# STA 674

Regression Analysis And Design Of Experiments

Experiments with Multiple Factors – Lecture 1

# STA 674, RA Design Of Experiments:
## Experiments with Multiple Factors

- Where does it fit in?
- What is it?
- Where next?

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## Experiments with Multiple Factors

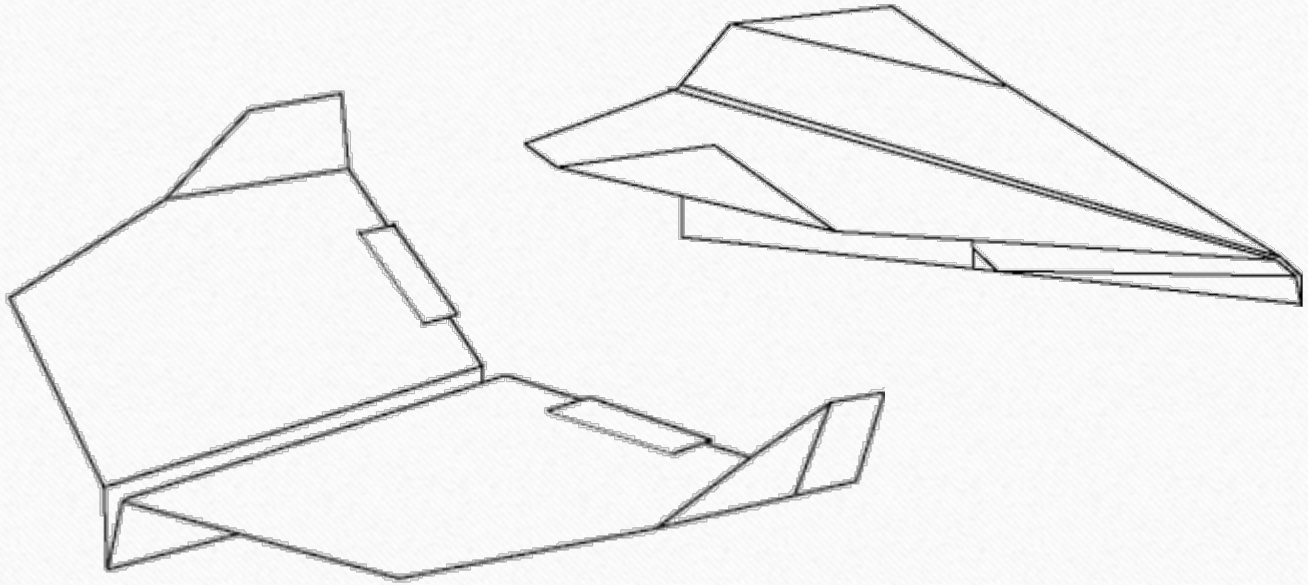
### Simple example

- Suppose that we are designing paper planes and we wish to consider two factors:
- Design of plane:

1. Type I
2. Type II

- Body Modification:

1. No flaps
2. With flaps



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## Experiments with Multiple Factors

### Factorial Treatment Design

- A **factorial treatment design** (or factorial experiment) is one in which every possible treatment created from combining the levels of several factors is assigned to at least one experimental unit.
- A $2 \times 2$ (or $2^2$, or $2^K$ in general for a $K$-factor experiment where each of the factors has two levels) factorial design includes 2 factors with 2 levels for a total of 4 treatments

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### Objectives

- Study the effect of multiple factors on a response.
- Study how the effect of one factor (or more) depends on the conditions determined by the remaining factors.

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### Cell Means Model

- The cell means model for the $2 \times 2$ factorial design is:

$$y_{ijk} = \mu_{ij} + e_{ijk}$$

where

- $\mu_{ij}$ is the mean response units treated with level $i$ of factor 1 and level $j$ of factor 2
- $e_{ijk}$ is the error for the $k^{th}$ unit treated with level $i$ of factor 1 and level $j$ of factor 2, that is $e_{ijk} = y_{ijk} - \mu_{ij}$
- $y_{ijk}$ is the response from the $k^{th}$ unit treated with level $i$ of factor 1 and level $j$ of factor 2

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### Cell Means Model

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| | **Factor 2** | |
|---|---|---|
| Factor 1 | Level 1 | Level 2 |
| Level 1 | $\mu_{11}$ | $\mu_{12}$ |
| Level 2 | $\mu_{21}$ | $\mu_{22}$ |

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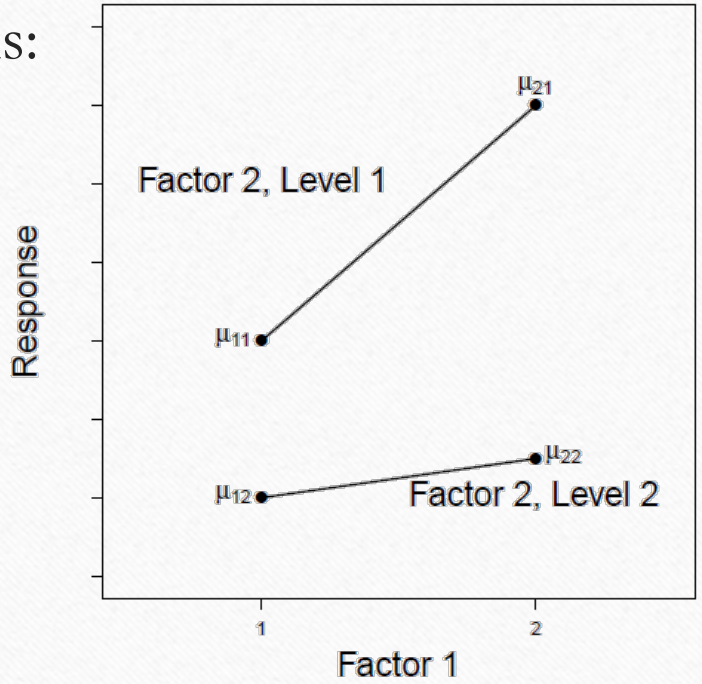

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### Simple Effects

- **Simple effects** are the differences in the mean response between two levels of one factor while the other factors remain fixed.

Examples:

The difference in mean flying distance:

- between Type I planes thrown with or without flaps,
- between Type I planes and Type II planes thrown with flaps.

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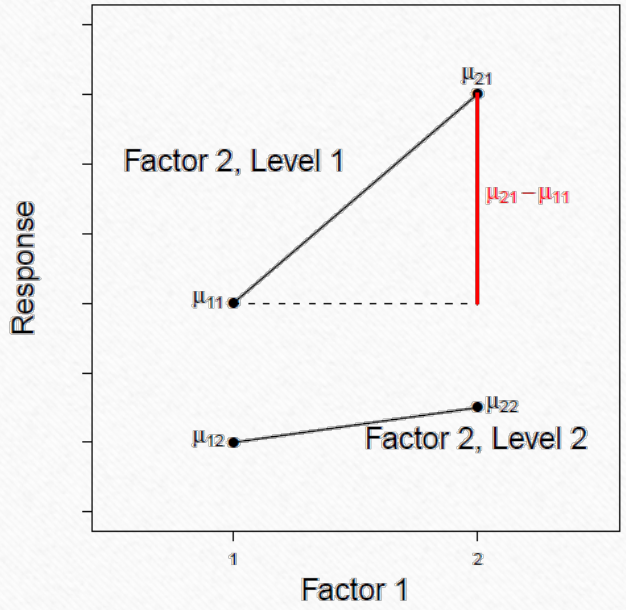
### Simple Effects

- Simple effects are differences between two means in the same row or the same column of the table of means.
- Simple effect of Factor 1 for Level 1 of Factor 2:

$$\mu_{21} - \mu_{11}$$

| | **Factor 2** | |
|---|---|---|
| Factor 1 | Level 1 | Level 2 |
| Level 1 | $\mu_{11}$ | $\mu_{12}$ |
| Level 2 | $\mu_{21}$ | $\mu_{22}$ |



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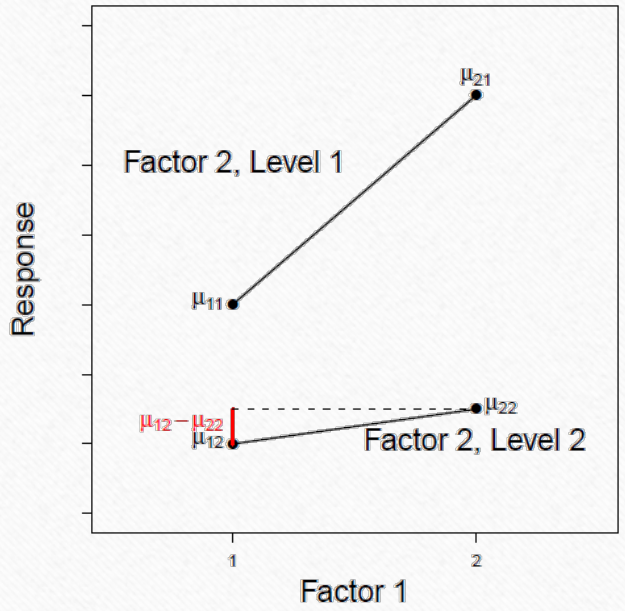
## Experiments with Multiple Factors

### Simple Effects

- Simple effects are differences between two means in the same row or the same column of the table of means.
- Simple effect of Factor 1 for Level 2 of Factor 2:

$$\mu_{22} - \mu_{12}$$

| | **Factor 2** | |
|---|---|---|
| Factor 1 | Level 1 | Level 2 |
| Level 1 | $\mu_{11}$ | $\mu_{12}$ |
| Level 2 | $\mu_{21}$ | $\mu_{22}$ |



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### Main Effects

- **Main effects** are the differences in the mean response between two levels of one factor averaged over all levels of the other factor(s.)

Examples:

The difference in mean flying distance for:

- planes of Type I and planes of Type II,
- planes without flaps and planes with flaps.

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### Main Effects

- Main effect are differences between the overall means of two columns or the overall means of two rows.
- Main effect of Factor 1:

$$\frac{\mu_{21}+\mu_{22}}{2} - \frac{\mu_{11}+\mu_{12}}{2}$$

| | Factor 2 | |
|---|---|---|
| Factor 1 | Level 1 | Level 2 |
| Level 1 | $\mu_{11}$ | $\mu_{12}$ |
| Level 2 | $\mu_{21}$ | $\mu_{22}$ |