# STA 674

Regression Analysis And Design Of Experiments

Fitting Simple Linear Regression Models – Lecture 8

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## Fitting Simple Linear Regression Models

- Last time: we finished our confidence interval discussion by building confidence intervals for the linear regression parameters, $\beta_0, \beta_1$
- Now we continue with the major form of statistical inference, significance (or hypothesis) testing.

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- A **confidence interval** answers the question: what values of the parameter(s) are plausible given the data?
- A **significance test** answers the question: are specific values of the parameter(s) plausible given the data?

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- A **confidence interval** answers the question: what values of the parameter(s) are plausible given the data?
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- A **confidence interval** answers the question: what values of the parameter(s) are plausible given the data?
- A **significance test** answers the question: ~~are specific values of the parameter(s) plausible given the data?~~ are the data plausible given specific values of the parameter(s)?
    - If the answer is ***no*** then we rule out that value of the parameter as a plausible value of the truth.
    - If the answer is ***yes*** then we cannot rule out that value of the parameter as a plausible value of the truth.

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Review: Significance test for the population mean

- Suppose that $X_1, X_2, \ldots, X_n$ are normally distributed with mean $\mu$ and variance $\sigma^2$.
- We might ask if there is sufficient evidence in the date to conclude that:
    1. $\mu$ is different from the specific value $\mu_0$ $(\mu \neq \mu_0)$
    2. $\mu$ is greater than the specific value $\mu_0$ $(\mu > \mu_0)$
    3. $\mu$ is less than the specific value $\mu_0$ $(\mu < \mu_0)$

  at a specified level of significance, $\alpha$.

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Review: Significance test for the population mean

- The logic of the significance test is:

1. If $X_1, X_2, \dots, X_n$ are normally distributed with common variance and the population mean really is $\mu_0$ then:

mean of sample

assumption of population mean...used to compare sample to

$$t = \frac{\bar{x} - \mu_0}{s_x/\sqrt{n}} \sim t_{n-1}$$

(Read "~" as "is distributed as".)

2. If the value of $t$ we observe from our sample is very unlikely when $\mu = \mu_0$ then it is more reasonable to conclude that $\mu \neq \mu_0$ than to believe that a very unlikely event has occurred.

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Example: Significance test for the population mean

A quality control inspector is concerned with the average amount of weight that can be held by a type of steel beam. A random sample of five beams is tested with the following amounts of weight added before the beams begin to show stress (in thousands of pounds):

9, 11, 10, 10, 8

The factory aims to produce beams that can hold 10 thousand pounds on average. Is there sufficient evidence to reject this hypothesis at the $\alpha = 0.05$ level of significance?

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- Definition: ***P*-value**
- The $p$-value is the probability of getting a value of the test statistic that is as extreme or more extreme than what we calculated. The $p$-values is an alternative way to measure whether or not the data were likely given the value of the parameter in the null hypothesis.
- If the $p$-value is smaller than $\alpha$, then we reject $H_0$ at the $\alpha$ level of significance. Otherwise, we *cannot reject* $H_0$.