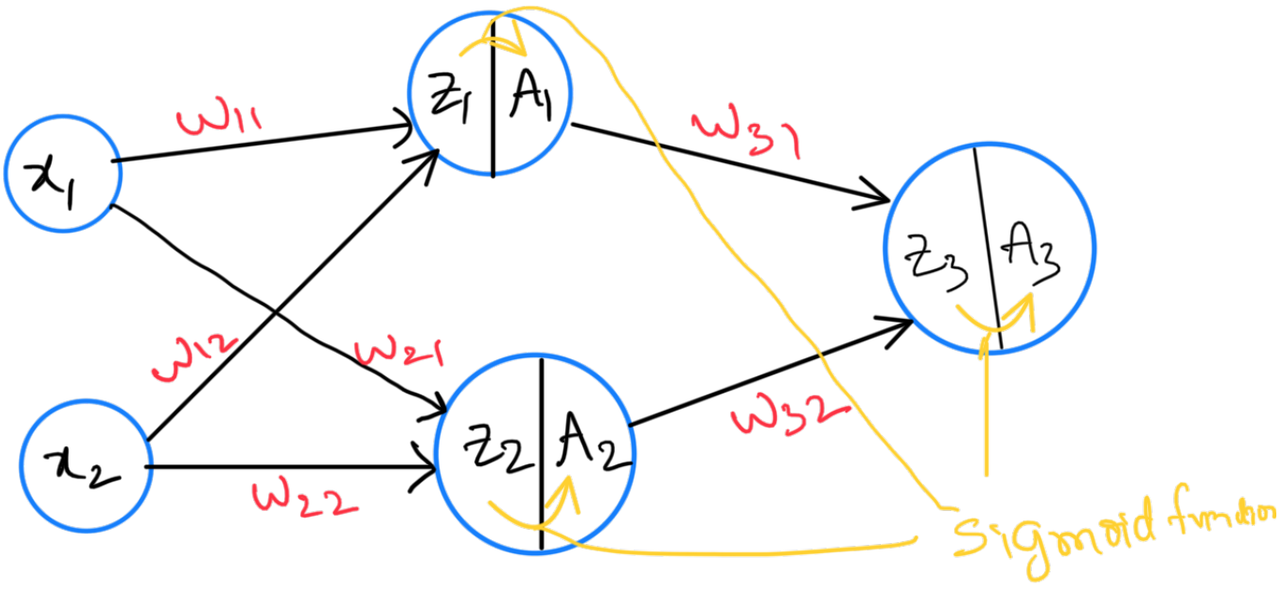
Backpropagation

① Example

i/p hidden o/p

② example code.



$$z_1 = x_1 w_{11} + x_2 w_{12}$$

$$A1 = \sigma(z1) = \frac{1}{1+e^{-z_1}}$$

$$z_2 = x_1 w_{21} + x_2 w_{22}$$

$$A2 = \sigma(z2) = \frac{1}{1+e^{-z_2}}$$

$$z3 = A_1 w_{31} + A2\, w_{32}$$

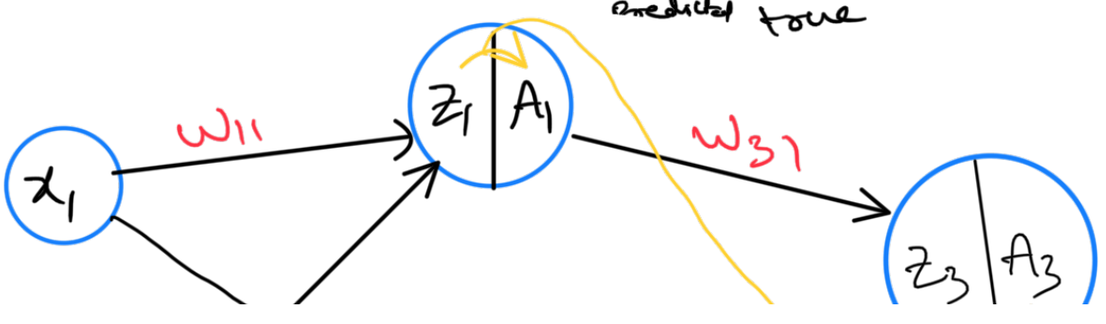
predicted $\Leftarrow$ $$A3 = \sigma(z3) = \frac{1}{1+e^{-z_3}}$$

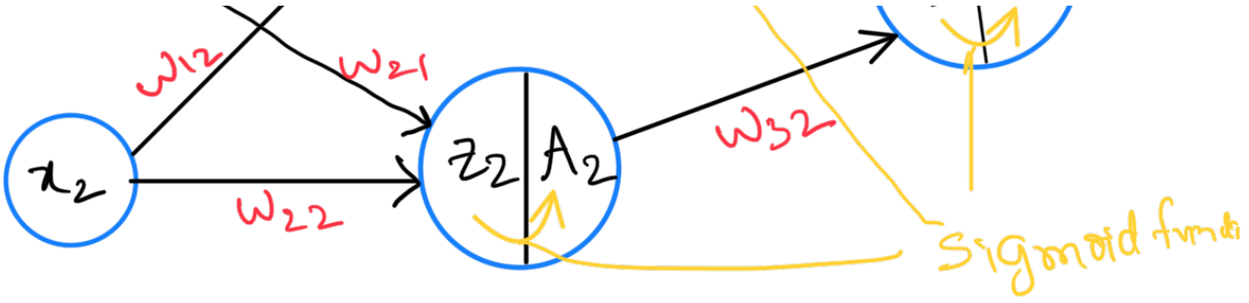
Comparing i.e. error

mean Square error

$$E = \frac{1}{2}(A_3 - y_t)^2$$

(A_3: predicted, y_t: true)





$$\frac{\partial E}{\partial w_{31}} = \frac{\partial E}{\partial A_3} \cdot \frac{\partial A_3}{\partial z_3} \cdot \frac{\partial z_3}{\partial w_{31}} \quad -④$$

$$\frac{\partial E}{\partial A_3} = \frac{\partial \left(\frac{1}{2}(A_3 - y_t)^2\right)}{\partial A_3} = \frac{1}{2} \times 2 (A_3 - y_t)$$

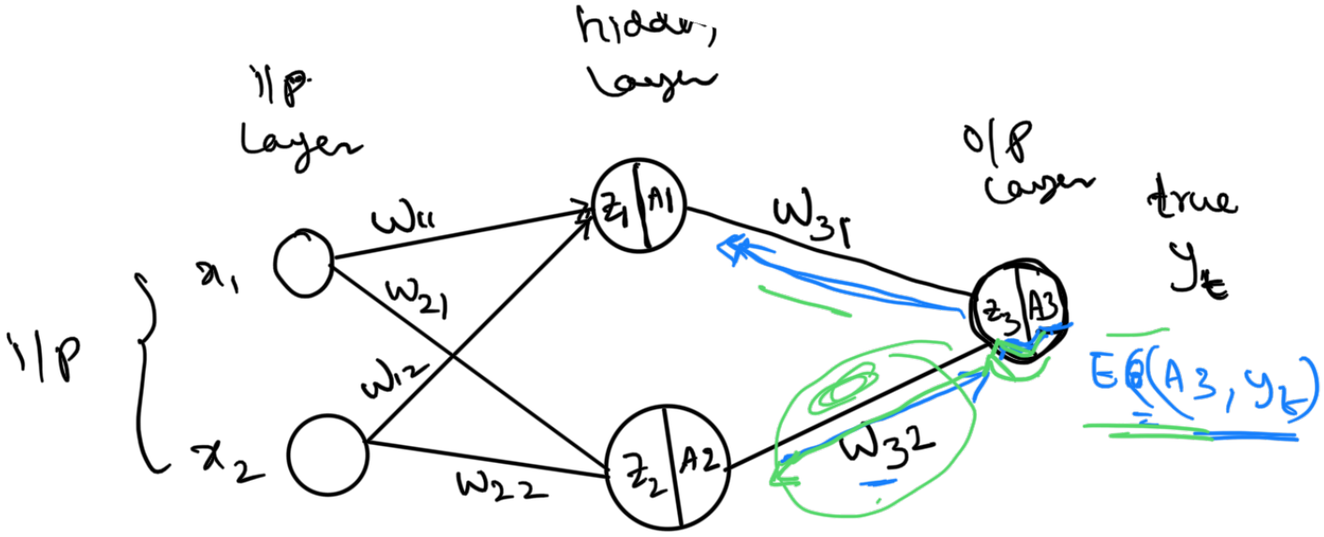
$$= (A_3 - y_t) \quad -①$$

$$\frac{\partial A_3}{\partial z_3} = \frac{\partial (\sigma(z_3))}{\partial z_3} = \frac{\partial \left(\frac{1}{1+e^{-z_3}}\right)}{\partial z_3}$$

$$\frac{\partial A_3}{\partial z_3} = \sigma(z_3)(1 - \sigma(z_3)) = A_3(1 - A_3) \quad -②$$

$$\frac{\partial z_3}{\partial w_{31}} = \frac{\partial (A_1 w_{31} + A_2 w_{32})}{\partial w_{31}} = A_1 \quad -③$$

$$\frac{\partial E}{\partial w_{31}} = (A_3 - y_t)(A_3(1 - A_3))(A_1) \quad -⑤$$



$$\frac{\partial E}{\partial w_{32}} = \frac{\partial E}{\partial A_3} \cdot \frac{\partial A_3}{\partial z_3} \cdot \frac{\partial z_3}{\partial w_{32}}$$

$$\frac{\partial E}{\partial A_3} = \frac{\partial \left(\frac{1}{2}(A_3 - y_t)^2\right)}{\partial A_3} = (A_3 - y_t)$$

$A_3 = \sigma(z_3)$

$$\frac{\partial A_3}{\partial z_3} = \frac{\partial}{\partial z_3}(\sigma(z_3)) = \sigma(z_3)(1-\sigma(z_3))$$

$$= A_3(1-A_3)$$

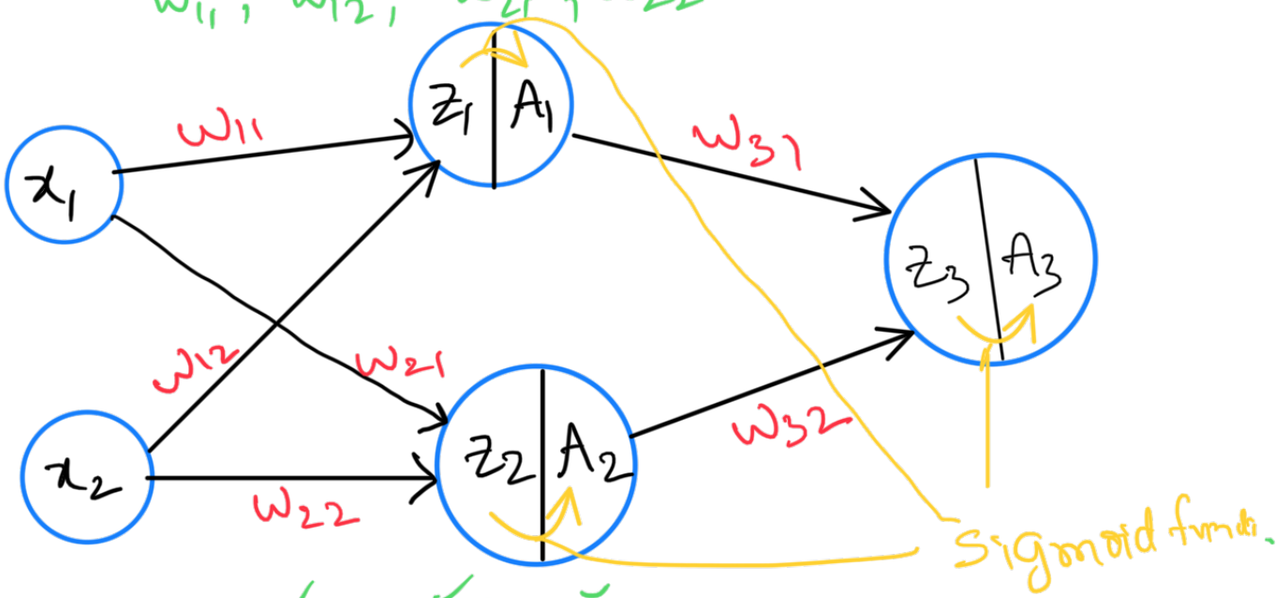
$$\frac{\partial z_3}{\partial w_{32}} = \frac{\partial}{\partial w_{32}}(A_2 w_{32} + A_1 w_{31}) = A_2$$

$$\frac{\partial E}{\partial w_{32}} = (A_3 - y_t)(A_3(1-A_3))A_2$$

$$\begin{bmatrix} \frac{\partial E}{\partial w_{31}} \\ \frac{\partial E}{\partial w_{32}} \end{bmatrix} = \begin{bmatrix} (A_3-y_t)(A_3(1-A_3)) & 0 \\ 0 & (A_3-y_t)(A_3(1-A_3)) \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$$

Weights from i/p to hidden

$w_{11}, w_{12}, w_{21}, w_{22}$



$$\frac{\partial E}{\partial w_{11}} = \frac{\partial E}{\partial A_3} \cdot \frac{\partial A_3}{\partial z_3} \cdot \frac{\partial z_3}{\partial A_1} \cdot \frac{\partial A_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_{11}} \quad -(5)$$

$$\frac{\partial E}{\partial A_3} = (A_3 - y_t) \quad \left| \quad \frac{\partial A_3}{\partial z_3} = A_3(1-A_3) \right.$$

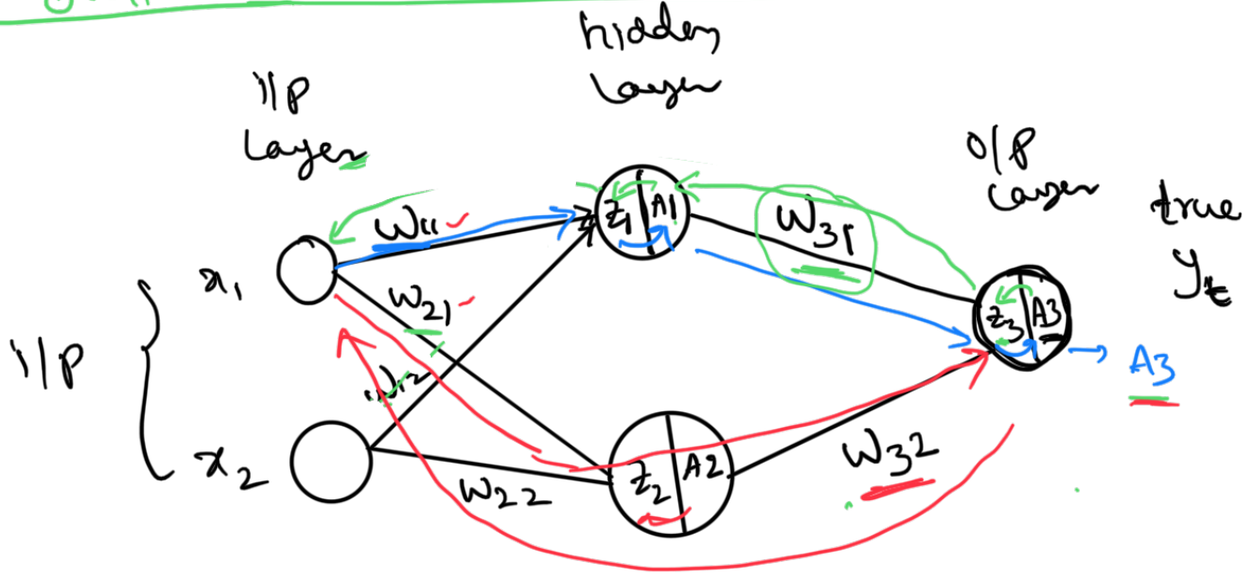
$$\frac{\partial z_3}{\partial A_1} = \frac{\partial}{\partial A_1}(A_1 w_{31} + A_2 w_{32}) = w_{31}$$

$$A_1 = \sigma(z_1)$$

$$\frac{\partial A_1}{\partial z_1} = \frac{\partial(\sigma(z_1))}{\partial z_1} = \sigma(z_1)(1-\sigma(z_1)) = A_1(1-A_1)$$

$$\frac{\partial z_1}{\partial w_{11}} = \frac{\partial(x_1 w_{11} + x_2 w_{12})}{\partial w_{11}} = x_1$$

$$\boxed{\frac{\partial E}{\partial w_{11}} = (A_3 - y_t)(A_3(1-A_3))\, w_{31} A_1(1-A_1)\, x_1}$$



$$\frac{\partial E}{\partial w_{21}} = \frac{\partial E}{\partial A_3} \cdot \frac{\partial A_3}{\partial z_3} \cdot \frac{\partial z_3}{\partial A_2} \cdot \frac{\partial A_2}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_{21}}$$

$$\frac{\partial E}{\partial A_3} = (A_3 - y_t) \cdot \frac{\partial A_3}{\partial z_3} = A_3(1-A_3)$$

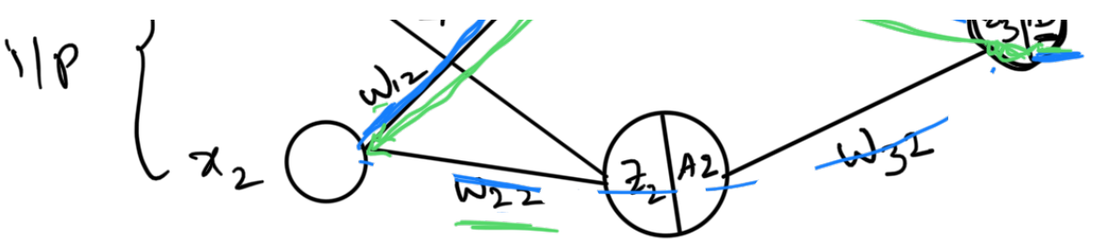
$$\frac{\partial z_3}{\partial A_2} = \frac{\partial(A_1 w_{31} + A_2 w_{32})}{\partial A_2} = w_{32}$$

$$\frac{\partial A_2}{\partial z_2} = \frac{\partial(\sigma(z_2))}{\partial z_2} = A_2(1-A_2)$$

$$\frac{\partial z_2}{\partial w_{21}} = \frac{\partial(x_1 w_{21} + x_2 w_{22})}{\partial w_{21}} = x_1$$

$$\boxed{\frac{\partial E}{\partial w_{21}} = (A_3 - y_t)(A_3(1-A_3))\, w_{32} A_2(1-A_2)\, x_1}$$

$$\frac{\partial E}{\partial w_{12}}$$



$$\frac{\partial E}{\partial w_{12}} = \frac{\partial E}{\partial A_3} \cdot \frac{\partial A_3}{\partial z_3} \cdot \frac{\partial z_3}{\partial A_1} \cdot \frac{\partial A_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_{12}}$$

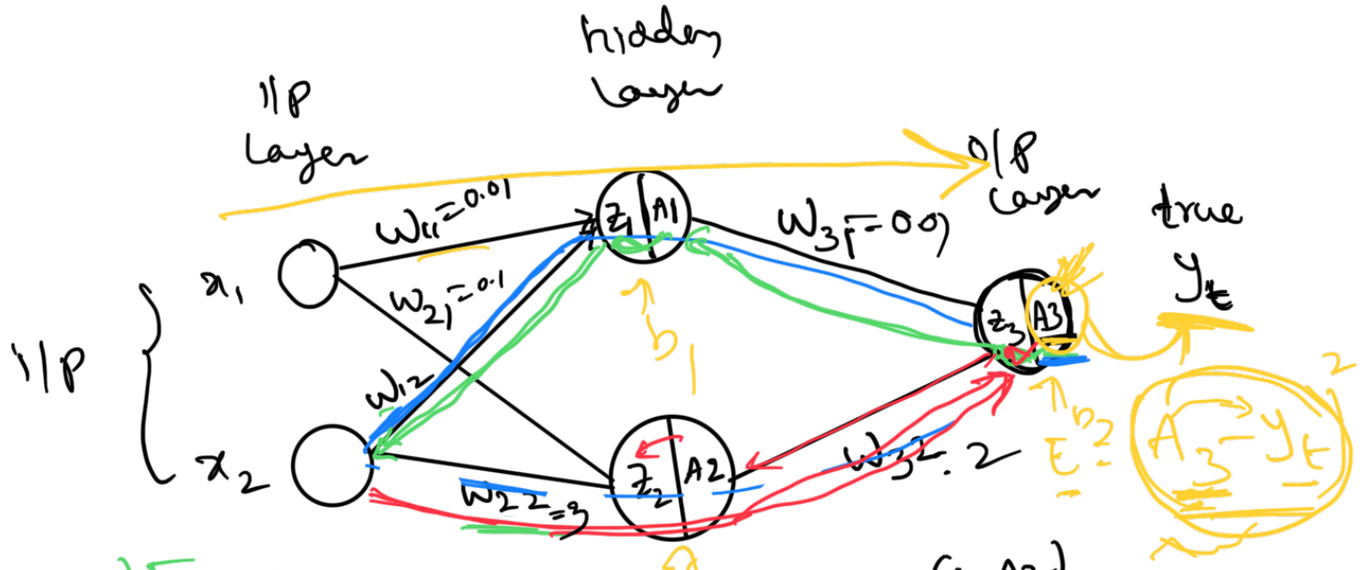
$$\frac{\partial E}{\partial A_3} = (A_3 - y_t) \quad \left| \quad \frac{\partial A_3}{\partial z_3} = A_3(1-A_3) \right.$$

$$\frac{\partial z_3}{\partial A_1} = \frac{\partial (A_1 w_{31} + A_2 w_{32})}{\partial A_1} = w_{31} \qquad (A_2 \text{ term} \to 0)$$

$$\frac{\partial A_1}{\partial z_1} = \frac{\partial (\sigma(z_1))}{\partial z_1} = \frac{\partial}{\partial z_1} \sigma(z_1)(1-\sigma(z_1)) = A_1(1-A_1)$$

$$\frac{\partial z_1}{\partial w_{12}} = \frac{\partial (x_1 w_{11} + x_2 w_{12})}{\partial w_{12}} = x_2 \qquad (x_1 w_{11} \to 0)$$

$$\frac{\partial E}{\partial w_{12}} = (A_3 - y_t)(A_3(1-A_3)) w_{31} A_1(1-A_1) x_2$$



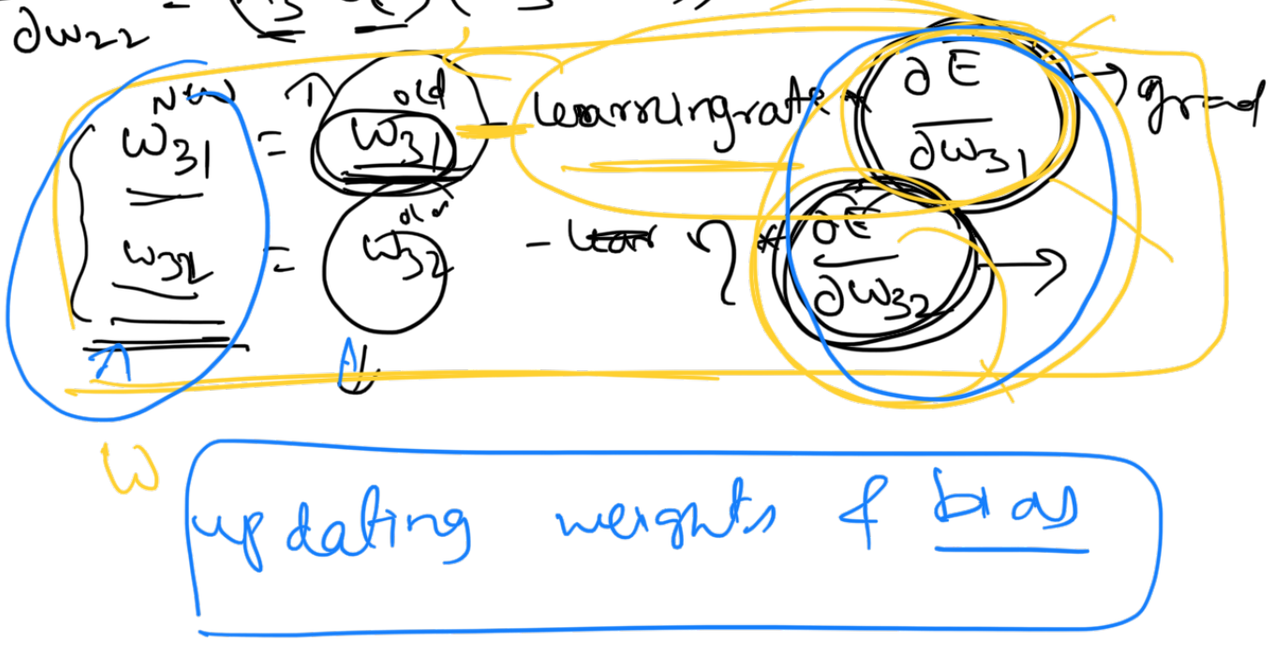
$$\frac{\partial E}{\partial w_{22}} \; ?$$

$$\frac{\partial E}{\partial w_{22}} = \frac{\partial E}{\partial A_3} \cdot \frac{\partial A_3}{\partial z_3} \cdot \frac{\partial z_3}{\partial A_2} \cdot \frac{\partial A_2}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_{22}}$$

$\frac{\partial E}{\partial A_3} = A_3 - y_t$; $\frac{\partial A_3}{\partial z_3} = A_3(1-A_3)$; $\frac{\partial z_3}{\partial A_2} = w_{32}$; $\frac{\partial A_2}{\partial z_2} = A_2(1-A_2)$; $\frac{\partial z_2}{\partial w_{22}} = x_2$

$$\frac{\partial E}{\partial}$$

$$\frac{\partial E}{\partial w_{22}} = (A_3 - y_k)\left(A_3(1-A_3)\right) w_{32} \cdot A_2(1-A_2) \cdot x_2$$



updating weights of bias