Esercizio

$$\int \frac{e^x}{\sqrt{e^x-3}}$$

metodo di sostituzione

$$\sqrt{e^x-3} = t$$

$$e^x - 3 = t^2$$

$$e^x = t^2+3$$

$$e^x dx = 2t\,dt$$

$$\int \frac{2\cancel{t}dt}{\cancel{t}} = \int 2\,dt = 2dt + c = 2\sqrt{e^x-3} + c, \quad c\in\mathbb{R}$$

Esercizio

$$\int \frac{e^{\operatorname{arctg} x}}{1+x^2}\,dx$$

$$\operatorname{arctg} x = t$$

$$\frac{1}{1+x^2}\,dx = dt$$

$$= \int e^{\operatorname{arctg} x} \cdot \frac{1}{1+x^2}\,dx = \int e^t\,dt = e^t + c =$$

$$= e^{\operatorname{arctg} x} + c, \quad c\in\mathbb{R}$$

Esercizio

$$\int \frac{1}{x(1+\ln x)}\,dx$$

$$\ln x = t$$

$$\frac{1}{x}\,dx = dt$$

$$= \int \frac{1}{x}\cdot\frac{1}{(1+\ln x)}\,dx = \int \frac{1}{1+t}\,dt = \ln|1+t| + c = \ln|1+\ln x| + c, \quad c\in\mathbb{R}$$

Esercizio

$$\int \frac{1}{x\sqrt{1-\ln^2 x}}\,dx$$

$$\ln x = t$$

$$\frac{1}{x}\,dx = dt$$

$$= \int \frac{1}{\sqrt{1-t^2}}\,dt = \operatorname{arcsen} t + c = \operatorname{arcsen}(\ln x) + c, \quad c\in\mathbb{R}$$

Esercizio

$$\int tg^4 x \, dx \qquad \begin{aligned} tg\,x &= t \\ x &= arctg\, t \\ dx &= \frac{1}{1+t^2}\,dt \end{aligned}$$

$$= \int t^4 \cdot \frac{1}{1+t^2}\,dt = \int \frac{t^4}{1+t^2}\,dt = \int \frac{t^4+1-1}{1+t^2}\,dt = \int \frac{t^4-1}{1+t^2}\,dt + \int \frac{1}{1+t^2}\,dt =$$

$$= \int \frac{\cancel{(t^2+1)}(t^2-1)}{\cancel{1+t^2}}\,dt + \int \frac{1}{1+t^2}\,dt = \int t^2\,dt - \int dt + \int \frac{1}{1+t^2}\,dt =$$

$$= \frac{t^3}{3} - t + arctg\, t + c = \frac{1}{3} tg^3 x - tg\,x + arctg(tg\,x) + c$$

$$= \frac{1}{3} tg^3 x - tg\,x + x + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int \frac{e^x\,dx}{1+\sqrt{e^x}} \qquad \begin{aligned} \sqrt{e^x} &= t \\ e^x &= t^2 \\ e^x dx &= 2t\,dt \end{aligned}$$

$$= \int \frac{1}{1+t} \cdot 2t\,dt = 2\int \frac{t}{1+t}\,dt = 2\int \frac{t+1-1}{1+t}\,dt$$

$$= 2\int \frac{t+1}{1+t}\,dt - 2\int \frac{1}{1+t}\,dt = 2\int dt - 2\int \frac{1}{1+t}\,dt = 2t - 2\ln|1+t| + c =$$

$$= 2\sqrt{e^x} - 2\ln|1+\sqrt{e^x}| + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int \sqrt{e^x - 1}\, dx \qquad \sqrt{e^x-1} = t$$

$$e^x - 1 = t^2$$

$$e^x = t^2 + 1$$

$$x = \ln(t^2+1)$$

$$dx = 2t \cdot \frac{1}{t^2+1}\, dt$$

$$= \int t \cdot \frac{2t}{t^2+1}\, dt = 2\int \frac{t^2}{t^2+1}\, dt = 2\int \frac{t^2+1-1}{t^2+1}\, dt = 2\int \frac{\cancel{t^2+1}}{\cancel{t^2+1}}\, dt - 2\int \frac{1}{t^2+1}\, dt =$$

$$= 2\int dt - 2\int \frac{1}{t^2+1}\, dt = 2t - 2\operatorname{arctg} t + c = 2\sqrt{e^x-1} - 2\operatorname{arctg}\sqrt{e^x-1} + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int \frac{1+e^{\sqrt{x}}}{\sqrt{x}}\, dx \qquad \sqrt{x} = t$$

$$x = t^2$$

$$dx = 2t\, dt$$

$$= \int \frac{1+e^t}{\cancel{t}} \cdot 2\cancel{t}\, dt = 2\int 1 + e^t\, dt = 2\int dt + 2\int e^t\, dt = 2t + 2e^t + c$$

$$= 2\sqrt{x} + 2e^{\sqrt{x}} + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int \frac{1}{x\sqrt{2x-1}}\, dx \qquad \sqrt{2x-1} = t$$

$$2x - 1 = t^2$$

$$2x = t^2 + 1$$

$$x = \frac{t^2+1}{2} \qquad \frac{t^2}{2} + \frac{1}{2} \xrightarrow{f'} \frac{1}{\cancel{2}} \cdot \cancel{2}t\, dt$$

$$dx = t\, dt$$

$$= \int \frac{1}{\frac{t^2+1}{2} \cdot \cancel{t}} \cdot \cancel{t}\, dt = \int \frac{2}{t^2+1}\, dt = 2\int \frac{1}{t^2+1}\, dt = 2\operatorname{arctg} t + c$$

$$= 2\operatorname{arctg}(\sqrt{2x-1}) + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int \cos^4 x \, \text{sen}^3 x \, dx$$

$$\cos x = t$$
$$-\text{sen}\, x \, dx = dt$$

$$= \int \cos^4 x \cdot \text{sen}\, x \left(1-\cos^2 x\right) dx = \int \left(\cos^4 x \cdot \text{sen}\, x - \cos^6 x \, \text{sen}\, x\right) dx =$$

$$= \int t^4 \, \text{sen}\, x \, dx - \int t^6 \, \text{sen}\, x \, dx = \int -t^4 \, dt + \int t^6 \, dt = -\frac{t^5}{5} + \frac{t^7}{7} + c =$$

$$= -\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int \text{sen}^4 x \cdot \cos^3 x \, dx$$

$$\text{sen}\, x = t$$
$$\cos x \, dx = dt$$

$$= \int \text{sen}^4 x \cdot \cos x \left(1-\text{sen}^2 x\right) dx = \int \left(\text{sen}^4 x \cos x - \text{sen}^6 x \cos x\right) dx =$$

$$= \int \text{sen}^4 x \cos x \, dx - \int \text{sen}^6 x \cos x \, dx = \int t^4 \, dt - \int t^6 \, dt = \frac{t^5}{5} - \frac{t^7}{7} + c =$$

$$= \frac{\text{sen}^5 x}{5} - \frac{\text{sen}^7 x}{7} + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int \sqrt{5+x} \, dx$$

$$\sqrt{5+x} = t$$
$$5+x = t^2$$
$$x = t^2 - 5$$
$$dx = 2t \, dt$$

$$(5+x)^{\frac{1}{2} \cdot 3}$$

$$= \int t \cdot 2t \, dt = 2\int t^2 \, dt = 2\,\frac{t^3}{3} + c = \frac{2}{3}(5+x)^{\frac{3}{2}} + c = \frac{2}{3}\sqrt{(5+x)^3} + c = \frac{2}{3}(5+x)\sqrt{5+x}$$

Esercizio

$$\int \sqrt[3]{4-5x}\,dx \qquad \sqrt[3]{4-5x} = t$$

$$4-5x = t^3$$

$$-5x = t^3-4$$

$$x = -\frac{1}{5}(t^3-4)$$

$$dx = -\frac{3}{5}t^2\,dt$$

$$= \int t\left(-\frac{3}{5}t^2\right)dt = -\frac{3}{5}\int t^3\,dt = -\frac{3}{5}\cdot\frac{t^4}{4}+c = -\frac{3}{5}\cdot\frac{(4-5x)^{\frac{4}{3}}}{4}+c =$$

$$= \left(-\frac{3}{5}\right)\frac{\sqrt[3]{(4-5x)^4}}{4}+c,\quad c\in\mathbb{R}$$

Esercizio

$$\int \operatorname{sen}\sqrt{x}\,dx \qquad \sqrt{x} = t$$

$$x = t^2$$

$$dx = 2t\,dt$$

$$= \int \operatorname{sen} t \; 2t\,dt = 2\int t\cdot\operatorname{sen} t\,dt = t\cdot(-\cos t) - \int -\cos t\,dt = -t\cos t + \int \cos t\,dt =$$

$$= -2t\cos t + 2\operatorname{sen} t + c = -2\sqrt{x}\cos\sqrt{x} + 2\operatorname{sen}\sqrt{x} + c,\quad c\in\mathbb{R}$$

Esercizio

$$\int \frac{1}{\sqrt[3]{(5x-1)^2}}\,dx \qquad 5x-1 = t$$

$$5x = t+1$$

$$x = \frac{1}{5}(t+1)$$

$$dx = \frac{1}{5}\,dt$$

$$= \int \frac{1}{\sqrt[3]{t^2}}\cdot\frac{1}{5}\,dt = \frac{1}{5}\int t^{-\frac{2}{3}}\,dt = \frac{1}{5}\cdot\frac{t^{1/3}}{1/3}+c = \frac{1}{5}\cdot 3t^{1/3}+c =$$

$$= \frac{3}{5}\sqrt[3]{5x-1}+c,\quad c\in\mathbb{R}$$

Esercizio

$$\int \frac{2x+1}{(x-2)^2}\,dx \qquad \begin{aligned} x-2 &= t \\ x &= t+2 \\ dx &= dt \end{aligned}$$

$$= \int \frac{2(t+2)+1}{t^2}\,dt = \int \frac{2t+4+1}{t^2}\,dt = \int \frac{2t+5}{t^2}\,dt = \int \frac{2t}{t^2}\,dt + 5\int \frac{dt}{t^2} =$$

$$= \int \frac{2t}{t^2}\,dt + 5\int t^{-2}\,dt = \ln(t^2) + 5\cdot\left(\frac{t^{-1}}{-1}\right) + c = \ln(x-2)^2 - \frac{5}{x-2} + c =$$

$$= 2\ln|x+2| - \frac{5}{x-2} + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int \frac{1+\cos\sqrt{x}}{\sqrt{x}}\,dx \qquad \begin{aligned} \sqrt{x} &= t \\ x &= t^2 \\ dx &= 2t\,dt \end{aligned}$$

$$= \int \frac{1+\cos t}{\cancel{t}} \cdot 2\cancel{t}\,dt = 2\int (1+\cos t)\,dt = 2\int dt + 2\int \cos t\,dt =$$

$$= 2t + 2\,\mathrm{sen}\,t + c = 2\sqrt{x} + 2\,\mathrm{sen}\sqrt{x} + c, \quad c \in \mathbb{R}$$

Esercizio

Integrazione per parti

$$\int \ln x \, dx$$

$$= \int \ln x \cdot 1 dx =$$

$$= \ln x \cdot x - \int \cancel{x} \cdot \frac{1}{\cancel{x}} dx = x \ln x - x + c = x(\ln x + 1) + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int e^x \cdot x \, dx =$$

$$= x \cdot e^x - \int e^x dx = xe^x - e^x + c = e^x(x-1) + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int e^x \cdot x^2 \, dx =$$

$$= x^2 \cdot e^x - \int e^x \, 2x \, dx$$

$$= x^2 e^x - 2 \int e^x \cdot x dx$$

$$\hookrightarrow \int e^x \cdot x dx =$$

$$= x \cdot e^x - \int e^x \cdot 1 dx = xe^x - e^x$$

$$= x^2 e^x - 2\left[xe^x - e^x\right] + c$$

$$= x^2 e^x - 2xe^x - 2e^x + c$$

$$= e^x(x^2 - 2x - 2) + c, \quad c \in \mathbb{R}.$$

Esercizio

$$\int e^x \cdot x^3\,dx$$

$$= x^3 \cdot e^x - \int e^x \cdot 3x^2\,dx$$

$$= x^3 e^x - 3\int e^x \cdot x^2\,dx$$

$$\hookrightarrow x^2 \cdot e^x - \int e^x\, 2x\,dx = x^2 e^x - 2\int e^x \cdot x\,dx$$

$$\int e^x\, x\,dx = x\, e^x - \int e^x \cdot 1\,dx = x e^x - e^x$$

$$= x^3 e^x - 3\left[x^2 e^x - 2\left(x e^x - e^x\right)\right] + c$$

$$= x^3 e^x - 3\left[x^2 e^x - 2x e^x + 2e^x\right] + c$$

$$= x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + c$$

$$= e^x\left(x^3 - 3x^2 + 6x - 6\right) + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int x \cdot \operatorname{sen} x\,dx$$

$$= x \cdot (-\cos x) - \int -\cos x \cdot 1\,dx = -x\cos x + \int \cos x\,dx =$$

$$= -x\cos x + \operatorname{sen} x + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int x \cdot \cos x\,dx$$

$$= x \cdot (\operatorname{sen} x) - \int \operatorname{sen} x \cdot dx$$

$$= x \operatorname{sen} x + \cos x + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int e^x \cdot \operatorname{sen} x \, dx$$

$$= e^x \cdot (-\cos x) - \int -\cos x \, e^x \, dx = -e^x \cos x + \int \cos x \cdot e^x \, dx$$

$$\int \cos x \cdot e^x \, dx = e^x \cdot \operatorname{sen} x - \int \operatorname{sen} x \cdot e^x \, dx$$

$$\int e^x \operatorname{sen} x \, dx = -e^x \cos x + e^x \operatorname{sen} x - \int \operatorname{sen} x \cdot e^x \, dx$$

$$2 \int e^x \operatorname{sen} x \, dx = e^x (\operatorname{sen} x - \cos x)$$

$$\int e^x \operatorname{sen} x \, dx = \frac{e^x}{2} (\operatorname{sen} x - \cos x) + c, \quad c \in \mathbb{R}$$

Altro modo:

$$\int e^x \operatorname{sen} x \, dx = \operatorname{sen} x \, e^x - \int e^x \cos x \, dx$$

$$\int e^x \cos x \, dx = \cos x \cdot e^x - \int e^x (-\operatorname{sen} x) \, dx = \cos x \, e^x + \int e^x \operatorname{sen} x \, dx$$

$$\int e^x \operatorname{sen} x \, dx = \operatorname{sen} x \, e^x - \left[\cos x \cdot e^x + \int e^x \operatorname{sen} x \, dx \right]$$

$$\int e^x \operatorname{sen} x \, dx = \operatorname{sen} x \, e^x - \cos x \, e^x - \int e^x \operatorname{sen} x \, dx$$

$$2 \int e^x \operatorname{sen} x \, dx = \operatorname{sen} x \, e^x - \cos x \, e^x$$

$$\int e^x \operatorname{sen} x \, dx = \frac{e^x}{2} (\operatorname{sen} x - \cos x) + c, \quad c \in \mathbb{R}$$

Poiché entrambe le funzioni e^x ed $\operatorname{sen} x$ sono continue, possiamo vedere come funzione $g'(x)$ entrambe le due funzioni.

Esercizio

$$\int e^x \cdot \cos x \, dx$$

$$= \cos x \cdot e^x - \int e^x \left(-\operatorname{sen} x\right) dx = \cos x \, e^x + \int e^x \operatorname{sen} x \, dx$$

$$\int e^x \operatorname{sen} x \, dx = \operatorname{sen} x \cdot e^x - \int e^x \cos x \, dx$$

$$\int e^x \cos x \, dx = \cos x \, e^x + \operatorname{sen} e^x - \int e^x \cos x \, dx$$

$$2 \int e^x \cos x \, dx = e^x \left(\cos x + \operatorname{sen} x\right)$$

$$\int e^x \cos x \, dx = \frac{e^x}{2} \left(\cos x + \operatorname{sen} x\right) + c, \; c \in \mathbb{R}$$

Esercizio

$$\int \operatorname{arctg} x \, dx$$

$$= \int \operatorname{arctg} x \cdot 1 \, dx = \operatorname{arctg} x \cdot x - \int x \cdot \frac{1}{1+x^2} \, dx$$

$$= \operatorname{arctg} x \cdot x - \frac{1}{2} \int \frac{2x}{1+x^2} \, dx =$$

$$= x \operatorname{arctg} x - \frac{1}{2} \ln\left(1+x^2\right) + c, \; c \in \mathbb{R}$$

Esercizio

$$\int x \cdot \operatorname{arctg} x \, dx$$

$$= \operatorname{arctg} x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{1+x^2} \, dx$$

$$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{x^2 +1-1}{1+x^2} \, dx$$

$$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \cancel{\frac{x^2+1}{1+x^2}} \, dx + \frac{1}{2} \int \frac{1}{1+x^2} \, dx$$

$$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int dx + \frac{1}{2} \int \frac{1}{1+x^2} \, dx$$

$$= \frac{x^2}{2} \operatorname{arctg} x - \frac{x}{2} + \frac{1}{2} \operatorname{arctg} x + c = \frac{1}{2}\left(x^2 \operatorname{arctg} x - x + \operatorname{arctg} x\right) + c, \; c \in \mathbb{R}$$

Esercizio

$$\int e^{5x}\,\text{sen}\,3x\,dx$$

$$= \text{sen}\,3x \cdot \frac{e^{5x}}{5} - \int \frac{e^{5x}}{5} \cdot \cos 3x \cdot 3\,dx$$

$$= \text{sen}\,3x \cdot \frac{e^{5x}}{5} - \frac{3}{5}\int e^{5x} \cdot \cos 3x\,dx$$

$$\int e^{5x} \cos 3x\,dx = \cos 3x \cdot \frac{e^{5x}}{5} - \int \frac{e^{5x}}{5} \cdot (-\text{sen}\,3x) \cdot 3\,dx$$

$$= \cos 3x \cdot \frac{e^{5x}}{5} + \frac{3}{5}\int e^{5x} \cdot \text{sen}\,3x\,dx$$

$$\int e^{5x} \cdot \text{sen}\,3x\,dx = \text{sen}\,3x \cdot \frac{e^{5x}}{5} - \frac{3}{5}\left[\cos 3x \cdot \frac{e^{5x}}{5} + \frac{3}{5}\int e^{5x}\,\text{sen}\,3x\,dx\right]$$

$$\int e^{5x} \cdot \text{sen}\,3x\,dx = \text{sen}\,3x \cdot \frac{e^{5x}}{5} - \frac{3}{25}\cos 3x \cdot e^{5x} - \frac{9}{25}\int e^{5x}\,\text{sen}\,3x\,dx$$

$$\left(1+\frac{9}{25}\right)\int e^{5x}\,\text{sen}\,3x\,dx = \frac{e^{5x}}{5}\left(\text{sen}\,3x - \frac{3}{5}\cos 3x\right)$$

$$\frac{34}{25}\int e^{5x}\,\text{sen}\,3x\,dx = \frac{e^{5x}}{5}\left(\text{sen}\,3x - \frac{3}{5}\cos 3x\right)$$

$$\int e^{5x}\,\text{sen}\,3x\,dx = \frac{\cancel{25}^{\,5}}{34} \cdot \frac{e^{5x}}{\cancel{5}_{\,1}}\left(\text{sen}\,3x - \frac{3}{5}\cos 3x\right)$$

$$\int e^{5x}\,\text{sen}\,3x\,dx = \frac{5e^{x}}{34}\left(\text{sen}\,3x - \frac{3}{5}\cos 3x\right) + c,\quad c \in \mathbb{R}$$

Esercizio

$$\int \operatorname{arctg}\sqrt{x}\, dx \qquad \begin{aligned} \sqrt{x} &= t \\ x &= t^2 \\ dx &= 2t\,dt \end{aligned}$$

$$\int \operatorname{arctg} t \cdot 2t\, dt = 2\int \operatorname{arctg} t \cdot t\, dt$$

$$= 2\left(\operatorname{arctg} t \cdot \frac{t^2}{2} - \int \frac{t^2}{2} \cdot \frac{1}{1+t^2}\, dt \right) =$$

$$= 2\left(\operatorname{arctg} t \cdot \frac{t^2}{2} - \frac{1}{2}\int \frac{t^2+1-1}{1+t^2}\, dt \right) =$$

$$= 2\left(\operatorname{arctg} t \cdot \frac{t^2}{2} - \frac{1}{2}\int \frac{t^2+\cancel{1}}{\cancel{1}+t^2}\, dt + \frac{1}{2}\int \frac{1}{1+t^2}\, dt \right) =$$

$$= 2\left(\operatorname{arctg} t \cdot \frac{t^2}{2} - \frac{t}{2} + \frac{1}{2}\operatorname{arctg} t \right) =$$

$$= \operatorname{arctg} t \cdot t^2 - t + \operatorname{arctg} t$$

$$= x \cdot \operatorname{arctg}\sqrt{x} - \sqrt{x} + \operatorname{arctg}\sqrt{x} + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int x \cdot \ln x\, dx = \frac{x^2}{2} \cdot \ln x - \int \frac{x^2}{2} \cdot \frac{1}{x}\, dx$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\int x\, dx = \frac{1}{2} x \left(x \ln x - 1 \right) + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int x \cdot \operatorname{sen} x\, dx = x \cdot (-\cos x) - \int -\cos x\, dx =$$

$$= -x\cos x + \int \cos x\, dx = -x\cos x + \operatorname{sen} x + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int x^2 \cdot \ln x\, dx = \frac{x^3}{3} \ln x - \int \frac{x^3}{3} \cdot \frac{1}{x}\, dx$$

$$= \frac{x^3}{3} \cdot \ln x - \frac{1}{3}\int x^2\, dx = \frac{x^3}{3} \cdot \ln x - \frac{1}{3} \cdot \frac{x^3}{3} = \frac{x^3}{3}\left(\ln x - \frac{1}{3} \right) + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int x \cdot e^{2x} = x \cdot \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2}\, dx = \frac{xe^{2x}}{2} - \frac{1}{2}\int e^{2x}\, dx = \frac{xe^{2x}}{2} - \frac{1}{2} \cdot \frac{e^{2x}}{2} =$$

$$= \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int x^2 \cdot \operatorname{sen} x\, dx = x^2 \cdot (-\cos x) - \int -\cos x \cdot 2x\, dx$$

$$= -x^2 \cos x + 2 \int \cos x \cdot x\, dx$$

$$\hookrightarrow x \cdot \operatorname{sen} x - \int \operatorname{sen} x\, dx = x \operatorname{sen} x + \cos x$$

$$= -x^2 \cos x + 2\left(x \operatorname{sen} x + \cos x \right)$$

$$= -x^2 \cos x + 2x \operatorname{sen} x + 2\cos x + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int \operatorname{sen}^2 x\, dx = \int \operatorname{sen} x \cdot \operatorname{sen} x\, dx$$

$$= \operatorname{sen} x \cdot (-\cos x) - \int (-\cos x) \cdot \cos x\, dx$$

$$= -\operatorname{sen} x \cos x + \int \cos^2 x\, dx$$

$$= -\operatorname{sen} x \cos x + \int \left(1 - \operatorname{sen}^2 x \right) dx$$

$$= -\operatorname{sen} x \cos x + \int dx - \int \operatorname{sen}^2 x\, dx$$

$$\int \operatorname{sen}^2 x\, dx = -\operatorname{sen} x \cos x + x - \int \operatorname{sen}^2 x\, dx$$

$$2\int \operatorname{sen}^2 x\, dx = -\operatorname{sen} x \cos x + x$$

$$\int \operatorname{sen}^2 x\, dx = -\frac{1}{2}\left(\operatorname{sen} x \cos x - x \right) + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int e^{3x}\cdot \cos x\, dx$$

$$= \cos x \cdot \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3}\cdot(-\operatorname{sen} x)\, dx = \cos x \cdot \frac{e^{3x}}{3} + \frac{1}{3}\int \underline{e^{3x}\cdot \operatorname{sen} x\, dx}$$

$$\left(\operatorname{sen} x \cdot \frac{e^{3x}}{3} - \frac{1}{3}\int e^{3x}\cdot \cos x\, dx\right)$$

$$= \cos x \frac{e^{3x}}{3} + \frac{1}{3}\left[\operatorname{sen} x \frac{e^{3x}}{3} - \frac{1}{3}\int e^{3x}\cos x\, dx\right] =$$

$$\int e^{3x}\cos x\, dx = \cos x \frac{e^{3x}}{3} + \operatorname{sen} x \frac{e^{3x}}{9} - \frac{1}{9}\int e^{3x}\cos x\, dx =$$

$$\left(1+\frac{1}{9}\right)\int e^{3x}\cos x\, dx = \cos x \frac{e^{3x}}{3} + \operatorname{sen} x \frac{e^{3x}}{9}$$

$$\int e^{3x}\cos x\, dx = \frac{\cancel{9}^{\,3}}{10}\left(\cos x \frac{e^{3x}}{\cancel{3}_{\,1}} + \operatorname{sen} x \frac{e^{3x}}{\cancel{9}_{\,3}}\right)$$

$$= \frac{3}{10}\cos x\, e^{3x} + \frac{1}{30}\operatorname{sen} x\, e^{3x}$$

Esercizio

$$\int \operatorname{sen}(\ln x)\, dx = \operatorname{sen}(\ln x)\cdot x - \int x \cdot \frac{1}{x}\cdot \cos(\ln x)\, dx$$

$$= x \operatorname{sen}(\ln x) - \int \cos(\ln x)\, dx$$

$$\hookrightarrow \cos(\ln x)\cdot x - \int x \cdot \frac{1}{x}\left(-\operatorname{sen}(\ln x)\right) dx$$

$$x\cdot \cos(\ln x) + \int \operatorname{sen}(\ln x)\, dx$$

$$\int \operatorname{sen}(\ln x)\, dx = x \operatorname{sen}(\ln x) - x\cos(\ln x) - \int \operatorname{sen}(\ln x)\, dx$$

$$2\int \operatorname{sen}(\ln x)\, dx = x\left(\operatorname{sen}(\ln x) - \cos(\ln x)\right)$$

$$\int \operatorname{sen}(\ln x)\, dx = \frac{x}{2}\left(\operatorname{sen}(\ln x) - \cos(\ln x)\right) + c,\ c \in \mathbb{R}$$

Esercizio

$$\int e^{2x} \cdot \operatorname{sen} x \, dx$$

$$= \operatorname{sen} x \cdot \frac{e^{2x}}{2} - \frac{1}{2}\int e^{2x} \cdot \cos x \, dx$$

$$\cos x \cdot \frac{e^{2x}}{2} + \frac{1}{2}\int e^{2x} \operatorname{sen} x \, dx$$

$$\int e^{2x} \operatorname{sen} x \, dx = \operatorname{sen} x \cdot \frac{e^{2x}}{2} - \frac{1}{2}\left[\cos x \, \frac{e^{2x}}{2} + \frac{1}{2}\int e^{2x} \operatorname{sen} x \, dx\right]$$

$$= \frac{1}{2}\operatorname{sen} x \cdot e^{2x} - \frac{1}{4}\cos x \, e^{2x} - \frac{1}{4}\int e^{2x} \operatorname{sen} x \, dx$$

$$\left(1+\frac{1}{4}\right)\int e^{2x} \operatorname{sen} x \, dx = \frac{1}{2}\operatorname{sen} x \cdot e^{2x} - \frac{1}{4}\cos x \, e^{2x}$$

$$\int e^{2x} \operatorname{sen} x \, dx = \frac{4}{5}\left(\frac{1}{2}\operatorname{sen} x \cdot e^{2x} - \frac{1}{4}\cos x \, e^{2x}\right)$$

$$= \frac{2}{5} \operatorname{sen} x \, e^{2x} - \frac{1}{10}\cos x \, e^{2x}$$

Esercizio

$$\int \ln^2 x \, dx =$$

$$\int \ln^2 x \cdot 1 \, dx = \ln^2 x \cdot x - \int x \cdot 2\ln x \cdot \frac{1}{x} \, dx$$

$$= \ln^2 x \cdot x - 2\int \ln x \, dx$$

$$\ln x \cdot x - \int x \cdot \frac{1}{x} dx$$

$$x \cdot \ln x - \int dx = x \ln x - x = x(\ln x - 1)$$

$$\int \ln^2 x \, dx = x\ln^2 x - 2x(\ln x - 1) + c, \; c \in \mathbb{R}$$

Esercizio

$$\int \arccos x \, dx = \arccos x \cdot x - \int x \cdot \left(-\frac{1}{\sqrt{1-x^2}}\right) dx = \arccos x \cdot x + \int \frac{x}{\sqrt{1-x^2}} dx =$$

$$= \arccos x \cdot x + \frac{1}{2}\int 2x \cdot (1-x^2)^{-\frac{1}{2}} dx =$$

$$= x\arccos x - \frac{1}{2}\left(\frac{(1-x^2)^{\frac{1}{2}}}{1/2}\right) + c = x\arccos x - \frac{1}{2} \cdot 2\sqrt{1-x^2} = x\arccos x - \sqrt{1-x^2} + c$$

Esercizio

$$\int \cos \ln x \, dx$$

$$= \cos \ln x \cdot x - \int x \cdot \frac{1}{x} \cdot (-\operatorname{sen}(\ln x))\, dx$$

$$= \cos(\ln x) \cdot x + \int \operatorname{sen}(\ln x)\, dx$$

$$\operatorname{sen}(\ln x) \cdot x - \int x \cdot \frac{1}{x} \cdot \cos(\ln x)\, dx$$

$$= \cos(\ln x) \cdot x + x \operatorname{sen}(\ln x) - \int \cos(\ln x)\, dx$$

$$2\int \cos(\ln x)\, dx = x\left(\cos(\ln x) + \operatorname{sen}(\ln x)\right)$$

$$\int \cos(\ln x)\, dx = \frac{x}{2}\left(\cos \ln x + \operatorname{sen} \ln x\right) + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int x^2 \cdot e^{3x}\, dx = x^2 \frac{e^{3x}}{3} - \frac{2}{3}\int e^{3x} \cdot x\, dx$$

$$x \cdot \frac{e^{3x}}{3} - \frac{1}{3}\int e^{3x}\, dx$$

$$= x^2 \frac{e^{3x}}{3} - \frac{2}{3}\left(x \frac{e^{3x}}{3} - \frac{1}{3} \cdot \frac{e^{3x}}{3}\right)$$

$$= \frac{x^2 e^{3x}}{3} - \frac{2x e^{3x}}{9} + \frac{2x e^{3x}}{27}$$

$$\frac{e^{3x}}{3}\left(x^2 - \frac{2}{3}x + \frac{2}{9}x\right)$$

$$\frac{e^{3x}}{3}\left(x^2 + \frac{-6+2}{9}x\right) = \frac{e^{3x}}{3}\left(x^2 + \frac{4}{9}x\right) + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int x^3 \ln x \, dx = \ln x \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x} \, dx$$

$$\ln x \cdot \frac{x^4}{4} - \frac{1}{4} \int x^3 \, dx$$

$$= \ln x \cdot \frac{x^4}{4} - \frac{1}{4} \cdot \frac{x^4}{4} = \ln x \cdot \frac{x^4}{4} - \frac{x^4}{16} = \frac{x^4}{4}\left(\ln x - \frac{1}{4}\right) + c, \; c \in \mathbb{R}$$

Esercizio

$$\int x^3 \cdot \operatorname{sen} x \, dx$$

$$= x^3 \cdot (-\cos x) - \int (-\cos x)\, 3x^2 \, dx$$

$$= -x^3 \cos x + 3 \int \cos x \cdot x^2 \, dx$$

$$x^2 \cdot \operatorname{sen} x - \int \operatorname{sen} x \cdot 2x \, dx$$

$$x^2 \cdot \operatorname{sen} x - 2 \int \operatorname{sen} x \cdot x \, dx$$

$$x \cdot (-\cos x) - \int (-\cos x) \, dx$$

$$-x \cos x + \int \cos x \, dx$$

$$-x \cos x + \operatorname{sen} x$$

$$= -x^3 \cos x + 3\left[x^2 \operatorname{sen} x - 2\left(-x \cos x + \operatorname{sen} x\right)\right]$$

$$= -x^3 \cos x + 3x^2 \operatorname{sen} x + 6x \cos x - 6 \operatorname{sen} x + c, \; c \in \mathbb{R}$$

Esercizio

$$\int \operatorname{sen}^3 x \, dx = \int \operatorname{sen}^2 x \cdot \operatorname{sen} x \, dx$$

$$= \operatorname{sen}^2 x \cdot (-\cos x) - \int -\cos x \cdot 2 \operatorname{sen} x \cos x \, dx$$

$$-\operatorname{sen}^2 x \cos x + 2 \int \operatorname{sen} x \cdot \cos^2 x \, dx$$

NB possiamo vedere $\cos^2 x = \operatorname{sen}^2 x - 1$

$$\cos^2 x \cdot (-\cos x) - \int (-\cos x) \cdot 2 \cos x \, (-\operatorname{sen} x) \, dx$$

$$-\cos^3 x - 2 \int \cos^2 x \cdot \operatorname{sen} x \, dx$$

$$-\operatorname{sen}^2 x \cos x - 2\cos^3 x - 4 \quad \frac{\cos^3 x}{3}$$

Esercizio

$$\int \operatorname{sen}^4 x \, dx =$$

$$\int \operatorname{sen}^3 x \cdot \operatorname{sen} x \, dx = \operatorname{sen}^3 x \cdot (-\cos x) + \int (-\cos x) \cdot 3\operatorname{sen}^2 x \cdot \cos x \, dx$$

$$= -\operatorname{sen}^3 x \cdot \cos x - 3 \int \operatorname{sen}^2 x \cdot \cos^2 x \, dx$$

$$\int \operatorname{sen}^2 x \cdot (1 - \operatorname{sen}^2 x) \, dx$$

$$-\operatorname{sen}^3 x \cdot \cos x - 3 \left(\int \operatorname{sen}^2 x \, dx - \int \operatorname{sen}^4 x \, dx \right)$$

$$-\operatorname{sen}^3 x \cos x - 3 \int \operatorname{sen}^2 dx - 3 \int \operatorname{sen}^4 x \, dx$$

$$\int \operatorname{sen}^2 dx = \int \operatorname{sen} x \cdot \operatorname{sen} x \, dx$$

$$\operatorname{sen} x \cdot (-\cos x) - \int (-\cos x) \cdot \cos x \, dx$$

$$-\operatorname{sen} x \cos x + \int \cos^2 x \, dx$$

$$-\operatorname{sen} x \cos x + \int dx - \int \operatorname{sen}^2 x \, dx$$

$$2 \int \operatorname{sen}^2 x \, dx = -\operatorname{sen} x \cos x + x$$

$$= -\frac{1}{2} \operatorname{sen} x \cos x + \frac{x}{2}$$

$$\int \operatorname{sen}^4 x \, dx = -\operatorname{sen}^3 x \cos x + \frac{3}{2} \operatorname{sen} x \cos x - \frac{3}{2} x - 3 \int \operatorname{sen}^4 x \, dx$$

$$4 \int \operatorname{sen}^4 x \, dx = -\operatorname{sen}^3 x \cos x + \frac{3}{2} \operatorname{sen} x \cos x - \frac{3}{2} x$$

$$\int \operatorname{sen}^4 x \, dx = -\frac{1}{4} \operatorname{sen}^3 x \cos x + \frac{3}{8} \operatorname{sen} x \cos x - \frac{3}{8} x + c, \ c \in \mathbb{R}$$

Esercizio

$$\int \operatorname{sen}^2 x \, dx = \int \left(\frac{1-\cos 2x}{2} \right)^2 dx$$

$$= \int \frac{1+\cos^2 2x - 2\cos 2x}{4} \, dx = \frac{1}{4}\int dx + \frac{1}{4}\int \cos^2 2x \, dx - \frac{2}{4}\int \cos 2x \, dx =$$

$$= \frac{1}{4}\int dx + \frac{1}{4}\int \frac{1+\cos 4x}{2} \, dx - \frac{1}{4}\int 2\cos 2x \, dx$$

$$= \frac{1}{4}\int dx + \frac{1}{8}\int dx + \frac{1}{8}\int \cos 4x \, dx - \frac{1}{4}\int 2\cos 2x \, dx$$

$$= \frac{1}{4}x + \frac{1}{8}x + \frac{1}{32}\int 4\cos 4 \, dx - \frac{1}{4}\int 2\cos 2x \, dx$$

$$= \frac{12}{32}x + \frac{1}{32}\operatorname{sen}^4 x - \frac{1}{4}\operatorname{sen} 2x + c$$

ho utilizzato la formula di bisezione

$$\operatorname{sen}^2 x = \frac{1-\cos 2x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

Esercizio

$$\int \sqrt{1-x^2}\,dx = \int (1-x^2)^{\frac{1}{2}}\,dx$$

$$= (1-x^2)^{\frac{1}{2}} \cdot x - \int x \cdot \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot (-2x)\,dx$$

$$= (1-x^2)^{\frac{1}{2}} \cdot x + \frac{\cancel{2}}{\cancel{2}} \int x^2 \cdot \frac{1}{\sqrt{1-x^2}}\,dx$$

$$x^2 \cdot \arcsin x - \int \arcsin x \cdot 2x\,dx$$

$$x^2 \cdot \arcsin x - 2 \int \arcsin x \cdot x\,dx$$

$$\int \arcsin x\,dx = \arcsin x \cdot x - \left(-\frac{1}{2}\right) \int -2x \cdot \frac{1}{\sqrt{1-x^2}}\,dx =$$

$$= \arcsin x \cdot x + \frac{1}{2} \cdot \frac{(1-x^2)^{1/2}}{1/2}$$

$$= x \arcsin x + (1-x^2)^{1/2}$$

$$\int \arcsin x \cdot x = x\left(x \arcsin x + (1-x^2)^{\frac{1}{2}}\right) - \int x \cdot \arcsin x + (1-x^2)^{\frac{1}{2}} \cdot 1\,dx =$$

$$= \frac{1}{2}x\left(x \arcsin x + (1-x^2)^{\frac{1}{2}}\right) - \arcsin x$$

$$\int \sqrt{1-x^2}\,dx = (1-x^2)^{\frac{1}{2}} \cdot x + \arcsin x \cdot x^2 - 2\left[\frac{1}{2}x\left(\arcsin x \cdot x \cdot (1-x^2)^{\frac{1}{2}}\right) - \arcsin x\right] + c$$

Esercizio

$$\int \cos^4 x \, dx = \int \cos^3 x \cdot \cos x \, dx$$

$$= \cos^3 x \cdot \operatorname{sen} x - \int \operatorname{sen} x \cdot 3\cos^2 x \cdot (-\operatorname{sen} x) \, dx$$

$$= \cos^3 x \cdot \operatorname{sen} x + 3 \int \operatorname{sen}^2 x \cos^2 x \, dx$$

$$\int \operatorname{sen}^2 x \cos^2 x = \int (1-\cos^2 x)\cos^2 x \, dx = \int \cos^2 x \, dx - \int \cos^4 x \, dx$$

$$\int \cos^2 x \, dx = \int \cos x \cdot \cos x \, dx$$

$$\cos x \cdot \operatorname{sen} x - \int \operatorname{sen} x \cdot (-\operatorname{sen} x) \, dx = \cos x \operatorname{sen} x + \int \operatorname{sen}^2 x \, dx$$

$$= \cos x \operatorname{sen} x + \int (1-\cos^2 x) \, dx =$$

$$= \cos x \operatorname{sen} x + \int dx - \int \cos^2 x \, dx$$

$$2\int \cos^2 x \, dx = \cos x \operatorname{sen} x + x$$

$$\int \cos^2 x \, dx = \frac{1}{2} \cos x \operatorname{sen} x + \frac{x}{2}$$

$$\int \cos^4 x \, dx = \cos^3 x \operatorname{sen} x + 3\left[\frac{1}{2} \cos x \operatorname{sen} x + \frac{1}{2} x - \int \cos^4 x \, dx \right]$$

$$4\int \cos^4 x \, dx = \cos^3 x \operatorname{sen} x + \frac{3}{2} \cos x \operatorname{sen} x + \frac{3}{2} x$$

$$\int \cos^4 x \, dx = \frac{1}{4} \cos^3 x \operatorname{sen} x + \frac{3}{8} \cos x \operatorname{sen} x + \frac{3}{8} x + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int e^{-4x} \cdot \cos\frac{x}{3}\,dx$$

$$= \cos\frac{x}{3} \cdot \left(\frac{e^{-4x}}{-4}\right) + \int \frac{e^{-4x}}{4}\left(-\operatorname{sen}\frac{x}{3}\cdot\frac{1}{3}\right)dx =$$

$$= -\cos\frac{x}{3}\cdot\frac{e^{-4x}}{4} - \frac{1}{12}\int e^{-4x}\operatorname{sen}\frac{x}{3}\,dx$$

$$\operatorname{sen}\frac{x}{3}\cdot\left(-\frac{e^{-4x}}{4}\right) - \int\left(-\frac{e^{-4x}}{4}\right)\cdot\frac{1}{3}\cos\frac{x}{3}\,dx$$

$$-\operatorname{sen}\frac{x}{3}\cdot\frac{e^{-4x}}{4} + \frac{1}{12}\int \underline{\underline{e^{-4x}\cdot\cos\frac{x}{3}\,dx}}$$

$$\int e^{-4x}\cos\frac{x}{3}\,dx = -\frac{e^{-4x}}{4}\cos\frac{x}{3} - \frac{1}{12}\left[-\frac{e^{-4x}}{4}\operatorname{sen}\frac{x}{3} + \frac{1}{12}\int e^{-4x}\cos\frac{x}{3}\,dx\right]$$

$$= \frac{-e^{-4x}}{4}\cos\frac{x}{3} + \frac{1}{48}e^{-4x}\operatorname{sen}\frac{x}{3} - \frac{1}{144}\int e^{-4x}\cos\frac{x}{3}\,dx$$

$$\left(1+\frac{1}{144}\right)\int e^{-4x}\cos\frac{x}{3}\,dx = \left(-\frac{e^{-4x}}{4}\cos\frac{x}{3} + \frac{1}{48}e^{-4x}\operatorname{sen}\frac{x}{3}\right)$$

$$\int e^{-4x}\cos\frac{x}{3}\,dx = \frac{144}{145}\left(\ldots\ldots\right) + c,\ c\in\mathbb{R}$$

Formule parametriche (dell'arcometà)

$$\operatorname{sen} x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$\operatorname{tg} x = \frac{2t}{1-t^2}$$

dove $t = \operatorname{tg}\frac{x}{2}$

Esercizio

$$\int \frac{1}{\operatorname{sen} x}\, dx$$

$$t = \operatorname{tg}\frac{x}{2}$$
$$\operatorname{arctg} t = \frac{x}{2}$$
$$2 \operatorname{arctg} t = x$$
$$2 \cdot \frac{1}{1+t^2}\, dt = dx$$

$$= \int \frac{1}{\frac{2t}{1+t^2}} \cdot 2 \cdot \frac{1}{1+t^2}\, dt$$

$$= \int \frac{1}{t}\, dt = \ln|t| + c = \ln\left|\operatorname{tg}\frac{x}{2}\right| + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int \frac{1}{1+\operatorname{sen} x}\, dx$$

$$= \int \frac{1}{1+\frac{2t}{1+t^2}} \cdot \frac{2\,dt}{1+t^2} = 2\int \frac{1}{\frac{1+t^2+2t}{1+t^2}} \cdot \frac{dt}{1+t^2} = 2\int \frac{1+t^2}{(1+t)^2} \cdot \frac{dt}{1+t^2} =$$

$$= 2\int \frac{1}{(1+t)^2}\, dt = \frac{-2}{1+t} + c = \frac{-2}{1+\operatorname{tg}\frac{x}{2}} + c, \quad c \in \mathbb{R}$$

$$\left[\int (1+t)^{-2}\, dt = \frac{(1+t)^{-1}}{-1} = \frac{-1}{(1+t)} \right]$$

Esercizio

$$\int \sqrt{1-x^2}\, dx$$

$$x = \operatorname{sen} t$$
$$dx = \cos t \cdot dt$$

$$\int \sqrt{1-\operatorname{sen}^2 t} \cdot \cos t \, dt =$$

$$= \int \cos t \cdot \cos t \, dt \qquad \left(NB \ \sqrt{a^2+b^2} = a \operatorname{sen} t\right)$$

$$= \int \cos^2 t \, dt = \int \frac{1+\cos 2t}{2} dt \qquad (\text{Formule Goniometriche})$$

$$= \frac{1}{2}\int dt + \frac{1}{2}\int \cos 2t \, dt = \frac{1}{2}\int dt + \frac{1}{4}\int 2\cos 2t \, dt =$$

$$= \frac{1}{2}t + \frac{1}{4}\operatorname{sen} 2t + c \qquad \int f'(x) \cdot \cos f(x) \, dx = \operatorname{sen} f(x) + c$$

$$= \frac{1}{2}t + \frac{1}{4} \cdot 2 \operatorname{sen} t \cdot \cos t$$

$$= \frac{1}{2}t + \frac{1}{2} \operatorname{sen} t \cos t + c$$

$$= \frac{1}{2} \operatorname{arcsen} x + \frac{1}{2} x \sqrt{1-x^2} + c, \ c \in \mathbb{R}$$

$$t = \operatorname{arcsen} x$$

Esercizio

$$\int \frac{1}{\operatorname{sen}^2 x \cos^2 x} dx$$

$$tg\, x = t$$
$$\frac{1}{\cos^2 x} dx = dt$$

$$NB. \ \operatorname{sen}^2 x = \frac{tg^2 x}{1+tg^2 x} = \frac{t^2}{1+t^2}$$

Vedi formule goniometriche che esprimono il seno in funzione della tangente

$$= \int \frac{1}{\frac{t^2}{1+t^2}} dt$$

$$= \int \frac{1+t^2}{t^2} dt = \int \frac{1}{t^2} dt + \int \frac{t^2}{t^2} dt = \frac{t^{-2+1}}{-2+1} + t + c = -\frac{1}{t} + t + c =$$

$$= -\frac{1}{tg\, x} + tg\, x + c, \ c \in \mathbb{R}$$

Esercizio

$$\int \sqrt[5]{6-7x}\ dx$$

$$\sqrt[5]{6-7x} = t$$
$$6-7x = t^5$$
$$-7x = t^5 - 6$$
$$7x = 6 - t^5$$
$$x = \frac{6}{7} - \frac{t^5}{7}$$

$$dx = -\frac{5}{7}t^4\,dt$$

$$\int t\cdot\left(-\frac{5}{7}t^4\right)dt = \int -\frac{5}{7}t^5\,dt = -\frac{5}{7}\int t^5\,dt = -\frac{5}{7}\cdot\frac{t^6}{6} + c$$

$$= -\frac{5}{42}(6-7x)^{\frac{6}{5}} + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{1}{x\sqrt{1-\ln^2 x}}\,dx$$

$$\ln x = t$$
$$\frac{1}{x}dx = dt$$

$$= \int \frac{1}{\sqrt{1-t^2}}\,dt = \operatorname{arcsen} t + c = \operatorname{arcsen}(\ln x) + c,\ c\in\mathbb{R}$$

Altro modo:

$$\ln x = t$$
$$e^{\ln x} = e^t$$
$$x = e^t$$
$$dx = e^t\,dt$$

$$= \int \frac{1}{\cancel{e^t}\sqrt{1-t^2}}\cancel{e^t}\,dt = \operatorname{arcsen} t + c = \operatorname{arcsen}(\ln x) + c,\ c\in\mathbb{R}$$

Integrali Indefiniti

Esercizio

$$\int dx = \int 1\, dx = x + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int \operatorname{sen} x\, dx = -\cos x + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int e^x\, dx = e^x + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int \operatorname{tg} x\, dx = -\ln|\cos x| + c, \quad c \in \mathbb{R}$$

$$\int \operatorname{tg} x\, dx = \int \frac{\operatorname{sen} x}{\cos x}\, dx = -\int \frac{-\operatorname{sen} x}{\cos x}\, dx = -\ln|\cos x| + c = \ln|\cos x|^{-1} + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int \frac{x}{x^2+1}\, dx = \frac{1}{2}\int \frac{2x}{x^2+1}\, dx = \frac{1}{2}\ln|x^2+1| + c = \ln\sqrt{x^2+1} + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \operatorname{arcsen} x + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int \frac{1}{1+x^2}\, dx = \operatorname{arctg} x + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int \ln^4 x \cdot \frac{1}{x}\, dx = \frac{1}{5}\ln^5 x + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int \frac{e^{2x}}{\sqrt{1-e^{4x}}}\, dx = \frac{1}{2}\int \frac{2e^{2x}}{\sqrt{1-(e^{2x})^2}}\, dx = \frac{1}{2}\operatorname{arcsen}(e^{2x}) + c, \quad c \in \mathbb{R}$$

Esercizio

$$\int 5\,\text{sen}\,5x\,dx = -\cos 5x + c,\ c \in \mathbb{R}$$

Esercizio

$$\int \cos 3x \cdot 3\,dx = \text{sen}\,3x + c,\ c \in \mathbb{R}$$

Esercizio

$$\int \cos 3x\,dx = \frac{1}{3}\,\text{sen}\,3x + c,\ c \in \mathbb{R}$$

Esercizio

$$\int \text{sen}\,5x\,dx = -\frac{1}{5}\cos 5x + c,\ c \in \mathbb{R}$$

Esercizio

$$\int e^{-x}\,dx = -e^{-x} + c,\ c \in \mathbb{R}$$

Esercizio

$$\int (x^2+1)^2 \cdot 2x\,dx = \frac{(x^2+1)^3}{3} + c,\ c \in \mathbb{R}$$

Esercizio

$$\int \frac{3x}{x^2+1}\,dx = \frac{3}{2}\int \frac{2x}{x^2+1}\,dx = \frac{3}{2}\ln(x^2+1) + c,\ c \in \mathbb{R}$$

Esercizio

$$\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \frac{x^{3/2}}{3/2} + c = \frac{2}{3}x^{3/2} + c,\ c \in \mathbb{R}$$

Esercizio

$$\int (x^2+5x-6)\,dx = \int x^2\,dx + 5\int x\,dx - 6\int dx = \frac{x^3}{3} + \frac{5}{2}x^2 - 6x + c,\ c \in \mathbb{R}$$

Esercizio

$$\int \frac{\frac{1}{x}}{\sqrt{1-(\ln x)^2}}\,dx = \text{arcsen}(\ln x) + c,\ c \in \mathbb{R}$$

Esercizio

$$\int \frac{2x}{x^2+3}\,dx = \ln(x^2+3) + c,\ c \in \mathbb{R}$$

Esercizio

$$\int \frac{3x}{2x^2+5}\,dx = 3\int \frac{x}{2x^2+5}\,dx = \frac{3}{4}\int \frac{4x}{2x^2+5}\,dx = \frac{3}{4}\ln(2x^2+5)+c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{x}{x+1}\,dx = \int \frac{x+1-1}{x+1}\,dx = \int\left(\frac{x+1}{x+1}-\frac{1}{x+1}\right)dx = \int \frac{x+1}{x+1}\,dx - \int \frac{1}{x+1}\,dx = \int dx - \int \frac{1}{x+1}\,dx =$$

$$= x - \ln|x+1| + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{1}{e^x+1}\,dx = \int \frac{1+e^x-e^x}{e^x+1}\,dx = \int\left(\frac{1+e^x}{e^x+1}-\frac{e^x}{e^x+1}\right)dx = \int \frac{1+e^x}{e^x+1}\,dx - \int \frac{e^x}{e^x+1}\,dx =$$

$$= \int dx - \int \frac{e^x}{e^x+1}\,dx = x - \ln(e^x+1) + c,\ c\in\mathbb{R}$$

Esercizio

$$\int\left(\frac{3+2x}{x}\right)^2 dx = \int\left(\frac{3}{x}+2\right)^2 dx = \int\left(\frac{9}{x^2}+4+\frac{12}{x}\right)dx = 9\int x^{-2}\,dx + 4\int dx + 12\int x^{-1}\,dx =$$

$$= 9\cdot\frac{x^{-2+1}}{-2+1} + 4\cdot x + 12\ln|x| = -\frac{9}{x} + 4x + 12\ln|x| + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \sqrt{x}\,(1-x^2)\,dx = \int x^{\frac{1}{2}}(1-x^2)\,dx = \int\left(x^{\frac{1}{2}} - x^{\frac{5}{2}}\right)dx = \frac{x^{3/2}}{3/2} - \frac{x^{7/2}}{7/2} =$$

$$= \frac{2}{3}x^{3/2} - \frac{2}{7}x^{7/2} + c = \frac{2}{3}\sqrt{x^3} - \frac{2}{7}\sqrt{x^7} + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{1}{\operatorname{sen}^2 x\cos^2 x}\,dx = \int \frac{\operatorname{sen}^2 x + \cos^2 x}{\operatorname{sen}^2 x\cdot\cos^2 x}\,dx = \int \frac{\cancel{\operatorname{sen}^2 x}}{\cancel{\operatorname{sen}^2 x}\cos^2 x}\,dx + \int \frac{\cancel{\cos^2 x}}{\operatorname{sen}^2 x\cancel{\cos^2 x}}\,dx =$$

$$= \int \frac{1}{\cos^2 x}\,dx + \int \frac{1}{\operatorname{sen}^2 x}\,dx = \operatorname{tg} x - \operatorname{cotg} x + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{e^{\operatorname{arctg} x}}{1+x^2}\,dx = \int e^{\operatorname{arctg} x}\cdot\frac{1}{1+x^2}\,dx = e^{\operatorname{arctg} x} + c$$

Esercizio

$$\int \frac{1+\ln^3 x}{x}\,dx = \int \frac{1}{x}\,dx + \int \frac{\ln^3 x}{x}\,dx = \ln|x| + \frac{1}{4}\ln^4|x| + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{e^{tg\,x}-1}{\cos^2 x}\,dx = \int \frac{e^{tg\,x}}{\cos^2 x}\,dx - \int \frac{1}{\cos^2 x}\,dx = e^{tg\,x} - tg\,x + c,\ c \in \mathbb{R}$$

Esercizio

$$\int \frac{x^4}{1+x^2}\,dx = \int \frac{x^4-1+1}{1+x^2}\,dx = \int \frac{x^4-1}{1+x^2}\,dx + \int \frac{1}{1+x^2}\,dx = \int \frac{\cancel{(x^2+1)}(x^2-1)}{\cancel{x^2+1}}\,dx + \int \frac{1}{1+x^2}\,dx =$$

$$= \int x^2\,dx - \int dx + \int \frac{1}{1+x^2}\,dx = \frac{x^3}{3} - x + arctg\,x + c,\quad c \in \mathbb{R}$$

Esercizio

$$\int \sqrt{1-4x+x^3}\,(3x^2-4)\,dx = \int (1-4x+x^3)^{\frac{1}{2}}\,(3x^2-4)\,dx = \frac{(1-4x+x^3)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c =$$

$$= \frac{(1-4x+x^3)^{\frac{3}{2}}}{3/2} = \frac{2\sqrt{(1-4x+x^3)^3}}{3} + c,\ c \in \mathbb{R}$$

Esercizio

$$\int \cos^3 x\,dx = \int \cos^2 x\,\cos x\,dx = \int (1-sen^2 x)\cos x\,dx = \int \cos x\,dx - \int sen^2 x \cos x\,dx =$$

$$= sen\,x - \frac{1}{3} sen^3 x + c,\ c \in \mathbb{R}$$

Esercizio

$$\int tg^2 x\,dx = \int (tg^2 x + 1 - 1)\,dx = \int (tg^2 x + 1)\,dx - \int dx = tg\,x - x + c,\ c \in \mathbb{R}$$

Esercizio

$$\int \sqrt{\frac{1+x}{1-x}}\,dx = \int \frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot \frac{\sqrt{1+x}}{\sqrt{1+x}} = \int \frac{1+x}{\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}}\,dx + \int \frac{x}{\sqrt{1-x^2}}\,dx =$$

$$= \int \frac{1}{\sqrt{1-x^2}}\,dx - \frac{1}{2}\int -2x\,(1-x^2)^{-\frac{1}{2}}\,dx = arcsen\,x - \frac{1}{2}\,\frac{(1-x^2)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c = arcsen\,x - (1-x^2)^{\frac{1}{2}} + c$$

Esercizio

$$\int \frac{x+2}{x^2-2x+10}\,dx = \frac{1}{2}\int \frac{2x+4}{x^2-2x+10}\,dx = \frac{1}{2}\int \frac{2x+4+2-2}{x^2-2x+10}\,dx = \frac{1}{2}\int \frac{2x-2}{x^2-2x+10}\,dx + \frac{1}{2}\int \frac{6}{x^2-2x+10}\,dx =$$

$$= \frac{1}{2}\int \frac{2x-2}{x^2-2x+10}\,dx + \frac{\cancel{6}^{\,3}}{\cancel{2}}\int \frac{1}{x^2-2x+10}\,dx \Rightarrow 3\int \frac{1}{(x-1)^2+9}\,dx \Rightarrow \frac{3}{9}\int \frac{\frac{1}{9}}{\left(\frac{x-1}{3}\right)^2+1}\,dx \Rightarrow \frac{1}{3}\int \frac{1}{\left(\frac{x-1}{3}\right)^2+1}\,dx$$

$10 = +1+9$

$$\Downarrow$$

$$\int \frac{\frac{1}{3}}{\left(\frac{x-1}{3}\right)^2+1}\,dx$$

$$= \frac{1}{2}\ln(x^2-2x+10) + arctg\left(\frac{x-1}{3}\right) + c$$

Esercizio

$$\int \frac{x+2}{x^2-4x+6}\,dx = \frac{1}{2}\int \frac{2x+4}{x^2-4x+6}\,dx = \frac{1}{2}\int \frac{2x-4+4+4}{x^2-4x+6}\,dx =$$

$$= \frac{1}{2}\int \frac{2x-4}{x^2-4x+6}\,dx + 4\int \frac{1}{x^2-4x+6}\,dx = \frac{1}{2}\int \frac{2x-4}{x^2-4x+6}\,dx + 4\int \frac{1}{\underbrace{x^2-4x-2}_{(x-2)^2}+8}\,dx =$$

$$4\int \frac{1}{(x-2)^2+8}\,dx \Rightarrow 4\int \frac{\frac{1}{8}}{\left(\frac{x-2}{\sqrt{8}}\right)^2+1}\,dx \Rightarrow \frac{1}{2}\,\cancel{\frac{4}{8}}\int \frac{1}{\left(\frac{x-2}{\sqrt{8}}\right)^2+1}\,dx \Longrightarrow \frac{\sqrt{8}}{2}\int \frac{\frac{1}{\sqrt{8}}}{\left(\frac{x-2}{\sqrt{8}}\right)^2+1}\,dx$$

$$\frac{\cancel{2}\sqrt{2}}{\cancel{2}}$$

$$= \frac{1}{2}\ln\left(x^2-4x+6\right) + \sqrt{2}\,\operatorname{arctg}\left(\frac{x-2}{\sqrt{8}}\right) + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{2e^x}{4+e^{2x}}\,dx = 2\int \frac{e^x}{4+(e^x)^2}\,dx = 2\int \frac{e^x\cdot\frac{1}{4}}{1+\left(\frac{e^x}{2}\right)^2}\,dx = \int \frac{e^x\cdot\frac{1}{2}}{1+\left(\frac{e^x}{2}\right)^2}\,dx = \operatorname{arctg}\left(\frac{e^x}{2}\right)+c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{13x^2+1}{x^2+1}\,dx = \int \frac{13x^2}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx = 13\int \frac{x^2+1-1}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx =$$

$$= 13\int \frac{x^2+1}{x^2+1}\,dx - 13\int \frac{1}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx = 13\int dx - 12\int \frac{1}{x^2+1}\,dx = 13x - 12\operatorname{arctg} x + c$$

Esercizio

$$\int \frac{-7x^2}{x^2+1}\,dx = -7\int \frac{x^2}{x^2+1}\,dx = -7\int \frac{x^2+1}{x^2+1}\,dx + 7\int \frac{1}{x^2+1}\,dx = -7\int dx + 7\int \frac{1}{x^2+1}\,dx =$$

$$= -7x + 7\operatorname{arctg} x + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{-5x^2}{3x^2+4}\,dx = -5\int \frac{x^2}{3x^2+4}\,dx = -5\int \frac{x^2}{(\sqrt{3}x)^2+4}\,dx = -5\int \frac{\frac{1}{4}x^2}{\left(\frac{\sqrt{3}}{2}x\right)^2+1}\,dx = -\frac{5}{3}\int \frac{\frac{3}{4}x^2}{\left(\frac{\sqrt{3}}{2}x\right)^2+1}\,dx$$

$$= -\frac{5}{3}\int \frac{\left(\frac{\sqrt{3}}{2}x\right)^2+1-1}{\left(\frac{\sqrt{3}}{2}x\right)^2+1}\,dx = -\frac{5}{3}\int \cancel{\frac{\left(\frac{\sqrt{3}}{2}x\right)^2+1}{\left(\frac{\sqrt{3}}{2}x\right)^2+1}}\,dx + \frac{5}{3}\int \frac{1}{\left(\frac{\sqrt{3}}{2}x\right)^2+1}\,dx =$$

$$= -\frac{5}{3}\int dx + \frac{10}{3\sqrt{3}}\int \frac{\frac{\sqrt{3}}{2}}{\left(\frac{\sqrt{3}}{2}x\right)^2+1}\,dx = -\frac{5}{3}x + \frac{10}{3\sqrt{3}}\operatorname{arctg}\left(\frac{\sqrt{3}}{2}x\right) + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{1}{1-x^2}\,dx$$

$$\frac{1}{1-x^2} = \frac{1}{(1+x)(1-x)} = \frac{A}{(1+x)} + \frac{B}{(1-x)} = \frac{A+Ax+B+Bx}{(1-x)(1+x)}$$

$$\Delta > 0 \Rightarrow 1 = \frac{A+B+x(A-B)}{(1-x)(1+x)} \Longleftrightarrow 1 = A+B+x(A-B)$$

$$\begin{cases} A+B=1 \\ A-B=0 \end{cases} \Rightarrow A=B=\frac{1}{2}$$

$$A+B=1$$
$$B+B=1$$
$$2B=1 \Rightarrow B=\frac{1}{2}$$

$$= \int \frac{\frac{1}{2}}{1-x}\,dx + \int \frac{\frac{1}{2}}{1+x}\,dx$$

$$= -\frac{1}{2}\int \frac{-1}{1-x}\,dx + \frac{1}{2}\int \frac{1}{1+x}\,dx$$

$$= -\frac{1}{2}\ln|1-x| + \frac{1}{2}\ln|1+x| + c,\ c\in\mathbb{R}$$

$$= \ln\sqrt{\left|\frac{1+x}{1-x}\right|} + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{1}{4-x^2}\,dx \qquad \frac{1}{(2-x)(2+x)} = \frac{A(2-x)+B(2+x)}{(2-x)(2+x)} = \frac{2A-Ax+2B+Bx}{(2-x)(2+x)}$$

$$= \int \frac{\frac{1}{4}}{2-x}\,dx + \int \frac{\frac{1}{4}}{2+x}\,dx$$

$$2A+2B+x(B-A)=1$$

$$\begin{cases} B-A=0 \Rightarrow B=A \\ 2A+2B=1 \end{cases}$$

$$2A+2A=1$$
$$4A=1$$
$$A=\frac{1}{4}$$

$$= -\frac{1}{4}\int \frac{-1}{2-x}\,dx + \frac{1}{4}\int \frac{1}{2+x}\,dx$$

$$= -\frac{1}{4}\ln|2-x| + \frac{1}{4}\ln|2+x| + c$$

$$= \ln\sqrt[4]{\left|\frac{2+x}{2-x}\right|} + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{2x+5}{x^2-6x-7}\,dx = \qquad \frac{2x+5}{(x-7)(x+1)} = \frac{A}{(x-7)} + \frac{B}{(x+1)}$$

$$2x+5 = A(x+1) + B(x-7)$$

$$2x+5 = Ax + A + Bx - 7B$$

$$2x+5 = x(A+B) + A - 7B$$

$$\begin{cases} A - 7B = 5 \\ A + B = 2 \Rightarrow A = 2 - B \Rightarrow A = 2 + \frac{3}{8} = \frac{19}{8} \end{cases}$$

$$(2-B) - 7B = 5$$

$$2 - 8B = 5$$

$$-8B = 3 \Rightarrow B = -\frac{3}{8}$$

$$= \int \frac{\frac{19}{8}}{x-7}\,dx + \int \frac{-\frac{3}{8}}{x+1}\,dx$$

$$= \frac{19}{8}\int \frac{1}{x-7}\,dx - \frac{3}{8}\int \frac{1}{x+1}\,dx$$

$$= \frac{19}{8}\ln|x-7| - \frac{3}{8}\ln|x+1| + c$$

$$= \ln\sqrt[8]{|x-7|^{19}} - \ln\sqrt[8]{|x+1|^{3}} + c$$

Esercizio

$$\int \frac{\cos 2x}{\operatorname{sen}x - \cos x}\,dx = \int \frac{\cos^2 x - \operatorname{sen}^2 x}{\operatorname{sen}x - \cos x}\,dx = -\int \frac{\cancel{(\cos x - \operatorname{sen}x)}(\cos x + \operatorname{sen}x)}{\cancel{\cos x - \operatorname{sen}x}}\,dx =$$

$$= -\int \cos x\,dx - \int \operatorname{sen}x\,dx = -\operatorname{sen}x + \cos x + c,\ c \in \mathbb{R}$$

Esercizio

$$\int \frac{e^x}{\cos^2(e^x-1)}\,dx = \operatorname{tg}(e^x-1) + c,\ c \in \mathbb{R}$$

Esercizio

$$\int \frac{1}{1+x^3}\,dx \qquad \frac{1}{(1+x)(1-x+x^2)} = \frac{A}{1+x} + \frac{Bx+C}{1-x+x^2} \qquad [\Delta<0]$$

$$\frac{1}{(1+x)(1-x+x^2)} = \frac{A - Ax + Ax^2 + Bx + Bx^2 + C + Cx}{(1+x)(1-x+x^2)}$$

$$1 = x(-A+B+C) + x^2(A+B) + A + C$$

$$\begin{cases} -A+B+C = 0 \\ A+B = 0 \Rightarrow A = -B \\ A+C = 1 \end{cases}$$

$$-B+C = 1 \Rightarrow C = 1+B$$

$$-A+B+C = 0$$

$$B+B+1+B = 0 \Rightarrow 3B = -1 \Rightarrow B = -\frac{1}{3}$$

$$A = \frac{1}{3},\ C = \frac{2}{3}$$

$$= \int \frac{\frac{1}{3}}{1+x}\,dx + \int \frac{-\frac{1}{3}x+\frac{2}{3}}{1-x+x^2}\,dx = \frac{1}{3}\int \frac{1}{1+x}\,dx - \frac{1}{3}\int \frac{x-2}{1-x+x^2}\,dx$$

$$-\frac{1}{6}\int \frac{2x-4}{1-x+x^2}\,dx \Rightarrow -\frac{1}{6}\int \frac{2x-1-3}{1-x+x^2}\,dx$$

$$-\frac{1}{6}\int \frac{2x-1}{1-x+x^2}\,dx + \frac{\cancel{3}}{\cancel{6}}\,\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx$$

$$\Rightarrow \frac{1}{2}\int \frac{1}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}\,dx \Rightarrow \frac{1}{2}\int \frac{\frac{4}{3}}{\left(\frac{2x-1}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{3}}\right)^2+1}\,dx$$

$$\Rightarrow \frac{\cancel{4}}{\cancel{6}}\,\frac{2}{3}\int \frac{1}{\left(\frac{2x-1}{\sqrt{3}}\right)^2+1}\,dx \Rightarrow \frac{1}{3\sqrt{3}}\int \frac{\frac{2}{\sqrt{3}}}{\left(\frac{2x-1}{\sqrt{3}}\right)^2+1}\,dx$$

$$= \frac{1}{3}\ln|1+x| - \frac{1}{6}\ln\left(1-x+x^2\right) + \frac{1}{3\sqrt{3}}\operatorname{arctg}\left(\frac{2x-1}{\sqrt{3}}\right) + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \operatorname{sen}^2 x\cdot\cos^3 x\,dx = \int \operatorname{sen}^2 x\cdot\left(1-\operatorname{sen}^2 x\right)\cos x\,dx$$

$$= \int \operatorname{sen}^2 x\cos x - \operatorname{sen}^4 x\cos x\,dx = \int (\operatorname{sen} x)^2\cos x\,dx - \int (\operatorname{sen} x)^4\cos x\,dx =$$

$$= \frac{\operatorname{sen}^3 x}{3} - \frac{\operatorname{sen}^5 x}{5} + c,\ c\in\mathbb{R}$$

Esercizio

$$\int (1-\operatorname{sen} x)^2\,dx = \int dx - \int \operatorname{sen}^2 x\,dx - 2\int \operatorname{sen} x\,dx =$$

$$= \int dx + \frac{1}{2}\int dx - \frac{1}{2}\int \cos 2x\,dx - 2\int \operatorname{sen} x\,dx =$$

$$= x + \frac{1}{2}x - \frac{1}{4}\operatorname{sen} 2x + 2\cos x + c = \frac{3}{2}x - \frac{1}{4}\operatorname{sen} 2x + 2\cos x + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{1}{\cos^2 x}\sqrt[5]{\operatorname{tg}^2 x}\,dx =$$

$$= \int \frac{1}{\cos^2 x}\cdot(\operatorname{tg} x)^{2/5}\,dx = \frac{(\operatorname{tg} x)^{\frac{2}{5}+1}}{\frac{2}{5}+1} + c = \frac{5(\operatorname{tg} x)^{\frac{7}{5}}}{7} + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{1}{\operatorname{sen} x}\,dx = \int \frac{\operatorname{sen}^2\frac{x}{2} + \cos^2\frac{x}{2}}{2\operatorname{sen}\frac{x}{2}\cos\frac{x}{2}}\,dx =$$

$$= \frac{1}{2}\int \frac{\operatorname{sen}^{\cancel{2}}\frac{x}{2}}{\cancel{\operatorname{sen}\frac{x}{2}}\cos\frac{x}{2}}\,dx + \frac{1}{2}\int \frac{\cos^{\cancel{2}}\frac{x}{2}}{\operatorname{sen}\frac{x}{2}\,\cancel{\cos\frac{x}{2}}}\,dx = \frac{1}{2}\int tg\,\frac{x}{2}\,dx + \frac{1}{2}\int ctg\,\frac{x}{2}\,dx$$

$$= -\ln\left|\cos\frac{x}{2}\right| + \ln\left|\operatorname{sen}\frac{x}{2}\right| + c = \ln\left|tg\,\frac{x}{2}\right| + c,\quad c \in \mathbb{R}$$

Esercizio

$$\int \frac{dx}{1+\cos x} = \int \frac{1}{2\cos^2\frac{x}{2}}\,dx$$

$$= \frac{1}{2}\int \frac{1}{\cos^2\frac{x}{2}}\,dx = tg\,\frac{x}{2} + c,\quad c \in \mathbb{R}$$

Esercizio

$$\int \frac{\operatorname{sen} x - 1}{(x+\cos x)^2}\,dx = \int (\operatorname{sen} x - 1)(x+\cos x)^{-2}\,dx = -\int (1-\operatorname{sen} x)(x+\cos x)^{-2}\,dx =$$

$$= -\frac{(x+\cos x)^{-2+1}}{-2+1} = +\frac{1}{x+\cos x} + c,\ c \in \mathbb{R}$$

Esercizio

$$\int e^x \cdot \frac{e^x - 1}{e^x + 1}\,dx = \int e^x \cdot \frac{e^x + 1 - 2}{e^x + 1}\,dx = \int e^x \cdot \frac{e^x+1}{e^x+1}\,dx - 2\int \frac{e^x}{e^x+1}\,dx =$$

$$= e^x - 2\ln(e^x+1) + c,\ c \in \mathbb{R}$$

Esercizio

$$\int \frac{x^3+8}{x-2}\,dx \qquad \frac{x^3+8}{x-2} = \frac{(x-2)(x^2+2x+4)}{x-2} + \frac{16}{x-2}$$

divisione euclidea:

$$\begin{array}{r|l} x^3 \qquad + 8 & x-2 \\ \hline -x^3 + 2x^2 \qquad & x^2+2x+4 \\ \hline / \ \ 2x^2 + 8 & \\ -2x^2 + 4x & \\ \hline / \ \ 4x + 8 & \\ -4x + 8 & \\ \hline / \ \ 16 & \end{array}$$

$$= \int (x^2+2x+4)\,dx + 16\int \frac{1}{x-2}\,dx = \frac{x^3}{3} + \cancel{2}\frac{x^2}{\cancel{2}} + 4x + 16\ln|x-2| + c,\ c \in \mathbb{R}$$

Esercizio

$$\int \frac{x^2-2}{2x+1}\,dx = \int \frac{(2x+1)\left(\frac{1}{2}x-\frac{1}{4}\right)}{2x+1}\,dx + \int \frac{-\frac{7}{4}}{2x+1}\,dx$$

$$\begin{array}{l|l} x^2-2 & 2x+1 \\ \hline -x^2-\frac{x}{2} & \frac{1}{2}x-\frac{1}{4} \\ \hline \;/\;-\frac{x}{2}-2 & \\ \;\;\frac{x}{2}+\frac{1}{4} & \\ \hline \;\;/\;-\frac{7}{4} & \end{array}$$

$$= \int \left(\frac{1}{2}x-\frac{1}{4}\right)dx - \frac{7}{4}\cdot\frac{1}{2}\int \frac{2}{2x+1}\,dx$$

$$= \frac{1}{4}x^2 - \frac{1}{4}x - \frac{7}{8}\ln|2x+1| + c,\ c\in\mathbb{R}$$

Esercizio

$$\int\left(\sqrt[8]{x^3}+5\,\mathrm{sen}\,x\right)dx = \int x^{\frac{3}{8}}\,dx + 5\int \mathrm{sen}\,x\,dx = \frac{8}{11}x^{\frac{11}{8}} - 5\cos x + c,\ c\in\mathbb{R}$$

Esercizio

$$\int\left(\frac{1}{6x^2}+3e^{-x}-5\,\mathrm{sen}\,x\right)dx = \frac{1}{6}\int x^{-2}\,dx - 3\int -e^{-x}\,dx - 5\int \mathrm{sen}\,x\,dx =$$

$$= -\frac{1}{6x} - 3e^{-x} + 5\cos x + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{e^x}{\mathrm{sen}^2(e^x+4)}\,dx = -\mathrm{cotg}\,(e^x+4) + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{x}{\sqrt[6]{6-x^2}}\,dx = \int x\cdot(6-x^2)^{-\frac{1}{6}}\,dx = -\frac{1}{2}\int -2x\,(6-x^2)^{-\frac{1}{6}}\,dx = -\frac{1}{\cancel{2}_{1}}(6-x^2)^{\frac{5}{6}}\,\frac{\cancel{6}^{3}}{5} =$$

$$= -\frac{3}{5}(6-x^2)^{\frac{5}{6}} + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{x^3}{5x^4+3}\,dx = \frac{1}{20}\int \frac{20x^3}{5x^4+3}\,dx = \frac{1}{20}\ln(5x^4+3) + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{2}{1+e^x}\,dx = 2\int \frac{1}{e^x+1}\,dx = 2\int \frac{1+e^x-e^x}{e^x+1}\,dx = 2\int \frac{\cancel{1+e^x}}{\cancel{1+e^x}}\,dx - 2\int \frac{e^x}{1+e^x}\,dx =$$

$$= 2x - 2\ln|1+e^x| + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{dx}{x \ln^3 x} = \int \frac{1}{x} \cdot \ln^{-3} x \, dx = \frac{\ln^{-2} x}{-2} + c, \ c \in \mathbb{R}$$

Esercizio

$$\int \frac{1}{\sqrt{1-16x^2}} \, dx = \frac{1}{14} \int \frac{14}{\sqrt{1-16x^2}} \, dx = \frac{1}{14} \operatorname{arcsen} 4x + c, \ c \in \mathbb{R}$$

Esercizio

$$\int \frac{e^{\operatorname{arctg} x + 2}}{x^2+1} \, dx = \int e^{2(\operatorname{arctg} x + 1)} \cdot \frac{1}{x^2+1} \, dx = \frac{1}{2} e^{2\operatorname{arctg} x + 2} + c, \ c \in \mathbb{R}$$

Esercizio

$$\int \frac{\operatorname{sen}(3-\sqrt{x})}{\sqrt{x}} \, dx = \int \operatorname{sen}(3 - x^{\frac{1}{2}}) \cdot x^{-\frac{1}{2}} \, dx = \frac{1}{2} x^{-\frac{1}{2}} + c, \ c \in \mathbb{R}$$

Esercizio

$$\int \operatorname{tg}^2 x \, dx = \int \operatorname{tg}^2 x + 1 - 1 \, dx = \int \operatorname{tg}^2 x + 1 \, dx - \int dx = \operatorname{tg} x - x + c, \ c \in \mathbb{R}$$

Esercizio

$$\int \frac{1-\cos^2 x}{1-\cos x} \, dx = \int \frac{\cancel{(1-\cos x)}(1+\cos x)}{\cancel{1-\cos x}} \, dx = \int dx + \int \cos x \, dx = x + \operatorname{sen} x + c, \ c \in \mathbb{R}$$

Esercizio

$$\int \frac{dx}{5 + (2x-1)^2} = \int \frac{\frac{1}{5}}{1 + \left(\frac{2x-1}{\sqrt{5}}\right)^2} \, dx = \frac{\sqrt{5}}{10} \int \frac{\frac{2}{\sqrt{5}}}{1 + \left(\frac{2x-1}{\sqrt{5}}\right)^2} \, dx = \frac{\sqrt{5}}{10} \operatorname{arctg} \left(\frac{2x-1}{\sqrt{5}}\right)^2 + c, \ c \in \mathbb{R}$$

Esercizio

$$\int (1 + 2\operatorname{sen} x)^2 \, dx = \int dx + 4 \int \operatorname{sen} x \, dx + 4 \int \operatorname{sen}^2 x \, dx$$

$$4 \int \operatorname{sen}^2 x \, dx = 4 \int \frac{1-\cos 2x}{2} \, dx = \frac{4}{2} \int dx - \frac{4}{2} \int \cos 2x \, dx = 2 \int dx - \frac{2}{2} \int 2 \cdot \cos 2x \, dx$$

$$= x - 4\cos x + 2x - \operatorname{sen} 2x + c, \ c \in \mathbb{R}$$

Esercizio

$$\int \frac{1}{2x^2 + 8x + 20} \, dx = \frac{1}{2} \int \frac{1}{x^2 + 4x + 10} \, dx = \frac{1}{2} \int \frac{1}{x^2 + 4x + 4 + 6} \, dx = \frac{1}{2} \int \frac{1}{(x+2)^2 + 6} \, dx =$$

$$= \frac{1}{2} \int \frac{\frac{1}{6}}{\left(\frac{x+2}{\sqrt{6}}\right)^2 + 1} \, dx \overset{(*)}{=} \frac{1}{2\sqrt{6}} \int \frac{\frac{1}{\sqrt{6}}}{\left(\frac{x+2}{\sqrt{6}}\right)^2 + 1} \, dx = \frac{1}{2\sqrt{6}} \operatorname{arctg}\left(\frac{x+2}{\sqrt{6}}\right)$$

$$(*) \left(\frac{1}{2} \cdot \frac{1}{6} \cdot \sqrt{6} = \frac{\sqrt{6}}{2(\sqrt{6})^2}\right)$$

$$\int \frac{1}{2x^2+8x+20}dx = \frac{1}{2}\int \frac{1}{x^2+4x+10}dx = \frac{1}{2}\int \frac{1}{x^2+4x+4+6}dx = \frac{1}{2}\int \frac{1}{(x+2)^2+6}dx =$$

$$= \frac{1}{2}\int \frac{\frac{1}{6}}{\left(\frac{x+2}{\sqrt{6}}\right)^2+1}dx \overset{*}{=} \frac{1}{2\sqrt{6}}\int \frac{\frac{1}{\sqrt{6}}}{\left(\frac{x+2}{\sqrt{6}}\right)^2+1}dx = \frac{1}{2\sqrt{6}}\operatorname{arctg}\left(\frac{x+2}{\sqrt{6}}\right)$$

$$*\left(\frac{1}{2}\cdot\frac{1}{6}\cdot\sqrt{6} = \frac{\sqrt{6}}{2(\sqrt{6})^2}\right)$$

Esercizio

$$\int \frac{1}{(x^2+9)(x-1)}dx \qquad \frac{Ax+B}{x^2+9}+\frac{C}{x-1} = \frac{1}{(x^2-9)(x-1)}$$

$$(Ax+B)(x-1)+(x^2+9)C = 1$$

$$Ax^2-Ax+Bx-B+Cx^2+9C = 1$$

$$x^2(A+C)+x(-A+B)-B+9C = 1$$

$$\begin{cases} A+C=0 \\ -A+B=0 \\ -B+9C=1 \end{cases}$$

$$A=-C$$
$$B=A=-C$$
$$-B+9C=1 \Rightarrow C+9C=1 \Rightarrow 10C=1 \Rightarrow C=\frac{1}{10}$$
$$A=-\frac{1}{10}\ ;\ B=-\frac{1}{10}$$

$$= \int \frac{-\frac{1}{10}x-\frac{1}{10}}{x^2+9}dx + \int \frac{\frac{1}{10}}{x-1}dx$$

$$= \int \frac{-\frac{1}{10}(x+1)}{x^2+9}dx + \frac{1}{10}\int \frac{1}{x-1}dx$$

$$= -\frac{1}{10}\int \frac{x+1}{x^2+9}dx + \frac{1}{10}\int \frac{1}{x-1}dx$$

$$-\frac{1}{10}\int \frac{x+1}{x^2+9}dx = -\frac{1}{20}\int \frac{2x}{x^2+9}dx - \frac{1}{10}\int \frac{1}{x^2+9}dx$$

$$-\frac{1}{10}\int \frac{1}{x^2+9}dx = -\frac{1}{10}\int \frac{\frac{1}{9}}{\left(\frac{x}{3}\right)^2+1}dx = -\frac{\cancel{3}^{1}}{\cancel{90}_{30}}\int \frac{\frac{1}{3}}{\left(\frac{x}{3}\right)^2+1}dx$$

$$= -\frac{1}{20}\ln(x^2+9) - \frac{1}{30}\operatorname{arctg}\left(\frac{x}{3}\right) + \frac{1}{10}\ln|x-1| + c,\ c\in\mathbb{R}$$

Esercizio

$$\int\left(\sqrt[11]{x^5}+\frac{1}{3}\cos x\right)dx=\int\left(x^{\frac{5}{11}}+\frac{1}{3}\cos x\right)dx=\int x^{\frac{5}{11}}dx+\frac{1}{3}\int\cos x\,dx=\frac{11}{16}x^{\frac{16}{11}}+\frac{1}{3}\operatorname{sen}x+c,\ c\in\mathbb{R}$$

Esercizio

$$\int\left(2\operatorname{sen}x-3e^{2x}+\frac{1}{4x^2}\right)dx=2\int\operatorname{sen}x\,dx-3\int e^{2x}dx+\frac{1}{4}\int x^{-2}dx=-2\cos x-\frac{3}{2}e^{2x}-\frac{1}{4}x^{-1}+c$$

Esercizio

$$\int\frac{x}{\sqrt{1-x^4}}dx=\int\frac{x}{\sqrt{1-(x^2)^2}}dx=\frac{1}{2}\int\frac{2x}{\sqrt{1-(x^2)^2}}dx=\operatorname{arcsen}(x^2)+c,\ c\in\mathbb{R}$$

Esercizio

$$\int\frac{12x^2+1}{5x^2+5}dx=\frac{1}{5}\int\frac{12x^2+1}{x^2+1}dx=\frac{12}{5}\int\frac{x^2+1-1}{x^2+1}dx+\frac{1}{5}\int\frac{1}{x^2+1}dx=\frac{12}{5}\int\frac{x^2+1}{x^2+1}dx-\frac{12}{5}\int\frac{1}{x^2+1}dx+\frac{1}{5}\int\frac{1}{x^2+1}dx$$

$$=\frac{12}{5}x-\frac{12}{5}\operatorname{arctg}x+\frac{1}{5}\operatorname{arctg}x+c,\ c\in\mathbb{R}$$

Esercizio

$$\int\frac{3\cos(\ln x+2)}{x}dx=3\int\cos(\ln x+2)\cdot\frac{1}{x}dx=3\operatorname{sen}(\ln x+2)+c,\ c\in\mathbb{R}$$

Esercizio

$$\int\frac{5\operatorname{sen}^2x}{4+4\cos x}dx=\frac{5}{4}\int\frac{\operatorname{sen}^2x}{1+\cos x}dx=\frac{5}{4}\int\frac{1-\cos^2x}{1+\cos x}dx=\frac{5}{4}\int\frac{(1-\cos x)\cancel{(1+\cos x)}}{\cancel{1+\cos x}}dx=\frac{5}{4}\int 1-\cos x\,dx=$$

$$=\frac{5}{4}\int dx-\frac{5}{4}\int\cos x\,dx=\frac{5x}{4}-\frac{5}{4}\operatorname{sen}x+c,\ c\in\mathbb{R}$$

Esercizio

$$\int\frac{dx}{e^x+5}=\int\frac{\frac{1}{5}}{\frac{e^x}{5}+1}dx=\frac{1}{5}\int\frac{1+\frac{e^x}{5}-\frac{e^x}{5}}{\frac{e^x}{5}+1}dx=\frac{1}{5}\int\cancel{\frac{1+\frac{e^x}{5}}{\frac{e^x}{5}+1}}dx-\frac{1}{5}\int\frac{\frac{e^x}{5}}{\frac{e^x}{5}+1}dx=\frac{1}{5}x-\frac{1}{5}\ln\left(\frac{e^x}{5}+1\right)+c$$

Esercizio

$$\int 5x\operatorname{sen}x^2\,dx=5\int x\cdot\operatorname{sen}x^2\,dx=\frac{5}{2}\int 2x\operatorname{sen}x^2\,dx=-\frac{5}{2}\cos x^2+c,\ c\in\mathbb{R}$$

Esercizio

$$\int\frac{dx}{\sqrt{9-36x^2}}=\int\frac{\frac{1}{3}}{\sqrt{1-4x^2}}dx=\frac{1}{3}\int\frac{1}{\sqrt{1-(2x)^2}}dx=\frac{1}{6}\int\frac{2}{\sqrt{1-(2x)^2}}dx=\frac{1}{6}\operatorname{arcsen}(2x)+c,\ c\in\mathbb{R}$$

Esercizio

$$\int\frac{dx}{\sqrt{1-x^2}\arccos x}=-\int\frac{-\frac{1}{\sqrt{1-x^2}}}{\arccos x}dx=-\int\frac{(1-x^2)^{-\frac{1}{2}}}{\arccos x}dx=-\ln|\arccos x|+c,\ c\in\mathbb{R}$$

Esercizio

$$\int\sqrt{\frac{1+x}{1-x}}\,dx = \int\frac{\sqrt{1+x}}{\sqrt{1-x}}\cdot\frac{\sqrt{1+x}}{\sqrt{1+x}}\,dx = \int\frac{1+x}{\sqrt{1-x^2}}\,dx = \int\frac{1}{\sqrt{1-x^2}}\,dx + \int\frac{x}{\sqrt{1-x^2}}\,dx =$$

$$= \int\frac{1}{\sqrt{1-x^2}}\,dx - \frac{1}{2}\int -2x\,(1-x^2)^{-\frac{1}{2}}\,dx = \text{arcsen}\,x - \frac{1}{2}\,\frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}} = \text{arcsen}\,x - \sqrt{1-x^2} + c,\ c\in\mathbb{R}$$

Esercizio

$$\int\cos^3 x\,dx = \int\cos^2 x\,\cos x = \int(1-\text{sen}^2 x)\cdot\cos x\,dx = \int(\cos x - \text{sen}^2 x\cos x)\,dx = \int\cos x\,dx - \int\text{sen}^2 x\cos x\,dx =$$

$$= \int\cos x\,dx - \int(\text{sen}\,x)^2\cos x\,dx = \text{sen}\,x - \frac{\text{sen}^3 x}{3} + c,\ c\in\mathbb{R}$$

Esercizio

$$\int e^x\,\frac{e^x-1}{e^x+1}\,dx = \int e^x\,\frac{e^x-1+2-2}{e^x+1}\,dx = \int e^x\,\frac{\cancel{e^x+1}}{\cancel{e^x+1}}\,dx - 2\int\frac{e^x}{e^x+1}\,dx = e^x - \ln(e^x+1) + c$$

Esercizio

$$\int\frac{3}{9+\frac{x^2}{11}}\,dx = \int\frac{\frac{1}{3}}{1+\frac{x^2}{99}}\,dx = \frac{1}{3}\int\frac{1}{1+\left(\frac{x}{\sqrt{99}}\right)^2}\,dx = \frac{1}{3\sqrt{99}}\int\frac{\frac{1}{\sqrt{99}}}{1+\left(\frac{x}{\sqrt{99}}\right)^2}\,dx = \frac{1}{3\sqrt{99}}\,\text{arctg}\left(\frac{x}{\sqrt{99}}\right) + c,\ c\in\mathbb{R}$$

Esercizio

$$\int\frac{2x+3}{x^2-4x+8}\,dx = \int\frac{2x+3+7-7}{x^2-4x+8}\,dx = \int\frac{2x-4}{x^2-4x+8}\,dx + 7\int\frac{1}{\underbrace{x^2-4x+8}_{(x-2)^2+4}}\,dx$$

$$= \int\frac{2x-4}{x^2-4x+8}\,dx + 7\int\frac{\frac{1}{4}}{\left(\frac{x-2}{2}\right)^2+1}\,dx = \int\frac{2x-4}{x^2-4x+8}\,dx + \frac{\overset{7}{\cancel{14}}}{\underset{2}{\cancel{4}}}\int\frac{\frac{1}{2}}{\left(\frac{x-2}{2}\right)^2+1}\,dx =$$

$$= \ln(x^2-4x+8) + \frac{7}{2}\,\text{arctg}\left(\frac{x-2}{2}\right) + c,\ c\in\mathbb{R}$$

Esercizio

$$\int\frac{dx}{(x-3)^2(x+5)} \qquad \frac{1}{(x-3)^2(x+5)} = \frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{C}{x+5}$$

$$\frac{1}{\cancel{(x-3)^2(x+5)}} = \frac{A(x-3)(x+5) + B(x+5) + C(x-3)^2}{\cancel{(x-3)^2(x+5)}}$$

$$1 = (Ax-3A)(x+5) + Bx + 5B + C(x^2+9-6x)$$

$$1 = Ax^2 + 5Ax - 3Ax - 15A + Bx + 5B + Cx^2 + 9C - 6Cx$$

$$1 = x^2(A+C) + x(2A+B-6C) - 15A + 5B + 9C$$

$$\begin{cases} A+C=0 \\ 2A+B-6C=0 \\ -15A+5B+9C=1 \end{cases}$$

$$A = -C$$

$$2(-C) + B - 6C = 0$$

$$-2C + B - 6C = 0$$

$$-8C = -B \Rightarrow C = \frac{1}{8}B$$

$$\begin{cases} A+C=0 \\ 2A+B-6C=0 \\ -15A+5B+9C=1 \end{cases}$$

$A=-C$

$2(-C)+B-6C=0$

$-2C+B-6C=0$

$-8C=-B \Rightarrow C=\frac{1}{8}B$

$-15A+5B+9C=1$

$-\frac{15}{8}B+5B+\frac{9}{8}B=1$

$\frac{15B+40B+9B}{\cancel{8}}=\frac{8}{\cancel{8}}$

$64B=8$

$8B=1 \Rightarrow B=\frac{1}{8}$

$\Rightarrow C=\frac{1}{8}B \Rightarrow C=\frac{1}{64}$ $\qquad$ $A=-C \Rightarrow -\frac{1}{64}$

$\left(A=-\frac{1}{64}\ ;\ B=\frac{1}{8}\ ;\ C=\frac{1}{64}\right)$

$$=\int \frac{-\frac{1}{64}}{x-3}dx+\int\frac{\frac{1}{8}}{(x-3)^2}dx+\int\frac{\frac{1}{64}}{x+5}dx=$$

$$=-\frac{1}{64}\int\frac{1}{x-3}dx+\frac{1}{8}\int(x-3)^{-2}dx+\frac{1}{64}\int\frac{1}{x+5}dx=$$

$$=\frac{1}{64}\ln|x-3|+\frac{1}{8}\frac{(x-3)^{-1}}{-1}+\frac{1}{64}\ln|x+5|+c$$

$$=\frac{1}{64}\ln|x-3|-\frac{1}{8(x-3)}+\frac{1}{64}\ln|x+5|+c\ ,\ c\in\mathbb{R}$$

Esercizio

$$\int\frac{e^{\frac{1}{x}}}{x^2}dx=-\int e^{(x^{-1})}\cdot(-x)^{-2}dx=-e^{\frac{1}{x}}+c\ ,\ c\in\mathbb{R}$$

Esercizio

$$\int\frac{\ln x}{x}dx=\int\ln x\cdot\frac{1}{x}dx=\frac{\ln^2 x}{2}+c\ ,\ c\in\mathbb{R}$$

Esercizio

$$\int\frac{1}{x\sqrt{\ln x}}dx=\int\frac{1}{x}\cdot(\ln x)^{-\frac{1}{2}}dx=2\sqrt{\ln x}+c\ ,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{x}{1+x^3}\,dx \qquad \frac{x}{1+x^3} = \frac{A}{1+x} + \frac{Bx+C}{1-x+x^2}$$

$$x = A(1-x+x^2)+(Bx+c)(1+x)$$

$$x = A - Ax + Ax^2 + Bx + c + Bx^2 + Cx$$

$$x = x^2(A+B) + x(B-A+c) + A + C$$

$$\begin{cases} A+B=0 \Rightarrow B=A+1+2A=0 \Rightarrow 3A=-1 \Rightarrow A=-\frac{1}{3} \\ B-A+C=1 \Rightarrow B=1-C+A \Rightarrow B=1+A+A \Rightarrow B=1+2A \Rightarrow B=1-\frac{2}{3}=\frac{1}{3} \\ A+C=0 \Rightarrow C=-A \end{cases}$$

$$\left(A=-\frac{1}{3}\,;\ B=\frac{1}{3}\,;\ C=\frac{1}{3}\right)$$

$$= \int \frac{-\frac{1}{3}\,dx}{1+x} + \int \frac{\frac{1}{3}x+\frac{1}{3}}{1-x+x^2}\,dx = -\frac{1}{3}\int \frac{dx}{1+x} + \frac{1}{3}\int \frac{x+1}{1-x+x^2}\,dx$$

$$\frac{1}{3}\int \frac{x+1}{1-x+x^2} = \frac{1}{6}\int \frac{2x+2}{1-x+x^2}\,dx = \frac{1}{6}\int \frac{2x+2-3+3}{1-x+x^2}\,dx = \frac{1}{6}\int \frac{2x+2-3}{1-x+x^2}\,dx + \frac{3}{6}\int \frac{1}{\underbrace{1-x+x^2}_{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}}\,dx$$

$$\frac{3}{6}\int \frac{\frac{4}{3}}{\left(\frac{2x-1}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{3}}\right)+1}\,dx$$

$$\frac{12}{18}\int \frac{1}{\left(\frac{2x-1}{\sqrt{3}}\right)^2+1}\,dx$$

$$\frac{4}{3\sqrt{3}}\int \frac{\frac{2}{\sqrt{3}}\,dx}{\left(\frac{2x-1}{\sqrt{3}}\right)^2+1}$$

$$= -\frac{1}{3}\ln|1+x| + \frac{1}{6}\ln(1-x+x^2) + \frac{4}{3\sqrt{3}}\operatorname{arctg}\left(\frac{2x-1}{\sqrt{3}}\right) + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{9x^3-6x^2-8x+3}{3x-4}\,dx$$

divisione euclidea:

$$\begin{array}{l|l} 9x^3-6x^2-8x+3 & 3x-4 \\ \hline -9x^3+12x^2 & 3x^2+2x \\ \quad /\quad 6x^2-8x+3 & \\ \quad\quad -6x^2+8x & \\ \quad\quad /\quad /\quad +3 & \end{array}$$

$$= \int (3x^2+2x)\,dx + \int \frac{3}{3x-4}\,dx = \int 3x^2 dx + 2\int x\,dx + \int \frac{3}{3x-4}\,dx =$$

$$= x^3 + x^2 + \ln|3x-4| + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \left(\cos x + \frac{\operatorname{sen}^3 x - 2}{\operatorname{sen}^2 x}\right) dx = \int \cos x\,dx + \int \frac{\operatorname{sen}^{\cancel{3}\,1} x}{\cancel{\operatorname{sen}^2 x}}\,dx - 2\int \frac{1}{\operatorname{sen}^2 x}\,dx =$$

$$= \operatorname{sen} x - \cos x - 2\cot g\, x + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{e^x\,dx}{\sqrt{1-e^{2x}}} = \int \frac{e^x}{\sqrt{1-(e^x)^2}}\,dx = \operatorname{arcsen}(e^x) + c$$

Esercizio

$$\int \frac{(2\cos x+1)(2\cos x-1)}{\cos^2 x}\,dx = \int \frac{4\cos^2 x - 1}{\cos^2 x}\,dx = 4\int \frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x}}\,dx - \int \frac{1}{\cos^2 x}\,dx = 4x - tg\,x + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{1}{1+2x^2}\,dx = \int \frac{1}{1+(\sqrt{2}x)^2}\,dx = \frac{1}{\sqrt{2}}\int \frac{\sqrt{2}}{1+(\sqrt{2}x)^2}\,dx = \frac{1}{\sqrt{2}}\operatorname{arctg}(\sqrt{2}x) + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{\operatorname{sen} x}{1+\cos^2 x}\,dx = -\int \frac{-\operatorname{sen} x}{1+(\cos x)^2}\,dx = -\operatorname{arctg}(\cos x) + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{1+xe^x}{x}\,dx = \int \frac{1}{x}\,dx + \int e^x\,dx = \ln|x| + e^x + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{1}{1+tg\,x}\cdot\frac{1}{\cos^2 x}\,dx = \int \frac{1}{1+tg\,x}\cdot(\cos x)^{-2}\,dx = \ln|1+tg\,x| + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{e^x dx}{\sqrt{2e^x+1}} = \frac{1}{2}\int 2e^x \cdot (2e^x+1)^{-\frac{1}{2}} dx = \frac{1}{2} \frac{(2e^x+1)^{\frac{1}{2}}}{\frac{1}{2}} + c = \sqrt{2e^x+1} + c, \ c \in \mathbb{R}$$

Esercizio

$$\int \frac{1}{tg^4 x} \cdot \frac{1}{\cos^2 x} dx = \int (tg x)^{-4} \cdot \frac{1}{\cos^2 x} dx = \frac{tg^{-3} x}{-3} + c, \ c \in \mathbb{R}$$

Esercizio

$$\int \frac{x^2}{x+1} dx = \int \frac{x^2+1-1}{x+1} dx = \int \frac{x^2-1}{x+1} dx + \int \frac{1}{x+1} dx = \int \frac{\cancel{(x+1)}(x-1)}{\cancel{x+1}} dx + \int \frac{1}{x+1} dx =$$

$$= \int x dx - \int dx + \int \frac{1}{x+1} dx = \frac{x^2}{2} - x + \ln|x+1| + c, \ c \in \mathbb{R}$$

Esercizio

$$\int \frac{2dx}{9+x^2} = 2\int \frac{\frac{1}{9}}{1+\left(\frac{x}{3}\right)^2} dx = \frac{6}{9}\int \frac{\frac{1}{3}}{1+\left(\frac{x}{3}\right)^2} dx = \frac{6}{9} \operatorname{arctg}\left(\frac{x}{3}\right) + c, \ c \in \mathbb{R}$$

Esercizio

$$\int \left(\frac{x}{3}+1\right) 4e^{\frac{x^2}{6}+x} dx = 4\int \left(\frac{x}{3}+1\right) \cdot e^{\frac{x^2}{6}+x} dx = e^{\left(\frac{x^2}{6}+x\right)} + c, \ c \in \mathbb{R}$$

Esercizio

$$\int \frac{2e^x}{4+e^{2x}} dx = 2\int \frac{e^x}{4+e^{2x}} dx = 2\int \frac{e^x}{4+(e^x)^2} dx = 2\int \frac{\frac{e^x}{4}}{1+\left(\frac{e^x}{2}\right)^2} dx = \int \frac{\frac{e^x}{2}}{1+\left(\frac{e^x}{2}\right)^2} dx =$$

$$= \operatorname{arctg}\left(\frac{e^x}{2}\right) + c, \ c \in \mathbb{R}$$

Esercizio

$$\int \frac{4+x}{\sqrt{1-16x^2}} dx = \int \frac{4+x}{\sqrt{1-(4x)^2}} dx = \int \frac{4}{\sqrt{1-(4x)^2}} dx - \frac{1}{16}\int -16x \cdot (1-16x^2)^{-\frac{1}{2}} dx = \operatorname{arcsen}(4x) - \frac{\cancel{2}^{1}}{\cancel{16}_{8}} \sqrt{1-16x^2} + c$$

Esercizio

$$\int \frac{x+2}{x^2-4x+6} dx = \frac{1}{2}\int \frac{2x+4-8+8}{x^2-4x+6} dx = \frac{1}{2}\int \frac{2x+4-8}{x^2-4x+6} dx + 8\int \frac{1}{(x-2)^2+2} dx$$

$$8\int \frac{1}{(x-2)^2+2} dx = 8\int \frac{\frac{1}{2}}{\left(\frac{x-2}{\sqrt{2}}\right)^2+1} dx = \frac{4}{\sqrt{2}}\int \frac{\frac{1}{\sqrt{2}}}{\left(\frac{x-2}{\sqrt{2}}\right)^2+1} dx$$

$$= \frac{1}{2}\ln(x^2-4x+6) + \frac{4}{\sqrt{2}} \operatorname{arctg}\left(\frac{x-2}{\sqrt{2}}\right) + c, \ c \in \mathbb{R}$$

Esercizio

$$\int \frac{1}{e^x+e^{-x}} dx = \int \frac{1}{e^x+\frac{1}{e^x}} dx = \int \frac{1}{\frac{e^{2x}+1}{e^x}} dx = \int \frac{e^x}{e^{2x}+1} dx = \int \frac{e^x}{(e^x)^2+1} dx = \operatorname{arctg} e^x + c, \ c \in \mathbb{R}$$

Esercizio

$$\int \frac{x^3-4x^2+3x+1}{x-1}\,dx =$$

$$\begin{array}{l|l} x^3-4x^2+3x+1 & x-1 \\ \hline -x^3+x^2 & x^2-3x \\ \quad / \; -3x^2+3x & \\ \qquad +3x^2-3x & \\ \qquad\; / \quad / \quad +1 & \end{array}$$

$$= \int (x^2-3x)\,dx + \int \frac{1}{x-1}\,dx = \int x^2\,dx - 3\int x\,dx + \int \frac{1}{x-1}\,dx = \frac{x^3}{3} - \frac{3x^2}{2} + \ln|x-1| + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{e^x-e^{-x}}{e^x+e^{-x}}\,dx = \ln(e^x+e^{-x}) + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{\ln^2 x+1}{x}\,dx = \int \frac{\ln^2 x}{x}\,dx + \int \frac{1}{x}\,dx = \int (\ln x)^2 \cdot \frac{1}{x}\,dx + \int \frac{1}{x}\,dx = \frac{\ln^3 x}{3} + \ln|x| + c$$

Esercizio

$$\int \frac{x-5}{x(x^2+4)}\,dx \qquad \frac{x-5}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$$x-5 = A(x^2+4) + (Bx+C)x$$

$$x-5 = Ax^2 + 4A + Bx^2 + Cx$$

$$x-5 = x^2(A+B) + Cx + 4A$$

$$\begin{cases} 4A = -5 \Rightarrow A = -\frac{5}{4} \\ C = 1 \\ A+B = 0 \Rightarrow B = \frac{5}{4} \end{cases}$$

$$= \int \frac{-\frac{5}{4}}{x}\,dx + \int \frac{\frac{5}{4}x+1}{x^2+4}\,dx = -\frac{5}{4}\int \frac{1}{x}\,dx + \frac{5}{4}\int \frac{x}{x^2+4}\,dx + \int \frac{1}{x^2+4}\,dx =$$

$$\hookrightarrow \int \frac{\frac{1}{4}}{\left(\frac{x}{2}\right)^2+1}\,dx = \frac{1}{2}\int \frac{\frac{1}{2}\,dx}{\left(\frac{x}{2}\right)^2+1}$$

$$= -\frac{5}{4}\ln|x| + \frac{5}{8}\ln(x^2+4) + \frac{1}{2}\operatorname{arctg}\left(\frac{x}{2}\right) + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{x-7}{x^2-5x+6}\,dx \qquad \frac{x-7}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$x-7 = A(x-3) + B(x-2)$$

$$x-7 = Ax - 3A + Bx - 2B$$

$$x-7 = x(A+B) - 3A - 2B$$

$$\begin{cases} A+B = 1 \Rightarrow B = 1-A \Rightarrow B = -4 \\ -3A-2B = -7 \\ -3A - 2(1-A) = -7 \\ -3A - 2 + 2A = -7 \\ -A = -5 \Rightarrow A = 5 \end{cases}$$

$$= \int \frac{5\,dx}{x-2} + \int \frac{-4\,dx}{x-3} =$$

$$= 5\int \frac{dx}{x-2} - 4\int \frac{1}{x-3}\,dx = 5\ln|x-2| - 4\ln|x-3| + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{x+2}{x^2(x-3)}\,dx \qquad \left[\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-3}\right]$$

$$\frac{x+2}{x^2(x-3)} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-3}$$

$$x+2 = Ax(x-3)+B(x-3)+Cx^2$$

$$x+2 = Ax^2-3Ax+Bx-3B+Cx^2$$

$$x+2 = x^2(A+C)+x(-3A+B)-3B$$

$$\begin{cases} A+C=0 \Rightarrow C=-A \Rightarrow C=+\frac{5}{9} \\ -3A+B=1 \Rightarrow -3A-\frac{2}{3}=1 \Rightarrow -3A=\frac{2}{3}+1 \Rightarrow -3A=\frac{5}{3} \Rightarrow A=-\frac{5}{9} \\ -3B=2 \Rightarrow B=-\frac{2}{3} \end{cases}$$

$$= \int \frac{-\frac{5}{9}}{x}\,dx + \int \frac{-\frac{2}{3}}{x^2}\,dx + \int \frac{\frac{5}{9}}{x-3}\,dx =$$

$$= -\frac{5}{9}\int \frac{1}{x}\,dx + -\frac{2}{3}\int x^{-2}\,dx + \frac{5}{9}\int \frac{1}{x-3}\,dx =$$

$$= -\frac{5}{9}\ln|x| + \frac{2}{3}\cdot\frac{x^{-1}}{-1} + \frac{5}{9}\ln|x-3| + c,\ c\in\mathbb{R}$$

$$= -\frac{5}{9}\ln|x| - \frac{2}{3x} + \frac{5}{9}\ln|x-3| + c,\ c\in\mathbb{R}$$

$$= -\frac{2}{3x}\ \ln\sqrt[9]{\left(\frac{x-3}{x}\right)^5} + c,\ c\in\mathbb{R}$$

Esercizio

$$\int x\cdot \operatorname{sen} x^2\,dx = \frac{1}{2}\int 2x\cdot \operatorname{sen} x^2\,dx = -\frac{1}{2}\cos x^2 + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{x^2+3}{1+x^2}\overset{-2+2}{}\,dx = \int \frac{x^2+3-2}{x^2+1}\,dx + 2\int \frac{1}{1+x^2}\,dx = \int dx + 2\int \frac{1}{1+x^2}\,dx = x + 2\operatorname{arctg} x + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{2}{9+\frac{x^2}{7}}\,dx = 2\int \frac{1}{9+\left(\frac{x}{\sqrt{7}}\right)^2}\,dx = 2\int \frac{\frac{1}{9}}{1+\left(\frac{x}{\sqrt{7}}\right)^2}\,dx = \frac{2}{9}\int \frac{1}{1+\left(\frac{x}{\sqrt{7}}\right)^2}\,dx = \frac{2\sqrt{7}}{9}\int \frac{\frac{1}{\sqrt{7}}}{1+\left(\frac{x}{\sqrt{7}}\right)^2}\,dx$$

$$= \frac{2\sqrt{7}}{9}\operatorname{arctg}\left(\frac{x}{\sqrt{7}}\right) + c,\ c\in\mathbb{R}$$

Esercizio

$$\int \frac{2x+1}{x^2-3x+7}\,dx = \int \frac{2x+1-4+4}{x^2-3x+7}\,dx = \int \frac{2x+1-4}{x^2-3x+7}\,dx + 4\int \frac{1}{x^2-3x+7}\,dx$$

Esercizio

$$\int\left(\sqrt[9]{x^4} + \frac{1}{2}\cos x\right)dx = \int x^{\frac{4}{9}}\,dx + \frac{1}{2}\int \cos x\,dx = \frac{9x^{\frac{13}{9}}}{13} + \frac{1}{2}\operatorname{sen} x + c,\ c \in \mathbb{R}$$

Esercizio

$$\int\left(\operatorname{sen} x - 5e^{2x} + \frac{1}{3x^4}\right)dx = \int \operatorname{sen} x\,dx - 5\int e^{2x}\,dx + \frac{1}{3}\int x^{-4}\,dx =$$

$$= \int \operatorname{sen} x\,dx - \frac{5}{2}\int 2\cdot e^{2x}\,dx + \frac{1}{3}\int x^{-4}\,dx = -\cos x - \frac{5}{2}e^{2x} + \frac{1}{3}\,\frac{x^{-3}}{-3} + c =$$

$$= -\cos x - \frac{5}{2}e^{2x} - \frac{1}{9x^3} + c,\ c \in \mathbb{R}$$

Esercizio

$$\int \frac{x^4-2}{x^2+1}\,dx = \int \frac{x^4-2+1-1}{x^2+1}\,dx = \int \frac{x^4-2+1}{x^2+1} - \int \frac{1}{x^2+1}\,dx = \int \frac{x^4-1}{x^2+1}\,dx - \int \frac{1}{x^2+1}\,dx =$$

$$= \int \frac{\cancel{(x^2+1)}(x^2-1)}{\cancel{x^2+1}}\,dx - \int \frac{1}{x^2+1}\,dx = \int x^2\,dx - \int dx - \int \frac{1}{x^2+1}\,dx = \frac{x^3}{3} - x - \operatorname{arctg} x + c$$

Esercizio

$$\int \frac{14x^2+1}{3x^2+3}\,dx = \frac{1}{3}\int \frac{14x^2+1}{x^2+1}\,dx = \frac{1}{3}\int \frac{14x^2}{x^2+1}\,dx + \frac{1}{3}\int \frac{1}{x^2+1}\,dx = \frac{14}{3}\int \frac{x^2}{x^2+1}\,dx + \frac{1}{3}\int \frac{1}{x^2+1}\,dx$$

$$\frac{14}{3}\int \frac{x^2+1-1}{x^2+1}\,dx = \frac{14}{3}\int \frac{x^2+1}{x^2+1}\,dx - \frac{14}{3}\int \frac{1}{x^2+1}\,dx = \frac{14}{3}\int dx - \frac{14}{3}\int \frac{1}{x^2+1}\,dx$$

$$= \frac{14}{3}x - \frac{14}{3}\operatorname{arctg} x + \frac{1}{3}\operatorname{arctg} x + c = \frac{14}{3}x - \frac{13}{3}\operatorname{arctg} x + c,\ c \in \mathbb{R}$$

Esercizio

$$\int \frac{2\cos(\ln x+2)}{x}\,dx = 2\int\left[\cos(\ln x+2)\cdot\frac{1}{x}\right]dx = 2\operatorname{sen}(\ln x+2) + c,\ c \in \mathbb{R}$$

Esercizio

$$\int \frac{3\operatorname{sen}^2 x}{2+2\cos x}\,dx = \frac{3}{2}\int \frac{\operatorname{sen}^2 x}{1+\cos x}\,dx = \frac{3}{2}\int \frac{1-\cos^2 x}{1+\cos x}\,dx = \frac{3}{2}\int \frac{\cancel{(1+\cos x)}(1-\cos x)}{\cancel{1+\cos x}}\,dx =$$

$$= \frac{3}{2}\int dx - \frac{3}{2}\int \cos x\,dx = \frac{3}{2}x - \frac{3}{2}\operatorname{sen} x + c,\ c \in \mathbb{R}$$

Esercizio

$$\int \frac{1}{e^x+3}\,dx = \int \frac{1+e^x-e^x}{e^x+3}\,dx = \int \frac{e^x+1}{e^x+3}\,dx - \int \frac{e^x}{e^x+3}\,dx =$$

$$\int \frac{e^x+1+2-2}{e^x+3}\,dx = \int \frac{e^x+3}{e^x+3}\,dx - 2\int \frac{1}{e^x+3}\,dx$$

Esercizio

$y = (3+x)^{2x^2}$