Chapter 5

5.1: Hypothesis Tests Using Normal Distributions

Previously we have looked at 3 different distributions:

- Sampling Distribution: Helped us visualize the behavior of statistics, where each statistic was calculated from a random sample taken from the population.
- Bootstrap Distribution: Helped us estimate the S.E. of the statistic by plotting bootstrap statistics.
- Randomization Distribution: Helped us calculate a p-value, by helping us visualize what statistics look like when the null hypothesis is true.

What did all 3 distributions have in common?

Symmetric and Bell Shaped

In this section, we are going to look at distributions that are symmetric and bell shaped, ie. Normal.

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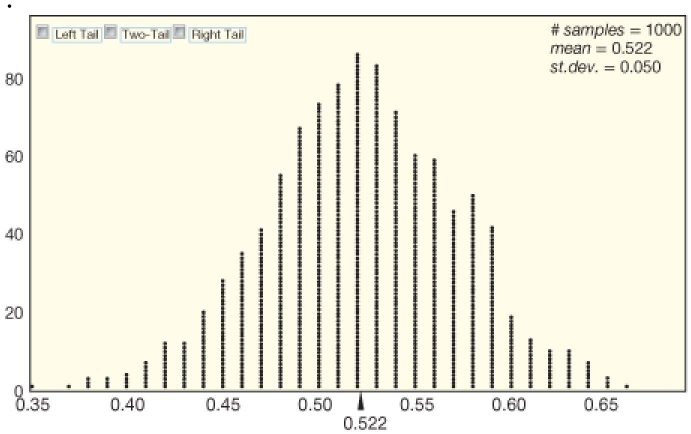
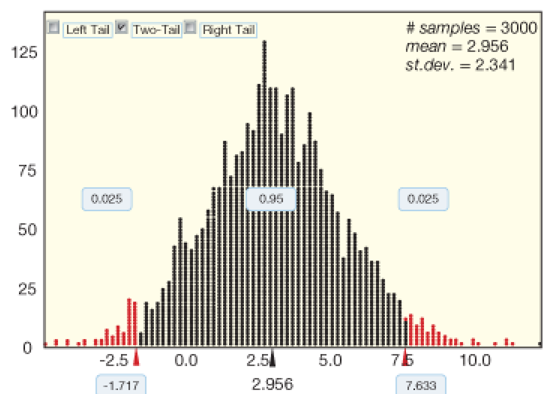
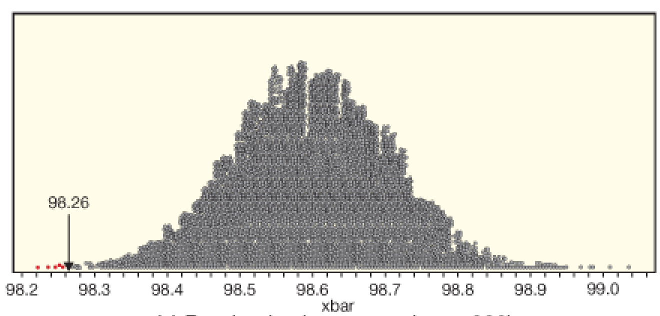


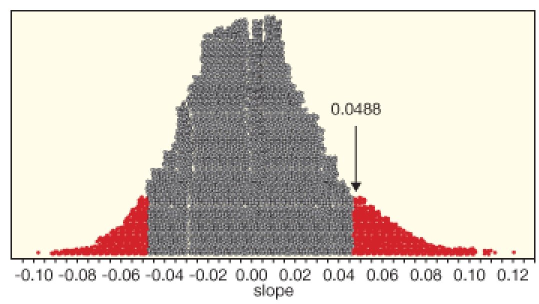
(a) Bootstrap proportions (page 233)



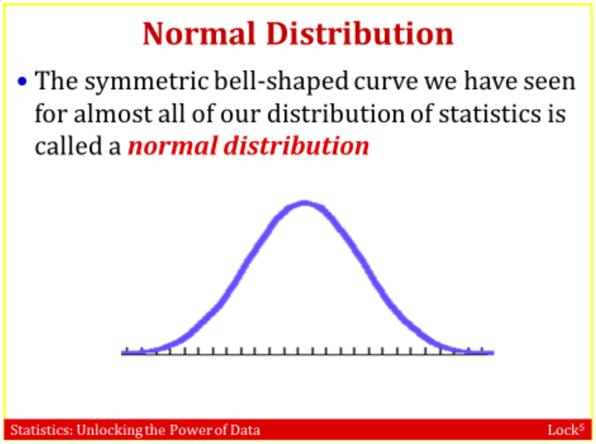
(b) Bootstrap differences in means (page 245)

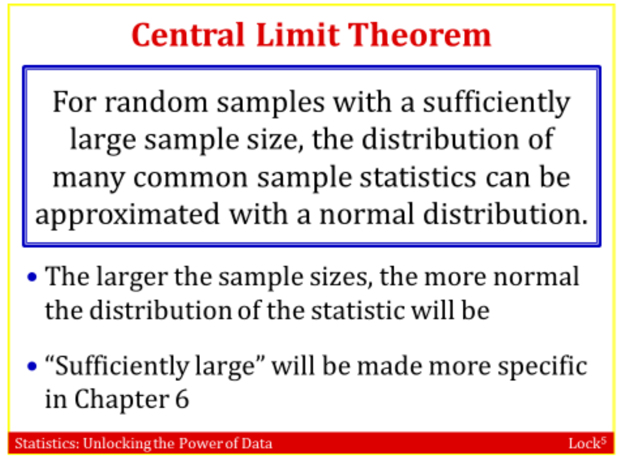


(c) Randomization means (page 330)



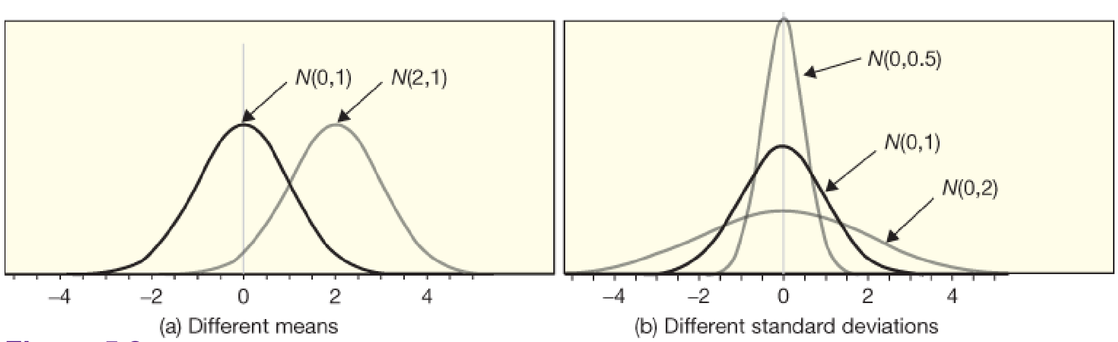
(d) Randomization slopes (page 347)





- **The normal distribution** is the Probability Distribution of a Continuous random variable that has a symmetric, bell-shaped and characterized by its mean μ and standard deviation σ.
- For shorthand we often use the notation $N(\mu, \sigma)$ to specify that a distribution is normal (N) with some mean (μ) and standard deviation (σ).

Figure 5.2 shows how the normal distribution changes as the mean μ is shifted to move the curve horizontally or the standard deviation σ is changed to stretch or shrink the curve.



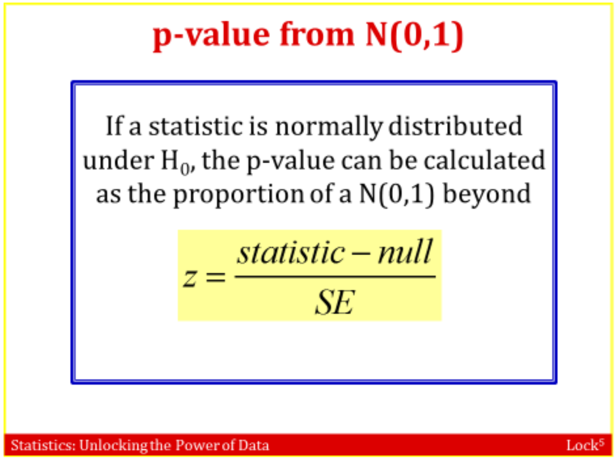
- **Figure 5.2** ***Comparing normal curves***

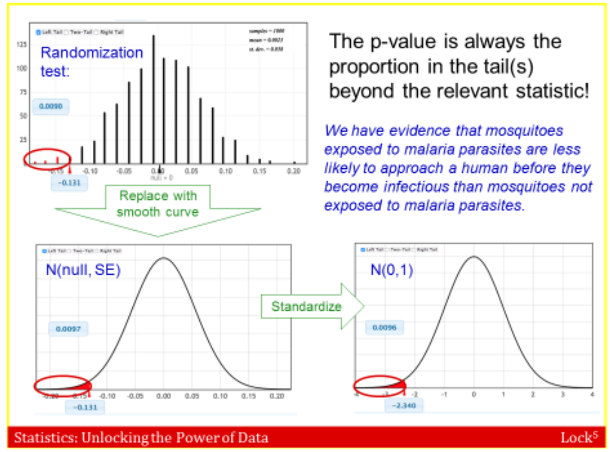
The Standard Normal Distribution

Because all the normal distributions look the same except for the horizontal scale, another common way to use normal distributions is to convert everything to one specific ***standard normal*** scale. The standard normal, $N(0, 1)$, has a mean of 0 and a standard deviation of 1. We often use the letter Z to denote a standard normal distribution.

To convert from an X value on a $N(\mu, \sigma)$ scale to a Z value on a $N(0, 1)$ scale, we standardize values with the z-score:

$$z = \frac{x - mean}{sd} = \frac{x - \mu}{\sigma}$$





P-value from a Standard Normal Distribution

The p-value for the test is the proportion of a standard normal distribution beyond this standardized test statistic, depending on the direction of the alternative hypothesis.

How to find probabilities using the TI-83(given Z value, find Probability or area under the standard normal curve)

1. Press **2nd VARS**.
2. Select **normalcdf(**.
3. Complete the entry to obtain **normalcdf(*lower bound, upper bound*)**, substituting the appropriate values in.

Example 1: Find the specified areas for a standard normal distribution, and sketch the area.

(a) The area to the left of $z = -0.8$

0.212

(b) The area to the right of $z = 1.2$

0.115

Quick Self-Quiz: Standard Normal Distribution

Find the specified areas for a standard normal distribution, and sketch the area.

(a) The area to the right of $z = 2.58$

0.0049 (more than two and half standard deviations above the mean so it is not surprising that it is so small.)

(b) The area to the left of $z = -1.32$

0.093

Example 2: Multiple Ways to Find a P-value

Suppose we want to test H_0: p = 0.5 vs H_a: p > 0.5 using a sample proportion of 520 out of 1000, or 0.52.

- Use a randomization distribution to find the p-value:

  p-value is 0.102 (Answers will vary slightly.)

- What is the standard error from the randomization distribution? ____*SE = 0.016* ______
- Model the randomization distribution with a normal distribution with mean 0.5 from the null hypothesis and standard deviation equal to this standard error. Find the p-value by finding the area to the right of 0.52 in this normal distribution.

  p-value is 0.106
- Find the standardized test statistic. Then find the p-value by finding the area beyond this standardized value in a standard normal distribution.

$$z \text{ test statistic} = \frac{\text{Sample statistic} - \text{Null parameter}}{SE} = \frac{0.52 - 0.5}{0.016} = 1.25$$

  This is a right-tail test and the p-value is 0.106.
- Compare the three p-values!

  They will be almost the same (up to round-off error.)

Example 4: Is Divorce Morally Acceptable?

In a study conducted by the Pew Foundation, we learn that 67% of women in a random sample view divorce as morally acceptable. Does this provide evidence that more than 60% of women view divorce as morally acceptable? The standard error for the estimate assuming the null hypothesis is true is 0.021.

(a) What are the null and alternative hypotheses for this test?

H_0: $p = 0.6$

H_a: $p > 0.6$

(b) What is the standardized test statistic?

$$\frac{\text{Sample statistic} - \text{Null parameter}}{SE} = \frac{0.67 - 0.6}{0.021} = 3.333$$

(c) Use the standard normal distribution to find the p-value.

This is a right tail test.

We see that the p-value is 0.00043

(d) What is the conclusion of the test?

The p-value is very small so we have strong evidence that more than 60% of all women view divorce as morally acceptable.

Example 5: Do Men and Women Differ in Opinions about Divorce?

In the same study described above, we find that 71% of men view divorce as morally acceptable. Use this and the information in the previous example to test whether there is a significant difference between men and women in how they view divorce. The standard error for the difference in proportions under the null hypothesis that the proportions are equal is 0.029.

(a) What are the null and alternative hypotheses for this test?

H_0: $p_M = p_F$

H_a: $p_M \neq p_F$

(b) What is the standardized test statistic?

$$\frac{\text{Sample statistic} - \text{Null parameter}}{SE} = \frac{(0.71-0.67) - 0}{0.029} = 1.379$$

(c) Use the standard normal distribution to find the p-value.

This is a two-tail test.

We see that the p-value is 2(0.084) = 0.168

(d) What is the conclusion of the test?

The p-value is larger than any reasonable significance level, so we do not find evidence of a difference between men and women in the proportion that view divorce as morally acceptable.

5.2: Confidence Intervals Using Normal Distributions

Bootstrap Distributions

A bootstrap distribution is

- centered at the sample statistic
- has standard deviation equal to the standard error of the statistic

If a bootstrap distribution is normally distributed, we can write it as

N(sample statistic, SE)

(Un)-standardization

- Standardized scale: $z = \dfrac{x - mean}{sd}$
- To un-standardize: $z = \dfrac{x - mean}{sd}$

$$z \cdot sd = x - mean$$

$$x = mean + z \cdot sd$$

(Un)-standardization

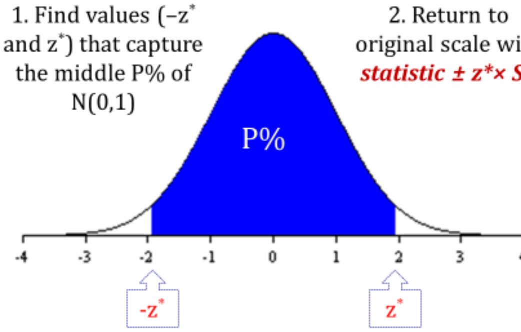
- In testing, we go to a standardized statistic
- In intervals, we find (-z*, z*) for a standardized distribution, and return to the original scale
- Un-standardization (reverse of z-scores):

$$x = \overset{\boxed{\text{statistic}}}{mean} + \overset{\boxed{\pm z^*}}{z} \cdot \overset{\boxed{\text{SE}}}{sd}$$

- What's the equivalent for the distribution of the statistic? (bootstrap distribution)

P% Confidence Interval

1. Find values ($-z^*$ and z^*) that capture the middle P% of N(0,1)



2. Return to original scale with ***statistic ± z*× SE***

Confidence Interval using N(0,1)

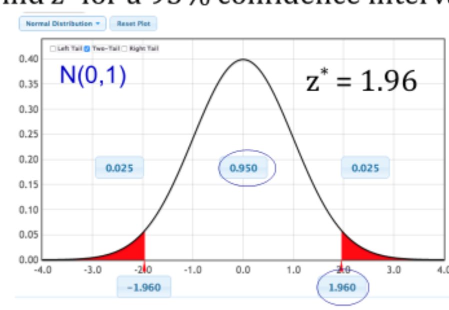
If a statistic is normally distributed, we find a confidence interval for the parameter using

statistic ± z*× SE

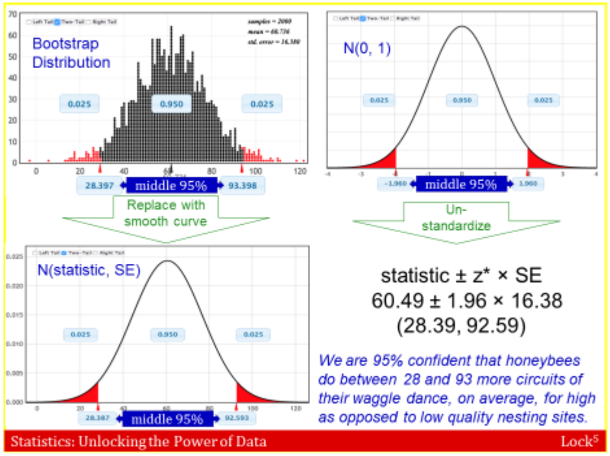
where the proportion between $-z^*$ and $+z^*$ in the standard normal distribution is the desired level of confidence.

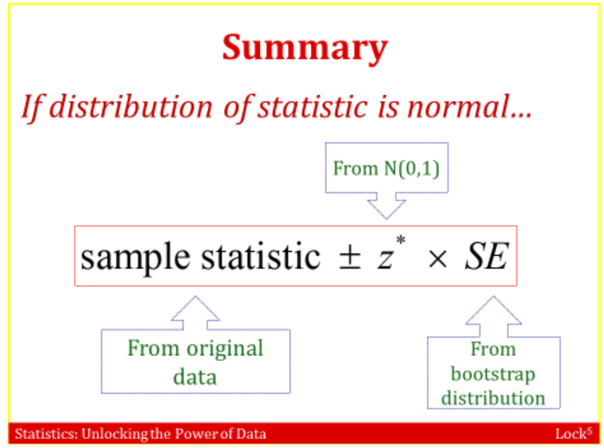
Confidence Intervals

Find z^* for a 95% confidence interval.



(rounds to 2, as we've been using)





- **Why use the standard normal?**
- **z^* is always the same, regardless of the data!**
- **Common confidence levels:**
    - **95%: $z^* = 1.96$ (but 2 is close enough)**
    - **90%: $z^* = 1.645$**
    - **99%: $z^* = 2.575$**
    - **98%: $z^* = 2.33$**

Example 1: Find endpoints on a standard normal distribution with the given property, and sketch the area.

(a) The area between $\pm z$ is 0.95.

-1.96 and 1.96

(b) The area between $\pm z$ is 0.80.

-1.28 and 1.28

Quick Self-Quiz: Standard Normal Distribution

Find endpoints on a standard normal distribution with the given property, and sketch the area.

The area between $\pm Z$ is 0.90

-1.645 and 1.645

Example 2: Confidence Intervals Three Ways

A survey conducted by the Pew Foundation found that 536 of 908 Twitter users say they use Twitter for get news. We wish to find a 95% confidence interval for the proportion of Twitter users who use it to get news.

What is the sample statistic? 536/908 = 0.590

Use percentiles on a bootstrap distribution to find the 95% confidence interval.

0.559 to 0.621 (Answers may vary slightly)

We can model the bootstrap distribution with a normal distribution with mean equal to the sample statistic and standard deviation equal to the standard error of the bootstrap distribution. Give the mean __0.590_ and standard deviation _0.016__ for this normal distribution. Use percentiles on this normal distribution to find the 95% confidence interval.

0.559 to 0.621

What is z^* for a 95% confidence interval? 1.96

Use the formula "Statistic ± z^* · SE" to find the 95% confidence interval.

$$\textit{Sample statistic} \pm z^* \cdot SE$$
$$0.590 \pm 1.960\,(0.016)$$
$$0.590 \pm 0.031$$
$$0.559 \quad \text{to} \quad 0.621$$

Compare the three answers. *They are the same (up to minor variations in the simulations).*

Example 3: Obesity in America

In Chapter 3, we see that the mean BMI (Body Mass Index) for a large sample of US adults is 27.655. We are told that the standard error for this estimate is 0.009. If we use the normal distribution to find a 99% confidence interval for the mean BMI of US adults:

(c) What is z^*? *2.575*

(d) Find and interpret the 99% confidence interval.

$$\textit{Sample statistic} \pm z^* \cdot SE$$
$$27.655 \pm 2.575(0.009)$$
$$27.632 \quad \text{to} \quad 27.678$$

We are 99% confident that the mean BMI of all US adults is between 27.632 and 27.678.

Example 4: Obesity in America: Exercises vs Non-exercisers

Also in Chapter 3, we see that the difference in mean BMI between non-exercisers (those who said they had not exercised at all in the last 30 days) and exercisers (who said they had exercised at least once in the last 30 days) is $\bar{x}_N - \bar{x}_E = 1.915$, with a standard error for the estimate of SE = 0.016. If we use the normal distribution to find a 90% confidence interval for the difference in mean BMI between the two groups:

(a) What is z^*? *1.645*

(b) Find and interpret the 90% confidence interval.

$$\textit{Sample statistic} \pm z^* \cdot SE$$
$$1.915 \pm 1.645(0.016)$$
$$1.889 \quad \text{to} \quad 1.941$$

We are 90% confident that the mean BMI of non-exercisers is between 1.889 and 1.941 higher than the mean BMI of exercisers, among all US adults.