

Important: Before reading WORD\_COMPONENTS, please read or at least skim the program for GB\_WORDS.

**1. Components.** This simple demonstration program computes the connected components of the Graph-Base graph of five-letter words. It prints the words in order of decreasing weight, showing the number of edges, components, and isolated vertices present in the graph defined by the first  $n$  words for all  $n$ .

```
#include "gb_graph.h"    /* the GraphBase data structures */
#include "gb_words.h"    /* the words routine */
<Preprocessor definitions>

main()
{
  Graph *g = words(0_L, 0_L, 0_L, 0_L); /* the graph we love */
  Vertex *v; /* the current vertex being added to the component structure */
  Arc *a; /* the current arc of interest */
  long n = 0; /* the number of vertices in the component structure */
  long isol = 0; /* the number of isolated vertices in the component structure */
  long comp = 0; /* the current number of components */
  long m = 0; /* the current number of edges */

  printf("Component analysis of %s\n", g->id);
  for (v = g->vertices; v < g->vertices + g->n; v++) {
    n++, printf("%4ld: %5ld %s", n, v->weight, v->name);
    <Add vertex v to the component structure, printing out any components it joins 2>;
    printf("; c=%ld, i=%ld, m=%ld\n", comp, isol, m);
  }
  <Display all unusual components 5>;
  return 0; /* normal exit */
}
```

**2.** The arcs from  $v$  to previous vertices all appear on the list  $v$ -arcs after the arcs from  $v$  to future vertices. In this program, we aren't interested in the future, only the past; so we skip the initial arcs.

```
<Add vertex v to the component structure, printing out any components it joins 2> ≡
<Make v a component all by itself 3>;
a = v->arcs;
while (a & a->tip > v) a = a->next;
if (!a) printf("[1]"); /* indicate that this word is isolated */
else { long c = 0; /* the number of merge steps performed because of v */
  for (; a; a = a->next) { register Vertex *u = a->tip;
    m++;
    <Merge the components of u and v, if they differ 4>;
  }
  printf("in %s [%ld]", v->master->name, v->master->size); /* show final component */
}
```

This code is used in section 1.

3. We keep track of connected components by using circular lists, a procedure that is known to take average time  $O(n)$  on truly random graphs [Knuth and Schönhage, *Theoretical Computer Science* **6** (1978), 281–315].

Namely, if  $v$  is a vertex, all the vertices in its component will be in the list

$$v, v\text{-link}, v\text{-link}\text{-link}, \dots,$$

eventually returning to  $v$  again. There is also a master vertex in each component,  $v\text{-master}$ ; if  $v$  is the master vertex,  $v\text{-size}$  will be the number of vertices in its component.

```
#define link z.V /* link to next vertex in component (occupies utility field z) */
#define master y.V /* pointer to master vertex in component */
#define size x.I /* size of component, kept up to date for master vertices only */
```

⟨ Make  $v$  a component all by itself 3 ⟩  $\equiv$

```
v-link = v;
v-master = v;
v-size = 1;
isol++;
comp++;
```

This code is used in section 2.

4. When two components merge together, we change the identity of the master vertex in the smaller component. The master vertex representing  $v$  itself will change if  $v$  is adjacent to any prior vertex.

⟨ Merge the components of  $u$  and  $v$ , if they differ 4 ⟩  $\equiv$

```
u = u-master;
if (u ≠ v-master) { register Vertex *w = v-master, *t;
  if (u-size < w-size) {
    if (c++ > 0) printf("%s_s[%ld]", (c ≡ 2 ? "with" : ","), u-name, u-size);
    w-size += u-size;
    if (u-size ≡ 1) isol--;
    for (t = u-link; t ≠ u; t = t-link) t-master = w;
    u-master = w;
  } else {
    if (c++ > 0) printf("%s_s[%ld]", (c ≡ 2 ? "with" : ","), w-name, w-size);
    if (u-size ≡ 1) isol--;
    u-size += w-size;
    if (w-size ≡ 1) isol--;
    for (t = w-link; t ≠ w; t = t-link) t-master = u;
    w-master = u;
  }
  t = u-link;
  u-link = w-link;
  w-link = t;
  comp--;
}
```

This code is used in section 2.

5. The *words* graph has one giant component and lots of isolated vertices. We consider all other components unusual, so we print them out when the other computation is done.

```

⟨Display all unusual components 5⟩ ≡
    printf("\nThe following non-isolated words didn't join the giant component:\n");
    for (v = g-vertices; v < g-vertices + g-n; v++)
        if (v-master ≡ v ∧ v-size > 1 ∧ v-size + v-size < g-n) { register Vertex *u;
            long c = 1; /* count of number printed on current line */
            printf("%s", v-name);
            for (u = v-link; u ≠ v; u = u-link) {
                if (c++ ≡ 12) putchar('\n'), c = 1;
                printf(" %s", u-name);
            }
            putchar('\n');
        }
    }

```

This code is used in section 1.

**6. Index.** We close with a list that shows where the identifiers of this program are defined and used.

*a*: 1.

**Arc**: 1.

*arcs*: 2.

*c*: 2, 5.

*comp*: 1, 3, 4.

*g*: 1.

**Graph**: 1.

*id*: 1.

*isol*: 1, 3, 4.

Knuth, Donald Ervin: 3.

*link*: 3, 4, 5.

*m*: 1.

*main*: 1.

*master*: 2, 3, 4, 5.

*n*: 1.

*name*: 1, 2, 4, 5.

*next*: 2.

*printf*: 1, 2, 4, 5.

*putchar*: 5.

Schönhage, Arnold: 3.

*size*: 2, 3, 4, 5.

*t*: 4.

*tip*: 2.

*u*: 2, 5.

*v*: 1.

**Vertex**: 1, 2, 4, 5.

*vertices*: 1, 5.

*w*: 4.

*weight*: 1.

*words*: 1, 5.

- ⟨ Add vertex  $v$  to the component structure, printing out any components it joins 2 ⟩ Used in section 1.
- ⟨ Display all unusual components 5 ⟩ Used in section 1.
- ⟨ Make  $v$  a component all by itself 3 ⟩ Used in section 2.
- ⟨ Merge the components of  $u$  and  $v$ , if they differ 4 ⟩ Used in section 2.

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