STA 674

Regression Analysis And Design Of Experiments

Treatment Comparisons – Lecture 1

STA 674, RA Design Of Experiments:
Treatment Comparisons

- Where does it fit in?
- What is it?
- What's next?

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Introduction

The overall F-test asks the question: F-test has to be significant before move onto other questions...if not significant then that's it

Is there any difference between the treatment means?

But there are lots of other questions we could ask:

1. Which pairs of treatments have different mean responses?
2. Which treatment or treatments produces the highest mean response?
3. Which treatments have mean response that are lower than the control?
4. Is the mean response from one treatment higher than the mean from all other treatments?
5. …

To answer these questions we need to write out the hypotheses in terms of the treatment means.

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Example exercise: tomato growth (from Kuehl)

An experiment was designed to test the effects of different types of sugars on the growth of tomato plants. Tissue culture from tomato plants were grown in a completely randomized design with 5 replicates of each of 4 treatments: control (no sugar), 3% glucose solution, 3% fructose solution, and 3% sucrose solution. Let μ_1, μ_2, μ_3 and μ_4 denote the mean growth from the control, glucose, fructose, and sucrose solutions respectively.

Example exercise: tomato growth

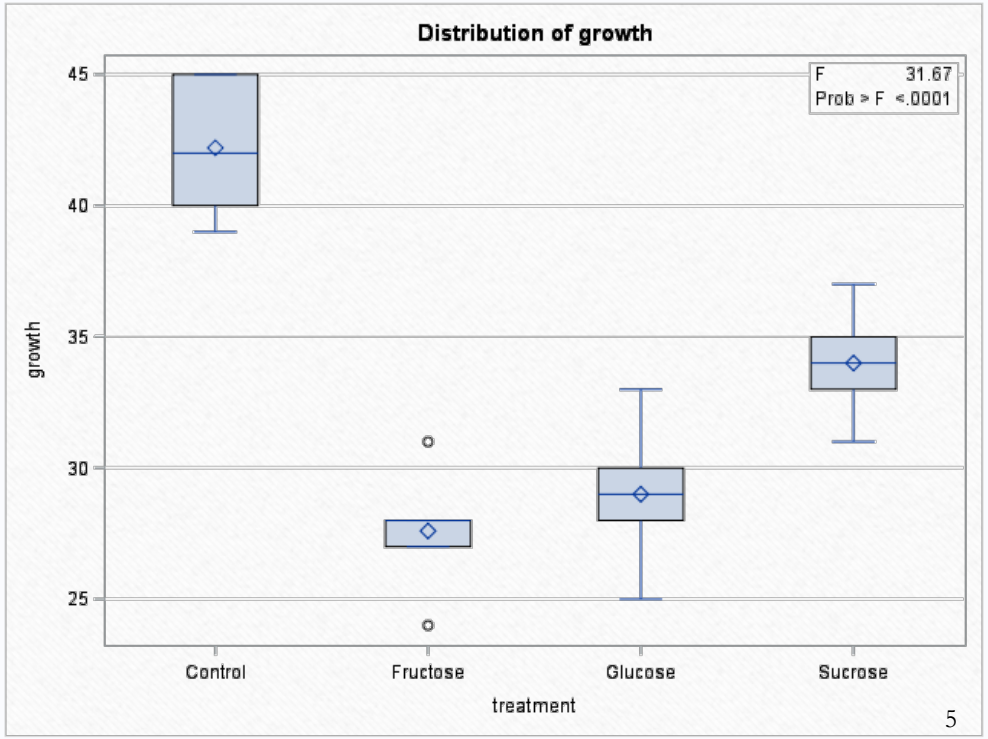
```
/* Initial model with no adjustment */;
PROC GLM DATA=TOMATO;
     CLASS treatment;
     MODEL growth=treatment;
     LSMEANS treatment / CL;
RUN;
```

F-test is significant

The GLM Procedure

Dependent Variable: growth

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	653.2000000	217.7333333	31.67	<.0001
Error	16	110.0000000	6.8750000		
Corrected Total	19	763.2000000			



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Example exercise: tomato growth

Let μ_1, μ_2, μ_3 and μ_4 denote the mean growth from the control, glucose, fructose, and sucrose solutions respectively.

Match each pair of hypotheses in the first list with the appropriate research question in the second list.

A. $H_0: \mu_1 = \mu_2$ vs. $H_a: \mu_1 \neq \mu_2$

B. $H_0: \mu_3 \leq \mu_1$ vs. $H_a: \mu_3 > \mu_1$

C. $H_0: \mu_1 \geq \frac{\mu_2}{3} + \frac{\mu_3}{3} + \frac{\mu_4}{3}$ vs. $H_a: \mu_1 < \frac{\mu_2}{3} + \frac{\mu_3}{3} + \frac{\mu_4}{3}$

D. $H_0: \mu_2 \geq \frac{\mu_3}{2} + \frac{\mu_4}{2}$ vs. $H_a: \mu_2 < \frac{\mu_3}{2} + \frac{\mu_4}{2}$

1. Is the mean growth from the glucose solution different than the mean growth from the control solution?

2. Is the combined mean growth from the glucose, fructose, and sucrose solutions higher than the mean growth from the control solution?

3. Is the mean growth from the fructose solution higher than the mean growth from control solution?

4. Is the combined mean growth from the fructose and sucrose solutions higher than the mean growth from the glucose solution alone?

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Example exercise: tomato growth

Consider the four hypotheses for the tomato experiment:

A. $H_0: \mu_1 = \mu_2$	**A.** $H_a: \mu_1 - \mu_2 \neq 0$
B. $H_0: \mu_3 \leq \mu_1$	**B.** $H_a: \mu_3 - \mu_1 > 0$
C. $H_0: \mu_1 \geq \frac{\mu_2}{3} + \frac{\mu_3}{3} + \frac{\mu_4}{3}$	**C.** $H_a: \mu_1 - \frac{\mu_2}{3} - \frac{\mu_3}{3} - \frac{\mu_4}{3} < 0$
D. $H_0: \mu_2 \geq \frac{\mu_3}{2} + \frac{\mu_4}{2}$	**D.** $H_a: \mu_2 - \frac{\mu_3}{2} - \frac{\mu_4}{2} < 0$

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Contrasts

- A contrast is a weighted sum of the treatment means for which the sum of the weights is 0. Contrasts are used to express hypotheses about the treatment means that we wish to test in the experiment.

Mathematically:

- A contrast for an experiment with t treatments is a sum:

$$C = k_1\mu_1 + \cdots + k_t\mu_t$$

such that

$$k_1 + \cdots + k_t = 0.$$

The values $k_1, \ldots, k_t$ are the weights and determine the meaning of the contrast.

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Example exercise: tomato growth

Which of the following represent valid contrasts for the tomato experiment with 4 treatments. For each valid contrast, write down the meaning of the test with hypotheses:

$$H_0: C = 0 \text{ versus } H_a: C \neq 0$$

A. $C = 1\mu_1 + 1\mu_2 + 0\mu_3 - 1\mu_4$ sum does not equal 0

B. $C = 1\mu_1 + 1\mu_2 - 1\mu_3 - 1\mu_4$ control and glucose vs fructose and sucrose

C. $C = 3\mu_1 - 1\mu_2 - 1\mu_3 - 1\mu_4$ control vs glucose, fructose, and sucrose

D. $C = 1\mu_1 - 1\mu_2 + 1\mu_3 - 1\mu_4$ control and fructose vs glucose and sucrose

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Note:

- Contrasts are not unique – the same hypothesis can be represented by many different contrasts.
- Two contrasts represent the same hypotheses if the weights in one contrast are a constant multiple of the weights in the other contrast.

Example:

- Consider the contrasts:

$$C_1 = 1\mu_1 - 0.5\mu_2 - 0.5\mu_3 + 0\mu_4$$

and

$$C_2 = 10\mu_1 - 5\mu_2 - 5\mu_3 + 0\mu_4$$

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Example exercise: tomato growth

Which of the following contrasts compares the mean growth of the control (μ_1) with the mean growth of the three sugar solutions:

A. $C_1 = \mu_1 - \frac{\mu_2}{3} + \frac{\mu_3}{3} + \frac{\mu_4}{3}$

B. $C_2 = 3\mu_1 - \mu_2 - \mu_3 - \mu_4$

C. $C_3 = -6\mu_1 + 2\mu_2 + 2\mu_3 + 2\mu_4$

D. All of these contrasts are equivalent.