STA 674

Regression Analysis And Design Of Experiments

Fitting Simple Linear Regression Models – Lecture 9

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Fitting Simple Linear Regression Models

- Last time: we shook the cobwebs loose on significance (or hypothesis) testing by looking at the major concepts and doing an example for the population mean
- This time, we'll give the setup for the population slope in the linear regression setting (and talk about how you *could* do it for the intercept, but it's not often done.)

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Significance test for the population slope (Testing that $\beta_1 \neq \beta_1^*$)

- **Hypotheses**: $H_0 : \beta_1 = \beta_1^*$ vs. $H_a : \beta_1 \neq \beta_1^*$
- **Test Statistic**: $t = (b_1 - \beta_1^*)/s_{b_1}$
- **Critical Values**: $\pm t_{\alpha/2, n-2}$
- **Rule**: Reject H_0 and conclude that $\beta_1 \neq \beta_1^*$ at the α level of significance if $t < -t_{\alpha/2,n-2}$ or $t > t_{\alpha/2,n-2}$. Otherwise, we fail to reject H_0.
- ***P*-value**: $2 \times P(|t| > t_{n-2})$=Probability that t_{n-2} is "farther away from 0" than TS t.

Beta 1 = parameter of interest
H0 = null hypothesis = Beta1=0
Ha = alternative hypothesis = beta 1 does not equal 0; two tail test
n=60
df = 58 (we're estimating two parameters - slope and intercept...so it's n-2); also because df is so high...it's close to Z distribution

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t = (15.1-0) / 6.4 = 2.35
critical value = +/- 2.002 (from t-distribution with df=58)
t falls within rejection region (beyond critical value)...so reject null hypothesis
P value = 0.022 (plotting t on t-distribution...doubling because two tail test...

Test statistic and P suggest that mortality does change with nitrous oxide potential

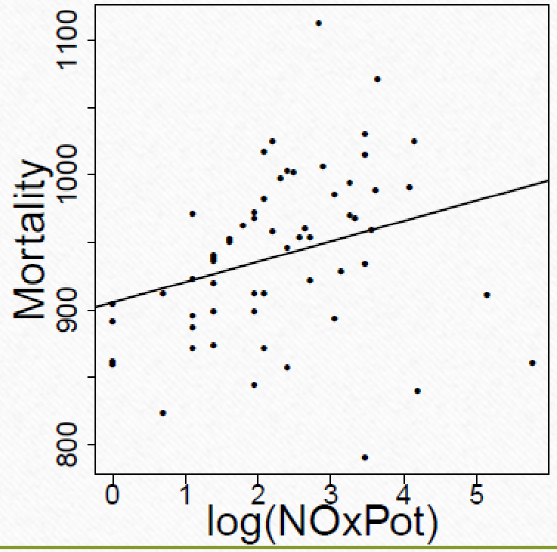
Example: Significance test for the population slope: air pollution and mortality

The following plot depicts mortality rates in 60 US cities versus the nitrous oxide potential (NOxPot). Is there evidence that mortality changes with differing NOxPot? Use $\alpha = 0.05$.

$n = 60$ $\quad b_0 = 905.6$ $\quad s_{b_0} = 16.7$

$s_e = 59.96$ $\quad b_1 = 15.1$ $\quad s_{b_1} = 6.4$

Recall: $b_1 \sim \text{Normal}\left(\beta_1, \frac{s_e^2}{(n-1)s_x^2}\right)$*



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Example: Significance test for the population slope: air pollution and mortality

SAS code and output:

```
PROC REG DATA=NOXPOT;
MODEL mortality = lognoxpot / CLB;
RUN;
```

Parameter Estimates							
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	95% Confidence Limits	
Intercept	1	905.61317	16.67218	54.32	<.0001	872.24018	938.98616
lognoxpot	1	15.09896	6.41871	2.35	0.0221	2.25052	27.94741

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Tests for $\boldsymbol{\beta_0}$:

- Tests can also be conducted for the intercept—but this is usually of less interest.
- To conduct a test for the intercept you simply need to replace:
  - β_1 with β_0,
  - b_1 with b_0 ,
  - β_1^* withβ_0^*, and
  - s_{b_1} with s_{b_0}
- in the formulas on the previous slides.