

June 25, 2025 at 17:35

1. Introduction. This program inputs an undirected graph and the names of two vertices in that graph (the “source” and “target” vertices). It outputs a not-necessarily-reduced binary decision diagram for the family of all simple paths from the source to the target.

The format of the output is described in another program, SIMPATH-REDUCE. Let me just say here that it is intended only for computational convenience, not for human readability.

I’ve tried to make this program simple, whenever I had to choose between simplicity and efficiency. But I haven’t gone out of my way to be inefficient.

```
#define maxn 255    /* maximum number of vertices; at most 255 */
#define maxm 2000   /* maximum number of edges */
#define logmemsize 27
#define memsize (1 << logmemsize)
#define loghtsize 25
#define htsize (1 << loghtsize)
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include "gb_graph.h"
#include "gb_save.h"
unsigned char mem[memsize];    /* the big workspace */
unsigned long long tail, boundary, head;    /* queue pointers */
unsigned int htable[htsize];    /* hash table */
unsigned int htid;    /* “time stamp” for hash entries */
int htcount;    /* number of entries in the hash table */
int wrap = 1;    /* wraparound counter for hash table clearing */
Vertex *vert[maxn + 1];
int arcto[maxm];    /* destination number of each arc */
int firstarc[maxn + 2];    /* where arcs from a vertex start in arcto */
unsigned char mate[maxn + 3];    /* encoded state */
int serial, newserial;    /* state numbers */
<Subroutines 13*>
main(int argc, char *argv[])
{
    register int i, j, jj, jm, k, km, l, ll, m, n, t, hash;
    register Graph *g;
    register Arc *a, *b;
    register Vertex *u, *v;
    Vertex *source =  $\Lambda$ , *target =  $\Lambda$ ;
    <Input the graph 2>;
    <Renumber the vertices 3>;
    <Reformat the edges 4>;
    <Do the algorithm 5>;
}
```

```

2. <Input the graph 2> ≡
if (argc ≠ 4) {
    fprintf(stderr, "Usage: %s foo.gb source target\n", argv[0]);
    exit(-1);
}
g = restore_graph(argv[1]);
if (¬g) {
    fprintf(stderr, "I can't input the graph %s (panic code %ld)!\n", argv[1], panic_code);
    exit(-2);
}
n = g→n;
if (n > maxn) {
    fprintf(stderr, "Sorry, that graph has %d vertices;\n", n);
    fprintf(stderr, "I can't handle more than %d!\n", maxn);
    exit(-3);
}
if (g→m > 2 * maxm) {
    fprintf(stderr, "Sorry, that graph has %ld edges;\n", (g→m + 1)/2);
    fprintf(stderr, "I can't handle more than %d!\n", maxm);
    exit(-3);
}
for (v = g→vertices; v < g→vertices + n; v++) {
    if (strcmp(argv[2], v→name) ≡ 0) source = v;
    if (strcmp(argv[3], v→name) ≡ 0) target = v;
    for (a = v→arcs; a; a = a→next) {
        u = a→tip;
        if (u ≡ v) {
            fprintf(stderr, "Sorry, the graph contains a loop %s--%s!\n", v→name, v→name);
            exit(-4);
        }
        b = (v < u ? a + 1 : a - 1);
        if (b→tip ≠ v) {
            fprintf(stderr, "Sorry, the graph isn't undirected!\n");
            fprintf(stderr, "(%s->%s has mate pointing to %s)\n", v→name, u→name, b→tip→name);
            exit(-5);
        }
    }
}
if (¬source) {
    fprintf(stderr, "I can't find source vertex %s in the graph!\n", argv[2]);
    exit(-6);
}
if (¬target) {
    fprintf(stderr, "I can't find target vertex %s in the graph!\n", argv[3]);
    exit(-7);
}

```

This code is used in section 1.

3. If the source vertex is the first vertex in the graph, I'll process vertices according to the graph's own ordering.

Otherwise, I use a simple breadth-first strategy to number the vertices: The source is vertex 1. Then, for each $j \geq 1$, I run through the arcs from vertex j and assign the first unused number to any of its neighbors that haven't already got one.

#define *num* *z.I*

\langle Renumber the vertices 3 $\rangle \equiv$

```

if (source  $\equiv$  g-vertices) {
    for (k = 0; k < n; k++) (g-vertices + k)-num = k + 1, vert[k + 1] = g-vertices + k;
} else {
    for (k = 0; k < n; k++) (g-vertices + k)-num = 0;
    vert[1] = source, source-num = 1;
    for (j = 0, k = 1; j < k; j++) {
        v = vert[j + 1];
        for (a = v-arcs; a; a = a-next) {
            u = a-tip;
            if (u-num  $\equiv$  0) u-num = ++k, vert[k] = u;
        }
    }
    if (target-num  $\equiv$  0) {
        fprintf(stderr, "Sorry, there's no path from %s to %s in the graph!\n", argv[2], argv[3]);
        exit(-8);
    }
    if (k < n) {
        fprintf(stderr, "The graph isn't connected (%d<%d)!\n", k, n);
        fprintf(stderr, "But that's OK; I'll work with the component of %s.\n", argv[2]);
        n = k;
    }
}

```

This code is used in section 1.

4. The edges will be considered as arcs $j \rightarrow k$ between vertex number j and vertex number k , when $j < k$ and those vertices are adjacent in the graph. We process them in order of increasing j ; but for fixed j , the values of k aren't necessarily increasing.

The k values appear in the *arcto* array. The edges for fixed j occur in positions *firstarc*[j] through *firstarc*[$j + 1$] - 1 of that array.

After this step, we forget the GraphBase data structures and just work with our homegrown integer-only representation.

⟨ Reformat the edges 4 ⟩ \equiv

```

for ( $m = 0, k = 1; k \leq n; k++$ ) {
    firstarc[ $k$ ] =  $m$ ;
     $v = \text{vert}[k]$ ;
    printf ("%ld(%s)\n",  $v\text{-num}$ ,  $v\text{-name}$ );
    for ( $a = v\text{-arcs}; a; a = a\text{-next}$ ) {
         $u = a\text{-tip}$ ;
        if ( $u\text{-num} > k$ ) {
            arcto[ $m++$ ] =  $u\text{-num}$ ;
            if ( $a\text{-len} \equiv 1$ ) printf ("□->□%ld(%s)□#%d\n",  $u\text{-num}$ ,  $u\text{-name}$ ,  $m$ );
            else printf ("□->□%ld(%s,%ld)□#%d\n",  $u\text{-num}$ ,  $u\text{-name}$ ,  $a\text{-len}$ ,  $m$ );
        }
    }
}
firstarc[ $k$ ] =  $m$ ;

```

This code is used in section 1.

5. The algorithm. Now comes the fun part. We systematically construct a binary decision diagram for all simple paths by working top-down, considering the arcs in *arcto*, one by one.

When we're dealing with arc *i*, we've already constructed a table of all possible states that might arise when each of the previous arcs has been chosen-or-not, except for states that obviously cannot be part of a simple path.

Arc *i* runs from vertex *j* to vertex $k = \text{arcto}[i]$. Let *l* be the maximum vertex number in arcs less than *i*. (If the breadth-first ordering option was taken above, we'll always have $k \leq l + 1$, because of the way we did the numbering and reformatting; but that method is not always best.)

The state before we decide whether or not to include arc *i* is represented by a table of values *mate*[*t*], for $j \leq t \leq l$, with the following significance: If *mate*[*t*] = *t*, the previous arcs haven't touched vertex *t*. If *mate*[*t*] = *u* and $u \neq t$, the previous arcs have connected *t* with *u* by a simple path. If *mate*[*t*] = 0, the previous arcs have "saturated" vertex *t*; we can't touch it again.

We also use a (slick?) trick: We imagine that an edge between the source and target has already been included. Then the final arc of a simple path will be an arc that completes a cycle, when no other incomplete paths are present. (Think about it.)

The *mate* information is all that we need to know about the behavior of previous arcs. And it's easily updated when we add the *i*th arc (or not). So each "state" is equivalent to a *mate* table, consisting of $l + 1 - j$ numbers.

The states are stored in a queue, indexed by 64-bit numbers *tail*, *boundary*, and *head*, where $\text{tail} \leq \text{boundary} \leq \text{head}$. Between *tail* and *boundary* are the pre-arc-*i* states that haven't yet been processed; between *boundary* and *head* are the post-arc-*i* states that will be considered later. The states before *boundary* are sequences of $s = l + 1 - j$ bytes each, and the states after *boundary* are sequences of $ss = ll + 1 - jj$ bytes each, where *ll* and *jj* are the values of *l* and *j* for arc *i* + 1.

Bytes of the queue are stored in *mem*, which wraps around modulo *memsize*. We ensure that $\text{head} - \text{tail}$ never exceeds *memsize*.

```

<Do the algorithm 5> ≡
  <Initialize the mate table 6>;
  <Initialize the queue 7>;
  for (i = 0; i < m; i++) {
    printf("#%d:\n", i + 1); /* announce that we're beginning a new arc */
    fprintf(stderr, "Beginning arc %d (serial=%d, head-tail=%lld)\n", i + 1, serial, head - tail);
    fflush(stderr);
    <Process arc i 8>;
  }

```

This code is used in section 1.

```

6. <Initialize the mate table 6> ≡
  for (t = 2; t ≤ n; t++) mate[t] = t;
  mate[target-num] = 1, mate[1] = target-num;

```

This code is used in section 5.

```

7. <Initialize the queue 7> ≡
  jj = ll = 1;
  mem[0] = mate[1];
  tail = 0, head = 1;
  serial = 2;

```

This code is used in section 5.

8. Each state for a particular arc gets a distinguishing number. Two states are special: 0 means the losing state, when a simple path is impossible; 1 means the winning state, when a simple path has been completed. The other states are 2 or more.

The output format on *stdout* simply shows the identifying numbers of a state and its two successors, in hexadecimal.

```
#define trunc(addr) ((addr) & (memsize - 1))
⟨Process arc  $i$  8⟩ ≡
    boundary = head, htcount = 0, htid = (i + wrap) << logmemsize;
    if (htid ≡ 0) {
        for (hash = 0; hash < htsize; hash++) htable[hash] = 0;
        wrap++, htid = 1 << logmemsize;
    }
    newserial = serial + ((head - tail)/(ll + 1 - jj));
    j = jj, k = arcto[i], l = ll;
    while (jj ≤ n ∧ firstarc[jj + 1] ≡ i + 1) jj++;
    ll = (k > l ? k : l);
    while (tail < boundary) {
        printf("%x:", serial);
        serial++;
        ⟨Unpack a state, and move tail up 9⟩;
        ⟨Print the successor if arc  $i$  is not chosen 11⟩;
        printf(",");
        ⟨Print the successor if arc  $i$  is chosen 10⟩;
        printf("\n");
    }
```

This code is used in section 5.

9. If the target vertex hasn't entered the action yet (that is, if it exceeds l), we must update its *mate* entry at this point.

```
⟨Unpack a state, and move tail up 9⟩ ≡
    for (t = j; t ≤ l; t++, tail++) {
        mate[t] = mem[trunc(tail)];
        if (mate[t] > l) mate[mate[t]] = t;
    }
```

This code is used in section 8.

10. Here's where we update the mates. The order of doing this is carefully chosen so that it works fine when $mate[j] = j$ and/or $mate[k] = k$.

```
⟨Print the successor if arc  $i$  is chosen 10⟩ ≡
    jm = mate[j], km = mate[k];
    if (jm ≡ 0 ∨ km ≡ 0) printf("0"); /* we mustn't touch a saturated vertex */
    else if (jm ≡ k) ⟨Print 1 or 0, depending on whether this arc wins or loses 12*⟩
    else {
        mate[j] = 0, mate[k] = 0;
        mate[jm] = km, mate[km] = jm;
        printstate(j, jj, ll);
        mate[jm] = j, mate[km] = k, mate[j] = jm, mate[k] = km; /* restore original state */
    }
done:
```

This code is used in section 8.

11. \langle Print the successor if arc i is not chosen 11 $\rangle \equiv$
`printstate(j, jj, ll);`

This code is used in section 8.

12* See the note below regarding a change that will restrict consideration to Hamiltonian paths. A similar change is needed here.

\langle Print 1 or 0, depending on whether this arc wins or loses 12* $\rangle \equiv$
`{`
`for ($t = j + 1$; $t \leq ll$; $t++$)`
`if ($t \neq k$) {`
`if ($mate[t]$) break;`
`}`
`if ($t > n$) printf("1"); /* we win: this cycle is all by itself */`
`else printf("0"); /* we lose: there's junk outside this cycle */`
`}`

This code is used in section 10.

13* The *printstate* subroutine does the rest of the work. It makes sure that no incomplete paths linger in positions j through $jj - 1$, which are about to disappear; and it puts the contents of $mate[jj]$ through $mate[ll]$ into the queue, checking to see if it was already there.

If ' $mate[t] \neq t$ ' is removed from the condition below, we get Hamiltonian paths only (I mean, simple paths that include every vertex).

\langle Subroutines 13* $\rangle \equiv$
`void printstate(int j , int jj , int ll)`
`{`
`register int $h, hh, ss, t, tt, hash$;`
`for ($t = j$; $t < jj$; $t++$)`
`if ($mate[t]$) break;`
`if ($t < jj$) printf("0"); /* incomplete junk mustn't be left hanging */`
`else if ($ll < jj$) printf("0"); /* nothing is viable */`
`else {`
`$ss = ll + 1 - jj$;`
`if ($head + ss - tail > memsize$) {`
`fprintf(stderr, "Oops, I'm out of memory (memsize=%d, serial=%d)!\n", memsize, serial);`
`fflush(stdout);`
`exit(-69);`
`}`
`\langle Move the current state into position after $head$, and compute $hash$ 14 \rangle ;`
`\langle Find the first match, hh , for the current state after $boundary$ 15 \rangle ;`
`$h = trunc(hh - boundary) / ss$;`
`printf("%x", newserial + h);`
`}`
`}`

This code is used in section 1.

14. \langle Move the current state into position after $head$, and compute $hash$ 14 $\rangle \equiv$
`for ($t = jj, h = trunc(head), hash = 0$; $t \leq ll$; $t++, h = trunc(h + 1)$) {`
`$mem[h] = mate[t]$;`
`$hash = hash * 31415926525 + mate[t]$;`
`}`

This code is used in section 13*.

15. The hash table is automatically cleared whenever *htid* is increased, because we store *htid* with each relevant table entry.

```

⟨Find the first match, hh, for the current state after boundary 15⟩ ≡
  for (hash = hash & (htsize - 1); ; hash = (hash + 1) & (htsize - 1)) {
    hh = htable[hash];
    if ((hh ⊕ htid) ≥ memsize) ⟨Insert new entry and goto found 16⟩;
    hh = trunc(hh);
    for (t = hh, h = trunc(head), tt = trunc(t + ss - 1); ; t = trunc(t + 1), h = trunc(h + 1)) {
      if (mem[t] ≠ mem[h]) break;
      if (t ≡ tt) goto found;
    }
  }
found:

```

This code is used in section 13*.

```

16. ⟨Insert new entry and goto found 16⟩ ≡
{
  if (++htcount > (htsize ≫ 1)) {
    fprintf(stderr, "Sorry, the hash table is full (%d, serial=%d)!\n", htsize, serial);
    exit(-96);
  }
  hh = trunc(head);
  htable[hash] = htid + hh;
  head += ss;
  goto found;
}

```

This code is used in section 15.

17* Index.

The following sections were changed by the change file: 12, 13, 17.

a: 1.
addr: 8.
Arc: 1.
arcs: 2, 3, 4.
arcto: 1, 4, 5, 8.
argc: 1, 2.
argv: 1, 2, 3.
b: 1.
boundary: 1, 5, 8, 13*.
done: 10.
exit: 2, 3, 13*, 16.
fflush: 5, 13*.
firstarc: 1, 4, 8.
found: 15, 16.
fprintf: 2, 3, 5, 13*, 16.
g: 1.
Graph: 1.
h: 13*.
hash: 1, 8, 13* 14, 15, 16.
head: 1, 5, 7, 8, 13*, 14, 15, 16.
hh: 13* 15, 16.
htable: 1, 8, 15, 16.
htcount: 1, 8, 16.
htid: 1, 8, 15, 16.
htsize: 1, 8, 15, 16.
i: 1.
j: 1, 13*.
jj: 1, 5, 7, 8, 10, 11, 13* 14.
jm: 1, 10.
k: 1.
km: 1, 10.
l: 1.
len: 4.
ll: 1, 5, 7, 8, 10, 11, 12*, 13* 14.
loghtsize: 1.
logmemsize: 1, 8.
m: 1.
main: 1.
mate: 1, 5, 6, 7, 9, 10, 12*, 13*, 14.
maxm: 1, 2.
maxn: 1, 2.
mem: 1, 5, 7, 9, 14, 15.
memsize: 1, 5, 8, 13*, 15.
n: 1.
name: 2, 4.
newserial: 1, 8, 13*.
next: 2, 3, 4.
num: 3, 4, 6.
panic_code: 2.
printf: 4, 5, 8, 10, 12*, 13*.
printstate: 10, 11, 13*.
restore_graph: 2.
serial: 1, 5, 7, 8, 13*, 16.
source: 1, 2, 3.
ss: 5, 13* 15, 16.
stderr: 2, 3, 5, 13*, 16.
stdout: 8, 13*.
strcmp: 2.
t: 1, 13*.
tail: 1, 5, 7, 8, 9, 13*.
target: 1, 2, 3, 6.
tip: 2, 3, 4.
trunc: 8, 9, 13*, 14, 15, 16.
tt: 13* 15.
u: 1.
v: 1.
vert: 1, 3, 4.
Vertex: 1.
vertices: 2, 3.
wrap: 1, 8.

- ⟨ Do the algorithm 5 ⟩ Used in section 1.
- ⟨ Find the first match, *hh*, for the current state after *boundary* 15 ⟩ Used in section 13*.
- ⟨ Initialize the queue 7 ⟩ Used in section 5.
- ⟨ Initialize the *mate* table 6 ⟩ Used in section 5.
- ⟨ Input the graph 2 ⟩ Used in section 1.
- ⟨ Insert new entry and **goto** *found* 16 ⟩ Used in section 15.
- ⟨ Move the current state into position after *head*, and compute *hash* 14 ⟩ Used in section 13*.
- ⟨ Print 1 or 0, depending on whether this arc wins or loses 12* ⟩ Used in section 10.
- ⟨ Print the successor if arc *i* is chosen 10 ⟩ Used in section 8.
- ⟨ Print the successor if arc *i* is not chosen 11 ⟩ Used in section 8.
- ⟨ Process arc *i* 8 ⟩ Used in section 5.
- ⟨ Reformat the edges 4 ⟩ Used in section 1.
- ⟨ Renumber the vertices 3 ⟩ Used in section 1.
- ⟨ Subroutines 13* ⟩ Used in section 1.
- ⟨ Unpack a state, and move *tail* up 9 ⟩ Used in section 8.

SIMPATH-HAM

	Section	Page
Introduction	1	1
The algorithm	5	5
Index	17	9