STA 674

Regression Analysis And Design Of Experiments

Treatment Comparisons – Lecture 3

STA 674, RA Design Of Experiments:
Treatment Comparisons

- Last time, we did some inference in SAS with contrasts and brought to consciousness the increased risk of Type I error when doing multiple comparisons.
- This time, we will look at one mathematically simple (but conservative) method of working against this phenomenon.

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Bonferroni's Inequality

- If n tests are conducted each with a comparisonwise error rate of α_C then the experimentwise error rate always satisfies:

$$n\alpha_C \leq \alpha_E$$

Bonferroni Correction

The experimentwise error rate for a set of n tests will be less than or equal to a desired level, α_E, if:

$$\alpha_C \leq \frac{\alpha_E}{n}$$

We can ensure that the experimentwise error rate is no bigger than α_E by setting $\alpha_C = \frac{\alpha_E}{n}$.

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Example exercise: tomato growth

Consider the experiment tomato plant experiment with four treatments and suppose that the researcher wants to compare every treatment with every other treatment. This entails

$$n = (4 \times 3)/2 \ = \ 6$$

different tests.

1. Explain the difference between the comparison-wise and experiment-wise error rates. Experiment-wise error = out of 6 comparisons, the likelihood that any of the 6 (1 or more) comparisons were type 1 errors (mistakenly rejected null hypothesis) was less than 0.05

2. Use Bonferroni's method to compute a comparison-wise error rate that ensures an experiment-wise error rate of $\alpha_E = 0.05$. 0.05/6 = ~1% = alpha c = prob. that mistakenly reject null on any individual comparison

Example exercise: tomato growth

comparing P to
bonferoni's
estimate of alpha c

```
/* all possible comparisons -- Bonferroni correction */;
PROC GLM DATA=TOMATO ;
    CLASS treatment;
    MODEL growth=treatment;
    MEANS treatment / clm lsd;
    CONTRAST "Control vs Glucose" treatment -1 0 1 0;
    CONTRAST "Control vs Fructose" treatment -1 1 0 0;
    CONTRAST "Control vs Sucrose" treatment -1 0 0 1;
    CONTRAST "Fructose vs Glucose" treatment 0 -1 1 0;
    CONTRAST "Fructose vs Sucrose" treatment 0 -1 0 1;
    CONTRAST "Glucose vs Sucrose" treatment 0 0 -1 1;

RUN;
```

The GLM Procedure

Dependent Variable: growth

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
Control vs Glucose	1	435.6000000	435.6000000	63.36	<.0001
Control vs Fructose	1	532.9000000	532.9000000	77.51	<.0001
Control vs Sucrose	1	168.1000000	168.1000000	24.45	0.0001
Fructose vs Glucose	1	4.9000000	4.9000000	0.71	0.4110
Fructose vs Sucrose	1	102.4000000	102.4000000	14.89	0.0014
Glucose vs Sucrose	1	62.5000000	62.5000000	9.09	0.0082

not significant

significant but
close to
insignificant

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Simultaneous Confidence Intervals

- Suppose that we are going to conduct an experiment with t treatments and plan to compute 95% confidence intervals for the mean response of each treatment.
- What is the probability that the confidence intervals will *all* cover their respective treatment means?

For regular CI of each mean...the probability of missing the mean (e.g., for 95% CI, 5% probability that mean is outside that range) adds up for each mean...so for 3 95% means/CI = 0.05+0.05+0.05 = 0.15 probability of at least one missing the mean

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Simultaneous Confidence Intervals

- If we wish to construct confidence intervals for the treatment means such that the probability that **all t intervals cover their respective means** is $(1-\alpha_E)$, then we need to increase the probability that any one interval covers its respective mean, $(1-\alpha_C)$.
- Confidence intervals constructed so that *all* treatment means are covered with probability $(1-\alpha_E)$ are called $(1-\alpha_E)$ 100% simultaneous confidence intervals.
- If we repeated the experiment many, many times and constructed confidence intervals in this way then $(1-\alpha_E)$ 100% of the time all t intervals would cover their respective means.

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Simultaneous Confidence Intervals with Bonferroni's Correction

- We can construct a set of $(1 - \alpha_E)$ 100% simultaneous confidence intervals for t treatment means by constructing $(1 - \alpha_C)$ 100% for each mean where:

$$\alpha_C = \frac{\alpha_E}{n}$$

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Example exercise: tomato growth

Consider the tomato experiment again and suppose we wish to construct 95% simultaneous confidence intervals for the means of each treatment.

Use the Bonferroni correction to compute a value of α_C such that if we compute $(1 - \alpha_C)$ 100% confidence intervals for each mean then the intervals will form simultaneous 95% confidence intervals. ac = 0.05 / 4 = 0.0125...use this for P value for individual CI

2. Provide an interpretation of the single interval for the control treatment.

3. Provide an interpretation for all 4 intervals together.

Example exercise: tomato growth

```
/* Comparisons of each alternative with control -- Bonferroni correction */;
PROC GLM DATA=TOMATO;
    CLASS treatment;
    MODEL growth=treatment;
    MEANS treatment / CLM LSD alpha = .016666666;
    CONTRAST "Control vs Glucose" treatment -1 0 1 0;
    CONTRAST "Control vs Fructose" treatment -1 1 0 0;
    CONTRAST "Control vs Sucrose" treatment -1 0 0 1;
RUN;
```

should be 0.0125

treatment	N	Mean	95% Confidence Limits	
Control	5	42.200	39.714	44.686
Sucrose	5	34.000	31.514	36.486
Glucose	5	29.000	26.514	31.486
Fructose	5	27.600	25.114	30.086

The SAS System

The GLM Procedure

Dependent Variable: growth

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
Control vs Glucose	1	435.6000000	435.6000000	63.36	<.0001
Control vs Fructose	1	532.9000000	532.9000000	77.51	<.0001
Control vs Sucrose	1	168.1000000	168.1000000	24.45	0.0001

treatment	N	Mean	98.33333% Confidence Limits	
Control	5	42.200	39.066	45.334
Sucrose	5	34.000	30.866	37.134
Glucose	5	29.000	25.866	32.134
Fructose	5	27.600	24.466	30.734

The SAS System

The GLM Procedure

Dependent Variable: growth

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
Control vs Glucose	1	435.6000000	435.6000000	63.36	<.0001
Control vs Fructose	1	532.9000000	532.9000000	77.51	<.0001
Control vs Sucrose	1	168.1000000	168.1000000	24.45	0.0001