

# **Absolute Value of the Magnetic Penetration Depth and Field Profile in the Meissner State of Exotic Superconductors**

**With concentration on YBCO and Pnictides**

by

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# Abstract

One of the fundamental quantities of a superconductor is the London penetration depth,  $\lambda$ , which is the characteristic length scale that a magnetic field penetrates into the surface of a superconductor while in the Meissner state. In the clean limit the absolute value of  $\lambda$  is directly related to the superfluid density  $n_s$  via  $1/\lambda^2 = \mu_0 e^2 n_s / m$  and consequently its variation as a function of temperature, doping and orientation are of central importance in testing microscopic theories of exotic superconductors. Low energy ( $\leq 30$  keV)  $\mu$ SR beam of muon ( $\mu^+$ ) such as in Paul Scherrer Institut (PSI), Switzerland, is ideal to measure London penetration depth  $\lambda$ . When a muon ( $\mu^+$ ) decays, it emits a fast decay positron preferentially along the direction of its spin due to the parity violating decay. The time evolution of statistical average direction of the spin polarization of the muon ensemble depends very sensitively on the spatial distribution and dynamical fluctuations of the muons' magnetic environment.

In this thesis, accurate measurements of  $\lambda$  and the anisotropies ( $\equiv \lambda_a / \lambda_b$ ) have been done for three different oxygen ( $x = 6.52, 6.92, 6.998$ ) contents of  $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$  and for  $\text{Ba}(\text{Co}_{0.07}\text{Fe}_{0.93})_2\text{As}_2$ . The measured values of  $\lambda$  and the anisotropies are considerably different from that of literature, often found with indirect methods. We observe an exponential decay of the magnetic field and corresponding supercurrent density deep inside the crystals. Small deviations from the London model are observed which indicate there is a suppression of the supercurrent density close to the surface. The measured ( $\lambda$ ) values are also found to depart substantially from the widely reported relation ( $T_c \propto 1/\lambda_a^2$ ).

# Preface

Results presented in section 4.1.1 has been published [1], under the title "Direct measurement of the London penetration depth in  $\text{YBa}_2\text{Cu}_3\text{O}_{6.92}$  using low-energy  $\mu\text{SR}$  " with me being the second author. Sections 4.1.2 and 4.1.3 are currently in the process of being published. The design of research methods, literature review, data analysis, were done by myself in consultation with my supervisor R. F. Kiefl. Manuscript of the published paper [1] was written primarily by R. F. Kiefl. The co-authors have been partly involved in taking the data and reviewing and commenting on the manuscripts, or supplying the studied samples. The results presented in section 4.2 is published in Physical Review B(R) [2]. A significant of part of data analysis was done by me. Manuscript was written by O. Ofer & microwave analysis was done by J. C. Baglo.

The large majority of figures presented in this thesis are vector graphics, i.e, can be zoomed to inspect specific areas without any loss of resolution. To avoid large whitespaces in figures, they are purposefully made compact.

This document is best viewed with freely available [Adobe Reader](#). Other suboptimal viewers (in order of preference) are Preview (Machintosh only), [Foxit Reader](#) (Windows only), [Evince](#) (Go Forward/Backward links nonresponsive), [Okular](#) (appearance of broken lines in figures).

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# List of Symbols

$\Delta(k, T)$	Momentum & energy dependent superconducting energy gap, page 16	eV
$\kappa$	Ginzburg-Landau parameter, page 3	
$T_c$	Superconducting critical temperature, page 1	K
$\xi$	Ginzburg-Landau coherence length, page 5	m
$\lambda$	London penetration depth, page 1	m
$\lambda_{ab}$	Average magnetic penetration depth, page 65	m
$\phi_0$	Unit of flux quantum, page 5	
$\psi$	Superconducting order parameter, page 1	
$\sigma$	Optical conductivity, page 10	
$\tau$	Relaxation time, page 18	
$H_c$	Critical magnetic field, page 3	Tesla
$n_s$	Superfluid density, page 5	
$T^*$	Strange metallic phase temperature, page 8	K
$\gamma_\mu$	Muon gyromagnetic ratio, page 26	MHz/Tesla
$\mathcal{A}(t)$	Time dependent muon asymmetry, page 33	
$\mathcal{R}$	Ratio of magnetic penetration depth, ie, $\frac{\lambda_a}{\lambda_b}$ , page 45	
$\tau_\mu$	Muon lifetime, page 26	s

# Glossary

UBC University of British Columbia

PSI [Paul Scherrer Institute, Villigen, Switzerland](#)

CIFAR [Canadian Institute for Advanced Research, Canada](#)

TRIUMF [TRI-University Meson Facility, Canada](#)

NSERC [Natural Sciences and Engineering Research Council of Canada](#)

YBCO Yttrium barium copper oxide/ $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$

FE-PNICTIDE Fe-As based superconductors

MUSR Muon spin [resonance/rotation/relaxation](#)

QCP Quantum Critical Point

HTSC High Temperature Superconductor

ODLRO Off Diagonal Long Range Order

YBCO-I  $\text{YBa}_2\text{Cu}_3\text{O}_{6.92}$

YBCO-II  $\text{YBa}_2\text{Cu}_3\text{O}_{6.998}$

YBCO-III  $\text{YBa}_2\text{Cu}_3\text{O}_{6.52}$

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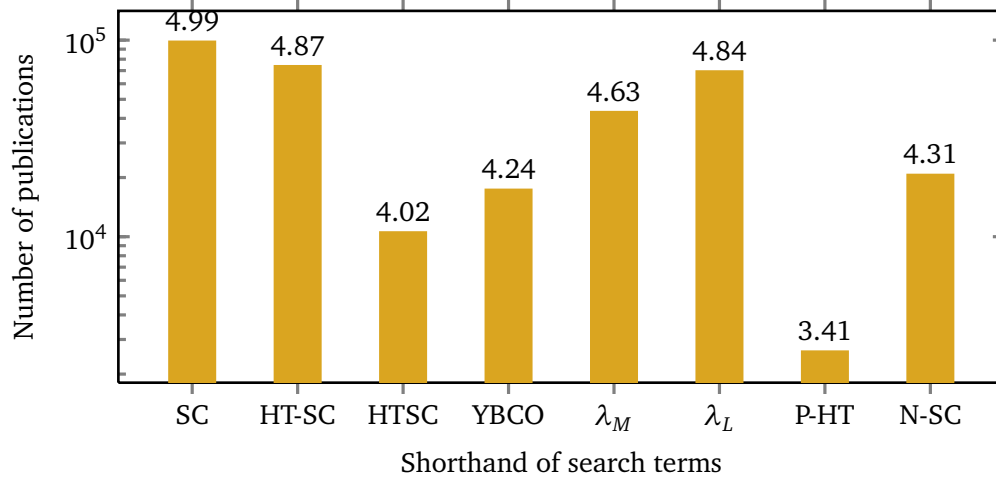
Finally, thanks to the writers of  $\text{\LaTeX}$  and many accompanying packages, used in typesetting this thesis.

## 2 Introduction

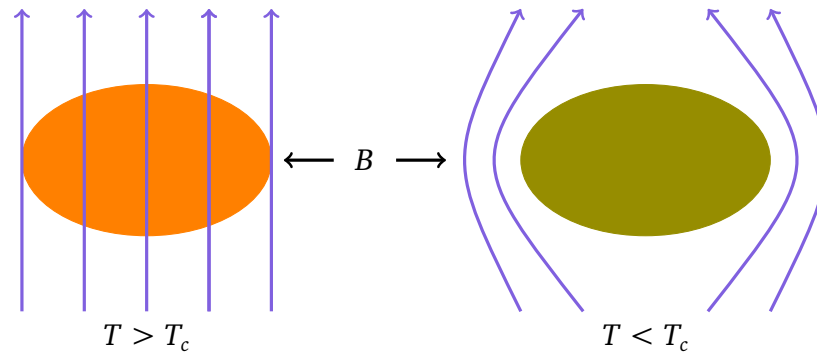
### 1.1 Brief History Of Superconductivity

4 HISTORICALLY superconductivity has played an important role in condensed matter  
5 physics. With the discovery of superconductivity in Hg [3], hundred years ago, it  
6 remains a very active area of research and continuing surprises. Figure 1.1 roughly  
7 shows the number of publications on the phenomenon of superconductivity in the last decade.  
8 Before the discovery of the phenomena of superconductivity, it was known that the resistivity  
9 of a metal drops with decreasing temperature. Resistivity in metals is generally attributed  
10 to electron-phonon scattering, the rate of which is proportional to the thermally excited  
11 phonons. However, the number of thermally excited phonons is finite above absolute zero  
12 and thus the resistivity is expected to be non zero at any finite temperature. Consequently, K.  
13 Onnes' discovery of virtual absence of resistivity in Mercury below 4.15K, in 1911 [4] was  
14 rather surprising. Soon after, in 1913, Lead was found to be superconducting below 7.2K  
15 and after 17 years of this discovery, niobium was found to be superconducting at 9.2K. The  
16 virtual absence of resistance in superconductor has been demonstrated by experiments with  
17 persistent currents in superconducting rings. Such currents have a decay time of magnitude  
18 of  $10^5$  years. The other important characteristic beyond zero resistivity is the phenomenon of  
19 the Meissner effect in which magnetic field is expelled ([5] figure 1.2) out of a sample when  
20 it's cooled below the so called critical temperature  $T_c$ . The phenomenon of the Meissner effect  
21 is different from perfect diamagnetism. In perfect diamagnetism, currents are generated to  
22 oppose any change in applied field. However, if the sample already had non-zero magnetic  
23 flux through it, cooling through  $T_c$  wouldn't make any change in the field whereas, in  
24 the Meissner effect, the field would be expelled from the sample when cooled below  $T_c$ .  
25 This phenomenon of the Meissner effect led London brothers [6] to propose equations to  
26 predict how the field is excluded from the sample and in particular, the field penetration  
27 near the surface. Londons' theory was later (1950) derived from the phenomenological  
28 theory of Ginzburg and Landau [7] (GL), who described superconductivity in terms of a  
29 macroscopic complex order parameter  $\psi$  which roughly dictates the extent to which a system  
30 is ordered. In the case of superconductivity, the amplitude of order parameter is proportional  
31 to superconducting electron density.

32 Although the phenomenological GL theory had been successful, the microscopic theory



**Figure 1.1:** Number of publications (year 2000 onwards) in log scale, for different search terms from a prominent search engine's scholar edition, done on January 17, 2012. Expansion of the shorthand terms: SC: superconductivity; HT-SC: high temperature superconductivity; HTSC: HTSC; YBCO: YBCO;  $\lambda_M$ : magnetic penetration depth;  $\lambda_L$ : London penetration depth; P-HT: pseudogap in high temperature superconductivity; N-SC: normal state in high temperature superconductivity. As may be seen, an enormous scholarly interest in the phenomenon of superconductivity exists in the contemporary physics.



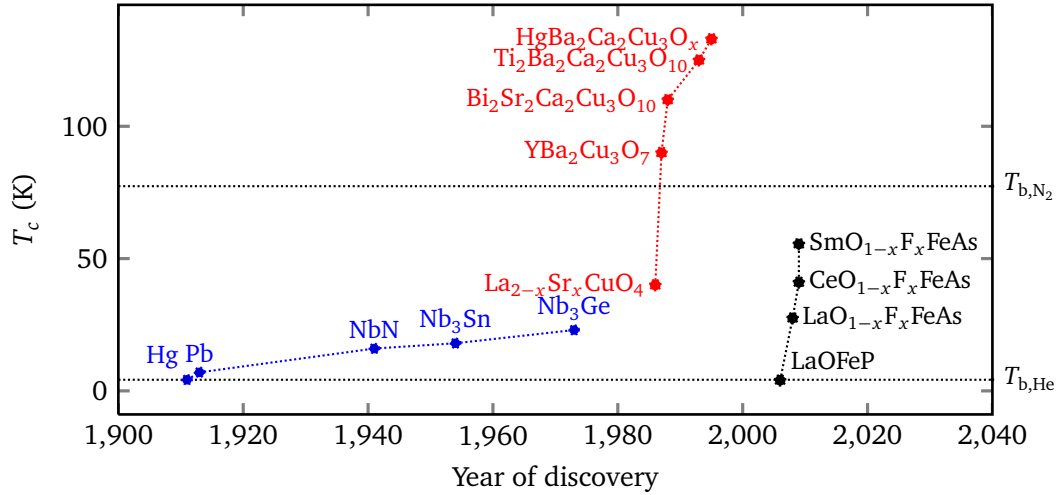
**Figure 1.2:** Meissner effect for a type I superconductor. When a superconductor is placed in an external magnetic field  $H$  and cooled below its superconducting temperature  $T_c$ , the magnetic flux is abruptly expelled. For  $B < B_c$ , it penetrates the surface of the superconductor within the penetration depth  $\lambda$ .

only came in 1957 from J. Bardeen, Leon Cooper and John Schrieffer [8, 9], now famously known as BCS theory. BCS theory explains superconductivity in terms of electron-electron interaction mediated by sound waves (phonons) and predicted that superconductivity may be found with critical temperature  $T_c \leq 23$  K. The carriers of supercurrents were shown to be a pair of electrons (“Cooper pairs” [10]) with opposite spin and momentum. Many new metals and alloys with superconducting properties, at low temperatures, were found by 1980, with the noted exceptions of ferromagnets such as Fe, Ni. It was later realized that magnetic order is antagonistic to superconductivity.

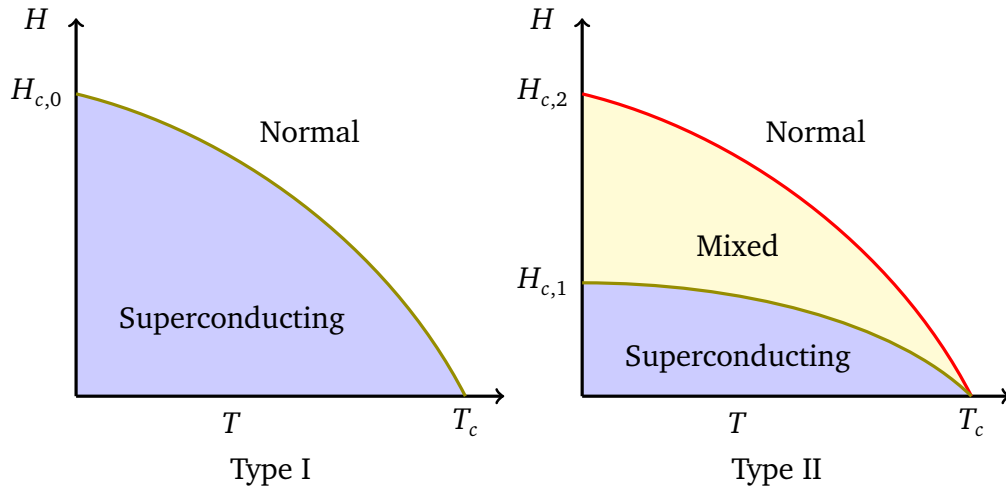
In 1986, J.G. Bednorz and K.A. Muller [11] discovered superconductivity in  $\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$  at 35K, thus initiating the era of high-temperature superconductivity. Building on that, Maw-Kuen Wu and his graduate students, Ashburn and Torng [12] at the University of Alabama discovered YBCO has a  $T_c$  of 93 K. Their work led to a rapid succession of new high temperature superconducting materials, ushering a new era in material science and chemistry. YBCO was the first family of materials to become superconducting above 77 K, the boiling point of liquid nitrogen. All materials developed before 1986 became superconducting only at temperatures near the boiling points of liquid helium ( $T_b = 4.2$  K) or liquid hydrogen ( $T_b = 20.28$  K) the highest being  $\text{Nb}_3\text{Ge}$  at 23 K. Although met with initial skepticism, the observations were validated when Uchida *et. al.* and Chu *et. al.* reproduced original results in 1987. In 2008, one new family of Fe-based superconductors were discovered. Due to the typical antagonistic relationship of superconductivity and magnetism, this was quite surprising. Remarkable progress has been made in discovering high- $T_c$  superconductors as shown in the figure 1.3. As superconductivity is found in so many different material families, it is considered a robust phenomenon; however high- $T_c$  superconductivity has many open questions.

## 1.2 Brief Review Of Superconducting Properties

Besides having a critical temperature  $T_c$ , superconductors also have critical magnetic fields ( $H_c$ ), above which their properties change. In this respect, superconductors are classified in two broad categories (figure 1.4), i) Type I, in which the material becomes normal above a critical magnetic field  $H_{c,0}$ . ii) Type II, in which the material has two critical magnetic fields  $H_{c1}$  and  $H_{c2}$ . In type II, at  $H < H_{c1}$ , the material remains in the Meissner state and at  $H_{c1} < H < H_{c2}$ , magnetic field penetrate the material in quantized vortices (for a very detailed review, consult [14]) and for  $H > H_{c2}$ , it becomes normal. Two other parameters characterize superconductivity in general, namely the coherence length  $\xi$  and the magnetic penetration depth  $\lambda$ . The coherence length  $\xi$  is the distance over which order parameter  $\psi$  varies appreciably and penetration depth  $\lambda$  is the depth over which shielding currents circulate to expel the applied external field.  $\lambda$  and  $\xi$  are two fundamental length scales in superconductivity. Other parameters of interest such as Ginzburg-Landau parameter  $\kappa = \frac{\lambda}{\xi}$ , two critical fields  $H_{c1}$ ,  $H_{c2}$ , thermodynamical critical field  $H_c$  may be derived from them.



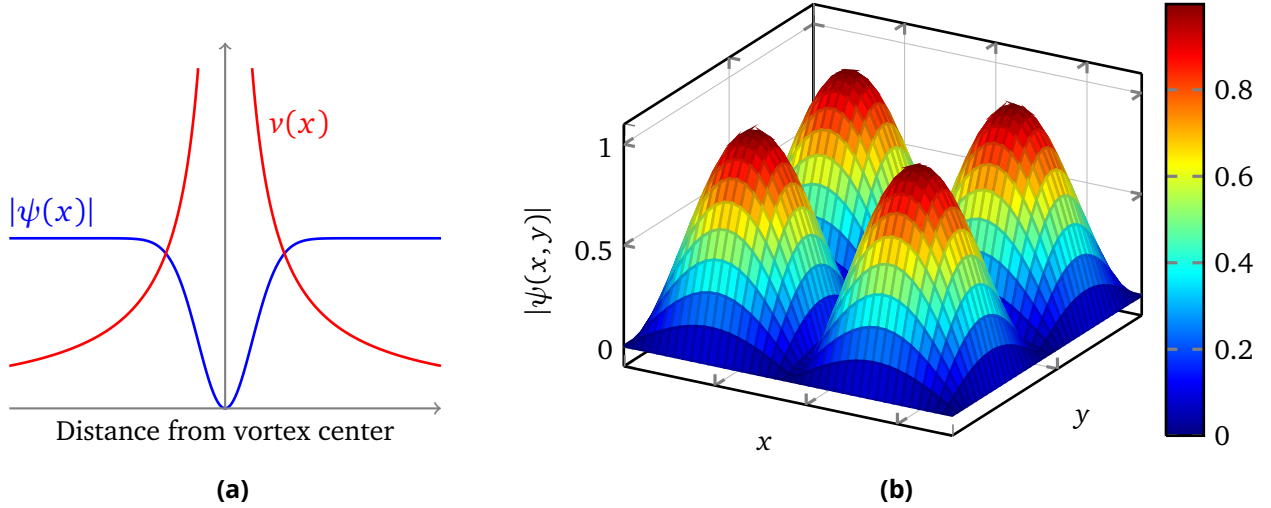
**Figure 1.3:** Superconducting critical temperature ([13])  $T_c$  has risen almost linearly with time, from 4 K to 40 K till about 1986. Around 1987, one of the CuO based high temperature superconductor family was found. In 2008, one new family of Fe-based superconductors were discovered. Due to the typical antagonistic relationship of superconductivity and magnetism, this was a significant surprise for science.



**Figure 1.4:** Superconductivity is destroyed when external field is too large or temperature too high. Superconductors are divided in two classes depending on the manner of this destruction. For type I superconductors, superconductivity is abruptly destroyed in a first order phase transition if  $H > H_c$  or  $T > T_c$ . Type II superconductivity has a complete Meissner region (below  $H < H_c$ ) in  $H-T$  phase diagram, however, in the “Mixed” ( $H_{c1} < H < H_{c2}$ ) state, laminar vortices with normal state cores enter into superconductor and superconductivity is destroyed in a continuous 2nd order phase transition to a normal state. Most high- $T_c$  superconductors are type II.



1 The normal regions (“vortices”) in a “mixed state” of a type II superconductor are  
2 configured to maximize surface area and minimize volume while keeping the magnetic flux  
3 constant. Abrikosov showed that this occurs if vortices are cylindrical and parallel to the  
4 local field direction. At the center of a vortex, superconductivity is completely destroyed,  
5 i.e, order parameter  $|\psi|^2$  vanishes (figure 1.5). However, the velocity of the carriers tend to  
6 increase as we approach the core. Within a radial distance of  $\xi$ , carrier density  $n_s$  reaches its  
7 bulk value. The radius  $\xi$  is known as “vortex core” and contains exactly one quantum of  
8 magnetic flux  $\phi_0 = \frac{c\hbar}{2e}$ . The supercurrent flowing around the vortex produces a magnetic  
9 field which is maximum at the center and decays approximately exponentially, with a length  
10 scale of  $\lambda$  in the radial direction. The vortices are usually arranged in a periodic lattice  
11 known as the Abrikosov lattice, the flux lattice or the flux line lattice. Vortices may also be  
12 dynamic and interacting depending on the level of doping and the magnetic field [15].



**Figure 1.5:** Left: A schematic model of the electronic/magnetic structure of the HTSC vortex core. Superfluid velocity  $v(x)$  rises and the HTSC order parameter  $|\psi(x)|$  falls as the core is approached. Right: The superconducting order is suppressed at the cores of the vortices. The colored surface shows the envelope of this order parameter, superimposed on the vortex lattice. This type of order can be static or dynamically fluctuating depending on the level of doping and the magnetic field.

### 1.3 High Temperature Superconductivity: A Review

#### 1.3.1 Cuprates: $\text{CuO}_2$ Layer Based High- $T_c$ Superconductor

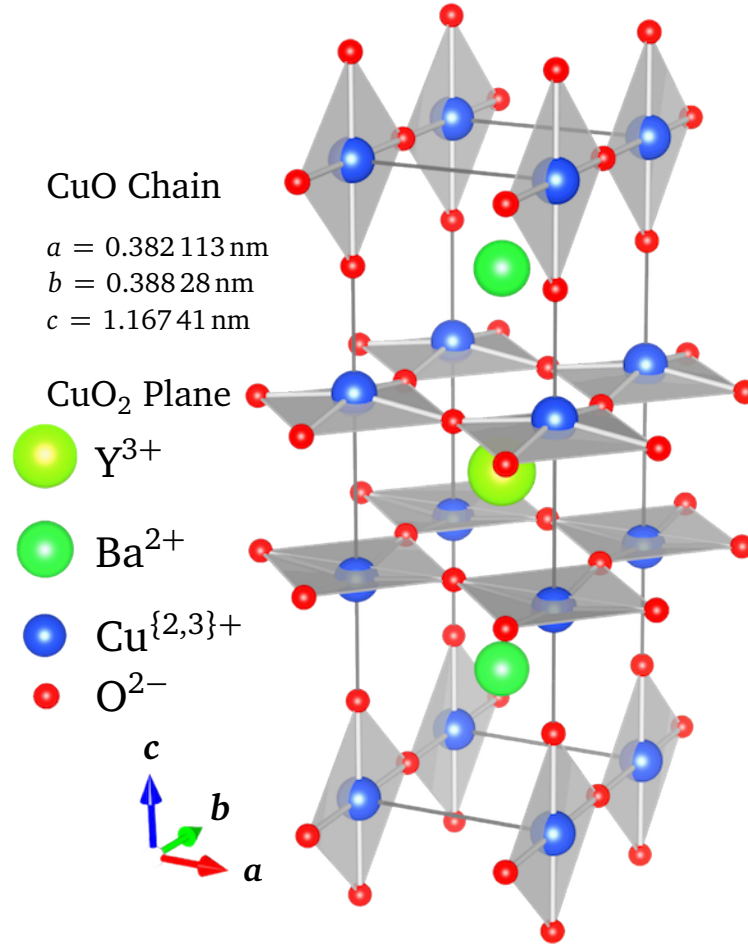
15 Discovery of superconductivity in the ceramic materials (with copper oxides) has led to  
16 a pursuit to understand this new phenomenon. This new type of superconductivity is  
17 considerably different from the “conventional” (i.e, BCS)-type superconductivity & exact  
18 microscopic mechanism is so far debated. However, significant inroads have been made in  
19 understanding different aspects of this “unconventional” superconductivity.

1 A traditional description of electronic behavior in solids is modeled after Drude, Sommer-  
 2 field, Wiedemann and Franz, where heavier positively charged cores of atoms form periodic  
 3 lattice and are immobile and electrons are almost free as in gas molecules in a jar, aptly  
 4 named as “free electron gas”. This theory is also known as Landau’s “Fermi-liquid theory”.  
 5 The Wiedemann-Franz (WF) law (an empirical observation) is one of the basic properties of  
 6 a Fermi liquid, reflecting the fact that the ability of a “free electron” to transport heat is the  
 7 same as its ability to transport charge, provided it cannot lose energy through collisions and  
 8 is written as

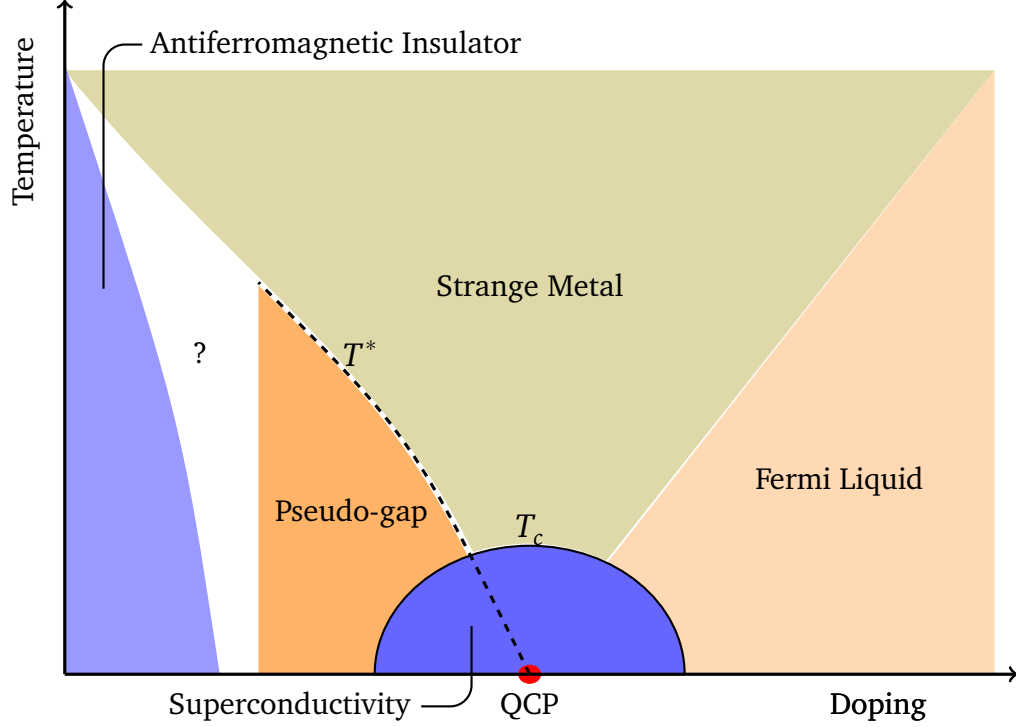
$$\frac{k}{\sigma T} = \frac{\pi^2}{3} \left( \frac{k_B}{e} \right)^2 = \text{constant} \quad (1.1)$$

9 where  $k$  and  $\sigma$  are the heat/electrical conductivity, respectively. In high- $T_c$  cuprate supercon-  
 10 ductor  $(\text{Pr,Ce})_2\text{CuO}_4$ , the WF law is violated in the normal state, suggesting that elementary  
 11 excitation that carry heat in this material are not fermions[16]. The Fermi-liquid description  
 12 is highly successful in explaining metallic, insulating and semiconducting behavior, however  
 13 fails to account for unconventional superconductivity where electron-electron interaction  
 14 is too strong. Strong Coloumb repulsion among electrons lead to antiferromagnetic Mott  
 15 insulating behavior in  $\text{CuO}$  materials at a composition where “free electron gas” theory  
 16 predicts a metal. Changes in composition (O doping) leads to many exotic phenomenon  
 17 such as superconductivity, charge ordering, strange metallicity, quantum criticality and Fermi  
 18 liquid phenomenon. A Mott insulator is very different from a regular (band) insulator. In  
 19 a band insulator, lack of conductivity arises due to Pauli exclusion principle as the highest  
 20 occupied band contains two electrons per unit cell and all the orbitals are filled. In a  
 21 Mott insulator, due to strong Coloumb repulsion, charge conduction is blocked, leaving  
 22 charge per unit cell fixed with electron spins fluctuating at each site. This fluctuation is  
 23 antiferromagnetic (figure 1.8) in nature. Doping (hole/electron) restores some electrical  
 24 conductivity by creating sites to which electrons can jump without having to gain additional  
 25 Coloumb energy.

26 High-temperature superconductivity arises in a family of layered copper oxides that all  
 27 feature weakly coupled square-planar sheets of  $\text{CuO}_2$ . Structure of one of the member of this  
 28 family,  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  (hereafter “YBCO”, possibly the most studied) is shown in figure 1.6,  
 29 as this material was a subject of this research. For  $\text{YBa}_2\text{Cu}_3\text{O}_7$ , three copper-oxygen layers  
 30 are stacked along the tetragonal  $\hat{c}$  axis. Two of these layers have oxygen atoms between  
 31 the copper ions in both the  $\hat{a}$  and  $\hat{b}$  directions, and are called  $\text{CuO}_2$  plane layers. The third  
 32 layer, called the  $\text{CuO}$  chain layer, has oxygen ions only along the  $\hat{b}$  direction [19]. The phase  
 33 diagram for YBCO, dependent on oxygen(hole) doping, is shown schematically in figure 1.7.  
 34 As may be noted from the phase diagram, with increased hole doping, antiferromagnetic  
 35 insulating state turns to be superconducting. The dependence of critical temperature  $T_c(p)$



**Figure 1.6:** YBCO consists of CuO<sub>2</sub> planes & CuO chains. Each plane layer consists of a single Cu atom sharing with four Oxygen vertices and perpendicular to these CuO<sub>2</sub> planes, are CuO chains where each Cu atom shares two oxygen vertices. The Yttrium atoms are found between CuO<sub>2</sub> planes, while the Barium atoms are found between CuO<sub>2</sub> planes and CuO chains. YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub> is a well-defined chemical compound with a specific stoichiometry. Non-stoichiometry is defined by oxygen vacancies as in YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7-x</sub>. With  $x = 1$ , O(1) sites in CuO<sub>2</sub> planes are vacant and the structure is tetragonal and insulating. For  $x < 0.65$ , CuO chains along  $b$ -axis start to form and the structure becomes orthorhombic. Maximum  $T_c \sim 95 \text{ K}$  occurs for  $x \sim 0.8$  [17, 18].



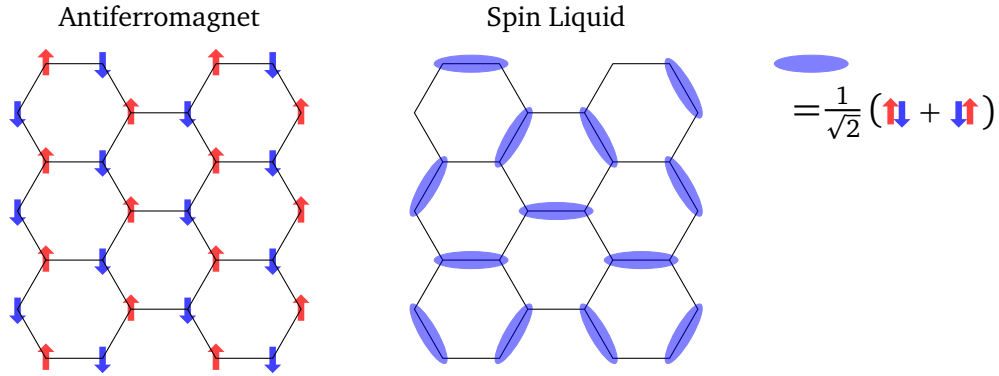
**Figure 1.7:** Schematic phase diagram: At very low levels of electron-hole doping, cuprates are insulating and antiferromagnetic (the materials' neighboring spins point in opposite directions). At increased doping levels, they become conducting, and the exact temperature and doping level determine which phase of matter they will be in. At temperatures below  $T_c$ , they become superconducting, and at temperatures above  $T_c$  but below  $T^*$  they fall into the pseudogap phase. The boundary of the pseudogap region at low doping levels is unknown. The transition between the Fermi-liquid phase and the strange-metal phase occurs gradually (by crossover). QCP denotes the quantum critical point at which the temperature  $T^*$  goes to absolute zero.

1 on doping is given by an empirical relationship [20],

$$T_c(p) = T_{c,\max} \left[ 1 - 82.6(p - 0.16)^2 \right], \quad (1.2)$$

2 where doping level  $p$  varies from 0.05 to 0.27. The proximity of antiferromagnetism and  
3 superconductivity gives rise to the conjecture that superconductivity is driven by magnetic  
4 interactions between electrons rather than pairing via phonons. Also important to note that,  
5 a signature of lattice vibration driving superconductivity, ie, the “isotope effect” has not  
6 been observed in high temperature superconductivity. However there has been renewed  
7 interest in the possible role of electron-lattice coupling [21–26] in high- $T_c$  superconductivity,  
8 although the role is suggested to be indirect [26] and small [23, 27]. A possible mechanism  
9 proposed by P. W. Anderson [28] is that “quantum fluctuations” may create instability in the

1 antiferromagnetic order and give rise to resonating valence bond [29–31] in which the spins  
 2 form a “spin-liquid” phase of singlet( $s = 0$ ) pairs. “Spin liquid” is defined to be aggregation  
 3 of pairs of antiparallel spins. The motion of such singlet pairs is similar to the resonance of  
 4  $\pi$  bonds in benzene, originating the term “resonating valence bond” (RVB), schematically  
 5 shown in figure 1.8. In this picture, electrons are paired up in antiparallel spin-formation  
 6 but cannot move due to Coloumb repulsion. Reducing average occupancy, from one, will  
 7 make these singlet pairs mobile, Anderson argued, giving rise to superconductivity. In  
 contrast, YBCO was found out to be antiferromagnets and not spin-liquid phase [32–34]. It



**Figure 1.8:** An example of a short range “resonating valence bond” (an aggregation of antiparallel neighboring spins). An oval represents a superposition of different possible spin configurations. This is a “spin-liquid” since there is no static order but their motions are highly correlated. Motions of singlet pairs are hindered due to Coloumb repulsion. Reducing average occupancy from one may make these singlet pairs mobile.

8  
 9 has been established that in cuprate systems, antiferromagnetic ordering resides entirely  
 10 on the  $\text{CuO}_2$  plane [35, 36], with a three-dimensional magnetic transition dictated by  
 11 very weak coupling between planes. As seen in the phase diagram in figure 1.7,  $T_c$  varies  
 12 (peaks at “optimal” doping) as a function of doping and is a well observed phenomenon in  
 13 all  $\text{CuO}_2$  layer based superconductors [37]. With increased doping 3D antiferromagnetic  
 14 ordering gives way to a disordered state with short range correlations [37], thereby retaining  
 15 some magnetism. At  $T > T_c$ , metallic behavior is observed for a broad range of dopings  
 16 & d.c electrical resistance is  $T$ -dependent rather than  $T^2$ -dependent as would have been  
 17 expected from a normal metal Fermi liquid behavior. In the overdoped regime, the copper  
 18 oxides behave more like ordinary metals with a  $T^2$  dependence of d.c resistivity [38]. A  
 19 naturally overdoped Copper oxide  $\text{TlBa}_2\text{Cu}_3\text{O}_{6+x}$  has been observed to show polar angular  
 20 magnetoresistance oscillation [39, 40], in high field, establishing the existence of a 3D Fermi  
 21 surface, consistent with the prediction from single electron band theory, i.e, metallic behavior.  
 22 However, the existence of 3D coherent Fermi surface poses a challenge to the widely held  
 23 belief and experimental evidence [41] that high- $T_c$  superconductivity arises from purely 2D

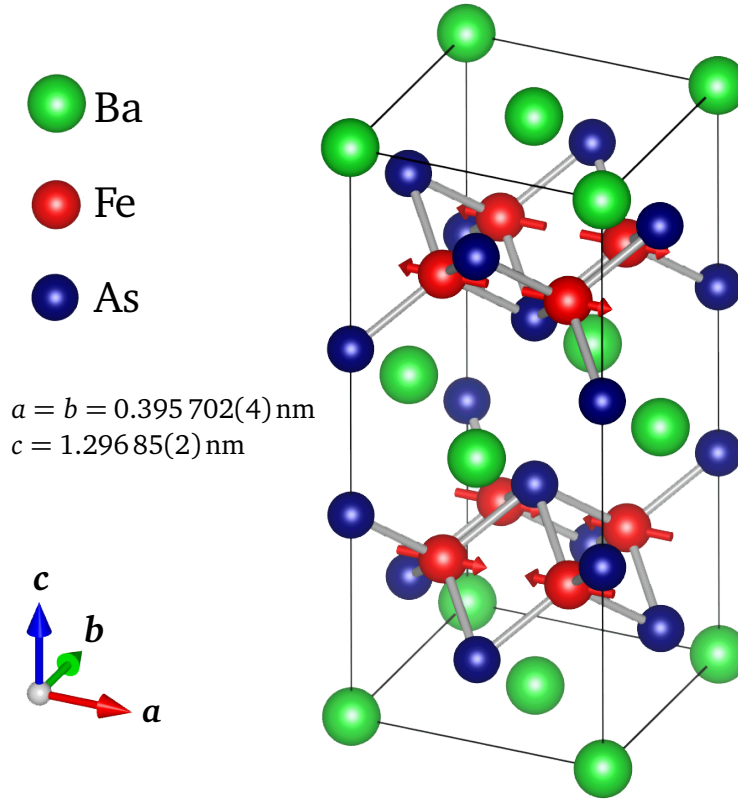
1 electron motion within the  $\text{CuO}_2$  planes. In underdoped regime, copper oxides have been  
2 found to show quantum oscillations [42], an indication of metallic behavior, in de Haas-van  
3 Alphen spectra. As may be noted in phase diagram Cu-O based superconductors includes a  
4 “pseudogap” region [41, 43], a precursor to the superconducting state. It has been shown  
5 that this phase originates in  $\text{CuO}_2$  planes and not in the CuO chains [44]. Whether this is  
6 a distinct phase of matter is still under debate [45]. It is metallic, however some parts of  
7 the Fermi surface show gaps [41, 43, 46]. If it exists, a quantum critical point [47–49] in  
8 the cuprates would also probably be the end point of a line( $T^*$ ) [50] of phase transitions  
9 that separates the pseudogap and strange-metal regions. It has also been suggested that  
10 the phase diagram is controlled by a quantum critical point [51, 52]. A QCP develops in a  
11 material at absolute zero temperature when a new form of order emerges from its ground  
12 state. QCP is a phenomenon of great interest because of their ability to influence the finite  
13 temperature properties of materials. The “normal” region (“strange metal”) above transition  
14 temperature  $T_c$  is of very unusual properties [53–55] (thermal conductivity  $k(T)$ , optical  
15 conductivity  $\sigma(\omega)$ , the nuclear relaxation rate  $T_1^{-1}(T)$ ), with large temperature-dependent  
16 resistivity implying a scattering rate linear in  $T$ , however several orders of magnitude of the  
17 average excitation energy  $k_B T/h$  [56, 57].

18 With all the significant differences from conventional superconductivity, supercurrent  
19 is still carried by electron pairs, shown via quantization of magnetic flux in units of  $\frac{h}{2e}$   
20 [58–60]. Most of the physical properties of the  $\text{CuO}_2$  have experimentally been established  
21 with a high degree of reliability and advances in preparing the materials are such that  
22 spurious effects and uncertainties in materials compositions, homogeneities and impurity  
23 content may be eliminated as hindrance to the understanding of the phenomenon of high- $T_c$   
24 superconductivity. In spite of substantial efforts in both experimental and theoretical  
25 research, there are many open questions regarding mechanism for high- $T_c$  superconductivity.

### 1.3.2 Pnictide: A New Type Of High- $T_c$ Superconductor

27 Because of the typically antagonistic relationship between superconductivity and magnetism  
28 has led researchers to avoid using magnetic elements (eg. Fe) in particular, as potential  
29 building blocks of new superconducting materials. The recent (2008) discovery of super-  
30 conductivity at  $T_c$ 's up to 55 K in iron pnictide systems [61–65] has sparked enormous  
31 interest in this class of materials. Even more surprising is that pnictide is the only material  
32 other than cuprates ( $\text{CuO}_2$  layer based superconductors) to have  $T_c$  higher than 40 K ( $\sim$ BCS  
33 theoretical maximum). The crystal structure of the parent compound  $\text{BaFe}_2\text{As}_2$  is shown in  
34 the figure 1.9 as a Co-doped pnictide ( $\text{Ba}(\text{Co}_{0.074}\text{Fe}_{0.926})_2\text{As}_2$ ) is a subject of this work.

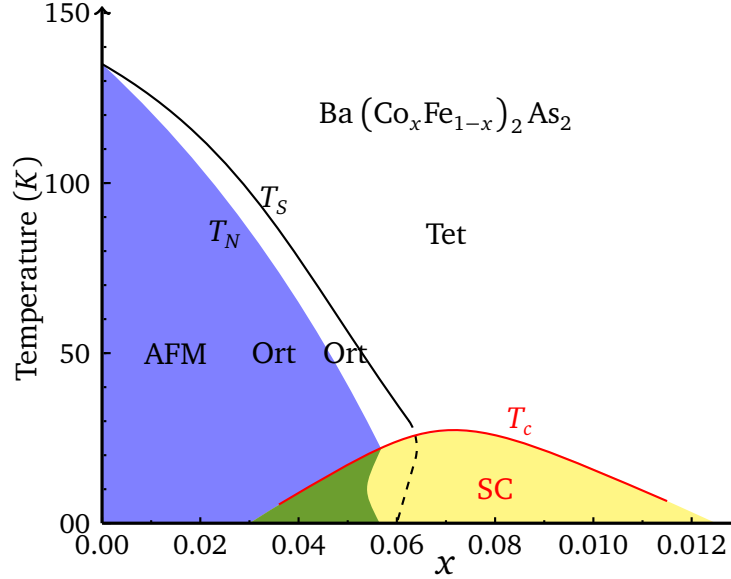
35 Cuprates and pnictides show similar behavior in many aspects, such as (i) both are  
36 layered structures, (ii) parent (non-superconducting) compounds are antiferromagnets,  
37 although with possibly different electronic correlation strengths, (iii) both materials show  
38 superconducting order upon doping. Striking dissimilarities are also abound: (i) parent



**Figure 1.9:** Left: Pnictide crystal structure [66] shows the antiferromagnetic alignment and magnetic moment (red arrows) both along the longer  $a$  axis in the FeAs plane. The magnetic unit cell is the same as the orthorhombic chemical unit cell.

1 compound for cuprates are Mott insulators while for pnictides, they are semimetals, (ii)  
 2 cuprates are essentially one-band while iron pnictides have multibands at the Fermi energy,  
 3 (iii) superconducting gap function is  $d$ -wave in cuprates whereas for pnictides, strong  
 4 contender is an “extended  $s$ -wave”, also called  $s_{\pm}$ .

5 Two families(parent materials) of pnictides have so far been discovered: originating from  
 6  $R\text{FeAsO}$  [64] ( $R$ =rare earth, abbreviated as 1111 for its 1:1:1:1 ratio of the four elements)  
 7 and  $A\text{Fe}_2\text{As}_2$  [67] ( $A$ =alkaline, the 122 compounds) earth metal, which are tetragonal at  
 8 room temperature but undergo an orthorhombic distortion in the range 100 K to 200 K that  
 9 is associated with the onset of antiferromagnetic order [66, 68–72]. Tuning the system  
 10 via element substitution [73, 74] or oxygen deficiency [75, 76] suppresses the magnetic  
 11 order and structural distortion in favor of superconducting  $T_c$ 's up to 55 K, with an overall  
 12 behavior strikingly similar to the high- $T_c$  copper oxide family of superconductors. However,  
 13 the induction of superconductivity by doping Co or other transition metals into the Fe site  
 14 indicates that atomic disorder in the superconducting Fe layer ostensibly does not suppress  
 15 superconductivity, contrary to the behaviors of layered cuprate high- $T_c$  superconductors



**Figure 1.10:** Phase diagram [77, 78] for  $\text{BaCo}_x\text{Fe}_{2-x}\text{As}_2$ . Yellow indicates the superconducting phase, which appears below the superconducting transition temperature  $T_c$ . A structural transition occurs at  $T_s$  from the tetragonal phase (Tet) at higher temperature to the orthorhombic phase (Ort). Blue represents the antiferromagnetic order (AFM), which appears at  $T_N$ , slightly below  $T_s$ . The stripes of enhanced superfluid density are observed only in the regime  $0.04 < x < 0.06$ .

1 where doping onto the Cu sublattice is always detrimental to  $T_c$ .  
2 A preliminary phase diagram [77, 78] of pnictide superconductors is shown in the  
3 figure 1.10. It may be noted that parent compound of superconducting iron arsenides exhibit  
4 spin density wave (SDW)-type long-range magnetic ordering at low temperatures [68, 79]  
5 just like the cuprates [32]. As it appears that in high- $T_c$  superconductivity, AF order needs  
6 to be suppressed before superconductivity may appear, leads many to the proposition that  
7 dynamic rather than static antiferromagnetism (or AF fluctuations) is favorable for high- $T_c$   
8 superconductivity. A recent neutron-scattering experiment found that, in  $\text{BaFe}_{1.85}\text{Co}_{0.15}\text{As}_2$ ,  
9 the AF fluctuation is as strong as that of  $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$  [80] and electron-phonon coupling  
10 is not the primary driver of superconductivity in pnictides. Origin of antiferromagnetic  
11 ordering in the pnictide parent compounds is a hotly debated topic, largely owing to its  
12 implications for the pairing mechanism: the electronic structure suggests that the same  
13 magnetic interactions that drive the AFM ordering also produce the pairing interaction for  
14 superconductivity [81]. Regardless of the exact nature of magnetic order, it is believed  
15 that magnetostructural coupling is prevalent throughout the Fe-based superconductors in  
16 the form of coupled magnetic and structural transitions [82, 83]. Competing presence of  
17 superconductivity and AF spin fluctuations has led to suggestions that quantum criticality



1 may play an important role [84–86], however, prominence of quantum critical behavior in  
 2 iron pnictides is disputed elsewhere [87]. Due to the large number of pnictides and the  
 3 nature of chemical substitution, one limitation so far is that many experiments have been  
 4 carried out on different systems or different chemical compositions of the same crystalline  
 5 system, and thus make comparisons difficult. However their generic features enables  
 6 general conclusions to be drawn from several experiments. For instance, NMR experiments  
 7 determined from Knight shift measurements that the superconducting state spin symmetry  
 8 is probably singlet [88–90], suggesting an even order parameter symmetry (eg. *s* wave, *d*  
 9 wave).

#### 11.4 Pairing Symmetry And Magnetic Penetration Depth Measurement

11 One fundamental quantity in characterization of superconductors is London penetration  
 12 depth  $\lambda$ , which is closely related to superfluid density ( $\rho_s \equiv \frac{1}{\lambda^2} \propto \frac{n_s}{m^*}$ ). In general, the  
 13 penetration depth  $\lambda$  is given as a function of  $n_s$ , effective mass  $m^*$ , Ginzburg-Landau  
 14 coherence length  $\xi$  and the mean free path  $l$  as [91]

$$\frac{1}{\lambda^2} = \frac{4\pi n_s e^2}{m^* c^2} \times \frac{1}{1 + \xi/l} \quad (1.3)$$

15 Close to the clean limit,  $\frac{\xi}{l} \rightarrow 0$  and the second term in (1.3) becomes unity.  $\lambda$ 's variation  
 16 as a function of temperature, doping and orientation are of central importance in testing  
 17 microscopic theories of exotic superconductors. For example, the linear variation of  $1/\lambda^2$   
 18 with respect to temperature was a key finding confirming the *d*-wave nature of the pairing  
 19 in  $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$  [92, 93]. Also, early  $\mu\text{SR}$  studies of the vortex phase in polycrystalline  
 20 samples found a linear correlation between  $1/\lambda^2$  and  $T_c$  in the under-doped region [94, 95].  
 21 The resulting Uemura plot has played a prominent role in theoretical efforts to understand  
 22 high  $T_c$  superconductivity [96]. Departure from Uemura scaling and the decline of the slope  
 23 as the  $T_c = 0$  quantum critical point is approached can be understood in terms of a 3D-QCP  
 24 model [97]. Scaling of  $T_c$  with  $n_s(0)$  in underdoped cuprates may also be due to quantum  
 25 fluctuations near a 2D quantum critical point [98].

26 It is widely believed that cuprate high- $T_c$  superconductivity originates in two-dimensional  
 27  $\text{CuO}_2$  layers [34, 35, 99].  $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$  also has one dimensional (1D)  $\text{CuO}$  chains which  
 28 contribute to superconductivity the mechanism for which is not fully understood [100].  
 29  $\text{CuO}$  chains are believed to act as quasi-1D system and charge reservoir [19]. Magnetic  
 30 ordering of Cu moments have been observed to be at different temperatures in plane and  
 31 chain layers [101]. Penetration depth anisotropy measurements indicate that chains become  
 32 superconducting at the same temperature as  $\text{CuO}_2$  planes [102]. Due to differences in  
 33 band structures between planes and chains [103], one natural explanation for the same  
 34 transition temperature is proximity effect [104–106] where by electron hopping between

1 chains and planes contribute to superfluidity along chain direction. The 1D nature of  
2 the chains themselves induces, in the filled chain compound,  $a - b$  anisotropy which has  
3 been observed in dc resistivity [107, 108] and optical conductivity and penetration depth  
4 measurements [102, 109] and is expected to affect the vortex core structure [104]. In  
5 this simple model of multiband superconductivity in  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ , there is an intrinsic  
6 pairing interaction in the plane, but the chains are intrinsically normal, which means that  
7 the superconducting order parameter is nonzero in the plane layer only [104]. The pairing  
8 mechanism of  $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$  is thought to be predominantly  $d$ -wave type. Other possible  
9 pairing states involving complicated gap functions, have been suggested [110–113]. Recently  
10 discovered  $\text{BaFe}_2\text{As}_2$  family of superconductors has yet to have a definitive pairing symmetry.  
11 An accurate determination of  $\lambda(T)$  is one way to probe the symmetry of the pairing state.  
12 It has been theorized [114] that only low temperature dependence of  $\lambda(T)$  is sensitive to  
13 pairing state of the superconductor. It is clear that accurate measurements of  $\lambda$  and  $a - b$   
14 anisotropy are essential in clarifying central questions in  $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ .

15 Unfortunately, accurate measurements of  $\lambda$  are difficult due to systematic uncertainties.  
16 For example, in any bulk measurement the assumption of an exponential decay of the  
17 field in the Meissner state is only valid in the local London limit of a perfect surface [115].  
18 Significant non-local effects are expected if the coherence length is comparable to the  
19 penetration depth [116] or if there are nodes in the superconducting gap function [117].  
20 Even within the London limit, there may be a non-exponential decay of the field, arising from  
21 any depth dependent change in the magnitude or symmetry of the order parameter. These  
22 add uncertainty to all conventional bulk measurements where the field profile is assumed  
23 and not measured. Alternatively, one can determine the absolute value of  $\lambda$  from  $\mu\text{SR}$  studies  
24 in the vortex state where the muon acts as a sensitive probe of the local magnetic field  
25 distribution. However, an accurate determination of  $\lambda$  requires there to be a well ordered  
26 vortex lattice with known symmetry. Also, there are substantial non-local and non-linear  
27 effects associated with vortices which complicate the theory and make it difficult to extract  
28 the true  $\lambda$  [117–120]. One approach is to fit the observed field distribution to a simple  
29 Ginzburg-Landau model involving an effective  $\lambda$  and then to extrapolate to zero magnetic  
30 field (or vortex density) [121]. Until now, the penetration depth has been measured in the  
31 vortex state via muon spin rotation [122] and using microwave techniques [92, 123–125].  
32 In vortex state measurement, Sonier *et al.* used a GL model for magnetic field distribution  
33 to extract  $\lambda$  as a function of applied magnetic field. However, it was mentioned that  $\lambda_{ab}$   
34 measured is an effective penetration depth which is model dependent. Consequently, one  
35 may expect some difference in  $\lambda$  measured in the Meissner state where there are no vortices.  
36 The microwave techniques used in [92, 123–125] reported London penetration depth for a  
37 number of high- $T_c$  superconductors. Microwave techniques are well-suited to measuring  
38 temperature dependence of  $\lambda$  but generally not very sensitive to the absolute value of  $\lambda$ .

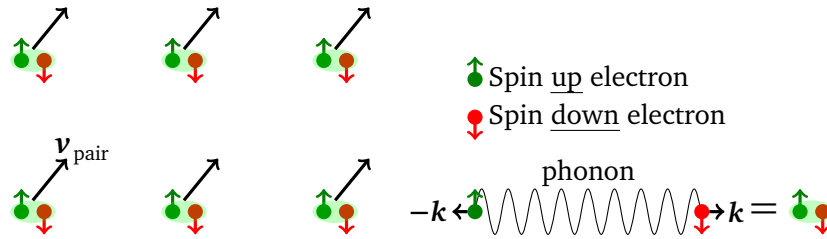
1        In this thesis, a reduction of magnetic field  $B(z)$  is measured as it enters the sample,  
2        via the reduction of muon spin precession frequency. The precession frequency contains all  
3        the information about muons' interaction with the local magnetic environment. Using a  
4        modified London model in the Meissner state, absolute value of magnetic penetration depths  
5        are obtained for three oxygen dopings of YBCO and a Co doped FE-PNICTIDE.

1

## 2 Theory

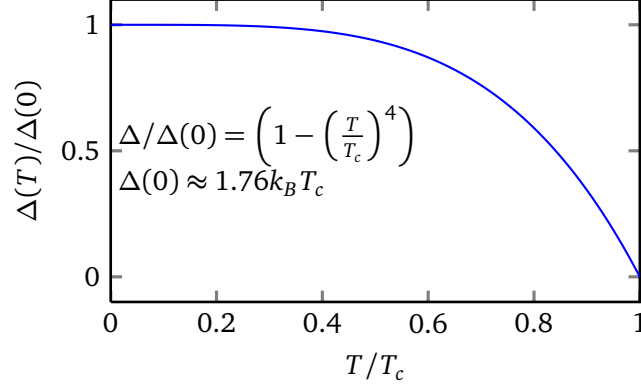
### 2.1 BCS Theory

4 The basic idea for BCS superconductivity is that an attractive interaction between electrons,  
 5 regardless of their strength, can bind the electrons into pairs [10]. We consider a case for  
 6 only two electrons added to the Fermi sea. The first electron attracts positive ions and these  
 7 ions, in turn, attract the second electron giving rise to an effective attractive interaction  
 8 between electrons. Due to the movement of ion cores, phonon waves are generated and  
 9 the interaction between electrons is thereby phonon mediated. The total energy of the  
 10 electron system is minimized when there are Cooper pairs compared to a Fermi gas with no  
 11 correlation. The center of mass of a Cooper pair is zero since the electrons have opposite  
 12 momenta and spin  $|\hbar\vec{k}, \uparrow\rangle$  and  $|\hbar\vec{k}, \downarrow\rangle$ . Due to this opposite momenta and spin, it is  
 13 labeled *s*-wave pairing since the relative angular momenta of the two electrons is zero. The  
 14 electron-phonon system is described by the single order parameter  $\psi$ . An schematic of  
 15 “in-phase” motion of the system is shown in the figure 2.1



**Figure 2.1:** In the superconducting state, electrons pair up in zero-spin composites. They all move “in phase” and are said to be “coherent”. This is considered to be a ordered state and the whole electron-phonon system may be described by a single wavefunction.

16 One important consequence of the BCS theory is that the presence of a momentum  
 17 dependent energy gap  $\Delta(k)$  at the Fermi surface so that an amount of  $2\Delta(k, T)$  energy is  
 18 required to break a Cooper pair. The energy gap is schematically shown in the figure 2.3.  
 19 The gap is opened at the Fermi energy as the temperature is lowered below the critical  
 20 temperature. A *d*-wave density of state is also shown in the figure 2.3. Unlike the *s*-wave  
 21 superconductors, some carriers are always available at the Fermi surface even at the lowest  
 22 temperatures. In the weak coupling limit, where the gap  $\Delta$  is much smaller than the



**Figure 2.2:** Temperature dependence of the superconducting energy gap in the weak coupling limit of BCS interaction. The superfluid density  $n_s \propto$  in a two-fluid model [126] implies the  $(T/T_c)$  dependence of energy gap. This gap model is also experimentally verified [127].

1 characteristic phonon energy  $\hbar\omega_D$ ,

$$\frac{2\Delta(0)}{k_B T_c} = 3.52. \quad (2.1)$$

2 The numerical factor 3.52 is well tested in experiments and found to be reasonable, in  
 3 purely BCS type interactions.  $\Delta(T)$  remains fairly constant until the phonon energy becomes  
 4 enough to thermally excite the quasiparticles. Near the transition temperature  $T_c$ ,  $\Delta(T)$   
 5 varies as

$$\frac{\Delta(T)}{\Delta(0)} \sim 1.74 \left(1 - \frac{T}{T_c}\right)^{\frac{1}{2}}, \quad T \sim T_c \quad (2.2)$$

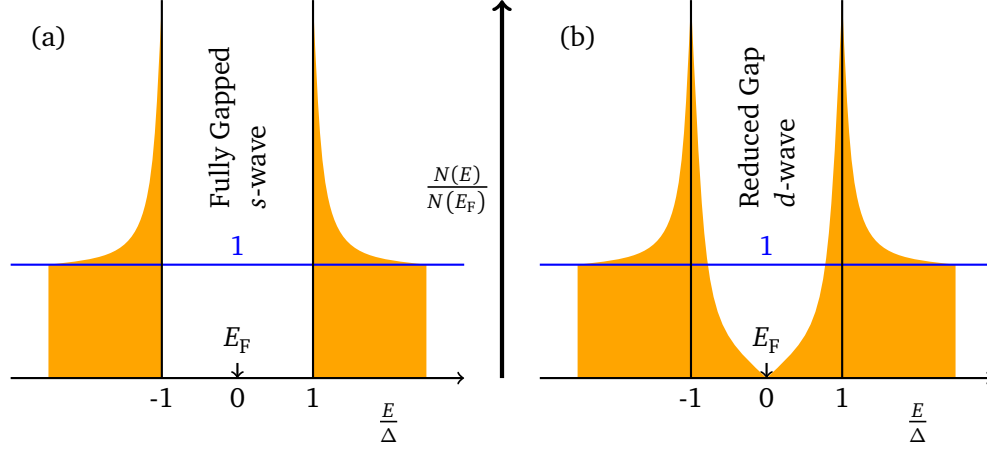
6 and is graphically shown in the figure 2.2.

7 The most important manifestation of the electron-phonon interaction is the super-  
 8 conducting state itself. According to our present understanding of Cooper pairing, the  
 9 electron-phonon induced attraction between two electrons would not overcome their direct  
 10 Coulomb repulsion, except for the fact that the former is retarded whereas the latter is not.  
 11 This gives rise to the pseudopotential effect; in some sense the pseudopotential effect is the  
 12 true mechanism of superconductivity, rather than the electron phonon interaction per se.

## 2.2 London Penetration Depth

14 We consider the penetration depth in the Meissner state of a type II superconductor. Below  
 15  $H_{c1}$ , the London equations provide a good description of the electromagnetic properties.  
 16 The relevant Maxwell's equation is

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}. \quad (2.3)$$



**Figure 2.3:** Left (s-wave): Density of states in a conventional superconductor such as Nb. The density of states is a measurement of how many electrons can reside at a specific energy level. In the figure, there is a region at the Fermi surface where no electrons can reside. This region comprises the superconducting energy gap. On either sides of this gap there are peaks in the density of states where large numbers of electrons can occupy the energy levels. Right (d-wave): Density of states in a high- $T_c$  superconductor like YBCO. The density of states is reduced at the Fermi surface, however there is no true gap.

1 In the classical Drude model of electrical conductivity, we have

$$\vec{F} = -m\frac{d\vec{v}}{dt} - e\vec{E} = m\frac{d\vec{v}}{dt}, \quad (2.4)$$

2 where  $\vec{v}$  is the average velocity of the electrons,  $m$  is the mass of an electron,  $\vec{E}$  is the  
 3 electric field the electrons are in and  $\tau$  is the relaxation time, i.e, roughly the time required  
 4 to bring the drift velocity to zero if electric field was suddenly set to zero. In a normal metal,  
 5 the competition between the scattering and the acceleration in (2.4) leads to a steady state  
 6 average velocity

$$\vec{v} = \frac{e\vec{E}\tau}{m}. \quad (2.5)$$

7 Assuming  $n$  conduction electrons per unit volume, we get the electric current density via  
 8 Ohm's Law,

$$\vec{J} = ne\vec{v} = \left(\frac{ne^2\tau}{m}\right)\vec{E} = \sigma\vec{E}. \quad (2.6)$$

9 To describe superconductivity, London assumed that a certain fraction of electron density  $n_s$   
 10 experience no relaxation i.e., letting  $\tau_s$  in (2.4) go to infinity. This leads to

$$\frac{d\vec{J}_s}{dt} = \left(\frac{n_s e^2}{m}\right)\vec{E}, \quad (2.7)$$

1 where  $n_s$  is density of the superconducting carriers. Taking curl on both side of the (2.7),  
 2 we get

$$\frac{m}{n_s e^2} \left( \vec{\nabla} \times \frac{d\vec{J}_s}{dt} \right) = \vec{\nabla} \times \vec{E}. \quad (2.8)$$

3 Substituting Maxwell (2.3) in (2.8), we obtain the second London equation

$$\frac{mc}{n_s e^2} \left( \vec{\nabla} \times \frac{d\vec{J}_s}{dt} \right) + \frac{d\vec{B}}{dt} = 0. \quad (2.9)$$

4 Interchanging the order of differentiation with respect to space and time in (2.9), London  
 5 postulated

$$\frac{mc}{n_s e^2} (\vec{\nabla} \times \vec{J}_s) + \vec{B} = 0. \quad (2.10)$$

6 Assuming no time varying electric field, another Maxwell equation connects  $\vec{J}_s$  with  $\vec{B}$  with  
 7 the equation

$$\vec{J}_s = \frac{c}{4\pi} (\vec{\nabla} \times \vec{B}) \quad (2.11)$$

Substituting (2.11) into (2.10), we get

$$\begin{aligned} \lambda^2 (\vec{\nabla} \times \vec{\nabla} \times \vec{B}) + \vec{B} &= 0, \\ \lambda^2 \nabla^2 \vec{B} + \vec{B} &= 0, \end{aligned} \quad (2.12)$$

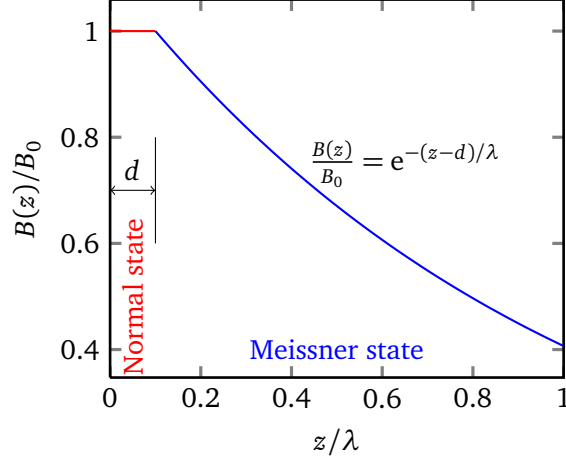
8 where

$$\frac{1}{\lambda^2} = \frac{4\pi n_s e^2}{mc^2}. \quad (2.13)$$

9 In a vacuum-superconductor interface (which is also the case in our experiment), the solution  
 10 of (2.12) is given by

$$B(x) = B_0 \exp\left(-\frac{x}{\lambda}\right) \quad (2.14)$$

11 where  $B_0$  is the magnitude of the external applied field. The quantity  $\lambda$  is known as London  
 12 penetration depth and  $\lambda^{-2} \propto n_s$ . The most important success of the London (2.11) and  
 13 (2.12) is that a static magnetic field is screened from the interior of a bulk superconductor  
 14 over a characteristic penetration depth  $\lambda$ . A simple estimate shows that this distance is a  
 15 macroscopical one that is much larger than the mean distance  $r$  between electrons in the  
 16 superconductor. As one approaches the critical temperature  $T_c$ ,  $n_s \rightarrow 0$  continuously and  
 17 as a consequence,  $\lambda(T)$  diverges as  $T \rightarrow T_c$ , according to (2.13). While the (2.14) may be  
 18 valid for a superconductor with an atomically flat surface, a rough surface might give rise  
 19 to a suppressed order parameter for few tens of nanometers and a modified London model



**Figure 2.4:** External magnetic field drops exponentially ((2.15)) as it enters a superconductor in Meissner state. The characteristic distance  $\lambda$  is called the London penetration depth

1 (figure 2.4)

$$B(z) = \begin{cases} B_0 \exp\left(-\frac{z-d}{\lambda}\right) & \text{if } z \geq d \\ B_0 & \text{if } z < d \end{cases} \quad (2.15)$$

2 may be more appropriate. Here  $B_0$  is the magnitude of the applied field,  $\lambda_{a,b}$  is the magnetic  
 3 penetration depth in the  $a$  or  $b$  direction, respectively,  $z$  is the depth into the crystal, and  $d$   
 4 is an effective dead layer inside of which the supercurrent density is suppressed.

### 2.3 Pairing Mechanism And Order parameter symmetry

The critical temperature  $T_c$  is the onset of long-range, macroscopic phase coherence in the Cooper pairs. Long range correlations between pairs are described by off-diagonal long-range order (ODLRO) [128, 129], with no classical analog, which implies non-zero value of the the pair correlation function

$$\begin{aligned} \rho(\vec{r}, \vec{r}') &= \langle \psi_{\downarrow}^{\dagger}(\vec{r}) \psi_{\uparrow}^{\dagger}(\vec{r}) \psi_{\downarrow}(\vec{r}') \psi_{\uparrow}(\vec{r}') \rangle \\ &= \langle \psi_{\downarrow}^{\dagger}(\vec{r}) \psi_{\uparrow}^{\dagger}(\vec{r}) \rangle \langle \psi_{\downarrow}(\vec{r}') \psi_{\uparrow}(\vec{r}') \rangle \end{aligned} \quad (2.16)$$

6 in the limit the pair separation  $|\vec{r} - \vec{r}'|$  is infinite. Here,  $\psi_{\uparrow}^{\dagger}(\vec{r})$  and  $\psi_{\uparrow}(\vec{r})$  are the particle  
 7 field operators <sup>1</sup> for creation and annihilation at a coordinate  $\vec{r}$ , with spin  $\vec{k} \uparrow$  From the  
 8 finite value of the pair correlation function ((2.16)), the local pair amplitude  $\langle \psi_{\downarrow}^{\dagger}(\vec{r}) \psi_{\uparrow}^{\dagger}(\vec{r}) \rangle$   
 9 must be non-zero, which is, in essence the amplitude squared of the GL order parameter,  
 10  $|\psi(\vec{r})|^2 \propto n_s$ , the superfluid density. It's important to note that, although  $n_s$  is “local pair”

<sup>1</sup> $\psi_{\uparrow}(\vec{r}) = \langle \psi | \psi_{\uparrow}(\vec{r}) | \psi \rangle$  is the real space representation, where  $|\psi\rangle$  is the ground state wavefunction.



1 amplitude, pairings are non-local, ie, partners of a single pair are in macroscopic distance.  
 2 With macroscopic ODLRO in effect, it's possible to derive Meissner effect [128, 130], flux  
 3 quantization [128]. It has been argued that that ODLRO is a property not only of BCS  
 4 superconductors but also of high- $T_c$  superconductivity [130, 131] and of recently discovered  
 5 superconductivity of Pnictide [132].

6 As in any appearance of order, superconducting order also reduces available symmetry  
 7 of the system. In case of 2nd order, continuous, superconducting transition, order parameter  
 8 is a measure of the amount of symmetry breaking in the ordered state. Symmetry group  
 9  $H$  describing the superconducting state must be a subgroup of the normal state symmetry  
 10 group  $G$ :

$$G = X \times R \times U(1) \times T \quad \text{for } T > T_c \quad (2.17)$$

11 and

$$H \subset G \quad \text{for } T < T_c \quad (2.18)$$

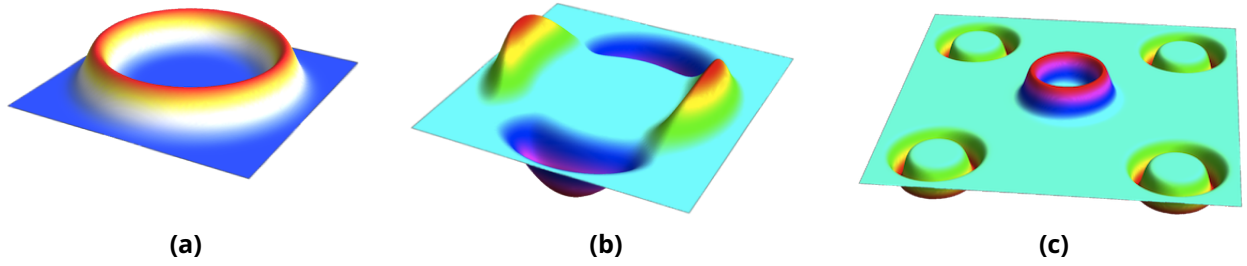
12 where  $X$  is the symmetry group of the crystal lattice,  $R$  the symmetry group of spin rotation,  
 13  $U(1)$  the one dimensional global gauge symmetry, and  $T$  the time reversal symmetry  
 14 operation. Nearly all group-theoretic classifications of superconducting states are based on  
 15 point-group symmetry. Point-group symmetry classification of pair states has been extensively  
 16 studied in cuprate superconductors [133–135].

17 Order parameter symmetry can give insight into the mechanism/nature of pair conden-  
 18 sate and limit the possible interactions that are possible. Crystal structures that have mirror  
 19 symmetry (eg,  $\text{CuO}_2$  layer based superconductors) can be described by parity of the the pair  
 20 state. It has been argued that the complex phase diagram of high- $T_c$  superconductors can be  
 21 deduced from a symmetry principle that unifies antiferromagnetism and order parameter  
 22 symmetry [136].

The superconducting energy gap  $\Delta(\vec{k}, T)$  for  $s$ -wave superconductors, are believed to  
 symmetric in momentum space (figure 2.5a). One other very important pairing symmetry is  
 of  $d$ -wave where the energy gap is thought to be of the form

$$\begin{aligned} \Delta(\vec{k}_F, T) &= \Delta_0(\cos k_x - \cos k_y) \\ &\approx \frac{\Delta_0}{2}(k_x^2 - k_y^2), \text{ along the nodes, } k_x, k_y \text{ small} \end{aligned} \quad (2.19)$$

23 where momentum  $\vec{k}_F$  is measured from the Fermi surface. It may be noted that for  $|\hat{k}_x| = |\hat{k}_y|$ ,  
 24 the gap is zero (figure 2.5b) meaning thermal excitations can easily destroy superconducting  
 25 carriers. High- $T_c$  superconductor family of YBCO are believed to be primarily of  $d$ -wave  
 26 [137–139]. Establishment of predominantly  $d$ -wave order in  $T_c$  materials, over a wide  
 27 range of doping and temperature range, entails the idea that  $d$ -wave symmetry is robust.  
 28 This also suggests that  $d$ -wave pairing in cuprates has a common origin. Newly discovered



**Figure 2.5:** A schematic representation of the superconducting order parameter in different cases: (a) a conventional,  $s$  wave superconductor (eg. Nb); (b) a  $d$  wave, as is the case in copper oxides; (c) an  $s_{\pm}$  wave, as is thought to be the case in iron-based superconductors. In (a) and (b), the two-dimensional Fermi surface is approximated by one circle. In (c), the Fermi surface is approximated by a small circle in the center (the first band) surrounded by four larger circles (to comply with the tetragonal symmetry [140]; the second band). In all cases, the height of the “rubber sheet” is proportional to the magnitude of the order parameter (including its sign).

- 1 family of FE-PNICTIDE superconductors are suggested to have  $s_{\pm}$  (figure 2.5c) symmetry of
- 2 order parameter from band structure calculations [81, 141]. Although a superconducting
- 3 mechanism isn't determined by the order parameter, the pairing hamiltonian must obey the
- 4 point-group symmetry of the gap function  $\Delta(\vec{k})$ .

## 2 Experimental Techniques

### 3.1 Introduction To $\mu$ SR

4 Muons were discovered in the 1930's and their properties learned in the 1940's and were  
 5 used as probes of magnetism in matter [Rasetti, 1944].  $\mu$ SR/ $\mu$ SR refers to muon Spin  
 6 [Rotation/Relaxation/Resonance](#) techniques which uses anisotropic decay of almost 100%  
 7 spin-polarized muons to investigate local magnetic environment of matter, both in bulk and  
 8 in thin films. This is significant improvement on other magnetic resonance probes such  
 9 as nuclear magnetic resonance (NMR) and electron spin resonance (ESR) methods that  
 10 must rely upon thermal equilibrium spin polarization in a magnetic field so that sufficient  
 11 polarization is often achieved only at low temperatures and/or in strong magnetic fields.

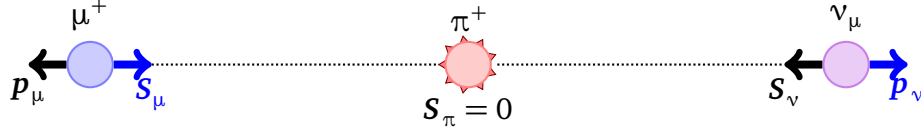
12 When a muon( $\mu^+$ , solely used in our experiments) decays, it emits a fast decay positron  
 13 preferentially along the direction of its spin due to the parity violation. From a single decay  
 14 positron one cannot be certain which direction the muon spin is pointing in the sample.  
 15 However, by measuring the anisotropic distribution of the decay positrons from a large  
 16 number of muons deposited at the same conditions, the statistical average direction of  
 17 the spin polarization of the muon ensemble can be determined. The time evolution of the  
 18 muon spin polarization depends very sensitively on the spatial distribution and dynamical  
 19 fluctuations of the muons' magnetic environment.

#### 3.1.1 Properties And Production Of Muons

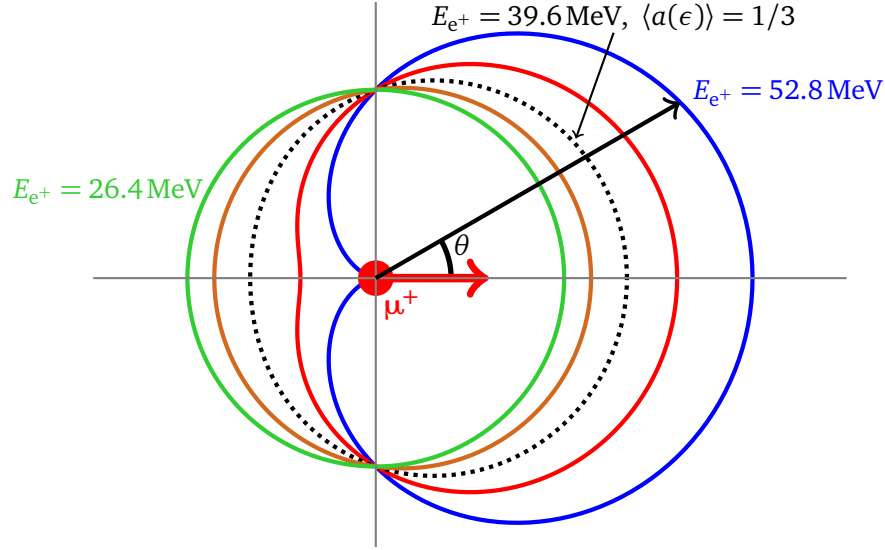
Muons are leptons, 207 times more massive than electrons. Muon properties are briefly mentioned in the following table 3.1. Muons are generated from the decay of charged pions ( $\pi^\pm$ ), produced via collision of high-energy protons with target nuclei, such as carbon or beryllium. The charged pions that are produced live for only about 26 ns and then decay into a muon and muon neutrino (antineutrino), as schematically shown in figure 3.1;



Negative pions in targets behaves as heavy electrons and are captured by nucleus instead of decaying to negative muons. Positive pions do, however, are repelled by nuclei and take up interstitial positions in target atoms and subsequently decay into positive muon and



**Figure 3.1:** Positively charged pions live for about 26 ns and then decay into muon and a neutrino. The muons are almost 100% spin-polarized and their spins are opposite to their momentum. The muons carry a kinetic energy of 4.12 MeV in the rest frame of pion. These muons are referred to as “surface muons” as they originate from pions decaying near the surface of the production target.



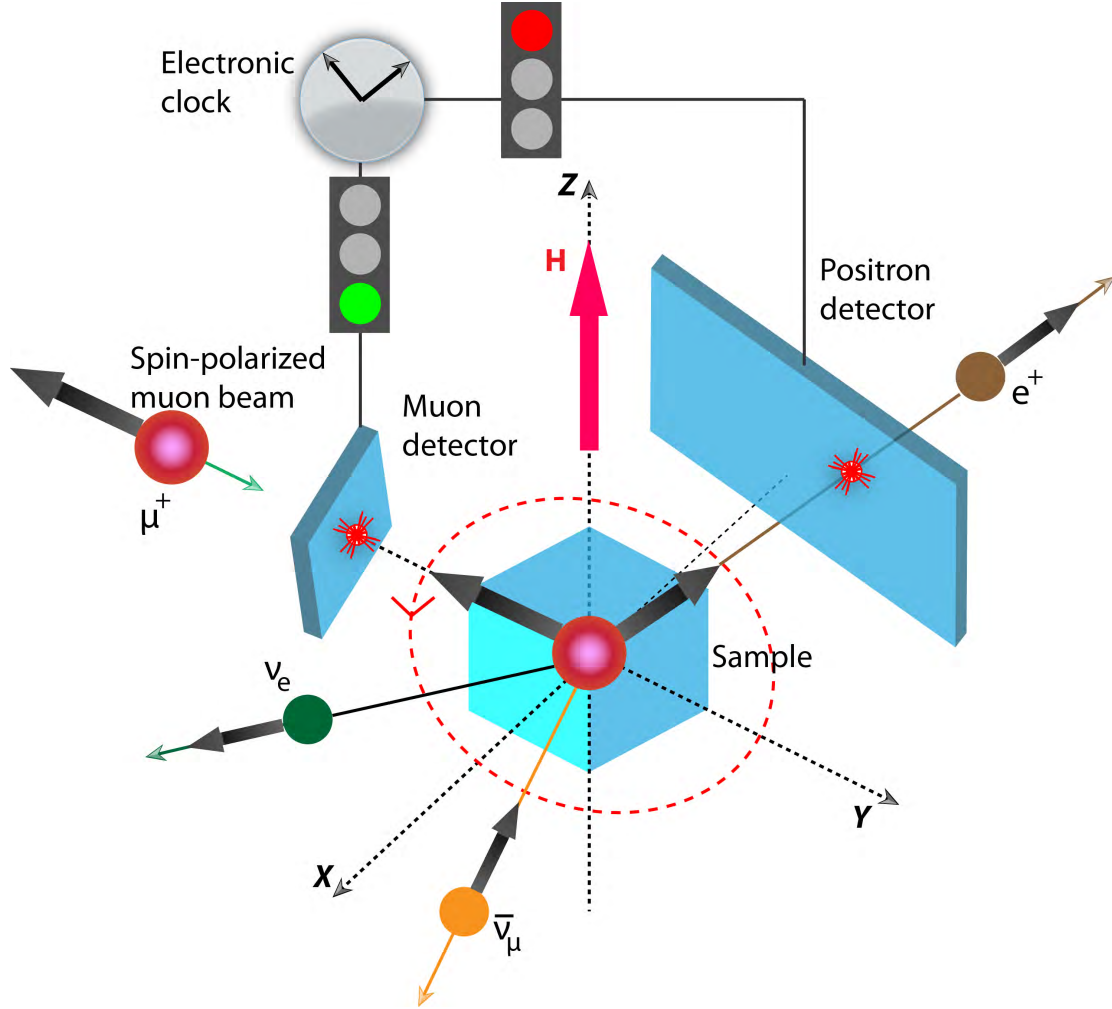
**Figure 3.2:** Angular probability distribution ( $K(\epsilon, \theta)$ , between 26.4 MeV and 52.8 MeV) of positrons emitted from muons with polarization along the red arrow direction is shown. The maximum average asymmetry for such a  $\beta$ -decay process is 1/3 (the dotted curve).

a neutrino. For this reason, only positive muons are used to investigate local magnetic environments in materials. Because of the parity violating nature of weak beta-decay, the positron in a  $\mu^+$  decay is correlated with direction of muon spin at that instant. Ensemble average polarization may be determined from the decay asymmetry of emitted positrons. The highly relativistic positron's probability per unit time, in decaying at an angle  $\theta$  with respect to the  $\mu^+$  spin polarization is given by

$$\frac{dW(\epsilon, \theta)}{dt} = \frac{e^{-t/\tau_\mu}}{\tau_\mu} [1 + a(\epsilon) \cos(\theta)] n(\epsilon) d\epsilon d(\cos(\theta)) \quad (3.2a)$$

$$\equiv \frac{e^{-t/\tau_\mu}}{\tau_\mu} K(\epsilon, \theta) n(\epsilon) d\epsilon d(\cos(\theta)) \quad (3.2b)$$

<sup>1</sup> where “reduced energy”  $\epsilon = E/E_{\max}$ , the **asymmetry**  $a(\epsilon) = (2\epsilon - 1)/(3 - 2\epsilon)$ , energy



**Figure 3.3:** Schematic of the arrangement for a TF-μSR experiment. The muon spin Larmor precesses about the local magnetic field  $\mathbf{B}$  at its stopping site in the sample, and subsequently undergoes the three-body decay  $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$ . The time evolution of the muon spin polarization is accurately determined by detection of the decay positrons from  $\sim 10^6$  muons implanted one at a time.

- 1 density function  $n(\epsilon) = 2\epsilon^2(3 - 2\epsilon)$ . The maximum positron energy  $E_{\max} \simeq 52.8 \text{ MeV}$ , about
- 2 half of the muon rest energy. Integrating over energy ( $\epsilon$ ), we get

$$dW(\theta) = \frac{e^{-t/\tau_\mu}}{\tau_\mu} [1 + \langle a(\epsilon) \rangle \cos(\theta)] d(\cos(\theta)) dt \quad (3.3)$$

- 3 where, the average of the asymmetry function is

$$\langle a(\epsilon) \rangle = \int_0^1 a(\epsilon) n(\epsilon) d\epsilon = \frac{1}{3} \quad (3.4)$$

**Table 3.1:** Properties of Muon [144]

Mass, $m_\mu$	105.658389(34) MeV/ $c^2$
Lifetime, $\tau_\mu$	2.197034(21) $\mu$ s
Charge, $q$	$\pm e$
Spin	$\frac{\hbar}{2}$
Magnetic moment	$4.49044786(16) \times 10^{-26}$ J/T
Spin $g$ -factor, $g_\mu$	2.0023318414(12)
Gyromagnetic ratio, $\gamma_\mu = g_\mu \mu_\mu / \hbar$	$2\pi \times 135.696\,82(5)$ MHz T $^{-1}$

1 The probability function  $K(\epsilon, \theta)$  as a function of polar angle  $\theta$  is plotted in figure 3.2  
2 for reduced energy values of  $\epsilon = 0.5, 0.625, 0.75, 0.875$  and  $1.0$ . For an ensemble of  
3 muons, the maximum theoretical  $\beta$ -decay asymmetry is therefore  $1/3$ . The asymmetry is  
4 determined in our experiments in a **transverse field (TF- $\mu$ SR)** arrangement as shown in  
5 figure 3.3. **TF- $\mu$ SR** refers to the case of incoming muon polarization being perpendicular to  
6 the external field direction For more details of muon production the reader is referred to  
7 these references [142, 143].

### 3.1.2 General $\mu$ SR Techniques

9 Detailed accounts of techniques may be found in the following references: the book of  
10 Schenck [145], the review article of Cox [146] & of H. Keller [147]. For general technical  
11 and statistical details, readers may consult these theses: Riseman [142], Chow [148] and  
12 Luke [149]. While conventional surface muon beams can be used to investigate rather small  
13 samples, there is a desire for still lower energy muons that can be stopped near sample  
14 surfaces (for example to determine depth dependent magnetic field), in thin films and near  
15 multi-layer interfaces (to determine exotic magnetic phenomenon). A number of innovative  
16 methods have been used in attempts to produce **ultra slow** muon beams. The results in  
17 this thesis were obtained using one such method employing ultra slow muons as probe,  
18 described in the next section.

### 3.2 Low Energy $\mu$ SR

20 The experiments detailed in this thesis were done in the low energy ( $\leq 30$  keV) **LEM- $\mu$ SR**  
21 beamline ( $\mu$ E4 [150]) in Paul Scherrer Institut (PSI). LEM group has developed a technique  
22 of slowing down a surface muon beam of 4 MeV and 100% polarization to a beam of low  
23 energy (0-30 keV), polarized muons. Figure 3.4 shows the schematic low energy beamline  
24  $\mu$ E4. The 4 MeV beam passes through a **moderator** consisting of a thin layer ( $\sim 100$  nm) of  
25 rare gas solid or solid nitrogen deposited on 125  $\mu$ m silver substrate. A very small fraction  
26 of the muons escape the moderator with a mean energy about 15 eV with an energy spread  
27 (FWHM) of  $\sim 20$  eV. The dominant fraction of the beam exits the moderator target as **fast**  
28 (degraded but not moderated) muons with a mean energy of 500 keV and a FWHM of the

1 same order. These fast muons are separated from the slow ones by a 90 degree deflection  
 2 by an electrostatic mirror. This deflection, of slow muons, necessarily changes momentum  
 3 direction of muons while keeping the spin direction unaltered. After deflection, muon  
 4 spin and momentum directions are perpendicular. Fast muons are little affected by the  
 5 electrostatic mirror and are monitored by multi channel plate[MCP] detector. The **low**  
 6 **energy muons** are clearly identified by a time-of-flight (TOF) measurement between the  
 7 start scintillator and the **trigger detector**. The trigger detector, made of an ultra-thin carbon  
 8 foil ( $2.2 \mu\text{g cm}^{-2}$ ), is used to set **time-zero**,  $t_0$  for the incoming low energy muons. The  
 9 muons traversing the foil emit a few electrons, which are deflected by 90 degrees and  
 10 detected by a MCP to give the start signal for the  $\mu\text{SR}$  measurement. Trigger detector causes  
 11 an energy loss of the muons  $\sim 1.6 \text{ keV}$  with a Gaussian energy spread  $\sim 500 \text{ eV}$ . Detection  
 12 efficiency of trigger detector is  $\sim 80\%$ . After passing trigger detector, the  $\mu^+$  beam is focused  
 13 on the sample by an einzel lens (L3) and a conically shaped electrostatic lens.

14 The sample and it's Ni-coated<sup>1</sup> Al holder are electrically insulated by a thick sapphire  
 15 crystal and can be biased  $-12.5 \text{ keV}$  to  $+12.5 \text{ keV}$ . Sapphire crystal also provides a good  
 16 thermal contact between the cold finger and the sample. The bias voltage, coupled with  
 17 the voltage at trigger detector (between 12 and 20 kV), determines the muon energy at  
 18 the sample ranging from  $0.5 \text{ keV}$  to  $30 \text{ keV}$ . A Helmholtz coil is used to generate external  
 19 magnetic field applied to the muons. External magnetic field( $B_{\text{ext}}$ ) can be applied in two  
 20 directions; either parallel to muon momentum direction or perpendicular to it, i.e, parallel to  
 21 sample plane ( $ab$ ). In the first case, muons precess in the sample plane and four scintillator  
 22 telescopes (Left, Right, Top and Bottom) are used to count emitted positrons. In the other  
 23 case, muon spins precess out of the plane and only left and right positron counters record  
 24 positron decays. The experiments detailed in this thesis uses the second scenario where  $B_{\text{ext}}$   
 25 is parallel to sample plane. Extensive details on low energy muon beamline may be found  
 26 elsewhere [151, 152].

### 3.2.1 Principles Of LE- $\mu\text{SR}$

28 Positron detectors help accumulating histograms of positron detection events having the  
 29 mathematical form

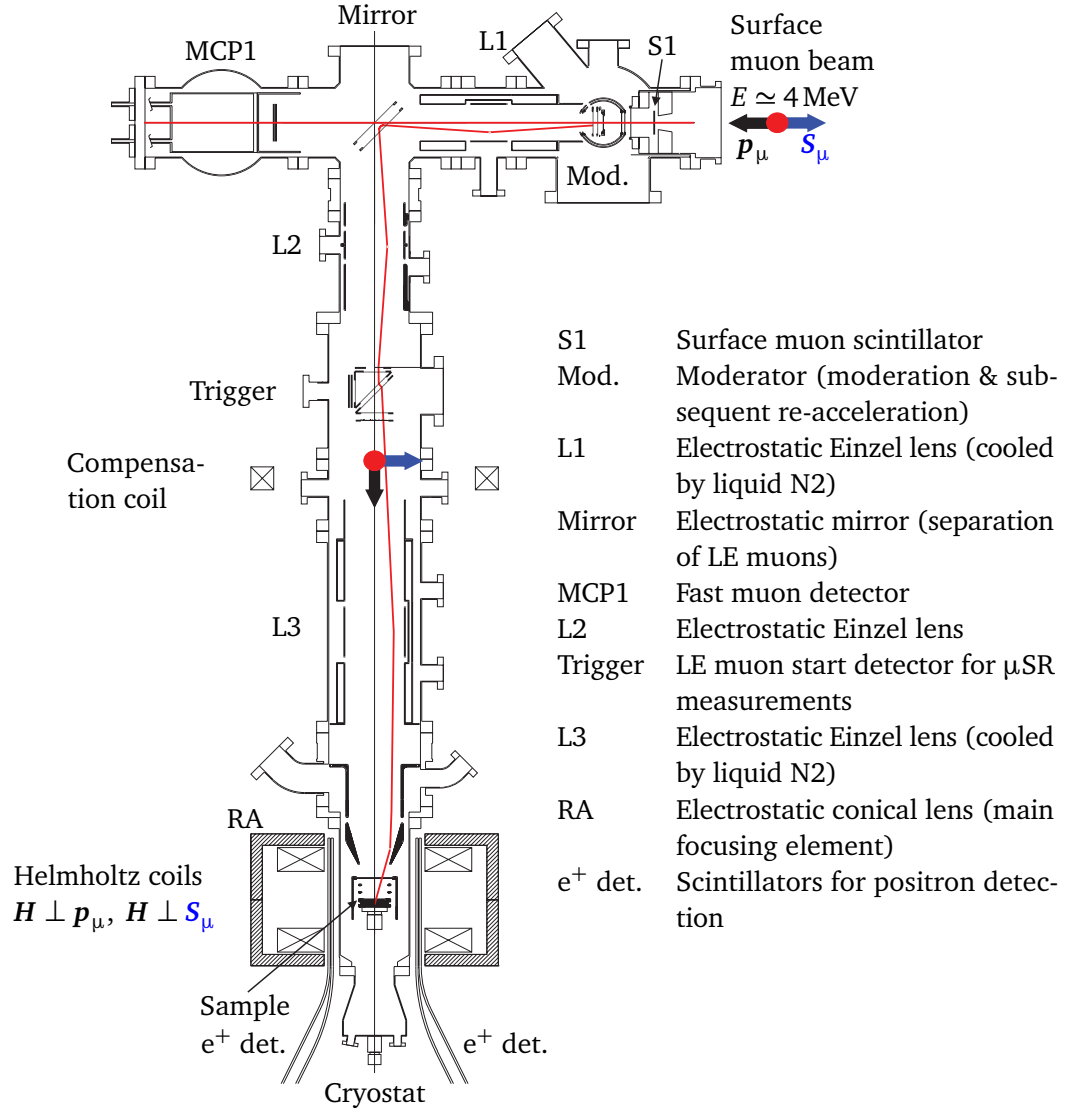
$$N(t) = N_{\text{bg}} + N_0 e^{-t/\tau_\mu} (1 + \mathcal{A}(t)), \quad (3.5)$$

30 where  $N_0$  is the normalization,  $N_{\text{bg}}$  is a time-independent background and  $\mathcal{A}(t)$  is the time  
 31 dependent asymmetry for the detector along  $\hat{n}$  direction, defined as

$$\mathcal{A}(t) = A_0 \mathcal{P}(t) = A_0 \hat{n} \cdot \mathcal{P}(t) \quad (3.6)$$

---

<sup>1</sup>The sample holder is coated with  $\sim 1 \mu\text{m}$  of Ni. Since Ni is ferromagnetic, muons that miss the sample, experience a big hyperfine field and disappears from the frequency window of interest. This very effective background suppression method was the critical step which allows low energy  $\mu\text{SR}$  to be applied to crystals much smaller than the beam diameter.



**Figure 3.4:** Adapted from [151]: A fraction of incoming muons yields very slow muons with a mean energy of about 15 eV. They are accelerated to energies up to 20 keV. The transverse magnetic field with respect to the muon spin is applied by use of a pair of Helmholtz coils. The positrons from the muons decaying in the sample are detected by four pairs of scintillators surrounding the vacuum tube.



1 where  $A_0$  is directly related to the theoretical maximum **asymmetry**  $a(\epsilon)$  referred in (3.4)  
 2 Muon polarization  $\mathcal{P}(t)$  reflects time-dependent spin-polarization and its modulus defined  
 3 as

$$\mathcal{P}(t) = \frac{\langle \mathbf{S}(t) \cdot \mathbf{S}(0) \rangle}{\langle S(0)^2 \rangle} \quad (3.7)$$

where  $\mathcal{P}(0) = \pm \hat{n}$ .  $\hat{n}$  refers to the direction of observation (detector). Experimentally observed asymmetry is lower than the maximum value of 1/3 as the initial spin polarization is less than 1 and emitted positrons are collected in a limited solid angle. Nevertheless, muon asymmetry (polarization) contains all information about muons interaction with the local magnetic environment. Upon entering the sample, the muon spins interact with local magnetic environment and ensemble average spin changes according to the Bloch equation,

$$\frac{d\mathbf{S}}{dt} = \boldsymbol{\mu} \times \mathbf{B} \quad (3.8)$$

where,  $\mathbf{B} = \mathbf{B}_{\text{ext}} + \mathbf{B}_{\text{int}}$ .  $\mathbf{B}_{\text{ext}} \gg \mathbf{B}_{\text{int}}$ , for the experiments detailed in this thesis. Without loss of generality, static external magnetic field may be assumed as  $\mathbf{B}_{\text{ext}} = B\hat{z}$ , yielding

$$\frac{dS_x}{dt} = \gamma S_y B \quad (3.9a)$$

$$\frac{dS_y}{dt} = -\gamma S_x B \quad (3.9b)$$

$$\frac{dS_z}{dt} = 0 \quad (3.9c)$$

4 The above equations (3.9a) and (3.9b) are solved by

$$\begin{aligned} S_x(t) &= S(0) \sin(\gamma B t + \varphi) \\ S_y(t) &= S(0) \cos(\gamma B t + \varphi) \end{aligned} \quad (3.10)$$

5 where  $S(0)$  and  $\varphi$  are determined by spin direction at  $t = 0$ .

6 So far, a big external field's effect (precession of polarization) has been discussed. Here  
 7 we discuss the effects of (small) internal fields that are present in any sample due to randomly  
 8 oriented magnetic moments.

$$\mathcal{P}(t) = \iiint P(\mathbf{B}) \mathcal{P}_B(t) d\mathbf{B} \quad (3.11)$$

9 where  $\mathbf{B}$  is the local magnetic field muon experiences,  $B = |\mathbf{B}|$ ,  $\mathcal{P}_B(t)$  is the time-dependent  
 10 (oscillating) muon polarization and  $P(B)$  is the normalized probability distribution of mag-  
 11 netic field inside the sample. Inside the sample, randomly oriented magnetic moments can  
 12 generate a distribution of magnetic field, at any given muon site, and may be approximated

1 by a three-dimensional Gaussian distribution,

$$P(\mathbf{B}) = \left( \frac{\gamma_\mu}{\sqrt{2\pi}\sigma} \right)^3 \exp \left( -\frac{\gamma_\mu^2 B^2}{2\sigma^2} \right) \quad (3.12)$$

2 For an external field perpendicular to muon spin direction, e.g.  $\mathbf{B}_{\text{ext}} = B_{\text{ext}} \hat{\mathbf{x}}$ , the Gaussian  
3 probability distribution (3.12) is centered around field  $\mathbf{B} = (B_{\text{ext}}, 0, 0)$ , and has the form

$$P(\mathbf{B}) = \left( \frac{\gamma_\mu}{\sqrt{2\pi}\sigma} \right)^3 \exp \left( -\frac{\gamma_\mu^2 \left( (B_x - B_{\text{ext}})^2 + B_y^2 + B_z^2 \right)}{2\sigma^2} \right) \quad (3.13)$$

With  $\mathcal{P}(0) \parallel \hat{\mathbf{z}}$ , the muon polarization may be written as,

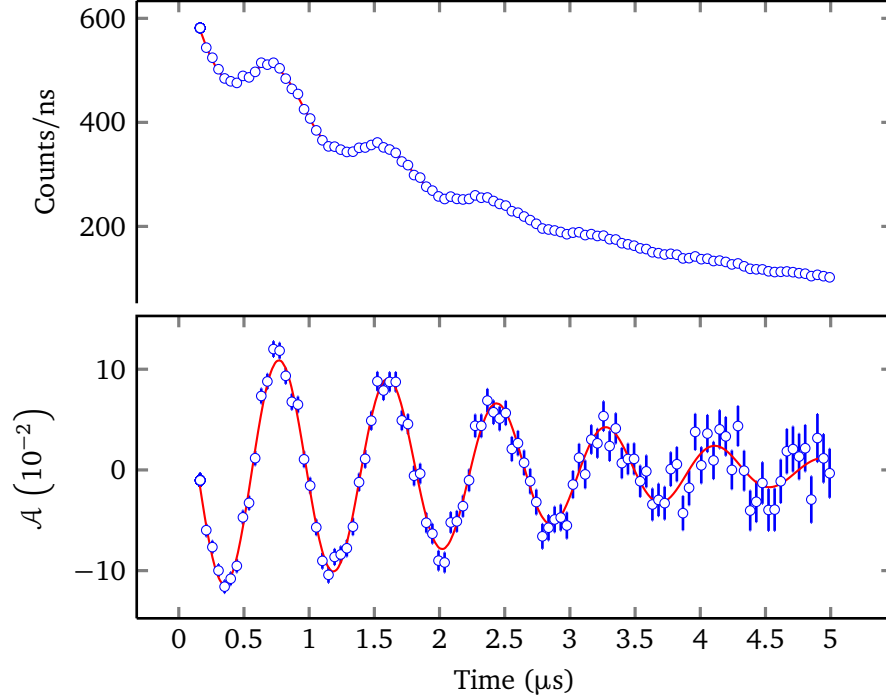
$$\begin{aligned} \mathcal{P}(t) &= \left( \frac{\gamma_\mu}{\sqrt{2\pi}\sigma} \right)^3 \int \int \int \exp \left( -\frac{\gamma_\mu^2 \left( (B_x - B_{\text{ext}})^2 + B_y^2 + B_z^2 \right)}{2\sigma^2} \right) \cos(\gamma_\mu B t) d\mathbf{B} \\ &\simeq \frac{\gamma_\mu}{\sqrt{2\pi}\sigma} \int_{B_x=-\infty}^{\infty} \exp \left( -\frac{\gamma_\mu^2 (B_x - B_{\text{ext}})^2}{2\sigma^2} \right) \cos(\gamma_\mu B_x t) dB_x \\ &= \left( \frac{\gamma_\mu}{\sqrt{2\pi}\sigma} \right) \left( \frac{\sqrt{2}\sigma}{\gamma_\mu} \right) \int_{n=-\infty}^{\infty} \exp(-n^2) \cos \left( \gamma_\mu \left( \frac{\sqrt{2}\sigma n}{\gamma_\mu} + B_{\text{ext}} \right) t \right) dn \\ &= \left( \frac{1}{\sqrt{\pi}} \right) \int_{n=-\infty}^{\infty} \exp(-n^2) \cos(\sqrt{2}\sigma t n + \gamma_\mu B_{\text{ext}} t) dn \\ &= \exp \left( -\frac{\sigma^2 t^2}{2} \right) \cos(\gamma_\mu B_{\text{ext}} t) \end{aligned} \quad (3.14)$$

4 As may be seen from (3.14), random moments in a sample give rise to damping in the  
5 muon polarization. Additionally, when muons are implanted into a sample with inequivalent  
6 magnetic sites (e.g. superconducting state), there will be a distribution ( $\rho(B)$ ), discussed in  
7 detail in the following section, of fields ( $B(z) \equiv B_{\text{ext}}(z)$ ) inside the sample & the polarization  
8 (3.14) takes the form

$$\mathcal{P}(t) = \exp \left( -\frac{\sigma^2 t^2}{2} \right) \int_0^\infty \rho(z) \cos(\gamma_\mu B(z) t) dz \quad (3.15)$$

9 The asymmetry  $\mathcal{A}(t) = A_0 \mathcal{P}(t)$  is fitted to the experimentally observed asymmetries to obtain  
10 physical parameters  $A_0$ ,  $\sigma$  and  $d, \lambda$  (in Meissner state).

11 An example histogram of raw data, from the **forward** counter and asymmetry  $\mathcal{A}(t)$  are  
12 given in figure 3.5. In this experiment, a transverse external field of 9.47 mT was applied  
13 and the resulting sinusoidal oscillations can be taken into account by the theoretical model in



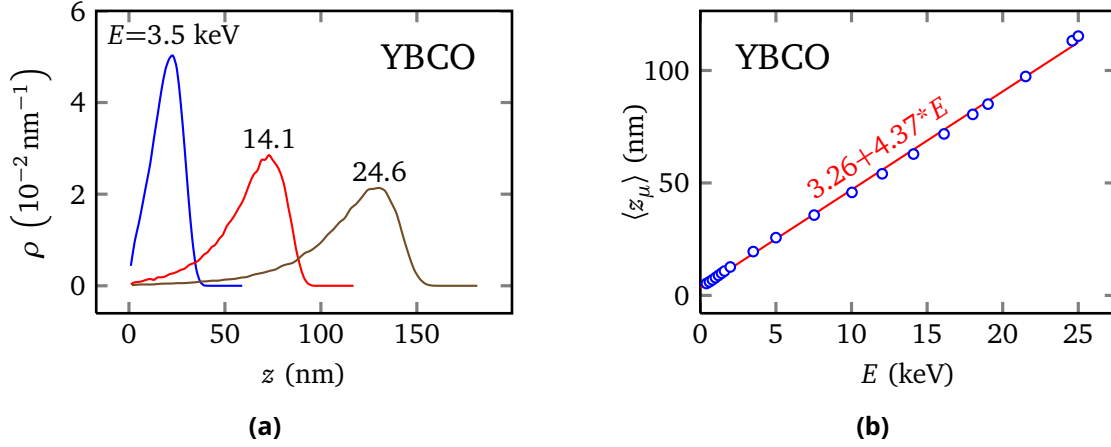
**Figure 3.5:** Top: A histogram from “forward” positron detector in a time-differential measurement on the sample of  $\text{YBa}_2\text{Cu}_3\text{O}_{6.92}$ , depicting the exponential decay of the muons and muon precession. Bottom: Larmor precession of muons’ spin magnetic moment may be noticed from the time dependence of asymmetry  $\mathcal{A}(t)$ , in an external applied field of 94.7G with an implantation energy of 22 KeV, with temperature of 90K.

- 1 (3.15). Probably the most important characteristic of the muon polarization decay spectrum
- 2 is the precession frequency which represents average magnetic field, including local magnetic
- 3 fluctuations, muons experience.

### 3.2.2 Stopping Distribution

- 5 Even at a specific energy, ions bombarded onto a material, end up at different depths
- 6 due to randomness in the collision process. The energy dependent stopping profiles can
- 7 be generated using “Transport of Ions in Solids” (TRIM) codes introduced by Ziegler *et*
- 8 *al* [153] based on the ideas of Eckstein [154]. The accuracy of TRIM in calculating ion range
- 9 distributions in various materials is well established, and they are routinely used in similar
- 10 depth controlled experiments such as Low-Energy  $\mu\text{SR}$ . By specifying the energy, charge, and
- 11 mass of the probe ions ( $\mu^+$  in this thesis), and the mass density and atomic numbers of the
- 12 elements of the probed material, one is able to simulate the implantation profile using TRIM.
- 13 Generated energy-dependent profiles of YBCO are shown in figure 3.6. Average depth of
- 14 muons increases with energy and an almost linear relationship ( $\langle z_\mu \rangle (E) = 3.26 + 4.37 * E \text{ nm}$ )

1 is obtained as shown in figure 3.6. For  $\text{Ba}(\text{Co}_{0.074}\text{Fe}_{0.926})_2\text{As}_2$ , a different energy dependent  
2 linear relationship ( $\langle z_\mu \rangle(E) = 4.26 + 4.53 * E \text{ nm}$ ) is obtained. The stopping profiles are only  
3 relevant for the  $\mu^+$  ions stopping in the probed material. A majority fraction of incoming  
4  $\mu^+$  ions stop at Ni coated sample plate where it quickly depolarizes and do not affect the  
5 calculation of penetration depth presented in this thesis. A review of depth resolved studies  
6 of materials may be found in [152, 155] and references therein.



**Figure 3.6:** Muon implantation profiles in YBCO: Implanted muons stop at differ depths even if the incoming beam energy is the same. By specifying the energy, charge, and mass of the  $\mu^+$  ions, and the mass density and atomic numbers of the elements, profiles are simulated via Monte-Carlo algorithm using TRIM.SP. The accuracy of TRIM.SP in calculating ion range distributions in various materials is well established, and they are routinely used in similar depth controlled experiments.

## Results and Analysis

In this chapter, measurements of  $\lambda$  and the anisotropies ( $\equiv \lambda_a/\lambda_b$ ) are presented for three different oxygen ( $x = 6.52, 6.92, 6.998$ ) contents of  $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$  and for  $\text{Ba}(\text{Co}_{0.07}\text{Fe}_{0.93})_2\text{As}_2$ . The measured values of  $\lambda$  and the anisotropies are considerably different from that of literature, often found with indirect methods.

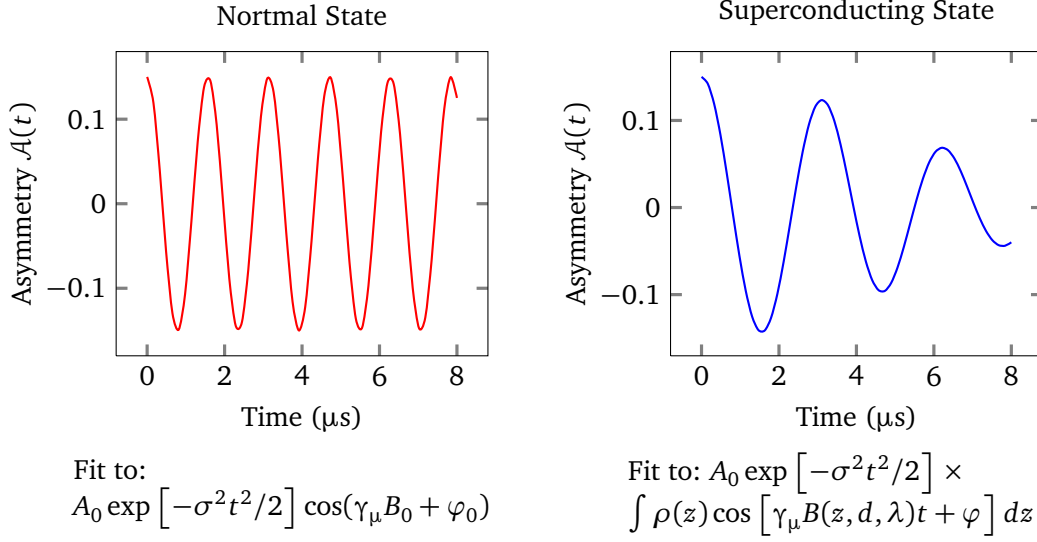
External magnetic field drops inside a superconductor according to London model, described in section 2.2, on the length scale of  $\lambda$ . Close to the surface, there may be deviations from a purely exponential  $B(z)$  as conjectured in (2.15). A reduction in magnetic field can be detected using LE- $\mu$ SR as muon precession frequency is proportional to the local magnetic field. An schematic representation of the fitting procedure to obtain  $\lambda$  and other parameters are shown in figure 4.1. In the normal state, external field ( $\mu_0 H$ ) penetrates the sample fully yielding an average muon precession frequency of  $(\gamma_\mu \mu_0 H)/2\pi$ . A distribution of the local fields, from randomly oriented nuclear moments (see (3.14)), is approximated by the broadening parameter  $\sigma$ . In the superconducting state, muon precession frequency depends on implantation depth ( $z$ ), for the muons landing between depth  $z$  and  $z + dz$  and the time dependent asymmetry may be written as

$$\mathcal{A}(t) = A_0 \exp \left[ -\sigma^2 t^2 / 2 \right] \times \int \rho(z) \cos \left[ \gamma_\mu B(z, d, \lambda) t + \varphi \right] dz \quad (4.1)$$

A slightly modified form (4.2) of the above equation (4.1) is used to fit **all** superconducting states and to extract parameters  $A_0, d, \sigma, \lambda, \varphi$ . Average magnetic field  $\langle B \rangle$  are computed from the above mentioned parameters for various externally applied magnetic fields ranging from 1.5 mT to 10 mT. All measurements in the superconducting state were carried out under **zero-field-cooled** conditions in order to avoid flux trapping at the surface.

The analysis in this chapter are presented chronologically:

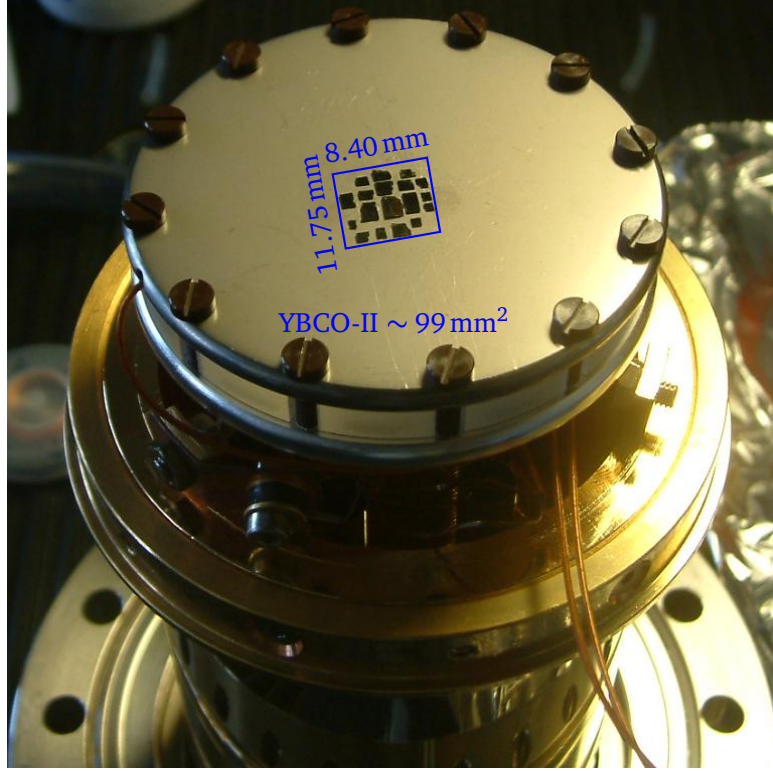
- (i)  $\text{YBa}_2\text{Cu}_3\text{O}_{6.92}$  i.e YBCO-I, section 4.1.1.
- (ii)  $\text{YBa}_2\text{Cu}_3\text{O}_{6.998}$  i.e YBCO-II, section 4.1.2
- (iii)  $\text{YBa}_2\text{Cu}_3\text{O}_{6.52}$  i.e YBCO-III, section 4.1.3
- (iv)  $\text{Ba}(\text{Co}_{0.07}\text{Fe}_{0.93})_2\text{As}_2$ , section 4.2



**Figure 4.1:** Quick outline of the fitting procedure: Left: In the normal state, oscillatory signal in muon polarization is fitted to a cosine function with a Gaussian broadening originating from randomly oriented local internal fields. Right: In the superconducting state, polarization is a sum of frequency ( $\gamma_\mu B$ ) dependent oscillation and an additional broadening  $\sigma$  taking into account expelled flux from neighboring crystals or from any other sources of random fields.

#### 4.1 LE- $\mu$ SR experiments on $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ single crystals

The LE- $\mu$ SR experiments on  $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$  have been conducted on freshly grown single crystals using the self-flux method [156]. The purity of the crystals is the same as for crystals in which quantum oscillations in resistivity have recently been reported [157]. Each of the crystals was approximately rectangular in shape with lateral dimensions in the  $a$ - $b$  plane ranging from 1 mm to 3 mm and a thickness in the  $c$ -direction ranging from 0.1 mm to 0.3 mm. They were detwinned to a level greater than 95 %. The mounted mosaic of YBCO-II on the coldfinger is shown in the figure 4.2. Two other oxygen doped crystals were mounted in very similar method. The crystal faces were mirror-like in appearance and atomic force microscopy indicates the roughness of the surface to be a few nm (cf. figure 4.3). A UHV compatible Ag epoxy was used to attach each crystal to the sample holder made of high purity Al coated with 1  $\mu\text{m}$  of Ni. Muons that miss the sample stop at the polycrystalline Ni coated sample holder and experience big hyperfine field from Ni moments; thereby quickly depolarizes and disappears from the frequency window of interest. Control experiments on a Ag disc the same size as the sample showed that such a thin layer of Ni has no effect on the precession signal in the sample. The maximum asymmetry  $A_0$  from all these experiments are significantly less than the theoretical maximum 1/3 as decay positrons are collected in a limited solid angle and only  $\sim 40\%$  muons land on  $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$  crystals.



**Figure 4.2:** YBCO-II Mosaic: The total area of the mosaic is about  $99 \text{ mm}^2$ . Each of the crystals was approximately 1 mm to 3 mm in the  $a - b$  plane and a  $c$ -axis thickness of 0.1 mm to 0.3 mm. An ultra high vacuum compatible Ag epoxy was used to attach each crystal to the sample holder of high purity Al coated with  $1 \mu\text{m}$  of Ni.

#### 4.1.1 $\text{YBa}_2\text{Cu}_3\text{O}_{6.92}$

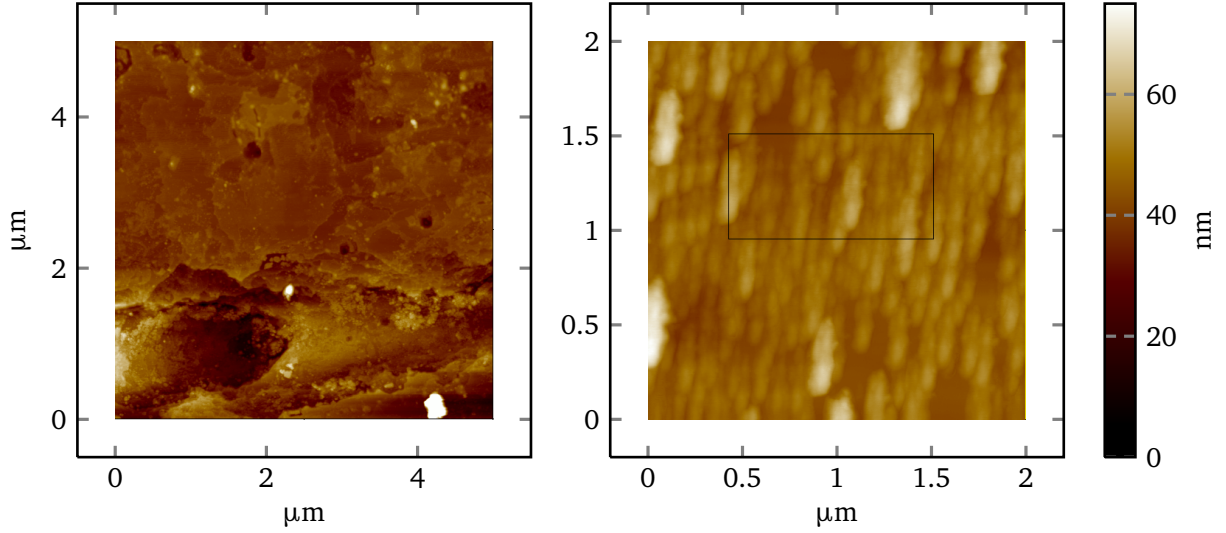
- 2 To analyze superconducting state asymmetry  $\mathcal{A}(t)$ , slightly modified version of (4.1) is used

$$\mathcal{A}(t) = A_0 \exp\left(-\frac{\sigma^2 t^2}{2}\right) \int \rho(B) \cos\left[\gamma_\mu B(z, d, \lambda)t + \varphi\right] dB, \quad (4.2)$$

- 3 where an integration over field distribution  $\rho(B)$  is done instead of an integral over  $\rho(z)$ .  
 4  $\rho(B)$  is defined as

$$\rho(B) = \rho(z) \left| \frac{dB}{dz} \right|^{-1} \quad (4.3)$$

- 5 Although equations (4.1) and (4.2) are mathematically equivalent for the analysis presented  
 6 in this thesis, the latter offers a more general approach for analyzing data using models  
 7 where a specific field distribution is believed to be present. To take into account any random



**Figure 4.3:** YBCO roughness: Left figure shows a  $5\mu\text{m}\times 5\mu\text{m}$  area of a YBCO-I crystal being scanned by tapping AFM. Right figure shows another region of a smaller size and even smaller sized box. A 2 nm to 3 nm of average roughness is found from these measurements.

- 1 local field present at a muon site, a Gaussian probability model is assumed as,

$$P(B', B) = \left( \frac{1}{\sqrt{2\pi}\Delta B} \right) \exp \left( -\frac{1}{2} \left( \frac{B' - B}{\Delta B} \right)^2 \right) \quad (4.4)$$

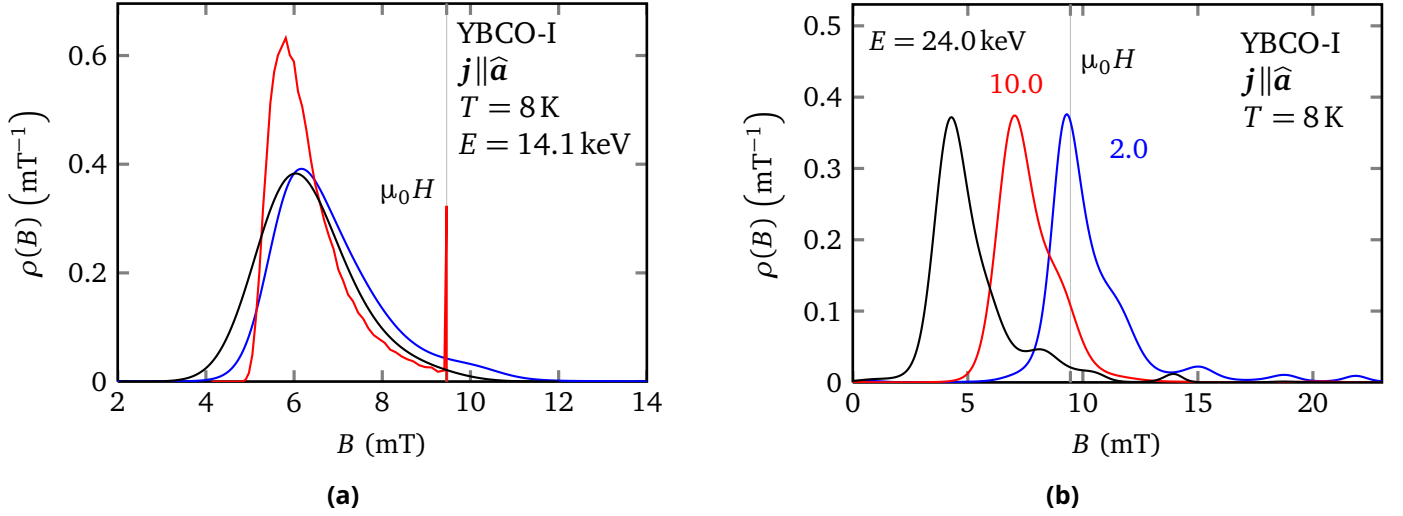
- 2 where  $P(B', B)$  is the probability function of the total field  $B'$  experienced by muon and  $\Delta B$
- 3 is the width of the probability function. Broadened total probability density is obtained by
- 4 convoluting (4.4) with the field distribution from the pure London model in the (4.3)

$$\rho'(B') \equiv \int_{B=0}^{\infty} P(B', B) \rho(B) dB \quad (4.5)$$

- 5 Since stopping distributions ( $\rho(z)$ ) are obtained as discrete numbers from TRIM.SP simula-
- 6 tions, the integral in the (4.5) is replaced by a summation to calculate the total probability
- 7 density. As may be seen from the figure 4.4a, the field distributions from maximum-entropy
- 8 (ME) [158–160] are wider than that expected from a pure London model ( $\sigma = 0$ ). For the
- 9 ME analysis a Gaussian apodization ( $\sigma_{\text{apod}} = 2.0\mu\text{s}$ ) is used for all the energy dependent
- 10 spectra. Apodization leads to very small symmetric broadening and smoothing of field
- 11 distribution but is not expected to change the average magnetic field  $\langle B \rangle$ . This may be noted
- 12 that there is considerable weight at higher fields (cf. figure 4.4b) than the applied field  $\mu_0 H$
- 13 which is out of the scope of the (4.3).

- 14 In the equations (4.1) and (4.2),  $\varphi$  should ideally depend on purely geometric parameters





**Figure 4.4:** (a) Magnetic field distribution as seen by muons at an implantation energy of  $E = 14.1$  keV and at  $T = 8$  K in an external magnetic field  $\mu_0 H = 9.5$  mT, applied parallel to the  $b$  axis of the crystals. The field distribution corresponds to the asymmetry spectrum (a) in the figure 4.5. The red line (—) is model field from pure London model with  $d = 10.3$  nm,  $\lambda_a = 128.9$  nm and  $\sigma = 0$ . The spike in  $\rho(B)$  at the applied field is from muons stopping in dead layer. The black line (—) is the theoretical distribution convoluted with a Gaussian with a second central moment  $\Delta B = 0.72$  mT corresponding to the average  $\sigma = 0.61 \mu\text{s}^{-1}$  from the figure 4.6b). The blue line (—) is obtained from maximum-entropy analysis. The higher weight at high fields in actual field distribution is likely to be from trapped vortices close to the surface and is not taken fully into account by our symmetrically broadened field distribution. (b) Maximum-entropy field distribution are shown at three different energies. At the lowest energy, there is significant contribution from possible demagnetization and trapped vortices.

1 and muon arrival time on the sample. Thereby, one way to analyze the spectra is to fix  $\varphi$  to  
2 values obtained at temperatures above  $T_c$ , as has been described in [161]. However,  $\varphi$  may  
3 be substantially different in the superconducting state compared to the values for  $T > T_c$ ,  
4 as we will see shortly.  $\varphi$  may also depend on muon implantation energy in the following  
5 manner<sup>1</sup>: in the normal state ( $T > T_c$ ) the angular part of the asymmetry is given by

$$\mathcal{A}(t) \sim \cos(\omega(t - t_0)) \quad (4.6)$$

6 assuming  $\varphi = 0$ , for simplicity. Here  $\omega = \gamma_\mu B$  is the frequency and  $t_0$  being the time muon  
7 enters the sample. If an error is made in determining  $t_0 \rightarrow t_0 - dt$ , then (4.6) becomes

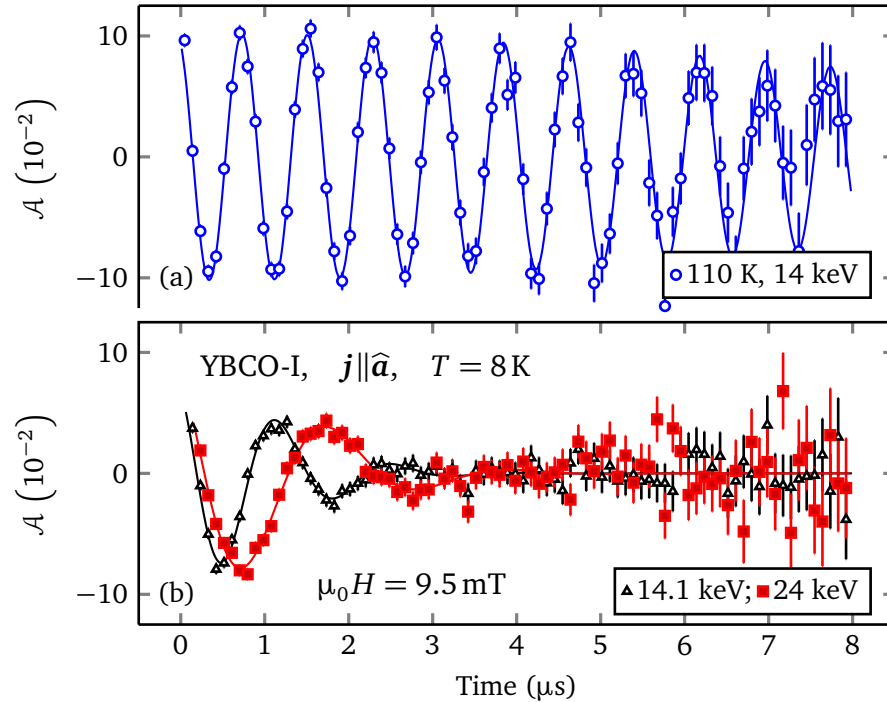
$$\mathcal{A}(t) \sim \cos(\omega(t - t_0) + \omega dt) \quad (4.7)$$

<sup>1</sup>This analysis was inspired by Rob Kiefl

Equation (4.7) implies that the fitted phase will be  $\omega dt$ . However, in superconducting state and in high implantation energy, muons deep inside the sample will see a reduced field and a corresponding reduced frequency  $\omega'_E < \omega$ , assuming for simplicity that all the muons precess in a single frequency. This implies that, in superconducting state, (4.7) may be written as

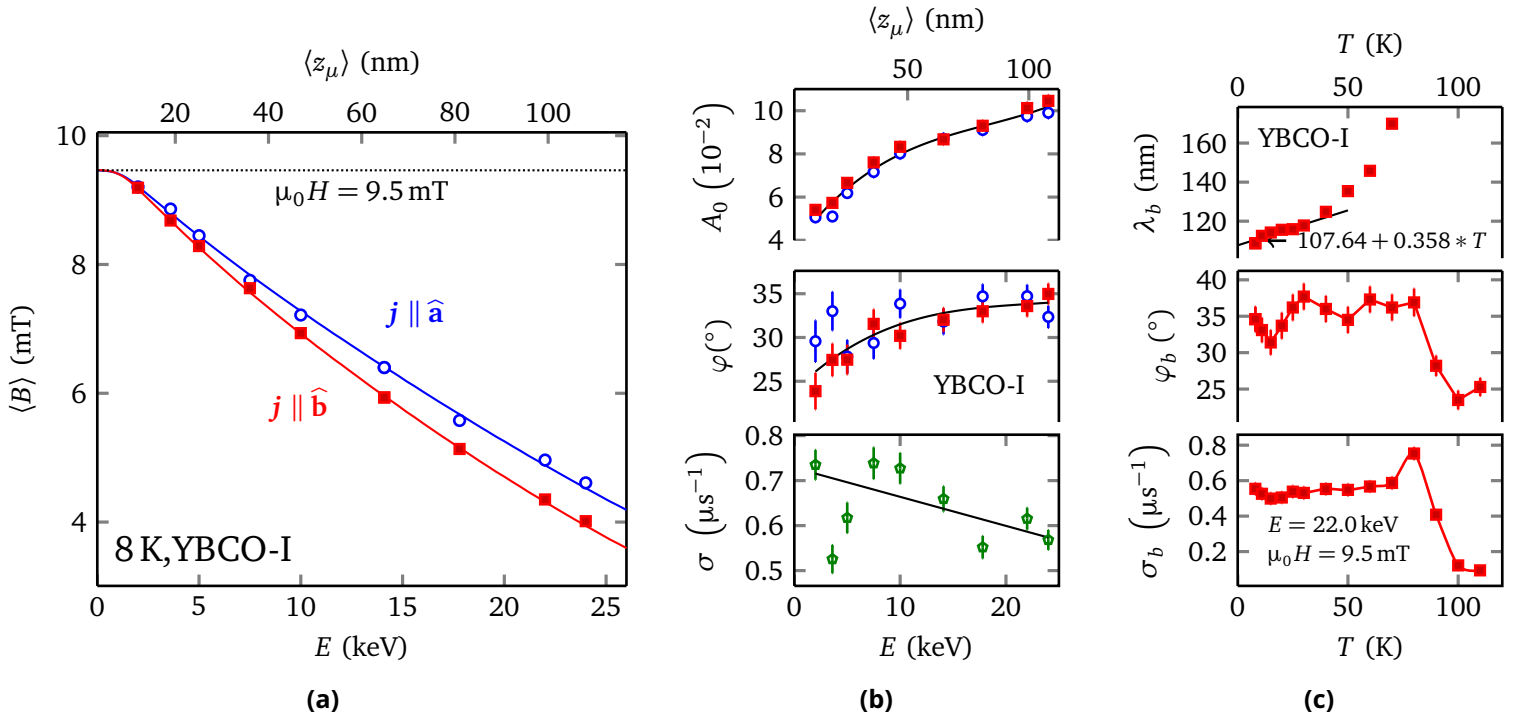
$$\mathcal{A}(t) \sim \cos(\omega'_E(t - t_0) + \omega'_E dt) \quad (4.8)$$

meaning a phase shift of  $(\omega'_E - \omega)dt$  will be detected. The possibility of actual frequency distribution being more asymmetric than our theoretical frequency distribution one can also introduce an apparent phase shift. To summarize, imperfections in fitting function, TOF distribution and detector geometry may all introduce energy dependent phase shift. For the reasons above, phases for individual energies were kept free (**individual phase model**) in the global fit. A shared phase global fit analysis (**shared phase model**) was also done which yields  $\lambda_{\{a,b\}}$  values within the systematic errors  $\sim 3$  nm. This is also a strong indication that the subtleties of muon arrival time, detector geometry and thereby the phases ( $\varphi$ ) & broadening parameter ( $\sigma$ ) do not significantly influence the extraction of the absolute values of London penetration depth.



**Figure 4.5:** Top: The muon spin precession signal in the normal state of  $\text{YBa}_2\text{Cu}_3\text{O}_{6.92}$  at 110 K in an external field of 9.46 mT applied parallel to the  $b$  direction. The mean implantation energy is  $E = 14.1$  keV which corresponds to a mean implantation depth of 65 nm. Bottom: the same conditions as top except in the superconducting state at  $T = 8$  K for two energies 14.1 keV and 24 keV.

Figure 4.5a shows the muon precession signal in the normal state at 110K in a small magnetic field of 9.5 mT applied along the  $a$  axis of the crystals with an implantation energy at 14.1 keV. Observed frequency in figure 4.5a also includes a damping rate of  $0.086(11)\mu\text{s}^{-1}$  which is consistent with the damping from randomly oriented Cu nuclear dipole moments. Meissner screening of the external field is apparent by comparing the normal state (figure 4.5a) with the superconducting state (figure 4.5b). As mentioned earlier, the superconducting state data are fitted to (4.2) with shared  $A_0, d$  and  $\lambda$  for all the spectra at 8 K in both  $a$  and  $b$  directions. The depth dependence of the average internal field is evident by comparing the two spectra ( $\circ$  and  $\blacksquare$ ) in figure 4.5b which have different implantation profiles. Exemplary profiles are shown in figure 3.6a.

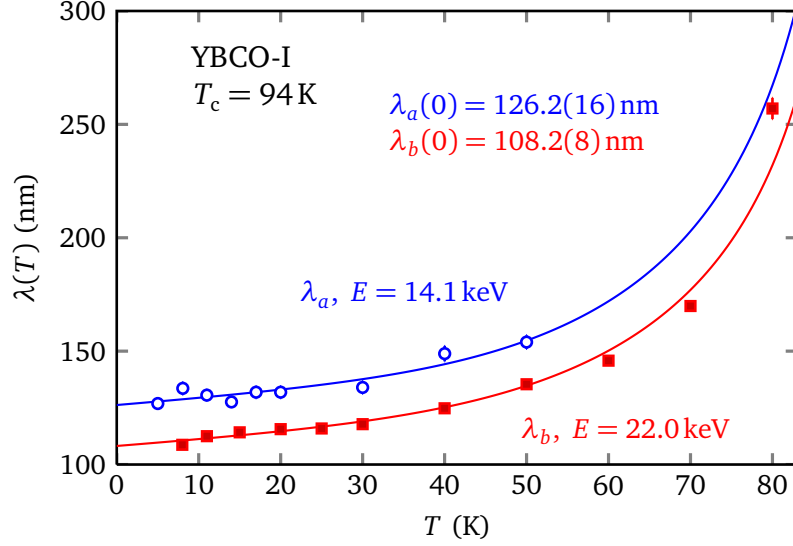


**Figure 4.6:** (a) The average magnetic field ( $\langle B \rangle$ ) versus mean stopping depth in an applied field of 9.46 mT such that the shielding currents are flowing in the  $a$  direction ( $\vec{j} \parallel \hat{a}$ ,  $\circ$ ) and  $b$  direction ( $\vec{j} \parallel \hat{b}$ ,  $\blacksquare$ ). The curves are the average fields generated from a global fit of all the spectra at 8 K taken at all energies and for both orientations. The common parameters are  $\lambda_a$ ,  $\lambda_b$  and  $d$ . The individual points are from a fit to the same model but at a single depth. The differences between the data points and curves reflect how close the data at a single energy agrees with the global fit. (b) The phase, asymmetry and broadening parameter versus  $E$ . (c) Temperature dependence of  $\lambda$ ,  $\varphi$  and  $\sigma$  when  $\mu_0 H \parallel \hat{b}$ . More details are in the text.

Figure 4.6a shows the average local field [ $\langle B \rangle = \int \rho(z)B(z)dz$ ] determined from fits

1 at a single energy as a function of beam energy (bottom scale) and corresponding mean  
 2 implantation depth (top scale). The filled circles (●) and filled squares (■) are from data  
 3 taken with the shielding currents flowing along the  $a$  and  $b$  axes respectively (or equivalently  
 4 the magnetic field along the  $b$  and  $a$  axes respectively). The corresponding London model  
 5 curves are generated from a global fit of runs taken at 8 K for both orientations and all  
 6 energies using the calculated TRIM.SP implantation profiles. The common parameters are  
 7  $\lambda_a = 128.9(12)$  nm,  $\lambda_b = 108.4(10)$  nm and  $d = 10.3(4)$  nm. In the **shared phase model**,  
 8  $d$  is fixed to 10.3 nm and the obtained  $\lambda_{a,b}$  differs by  $\sim 1$  nm as shown in the table 4.1.  
 9 Statistical uncertainties are determined from the global  $\chi^2$  surface and take into account  
 10 the strong correlation between  $\lambda_{a,b}$  and  $d$ . Since there is almost no correlation between  
 11  $d$  and  $\lambda_a/\lambda_b$ , this ratio is determined more accurately than the absolute values of  $\lambda_a$  and  
 12  $\lambda_b$  as may be seen in the table 4.1. As may be evidenced from figure 4.6a, the average  
 13 field falls exponentially from the surface for higher implantation energies however, close  
 14 to the surface, deviation from a simple exponential is modelled from the curvature at the  
 15 lowest energies. This implies that the supercurrent density is suppressed near the surface  
 16 relative to a London model. This is idealized but the nature of the data does not warrant  
 17 more complicated models. It is unclear to what extent this suppression of the supercurrent  
 18 density is intrinsic as a result of the discontinuous nature of electronic properties near a  
 19 surface. Higher average field from ME analysis is likely due to trapped vortices close to the  
 20 surface and from fields expelled from neighboring crystals. A suppression of the supercurrent  
 21 density was reported in a previous low energy  $\mu$ SR study of the field profile in a thin film  
 22 of  $\text{YBa}_2\text{Cu}_3\text{O}_{6.92}$  and attributed to surface roughness [162]. It is difficult to exclude such  
 23 extrinsic effects which could also lead to a suppression of the supercurrent density near a  
 24 surface. Measurements on atomically flat cleaved surfaces are needed to resolve the origin  
 25 of  $d$ . There are no other measurements of electromagnetic properties as a function of depth  
 26 in  $\text{YBa}_2\text{Cu}_3\text{O}_{6.92}$  crystals to compare with. Surface sensitive techniques such as STM and  
 27 ARPES are only sensitive to the top few unit cells where the properties can be very different  
 28 than in the bulk.

29 It may also be noted from the figure 4.6b, that the fitted asymmetry goes from 5 % (low  
 30 energy) to 10 % (high energy). This is expected since at low energies, incoming muons  
 31 have an angular distribution as they approach the sample surface and at lower energy,  
 32 many of them are deflected backwards, i.e., backscattering reduces the number of total  
 33 muons, reducing the signal strength and asymmetry. The average phase from **individual**  
 34 **phase model** is close to that from the **shared phase model**, indicating that the latter model  
 35 determines an “effective average phase”. The broadening parameter  $\sigma$ ’s energy dependence  
 36 reflects the random local fields and is also temperature independent below  $0.8 T_c$ , as expected  
 37 and evidenced from the figure 4.6c. This is due to the bulk magnetization effects, whereby  
 38 flux expelled from neighboring crystals broadens the magnetic field distribution at the



**Figure 4.7:** Temperature dependence of penetration depths.

- 1 surface of any given crystal. Since  $\sigma$  reflects the broadening of an **effective field** at a specific
- 2 energy, it is always kept as a free parameter both in **individual phase model** and **shared**
- 3 **phase model**.

The absolute value of  $\lambda_b$  as a function of temperature is shown in figure 4.6c (top panel). The data points are the fitted values of  $\lambda_b$  determined from a fit to the model at a single implantation energy of 22 keV. Since  $d$  is not temperature dependent it was fixed at 10.3 nm. The solid line is a linear fit of our data below 30 K and gives a slope of  $0.357(67) \text{ nm K}^{-1}$ . This was used to extrapolate our measurement of  $\lambda_b$  at 8 K down to zero temperature. The slope and extrapolated value depend slightly on the fitted temperature range adding an additional systematic error of about 1 nm. To obtain  $\lambda_a(0)$ , normalized superfluid density in  $a$  and  $b$  axis directions are taken as approximately equal as in the (4.9)

$$\begin{aligned} \left( \frac{\lambda_a(0)}{\lambda_a(T)} \right)^2 &\simeq \left( \frac{\lambda_b(0)}{\lambda_b(T)} \right)^2 \\ \Rightarrow \lambda_a(0) &= \lambda_b(0) \left( \frac{\lambda_a(T)}{\lambda_b(T)} \right), \end{aligned} \quad (4.9)$$

- 4 where  $\lambda_{a,b}(T)$  are the finite temperature penetration depths measured by our modified
- 5 London model.

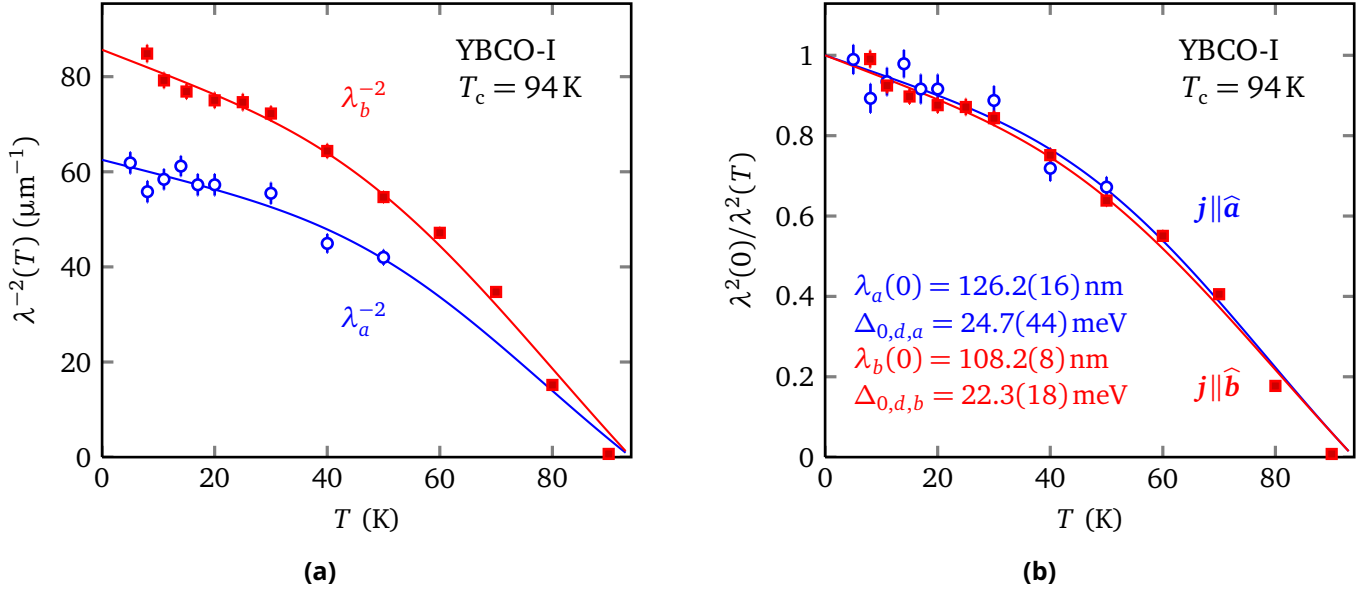
6 Table 4.1 gives our results for  $\lambda_a$  and  $\lambda_b$  measured at 8 K in  $\text{YBa}_2\text{Cu}_3\text{O}_{6.92}$ . The errors

7 reported here are just the statistical uncertainty. Systematic errors due to uncertainties in

8 the muon stopping distribution and due to extrapolation to 0 K is  $\sim 3\%$ . At the moment,

9 the latter dominates the overall uncertainty but should improve with refinements of the

10 stopping distribution calculations. More details about possible slightly different stopping



**Figure 4.8:** Temperature dependence of penetration depths and normalized superfluid density.

**Table 4.1:** Summary of results in  $\text{YBa}_2\text{Cu}_3\text{O}_{6.92}$ : Measured London penetration depths at 8 K are shown for two models: “phase individual” and “phase shared”, for individual energies. The errors are reported here are just statistical errors. An additional  $\sim 2\%$  ( $\sim 2$  nm) error is due to uncertainty in stopping distribution. It may be noted that the two phase models yield penetration depths within a nm of each other for both  $a$  and  $b$  axis, stressing that the determination of phase does not affect the measurement of absolute  $\lambda$ .

$B$ (mT)	$d$ (nm)	$\varphi$ ( $^\circ$ )	$\lambda_a(8\text{ K})$ (nm)	$\lambda_b(8\text{ K})$ (nm)	$\mathcal{R} \equiv \lambda_a/\lambda_b$	$\chi^2/\text{DF}$
9.46	10.3(5)	32.2(6) <sup>a</sup>	128.9(12)	108.4(10)	1.19(1)	1.058
	10.3 (fixed)	33.0(8) <sup>b</sup>	130.2(14)	109.2(12)	1.19(1)	1.069

<sup>a</sup> Average of energy specific phases from “individual phase” analysis.

<sup>b</sup> Global phase from “shared phase” analysis.

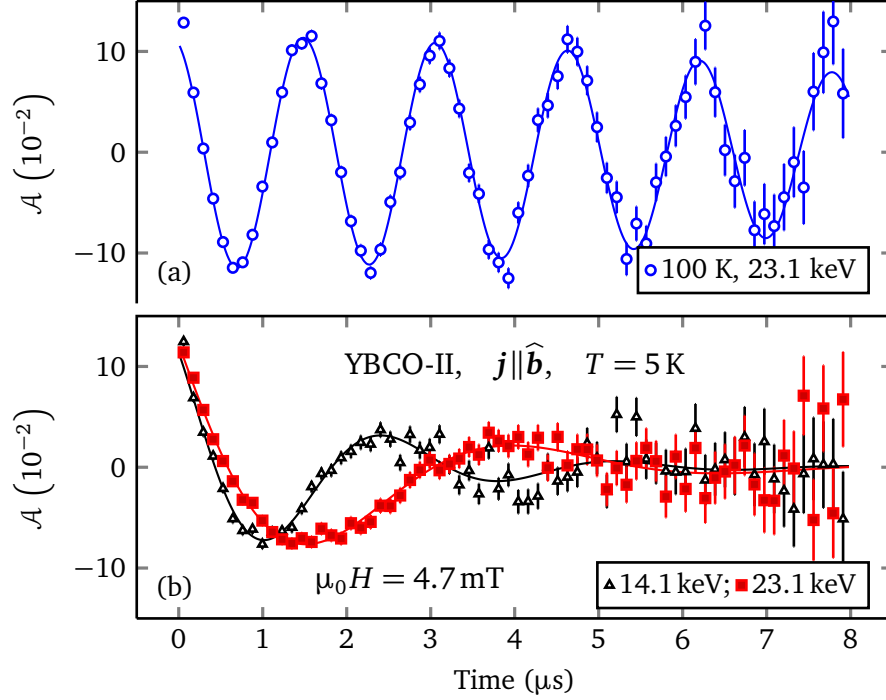
1 profiles is available in the appendix. In summary we have used low energy  $\mu\text{SR}$  to measure  
2 the magnetic field profiles in the Meissner state of a mosaic of detwinned single crystals  
3 of  $\text{YBa}_2\text{Cu}_3\text{O}_{6.92}$ . The comparison of  $\lambda_{a,b}$  values obtained here with those from different  
4 methods is deferred until a later section. Since the data analysis method for the next three  
5 sections will be very similar to this one, primarily differences will be mentioned.

#### 4.1.2 $\text{YBa}_2\text{Cu}_3\text{O}_{6.998}$

7 In this section, total four sets of data are analyzed and presented: (i) in an external field  
8 of 4.7 mT at three temperatures 4 K, 5 K and 12 K (ii) in an external field of 9.5 mT at the

1 temperature of 5 K.

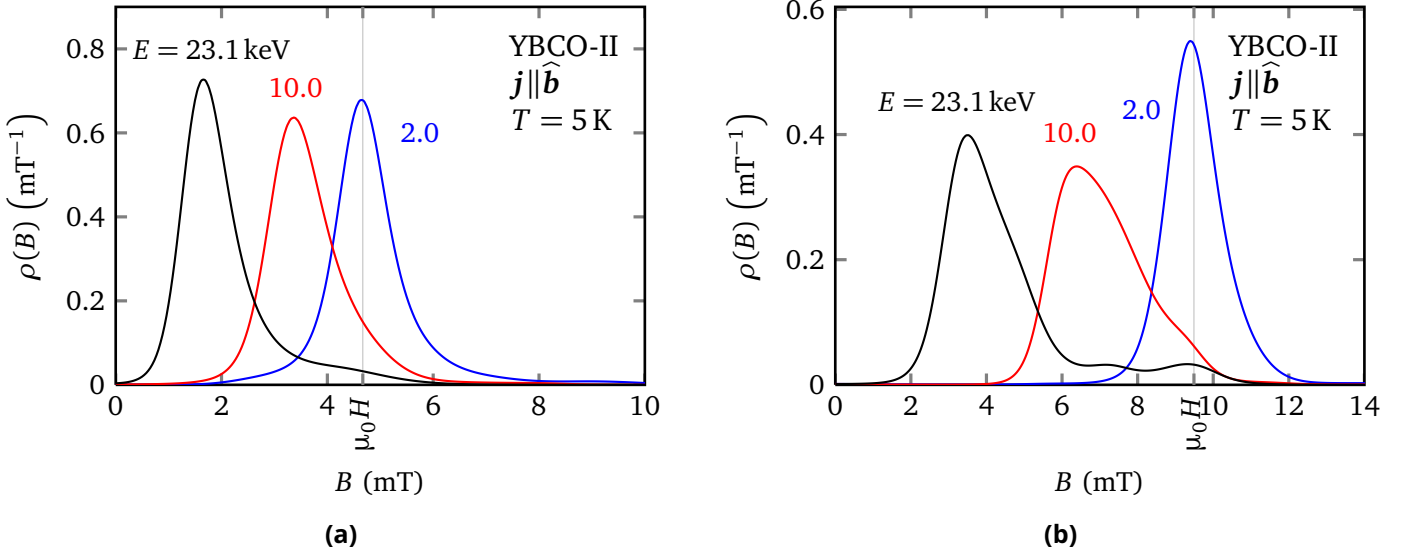
2 Examples of the muon precession signals in the crystal mosaic may be seen in figure 4.9.  
 3 Figure 4.9a shows the muon precession signal in the normal state at 100K in a small  
 4 magnetic field of 4.7 mT applied along the  $a$  axis of the crystals with an implantation energy  
 at 23.1 keV. In figure 4.9a, the average frequency corresponds to the applied field with a



**Figure 4.9:** (a) The muon spin precession signal in the normal state of  $\text{YBa}_2\text{Cu}_3\text{O}_{6.998}$  at 100 K, 23.1 keV in an external field of 4.7 mT applied parallel to the  $a$  direction. (b) The same conditions as (a) except in the superconducting state at  $T = 5$  K with an two implantation energies 14.1 keV and 23.1 keV. The solid lines are fits to a London model profile described in the previous section.

5  
 6 damping rate of  $0.110(10)\mu\text{s}^{-1}$ , which is slightly bigger than obtained in  $\text{YBa}_2\text{Cu}_3\text{O}_{6.92}$   
 7 but nonetheless consistent with a random local field distribution. All measurements in  
 8 the superconducting state were carried out under **zero-field-cooled** conditions in order to  
 9 avoid flux trapping at the surface. Meissner screening of the external field is apparent by  
 10 comparing the normal state in figure 4.9a with the superconducting states in figure 4.9b. The  
 11 depth dependence of the average internal field( $\omega$ ) is also perceptible by comparing the two  
 12 spectras in figure 4.9b which have different implantation profiles. The curves in figure 4.9b  
 13 are generated from fits to a London model profile as described in the previous section. Since,  
 14 the precession signal spectra bear similar characteristic as figure 4.9, further precession plots  
 15 are omitted for this sample.

16 Maximum entropy field distributions corresponding to the two external fields of 4.7 mT



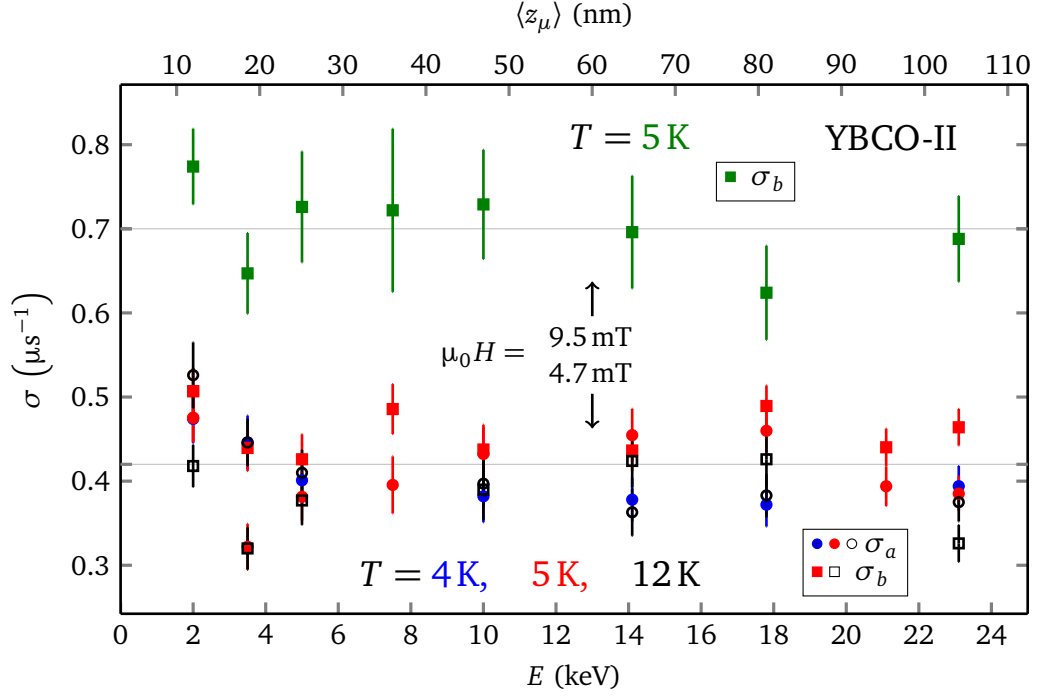
**Figure 4.10:** Magnetic field distribution as seen by muons at various implantation energies and at  $T = 5$  K in an external applied magnetic field ( $\mu_0 H$ ) of 4.7 mT and 9.5 mT, applied parallel to the  $a$  axis.

1 and 9.5 mT are shown in the figure 4.10. As may be noticed,  $\rho(B)$ , corresponding to the  
2 higher applied field (9.5 mT), is more asymmetric. However, in both cases, there are long  
3 tails, in field distributions, even at higher implantation energies possibly due to trapped  
4 vortices close to the surface.

5 The fitted asymmetry in figure 4.11 varies between 5% (low energy) to 10% (high  
6 energy). This is consistent with backscattering of muons at low energies. The phases for  
7 individual runs varies between  $\sim 15^\circ$  to  $40^\circ$ . This range is also consistent with phases noticed  
8 in the previous section, underlining that the systematic errors in determining a precise phase  
9 remains the same. For low temperatures significantly below the critical temperature  $T_c$ , the  
10 broadening parameter  $\sigma$  is about  $0.4 \mu s^{-1}$  (bottom panel, figure 4.11) which is about half of  
11 that observed in  $YBa_2Cu_3O_{6.92}$  in 9.5 mT. This is reasonable considering the magnetization  
12 effects coming from neighboring crystals will also be reduced with the reduced applied field  
13 of 4.8 mT. It may be also noted from figure 4.11 that the average  $\sigma$  at the external field,  
14  $B_o = 9.5$  mT, is about the same as that from  $YBa_2Cu_3O_{6.92}$ , figure 4.6 indicating that the  
15 broadening is due to expelled flux from crystals and is not very sensitive to the geometric  
16 arrangement of crystals in the mosaic.

17 Figure 4.12 shows the average local field  $\langle B \rangle = \int \rho(z)B(z)dz$  determined from fits  
18 at a single energy as a function of beam energy (bottom scale) and corresponding mean  
19 implantation depth (top scale). The open circles and filled squares are from data taken  
20 with the shielding currents flowing along the  $a$  and  $b$  axes respectively (or equivalently the  
21 magnetic field along the  $b$  and  $a$  axes respectively). As may be noticed in figure 4.12, the

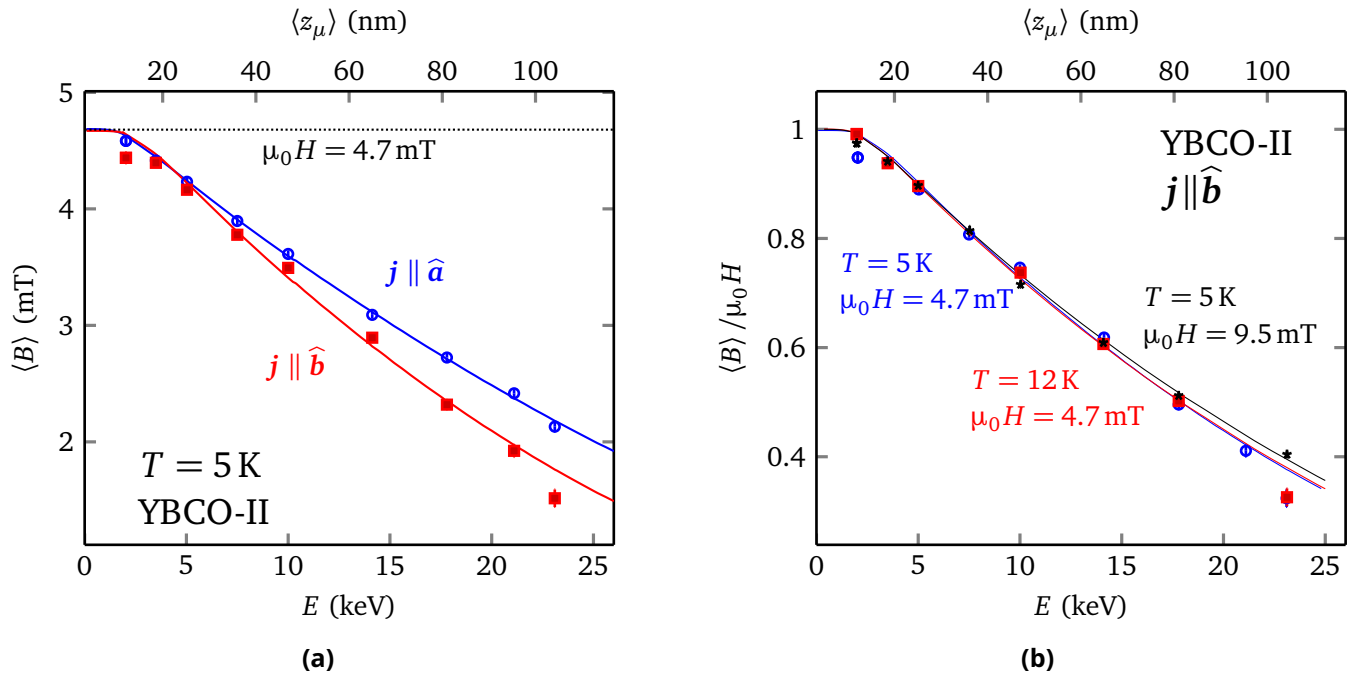




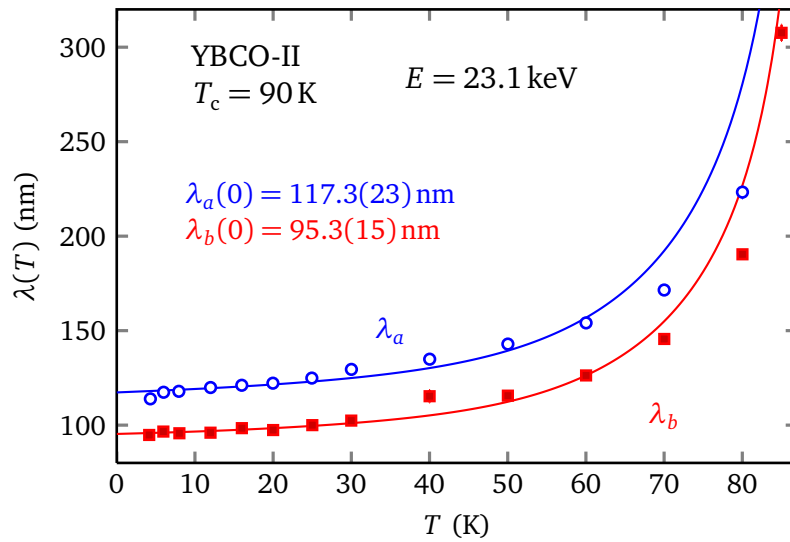
**Figure 4.11:** Broadening parameter: The three important parameters for individual energy runs in the global fit, with shared  $\lambda_a$ ,  $\lambda_b$  and  $d$  are shown. The broadening parameter ( $\sigma$ ) shows some random variation with an average  $\sim 0.42 \mu s^{-1}$  at the external field  $\mu_0 H = 4.7$  mT; the higher values at low energy may be due to vortices entering the samples close to the surface and broadening the field distribution.

1 theoretical global fit  $\langle B \rangle$  line(s) are in excellent agreement with the individually measured  
2  $\langle B \rangle$ . The common parameters are all shown in the table 4.2 and  $\lambda_{a,b}$  along with  $\chi^2/DF$   
3 are plotted in the figure 4.18. Since there is almost no correlation between  $d$  and  $\lambda_a/\lambda_b$ ,  
4 this ratio is determined more accurately than the absolute values of  $\lambda_a$  and  $\lambda_b$  as shown in  
5 the seventh column of the table 4.2. Slightly different penetration depths  $\lambda_{a,b}$  and other  
6 parameters are obtained in the **individual phase model** compared to the **shared phase**  
7 **model**. The differences in average field from the two models is negligible.

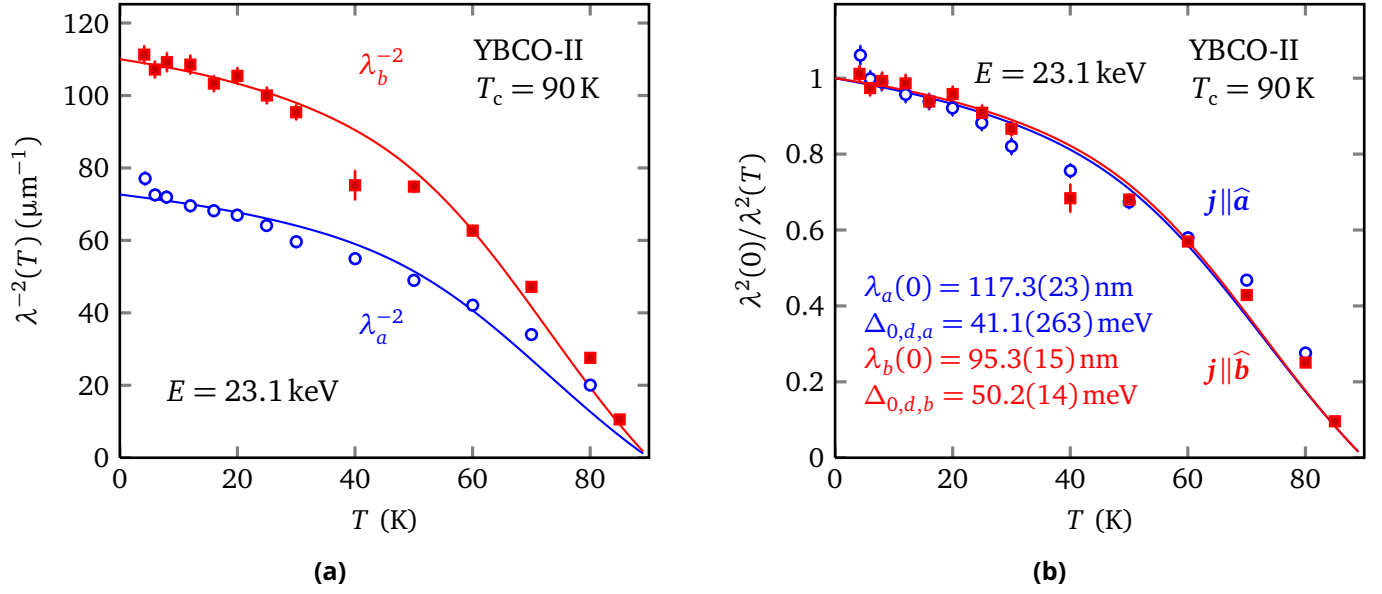
8 Figure 4.17 shows the temperature dependence of penetration depth  $\lambda_a$  at 4.7 mT  
9 and 23.1 keV. The  $\lambda_a$ 's are obtained via the method of "individual phase" analysis of  
10 measurements with a  $d$  fixed to 14.79 nm, obtained from global fit of measurements in both  
11  $a$  and  $b$  direction. The measured  $\lambda_a$ 's are fitted to a line giving  $\lambda_a(0 \text{ K}) = (112.26 \pm 1.46)$   
12 nm and slope of the line ( $\Delta\lambda/\Delta T = 0.495 \pm 0.078$  (nm/K)). Since these measurements were  
13 done at a single energy,  $\lambda_a(0 \text{ K})$  isn't a very good measure of penetration depth at zero  
14 temperature; however, note the closeness ( $\lambda_a(5 \text{ K}) \sim 115$  nm) with the absolute  $\lambda_a$ 's found  
15 from global fits table 4.2. The  $\Delta\lambda/\Delta T$  is a good measure of low temperature dependence



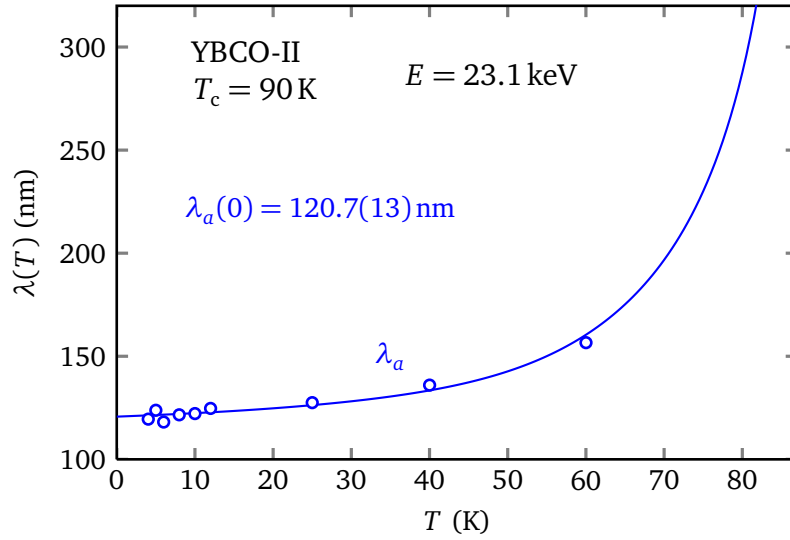
**Figure 4.12:** (a) The average magnetic field ( $\langle B \rangle$ ) versus mean stopping depth in an applied field of 4.7 mT. The curves are the average fields generated from a global fit of all the spectra at 5 K taken at all energies and for both orientations (cf. figure 4.6). (b) Relative average magnetic fields in this sample for two temperatures and two magnetic fields. It is apparent that the absolute London penetration depths measured here are, differ only slightly.



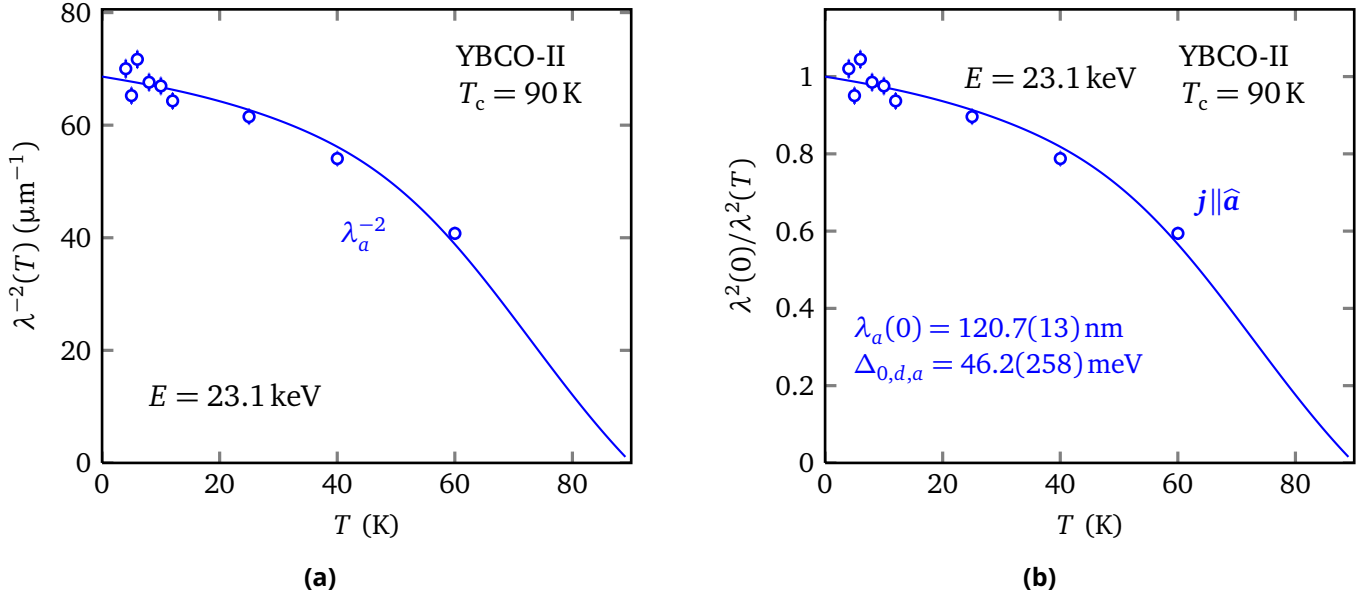
**Figure 4.13:** Temperature dependence of penetration depths.



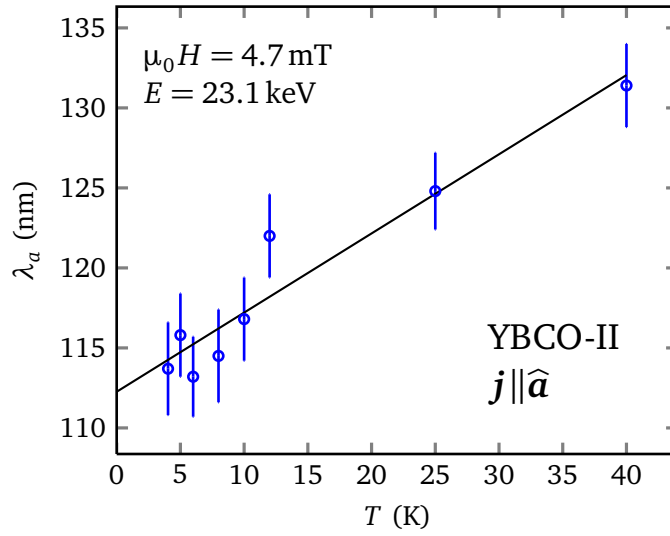
**Figure 4.14:** Temperature dependence of penetration depths and normalized superfluid density.



**Figure 4.15:** Temperature dependence of penetration depths.



**Figure 4.16:** Temperature dependence of penetration depths and normalized superfluid density.



**Figure 4.17:** Temperature dependence of London penetration depth with in low temperatures are shown. The muon beam energy for this set of runs is  $E=23.1$  keV and an external field of 4.7 mT is applied parallel to  $b$ -axis. A linear fit of  $\lambda_a(T)$  yields  $\lambda_a(0) = 112.26 \pm 1.46$  and  $\Delta\lambda/\Delta T = 0.495 \pm 0.078$ . The slope was used to extrapolate finite temperature penetration depth to 0 K.

**Table 4.2:** Summary of results for  $\text{YBa}_2\text{Cu}_3\text{O}_{6.998}$ : Measured London penetration depth in external fields of 4.7 mT & 9.5 mT are shown, for “phase individual” and “phase shared” models. There is some  $\varphi$ -dependence (of the order of systematic uncertainty) of measured penetration depths. It’s interesting to note that the “dead layer”  $d$  is within 10 to 20 nm indicating that it is a temperature and magnetic field invariant quantity. The shared phase is for 4.7 mT experiments is  $\sim 25^\circ$  and slightly higher for the 9.5 mT set of runs. The  $\chi^2/\text{DF}$  is slightly less for “individual phase” models, which indicates better fits.

$B$ (mT)	$T$ (K)	$d$ (nm)	$\varphi$ ( $^\circ$ )	$\lambda_a$ (nm)	$\lambda_b$ (nm)	$\mathcal{R} \equiv \lambda_a/\lambda_b$	$\chi^2/\text{DF}$
4.66	4	14.8(9)	27.0(16) <sup>a</sup>	115.6(18)	$\emptyset$	$\emptyset$	1.323
			25.9(13) <sup>b</sup>	119.2(24)	$\emptyset$	$\emptyset$	1.385
4.67	5	17.2(15)	27.0(16) <sup>a</sup>	107.5(39)	84.0(35)	1.28(2)	1.257
			23.0(13) <sup>b</sup>	114.2(26)	92.7(22)	1.23(1)	1.382
4.66	12	16.8(6)	27.9(15) <sup>a</sup>	117.2(25)	87.7(21)	1.34(3)	1.448
			25.4(6) <sup>b</sup>	120.3(18)	93.9(14)	1.28(1)	1.586
9.49	5	16.5(8)	31.1(20) <sup>a</sup>	$\emptyset$	91.9(22)	$\emptyset$	1.105
			30.1(16) <sup>b</sup>	$\emptyset$	94.4(17)	$\emptyset$	1.135

<sup>a</sup> Average of energy specific phases from “individual phase” analysis.

<sup>b</sup> Global phase from “shared phase” analysis.

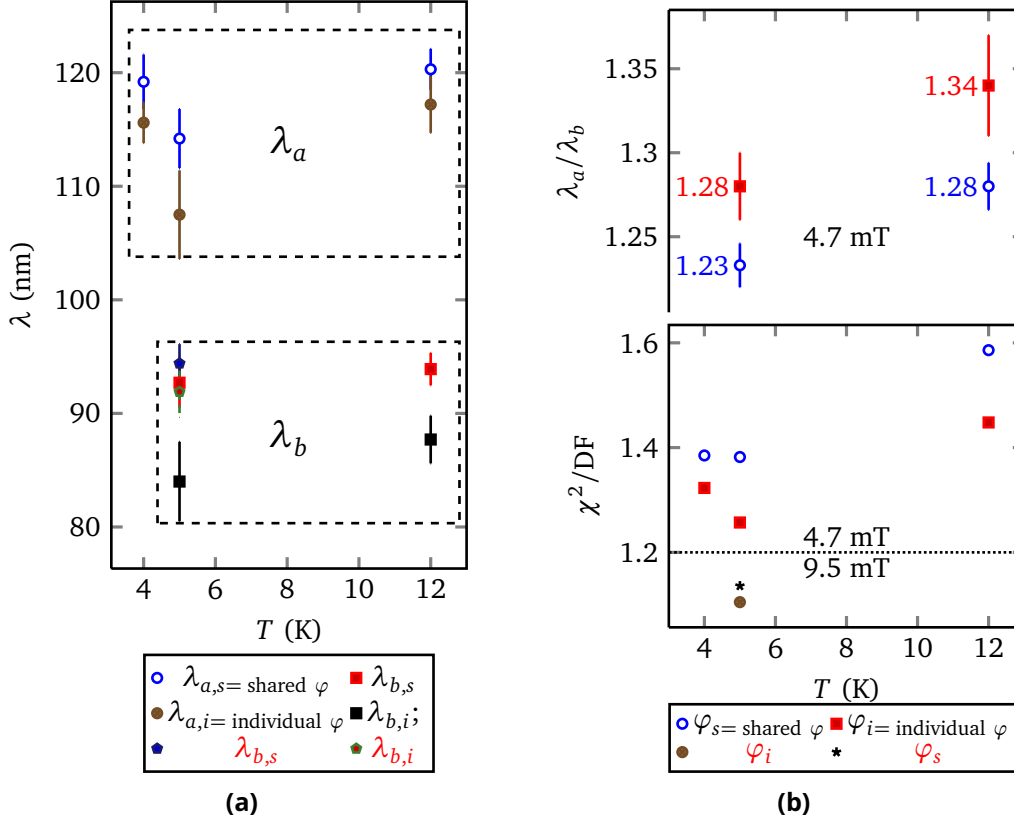
1 of  $\lambda_a$  and will be used to extrapolate zero temperature penetration depths from the finite  
2 temperature ones.

3 It is may be noted that  $\lambda_{a,b}$  increases slightly as temperature is increased (4K to 5K)  
4 which is consistent with  $\sim 0.5 \text{ nm K}^{-1}$  increase, as found from the temperature dependence  
5 in the figure 4.17. It may also be noted that the ratio between the two penetration depths,  
6  $\mathcal{R}$ , slightly ( $\sim 4\%$ ) depend on whether the **individual phase model** or the **shared phase**  
7 **model** was used. However, the difference is within statistical ( $\sim 1\%$  to  $2\%$ ) and systematic  
8 uncertainty ( $\sim 3\%$ ) in  $\lambda_{a,b}$ . The **goodness of fit**  $\chi^2/\text{DF}$  becomes slightly worse ( $\sim 10\%$ )  
9 with **shared phase model**, although, global phase is determined within a statistical error of  
10  $\sim 2\%$ .

11 We will now summarize the analysis of this section as found in the table 4.2 and fig-  
12 ure 4.18. It is interesting to note that the **dead layer**  $d$ , the layer close to the surface of  
13 the crystals where supercurrent is suppressed, varies only between 15 nm to 17 nm where  
14 in  $\text{YBa}_2\text{Cu}_3\text{O}_{6.92}$ , it was found to be 10.3 nm. Regardless of the temperature, oxygen dop-  
15 ing & applied external magnetic field, there seems to be an thin ( $\leq 10\%$ ) outside layer  
16 where the supercurrent is suppressed. Regardless of the **phase models**, the determination  
17 of phase is not very sensitive to whether we have data along both axis or just one axis.  
18 Determination of phase is important since fitted values of  $\lambda_{a,b}$  depends slightly on whether

1 phases are energy specific or a single phase should be assumed for the entire set of runs.  
 2 To discern between phase and frequency (and thereby average field), more oscillations in  
 3 signal is better, which is feasible closer to the surface. Deeper in the sample, field drops  
 4 by a significant fraction and thereby frequency becomes proportionately low which results  
 5 in phase & average frequency somewhat correlated. Yet, the correlation among  $\varphi$  and  $\omega$   
 6 doesn't produce significant uncertainty in  $d$  and  $\lambda_{a,b}$  as they are determined from a range  
 7 of implantation energies (i.e global fit). This may be observed from the plot of phases as  
 8 function of energy in figure 4.11, that the errors in phases are about the same in both low  
 9 (closer to surface) energies and high (deeper in the sample) energies. The sharp increase  
 10 in  $\sigma$  at lower implantation energies (close to surface) is possibly due to pinned vortices at  
 11 the top layers of the crystals. Broad field distribution close to the surface doesn't, however,  
 12 influence determination of  $\lambda$  as the latter depends primarily on the field distribution deeper  
 13 in the sample. The fitted phases are found to be almost temperature independent, however  
 14 weakly field dependent, as  $\varphi$  increases  $\sim 5^\circ$  as the applied field becomes doubled. With  
 15 the mosaic of  $\text{YBa}_2\text{Cu}_3\text{O}_{6.92}$ , global  $\varphi$  was found to be  $33.0 \pm 0.8$  at an external magnetic  
 16 field of 9.5 mT. While the origin of phases' dependence on applied magnetic field remains to  
 17 be understood, the determination of the London penetration depths  $\lambda_{a,b}$  and their ratio  $\mathcal{R}$   
 18 can be determined with with a few percent uncertainty. With our simple model of London  
 19 penetration depth, dead layer  $d$  and penetration depths  $\lambda_{a,b}$  are inversely correlated, as  $d$ 's  
 20 increase will decrease  $\lambda_{a,b}$  and vice versa. However, the ratio  $\mathcal{R}$  is immune to the variations  
 21 just mentioned and thereby has better accuracy. One important model (**phase**) dependent  
 22 variation is the biggest uncertainty in determination of  $\mathcal{R}$ . Although measured at finite  
 23 temperatures (4K, 5 K and 12K),  $\mathcal{R}$  is a good measure of the penetration depth anisotropy  
 24 down to 0 K, assuming the normalized superfluid density being equal at both directions at  
 25 any temperature as given in the (4.9).

26 As may be noticed the from table 4.2 and figure 4.18, that the  $\chi^2/\text{DF}$  ("goodness of  
 27 fit") depends on applied external magnetic field strongly and slightly on the (phase) model.  
 28 Higher external field induces more oscillations in the spectrum & thereby average magnetic  
 29 field(frequency) inside the sample can be determined with better accuracy. The  $\chi^2/\text{DF}$  for 9.5  
 30 mT set of measurements in  $\text{YBa}_2\text{Cu}_3\text{O}_{6.998}$  is very much comparable to that of  $\text{YBa}_2\text{Cu}_3\text{O}_{6.92}$ .  
 31  $\chi^2/\text{DF}$  also is smaller (better fit) for a fit with energy specific phases compared to a global  
 32 phase. This is expected as more free (fit) parameters mean more degrees of freedom for  
 33 the model and a better fit, however it introduces correlations among variables  $\varphi$ ,  $\sigma$  and  
 34 asymmetry. Nonetheless, statistical uncertainty in "shared phase" model is better as  $\chi^2/\text{DF}$   
 35 is inversely correlated with the number of parameters. Model dependent uncertainties can  
 36 be avoided if better measurements of phase may be done for a specific set of measurements.  
 37 In the following section, we have a better determination of phase in a novel way, which  
 38 avoids the uncertainty from a lack of knowledge of unique phase. However, it should still

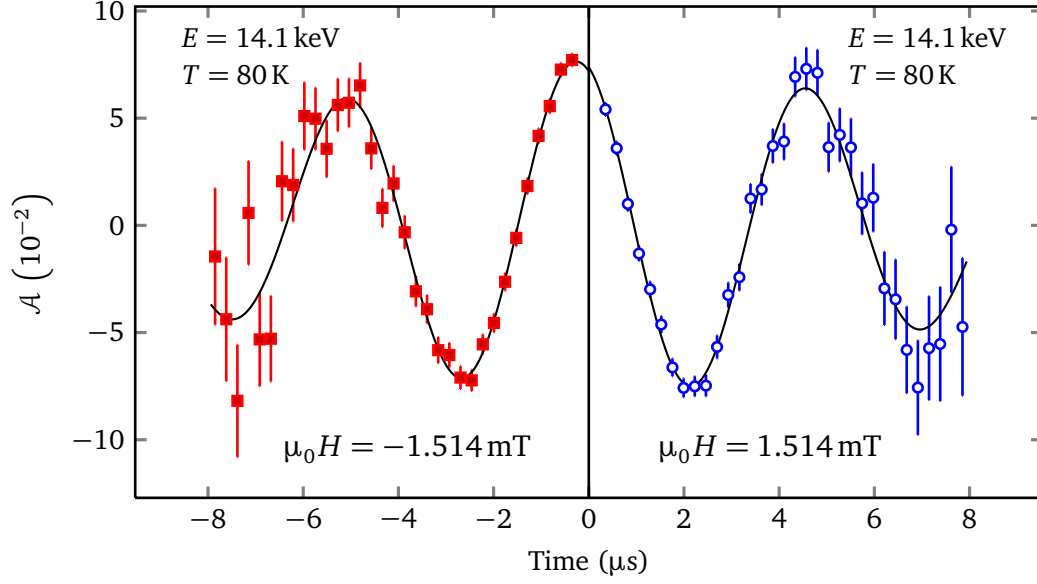


**Figure 4.18:** Summary of results for YBa<sub>2</sub>Cu<sub>3</sub>O<sub>6.998</sub>: Measured London penetration depth in external fields of 4.7 mT & 9.5 mT are shown, for **individual(shared) phase** models. There is some  $\varphi$ -dependence (of the order of systematic uncertainty) of measured penetration depths. The  $\lambda_a/\lambda_b$  and  $\chi^2/\text{DF}$  is slightly less for **individual phase model**, which indicates better fits. More discussion are in the text.

- 1 be noted that model dependence of  $\lambda_{a,b}$  only introduces an additional error of the order of
- 2 statistical uncertainty from a single model.

#### 4.1.3 YBa<sub>2</sub>Cu<sub>3</sub>O<sub>6.52</sub>

- 4 As has been discussed in the sections 4.1.1 and 4.1.2, determination of London penetration
- 5 depths  $\lambda_{a,b}$  although may be done with a few percent statistical uncertainty, the **shared**
- 6 **phase vs individual phase** models introduces additional systematic uncertainties in obtained
- 7  $\lambda_{a,b}$ . To have a simpler model having one effective phase (depending on when muon enters
- 8 into sample and partially on other geometric factors), a novel method was used to determine
- 9  $\varphi$ . Two normal state runs were taken at a single field (to be determined from fit) but one of
- 10 them having it's direction reversed. The resulting spectra is plotted in the figure 4.19. These
- 11 normal states runs were taken, as a standard procedure, *after* the superconducting state
- 12 runs in zero-field-cooled magnetic field, to avoid any flux trapping and to have an accurate



**Figure 4.19:** YBCO 6.52 in an applied field of 1.45 mT & -1.45 mT at 80K. Field reversing is like taking a measurement in negative time.  $\varphi = 20.1 \pm 1.0$  and  $\sigma = 0.14(1)$

- 1 determination of normal state field. Having a negative magnetic field  $-B$  is identical to
- 2 having a precession in negative time, as

$$\omega t \equiv \gamma_{\mu}(-B)t = (\gamma_{\mu}B)(-t).$$

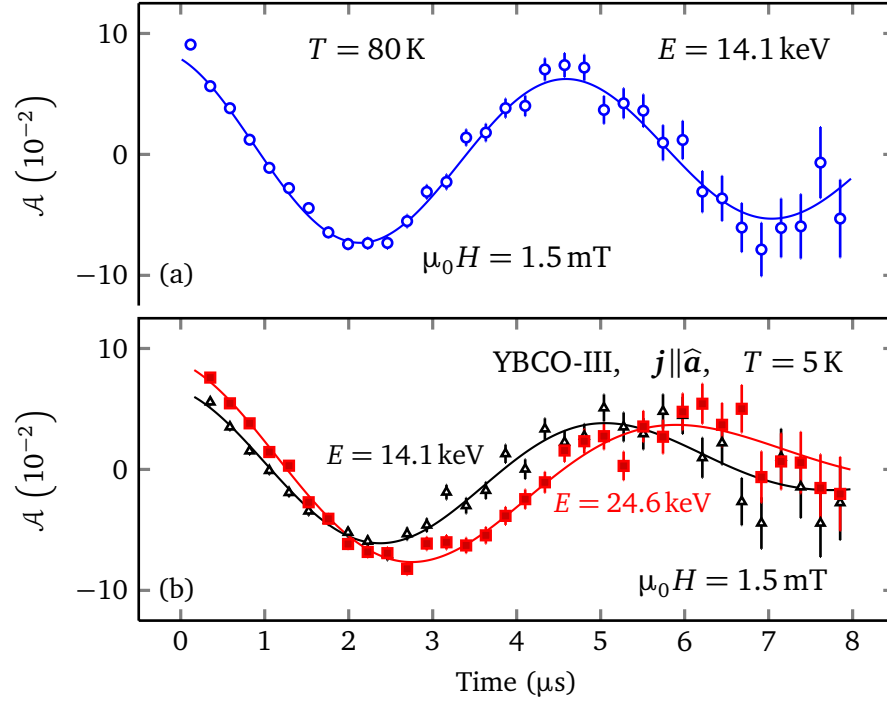
- 3 These precessions with  $B$  and  $-B$  fields essentially double the length of muon precession
- 4 time and fitting to the entire length with asymmetry spectrum function

$$\mathcal{A}(t) \equiv A_0 \exp \left[ -\sigma^2 t^2 / 2 \right] \cos(\gamma_{\mu} B_0 t + \varphi_0)$$

- 5 one can determine  $\mu_0 H$ ,  $\varphi$  and  $\sigma$  (for normal state). The fitted values from figure 4.19
- 6 were found to be  $\mu_0 H = 1.514(6)$  mT,  $\varphi = 20.1(10)^\circ$  and  $\sigma = 0.14(1) \mu s^{-1}$ . This unique
- 7 determination of phase is specially crucial at low field since, muon oscillation frequencies
- 8 ( $\omega = \gamma_{\mu} B$ ) are also lower and fewer full oscillations are available (thereby a large correlation
- 9 between  $\varphi$  and  $\omega$ ) in the spectrums as may be viewed in figure 4.20. Note the clear
- 10 reduction in precession frequency (thereby, internal field) in figure 4.20 from normal state to
- 11 superconducting states. This should be stressed that treating  $\varphi$  to be a geometric parameter
- 12 and to be equal to the normal state phase is a deviation from the analyses done so far.
- 13 However, with the set of runs with 1.5 mT external field, only a fixed phase (20.1, found
- 14 from normal states) analysis is shown in table 4.3 as a global fit with **shared phase model**
- 15 results in unrealistic average magnetic fields close to the surface. Fixing  $\varphi$  to the normal

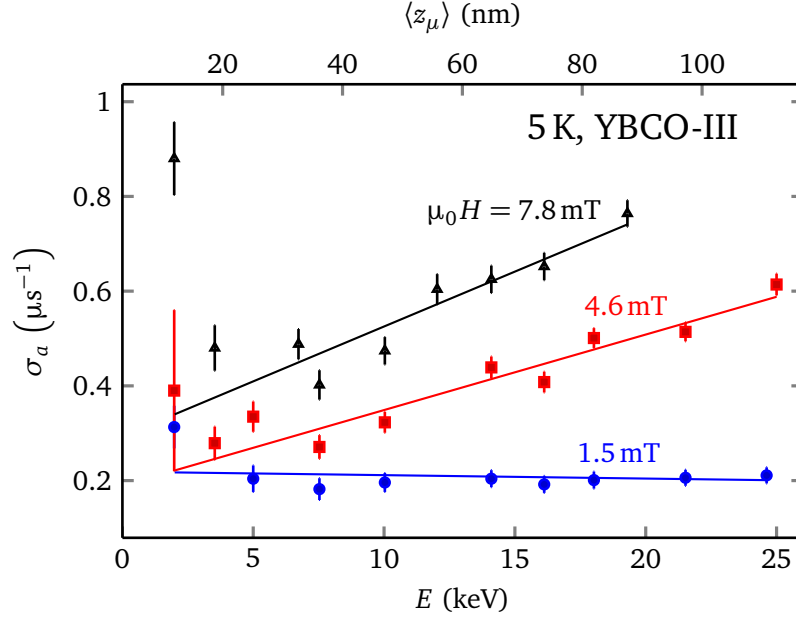


1 state values yields a better estimate of average frequencies, although usually a higher  $\chi^2/\text{DF}$   
2 However, it may be noticed from table 4.3 that 1.5 mT set of runs yields the lowest  $\chi^2/\text{DF}$  as  
3 well. The other two sets of measurements with higher external fields of 4.6 mT and 7.8 mT  
4 are analyzed with both a fixed phase (20.1) and with a **shared phase model**.



**Figure 4.20:** Top: The muon spin precession signal in the normal state of  $\text{YBa}_2\text{Cu}_3\text{O}_{6.52}$  at 80 K in an external field of 1.5 mT applied parallel to the  $b$ -direction. The mean implantation energy is  $E = 14.1\text{ keV}$  which corresponds to a mean implantation depth of 65 nm. Bottom: The same conditions as above except in the superconducting state at  $T = 5\text{ K}$  with energy 14.1 keV & 24.6 keV.

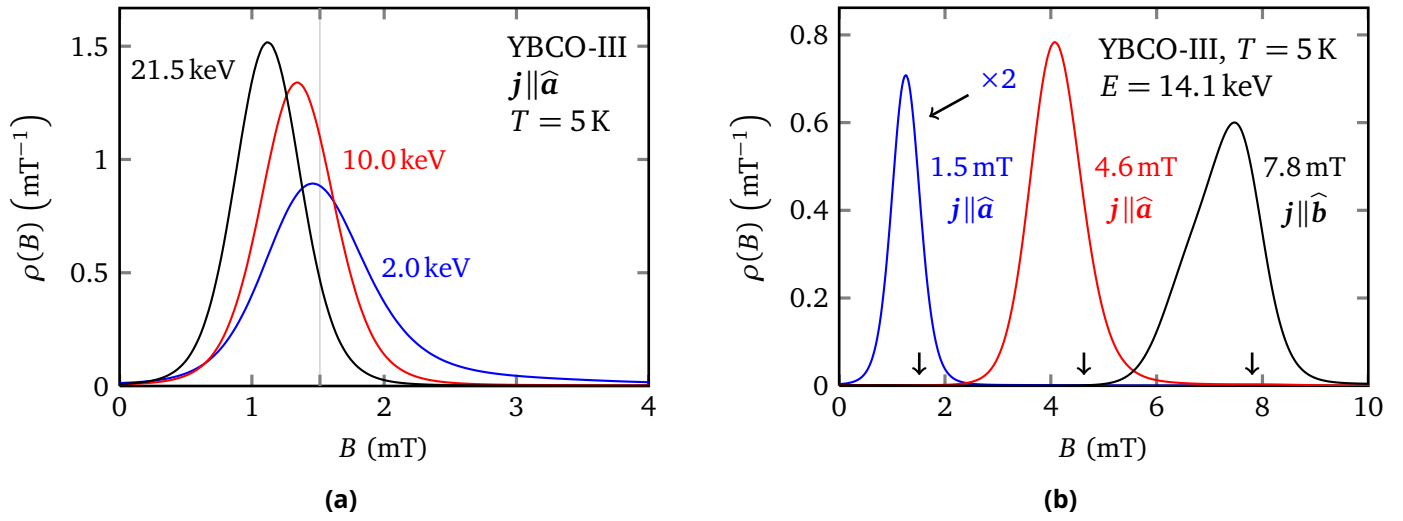
5 The fitted asymmetry and depolarization rate  $\sigma$  corresponding to the individual energies'  
6 in the global fit setup at  $\mu_0 H = 1.5\text{ mT}$  are shown in the figure 4.21. As may be noticed,  
7 the asymmetry rises with muon incident energy just like the previous experiments have  
8 shown, however, the very linear fashion of rise is interesting. It may also be noted that the  
9 range for asymmetry variation is  $\sim 4\% - 10\%$ , meaning that there is no significant loss of  
10 signal strength compared to the experiments done on  $\text{YBa}_2\text{Cu}_3\text{O}_{6.92}$  and  $\text{YBa}_2\text{Cu}_3\text{O}_{6.998}$ .  
11 The damping rate  $\sigma$  is about the same as the value obtained in previous sections at higher  
12 magnetic fields. This is reasonable damping from host Cu nuclear dipole moments. One  
13 other feature is that  $\sigma$  is almost invariant as a function of implantation energy, as is ideally  
14 expected, but close to the surface, it rises significantly. This suggests that the actual field  
15 distribution is broader than than expected from a purely stopping distribution  $\rho(z)$  related  
16 one. One possible source of broadening is field expelled from neighboring crystals. However,



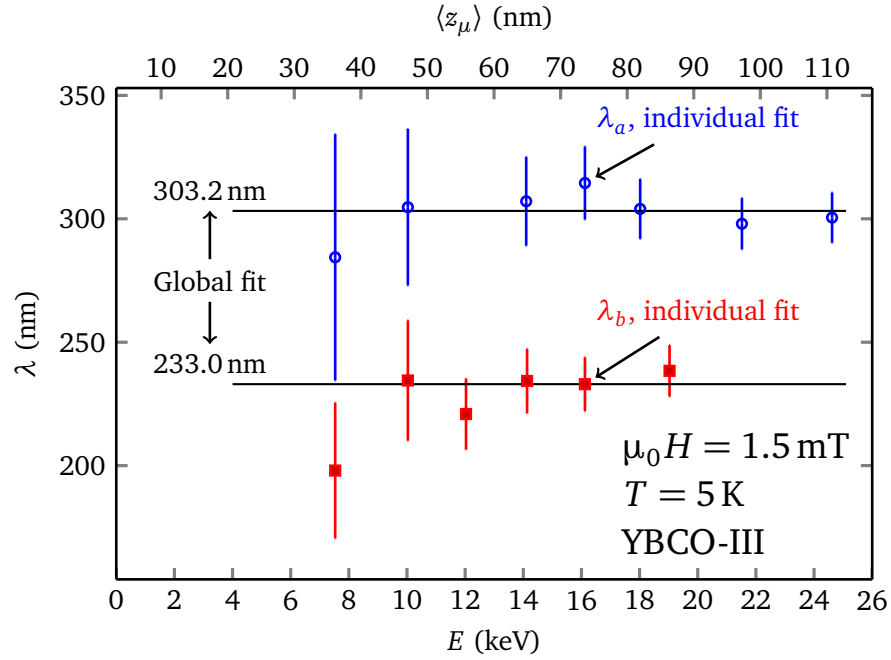
**Figure 4.21:** YBCO 6.52 broadening parameter are shown for an external applied field of 1.45 mT, 4.6 mT and 7.8 mT such that the shielding currents are flowing in the  $a$  direction ( $\mathbf{j} \parallel \hat{\mathbf{a}}$ ). The common parameters are  $\lambda_a$ ,  $\lambda_b$  and  $d$ . The upturn in  $\sigma$  at lower energies are possibly due to vortex entrance close to the surface. The solid lines are guides to the eye.

1 that would increase (not observed) the average field close to the surface. More feasible is the  
2 possible entrance of magnetic vortices close to surface which would introduce broadening in  
3 magnetic field while maintaining an average field close to the external applied field. Higher  
4 external fields in the figure 4.21, yields  $\sigma(E) \propto E$ . It may be speculated that vortices enter  
5 into significant percentage of area close to the surface and just below  $\sim \text{nm}$  of the sample  
6 surface, very few vortices remain, however the vortex core gets widened in as it gets deeper  
7 into the sample, thereby increasing the width of the field distribution.

8 Figure 4.23 shows the average  $\lambda$  and individually fitted (with  $d$  fixed)  $\lambda$  for 1.5 mT and  
9 4.6 mT external fields. It may be noted that the global fits well represent the individual  
10 energy specific fits except at low energies, where  $\langle B \rangle$  isn't very sensitive (thereby large error  
11 bars) to the penetration depths. Although  $\lambda$ 's show large error close to the surface, the  
12 error produced in  $\langle B \rangle$  is considerably smaller as only dead layer  $d$  dominates the average  
13 magnetic field computation in this region. Fitting the set of superconducting state runs  
14 at 1.5 mT with a London model, as described earlier, global  $\lambda_{a,b}$  and dead layer  $d$  are  
15 determined. The corresponding global and individual average magnetic fields are shown in  
16 the figure 4.24a. The closeness of the individual average field points to the global fit lines  
17 represent how close the data at single energies agree with global fit with shared  $\lambda_{a,b}$  and  
18  $d$ . A comparison of relative average magnetic field for 1.5 mT, 4.6 mT and 7.8 mT is shown



**Figure 4.22:** Magnetic field distribution as seen by muons at various implantation energies and at  $T = 5\text{ K}$  in an external applied magnetic field ( $\mu_0 H$ ) of 1.5 mT and 4.6 mT, applied parallel to the  $b$  axis.

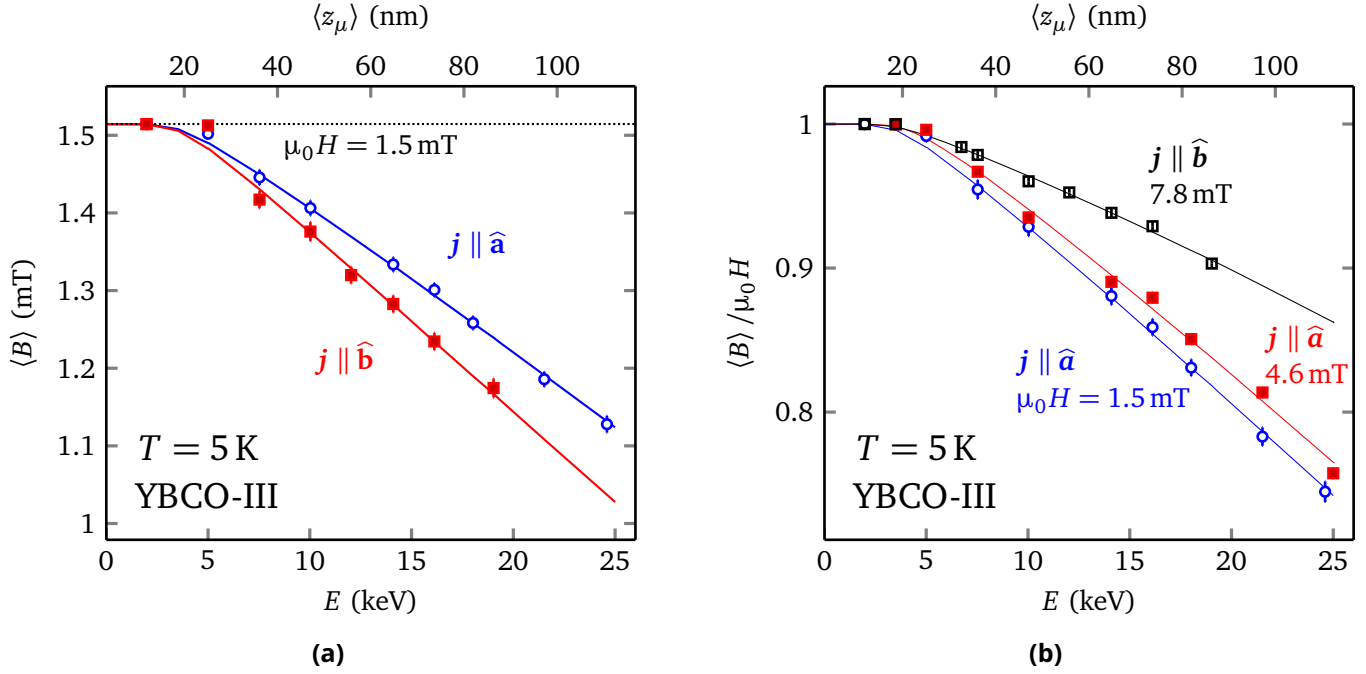


**Figure 4.23:** Global fit  $\lambda$  and individually fitted  $\lambda$ 's are shown for the external field of 1.5 mT. As may be noticed, individual penetration depths scatter around the global fit values except at lowest energies, since at these energies  $\lambda$  is less sensitive measure to the average magnetic field  $\langle B \rangle \approx B_0$ .

1 in the figure 4.24b. It is clearly evident that the lowest  $\lambda_{a,b}$  for this sample results from  
2 the 1.5 mT set of runs. As may be noticed the average field ( $\langle B \rangle$ ) in the figure 4.24 doesn't  
3 drop until  $\sim 5$  keV, corresponding to an implantation energy  $\sim 25$  nm which is also reflected  
4 in the dead layer  $d$  in the table 4.3. One interesting phenomenon consistently seen in all  
5 analyses is that the  $d$  is between 10 nm to 30 nm, irrespective of the external field magnitude  
6  $\mu_0 H$  or orientation, suggesting that some intrinsic mechanism being responsible for the  
7 reduced supercurrent close to the surface. To extract  $\lambda_{a,b}(0)K$ , temperature dependent  
8 measurements of penetration depths have been obtained as shown in the figure 4.25. It may  
9 be noticed that low temperature dependence of  $\lambda$  is linear only at an external field of 1.5 mT.  
10 The higher external field of 4.6 mT yields very nonlinear temperature dependence possibly  
11 indicating entrance of vortices in the mosaic, as already argued noting that  $\sigma(E)$  increases  
12 almost linearly, with higher energy, as shown in the figure 4.21. The unusual temperature  
13 dependence of  $\lambda_a(T)$  in the 4.6 mT set of measurements prompted the significantly lower  
14 1.5 mT set of measurements. The resulting slope from the 1.5 mT temperature dependence  
15 has been used to extrapolate  $\lambda_b(5\text{ K})$  to  $\lambda_b(0\text{ K})$ . Penetration depth in the other orientation  
16  $\lambda_a(0\text{ K})$  has been obtained using the (4.9).

17 Figure 4.26a shows effective penetration depth  $\lambda_b$ 's dependence on external field. The  
18 phase was left fixed at the normal state value at 1.5 mT, for all three analyses. As argued  
19 before, the two higher field of 4.6 mT and 7.8 mT have possible vortex penetration and  
20 obtained  $\lambda_a$  are just effective penetration depths which can be used to determine average  
21 magnetic field at various depths using the stopping distribution  $\rho(z)$ . Note that the effective  
22  $\lambda_a$ s from global fit are more reliable than the single run  $\lambda_b$ s (with a fixed  $d = 23.6$  nm)  
23 shown in figure 4.26a. Although the individually fitted  $\lambda_b$  values are less reliable, the almost  
24 linear field dependence strongly suggests magnetic vortex entrance at higher fields. Also may  
25 be noted from the figure 4.26b, normal state phases to be almost independent ( $\sim 25^\circ$ ) of  
26 applied external field while the superconducting state phase seems to be slightly dependent  
27 on applied field for  $\mu_0 H > 6$  mT. The broadening parameter  $\sigma$  in the superconducting state  
28 rises almost linearly with the applied external magnetic field due to the expelled flux being  
29 proportional to the applied field. Although the single fit  $\lambda_b$ s in the figure 4.26a are not very  
30 reliable, it does bring an important point that caution must be taken to measure the London  
31 penetration depth in the Meissner state by excluding possibility of vortices getting in the  
32 sample.

33 As may be seen from the table 4.3, a difference of  $\sim 20$  nm exist between the fixed  
34 phase and the shared phase models. A single normal state run at 4.6 mT yields a phase of  
35  $(23.5 \pm 1.8)^\circ$  &  $(0.039 \pm 0.013)^\circ$ . This is interesting because shared phase  $\varphi$  seems to suggest  
36 a different phase other than the one determined from 1.5 mT set of runs with a reversal of  
37 field. From the measured  $\lambda_{a,b}$  in table 4.3, it's very likely only minor vortex penetration  
38 happened in 4.6 mT set of runs whereas a significant amount of vortices entered in a 7.8 mT

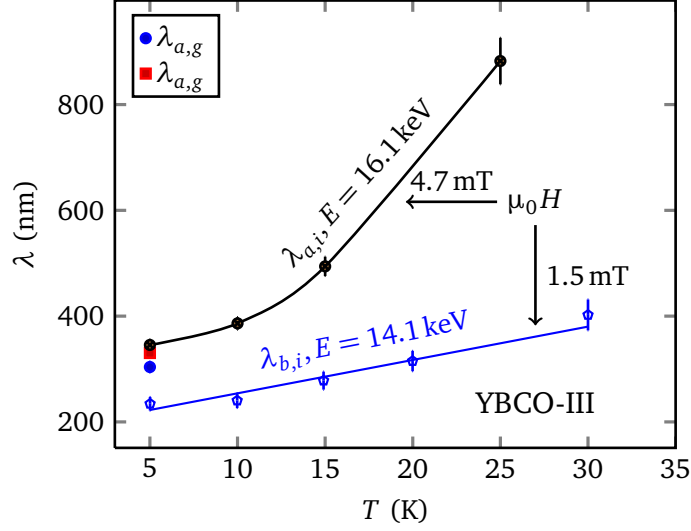


**Figure 4.24:** YBCO 6.52 average magnetic field  $\vec{B}$  inside the sample. The average magnetic field versus energy (mean stopping depth) in an applied field of 1.45 mT such that the shielding currents are flowing in the  $a$  direction ( $\vec{j} \parallel a$ , open circles) and  $b$  direction ( $\vec{j} \parallel b$ , filled squares). Relative average local magnetic field with respect to the applied field as a function of muon implantation energy for the two fields of 4.7 mT, and 9.5 mT applied parallel to the  $b$  axis of the YBCO-III. The depicted fields have been calculated in the same way as in Fig. and normalized to the applied fields.

1 set of runs, making the global  $\lambda$  very long. The “effective magnetic penetration” depth is  
2 also very sensitive to the muon phase  $\varphi$ , as may be seen in table 4.3. The phase in a “shared,  
3 global phase” analysis in this case  $\sim 26.5^\circ$  which is also close to the phase found from a  
4 single superconducting state analysis in figure 4.26. It is tempting to suggest that the “shared  
5 phase model” estimates the true phase. However, as may be seen from figure 4.26, the  
6 normal state phases vary little compared to the superconducting state phases and the latter  
7 has distinct field dependence, although with bigger error limits. The phases’ dependence  
8 of magnetic field at higher applied fields, although an interesting one, doesn’t affect the  
9 determination of absolute penetration depth at 0 K, extrapolated from measurements at 1.5  
10 mT.

## 4.2 Pnictide

12 In this section, measurements of the  $\lambda$  in the Meissner state on a single crystal of  $\text{Ba}(\text{Co}_x\text{Fe}_{1-x})_2\text{As}_2$   
13 are reported, using a combination of LE- $\mu$ SR and microwave perturbation. The combination



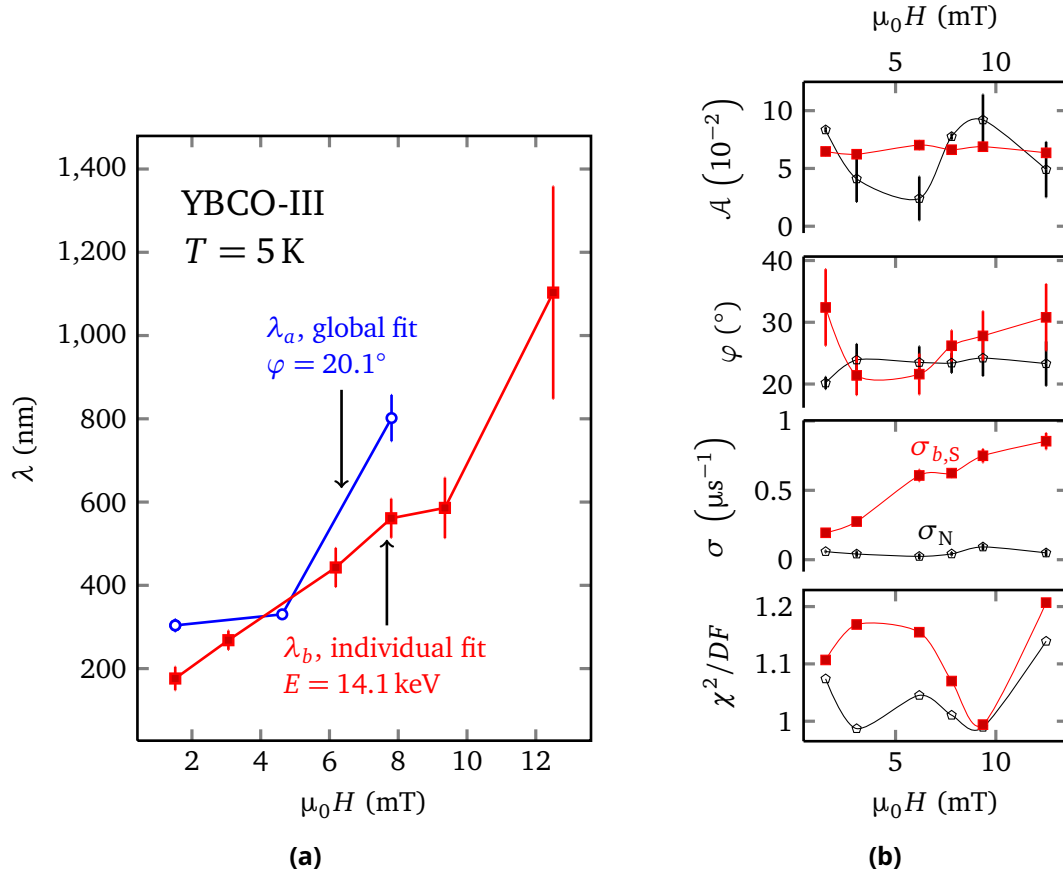
**Figure 4.25:** As may be noticed that the average field increases, with a corresponding large increase in  $\lambda$ , with temperature. Possible entrance of vortices is likely the reason of increase of “effective  $\lambda$ ”. This prompted us to take low field (1.6 mT) set of runs. The fit parameters at 1.5 mT external field are, intercept= $191 \pm 13.38$  nm. slope= $6.305 \pm 0.96$  nm/K. The slope is an order of magnitude bigger than that found in other oxygen dopings of YBCO.

**Table 4.3:** Summary of results in  $\text{YBa}_2\text{Cu}_3\text{O}_{6.52}$ : Measured London penetration depths at 5 K are shown for two models: “phase individual” and “phase shared”, for individual energies. The errors are reported here are just statistical errors. An additional  $\sim 2\%$  ( $\sim 2$  nm) error is due to uncertainty in stopping distribution. The deadlayer  $d \sim 25$  nm, is found from “individual phase” model global fit & is assumed to be the same for “shared phase” model. In the lowest of the fields, 1.5 mT, phase is kept fixed to the measured phase found in the normal state, via field reversing. The lowest penetration depths were obtained for both  $a$  and  $b$  axis in 1.5 mT external field. The penetration depths at higher fields of 4.6 mT & 7.8 mT include contributions from vortices entering the sample, as may be observed from increasing  $\sigma$  at higher energies.

$B$ (mT)	$T$ (K)	$d$ (nm)	$\varphi$ ( $^\circ$ )	$\lambda_a$ (nm)	$\lambda_b$ (nm)	$\mathcal{R} \equiv \lambda_a/\lambda_b$	$\chi^2/\text{DF}$
1.51	5	23.6(18)	$\varphi_n^a$	303.2(87)	233.0(92)	1.30(2)	1.027
4.63	5	26.6(12)	$\varphi_n$	330.3(90)	$\emptyset$	$\emptyset$	1.147
			24.6(10) <sup>b</sup>	310.0(75)	$\emptyset$	$\emptyset$	1.078
7.80	5	23.7(21)	$\varphi_n$	$\emptyset$	616.3(428)	$\emptyset$	1.098
			26.5(13) <sup>b</sup>	$\emptyset$	536.6(339)	$\emptyset$	1.069

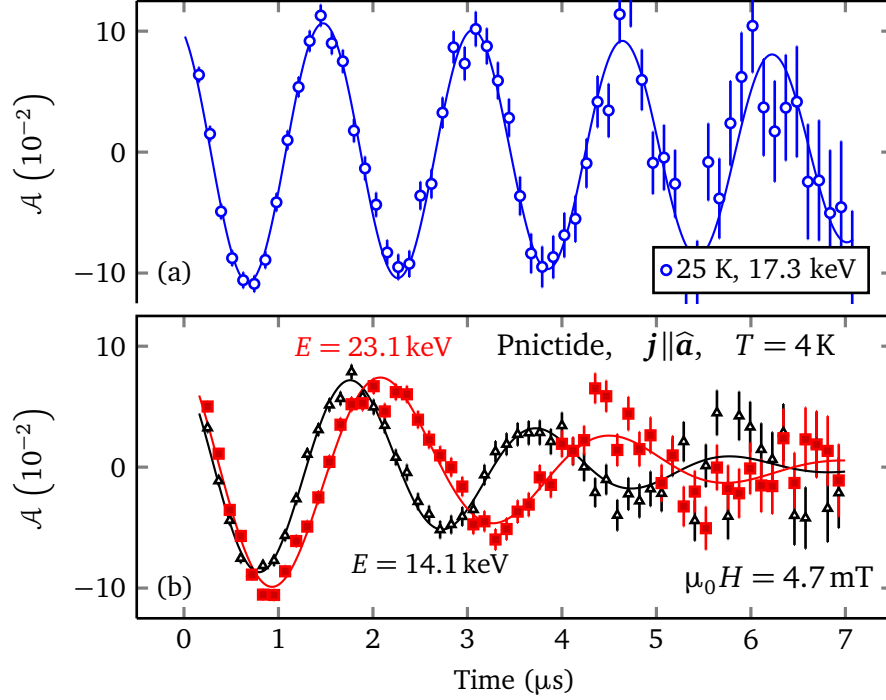
<sup>a</sup>  $\varphi_n = 20.1^\circ$  (fixed)

<sup>b</sup> Global phase from “shared phase” analysis.



**Figure 4.26:** (a) Single measurements of effective penetration depths in various external magnetic fields are shown here. The almost linear rise in  $\lambda_b$  with magnetic field stresses that lower external field must be used and also be made sure that the samples are indeed in the Meissner state. (b) The asymmetry in the normal states seem to show some unusual variance (should be constant) with respect to  $\mu_0 H$ . On the other hand, these single measurements may not accurately reflect the phase from global fits. The rise of  $\sigma$  with muon energy is due to the expelled flux ( $\propto \mu_0 H$ ) from the neighboring crystals.

1 of the two techniques allows a precise determination of the  $T$  dependence of the magnetic  
2 penetration depth, which depends on the symmetry of the superconducting gap.  
3 In the current study we measure the field profile directly on a freshly cleaved surface of  
4  $\text{Ba}(\text{Co}_{0.074}\text{Fe}_{0.926})_2\text{As}_2$  using the modified London model as described in the section 4.1.1.  
5 The single crystal of optimally-doped  $\text{Ba}(\text{Co}_{0.074}\text{Fe}_{0.926})_2\text{As}_2$  was grown using a self-flux  
6 method [163]. The crystal was approximately square-shaped with dimensions  $5.6 \text{ mm} \times$   
7  $4.5 \text{ mm} \times 0.3 \text{ mm}$  and exhibited a sharp transition at  $T_c = 21.7 \text{ K}$  as measured by SQUID  
8 magnetometry. The crystal was cleaved to a thickness of about  $0.3 \text{ mm}$  under flowing  $\text{N}_2$   
9 gas just prior to loading it into the ultra-high-vacuum sample chamber.  
10 Figure 4.27 shows typical  $\mu\text{SR}$  precession signals obtained with a small transverse

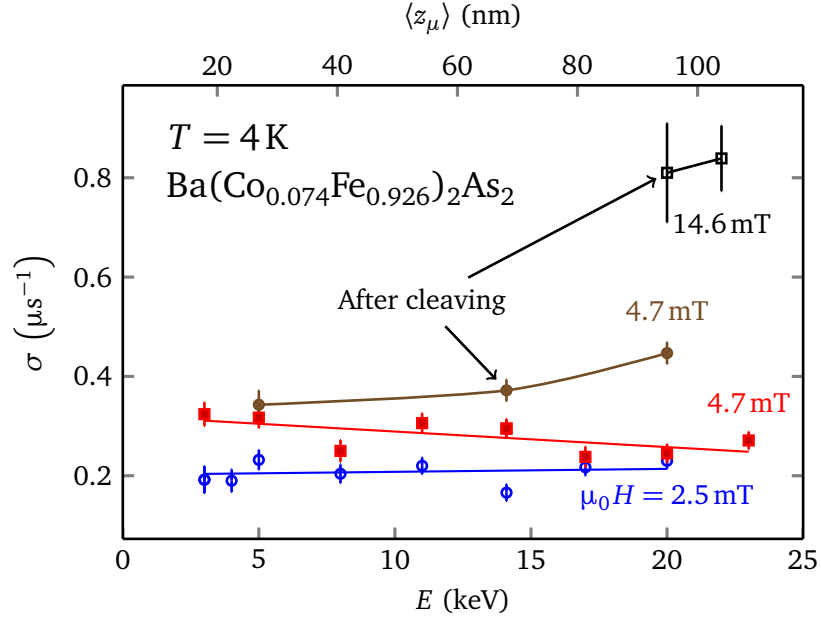


**Figure 4.27:** Muon precession signals in  $\text{Ba}(\text{Co}_x\text{Fe}_{1-x})_2\text{As}_2$  in an applied field of  $\mu_0 H = 4.7$  mT. (a) In the normal state at  $T = 25$  K. (b) In the superconducting state at  $T = 4$  K with  $E = 14$  keV and  $E = 23$  keV.

1 magnetic field applied parallel to the  $ab$  face of the crystal. The top panel shows the  
2 precession signal in the normal state where the mean internal field is equal to the applied  
3 field obtained via fitting the spectrum to a Gaussian. The normal state at  $\mu_0 H = 4.7$  mT  
4 yields a  $\sigma$  of  $0.12 \mu\text{s}^{-1}$ . Signals taken below  $T_c$  are shown in the bottom panel. All the  
5 measurements in the Meissner state were made in **zero field cooled** method as detailed  
6 earlier. The reduction of precession frequency, for a higher muon implantation energy, is  
7 clearly visible comparing the two spectras ( $\blacktriangle$  and  $\blacksquare$ ) in the figure 4.27. The asymmetry,  
8 phase and broadening parameter are shown in the figure 4.28. The asymmetry shows the  
9 usual energy dependent behavior. The  $\langle\sigma\rangle$  in the Meissner state of  $\mu_0 H = 4.7$  mT is slightly  
10 higher ( $0.28(1) \mu\text{s}^{-1}$ ) than that ( $0.21(1) \mu\text{s}^{-1}$ ) corresponding to the  $\mu_0 H = 2.5$  mT. The  
11 increase of  $\sigma$  with external field were attributed to **bulk magnetization effects**, where  
12 flux expelled from neighboring crystals broaden the effective field, in all three dopings of  
13 mosaic  $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ . The bulk magnetization effect is linearly dependent to the external  
14 field. However, the present Pnictide sample is a single crystal and the average additional  
15 (not accounted for by the stopping distribution  $\rho(z)$ ) broadening  $\langle\sigma\rangle$  increases by  $\sim 25\%$   
16 with the approximate doubling of  $\mu_0 H$ . Also may be noted from the figure 4.28, that  $\sigma$   
17 does not increase sharply close to the surface as has been seen previously in the two YBCO



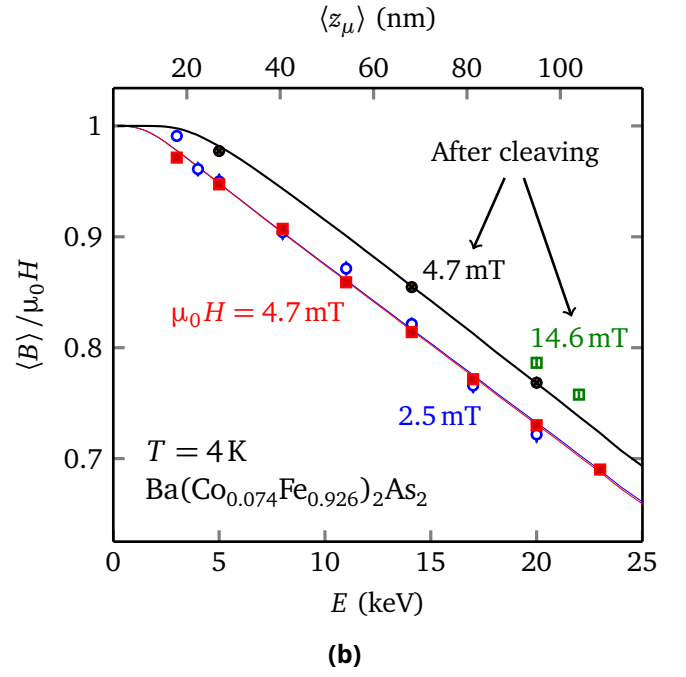
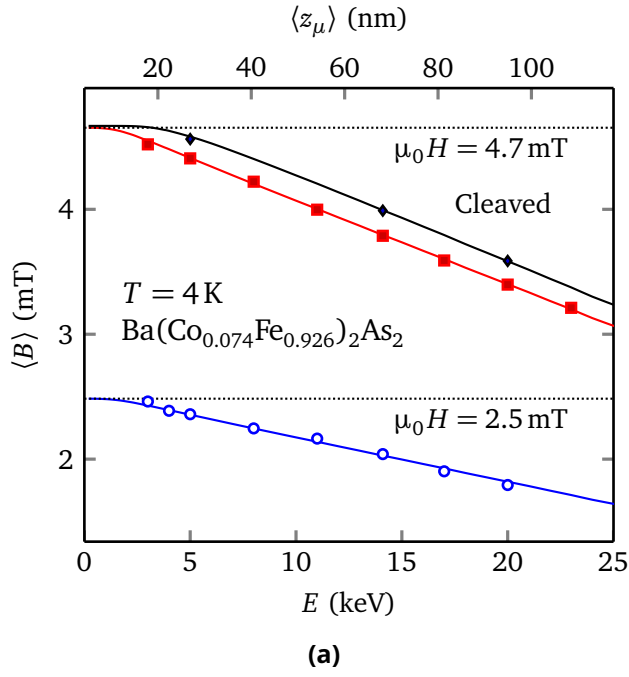
1 mosaics shown in the figures 4.11 and 4.21. This suggests that there is no vortices close to  
2 the surface and yet there is a suppression of superfluid density  $\sim 15$  nm, close to the sample  
3 surface, as may be seen from the curvatures of the global fits in the figure 4.29.



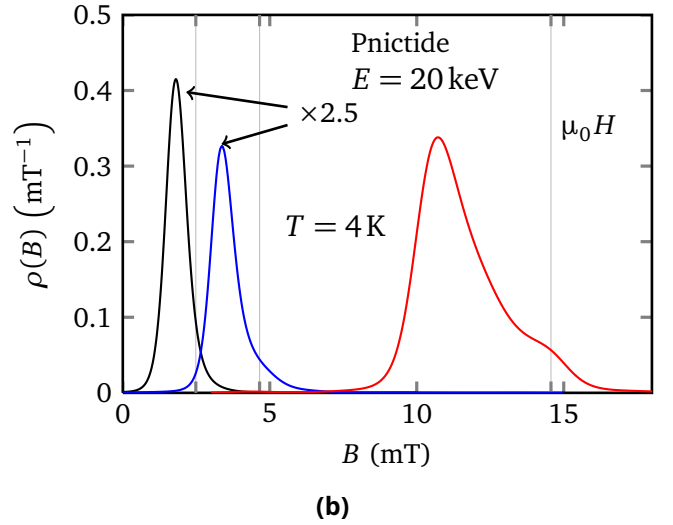
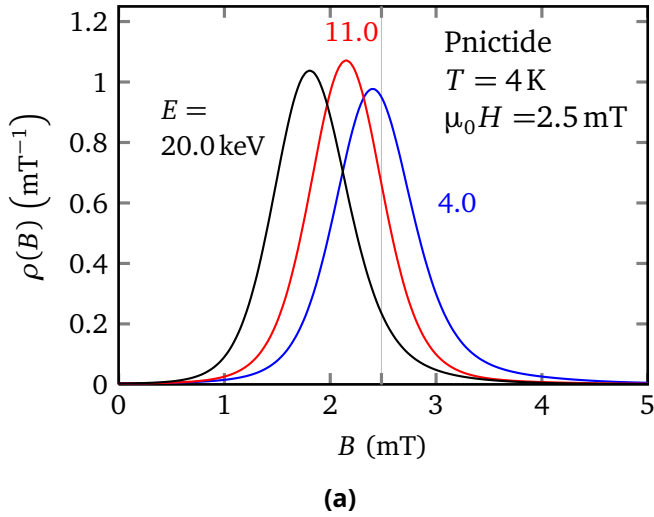
**Figure 4.28:** Broadening parameter ( $\sigma$ ) in  $\text{Ba}(\text{Co}_x\text{Fe}_{1-x})_2\text{As}_2$  in external applied fields of  $\mu_0 H = 2.5 \text{ mT}$  and  $4.7 \text{ mT}$ . The solid lines are guides to the eye.

4 Figure 4.29 shows the average local field  $\langle B \rangle$  as a function of the beam energy at  $T = 4 \text{ K}$ .  
5 The data are consistent with an exponential decrease as a function of increasing depth or  
6 implantation energy, as expected from a London model. Fit are  $\lambda(T = 4 \text{ K}) = 251.7(19) \text{ nm}$   
7 and  $d = 14.5(9) \text{ nm}$  where the uncertainties are purely statistical. There is also a 3%  
8 systematic uncertainty in  $\lambda$  due to uncertainties in the muon stopping distribution. Similar  
9 results were obtained at a magnetic field of  $2.5 \text{ mT}$ , where  $\lambda = 252.2(45) \text{ nm}$ , indicating  
10 there is little field dependence in  $\lambda$ , which is also reflected in the  $\langle B \rangle$  in the figure 4.29b.  
11 Results of analysis done on two phase (**independent and shared**) models are shown in  
12 the table 4.4. Notice that the average of phases from **individual phase model** is within  
13  $\sim 1^\circ$ , while **shared phase model** yields a  $\sim 3^\circ$ . Yet with all the model dependence of  $A_0$ ,  $\sigma$   
14 and  $\varphi$ , there is only  $\sim 2 \text{ nm}$  difference in the measured  $\lambda$  in the external fields of  $2.5 \text{ mT}$   
15 and  $4.7 \text{ mT}$ . This suggests the robustness of our method in obtaining the absolute London  
16 penetration depth in Pnictide.

17 The temperature dependence of  $\lambda$  measured with LE- $\mu\text{SR}$  ( $\blacksquare$ ) and with  $\mu\text{W}$  ( $\text{---}$ ), MFM  
18 ( $\circ$ ) and TDR ( $\text{---}$ ) are shown in the figure 4.31. The data points were obtained with a  
19 single energy of  $21.3 \text{ keV}$  with  $d$  fixed to the value determined from the global fit at  $T = 4 \text{ K}$   
20 (see figure 4.29). Microwave resonance measurements of  $\Delta\lambda$  were made on a piece of the

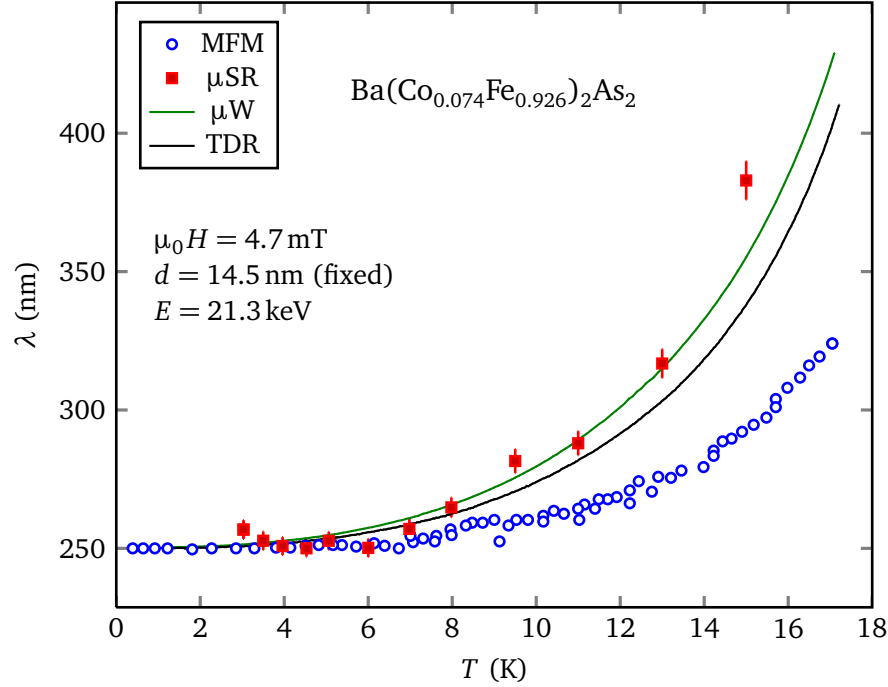


**Figure 4.29:** The average magnetic field versus the muon energy at  $T = 4$  K with a  $\mu_0 H = 2.5$  mT applied field. The dotted line indicate the applied field and the solid line indicates a fit to (2.15).



**Figure 4.30:** Magnetic field distribution as seen by muons at various implantation energies and at  $T = 5$  K in an external applied magnetic field ( $\mu_0 H$ ) of 2.5 mT and 4.7 mT, applied parallel to the  $ab$  plane.

same crystal, which was cleaved on both sides. The piece of  $\text{Ba}(\text{Co}_{0.074}\text{Fe}_{0.926})_2\text{As}_2$  was mounted on a temperature-controlled sapphire plate, and a 942 MHz loop-gap resonator, described in detail elsewhere[92], was used to obtain  $\Delta\lambda$ . It is clear the two methods are



**Figure 4.31:** The temperature dependence of  $\lambda$  plotted versus  $t = T/T_c$ . The red squares (■) are measurements of the absolute value of  $\lambda$  from LE- $\mu\text{SR}$  whereas the green line (—) are from microwave cavity perturbation on a piece of the same crystal shifted to overlap with the LE- $\mu\text{SR}$  at low temperature. For comparison, we also show recent TDR (—) and MFM (○) results for  $\Delta\lambda$ , all shifted to agree at  $T = 0$ .

in excellent agreement below 13 K. One can use the microwave data to extrapolate the 4 K  $\mu\text{SR}$  measurement to obtain  $\lambda(0) = 250.2(2.6)\text{nm}$ . Above 13 K there is some difference between the two methods which we attribute to a small amount of flux penetration in the  $\mu\text{SR}$  experiment as one approaches  $T_c$  and the applied magnetic field of 4.67 mT exceeds  $H_{c1}$ . Note the temperature dependence of  $\lambda$  at low temperatures is similar to recent TDR results [164] on a thin sample but considerably weaker than previous studies on thicker crystals [165]. This suggests early studies may have been affected by an anomalous temperature-dependent field penetration from the  $c$ -axis edges. It is interesting to note that all of these results are stronger than found by MFM also shown in figure 4.31 [166]. One difference is that the present measurements, as well as previous TDR results, measure an average over the surface whereas MFM is a point-like probe. Such differences between methods and crystals indicate there are considerable variations in the spectrum of low energy excitations depending on doping and/or surface quality. Combination of  $\mu\text{W}$  and LE- $\mu\text{SR}$

1 reduces allows one to determine the superfluid density and its variation as a function of  
 2 temperature with a high confidence level. The temperature dependence of the superfluid  
 3 density normalized to zero temperature is shown in the figure 4.32. Indeed, a similar quality  
 4 of fit for low temperatures, which also fits the whole  $T$  range, can be obtained using a  
 5 phenomenological two-gap  $s$ -wave model (“ $\alpha$  model”)[167], where

$$\rho = 1 - x \frac{\delta n_s(\Delta_S(T), T)}{n_s(0)} - (1 - x) \frac{\delta n_s(\Delta_L(T), T)}{n_s(0)} \quad (4.10)$$

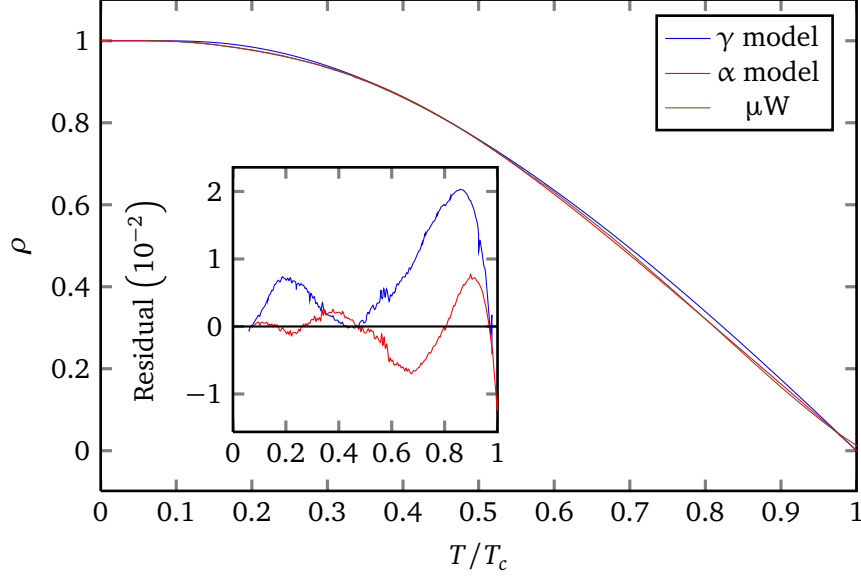
and,

$$\begin{aligned} \frac{\delta n_s(\Delta_i(T), T)}{n_s(0)} &= \frac{2}{k_B T} \int_0^\infty f(\varepsilon, \Delta_i(T), T) \\ &\times [1 - f(\varepsilon, \Delta_i(T), T)] d\varepsilon \end{aligned} \quad (4.11)$$

$$\Delta_{L,S}(T) = \Delta_{L,S}(0) \tanh \left( \frac{\pi k_B T_c}{\Delta_{L,S}(0)} \sqrt{a_{L,S} \left( \frac{T_c}{T} - 1 \right)} \right). \quad (4.12)$$

6 Here, the subscript  $i = L$  denotes the larger gap which dominates at  $T \sim T_c$  where the other  
 7 gap  $i = S$  dominates at lower temperatures.  $f(\varepsilon, \Delta_i(T), T)$  is the Fermi-Dirac distribution  
 8 at energy  $\varepsilon$  and gap  $\Delta$ . The free parameter  $a_{L,S}$  describes phenomenologically the shape  
 9 of the gap, e.g.  $a_{L,S} \equiv 1$  in the BCS limit; for the small gap we define  $a_S = 1$ [168] and  
 10 obtain  $a_L = 0.83(3)$ . We find the large gap  $2\Delta_{0,L}/k_B T_c = 3.46(0.10)$  is close to the BCS  
 11 weak-coupling limit whereas the small gap  $2\Delta_{0,S}/k_B T_c = 1.20(7)$  is much less. These  
 12 parameters are also close to those derived from vortex-state  $\mu$ SR measurements [168].  
 13 This is somewhat surprising given the high degree of vortex lattice disorder and the field  
 14 induced magnetism[169]. The data were also fit to a self-consistent two-gap model which  
 15 takes into account the interaction between bands (“ $\gamma$  model”)[170]. The quality of the  
 16 fit is similar to the phenomenological two-gap model over the full temperature range and  
 17 the fitted gap parameters are about 10% larger. The data fit well to a power law form  
 18  $\lambda^2(0)/\lambda^2(T) \approx 1 - \alpha(T/T_c)^n$  and are consistent with the superfluid density obtained directly  
 19 from LE- $\mu$ SR. A fit from base temperature to 12 K yields  $n = 2.51(2)$  and  $\alpha = 1.39(3)$  K and  
 20 is only weakly dependent of the fitting range up to 11 K.

21 In conclusion, we have investigated the magnetic field penetration in the Meissner  
 22 state of freshly-cleaved  $\text{Ba}(\text{Co}_{0.074}\text{Fe}_{0.926})_2\text{As}_2$  using both LE- $\mu$ SR and microwave cavity  
 23 perturbation. The absolute value of  $\lambda$  extrapolated to  $T = 0$  is 250(8) nm, where most of  
 24 this uncertainty originates from the muon stopping distribution. There is no evidence of  
 25 sharp rise in field broadening close to the sample surface. Also weaker field dependence  
 26 of  $\sigma$  indicates a different origin other than bulk magnetization effect. A two-gap  $s$ -wave



**Figure 4.32:** The temperature dependence of  $\rho$  plotted versus  $t = T/T_c$ . Solid black (dotted blue) line is a fit to the  $\alpha(\gamma)$  model. The dashed curve is the fit to a power law. The inset shows the residuals at low temperature.

**Table 4.4:** Pnictide Two Fields

State	4.7 mT			2.5 mT		
	$\lambda$ (nm)	$\varphi(^{\circ})$	$\chi^2/DF$	$\lambda$ (nm)	$\varphi(^{\circ})$	$\chi^2/DF$
Normal	–	25.5(18)	0.932	–	21.4(17)	0.972
Superconducting	252.8(19)	30.9(12) <sup>a</sup>	1.118	254.5(41)	27.6(7) <sup>a</sup>	1.030
	251.7(19)	28.3(6) <sup>b</sup>	1.052	252.2(45)	27.8(5) <sup>b</sup>	1.029
Superconducting, cleaved	254.4(52)	27.9(9) <sup>a</sup>	1.198			
	253.9(51)	27.5(2) <sup>b</sup>	1.178			

<sup>a</sup> Global phase from **shared phase model** analysis.

<sup>b</sup> Average of energy specific phases from **individual phase model** analysis.

- 1 model and a weak power law model describes the temperature dependence of the superfluid
- 2 density equally well. The latter model is characteristic of any non  $s$  wave gap. There is broad
- 3 agreement between methods at least at this one Co concentration, except for MFM which
- 4 probes  $\Delta\lambda$  on much smaller length scale.

### 4.3 Summary Of Results

- 6 Table 4.5 shows the summary of London penetration depths measurements for three oxygen
- 7 dopings of  $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$  and on  $\text{Ba}(\text{Co}_{0.074}\text{Fe}_{0.926})_2\text{As}_2$ . The first number is the measured
- 8  $\lambda_{a,b}$  extrapolated to  $T = 0$  K. The first error is the statistical uncertainty at the temperature
- 9  $\lambda_{a,b}$  are measured and the second error is the systematic uncertainty. For comparison,  $\lambda_{a,b}$

1 values from other measurements are also mentioned In  $\text{YBa}_2\text{Cu}_3\text{O}_{6.92}$ , “individual phase”  
2 and “shared phase” models yield  $\lambda_{a,b}$  within a nm of each other and anisotropy is the same  
3 for both models. This suggests that there is little ( $\varphi$ ) model dependence and accurate values  
4 of  $\lambda_{a,b}$  can be obtained by LE- $\mu$ SR method. The  $\lambda_{a,b}$  obtained here is in good agreement  
5 with bulk  $\mu$ SR measurement in vortex state. It may be noted that conventional  $\mu$ SR is very  
6 different type of measurement. The  $\lambda_{a,b}$  in  $\text{YBa}_2\text{Cu}_3\text{O}_{6.92}$  obtained here is shorter than that  
7 obtained in film. The difference is understandable considering the measurement in film was  
8 done at 20 K and the  $T_c$  of the film (87.5 K) is less than in crystals possibly indicating a  
9 different doping level than in crystals.

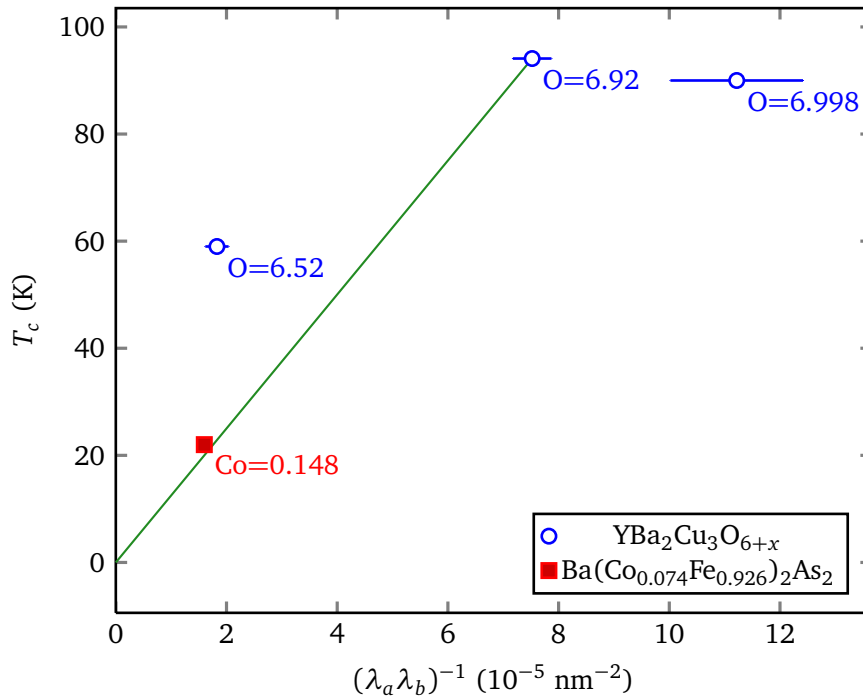
10 Row two of table 4.5 represent our results of  $\lambda_{a,b}$  in Ortho-I  $\text{YBa}_2\text{Cu}_3\text{O}_{6.998}$ , measured  
11 at 5 K & 4.7 mT, extrapolated to 0 K by the method described above. In our analysis of  
12 “individual phase” model, slightly different  $\lambda_{a,b}$  and anisotropy are obtained. However, the  
13 measurement of  $\lambda$  is dependent on our ability to determine phase ( $\varphi$ ) and frequency ( $\omega$ ) at  
14 the same time, which is difficult in lower fields. The lower external field has one significant  
15 advantage of reducing the chance of vortex entrance. It may be noted that in  $\text{YBa}_2\text{Cu}_3\text{O}_{6.998}$ ,  
16 both 4.7 mT & 9.5 mT measurements were done and the  $\lambda_b$  is about the same in “shared  
17 phase” model for 9.5 mT. However,  $\lambda$  at higher external field may have some contribution  
18 from vortices and “individual phase” model has significantly lower  $\chi^2/\text{DF}$ . Note that, there  
19 is very close agreement between our result and ESR Gd-doped Ortho-I  $\text{YBa}_2\text{Cu}_3\text{O}_{6.95}$ .

20 Row three of table 4.5 represent our results of  $\lambda_{a,b}$  in  $\text{YBa}_2\text{Cu}_3\text{O}_{6.52}$ , measured at 5 K  
21 & 1.5 mT, extrapolated to 0 K.  $\lambda_{a,b}$ ’s were measured in 1.5 mT, 4.7 mT & 7.8 mT, however,  
22 there are clear evidences that all external fields except the lowest 1.5 mT introduced  
23 vortices. In 1.5 mT external field, a unique determination of phase was made by reversing the  
24 field, essentially doubling the range of muon polarization’s oscillation time. Determination  
25 of phase is crucial specially at low field as frequency is low and very few full oscillations  
26 are observed in muon polarization. It is also interesting that the low temperature linear  
27 dependence of  $\lambda$  in  $\text{YBa}_2\text{Cu}_3\text{O}_{6.52}$  is an order of magnitude higher than those found in other  
28 two oxygen dopings. Note that  $\lambda_{a,b}$  in LE- $\mu$ SR is significantly longer than that measured  
29 in ESR on Gd-doped  $\text{YBa}_2\text{Cu}_3\text{O}_{6.52}$ . This is surprising considering the other two oxygen  
30 dopings have produced very similar results.

31 On eleventh row (just below the horizontal line) in table 4.5, our measurement of  $\lambda_{ab}$ ,  
32 extrapolated to 0 K is shown. Measurements were done in external field of 2.5 mT 4.7  
33 mT. The  $\lambda_{ab}$  obtained in for both external field in “shared phase” and “individual phase”  
34 models were close to each other within  $\sim 4\text{nm}$  i.e, introducing  $\sim 2\%$  systematic uncertainty.  
35 A two-gap  $s$ -wave model describes the temperature dependence of the superfluid density but  
36 an equally good fit at low temperatures can be obtained with a weak power-law behavior  
37 characteristic of point nodes in the gap function. There is broad agreement between methods  
38 at least at this one Co concentration, except for MFM which probes  $\Delta\lambda$  on much smaller

length scale. For comparison some results from other methods, especially MFM are also shown. Among the MFM results, one of the results with the exact same Co-doping as ours, yield the same  $\lambda$ , however with a much larger uncertainty.

Figure 4.33 shows the critical temperature vs effective superfluid density  $1/(\lambda_a\lambda_b)^{-1}$ . A linear relationship between the two quantities was first suggested by Uemura *et al.* LE- $\mu$ SR results significantly differ from a linear relationship. The relationship seems to be sublinear dependence of  $T_c$  on  $\lambda_{ab}^2$  since, for large  $\lambda_{ab}$ , superfluid density will be proportionately smaller and  $T_c$  will also be smaller. The linear relationship has been widely regarded as an evidence of order parameter's phase fluctuation is the parameter responsible for setting  $T_c$ . The probable sublinear relationship of  $T_c$  on  $\lambda_{ab}^2$  is an indication of other mechanisms being influential in determining  $T_c$ . One of the interesting aspect from figure 4.33 is that optimally doped cuprate & Co-doped pnictide almost falls on a line. Measurements on a range of dopings will be needed to determine the exact relationship of  $T_c$  on  $\lambda_{ab}^2$ .



**Figure 4.33:**  $T_c$  vs  $\Lambda_{ab}$ .

#### 4.4 Discussion On “Deadlayer”

As mentioned in earlier section, a superconductor carries no bulk magnetic field and applied external field decays exponentially inside the sample as it penetrates the surface, according to the London equations. However, as will be shown in a later chapter that, very close to surface, there is a distance over which magnetic field essentially remains constant, an effective dead layer. One possible explanation is surface roughness: small perturbations

**Table 4.5:** Measurements of the absolute value of the magnetic penetration depth ( $\lambda_{a,b}$ ) in  $\text{YBa}_2\text{Cu}_3\text{O}_{6.52}$ ,  $\text{YBa}_2\text{Cu}_3\text{O}_{6.92}$  and  $\text{YBa}_2\text{Cu}_3\text{O}_{6.998}$ . Average magnetic penetration depth  $\lambda_{ab} = \sqrt{\lambda_a \lambda_b}$ . Vortex state measurements are quoted without systematic errors.

$\lambda_a$ (nm)	$\lambda_b$ (nm)	$\lambda_{ab}$ (nm)	$\lambda_a/\lambda_b$	Comment
$125.6(17) \pm 3$	$105.5(11) \pm 3$	$115.1(10) \pm 3$	1.19(1)	$\text{YBa}_2\text{Cu}_3\text{O}_{6.92}$ <sup>a</sup>
$105.0(39) \pm 3$	$82.0(33) \pm 3$	$92.8(26) \pm 3$	1.28(2)	$\text{YBa}_2\text{Cu}_3\text{O}_{6.998}$ <sup>a</sup>
$261.9(141) \pm 3$	$201.5(104) \pm 3$	$229.7(86) \pm 3$	1.30(2)	$\text{YBa}_2\text{Cu}_3\text{O}_{6.52}$ <sup>a</sup>
		118.0(4)		$\mu\text{SR}$ in vortex state <sup>b</sup> [121]
		146(3)		LE- $\mu\text{SR}$ in thin film at 20 K [162]
160	100	126.5	1.6	IR reflectivity at 10 K [109]
103(8)	80(5)	91(7)	1.29(7)	ESR on $\text{YBa}_2\text{Cu}_3\text{O}_{6.995}$ [171]
202(22)	140(28)	168(19)	1.4(3)	ESR on $\text{YBa}_2\text{Cu}_3\text{O}_{6.52}$ [171]
		150(10)	1.16(2)	$\mu\text{SR}$ at 10 K [172]
		138(5)	1.18(2)	SANS at 10 K [173]
$250.0(26) \pm 5$	$250.0(26) \pm 5$	$250.0(26) \pm 5$	-	$\text{Ba}(\text{Co}_{0.074}\text{Fe}_{0.926})_2\text{As}_2$ <sup>a</sup>
		325(50)	-	MFM [166] in $\text{Ba}(\text{Co}_{0.05}\text{Fe}_{0.95})_2\text{As}_2$
		190(10)	-	Estimated for a range of dopings in $\text{BaCo}_x\text{Fe}_{2-x}\text{As}_2$ [174]
		250(36)	-	MFM [175] on $\text{Ba}(\text{Co}_{0.074}\text{Fe}_{0.926})_2\text{As}_2$

<sup>a</sup> This work

<sup>b</sup> Conventional  $\mu\text{SR}$



1 from a perfect flat geometry. A set of mathematical analysis have been done [176] assuming  
 2 uneven surface geometry

$$z = \epsilon \cos(\omega_x x) \cos(\omega_y y). \quad (4.13)$$

3 In these set of analysis, a length of 1 corresponds to a distance of  $\lambda$  in physical units; a  
 4 frequency  $\omega$  corresponds to a physical frequency  $\tilde{\omega} = \frac{2\pi}{\omega\lambda}$ ; a field strength of 1 corresponds  
 5 to the applied external field  $|\mathbf{B}_{\text{applied}}|$ ; roughness amplitude  $\epsilon$  is believed to be no bigger  
 6 than (1/10) of  $\lambda$ . A definition of an effective dead layer  $\delta$ , in dimensionless units, may be  
 7 adopted as

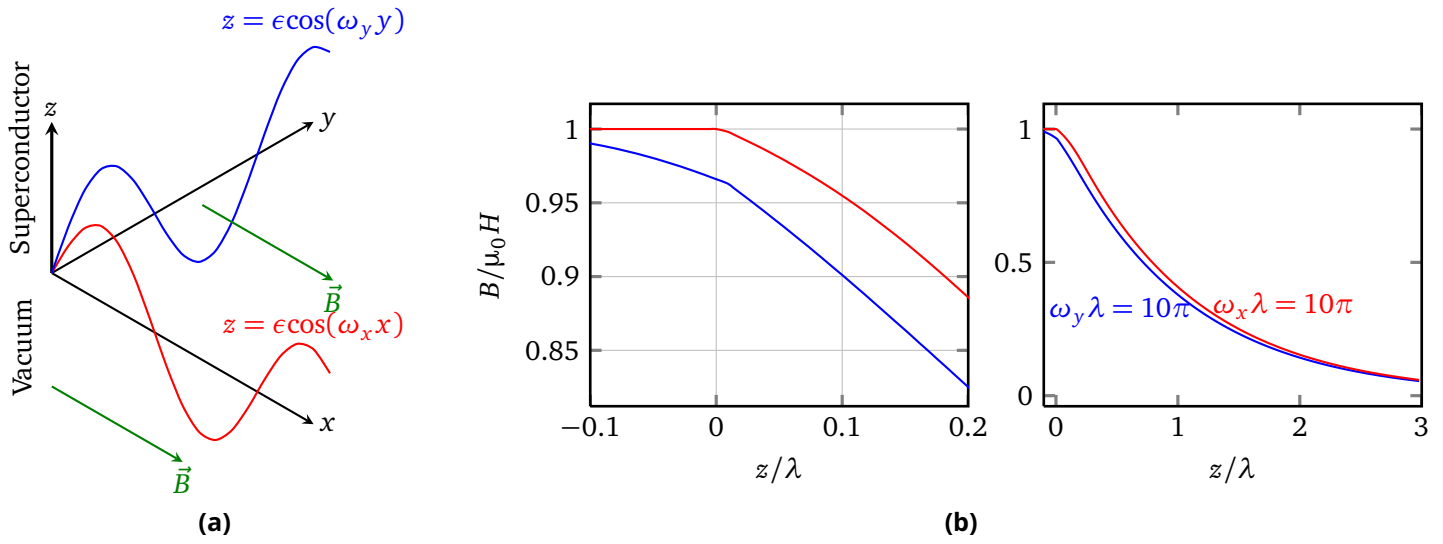
$$\delta = \int_0^\infty |\mathbf{b}|_{\text{avg}}(s) ds - 1, \quad (4.14)$$

8 where  $\mathbf{b}(s)$  is the depth dependent magnetic field & with the interpretation that for a true  
 9 dead layer of size  $\delta$ , the area under  $|\mathbf{b}|_{\text{avg}}$  from  $s = 0$  to  $\infty$  is precisely  $\delta + 1$ .

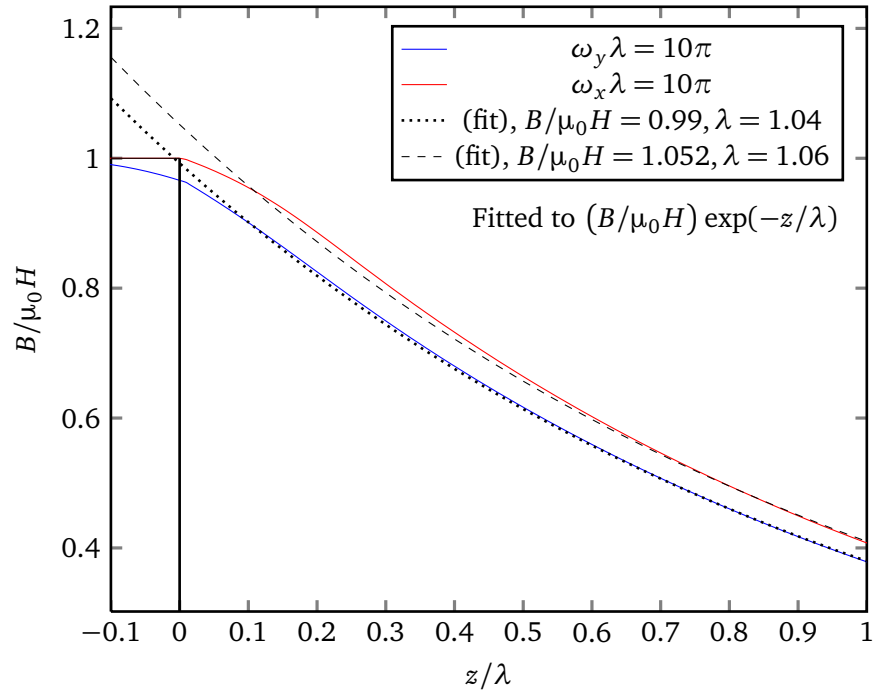
10 Without loss of generality, sample surface may be modelled as sinusoidal, as more  
 11 complicated surface may always be Fourier-transformed. For a surface like this, external  
 12 magnetic field will also be sinusoidal, close to the surface, as depicted in figure 4.34(a).

13 With an asymptotic expansion of magnetic field  $\mathbf{b} = \mathbf{b}_0 + \epsilon \mathbf{b}_1 + \epsilon^2 \mathbf{b}_2 + \dots$ , where  $\mathbf{b}_0$  is the  
 14 known solution for a flat surface &  $\mathbf{b}_i$ 's are the  $i$ -th smaller components M. Lindstrom et  
 15 al. [176] has found via linear analysis, that average magnetic field  $|\mathbf{b}|_{\text{avg}}$  may differ from  
 16 the London solution by as much as 1%.(figure 4.34). As may be noted from the figure  
 17 the external field starts decaying before entering the sample, a contrasting scenario with  
 18 the experimental observation of local fields is possible only inside the surface. Our results  
 19 suggest that for surfaces with roughness amplitudes in the ballpark of  $\lambda/10$  whose spatial  
 20 frequencies aren't too high, the dead layer is no bigger than  $\lambda/20$ .

21 One way the supercurrent at the surface may be suppressed is via vortex penetration  
 22 which can be facilitated by suppression of d-wave order, i.e a reduction of energy gap near  
 23 twin or grain boundaries [177, 178]. Also, surface roughness has been attributed to as the  
 24 cause of vortex nucleation at fields  $H \leq H_{c1}$  [179]. Surface vortices have been observed in  
 25 YBCO in fields as minute as 4G applied parallel to  $a - b$  plane [180]. It may be speculated  
 26 that full-strength external field near the vacuum-surface boundary may be a harbinger  
 27 of vortices. Also vortex-vortex interaction may be an additional element in the apparent  
 28 supercurrent suppression. Field inhomogeneities may result from local variations of current  
 29 close to surface due to surface roughness and twin and grain boundaries [181]. Further  
 30 experiments on atomically flat surfaces may help elucidate the origin of the reduction of  
 31 supercurrent in the surface vicinity.



**Figure 4.34:** (a) A rough surface geometry may be modelled as sinusoidal as any complex surface structure can be Fourier-transformed. External field direction is taken to be along  $a$  axis of the crystal. (b) Simulated relative field as it enters the sample.



**Figure 4.35:** Simulated external field fitted to the London model function for the depth range 0 to  $10\lambda$ . Only the region of  $0-\lambda$  is shown.

## Conclusions & Outlook

In this thesis, recent measurements of  $\lambda_{a,b}$  and the anisotropies ( $\equiv \lambda_a/\lambda_b$ ) have been done for three different oxygen ( $x = 6.52, 6.92, 6.998$ ) dopings of  $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$  and one on  $\text{Ba}(\text{Co}_{0.07}\text{Fe}_{0.93})_2\text{As}_2$ . The measured values of  $\lambda$  and the anisotropies are considerably different from that of literature, often found with bulk methods. An exponential decay of the magnetic field and corresponding supercurrent density  $\sim 100$  nm inside the crystals. Small deviations from the London model are observed which indicate there is a suppression of the supercurrent density close to the surface. The measured ( $\lambda$ ) values are also found to depart substantially from the widely reported Uemura relation ( $T_c \propto 1/\lambda_a^2$ ). Low energy  $\mu\text{SR}$  is a very sensitive depth-dependent probe close to the surface of the crystals. Using London model and simulated stopping profiles, one is able to extract a very precise measure of London penetration depth as a function of observable parameters such doping, temperature, impurity. The measured penetration depths slightly depend on the exact phase models used, as is the case with any model. However the uncertainties due to an exact determination of phase are included as systematic errors in the final results. In fields on the order of  $\sim 10$  mT, measured London depths have little model dependence compared to lower fields such as 5 mT or 1.5 mT. This is easily understood as we note that there are fewer full oscillations in asymmetry spectrum for lower fields and phase & frequency ( $\gamma_\mu B$ ) becomes correlated. However lower fields were required for one oxygen doping in  $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$  due to vortex entrance at external fields  $\sim 4.6$  mT. The measured  $\lambda_{a,b}$  have been found in YBCO good agreement with ESR and conventional  $\mu\text{SR}$ , with the exception of considerable difference oxygen doping 6.52. Low temperature linear dependence of  $\lambda_{a,b}$  have also been observed in all the oxygen dopings in  $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ , which is an important signature for a  $d$ -wave superconducting gap. One important parameter, penetration depth anisotropy  $\mathcal{R}$ , is also determined for all the dopings of YBCO. The anisotropy, being the ratio of  $\lambda$  in both directions, are determined with more accuracy than the penetration depths.

Pnictide penetration depth is also in very good agreement with a recent MFM (with a larger error) determination of  $\lambda$  and is somewhere between the earlier estimates. The temperature dependence of the superfluid density is obtained by combining low energy  $\mu\text{SR}$  and microwave resonance and a weak power law behavior for the superfluid density is found at low temperature while a two-gap model fits the whole temperature range.

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