STA 674

Regression Analysis And Design Of Experiments

Fitting Simple Linear Regression Models – Lecture 5

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- Last time: we flirted with inference (about the LR parameters)
- Now we'll use the sampling distribution information to differentiate between the two facets of the estimators' properties:
  - Accuracy
  - Precision

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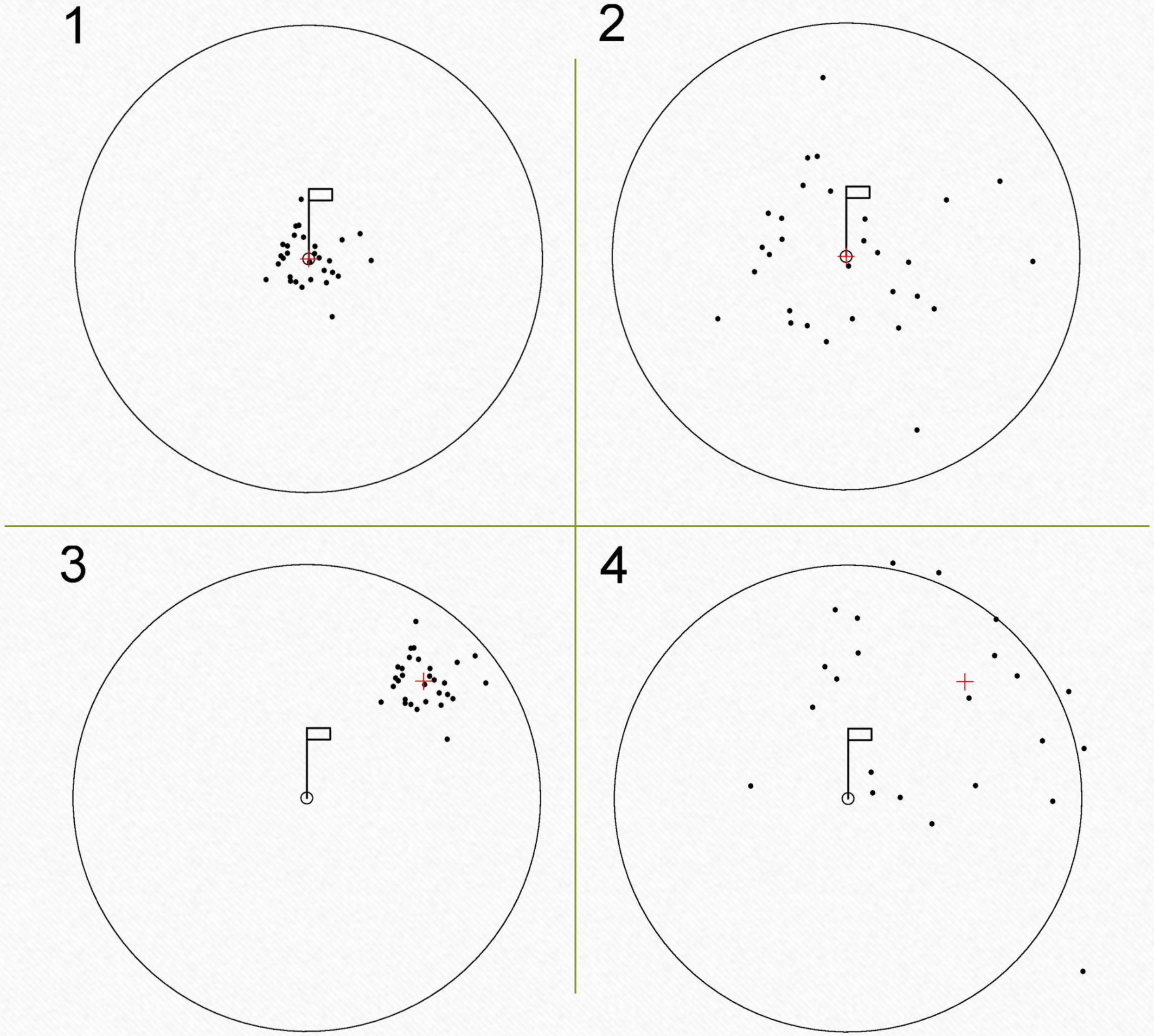
Accuracy, precision of the linear regression parameters' estimators

- IF the assumptions of the regression model are satisfied **THEN** …

$$b_0 \sim Normal\left(\beta_0, \sigma_e^2\left(\frac{1}{n} + \frac{\bar{x}^2}{(n-1)s_x^2}\right)\right)$$

$$b_1 \sim Normal\left(\beta_1, \frac{\sigma_e^2}{(n-1)s_x^2}\right)$$

- **Accuracy and Precision**
  - **Accuracy**: a parameter estimator is accurate if its average value is equal to the population parameter.
  - **Precision**: a parameter estimator is precise if its sampling variation is small.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- **Accuracy**: if the assumptions of the regression model are satisfied then the least squares estimates are accurate:
- the expected value of b_0 is β_0
- the expected value of b_1 is β_1
- **Precision**: if the assumptions of the regression model are satisfied then we can measure precision by the standard deviation of the sampling distributions:
- $SD(b_0) = \sqrt{\sigma_e^2\left(\frac{1}{n} + \frac{\bar{x}^2}{(n-1)s_x^2}\right)}$ and $SD(b_1) = \sqrt{\frac{\sigma_e^2}{(n-1)s_x^2}}$

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- **Standard errors**: if we approximate the values of $SD(b_0)$ and $SD(b_1)$ by replacing the unknown σ_e^2 by *its* estimate, s_e^2 , then we get the **standard errors** for these estimators:

Standard deviation requires knowledge of population parameter...
Standard error is far more practical and only requires knowledge of sample statistic

- $SD(b_0) \approx \sqrt{s_e^2\left(\frac{1}{n} + \frac{\bar{x}^2}{(n-1)s_x^2}\right)} = s_{b_0}$

- $SD(b_1) \approx \sqrt{\frac{s_e^2}{(n-1)s_x^2}} = s_{b_1}$

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- Example: Hooker's data:
- A (severely curtailed) SAS output is given at right.
- Identify the standard errors of b_0 and b_1.
- Provide an interpretation of these values.

Parameter Estimates			
Variable	DF	Parameter Estimate	Standard Error
Intercept	1	146.67290	0.77641
pressure	1	2.25260	0.03809

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Summary

- The least squares parameter estimates will vary from sample-to-sample.
- Under certain conditions the least squares estimates are normally distributed.
- The mean of b_0 is β_0 and the mean of b_1 is β_1 – this means that the least squares estimates are accurate.
- The precision of the estimates can be measured by the standard deviations of the distributions of b_0 and b_1.
- The variances of these distributions depend on the unknown value σ_e^2, but can be approximated by replacing σ_e^2 by its estimate s_e^2. The new values are called the standard errors of b_0 and b_1.