STA 674

Regression Analysis And Design Of Experiments

Advanced Concepts – Lecture 2

Split Plot Designs and Random Effects

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Split Plot Designs

- Last time, talked about analysis in the instance where we have one factor being randomly assigned within experimental units of a second factor—split plot designs.
- This time, we'll practice this concept by looking at a second example and consider the different nature of one of the factors in each of these.

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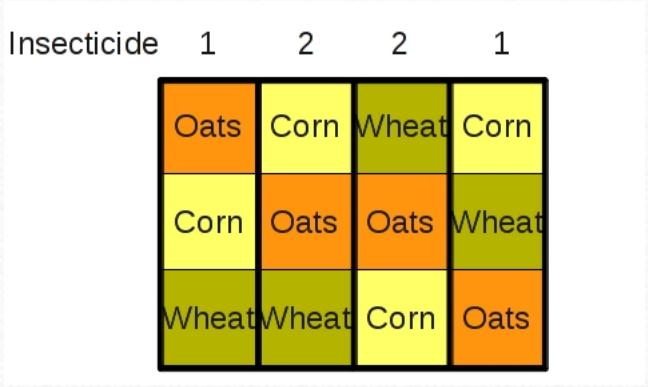
Example – aerial spraying of crop types



- δ_{ik} – whole plot (strip) effect

models differences in effectiveness between two strips sprayed with the same insecticide

- ϵ_{ijk} – subplot (square) effect

models differences in effectiveness of two plots within the same strip planted with the same grain

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Example – paper manufacture

- A paper manufacturer is interested in making the strongest paper possible. To optimize the paper produced in her factory, she decides to run an experiment to compare different settings for two stages in the process: the length of time that the pulp is processed (with three levels) and the temperature at which the paper is dried (with four possible levels). The factory produces one batch of pulp each day. However, the oven that is used to dry the paper can't fit the entire batch and the pulp produced on one day is separated into four loads that are dried separately one after the other.
- How can the manufacturer run an experiment in the shortest possible time that includes 3 replicates for each of the 12 possible treatments? Sketch out your design, explain how you would conduct the randomization, and identify your experimental units.

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Example – paper manufacture

The whole plots and whole plot factor are:

- method of pulp processing and drying temperature
- day and method of pulp processing
- load and drying temperature
- day*batch and method*temperature

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Example – paper manufacture

The subplots and subplot factor are:

- method of pulp processing and drying temperature
- day and method of pulp processing
- load and drying temperature
- day*batch and method*temperature

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Example – paper manufacture

$$y_{ijk} = \mu_{ij} + \delta_{ik} + \epsilon_{ijk}$$

- $i = 1,2,3$ for method of pulp production
- $j = 1,2,3,4$ for drying temperature
- $k = 1,2,3$ for day assigned each method of pulp production
- δ_{ik} – whole plot (day) effect

models differences in strength between two days that method i was used

- ϵ_{ijk} – subplot (load) effect

models error for the strength using temperature j on k^{th} day that method i was used

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- To account for the two levels of variation in the model we need to include a random effect that accounts for differences between the whole plots.
- **Fixed Effects**
  - Fixed effects model differences in experimental conditions that can be replicated exactly if the experiment were repeated.
  - For example, modeling differences in the treatment means.
- **Random Effects**
  - Random effects model differences in the experimental conditions that would change if the experiment were repeated.
  - E.g., modeling differences between days in the paper strength experiment.

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- **Fixed versus Random Effects**
  - Differences between experimental units that:
  - *you* control are modeled with fixed effects
  - *you do not* control are modeled random effects.

Example – paper strength

The manufacturer controls: (Fixed Effects)

- differences due to the pulp processing time, and
- differences due to the drying time.

The manufacturer does not control: (Random Effects)

- differences between days with the same pulp processing time.

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Random Effects in SAS

- Models that include both fixed and random effects are called mixed effects models.
- These models can be fit in SAS using the MIXED procedure.
- Random effects are specified using the RANDOM statement.

Example – paper strength

```
PROC MIXED DATA=strength;
CLASS temp method day;
MODEL strength = temp method temp*method;
RANDOM day(method);
RUN;
```