Exam & Seminar Selection Under Time and CFU Constraints

Binary Linear Optimization (MILIP): deterministic baseline + stochastic extension

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Motivation and Decision Problem

- Students face limited time and multiple possible activities (exams, seminars)
- Each activity has a “cost” in time and a “benefit” in CFU and/or performance
- Goal: select the best subset under constraints
- The decision is binary: an activity is either selected or not
- Approach: optimization replaces trial-and-error with a structured decision rule

Dataset Overview (Simulation)

- Activities: 10 exams + 4 seminars
- Key variables: type, CFU, semester (S1/S2), grade (exams only), passing probability (exams), time components
- Seminars: CFU contribution, no grade, always passed
- CFU vary across exams (6, 9, 12) to reflect heterogeneous workloads
- Purpose: generate realistic inputs to test the modeling framework

How the Data Were Generated

- Exam grades: simulated around typical values, bounded to a realistic range
- Exam pass probability p_pass: varies across exams (captures uncertainty)
- Time: lecture_hours linked to CFU; study_hours generated within a plausible interval
- Total_time = lecture_hours + study_hours (with one special heavy exam case)
- The dataset is an input to the optimization model, not the final goal

Derived Metrics Used in the Project

- expected_grade (exams): p_pass × grade
- expected_grade (seminars): 0 (no grade component)
- efficiency: (expected_grade × CFU) / total_time
- Interpretation: performance “return” per unit of time
- Efficiency is used for interpretation, not for optimization

Deterministic Model: Decision Variables

- Decision variable for each activity i:
  - $x_i = 1$ if activity i is selected
  - $x_i = 0$ otherwise
- Activities include both exams and seminars
- Binary structure turns the model into a MILP (knapsack-like selection)

Deterministic MILP: Objective and Constraints

- Assumption (baseline): passing is certain ($p_i = 1$ for all)
- Objective (performance): maximize total grade from selected exams max $\Sigma(\text{grade}_i \cdot x_i)$ [exams only; seminars add no grade]
- Constraints:
  - Minimum credits: $\Sigma(\text{CFU}_i \cdot x_i) \geq \text{CFU\_min}$
  - Time limit: $\Sigma(\text{total\_time}_i \cdot x_i) \leq \text{Time\_max}$
  - Integrality: $x_i \in \{0,1\}$

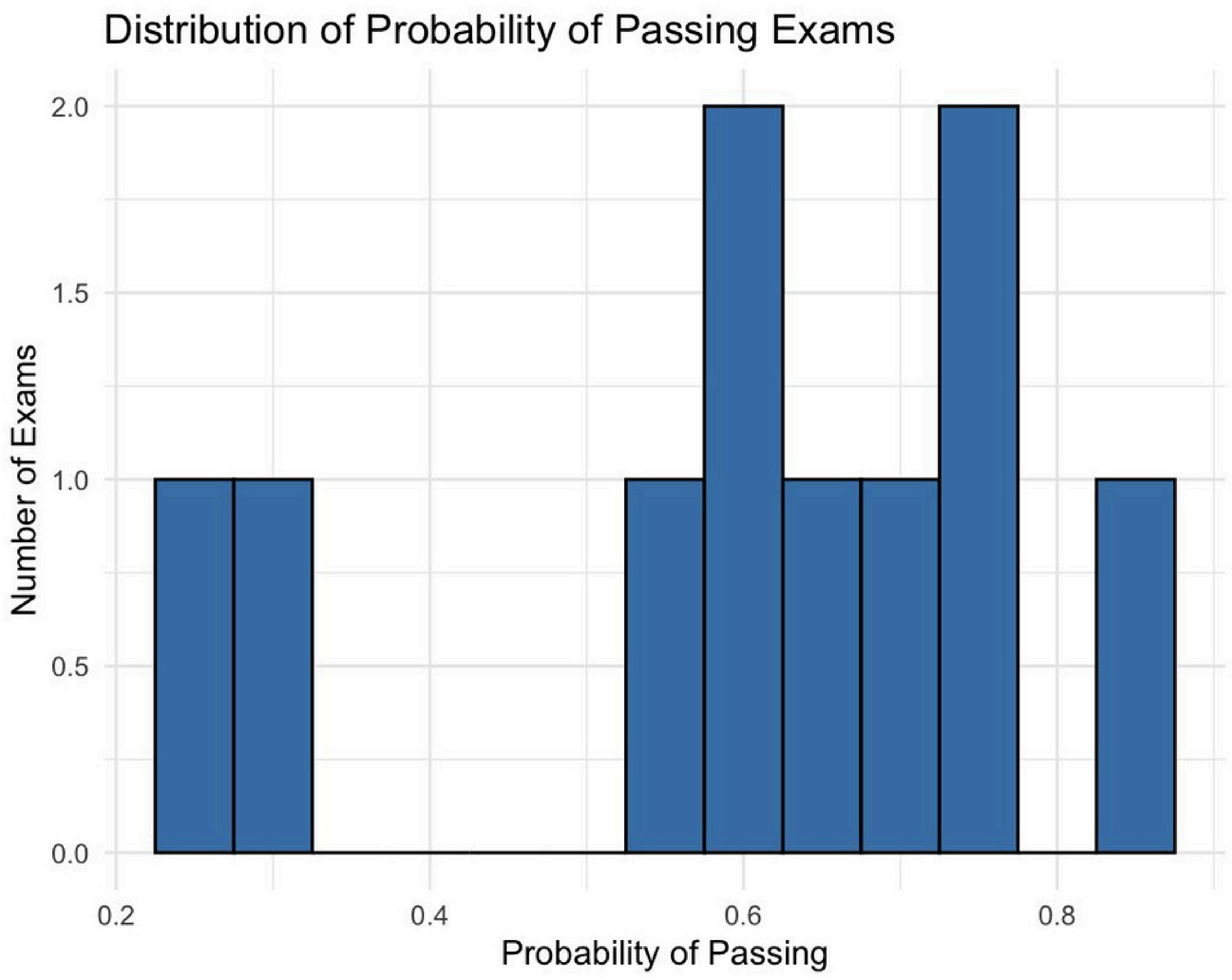
Why Introduce Uncertainty

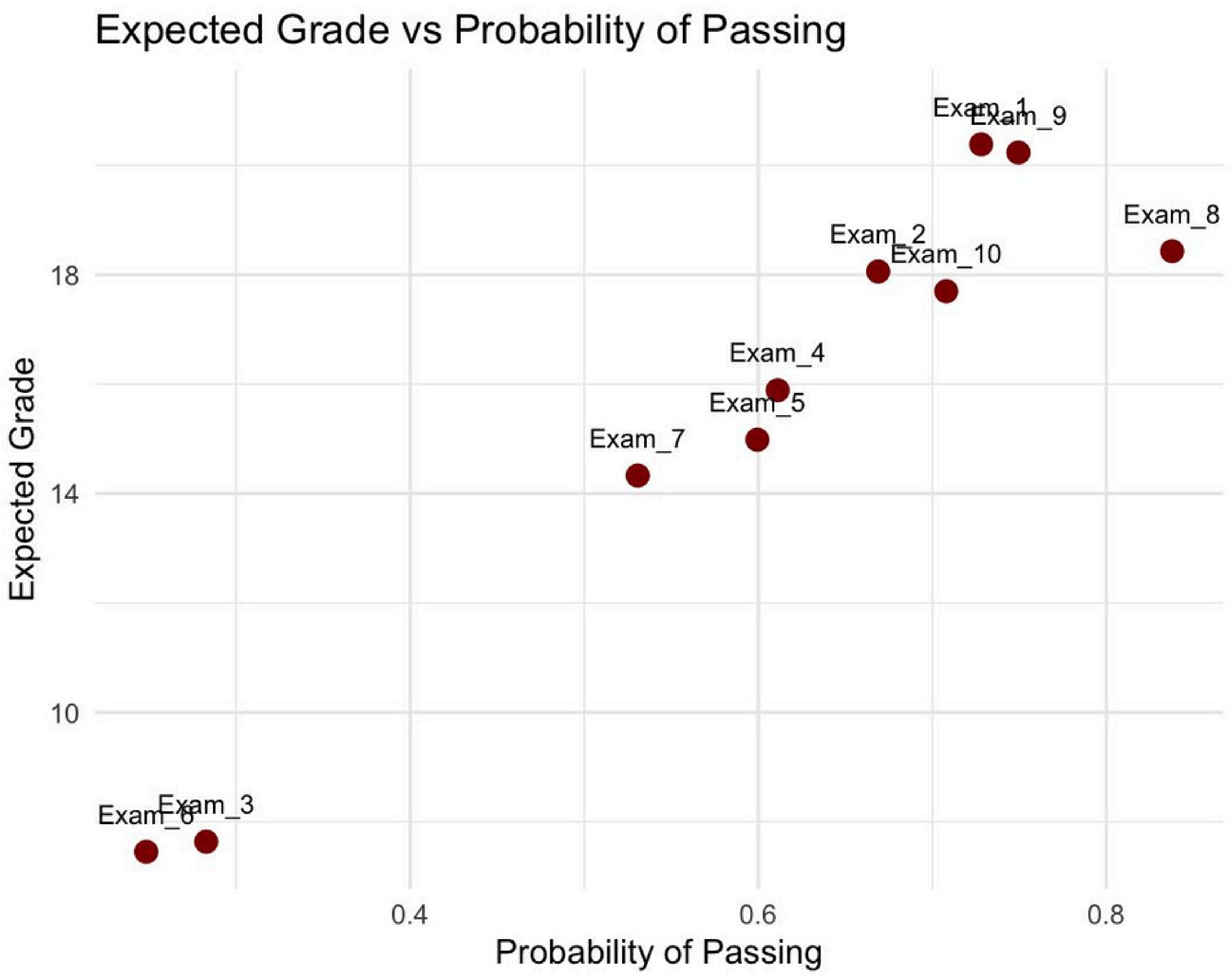
- Deterministic assumption is unrealistic: exams may not be passed
- Each exam has a probability of passing p_pass in (0,1)
- Risk: choosing only high-grade exams might reduce the chance of success
- Need a model that accounts for expected outcomes, not just nominal grades

Stochastic Extension (Expected Value Model)

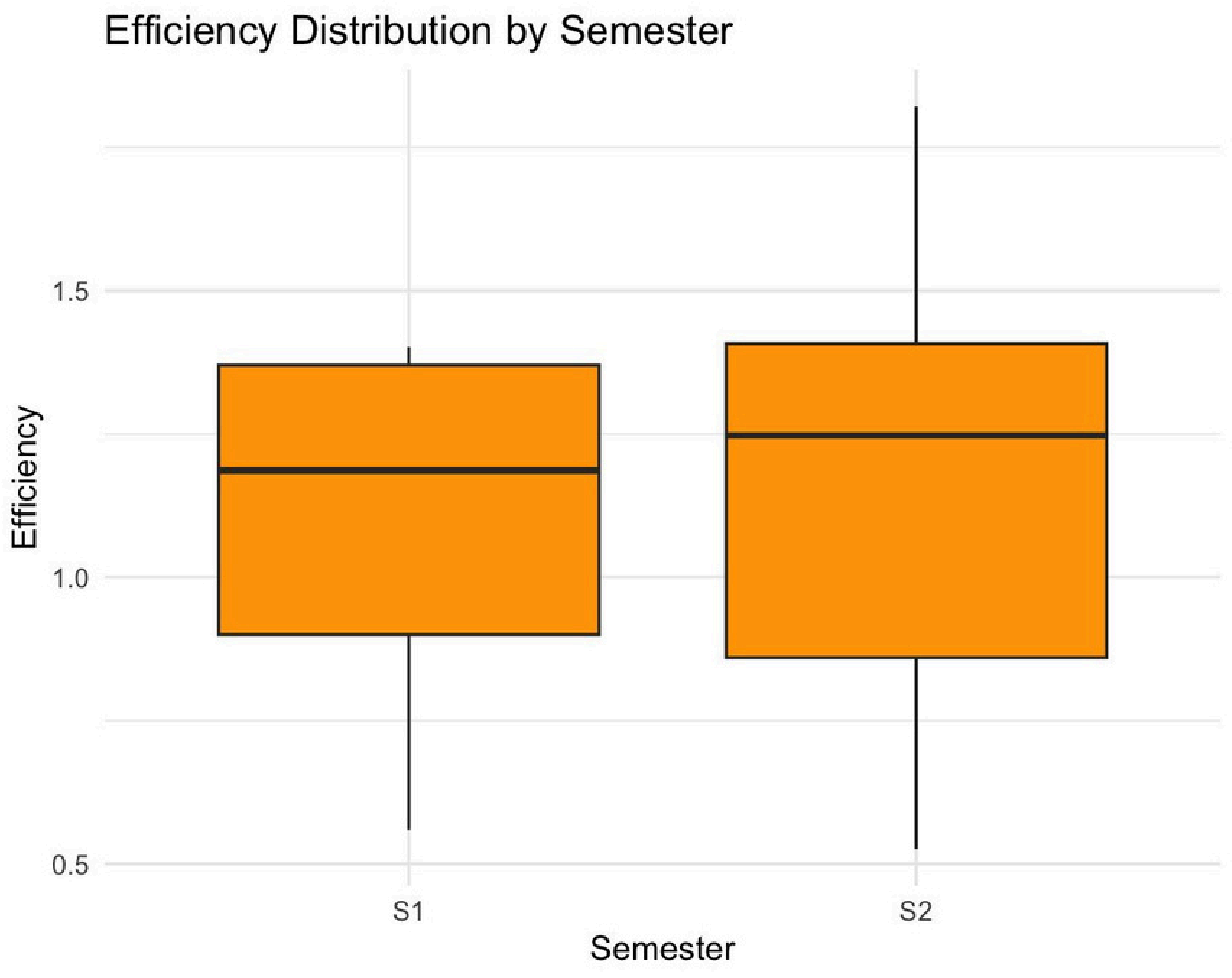
- Exams: p_i = p_pass_i; seminars: $p_i = 1$
- Expected contribution for an exam: $E[grade_i] = p_i \times grade_i$
- New objective (risk-adjusted performance): $\max \Sigma(p_i \cdot grade_i \cdot x_i)$
- Constraints unchanged: CFU_min, Time_max, x_i binary

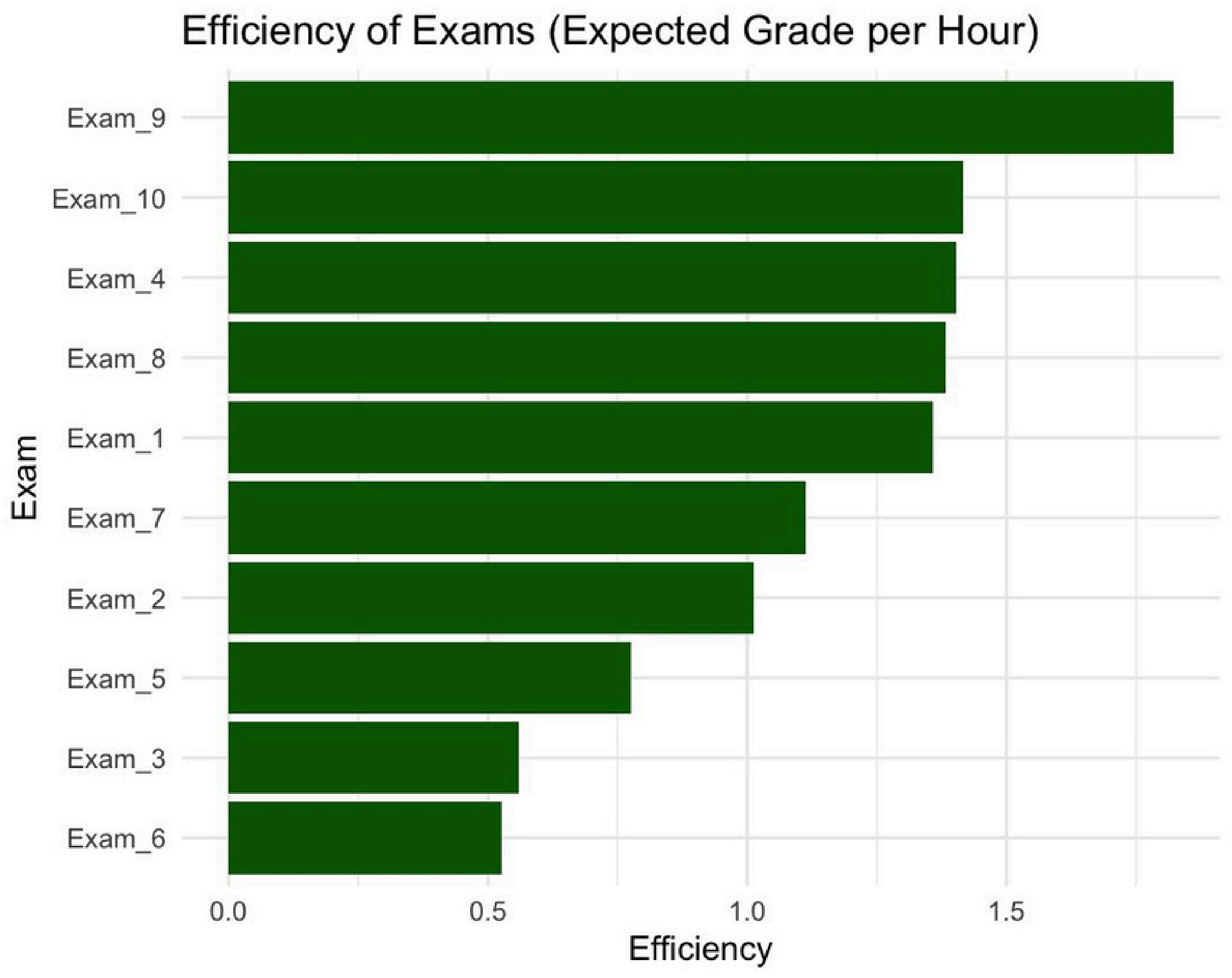
Exploratory Analysis: Uncertainty Patterns





Exploratory Analysis: Efficiency and Timing





Workflow, Conclusions, and Extensions

- Workflow:
  1. Simulate dataset (exams + seminars)
  2. Compute expected_grade and efficiency
  3. Produce descriptive plots and summaries
  4. Formulate deterministic MILP
  5. Extend to stochastic expected-value objective
- Conclusions: optimization provides a transparent rule for selection under constraints; uncertainty changes priorities vs the deterministic baseline
- Possible extensions: chance constraints (minimum probability of meeting CFU), scenario-based stochastic programming, robust optimization, multi-objective (grades + CFU + risk)