Iterative Energy Shaping of a Ball–Dribbling Robot

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1st IFAC Workshop on Robot Control
2019-09-18

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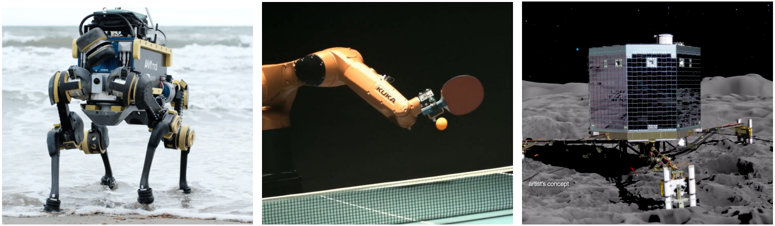
Modeling, **analysis** and **control** of *nonsmooth* mechanical systems
$\Longrightarrow$ **Open** and hard problem of **robotics** and **control theory**



(a) legged locomotion (b) manipulation (c) soft landing

Why is it so hard to deal with them?

Their behavior features interacting *continuous* and *discrete* time dynamics

Hybrid Dynamical System

Discontinuous (impacts)

Smooth (flows)

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Port–Hamiltonian Systems (energy–based modeling)

A breath of fresh air in nonlinear control!

Can we use them to control *nonsmooth* systems?

bring physical insight into **hybrid systems** for robotics

Common factor among interactions (**continuous** & **discrete**)?

ENERGY

Research Objective:

Test the potential of this approach by employing a prototype system:
the **ball–dribbling robot**

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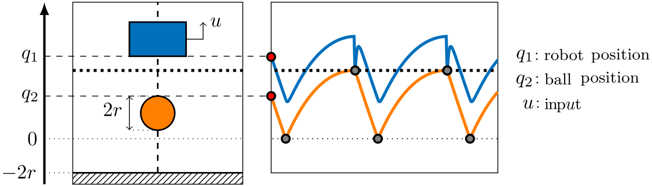
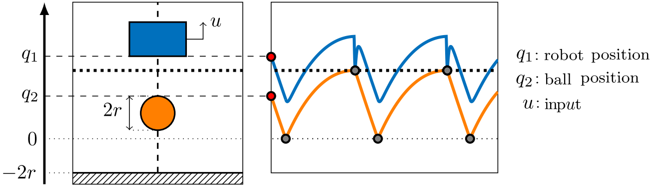
Ball–Dribbling Robotic System



Well, it may look simple but... it is quite challenging

Under–actuation → a (control) force can only be applied to the robot
Flow–decoupling → forces applied to the robot do **not** affect ball's motion
Impact coupling → robot and ball dynamics couple only during impacts
Chaos → chaotic behavior arises in the uncontrolled system

Ball–Dribbling Robotic System



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Problem setting:

Periodic regulation of a ball–dribbling system

Approach:

- **Energy–based** modeling of the hybrid system;
- Apply **passivity–based** control of port–Hamiltonian systems

Contribution:

- **Robust** regulation of the ball–dribbling robot;
- Simple and **insightful** control synthesis;
- **Avoid** the need of **model parameters** in the design.

Originality:

Casting *energy shaping* control in a *learning* context.

Modeling Approach: Background

Overall system is a *hybrid dynamical system*: we use *hybrid inclusions*

Single–flow hybrid inclusion

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}) & (\mathbf{x},\mathbf{u}) \in \mathcal{C} \times \mathcal{U} \\ \mathbf{x}^+ \in \mathcal{G}(\mathbf{x}) & \mathbf{x} \in \mathcal{D} \\ \mathbf{y} = \mathbf{h}(\mathbf{x}) & \end{cases}$$

flow map	$\mathbf{f}: \mathbb{R}^n \to \mathbb{R}^n$ (smooth)
flow set	$\mathcal{C} \subset \mathbb{R}^n$
jump map	$\mathcal{G}: \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ (set–valued)
jump set	$\mathcal{D} \subset \mathbb{R}^n$
output map	$\mathbf{h}: \mathbb{R}^n \to \mathbb{R}^m$ (smooth)

The *continuous–time* part is modeled as a *port–Hamiltonian system*:

Port–Hamiltonian system

$$\begin{cases} \dot{\mathbf{x}} = [\mathbf{J}(\mathbf{x}) - \mathbf{R}(\mathbf{x})]\,\partial\mathcal{H} + \mathbf{G}(\mathbf{x})\mathbf{u} \\ \mathbf{y} = \mathbf{G}^\top(\mathbf{x})\partial\mathcal{H} \end{cases}$$

Energy	$\mathcal{H}: \mathbb{R}^n \to \mathbb{R}$ (smooth)
Interconnection	$\mathbf{J} = -\mathbf{J}^\top$
Dissipation	$\mathbf{R} = \mathbf{R}^\top \succeq 0$
Power port	$\mathbf{G} \in \mathbb{R}^{m \times n}$

Port–Hamiltonian Model of the Flows

Generalized Inertia Matrix

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

System state

$$\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}, \quad \mathbf{p} = \mathbf{M}\dot{\mathbf{q}}, \quad \boxed{\mathbf{x} = (\mathbf{q}, \mathbf{p})}$$

Viscous Friction

$$\mathbf{B} = \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix}$$

State–Space Model

$$\boxed{\begin{cases} \dot{\mathbf{q}} = \mathbf{M}^{-1}\mathbf{p} \\ \dot{\mathbf{p}} = -\partial_{\mathbf{q}}\mathcal{V} - \mathbf{B}\mathbf{M}^{-1}\mathbf{p} + \mathbf{u} \end{cases}}$$

$\mathcal{V}(\mathbf{q}) = \gamma m_1 q_1 + \gamma m_2 q_2$ (gravitational potential)

γ (gravitational constant)

Port–Hamiltonian Model

$$\boxed{\begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & -\mathbf{B} \end{bmatrix} \begin{bmatrix} \partial_{\mathbf{q}}\mathcal{H} \\ \partial_{\mathbf{p}}\mathcal{H} \end{bmatrix} + \mathbf{G}u \\ y = \mathbf{G}^\top \partial\mathcal{H} = \dot{q}_1 \end{cases}}$$

$\mathcal{H} = \frac{1}{2}\mathbf{p}^\top\mathbf{M}^{-1}\mathbf{p} + \mathcal{V}(\mathbf{q})$ (energy)

$\mathbf{G} = [0, 0, 1, 0]^\top$ (input matrix)

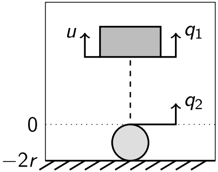
Ball-Ground Collision

Ball–ground collision causes a **sudden change** of the ball's momentum.

$$p_2^+ = -c_g p_2 \qquad c_g \in (0,1)$$

Jump Map:

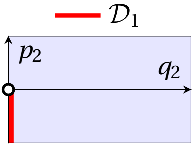
$$\mathbf{x}^+ = \mathbf{g}_1(\mathbf{x}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -c_g \end{bmatrix} \mathbf{x}$$



ball–ground collision
$\mathbf{x}^+ = \mathbf{g}_1(\mathbf{x})$ if $\mathbf{x} \in \mathcal{D}_1$

Jump Set:

$$\mathcal{D}_1 = \{\mathbf{x} : q_2 = 0 \wedge p_2 < 0\}$$

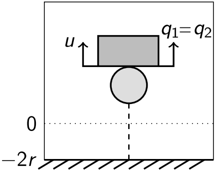


Ball-Robot Collision

In **ball–robot** collisions both the robot and ball momenta **change abruptly**.

$$\boxed{p_1^+ + p_2^+ = p_1 + p_2}$$ [conservation of momentum]

$$\boxed{\frac{p_1^+}{m_1} - \frac{p_2^+}{m_2} = c_i\left(\frac{p_1}{m_1} - \frac{p_2}{m_2}\right)}$$ [partial inelasticity $c_i \in (0,1)$]



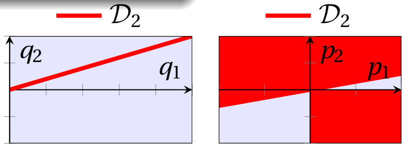
ball–robot collision
$\mathbf{x}^+ = \mathbf{g}_2(\mathbf{x})$ if $\mathbf{x} \in \mathcal{D}_2$

Jump Map:

$$\mathbf{x}^+ = \mathbf{g}_2(\mathbf{x}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1-\mu m_2 & \mu m_1 \\ 0 & 0 & \mu m_2 & 1-\mu m_1 \end{bmatrix}\mathbf{x}, \quad \mu = \frac{c_i + 1}{m_1 + m_2}$$

Jump Set:

$$\mathcal{D}_2 = \{\mathbf{x} : q_1 = q_2 \wedge (p_1 p_2 < 0 \vee m_2 p_1 < m_1 p_2)\}$$

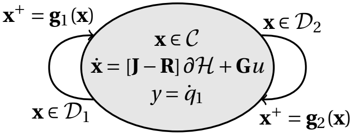


Overall *Hybrid* Model

Jump Set	**Jump Map**	**Admissible Inputs Space**
$\mathcal{D} = \mathcal{D}_1 \cup \mathcal{D}_2$	$\mathcal{G}(\mathbf{x}) = \{\mathbf{g}_1(\mathbf{x}),\ \mathbf{g}_2(\mathbf{x})\}$	$\mathcal{U} = \mathbb{R}$

Final Hybrid Model:

$$
\begin{cases}
\dot{\mathbf{x}} = [\mathbf{J} - \mathbf{R}]\partial\mathcal{H} + \mathbf{G}u & (\mathbf{x}, u) \in \mathcal{C} \times \mathcal{U} \\
\mathbf{x}^+ \in \mathcal{G}(\mathbf{x}) & \mathbf{x} \in \mathcal{D} \\
y = \dot{q}_1 &
\end{cases}
$$



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Ball–Dribbling Robot: uncontrolled and conservative

Trajectories with almost same initial conditions diverge with time

Ball–Dribbling Robot: sensitivity to initial condition

wave function (spatial distribution): $\boldsymbol{\Psi}(t=0) \triangleq \mathcal{N}(\mathbf{x}_0, \sigma\mathbb{I}) \xrightarrow{?} \boldsymbol{\Psi}(t)$

$\sigma = 10^{-6}$, q_1 : robot position, q_2 : ball position, p_1 : robot momentum, p_2 : ball momentum

After short time the state distribution is spread on the state–space

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Play/Pause

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Ball–Dribbling Robot: Control Problem

Control Task:

Continuously bounce the ball at the same height $q^*_{2,max}$.

Controller Structure

Two control states:

S_w **Wait state:** Wait for the ball reaching the peak of the bounce

$$\mathbf{x} \in \mathcal{D}_3 = \{\mathbf{x} : p_2 = 0\}$$

S_h **Hit state:** Move toward the ball and hit it

Control Law: energy–balancing passivity based control

We can only shape the energy of the robot: $\mathcal{H}_1(q_1, p_1) \to \mathcal{H}^*_1(q_1, p_1)$

Control law "u"

$$\boxed{u = \partial_{q_1}\mathcal{V} - k_p(q_1 - q^*_1) - k_d\dot{q}_1}$$

Control parameters "$\boldsymbol{\omega}$"

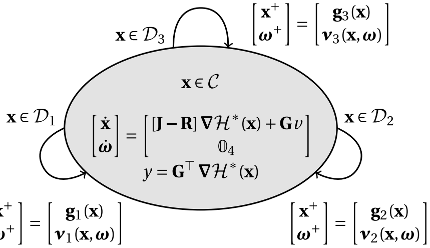
$$\begin{array}{llll} S_w & \to & \boldsymbol{\omega}^+ = \left(k_{p,w},\ k_{d,w},\ q_2 + \delta\right) & \mathbf{x} \in \mathcal{D}_2 \\ S_h & \to & \boldsymbol{\omega}^+ = \left(k_{p,h},\ k_{d,h},\ q_2\right) & \mathbf{x} \in \mathcal{D}_3 \end{array}$$

Ball–Dribbling Robot: Controlled System

"Shaped" Hamiltonian function: $\mathcal{H}^*(\mathbf{x},\boldsymbol{\omega}) = \frac{1}{2}\mathbf{p}^\top\mathbf{M}^{-1}\mathbf{p} + \frac{1}{2}k_p(q_1 - q_1^*)^2 + \gamma m_2 q_2$

Controlled Augmented System:

$$\begin{cases} \begin{bmatrix}\dot{\mathbf{x}}\\ \dot{\boldsymbol{\omega}}\end{bmatrix} = \begin{bmatrix}(\mathbf{J}-\mathbf{R})\,\partial\mathcal{H}^* + \mathbf{G}v\\ 0\end{bmatrix} & (\mathbf{x},\boldsymbol{\omega},v)\in\mathcal{C}\times\Omega\times\mathcal{U}\\ \begin{bmatrix}\mathbf{x}^+\\ \boldsymbol{\omega}^+\end{bmatrix} \in \mathcal{G}(\mathbf{x})\times\boldsymbol{\Lambda} & (\mathbf{x},\boldsymbol{\omega})\in\mathcal{D}\times\Omega\\ y = q_1 & \end{cases}$$



System behavior strongly depends on $k_{p,h}$.

No **priors** on the **correct gain** for the energy shaping $\Rightarrow$ **we can learn it!**

Iterative Learning Control Loop

1. Compute the *error* when the ball reaches the peak of the bounce:
$$e = q^*_{2,max} - q_{2,max}$$
2. Update the energy shaping gain as a function of the error:
$$\boxed{k_{p,h}(e,\xi) = k_0 + \alpha e(\xi) + \beta \sum_{i=1}^{\xi} e(i)}$$

(ξ: *bounce counter*)

Final Controller:

Energy Shaping + Iterative Learning = Iterative Energy Shaping

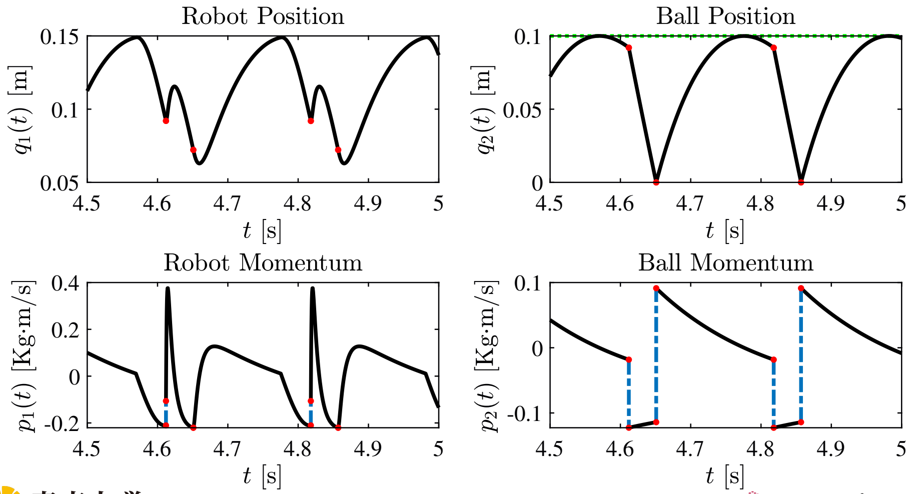
Ball–Dribbling Robot: Results

t : time, q_1 : robot position, q_2 : ball position

After a short transient, the system stabilized onto a periodic orbit.

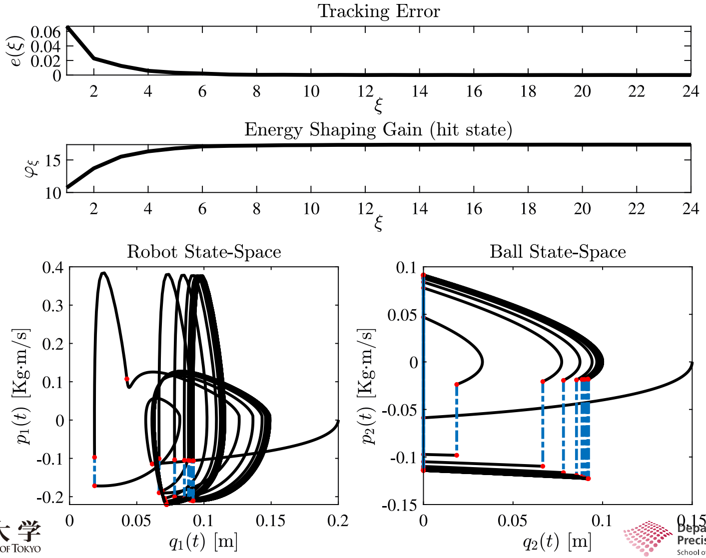
Detailed View at Steady–State

The system is on a periodic orbit where the error is zero.



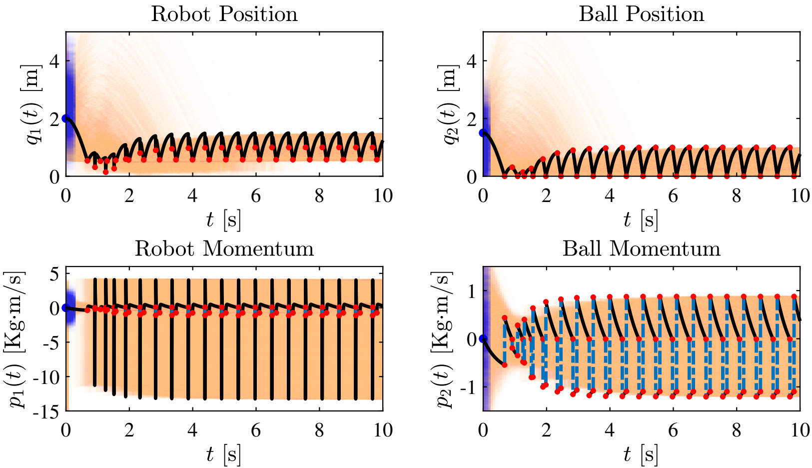
Iterative Learning Results

The gain and the error converge. The state approaches a limit cycle.



Robustness to Initial Condition

Regardless the initial condition, the system converges to the desired trajectory



blue dots: perturbed initial conditions, **orange:** trajectories traces, **black:** nominal trajectory

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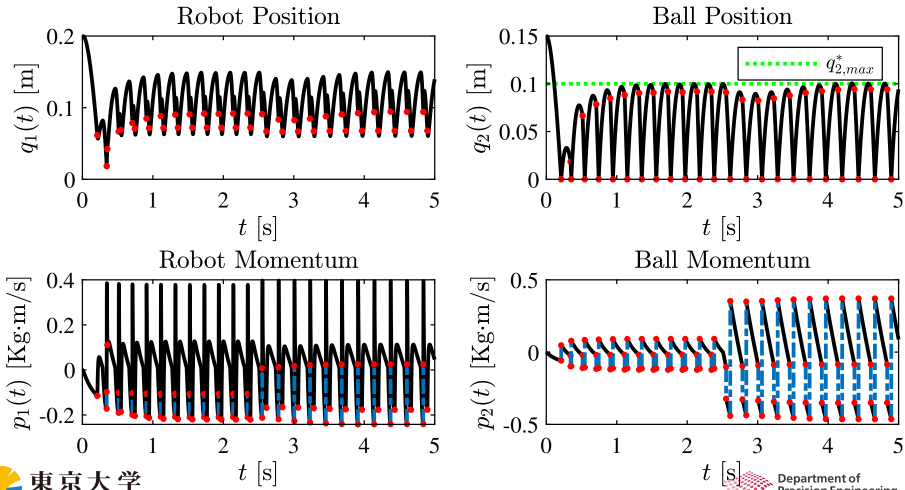
Robustness to Parameter Change

At $t = 2.5s$ the mass of the ball is changed from 0.05Kg to 0.25Kg. The system adapts to the change autonomously.



Conclusions and Future Work

Results:

- **Periodic regulation** of the ball–dribbling system has been achieved;
- The "**iterative energy shaping**" control was proposed to solve the problem;
- The novel approach **relaxes** most **assumptions** on the physical model;

Future Work:

- **Formal proof** of set **stability**;
- Additional **formalization** of energy–based modeling of hybrid systems;
- Generalization of iterative energy shaping to **other robotic systems**;
- **Experiments** with a real system.

Thank you for your attention

Any questions?

Comparisons with Previous Methods

	Proposed	Batz et al. 2010	Haddadin et al. 2018
friction	✓X	X	
interaction type	part. elastic	inelastic	inelastic
account for discontinuities	✓X	✓	
feedback	robot/ball pos.	robot/ball pos.	robot pos.
no parameters	✓	X	X
robustness test	✓X	X	
stability proof	numerical	formal	numerical/formal
real experiment	X ✓	✓	