Modeling Luxury Taxes in the NBA

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1 Introduction and Motivation

In the 2021-2022 NBA season, the Golden State Warriors won their fourth championship in the past eight years, marking the continued dominance of their historic dynasty. However, in the process, they racked up a huge bill. In addition to paying their players \$187.7M, they incurred a \$170.3M luxury tax for going \$78.4M over the salary cap.[1] [2] All of this spending resulted in an operating deficit of \$44M for the season.[3] This trend is projected to continue into the 2023-2024 NBA season, with the Warriors expecting to pay a salary of \$215M to their players and an additional \$268M in luxury tax to bring their total salary for the season to \$483M.[4] With such an enormous salary, the Warriors will be able to retain some of the top talent in the league and look well positioned to compete for another championship. This raises the question: why would the league allow the Golden State Warriors to seemingly "pay for a championship"?

The NBA is one of the largest professional sports organizations in the world. With both fans and team shareholders alike invested in season results, the NBA seeks to create rules that keep the league interesting and equitable. With no regulations in place, there's nothing stopping the teams with the deepest pockets from hoarding and retaining the most star-studded players, precipitating a breakdown of the league due to smaller teams being unable to keep up. To address this, the NBA's luxury tax rules incentivize a fair distribution of talent throughout the league so that no one season is exactly like another– after all, if the same team wins the NBA championship every year, this becomes stale for spectators and can hurt league revenues and viewership.

[1] Toporek, Bryan. "Sorry, Golden State Warriors, but the NBA's Luxury Tax Is Working as Intended." Forbes, 2 Aug. 2022, www.forbes.com/sites/bryantoporek/2022/08/02/sorry-golden-state-warriors-but-the-nbas-luxury-tax-is-working-as-intended.

[2] "NBA 2022-2023 Cap Tracker." Spotrac.com, www.spotrac.com/nba/cap. Accessed 7 Dec. 2022.

[3] Ozanian, Mike. "The NBA's Most Valuable Teams 2021-22: New York Knicks Lead a Trio Now Worth Over \$5 Billion Each." Forbes, 18 Oct. 2021, www.forbes.com/sites/mikeozanian/2021/10/18/the-nbas-most-valuable-teams-2021-22-new-york-knicks-lead-a-trio-now-worth-over-5-billion-each.

[4] Goldberg, Rob. "Warriors' Projected Salary, Luxury Tax Bill for '23-24 After Poole, Wiggins Contracts." Warriors' Projected Salary, Luxury Tax Bill for '23-24 After Poole, Wiggins Contracts — News, Scores, Highlights, Stats, and Rumors — Bleacher Report, 15 Oct. 2022, bleacherreport.com/articles/10052459-warriors-projected-salary-luxury-tax-bill-for-23-24-after-poole-wiggins-contracts.

However, if teams like the Golden State Warriors can operate on a multimillion dollar deficit and still profit in the long term, they have no incentive to stay below the luxury tax. These teams will continue to pay massive salaries to remain dominant, thus reducing the overall parity and competitiveness in the league. This could also lead to teams across other professional sports leagues emulating this strategy. In the future, we could see a situation in which sports teams are forced to take on large amounts of debt to retain and secure top players or are simply unable to compete against teams who are able and willing to pay extremely large salary expenses.

Whereas the NFL utilizes a hard cap through which no team under any circumstance can exceed the salary cap, the NBA utilizes a soft cap with a graduated luxury tax. This means that teams are charged for every dollar above the designated cap, and the surplus is redistributed to teams that pay less. Beginning in 1999, the NBA luxury tax was dollar-for-dollar. However, this saw smaller teams like the Milwaukee Bucks struggling to stomach the tax, whereas bigger teams like the NY Knicks would willingly go over the cap and routinely held 9-figure payrolls. This threatened the long-term system of a sustainable, competitive balance between small and big markets. To solve this, the league designed a graduated luxury tax system with the total payment amount depending on several factors. The first of these factors is how far over the salary cap a team is. The incremental tax for being above the salary cap is as follows: For a team $0-5MM above the cap line, the tax rate is $1.50 for every dollar over the cap, with an incremental max of $7.5MM. For a team $5-10MM above the cap line, the tax rate is $1.75 for every dollar over the cap, with an incremental max of $8.75MM. For a team $10-15MM above the cap line, the tax rate is $2.50 for every dollar over the cap, with an incremental max of $12.5MM. For a team $15-20MM above the cap line, the tax rate is $3.25 for every dollar over the cap, with an incremental max of $16.25MM. Furthermore, a team must pay $0.50 on every dollar for every additional $5MM above the cap line beyond $20MM. The other factor which determines the tax amount for a team is being a repeat offender, which means that a team has paid the luxury tax in 3 out of the last 4 years.The incremental tax for being a repeat offender is as follows: For a team $0-5MM above the cap line, the tax rate is $2.50 for every dollar over the cap, with an incremental max of $12.5MM. For a team $5-10MM above the cap line, the tax rate is $2.75 for every dollar over the cap, with an incremental max of $13.75MM. For a team $10-15MM above the cap line, the tax rate is $3.50 for every dollar over the cap, with an incremental max of $17.5MM. For a team $15-20MM above the cap line, the tax rate is $4.25 for every dollar over the cap, with an incremental max of $21.25MM. Furthermore, a team must pay $0.50 on every dollar for every additional $5MM above the cap line beyond $20MM.[5] Although this system certainly seems intuitive, it is still unable to stop teams like the Golden State Warriors from repeatedly spending well over the salary cap. This leads us to question whether there may be a more effective way of setting the tax.

[5] Urbina, Frank. “How Does the NBA’s Luxury Tax Work?” HoopsHype, 11 Oct. 2018, hoopshype.com/2018/10/11/nba-luxury-tax.

2 Research Question

We pose the following specific research question for our project: What is the optimal luxury tax rate and cap to maximize parity in a sports league like the NBA without significantly reducing overall revenue of the league?

To answer this question, we would like to develop a model for the NBA and simulate the effects of different luxury tax rates and caps on this model across a variety of conditions. Conditions we will consider include how strong team spending impacts win probabilities and the magnitude of the imbalance in revenue between teams.

3 Literature Review

The existing literature on our topic confirms that salary caps and luxury taxes do increase league parity. Dietl et. al (2008) [6] suggest that the implementation of luxury taxes increase parity and social welfare within the MLB and NBA. By using a game theoretical model, Dietl et. al. suggest that a luxury tax creates a more balanced league – by taxing large market teams, their revenue can be redistributed to smaller market teams and allow them to be more competitive which drives viewership and revenues. Outside of this finding, we do not know the degree to which salary caps and luxury taxes impact the NBA and how they can be optimized. Our other sources help us create assumptions for our model in which we seek to address this.

1. **Revenue is correlated with market size**: From analyzing data we quickly see that revenue of NBA teams is correlated with their market size. In our example of the NBA, this is the population of the city and greater metropolitan area that surrounds the team. This is something that we were able to estimate experimentally and found no confounding factors thus confirming this relationship. Moreover, this relationship has already been explored by Dietl et. al.

2. **Factors determining revenue in next year**: We also make assumptions surrounding the factors that predict revenue from one year to the next. The two more intuitive factors that influence the projected revenue of a team are the population of home city, which was already cited as well as previous year's revenue. This is intuitive because when teams do well financially, they are more likely continue this success the following year.

   (a) Population

   (b) Previous season's revenue

   (c) Success (measured as win percentage): It is important to clarify an NBA team's success on the court is the most important variable influencing their revenue. The results of a Berri et. al. [7] multiplicative model suggest that

[6] Dietl, Helmut M. and Lang, Markus and Werner, Stephan, The Effect of Luxury Taxes on Competitive Balance, Club Profits, and Social Welfare in Sports Leagues (June 8, 2009). International Journal of Sport Finance, Vol. 5, No. 1, pp. 41-51, February 2010.

[7] Berri, D. J., Schmidt, M. B., & Brook, S. L. (2004). Stars at the Gate: The Impact of Star Power on NBA Gate Revenues. Journal of Sports Economics, 5(1), 33–50.

higher win percentages are the main revenue driving factor for teams. Their initial hypothesis was that retention of top talent with loyal fan bases, aka "stars" would drive attendance, viewership and therefore revenues; however their model found that win percentage was the largest factor driving revenues. These findings largely allow us to assume that NBA teams are incentivized to allocate spending in a way that maximizes win percentage.

3. **Predictors of win percentage**:

   (a) Previous season's win percentage: Win percentage from year to year follows a trend. Teams are more likely to find success the year after a successful season as they are more likely to have the same good players and management. However, it is worth noting that if a team's win percentage were consistently high so that it reduced competitiveness within the league, this would reduce revenues and viewership. Dietl et al. [8] suggest that leagues must balance successful teams with league parity to maximize revenues.

   (b) Current season's spending: A factor that helps determine a team's revenue is how much money they spend in that season. However, because high spending can impact results in some sports more than others, the exact relationship between these variables is unclear. In some organizations, luxury taxes are more impactful than in others because spending is correlated with success in varying degrees. In the rest of this section, in the absence of more literature on the NBA, we explore the impact of spending on two majors sports leagues: Formula 1 and the MLB.

Formula 1 Racing:

An extreme example where spending and results are correlated would be Formula 1 racing. Revenue imbalances in the early 2000's created a situation in which "teams who spend the most tend[ed] to win the most." Ferrari outspent competitors every year and won 8 Constructor's Cup championships from 1999 to 2008.[9] The league has recently implemented a budget cap on car spending to retain viewership and boost parity. [10] Historically, teams with the highest revenues and budgets could outspend competitors to invest in the best engines, steering and technology that would objectively make their car perform better than competitors. In this case, spending is correlated with success and a league like this benefits significantly from implementing luxury taxes and salary caps.

Major League Baseball:

An example of luxury taxes having a less concrete impact on team success is in the MLB. The factors that lead to the success of a team can be random and hard to

[8] Dietl, Helmut M. and Lang, Markus and Werner, Stephan, The Effect of Luxury Taxes on Competitive Balance, Club Profits, and Social Welfare in Sports Leagues (June 8, 2009). International Journal of Sport Finance, Vol. 5, No. 1, pp. 41-51, February 2010.

[9] Judde, C., Booth, R., Brooks, R. (2013). Second Place Is First of the Losers: An Analysis of Competitive Balance in Formula One. Journal of Sports Economics, 14(4), 411–439.

[10] Knight, Brett. "A New Budget Cap Gave Small Teams a Reason to Stay in Formula 1. They're Thinking Much Bigger." Forbes, Forbes Magazine, 10 May 2022, https://www.forbes.com/sites/brettknight/2021/07/22/a-new-budget-cap-gave-small-teams-a-reason-to-stay-in-formula-1-theyre-thinking-much-bigger/?sh=1b65f0271ad0.

quantify. A team could spend millions on the best prospects and seasoned players, but due to factors like unexpected injuries and team cohesion, this spending might not translate to performance.

It can also be hard to determine how much spending leads to success because of the potential for irrational owners – it's possible that they might overspend on players that don't perform as well as anticipated because of bias.

Ajilore and Hendrickson's study sought to prove the parity implications of a luxury tax in the MLB. [11] Though their study suggests that higher spending improves a team's "competitiveness," they could not show a statistically significant relationship between a teams "payroll" and win percentage. Though we know that spending does impact league parity, it is hard to estimate the degree to which this happens because there is no concrete relationship between spending and results.

In many professional sports leagues, the relationship between spending and results is uncertain – it is only one of many factors that contribute to a team's success. For this reason, it is difficult to know at what figures and rates the NBA should set its salary cap and luxury tax to ensure maximum league parity. This is what will address in our model.

4 Proposed Methodology

In this section, we propose the initial model we will use to explore the effects of luxury tax rates and caps on parity and revenue in the NBA. We also propose a simulation-based experiment by which we will investigate these parameters.

4.1 The Model

Consider a league with two teams in two distinct cities. Let us denote these teams with indices $i \in \{0, 1\}$.

Let P_0 and P_1 denote the populations of the metro areas in which these teams reside. This is a rough approximation of the market size of these teams. We will assume that these populations are constant since we will perform each iteration of our simulation of our simulated experiments over a relatively small time horizon. Without loss of generality, let $P_0 \leq P_1$.

Let $C_{0,t}$ and $C_{1,t}$ denote the maximum capital each team can spend (i.e. the total revenue at their disposal) in season t.

For $t = 0$, let $C_{0,0} = \beta_0^P + \beta_1^P \log(P_0)$ and $C_{1,0} = \beta_0^P + \beta_1^P \log(P_1)$, where β_0^P, β_1^P are estimated from historical NBA data using a simple linear model. See Figure 7.1 in the Appendix for more information on these estimates.

[11] Ajilore, Gbenga and Hendrickson, Joshua, (2005), The impact of the luxury tax on competitive balance in Major League Baseball, No 0517, IASE Conference Papers, International Association of Sports Economists

Let $S_{0,t}$ and $S_{1,t}$ denote the actual amount each team spends in season t. Assume that all expenses are of the same category and contribute the same to team success (i.e. all teams are managed perfectly efficient).

Now, we will assume that the league has implement a simple fixed luxury tax $T_{\lambda,c}$ of the following form:

$$T_{\lambda,c}(x) = \lambda \max(0, x - c)$$

where $\lambda \geq 0$ and $c \geq 0$ are experimental parameters representing the luxury tax rate and cap.

Additionally, we will assume that teams can not run deficits or surpluses each year. Thus, for $i \in \{0, 1\}$ and for all $t \geq 0$:

$$T_c(S_{i,t}) + S_{i,t} = C_{i,t}$$

Under this assumption, given c, λ, and $C_{i,t}$, we can solve for $S_{i,t}$ analytically as follows:

$$S_{i,t} = \begin{cases} C_{i,t}, & \text{if } C_{i,t} \leq c \\ \frac{C_{i,t}+\lambda c}{1+\lambda}, & \text{otherwise} \end{cases}$$

Now, let $W_{0,t}$ and $W_{1,t}$ denote the win probability of each team in season t. For $t = 0$, we set $W_{0,0} = \frac{1}{2}$ and $W_{1,0} = \frac{1}{2}$. That is, we assume initial league parity.

Furthermore, for $t > 0$, we assume that:

$$W_{0,t} = qW_{0,t-1} + \frac{1-q}{1 + 10^{\frac{S_{1,t}-S_{0,t}}{100}}}$$

$$W_{1,t} = qW_{1,t-1} + \frac{1-q}{1 + 10^{\frac{S_{0,t}-S_{1,t}}{100}}}$$

where $0 \leq q \leq 1$ is an experimental parameter that approximates the strength of the relationship between a team's spending in a given season $t-1$ (relative to other teams) and their win probability in the next season t. $q = 0$ indicates that spending entirely predict future win probably and $q = 1$ indicates that win probabilities are constant. This approach to modeling win probabilities is adopted from the Elo rating system. [12]

Under this assumed relationship, $W_{0,t} + W_{1,t} = 1$ for all $t \geq 0$.

For each season t, we simulate $n_G = 82$ games (the same as the real NBA season) between the two teams using a binomially distributed random variable $G_t \sim \mathbf{Bin}(n_G, W_{0,t})$.

We calculate the resulting win percentages of each team for season t as $R_{0,t} = G_t/n_G$ and $R_{1,t} = 1 - R_{0,t} = 1 - G_t/n_G$.

[12] "Elo rating system." Wikipedia, en.wikipedia.org/wiki/Elo_rating_system.

Let j denote the index of the team with the greater win percentage. Let team j win the championship in season t with probability $R_{j,t}$. Thus, their championship indicator $I_{i,t} \sim \textbf{Bern}(R_{j,t})$ is a Bernoulli distributed random variable and $I_{1-i,t} = 1 - I_{i,t}$.

Finally, compiling all these results, we get the following function to update team capital in each season $t > 0$:

$$C_{0,t} = \beta_0^C + \beta_1^C I_{0,t-1} + \beta_2^C \log(\frac{R_{0,t-1}}{1 - R_{0,t-1}}) + \beta_3^C C_{0,t-1} + \beta_4^C P_0$$

$$C_{1,t} = \beta_0^C + \beta_1^C I_{1,t-1} + \beta_2^C \log(\frac{R_{1,t-1}}{1 - R_{1,t-1}}) + \beta_3^C C_{1,t-1} + \beta_4^C P_1$$

where where $\beta_0^C, \beta_1^C, \beta_2^C, \beta_3^C, \beta_4^C$ are estimated from historical NBA data using a simple linear model. The win percentages are transformed to normally distributed values using the logit function as shown above. The intercept β_0^C and constant population terms $\beta_4^C P_i$ account for the growth in revenue for each team over time regardless of performance. See Figure 7.2 in the Appendix for more information on these estimates.

4.2 Experimental Design

The model described above was implemented in a Jupyter Notebook using Python.

The effect of the following model parameters were explored:

1. λ: luxury tax rate
2. c: luxury tax cap
3. q: the relationship between spending and win percentage
4. $\frac{R_{1,0}}{R_{0,0}}$: the ratio of initial capital between the two teams

We explored every combination of parameters over the following ranges, for a total of over 166,992 combinations:

1. λ: $[0, 5]$, step size: 0.25
2. c: $[100, 800]$, step size: 10
3. q: $[0, 0.01, 0.05, 0.1, 0.25, 0.5, 0.6, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95, 0.975, 0.99, 1]$
4. $\frac{R_{1,0}}{R_{0,0}}$: $[1, 4]$, step size: 0.5

For every $\frac{R_{1,0}}{R_{0,0}}$, we set $P_0 = 10^6$ and calculate $R_{0,0}$, $R_{1,0}$, and P_1 using $\frac{R_{1,0}}{R_{0,0}}$, β_0^P, and β_1^P.

The search spaces for these parameters are grounded in real NBA data (see Data subsection of Appendix):

- The maximum revenue between any two teams in an NBA season (from 2011-2020) was 3.12. The maximum revenue ratio we chose to simulate is 4.
- The minimum population for an NBA Team (from 2011-2020) is 1.1M. The simulated population for the small-market team is 1M.
- The maximum population for an NBA Team (from 2011-2020) is 19.6M. The maximum simulated population for the large-market team is 106.5M (when $\frac{R_{1,0}}{R_{0,0}} = 4$).

For each combination of parameters, the model was run for 100 simulations of 30 seasons. We then compute the mean of the following 3 metrics for each combination over those 100 simulations:

1. Parity: the number of championship wins for Team 0 (i.e. the small market team) calculated as $\Sigma_{t=0}^{29} I_{0,t}$. The optimal value for this metric would be 15 (i.e. both teams win the same number of championships). Furthermore, under the formulation of our model, this value would rarely be greater than 15.
2. Total Taxes Paid (TTP): the total taxes paid (in million dollars) by both teams over 30 seasons calculated as $\Sigma_{t=0}^{29} \Sigma_{i=0}^{1} T_{\lambda,x}(S_{i,t})$. We seek to minimize this metric under the assumption that increasing luxury taxes decreases the efficiency of the NBA as a market and thus would decrease overall revenue for the league.
3. Luxury Tax Efficiency (LTE): improvement in parity per additional million dollars in TTP. This will serve as the objective function by which ultimately evaluate the efficacy of different tax rates and caps. To calculate this metric for each $\lambda, c, q, \frac{R_{1,0}}{R_{0,0}}$, we take the difference between parity under that combination of parameters and parity when we set $\lambda = 0$. Then, we divide by TTP. Thus, we will only calculate this metric for $\lambda > 0$. Since LTE represents improvement, we will seek to maximize this metric.

5 Results

5.1 Validating Model Assumptions

First, we must confirm that the model we designed is behaving as expected in the absence of luxury taxes. Some expectations we have for the model are:

1. Parity will be approximately 15 when $q = 1$ or $\frac{R_{1,0}}{R_{0,0}} = 1$. When spending doesn't matter or they have the same access to capital, we expect them to win the same number of championships.
2. Parity will decrease with $\frac{R_{1,0}}{R_{0,0}}$ when $q < 1$. If spending has some positive effect on win percentage, a larger imbalance in available capital to spend will lead to a larger gap in win percentage and thus championships won.

3. Parity will increase with q for $\frac{R_{1,0}}{R_{0,0}} > 1$. If there is an imbalance in available capital to spend, decreasing the importance of spending on win percentage will decrease the corresponding imbalance in championships won.

To confirm this, we can plot parity as function of q and $\frac{R_{1,0}}{R_{0,0}}$ when $\lambda = 0$ (i.e. there is no luxury tax). As seen in Figure 7.3, all three expectations are satisfied, with parity minimized when $\frac{R_{1,0}}{R_{0,0}}$ is large and q is small. We should note that, when $q = 1$ or $\frac{R_{1,0}}{R_{0,0}} = 1$, we observe a mean parity slightly below 15; however, this discrepancy can likely be attributed to randomness over the simulations.

5.2 Revenue

While annual revenue is not one of the core metrics we selected to evaluate in our experiments, understanding its relationship to the experimental parameters heavily informs our understanding of the upcoming metrics to be discussed.

When there are no luxury taxes ($\lambda = 0$), we plot the change in revenue over time across all simulations and all parameters $\frac{R_{1,0}}{R_{0,0}}$ and q. A selection of these plots can be found in 7.4. First and foremost, these plots emphasize the severity of the problem which luxury taxes seek to address. For most values of $\frac{R_{1,0}}{R_{0,0}}$ and q, the smaller-market team goes bankrupt, as shown in 7.4c. Furthermore, for large q, even if the initial capital advantage $\frac{R_{1,0}}{R_{0,0}}$ is large, both teams can win consistently and thus both their annual revenues grow over time as shown in 7.4a and 7.4b. However, for slightly smaller q, even if the capital advantage $\frac{R_{1,0}}{R_{0,0}}$ is relatively small, the smaller-market team will struggle to win championships and eventually go bankrupt (as seen in 7.4c), while the larger-market team will capture this lost revenue (as seen in 7.4d). It should be noted that the magnitude of $\frac{R_{1,0}}{R_{0,0}}$ determines the rate at which these phenomenon occur.

Another interesting finding from the simulations is how $\frac{R_{1,0}}{R_{0,0}}$ and q interact to influence the variance of revenue for both teams:

- For $\frac{R_{1,0}}{R_{0,0}} = 1$, we see that the variance of revenue for both teams is constant, as both teams, on average, have the same win probabilities regardless of how much spending matters.
- For slightly larger $\frac{R_{1,0}}{R_{0,0}} = 1.5$, variance is highest when q is small and decreases as q increases. That is, when spending impacts win probability, the effect of this slight edge in capital can have wildly varying effects on early team performance and thus future revenue. For example, if the smaller-market team had better performance in the first few seasons by chance, they may actually accumulate as much capital as the larger-market team in the short-term as a result of this success, achieving approximate parity and staving off bankruptcy. However, if the the larger-market team finds early success in their first few seasons, they will compound on their early edge and accumulate capital at the expense of the small-market team.

- For large $\frac{R_{1,0}}{R_{0,0}} \geq 2$, the value for q at which variance is highest becomes larger, ranging between $0.6 \leq q \leq 0.8$. Intuitively, for small q when spending is highly indicative of win probability, the large disparity in revenue results in the larger-market team consistently out-performing and out-earning the small-market team. Similarly, for large q when spending has little to no effect on performance, each team wins at about the same rate on average and the revenue growth rate becomes constant. Thus, there is a sweet spot, where spending has a moderate impact on performance, during which revenue growth varies most widely

These additional findings are drawn from the attached figures found in the `Figures` folder.

5.3 Parity

Next, let us investigate the effect of luxury taxes on parity. We expect that, when $q \neq 1$ and $\frac{R_{1,0}}{R_{0,0}} \neq 1$, increasing the luxury tax rate λ and decreasing the cap c will increase parity. As we approach $c \leq C_{0,0}$ (i.e. we set the cap to be less than our equal to the initial capital of the small-market team), both teams are penalized for spending more than c and thus are incentivized to spend the same. Furthermore, as λ increases, this penalty becomes much harsher, meaning the larger-market team can spend less over the cap. Thus, the luxury cap becomes a closer and closer approximation of a fixed cap, where teams can not possibly spend more than c.

The results of the simulation confirm our intuition. As seen in all three heatmaps in Figure 7.5 below, parity is maximized when the tax rate is largest and cap is smallest. Another interesting conclusion we are able to draw from the simulations is that, as q increases, the severity of rate and cap necessary to reach the same level of parity decreases. This is shown by the much larger mean parity μ in 7.5b as compared to 7.5a. Similarly, as $\frac{R_{1,0}}{R_{0,0}}$ increases, the severity of rate and cap necessary to reach the same level of parity also increases; this is shown by the much smaller mean parity μ in 7.5c. These results indicate that the true conditions of the NBA heavily dictate the intensity of luxury taxes necessary to achieve parity.

5.4 Total Taxes Paid (TTP)

Many of our findings about TTP under different experimental conditions follow intuitively from our discussion of annual revenue:

- Much like parity, TTP increases as the tax rate λ increases or cap c decreases (as seen in Figure 7.7). This is because more of each teams' spending is subject to being taxed at a higher rate.
- For a fixed rate λ and cap c, as $\frac{R_{1,0}}{R_{0,0}}$ increases, TTP increases (as seen from the much large mean TTP μ in 7.6c as compared to 7.6a). This is because more of the larger market team is able to spend more and thus spends more over the cap.

- For a fixed rate λ and cap c, as q increases, TTP decreases (as shown by the much smaller mean TTP μ in 7.6b). This is because, as discussed in Section 5.2, the larger-market team accumulates less wealth as q increases and thus there is less money being spent above the cap (since the small market team typically falls below the cap).

5.5 Luxury Tax Efficiency (LTE)

Building on the conclusions of the past sections, we use luxury tax efficient (LTE) to understand how to optimally set the luxury tax rate and cap across experimental parameters, answering our original question:

- When $\frac{R_{1,0}}{R_{0,0}}$ is high and q is low, LTE is maximized when the luxury tax rate is maximized and the cap is set just above the initial capital of the small market team $C_{0,0}$. We can interpret this as severely punishing the large-market team when they have a singificant advantage in a league heavily determined by spending. See Figure 7.7a.
- As q increases for large $\frac{R_{1,0}}{R_{0,0}}$, we see the top right corner of the LTE heatmaps become brighter (see Figure 7.7b), until LTE is maximized when the cap is set above both team's maximum capital and the tax rate is minimized. This suggests that, when their is disparity in capital between teams, but spending has a limited impact on performance, it is inefficient to enforce a luxury tax and instead the league should enact a laissez-faire approach.
- For smaller values of $\frac{R_{1,0}}{R_{0,0}}$, we see a breakdown in the optimization of LTE. For most values of λ and c, we observe an LTE of 0, indicating that the luxury tax provides no significant additional parity benefit. This is intuitive because the larger market team has a less significant advantage. However, for one arbitrary cap c, we observe extremely large LTE. See Figure 7.7c. More work must be done to develop an intuitive understanding of these results.
- For smaller values of $\frac{R_{1,0}}{R_{0,0}}$, as q increases, the aforementioned cap line at which we observe a strange unexpected maximization of LTE begins to move left along the heatmap (i.e. the cap decreases). See Figure 7.7d. More work must be done to develop an intuitive understanding of these results.

6 Conclusion and Future Directions

Returning to our motivation, we must now use our understanding of how to optimally set a luxury tax to make concrete policy recommendations to the NBA. First, we must identify where the NBA falls within our experimental landscape. As previously mentioned, the maximum ratio of revenue imbalance between two teams in the NBA was 3.12. Thus, we can place the NBA in the large $\frac{R_{1,0}}{R_{0,0}}$ regime with $\frac{R_{1,0}}{R_{0,0}} \approx 3$. Furthermore, as shown in our motivating example with the Golden State Warriors, it is assumed that spending has a strong positive impact on win percentage. Thus, we can assume that q is small; in future experiments, we could compute the mean squared error of our win percentage model on a validation set across a wide-range of values for q to attempt to estimate the true relationship. Under this assumption and based on our findings, the NBA should set the luxury cap just above the spending of the team with the lowest revenue and set the tax rate incredibly high.

But exactly how low should the NBA set the cap and how high should they set the rate?

As shown in 7.8, which displays the optimal cap as a function of $\frac{R_{1,0}}{R_{0,0}}$ and q, for large $\frac{R_{1,0}}{R_{0,0}}$ and small q, the optimal cap ranges from \$150-200M. which is \$33-83M (or 28-70%) above the initial capital of \$117M of the small-market team. While this may seem like a large buffer, this simple analysis fails to account for the growth in revenue of the small-market team. Further analysis should attempt to pinpoint an analytic approximation for determining this cap as a function of the revenue of the small-market team over all 30 seasons. It is likely that these caps closely approximate the mean or final revenue of the small market team. Ultimately, this cap policy closely reflects current NBA policy. In the 2015-2016 NBA season, the lowest team salary (i.e. amount spent on a team) was \$61M. The luxury tax threshold in that season was \$84.7M, \$23.7M or 39% above the lowest salary and firmly within the recommended range according to our experiments.[13] However, our model assumes a fixed cap, which is set at $t = 0$ for all 30 seasons. In contrast, the NBA utilizes a floating cap, which is updated prior to each season. If we were to include such a features in our model, it is likely that it would recommend a cap that is much tighter to the spending of the small market team each season. We could confirm this hypothesis by simulating this more complex cap structure and calculating its difference from the lower-market teams' spending each season.

For large $\frac{R_{1,0}}{R_{0,0}}$ and small q, the optimal tax rate is always the largest possible rate. This suggests that luxury tax for the NBA would be a hard cap, under which teams couldn't even compete if they exceeded it. To confirm this hypothesis, we would need to run our model on a wider range of tax rates (especially larger values of λ) as well as with a fixed cap and evaluate which yields the highest LTE under experimental conditions matching the NBA. One limitation of our analysis so far is that our model assumes a fixed rate, however, the NBA uses a graduate tax rate that is further increased for repeat offenders. It is possible that modelling these more complex tax rates could yield different conclusions, but given the results thus far, it is much likely to further support the implementation of strict hard cap.

7 Appendix

7.1 Data

Data on NBA teams was obtained from 2011-2020 to estimate the relationship between variables in our model. This data was obtained from the following sources:

[13]"Luxury tax (sports)." Wikipedia

1. Playoff performance and regular season win percentage: Basketball Reference
2. Metro-area population: U.S. Census
3. Revenue: RunRepeat

The data can be found in `nba.xlsx`

7.2 Code

The code for all the simulation experiments can be found in `simulations.ipynb`

7.3 Figures

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                     TR   R-squared:                       0.223
Model:                            OLS   Adj. R-squared:                  0.195
Method:                 Least Squares   F-statistic:                     8.023
Date:                Mon, 05 Dec 2022   Prob (F-statistic):            0.00846
Time:                        14:48:34   Log-Likelihood:                -144.91
No. Observations:                  30   AIC:                             293.8
Df Residuals:                      28   BIC:                             296.6
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept    114.2717      8.484     13.468      0.000      96.892     131.651
Population  3.343e-06   1.18e-06      2.833      0.008    9.25e-07    5.76e-06
==============================================================================
Omnibus:                        4.385   Durbin-Watson:                   2.168
Prob(Omnibus):                  0.112   Jarque-Bera (JB):                2.860
Skew:                          -0.507   Prob(JB):                        0.239
Kurtosis:                       4.122   Cond. No.                     1.06e+07
==============================================================================
```

Figure 7.1: Coefficient estimates for the relationship between population and revenue of NBA teams. As expected, both the intercept and slope estimates are positive and highly significant. While this simple linear model seems to significantly explain the relationship, a linear mixed effects model with a random effect corresponding to time (as indicated by NBA season) might be more appropriate.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                     FR   R-squared:                       0.813
Model:                            OLS   Adj. R-squared:                  0.811
Method:                 Least Squares   F-statistic:                     320.1
Date:                Mon, 05 Dec 2022   Prob (F-statistic):         8.92e-106
Time:                        14:48:36   Log-Likelihood:                -1459.3
No. Observations:                 299   AIC:                             2929.
Df Residuals:                     294   BIC:                             2947.
Df Model:                           4
Covariance Type:            nonrobust
=========================================================================================
                            coef    std err          t      P>|t|      [0.025      0.975]
-----------------------------------------------------------------------------------------
Intercept                37.3545      5.184      7.206      0.000      27.153      47.556
Championship              5.8283     10.940      0.533      0.595     -15.702      27.359
transformedWinPercent     4.9318      2.939      1.678      0.094      -0.852      10.715
TR                        0.8204      0.027     30.657      0.000       0.768       0.873
Population             1.273e-06   4.22e-07      3.015      0.003    4.42e-07     2.1e-06
==============================================================================
Omnibus:                       29.119   Durbin-Watson:                   1.373
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               77.297
Skew:                          -0.422   Prob(JB):                     1.64e-17
Kurtosis:                       5.344   Cond. No.                     4.25e+07
==============================================================================
```

Figure 7.2: Coefficient estimates for a simple linear model predicting future revenue. As expected, all the intercept and slope estimates are positive. However, the t-tests suggest that the effects of winning a championship and regular season win percentage on future revenue may not be significant.

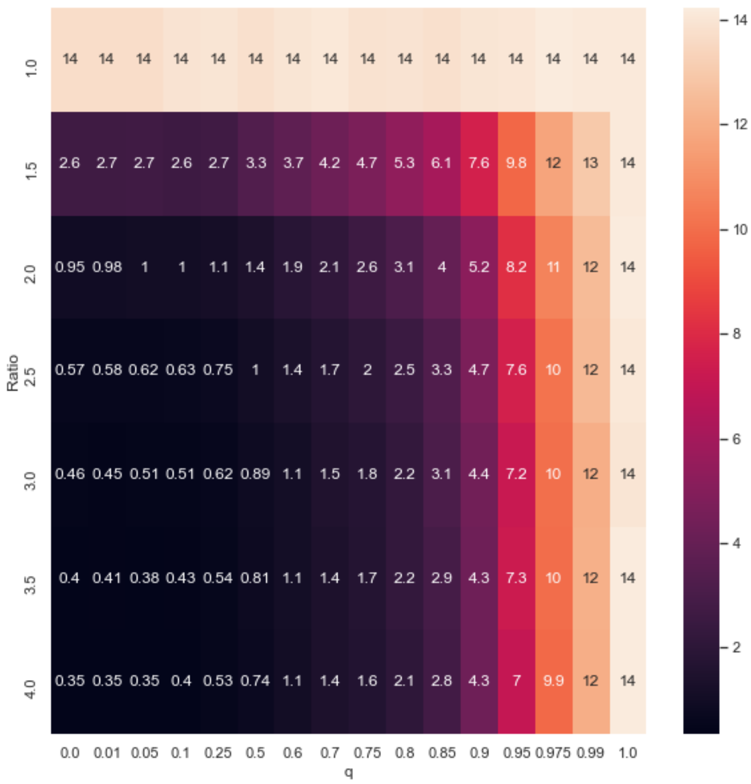
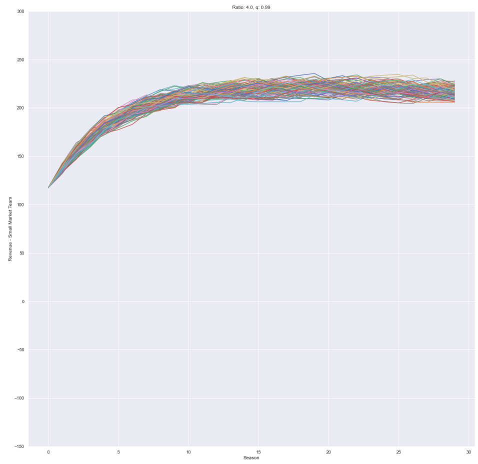
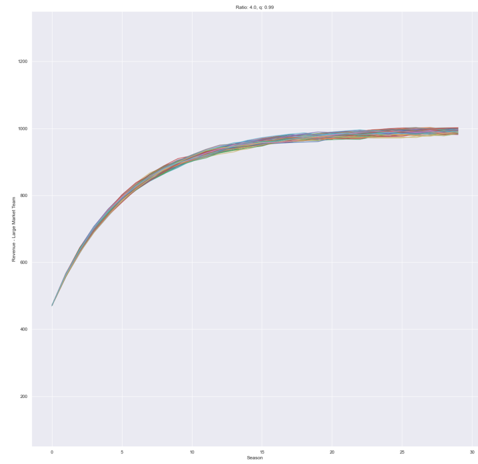


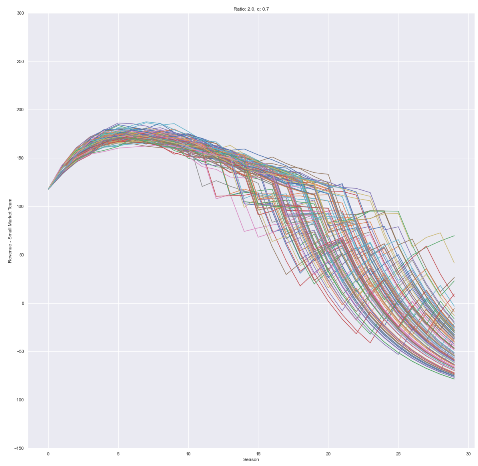
Figure 7.3: A heatmap of parity as a function of the initial revenue ratio $\frac{R_{1,0}}{R_{0,0}}$ and q when $\lambda = 0$



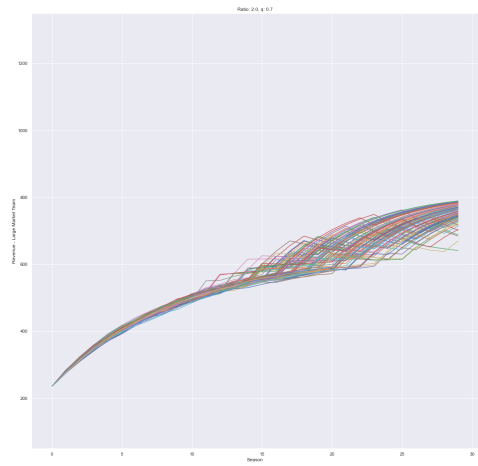
(a) Small Market Team, $\frac{R_{1,0}}{R_{0,0}} = 4$, $q = 0.99$



(b) Large Market Team, $\frac{R_{1,0}}{R_{0,0}} = 4$, $q = 0.99$

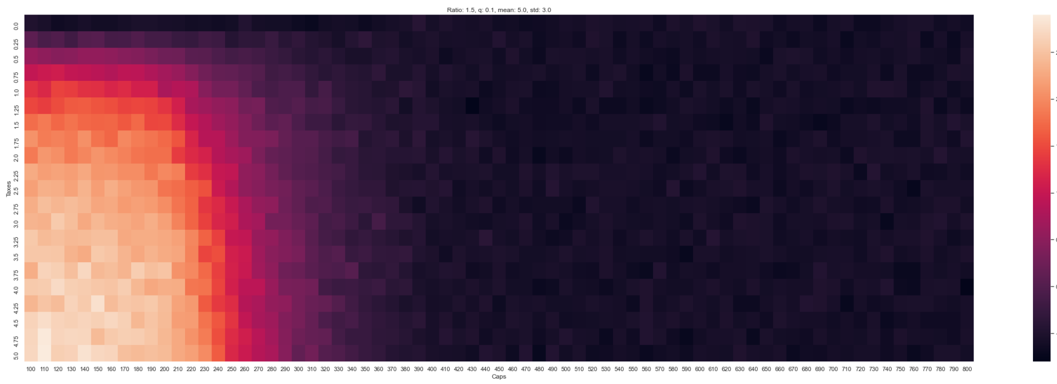


(c) Large Market Team, $\frac{R_{1,0}}{R_{0,0}} = 2$, $q = 0.7$

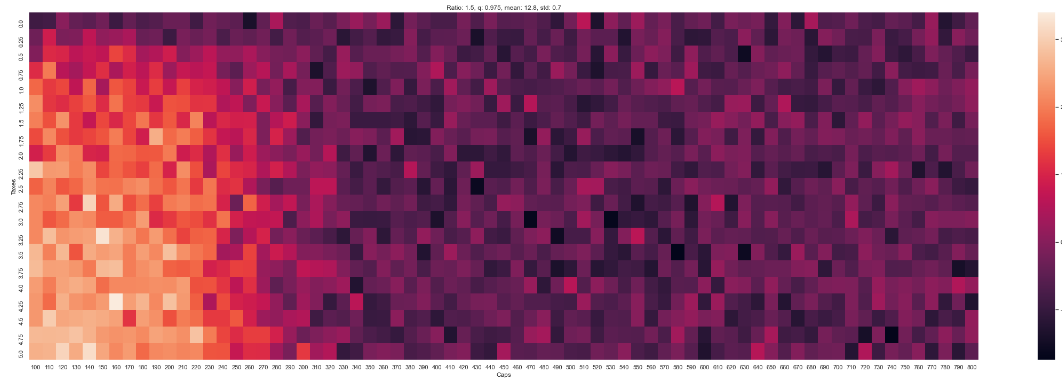


(d) Large Market Team, $\frac{R_{1,0}}{R_{0,0}} = 2$, $q = 0.7$

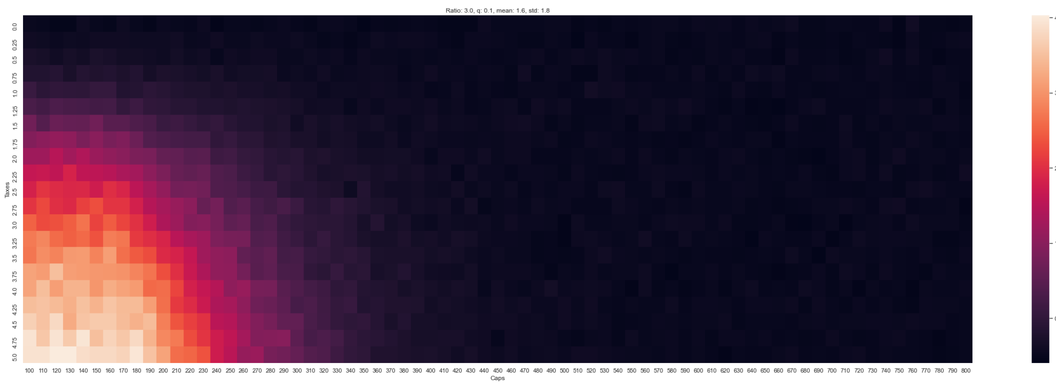
Figure 7.4: Plots of annual revenue as function of time across all simulations for a specific combination of $\frac{R_{1,0}}{R_{0,0}}$ and q. Full-sized images of these plots for each $r = \frac{R_{1,0}}{R_{0,0}}$ and q can be found in the accompanying folder `Figures/capital_large` and `Figures/capital_small` labelled as `rev_r_q.png`



(a) $\frac{R_{1,0}}{R_{0,0}} = 1.5$, $q = 0.1$, $\mu = 5.0$, $\sigma = 3.0$

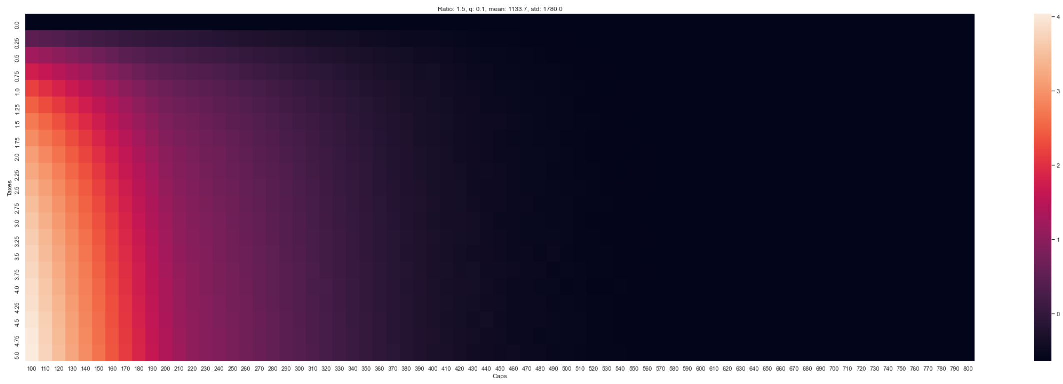


(b) $\frac{R_{1,0}}{R_{0,0}} = 1.5$, $q = 0.975$, $\mu = 12.8$, $\sigma = 0.7$

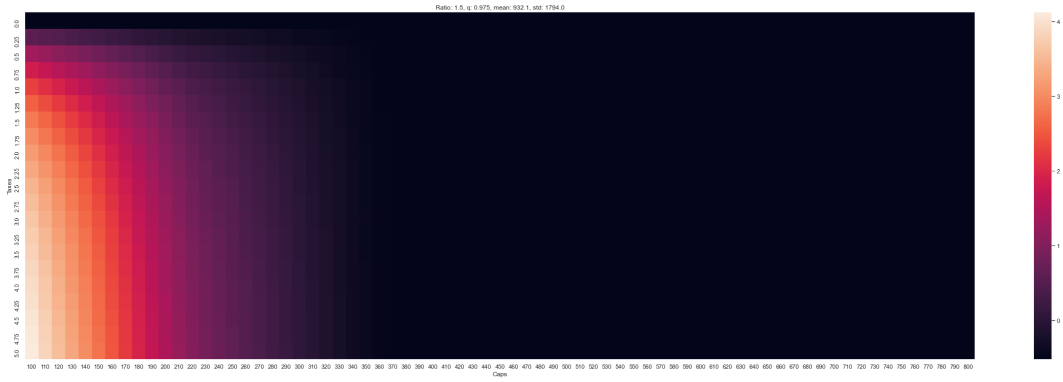


(c) $\frac{R_{1,0}}{R_{0,0}} = 3$, $q = 0.1$, $\mu = 1.6$, $\sigma = 1.8$

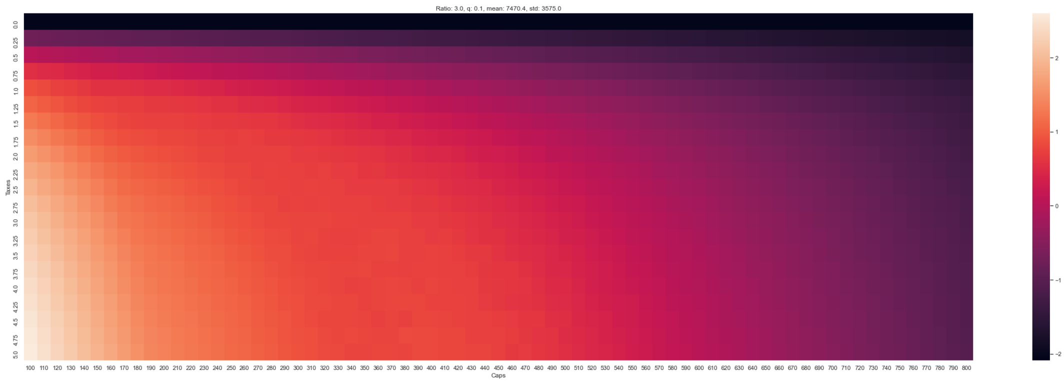
Figure 7.5: Heatmaps of parity as a function of tax rate λ and cap c across varying revenue ratios $\frac{R_{1,0}}{R_{0,0}}$ and spending weights q. The mean μ and standard deviation σ of each heatmap are also provided. Full-sized images of these heatmaps for each $r = \frac{R_{1,0}}{R_{0,0}}$ and q can be found in the accompanying folder `Figures/parity` labelled as `parity_r_q.png`



(a) $\frac{R_{1,0}}{R_{0,0}} = 1.5,\ q = 0.1,\ \mu = 1133.7,\ \sigma = 1780.0$

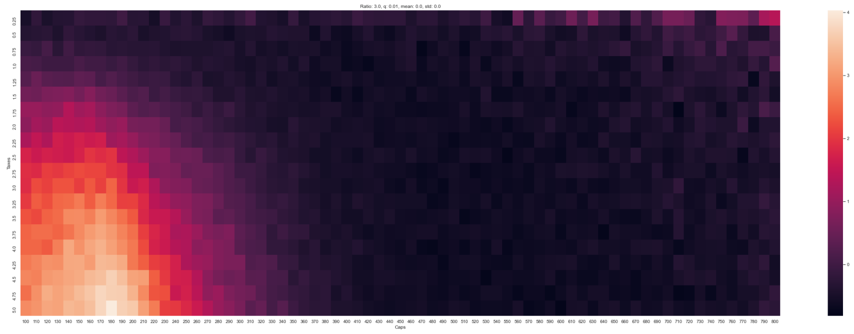


(b) $\frac{R_{1,0}}{R_{0,0}} = 1.5,\ q = 0.975,\ \mu = 932.1,\ \sigma = 1794.0$

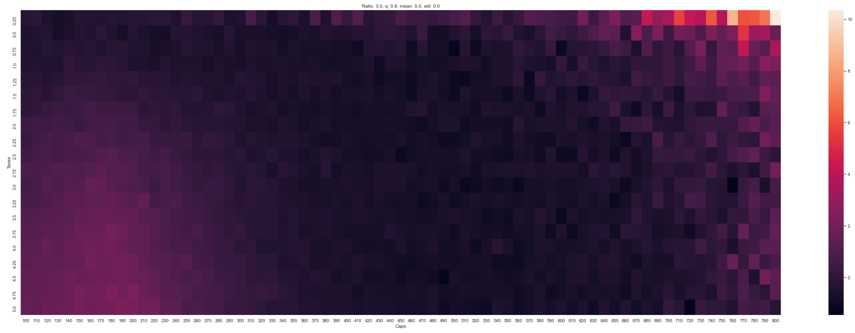


(c) $\frac{R_{1,0}}{R_{0,0}} = 3,\ q = 0.1,\ \mu = 7470.4,\ \sigma = 3575.0$

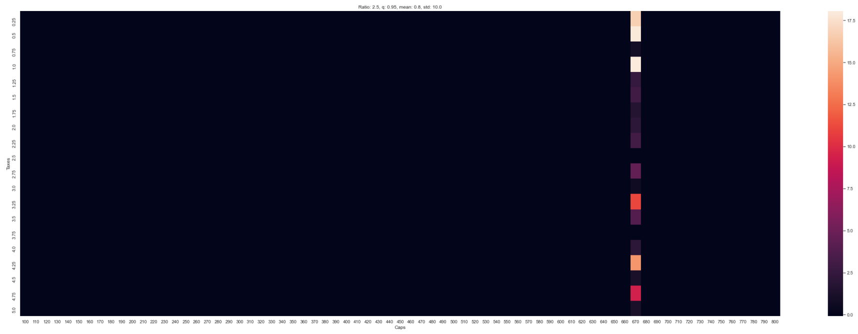
Figure 7.6: Heatmaps of total taxes paid (TTP) as a function of tax rate λ and cap c across varying revenue ratios $\frac{R_{1,0}}{R_{0,0}}$ and spending weights q. The mean μ and standard deviation σ of each heatmap are also provided. Full-sized images of these heatmaps for each $r = \frac{R_{1,0}}{R_{0,0}}$ and q can be found in the accompanying folder `Figures/taxes` labelled as `tp_r_q.png`



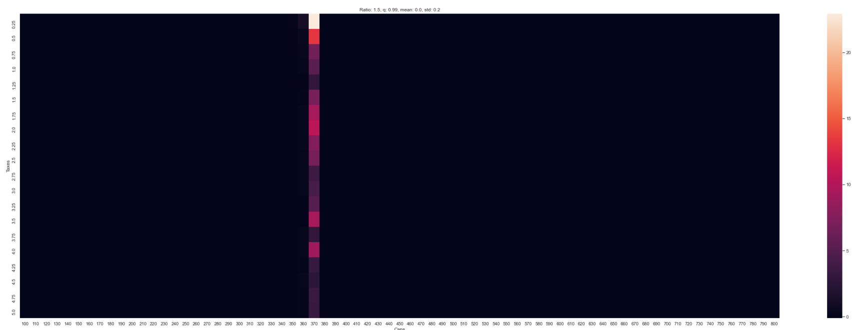
(a) $\frac{R_{1,0}}{R_{0,0}} = 3,\ q = 0.01,\ \mu = 0,\ \sigma \approx 0$



(b) $\frac{R_{1,0}}{R_{0,0}} = 3,\ q = 0.8,\ \mu = 0,\ \sigma \approx 0$



(c) $\frac{R_{1,0}}{R_{0,0}} = 2.5,\ q = 0.95,\ \mu = 0.8,\ \sigma = 10$



(d) $\frac{R_{1,0}}{R_{0,0}} = 1.5,\ q = 0.99,\ \mu = 0,\ \sigma = 0.2$

Figure 7.7: Heatmaps of luxury tax efficiency (LTE) as a function of tax rate λ and cap c across varying revenue ratios $\frac{R_{1,0}}{R_{0,0}}$ and spending weights q. The mean μ and standard deviation σ of each heatmap are also provided. Full-sized images of these heatmaps for each $r = \frac{R_{1,0}}{R_{0,0}}$ and q can be found in the accompanying folder `Figures/objective` labelled as `obj_r_q.png`

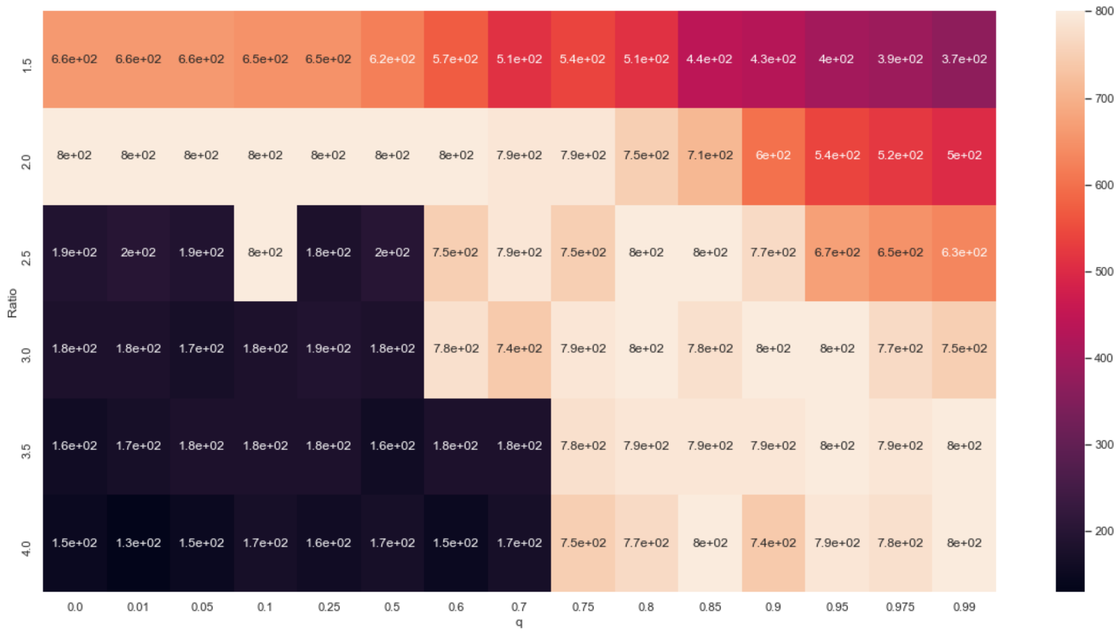


Figure 7.8: A heatmap of optimal cap as a function of the initial revenue ratio $\frac{R_{1,0}}{R_{0,0}}$ and q