i) subgradient descent
   (j) proximal methods
4. Theory for constrained optimization
   (a) optimality conditions
   (b) Fenchel duality
   (c) Lagrange duality
   (d) Karush-Kuhn-Tucker (KKT) conditions
5. Methods for constrained optimization
   (a) barrier techniques
   (b) projected gradient descent
   (c) splitting methods, alternating direction method of multipliers
   (d) ADMM
6. Applications/extensions
   (a) convex relaxation and nonconvex optimization
   (b) optimization for robotics
   (c) optimization for control
   (d) optimization for statistical inference

(e) optimization for machine learning

(f) optimization for inverse problems

Throughout the course, we will be using different applications to motivate the theory. These will cover some well-known (and not so well-known) problems in signal and image processing, communications, control, machine learning, and statistical estimation (among other things).