STA 674

Regression Analysis And Design Of Experiments

Experiments with a Single Factor – Lecture 2

STA 674, RA Design Of Experiments:
Experiments with a Single Factor

- Last time, we introduced the concept of the cell means model and illustrated this using an example.
- This time, we'll focus more on the formal statistical expression of the research question(s) involved and how do those point toward the proper inferential procedures.

STA 674, RADOE:
Experiments with a Single Factor

Research Questions

Consider the research questions in the experiments that we have looked at so far:

- Example 1: Is it better to run than to walk in the rain?
- Example 2: Cancer Drug Trial – does the new drug increase time to recurrence?
- Example 3: Reading Comprehension – do the different methods of instruction a affect reading comprehension scores?

What do these questions mean?

STA 674, RADOE:
Experiments with a Single Factor

Research Questions

We are *really* interested in are questions about the **treatment means**.

- Example 1: ***On average*** do people get less water on them if they run than if they walk in the rain?
- Example 2: Cancer Drug Trial – ***On average*** is the time to recurrence greater for patients on the drug than for patients on the placebo?
- Example 3: Reading Comprehension – Is there a difference between the ***average*** increase in reading comprehension for the three treatments?

STA 674, RADOE:
Experiments with a Single Factor

Cell Means Model

Notation:

1. Let t denote the number of treatments
2. Let r denote the number of experimental units for each treatment
3. Let y_{ij} denote the response for the j^{th} experimental unit assigned to the i^{th} treatment

- The cell or treatment means model states that:

$$y_{ij} = \mu_i + e_{ij}$$

where

- μ_i is the mean response for treatment i
- e_{ij} is the error for the j^{th} experimental unit assigned to treatment i and we assume $e_{ij} \sim N(0, \sigma^2)$, independent.

STA 674, RADOE:
Experiments with a Single Factor

Research Questions

We are *really* interested in are questions about the **treatment means**.

- Let μ_1 and μ_2 denote the average amount of water absorbed by people who walk and run in the rain. Does $\mu_1 = \mu_2$?
- Example 2: Cancer Drug Trial – Let μ_1 and μ_2 denote the average time to recurrence for patients on the placebo and patients on the drug. Does $\mu_1 = \mu_2$?
- Example 3: Reading Comprehension – Is there a difference between the ***average*** increase in reading comprehension for the three treatments?
- Let μ_1 , μ_2, and μ_3 denote the average change in reading comprehension scores for students in the three groups.

Does $\mu_1 = \mu_2$, $\mu_1 = \mu_3$, and $\mu_2 = \mu_3$?

STA 674, RADOE:
Experiments with a Single Factor

Research Questions

We are *really* interested in are questions about the **treatment means**.

- Let μ_1 and μ_2 denote the average amount of water absorbed by people who walk and run in the rain. Does $\mu_1 = \mu_2$?
- Example 2: Cancer Drug Trial – Let μ_1 and μ_2 denote the average time to recurrence for patients on the placebo and patients on the drug. Does $\mu_1 = \mu_2$?
- Example 3: Reading Comprehension – Is there a difference between the ***average*** increase in reading comprehension for the three treatments?
- Let μ_1 , μ_2, and μ_3 denote the average change in reading comprehension scores for students in the three groups.

$$\text{Does } \mu_1 = \mu_2 = \mu_3?$$

STA 674, RADOE:
Experiments with a Single Factor

Example: Reading Comprehension

Research Question

Is there a difference between the ***average*** increase in reading comprehension for the three treatments?

- The cell means model is:

$$y_{ij} = \mu_i + e_{ij}$$

where

- μ_1 is the mean for students in the Basal treatment,
- μ_2 is the mean for students in the DRTA group,
- μ_3 is the mean for students in the Strat group.

We want to test the hypothesis:

- $H_0: \mu_1 = \mu_2 = \mu_3$ versus $H_a: \mu_j \neq \mu_k$ for some j and k

STA 674, RADOE:

Experiments with a Single Factor

Example: Reading Comprehension

Fit model using PROC GLM

```
/* Fit model using PROC GLM */;
PROC GLM DATA=READING;
CLASS group;
MODEL score=group;
RUN;
```

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	84.9392399	42.4696199	7.24	0.0016
Error	56	328.6878788	5.8694264		
Corrected Total	58	413.6271186			

STA 674, RADOE:
Experiments with a Single Factor

Example: Reading Comprehension

Fit model using PROC GLM

```
/* Fit model using PROC GLM and compute treatment means*/;
PROC GLM DATA=READING;
    CLASS Group;
    MODEL score=group;
    LSMEANS group / CL;
RUN;
```

Group	Score LSMEAN	95% Confidence Limits	
Basal	5.533333	4.280234	6.786432
DRTA	6.227273	5.192560	7.261985
Strat	8.363636	7.328924	9.398349