Continuous data – test for equality of variances

Two Groups of Interest

Two-group t-test for paired and independent samples are for means!

Paired samples

- Each observation in the first group has a corresponding observation in the second group
- Oftentimes the two observations are within a single subject
- Examples:
  - Test scores before vs. after a course
  - Balance on leg with torn ACL vs. leg without torn ACL

Independent samples

- Observations in the two groups are not related to each other
- Cannot match up or pair the observations together in any way
- Examples:
  - LDL cholesterol level in the United States vs. Australia
  - Life expectancy in men vs. women

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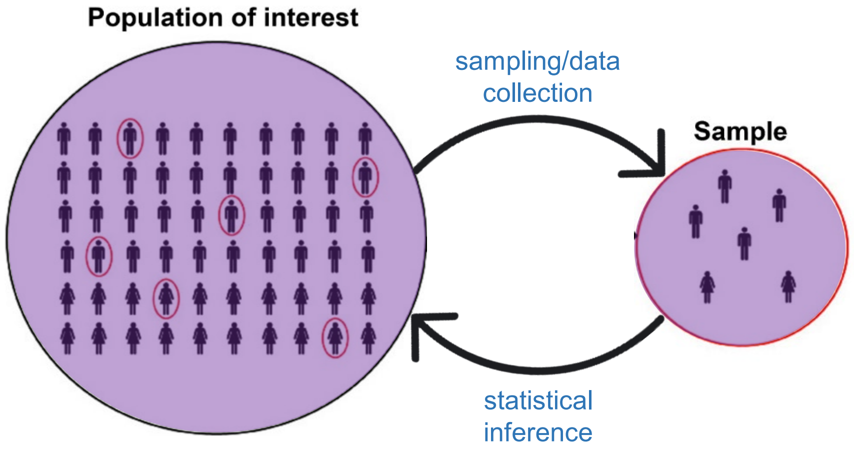
Equal vs. Unequal Variances

- For the two-group t-test, you have to specify if the two groups have equal variance or not
  - If equal: An estimate of the pooled standard deviation is used for the test
  - If unequal: The standard deviations of the two groups are used separately
- We can perform a separate hypothesis test *for the variances* to decide if the two population variances are equal

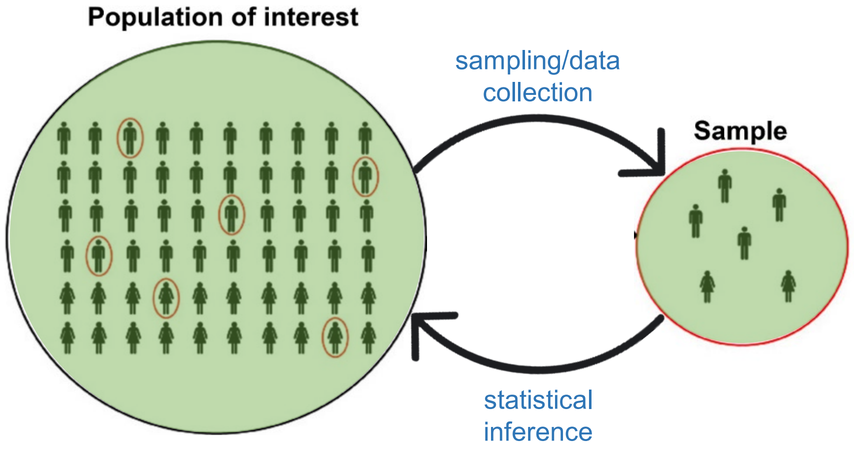
Two Independent Groups: Notation

- μ_1 = population mean in group 1
- σ_1 = population SD in group 1



- $\bar{x}_1$ = sample mean in group 1
- s_1 = sample SD in group 1

- μ_2 = population mean in group 2
- σ_2 = population SD in group 2



- $\bar{x}_2$ = sample mean in group 2
- s_2 = sample SD in group 2

Two Variances F-Test

- To test whether the variances in two groups are equal, we use the two variances F-test
- Test machinery uses the ratio of the two population variances
  - What value of $\frac{\sigma_1^2}{\sigma_2^2}$ would indicate that the two variances are equal?

Two Variances F-Test

Another way to write these...H0: sigma^2 sub1 = sigma^2 sub2 & HA: sigma^2 sub1 != sigma^2 sub2

- Hypotheses:

$$H_0: \frac{\sigma_1^2}{\sigma_2^2} = 1 \qquad\qquad H_A: \frac{\sigma_1^2}{\sigma_2^2} \neq 1$$

- Use p-value the same way we always have
  - p-value ≤ α → Reject H_0
  - p-value > α → Fail to reject H_0
- Test decision tell us which form of the two-group t-test (for means) to use
  - Reject H_0 → Assume unequal variances in two-group t-test
  - Fail to reject H_0 → Assume equal variances in the two-group t-test

Low Birth Weight Data

- Information on 100 low birth weight infants born in two teaching hospitals in Boston, Massachusetts

Variable	Description
sex	Sex of the baby (Male, Female)
gestage	Gestational age at time of birth (weeks)
length	Length of the baby (cm)
birthwt	Birth weight of the baby (g)
headcirc	Baby’s head circumference (cm)
apgar	Apgar score (integers, min=0, max=10). This is a scoring system used for assessing the clinical status of a newborn. 7 or higher is generally considered normal, 4-6 is low, and 3 or below is critically low.

Find the dataset (lowbwt.xlsx) and the full data dictionary (lowbwt Data Dictionary.pdf) in the Data Module on the Canvas site

Example: Birth Weight/Apgar Score

- Perform a hypothesis test to test whether the average birth weight among babies with a low Apgar score is different from the average birth weight among babies with a normal Apgar score. Use a significance level of 0.05.

mu1 = mean birthweight in low apgar score babies

mu2 = mean bw in normal apgar score babies

H0: mu1 = mu2

HA: mu1 != mu2

	mean	sd
Low	1015.625	292.8949
Normal	1154.333	240.3836

Two independent groups...so have to test for equal variance first with F test
sigma^2 sub1 = variance of birthweight in low apgar babies
sigma^2 sub2 = variance of birthweight in normal apgar babies

Example: Birth Weight/Apgar Score

$$H_0: \frac{\sigma_1^2}{\sigma_2^2} = 1 \qquad H_A: \frac{\sigma_1^2}{\sigma_2^2} \neq 1 \qquad \alpha = 0.05$$

Two variances F-test comparing variance of birth weight between Apgar groups:

```
        F test to compare two variances

data:  birthwt by apgarlow
F = 1.4846, num df = 39, denom df = 59, p-value = 0.1673
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
 0.8463593 2.6957564
sample estimates:
ratio of variances
          1.484615
```

$\text{p-value} = 0.167$

Since the p-value is greater than 0.05, we fail to reject H_0 and conclude that there is not sufficient evidence to say that the variance of birth weight in babies with a normal Apgar score is different from the variance of birth weight in babies with a low Apgar score. Thus, we will perform the two-group t-test assuming equal variances.

Example: Birth Weight/Apgar Score

- Perform a hypothesis test to test whether the average birth weight among babies with a low Apgar score is different from the average birth weight among babies with a normal Apgar score. Use a significance level of 0.05.

Let μ_1 be the mean birth weight among all infants with a low Apgar score
Let μ_2 be the mean birth weight among all infants with a normal Apgar score

$H_0\colon \mu_1 = \mu_2$

$H_A\colon \mu_1 \neq \mu_2$

When performing the two-group t-test, we'll specify that we want to assume <u>equal variances</u>

Example: Birth Weight/Apgar Score

H_0: $\mu_1 = \mu_2$ $\qquad$ H_A: $\mu_1 \neq \mu_2$ $\qquad$ $\alpha = 0.05$

Two-group t-test for difference in average birth weight between Apgar groups:

```
        Two Sample t-test

data:  birthwt by apgarlow
t = -2.5883, df = 98, p-value = 0.01111
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -245.05832  -32.35835
sample estimates:
   mean in group Low mean in group Normal
            1015.625             1154.333
```

$\bar{x}_1 = 1015.6 \quad \bar{x}_2 = 1154.3$

p-value $= 0.011$

Since the p-value is less than 0.05, we reject H_0 and conclude that there is sufficient evidence to say that the average birth weight among babies with a normal Apgar score is different from the average birth weight among babies with a low Apgar score.

Example: Length/Apgar Score

- Perform a hypothesis test to test whether the average length of babies with a low Apgar score is different from the average length of babies with a normal Apgar score. Use a significance level of 0.05.

	mean	sd
Low	35.75000	4.055702
Normal	37.53333	3.039105

Example: Length/Apgar Score

$H_0: \dfrac{\sigma_1^2}{\sigma_2^2} = 1$ $\quad H_A: \dfrac{\sigma_1^2}{\sigma_2^2} \neq 1$ $\quad \alpha = 0.05$

Two variances F-test comparing variance of length between Apgar groups:

```
        F test to compare two variances

data:  length by apgarlow
F = 1.7809, num df = 39, denom df = 59, p-value = 0.04416
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
 1.015270 3.233758
sample estimates:
ratio of variances
          1.780905
```

p-value $= 0.044$

Since the p-value is less than 0.05, we reject H_0 and conclude that there is sufficient evidence to say that the variance of length in babies with a normal Apgar score is different from the variance in length among babies with a low Apgar score. Thus, we will perform the two-group t-test assuming unequal variances.

Example: Length/Apgar Score

- Perform a hypothesis test to test whether the average length of babies with a low Apgar score is different from the average length of babies with a normal Apgar score. Use a significance level of 0.05.

Let μ_1 be the mean length among all infants with a low Apgar score
Let μ_2 be the mean length among all infants with a normal Apgar score

$$H_0\colon \mu_1 = \mu_2$$

$$H_A\colon \mu_1 \neq \mu_2$$

When performing the two-group t-test, we'll specify that we want to assume <u>unequal variances</u>

Example: Length/Apgar Score

$H_0\colon \mu_1 = \mu_2$ $\quad H_A\colon \mu_1 \neq \mu_2$ $\quad \alpha = 0.05$

Two-group t-test for difference in average length between Apgar groups:

```
        Welch Two Sample t-test

data:  length by apgarlow
t = -2.3722, df = 67.419, p-value = 0.02054
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -3.2836956 -0.2829711
sample estimates:
   mean in group Low mean in group Normal
            35.75000             37.53333
```

$\bar{x}_1 = 35.75$ $\quad \bar{x}_2 = 37.53$

p-value $= 0.021$

Since the p-value is less than 0.05, we reject H_0 and conclude that there is sufficient evidence to say that the average length of babies with a normal Apgar score is different from the average length of babies with a low Apgar score.

Important Points

- When performing a two-group t-test for means, you have to assume equal or unequal variances in the test
- Two variances F-test is used to test whether the variances in two groups are equal
- Set up and interpretation of two variances F-test
- Using results of two variances F-test to make the appropriate variance assumption in the two-group t-test for means