# Customer Booking Behavior Analysis Using Markov Chains and a Markov Decision Process

## 1. Introduction

The project analyzes customer booking behavior for a set of holiday houses managed by an agency. Customers interacting with the agency may either complete a booking (*Stay = S*) or not (*No stay = N*). The agency can promote its properties through three alternative booking channels: *Dirette*, *Napoleon*, and *Sardegna Travel.*
**objective**:
- **descriptive analysis** conducted using Markov chains in order to study the long-run behavior of customers under each booking channel.
- the model is extended to a **decision-oriented framework** by formulating a Markov Decision Process (MDP), which allows the agency to compare alternative booking strategies while accounting for operational costs.

## 2. Interpretation of the Booking Channels

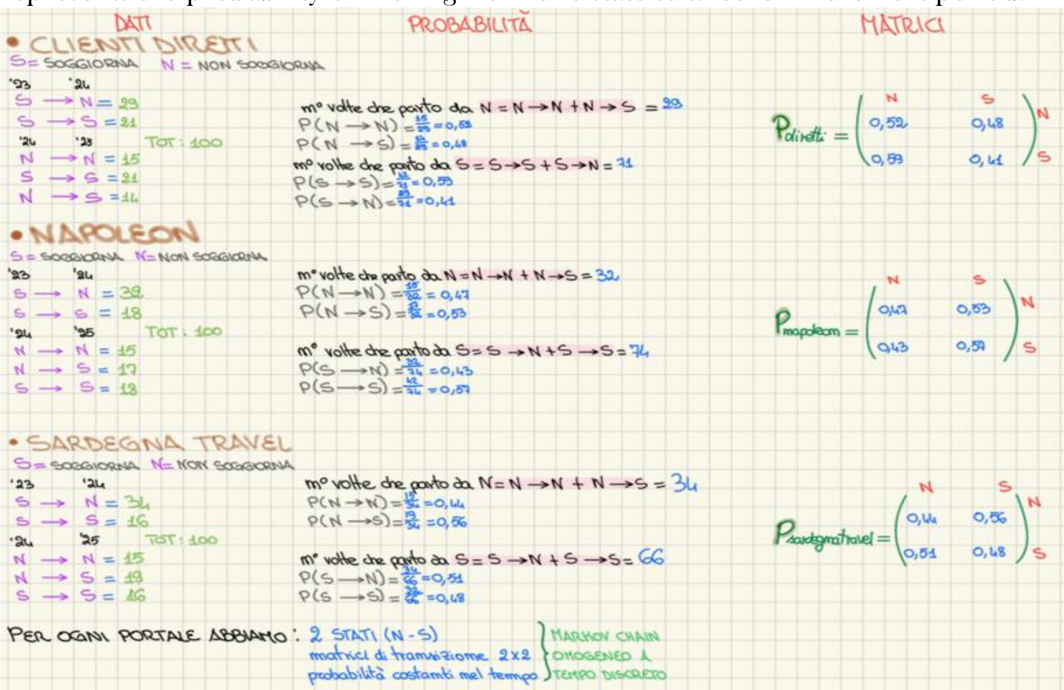
The Markov chains associated with *Dirette, Napoleon, and Sardegna Travel* do **not** represent independent stochastic systems. Instead, they describe **three alternative interaction strategies applied to the same underlying system**, namely the same holiday houses and the same pool of potential customers.
Each booking channel represents a different way for the agency to interact with customers, leading to different transition dynamics between the states *No stay* and *Stay*. Therefore, any comparison between the chains should be interpreted as a **comparison between booking channels**, rather than as a comparison between separate systems.

## 3. Markov Chains

customer behavior is modeled using a discrete-time homogeneous Markov chain. The state space (ss) is defined as:

- *No stay (N)*: the customer does not complete a booking
- *Stay (S)*: the customer completes a booking

For each booking channel, transition probabilities are estimated from observed data describing customer transitions between the two states. These probabilities are collected into a transition matrix, where each element represents the probability of moving from one state to another in the next period.



DATI | PROBABILITÀ | MATRICI

• CLIENTI DIRETTI
S = SOGGIORNA N = NON SOGGIORNA
'23 '24
S → N = 29
S → S = 21
'24 '25 TOT: 100
N → N = 15
S → S = 21
N → S = 14

n° volte che parto da N = N → N + N → S = 29
P(N → N) = 15/29 = 0,52
P(N → S) = 14/29 = 0,48
n° volte che parto da S = S → S + S → N = 71
P(S → S) = 42/71 = 0,59
P(S → N) = 29/71 = 0,41

$$P_{diretti} = \begin{array}{c|cc|c} & N & S & \\ \hline & 0,52 & 0,48 & N \\ & 0,59 & 0,41 & S \end{array}$$

• NAPOLEON
S = SOGGIORNA N = NON SOGGIORNA
'23 '24
S → N = 32
S → S = 18
'24 '25 TOT: 100
N → N = 15
N → S = 17
S → S = 18

n° volte che parto da N = N → N + N → S = 32
P(N → N) = 15/32 = 0,47
P(N → S) = 17/32 = 0,53
n° volte che parto da S = S → N + S → S = 74
P(S → N) = 32/74 = 0,43
P(S → S) = 42/74 = 0,57

$$P_{napoleon} = \begin{array}{c|cc|c} & N & S & \\ \hline & 0,47 & 0,53 & N \\ & 0,43 & 0,57 & S \end{array}$$

• SARDEGNA TRAVEL
S = SOGGIORNA N = NON SOGGIORNA
'23 '24
S → N = 34
S → S = 16
'24 '25 TOT: 100
N → N = 15
N → S = 19
S → S = 16

n° volte che parto da N = N → N + N → S = 34
P(N → N) = 15/34 = 0,44
P(N → S) = 19/34 = 0,56
n° volte che parto da S = S → N + S → S = 66
P(S → N) = 34/66 = 0,51
P(S → S) = 32/66 = 0,48

$$P_{sardegnatravel} = \begin{array}{c|cc|c} & N & S & \\ \hline & 0,44 & 0,56 & N \\ & 0,51 & 0,48 & S \end{array}$$

PER OGNI PORTALE ABBIAMO: 2 STATI (N - S)
matrici di transizione 2x2
probabilità costanti nel tempo
} MARKOV CHAIN OMOGENEO A TEMPO DISCRETO

Secondly, *"what happens in the long run if the agency does nothing and keeps using a given booking channel?"*: for each booking channel, the ***steady-state distribution*** of the Markov chain is computed. The steady-state distribution represents the **long-run proportion of time** the system spends in each state, assuming that the same booking channel is applied consistently over time.

DATO CHE, da N si può andare a S e viceversa (tutte le probabilità > 0); le probabilità sulla diagonale sono > 0 (APERIODICA)
POSSIAMO DIRE CHE, tutte e tre le catene ammettono una distribuzione stazionaria unica.

N.B. i tre canali di prenotazione sono stati analizzati separatamente in quanto ciascun portale presenta caratteristiche operative e commerciali differenti, che influenzano il comportamento dei clienti.
Di conseguenza, le probabilità di transizione tra gli stati N - S risultano diverse per ciascun canale. Per tale motivo è stata costruita una matrice di transizione specifica per ogni portale, modellando tre catene di Markov distinte.

Probabilità Stazionarie $(\pi_N, \pi_S)$

$$\pi_N + \pi_S = 1$$
$$\pi = \pi P$$
$$\begin{cases} \pi_N = \pi_N P(N \to N) + \pi_S P(S \to N) \\ \pi_S = \pi_N P(N \to S) + \pi_S P(S \to S) \end{cases}$$

$$P_{diretti} = \begin{array}{c} \\ N \\ S \end{array}\begin{array}{c} \begin{array}{cc} N & S \end{array} \\ \begin{pmatrix} 0{,}52 & 0{,}48 \\ 0{,}59 & 0{,}41 \end{pmatrix} \end{array} \longrightarrow \begin{array}{l} a = 0.52 \\ 1-a = 0.48 \\ b = 0.59 \end{array} \quad \text{denominatore} = b + (1-a) = 0.59 + 0.48 = 1.07 \quad \begin{array}{l} \pi_N = \frac{0.59}{1.07} = 0.55 \\ \pi_S = \frac{0.48}{1.07} = 0{,}45 \end{array}$$

$$P_{napoleon} = \begin{array}{c} \\ N \\ S \end{array}\begin{array}{c} \begin{array}{cc} N & S \end{array} \\ \begin{pmatrix} 0{,}47 & 0{,}53 \\ 0{,}43 & 0{,}57 \end{pmatrix} \end{array} \longrightarrow \begin{array}{l} a = 0{,}47 \\ 1-a = 0.53 \\ b = 0.43 \end{array} \quad \text{denominatore} = b + (1-a) = 0.43 + 0.53 = 0.96 \quad \begin{array}{l} \pi_N = \frac{0.43}{0.96} = 0.45 \\ \pi_S = \frac{0.53}{0.96} = 0.55 \end{array}$$

$$P_{sardegnatravel} = \begin{array}{c} \\ N \\ S \end{array}\begin{array}{c} \begin{array}{cc} N & S \end{array} \\ \begin{pmatrix} 0{,}44 & 0{,}56 \\ 0{,}51 & 0{,}48 \end{pmatrix} \end{array} \longrightarrow \begin{array}{l} a = 0.44 \\ 1-a = 0.56 \\ b = 0.81 \end{array} \quad \text{denominatore} = b + (1-a) = 0.81 + 0.56 = 1.37 \quad \begin{array}{l} \pi_N = \frac{0.81}{1.37} = 0.59 \\ \pi_S = \frac{0.56}{1.37} = 0.41 \end{array}$$

At this point, the results show that the booking channels differ in terms of their long-run customer retention rates. In particular, Napoleon exhibits the highest steady-state probability of *Stay*, followed by Dirette, while Sardegna Travel displays the lowest long-run probability of booking.

## 4. MDP Extension

While the Markov chain analysis provides valuable descriptive insights, it does not support **decision-making**. In practice, the agency must actively choose which booking channel to use, taking into account not only customer retention but also operational costs such as commissions or marketing expenses.
To address this limitation, the model is extended to a **Markov Decision Process (MDP)**, which explicitly incorporates decisions, costs, and rewards, allowing the agency to identify the booking strategy that maximizes long-run performance.

APPLICATION:

- **States**

The state space of the MDP coincides with that of the Markov chain: $ss$ = {No stay,Stay}

- **Actions**

The action space corresponds to the available booking channels: $A$ = {*Dirette,Napoleon,Sardegna Travel*}
At each decision epoch, the agency selects one booking channel to interact with customers.

- **Transition Probabilities**

For each action, transition probabilities are given by the previously estimated transition matrices. Each matrix captures the customer dynamics induced by a specific booking channel and is assumed to be stationary over time.

- **Action Costs**

To reflect differences in commission structures and operational expenses, a hypothetical monetary cost is associated with each booking channel. *These costs are assumed for modeling purposes and are not based on real data.*

| BOOKING CHANNEL | COST € | INTERPRETATION |
|---|---|---|
| Dirette | 0 | Direct channel, no external commission |

| | | |
|---|---|---|
| Napoleon | 15 | Approximately 15% OTA commission |
| Sardegna Travel | 20 | Higher commission and promotional cost |

- **Reward Function**

The reward function is defined to account for both customer retention and action costs. A positive reward is obtained when the system is in the *Stay* state, reflecting the economic value of a completed booking. No reward is obtained in the *No stay* state.
the reward function is defined as:

$$R(s,a) = \begin{cases} 100 - c(a) & \text{if } s = \text{Stay} \\ 0 - c(a) & \text{if } s = \text{No stay} \end{cases}$$

Where:
- $c(a)$denotes the cost associated with action $a$.
- the value of 100 € represents the assumed average booking revenue.

**OBJECTIVE:** the objective of the MDP is to identify the booking channel strategy that **maximizes the long-run expected reward**, balancing customer retention and monetary costs.

**RESULTS**: Although the Napoleon channel exhibits the highest long-run probability of customer stay, its commission cost significantly reduces the net expected reward.Under the stated assumptions, the direct booking channel (Dirette) maximizes the long-run expected reward. This result highlights the trade-off between customer retention and operational costs: a slightly lower retention rate can be more than compensated by the absence of commission fees.
**BETTER EXPLANATION →** Using the Markov Decision Process framework, the long-run expected reward was computed for each booking channel by combining customer retention dynamics with monetary costs expressed in euros.

Assuming an average booking value of 100 €, the resulting long-run expected rewards are:

- **Dirette:** approximately 44.9 €
- **Napoleon:** approximately 40.2 €
- **Sardegna Travel:** approximately 32.3 €

Although the Napoleon channel exhibits the highest long-run probability of customer stay, its commission cost significantly reduces the net expected reward. In contrast, the direct booking channel (Dirette) benefits from the absence of commission fees, which compensates for its lower customer retention rate.

Thus, ynder the stated assumptions, the **Dirette** channel maximizes the long-run expected reward and is therefore identified as the optimal booking strategy.

## 5. Conclusion

This project combines a descriptive Markov chain analysis with a decision-oriented Markov Decision Process framework to study customer booking behavior across alternative booking channels.

The Markov chain analysis provides a baseline understanding of the long-run behavior of customers when no active decision is taken. Extending the model to an MDP allows the agency to explicitly account for strategic choices and monetary costs.

Under the stated assumptions, including realistic commission costs expressed in euros, the direct booking channel (Dirette) maximizes the long-run expected reward. This result illustrates how lower operational costs can compensate for reduced customer retention, highlighting the importance of integrating economic considerations into decision-making models.

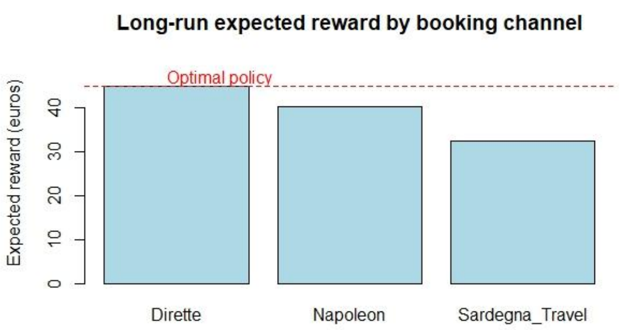
Overall, the analysis demonstrates how stochastic models can support managerial decisions by balancing customer behavior and cost efficiency.

R:

```
# ============================================================
# QM PROJECT — Markov Chains and Markov Decision Process
# ============================================================

# States:
# 1 = No stay
# 2 = Stay


# ============================================================
# (1) RAW DATA — OBSERVED COUNTS
# ============================================================

counts_dirette <- matrix(c(52, 48,
                           59, 41),
                         nrow = 2, byrow = TRUE)

counts_napoleon <- matrix(c(47, 53,
                            43, 57),
                          nrow = 2, byrow = TRUE)

counts_sardegna <- matrix(c(44, 56,
                            51, 49),
                          nrow = 2, byrow = TRUE)


# ============================================================
# (2) TRANSITION MATRICES
# ============================================================

transition_matrix <- function(counts) {
  counts / rowSums(counts)
}

P_dirette  <- transition_matrix(counts_dirette)
P_napoleon <- transition_matrix(counts_napoleon)
P_sardegna <- transition_matrix(counts_sardegna)


# ============================================================
# (3) STEADY-STATE DISTRIBUTIONS (BASELINE)
# ============================================================

steady_state <- function(P) {
  A <- matrix(c(1 - P[1,1], -P[2,1],
                1, 1),
              nrow = 2, byrow = TRUE)
  b <- c(0, 1)
  solve(A, b)
}

pi_dirette  <- steady_state(P_dirette)
pi_napoleon <- steady_state(P_napoleon)
pi_sardegna <- steady_state(P_sardegna)


# ============================================================
# (4) MDP PARAMETERS
# ============================================================
```

```
# Average booking revenue (euros)
booking_value <- 100

# State rewards
reward_state <- c(0, booking_value)  # No stay, Stay

# Action costs (euros)
costs <- c(
  Dirette = 0,
  Napoleon = 15,
  Sardegna_Travel = 20
)


# ==========================================================
# (5) LONG-RUN EXPECTED REWARD
# ==========================================================

expected_reward <- function(P, cost) {
  pi <- steady_state(P)
  sum(pi * reward_state) - cost
}

R_dirette  <- as.numeric(expected_reward(P_dirette,  costs["Dirette"]))
R_napoleon <- as.numeric(expected_reward(P_napoleon, costs["Napoleon"]))
R_sardegna <- as.numeric(expected_reward(P_sardegna, costs["Sardegna_Travel"]))

results <- c(
  Dirette = R_dirette,
  Napoleon = R_napoleon,
  Sardegna_Travel = R_sardegna
)

results


# ==========================================================
# (6) OPTIMAL POLICY
# ==========================================================

optimal_action <- names(which.max(results))

cat("\nOptimal booking channel under the stated assumptions:",
    optimal_action, "\n")
```

## Graphic representation




***Long-run expected reward by booking channel.*** *The figure summarizes the MDP results by comparing the long-run expected rewards associated with each booking channel. Long-run expected reward associated with each booking channel under the stated assumptions. The direct booking channel (Dirette) maximizes the expected reward once commission costs are taken into account.*