STA 674

Regression Analysis And Design Of Experiments

Treatment Comparisons – Lecture 2

STA 674, RA Design Of Experiments:
Treatment Comparisons

- Last time, we began our topic of treatment comparisons by introducing an important tool in DOE: the contrast.
- This time, we will look at practical inference about these contrasts and interpretation of the results. We will also discuss why you should be cautious when you are making many (as in more than one or two) comparisons based on the same data.

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Testing Hypotheses Defined by Contrasts

- Hypotheses defined through contrasts can be tested by t-tests or equivalently through F-tests with one degree of freedom.

Two-sided hypothesis tests:

Hypotheses: $H_0: C = 0$ versus $H_a: C \neq 0$

Test statistic: $F = \frac{SSC}{MSE}$

Decision Rule: Reject H_0 at the α level of significance if:

1. $F > F_{\alpha,1,N-t}$ or
2. the p-value is less than α comparing probab

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Example exercise: tomato growth

The treatments in the tomato experiment include a control (treatment 1) and three alternative solutions containing:

- glucose (treatment 2),
- fructose (treatment 3), and
- sucrose (treatment 4).

Let's construct contrasts to test whether the mean growth for each of the alternatives is different from the mean growth of the control.

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Example exercise: tomato growth

```
/* Comparisons of each alternative with control */;
PROC GLM DATA=TOMATO;
    CLASS treatment;
    MODEL growth=treatment;
    LSMEANS treatment / CL;
    CONTRAST "Control vs Glucose" treatment -1 0 1 0;
    CONTRAST "Control vs Fructose" treatment -1 1 0 0;
    CONTRAST "Control vs Sucrose" treatment -1 0 0 1;
RUN;
```

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
Control vs Glucose	1	435.6000000	435.6000000	63.36	<.0001
Control vs Fructose	1	532.9000000	532.9000000	77.51	<.0001
Control vs Sucrose	1	168.1000000	168.1000000	24.45	0.0001

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Example exercise: tomato growth

Consider the SAS code below. What are the appropriate labels for the 3 contrasts?

```
PROC GLM DATA=TOMATO;
CLASS treatment;
MODEL growth=treatment;
MEANS treatment / CLM LSD;
CONTRAST "?????" treatment 0 1 -1 0; fructose vs glucose
CONTRAST "?????" treatment 0 0 -1 1; glucose vs sucrose
CONTRAST "?????" treatment 0 -.5 -.5 1; mean of fructose and glucose vs sucrose
RUN;
```

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Multiple Comparisons

- Every time we conduct a hypothesis test there is a chance that we erroneously reject the null hypothesis if it is true (i.e., that we commit a "Type I error.")
- When many tests are conducted at the usual level of significance, there is a high probability that at least one of the tests rejects the null hypothesis by chance when all of the null hypotheses are all true – even if α is small.
- Rather than protecting against Type I Error for each test we would like to protect against ever making a Type I Error.

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- Suppose that we conduct an experiment and perform n comparisons between the treatment means.

Comparisonwise error rate

- The comparisonwise error rate (α_C) is the probability that a type I error occurs on *any specific* test.

Experimentwise error rate

- The experimentwise error rate (α_E) is the probability that a type I error occurs on *at least one* of the n tests.

Experimentwise error rate when $\alpha = 0.05$