

# STA 674

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Regression Analysis And Design Of Experiments

Experiments with a Single Factor – Lecture 1



# STA 674, RA Design Of Experiments: Experiments with a Single Factor

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- Where does it fit in?
- What is it?
- Where next?

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## Experiments with a Single Factor

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### Example: Testing a Cancer Drug

- Suppose that a new cancer drug is to be tested in a double-blind clinical trial. A cohort of 60 patients is recruited – 10 from each of 6 hospitals. Each patient is to be treated as an experimental unit. At the end of the study the time to recurrence for each patient will be compared.

How can we model the data that is collected?



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### Example: Testing a Cancer Drug

- The statistical model we will use to describe the recurrence times for the cancer patients is:

$$y_{ij} = \mu_i + e_{ij}; \quad i = 1, 2, j = 1, \dots, 30, \text{ where the terms represent:}$$

#### 1. Observed Values

$y_{1j}$  is the time to recurrence for the  $j^{\text{th}}$  patient given the placebo

$y_{2j}$  is the time to recurrence for the  $j^{\text{th}}$  patient given the drug

#### 2. Treatment Means

$\mu_1$  is the mean time to recurrence for the patients given the placebo

$\mu_2$  is the mean time to recurrence for the patients given the drug

#### 3. Errors

$e_{1j}$  is the error for the  $j^{\text{th}}$  patient given the placebo ( $e_{1j} = y_{1j} - \mu_1$ )

$e_{2j}$  is the error for the  $j^{\text{th}}$  patient given the drug ( $e_{2j} = y_{2j} - \mu_2$ )

number of subscripts on  $\mu$ , equal the number of factors in experiment

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### Cell Means Model

- The cell means model for the completely randomized design models the response from each observational unit as the sum of two components:
  1. the treatment mean, and
  2. error about the mean.
- For a study with  $t$  treatments and  $r$  replicates per treatment the model for the response from the  $j^{th}$  replicate in the  $i^{th}$  treatment is:

$$y_{ij} = \mu_i + e_{ij}; \quad i = 1, \dots, t, j = 1, \dots, r$$



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## Introduction to Comparative Experiments

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### Cell Means Model

- The assumptions of the model are that the errors:
  1. are normally distributed,
  2. are independent,
  3. have mean zero, and
  4. have constant variance ( $\sigma^2$ ) variance is same in both groups (2 groups for single factor experiment)

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## Experiments with a Single Factor

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### Exercise: Reading Comprehension

- An education researcher is interested in comparing the effects of different methods of instruction on the reading comprehension of students. A group of 66 students is enrolled in the study and 22 are randomly assigned to receive one of three methods of instruction: Basal, DRTA, or Strat. Students reading comprehension is tested before and after the instruction, and the response is the difference in their scores.
- Write down the cell means model for the data and explain the components of the model.

$y_{ij} = \mu_i + e_{ij}$  where  $i=1,2,3$  for basal, drta, and strat teaching methods; e.g.,  $\mu_1$ =mean difference of reading comprehension scores using basal method and  $j=1, 2, \dots, 22$ ...number of students in each group