

# **Exam & Seminar Selection Under Time and CFU Constraints**

*Binary Linear Programming: deterministic baseline + stochastic extension*

---

Quantitative Methods for Management  
Prof. Cannas

Andrea Dessalvi  
Daniele Tonarelli  
Jacopo Zoccheddu

# Motivation and Decision Problem

---

- Students face limited time and multiple possible activities (exams, seminars)
- Each activity has a “cost” in time and a “benefit” in CFU and/or performance
- Goal: select the best subset under constraints
- The decision is binary: an activity is either selected or not
- Approach: optimization replaces trial-and-error with a structured decision rule

# Dataset Overview (Simulation)

---

- Activities: 10 exams + 4 seminars
- Key variables: type, CFU, semester (S1/S2), grade (exams only), passing probability (exams), time components
- Seminars: CFU contribution, no grade, always passed
- CFU vary across exams (6, 9, 12) to reflect heterogeneous workloads
- Purpose: generate realistic inputs to test the modeling framework

# How the Data Were Generated

---

- Exam grades: simulated around typical values, bounded to a realistic range
- Exam pass probability  $p_{\text{pass}}$ : varies across exams (captures uncertainty)
- Time:  $\text{lecture\_hours}$  linked to CFU;  $\text{study\_hours}$  generated within a plausible interval
- $\text{Total\_time} = \text{lecture\_hours} + \text{study\_hours}$  (with one special heavy exam case)
- The dataset is an input to the optimization model, not the final goal

# Derived Metrics Used in the Project

---

- `expected_grade(exams)`:  $p_{\text{pass}} \times \text{grade}$
- `expected_grade(seminars)`: 0 (no grade component)
- `efficiency`:  $(\text{expected\_grade} \times \text{CFU}) / \text{total\_time}$
- Interpretation: performance “return” per unit of time
- Efficiency is used for interpretation, not for optimization

# Deterministic Model: Decision Variables

---

- Decision variable for each activity  $i$ :
  - $x_i = 1$  if activity  $i$  is selected
  - $x_i = 0$  otherwise
- Activities include both exams and seminars
- Binary structure turns the model into a MILP (knapsack-like selection)

# Deterministic MILP: Objective and Constraints

---

- Assumption (baseline): passing is certain ( $p_i = 1$  for all)
- Objective (performance): maximize total grade from selected exams  $\max \sum (\text{grade}_i \cdot x_i)$   
[exams only; seminars add no grade]
- Constraints:
  - Minimum credits:  $\sum (\text{CFU}_i \cdot x_i) \geq \text{CFU\_min}$
  - Time limit:  $\sum (\text{total\_time}_i \cdot x_i) \leq \text{Time\_max}$
  - Integrality:  $x_i \in \{0,1\}$

# Why Introduce Uncertainty

---

- Deterministic assumption is unrealistic: exams may not be passed
- Each exam has a probability of passing  $p_{\text{pass}}$  in  $(0,1)$
- Risk: choosing only high-grade exams might reduce the chance of success
- Need a model that accounts for expected outcomes, not just nominal grades

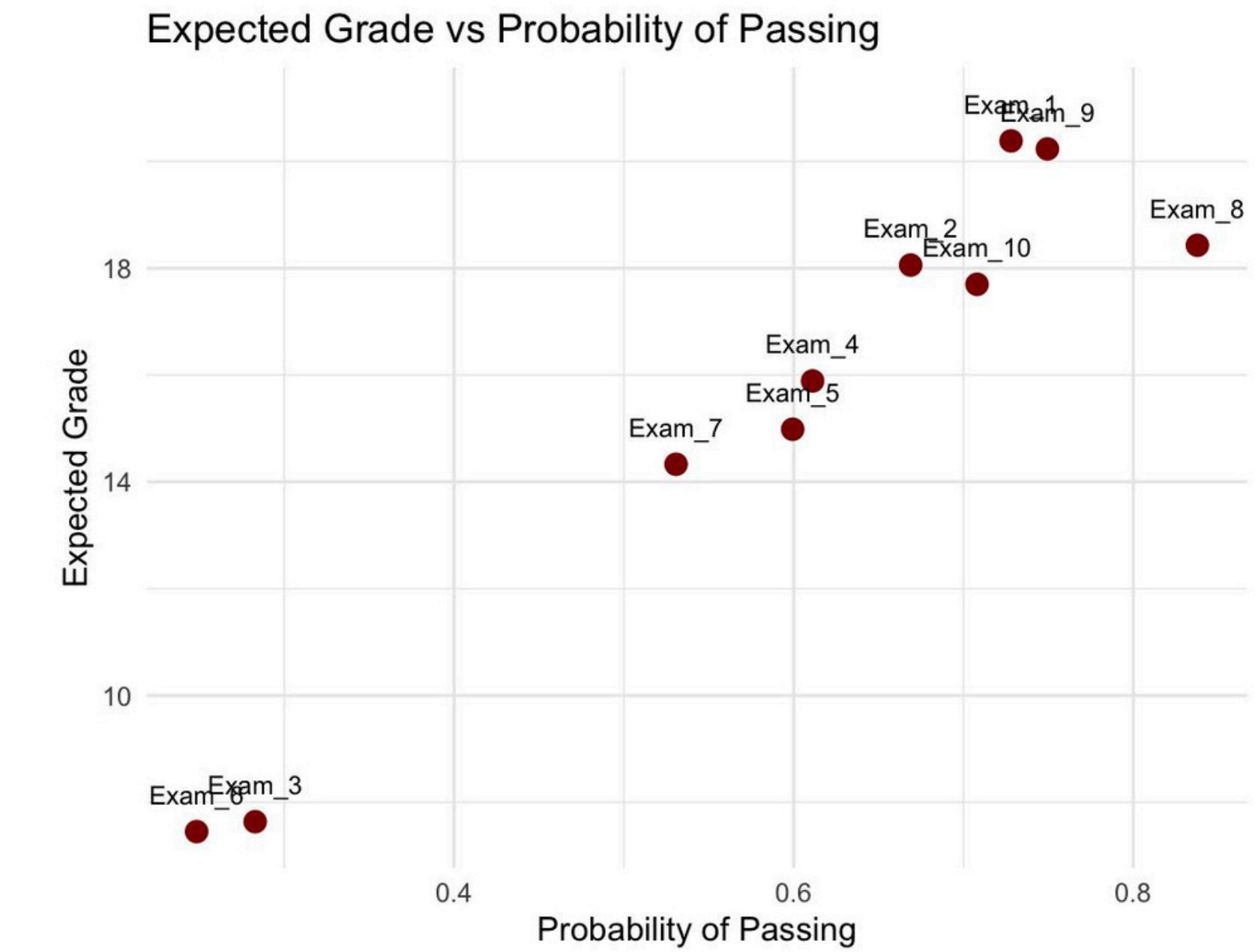
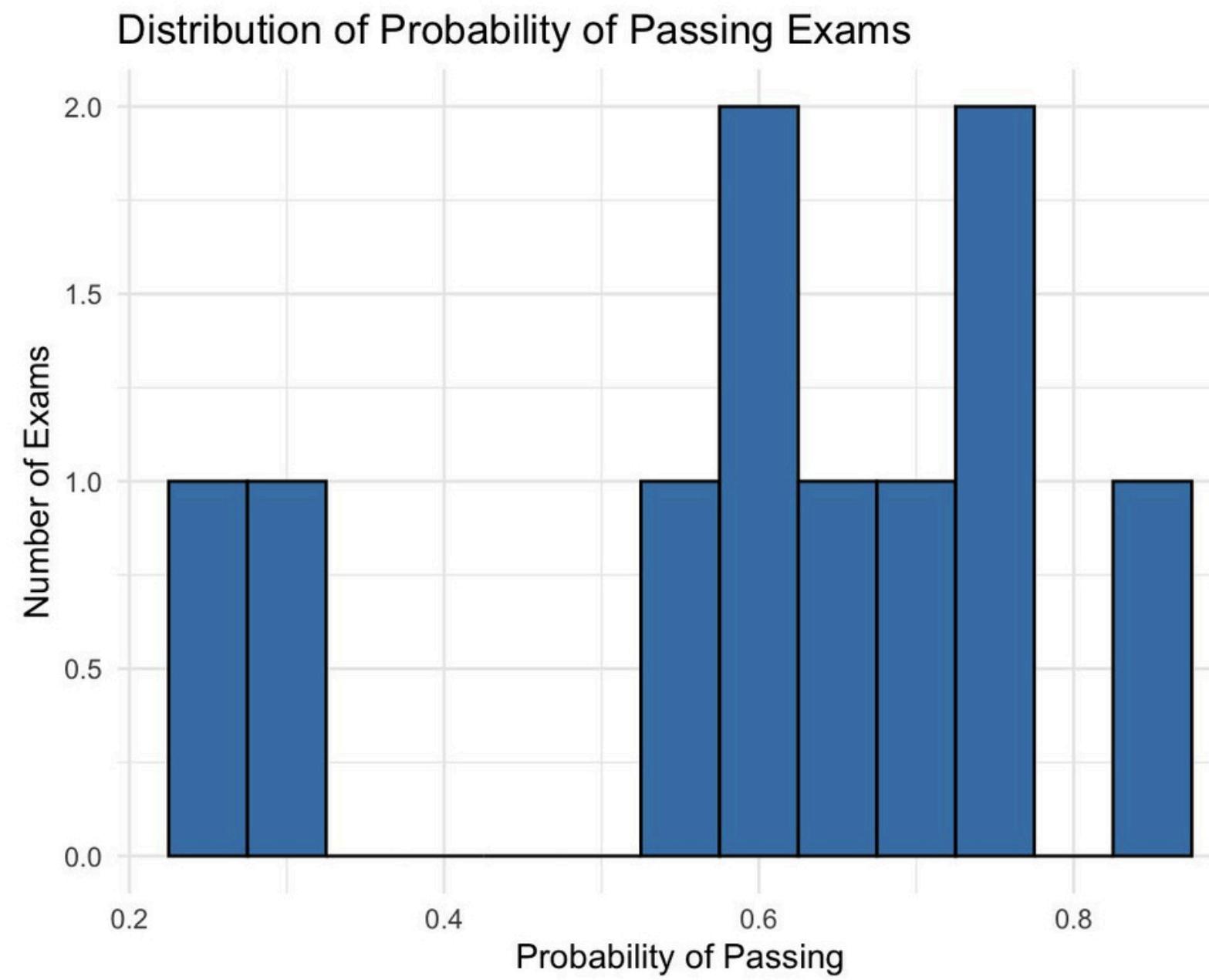
# Stochastic Extension (Expected Value Model)

---

- Exams:  $p_i = p_{\text{pass}_i}$ ; seminars:  $p_i = 1$
- Expected contribution for an exam:  $E[\text{grade}_i] = p_i \times \text{grade}_i$
- New objective (risk-adjusted performance):  $\max \sum (p_i \cdot \text{grade}_i \cdot x_i)$
- Constraints unchanged: CFU\_min, Time\_max,  $x_i$  binary

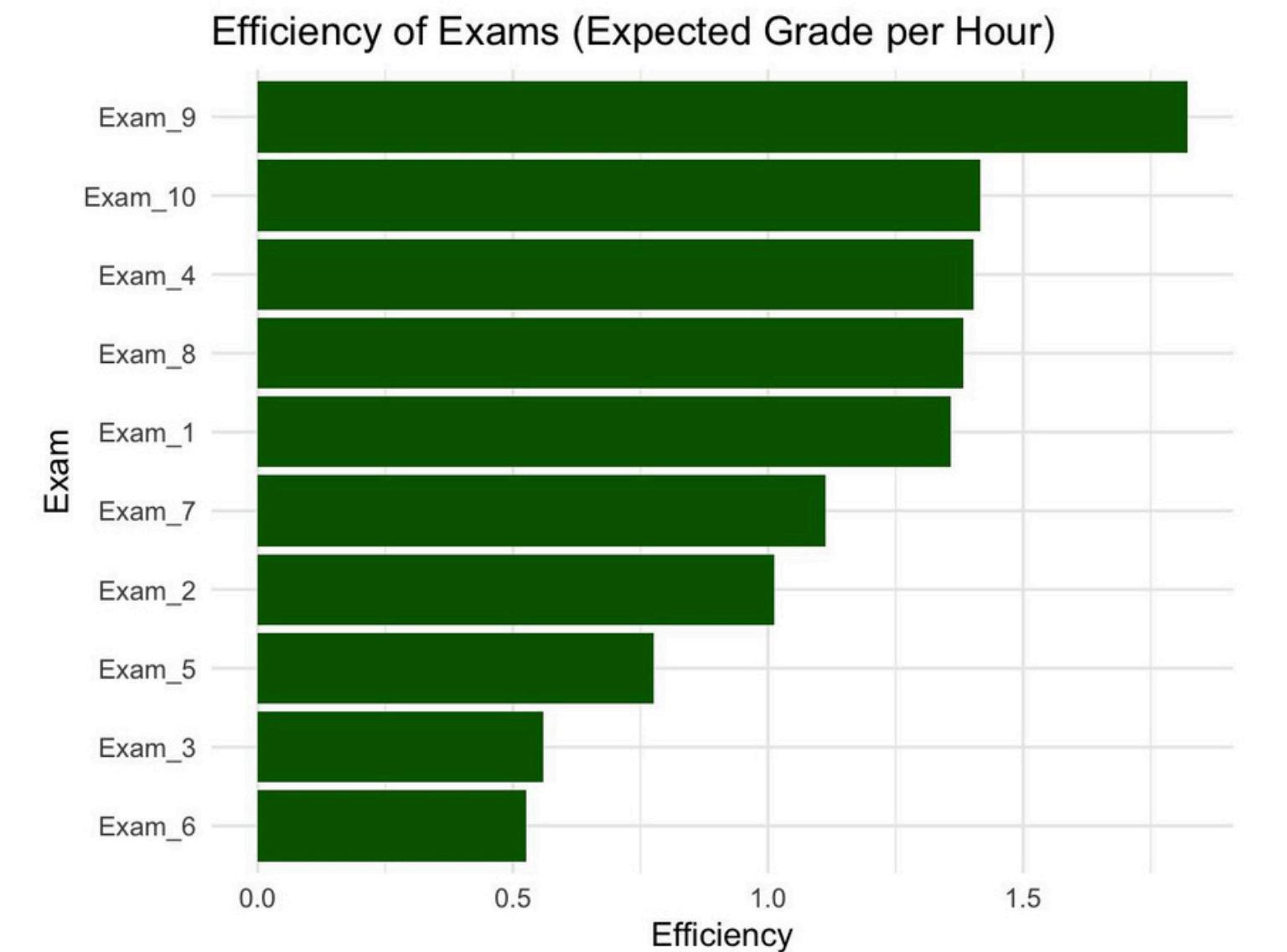
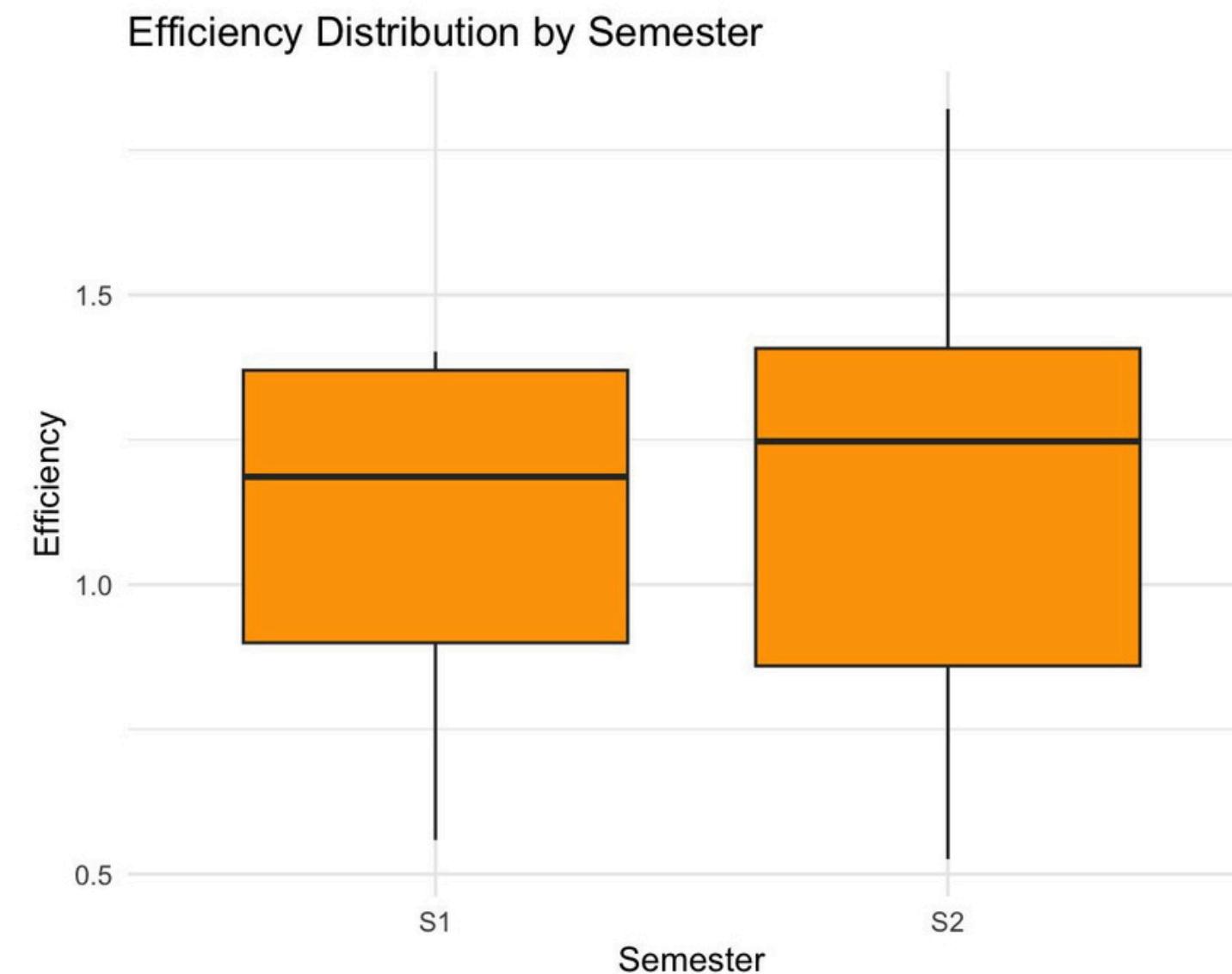
# Exploratory Analysis: Uncertainty Patterns

---



# Exploratory Analysis: Efficiency and Timing

---



# Workflow, Conclusions, and Extensions

---

- Workflow:
  1. Simulate dataset (exams + seminars)
  2. Compute expected\_grade and efficiency
  3. Produce descriptive plots and summaries
  4. Formulate deterministic MILP
  5. Extend to stochastic expected-value objective
- Conclusions: optimization provides a transparent rule for selection under constraints; uncertainty changes priorities vs the deterministic baseline
- Possible extensions: chance constraints (minimum probability of meeting CFU), scenario-based stochastic programming, robust optimization, multi-objective (grades + CFU + risk)