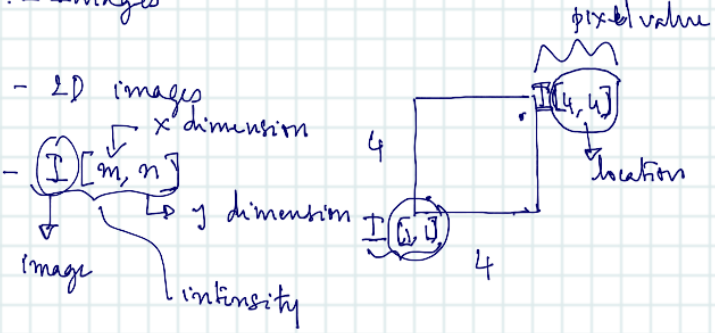
29/10/22

The Convolution Operation

- Signals and Systems
- Signal :- Images



- 2D images
- 1D Signals

Ball index	1	2	3	4	5	6
Runs scored	4	0	0	1	0	2

- $r[n]$, $r[3] = 1$ (assuming index starts at 0)

  $r[5] = 2$

- How is run rate calculated? R.P.O

$$rpo = \frac{\sum_{o=0}^{19} r[o]}{20} \quad \text{--- (1)}$$

← global average

# overs	0	1	. . .	19
# runs	10	2	. . .	5 ← $\sum_{o=0}^{19} r[o]$ = total

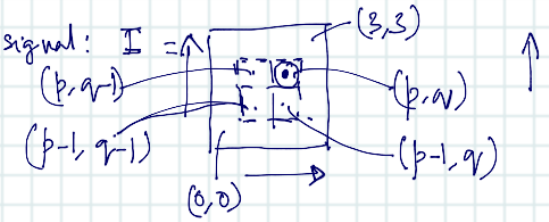
- digital System

$$rpo\text{-}past\text{-}five[N] = \frac{1}{5}\sum_{i=0}^{4} r[N-i]$$

← Averaging system ← moving average

- Digital system: A mapping from an input sequence to a desired output
  (digital / discrete)

- 2D signal: $I =$



- The average pixel intensity of I ?

$$I_{average} = \frac{1}{16}\sum_{m=0}^{3}\sum_{n=0}^{3} I[m,n]$$ ← global average

$$I_{local\ average}[p,q] = \frac{1}{4}\sum_{m=0}^{1}\sum_{n=0}^{1} I[p-m, q-n]$$ ← local average

$$= \frac{1}{4}I[p,q] + \frac{1}{4}I[p-1,q] + \frac{1}{4}I[p,q-1] + \frac{1}{4}I[p-1,q-1]$$

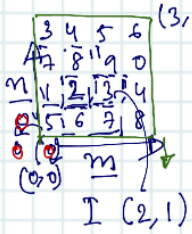
Gen. local avg. $I_{g,\text{local avg}} = W[0,0]\cdot I[p-0, q-0] + W[1,0]\, I[p-1, q-0] + \cdots + W[1,1]\cdot I[p-1, q-1]$

$$I_{g,\text{local avg}} = \sum_{m=0}^{1}\sum_{n=0}^{1} W[m,n]\, I[p-m, q-n] \quad \leftarrow \text{convolution sum.} \qquad I * W$$

Conv. kernel — filter

In a CNN $W[m,n]$ is learnt during training

Ex 1: $I =$ (3,3)



$W = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix}$ $\quad J = I * W$

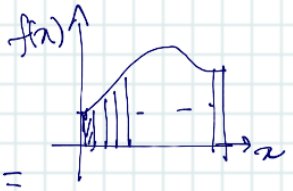
$$J[2,1] = \sum_{m=0}^{1}\sum_{n=0}^{1} I[2-m, 1-n]$$

$$= \frac{1}{4}[8 + 2 + 6 + 7]$$

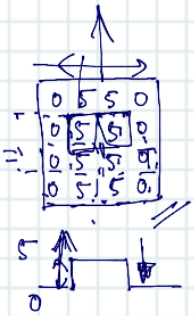
$$J[2,0] = \frac{1}{4}[21] = 5.25$$

$$J[0,0] = \frac{1}{4}[5 + 0 + 0 + 0] = 1.25$$

$$\int f(x)\,dx = \lim_{\Delta x \to 0} \sum f(x)\cdot\Delta x$$



Ex 2:



$W = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

Edge detector

$J[0,2] = 0$

$J[1,2] = -10$ ✓

$J[2,2] = 5\cdot 1 + 5\cdot 1 + 5\cdot -1 + 5\cdot -1 = 0$

$J[3,2] = 10$ ✓

$$\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Ex 2:

$W = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix}$

Averaging

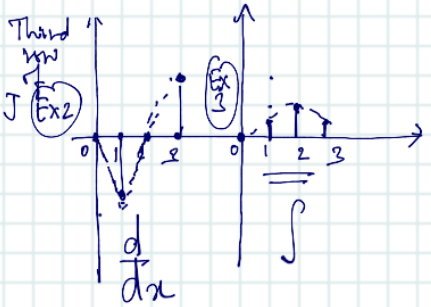
$J[0,2] = 0$

$J[1,2] = \frac{10}{4}$

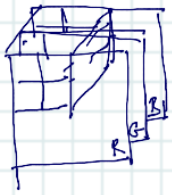
$J[2,2] = \frac{20}{4}$

$J[3,2] = \frac{10}{4}$

Let's plot the third row of J in Ex 2 and 3



$I_c =$



$W = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix}$