Georgia Institute of Technology

ECE 6254 - Assignment 8

Fall 2020 - v1.0

- There are 2 problems over 2 pages (including the cover page.
- The problems are not necessarily in order of difficulty.
- All problems are assigned the same weight in the overall grade. The questions in problems with many questions are worth less than the questions in problem with few questions.
- Each question is graded as follows: no credit without meaningful work, half credit for partial work, full credit if essentially correct.
- Unless otherwise specified, you should concisely indicate your reasoning and show all relevant work.
- The grade on each question is based on our judgment of your level of understanding as reflected by what you have written. If we cannot read it, we cannot grade it.
- Please use a pen and not a pencil if you handwrite your solution.
- **You must submit your assignment on Gradescope**.

Problem 1: Donut classifiers

Consider the set $\mathcal{H}$ of donut classifiers, functions on $\mathbb{R}^2$ of the form

$$h(\mathbf{x}) = \begin{cases} -1 \text{ if } \|\mathbf{x}\|_2^2 \leqslant a^2 \\ +1 \text{ if } a^2 < \|\mathbf{x}\|_2^2 \leqslant b^2 \\ -1 \text{ if } b^2 < \|\mathbf{x}\|_2^2 \end{cases} \tag{1}$$

for some $0 < a < b$.

[Q1] Why is it called a donut classifier?

[Q2] What is the growth function $m_{\mathcal{H}}(N)$ of this classifier?

[Q3] What is the VC dimension of $\mathcal{H}$?

Problem 2: Positive rectangles

Consider the set of classifiers defined by *positive rectangles* whose edges are parallel to the coordinate axes in $\mathbb{R}^2$, i.e., h such that for any $\mathbf{x} \in \mathbb{R}^2$,

$$h(\mathbf{x}) = \begin{cases} +1 & \text{if } a \leq x(1) \leq b \text{ and } c \leq x(2) \leq d \\ -1 & \text{otherwise,} \end{cases}$$

for some $a, b, c, d \in \mathbb{R}$. Our objective it to determine the VC dimension d_{VC} of the set of all such classifiers. For whatever d^* you think the answer is,

[Q1] Use an example to show that the VC dimension must be *at least* d^*.

[Q2] Provide an argument that the VC dimension cannot be any larger than d^*.