

Exam & Seminar Selection Under Time and CFU Constraints

Binary Linear Programming: deterministic baseline + stochastic extension

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Motivation and Decision Problem

- Students face limited time and multiple possible activities (exams, seminars)
- Each activity has a “cost” in time and a “benefit” in CFU and/or performance
- Goal: select the best subset under constraints
- The decision is binary: an activity is either selected or not
- Approach: optimization replaces trial-and-error with a structured decision rule

Dataset Overview (Simulation)

- Activities: 10 exams + 4 seminars
- Key variables: type, CFU, semester (S1/S2), grade (exams only), passing probability (exams), time components
- Seminars: CFU contribution, no grade, always passed
- CFU vary across exams (6, 9, 12) to reflect heterogeneous workloads
- Purpose: generate realistic inputs to test the modeling framework

How the Data Were Generated

- Exam grades: simulated around typical values, bounded to a realistic range
- Exam pass probability p_{pass} : varies across exams (captures uncertainty)
- Time: lecture_hours linked to CFU; study_hours generated within a plausible interval
- Total_time = lecture_hours + study_hours (with one special heavy exam case)
- The dataset is an input to the optimization model, not the final goal

Derived Metrics Used in the Project

- $\text{expected_grade (exams)}: p_{\text{pass}} \times \text{grade}$
- $\text{expected_grade (seminars)}: 0$ (no grade component)
- $\text{efficiency}: (\text{expected_grade} \times \text{CFU}) / \text{total_time}$
- Interpretation: performance “return” per unit of time
- Efficiency is used for interpretation, not for optimization

Deterministic Model: Decision Variables

- Decision variable for each activity i :
 - $x_i = 1$ if activity i is selected
 - $x_i = 0$ otherwise
- Activities include both exams and seminars
- Binary structure turns the model into a MILP (knapsack-like selection)

Deterministic MILP: Objective and Constraints

- Assumption (baseline): passing is certain ($p_i = 1$ for all)
- Objective (performance): maximize total grade from selected exams $\max \sum (\text{grade}_i \cdot x_i)$
[exams only; seminars add no grade]
- Constraints:
 - Minimum credits: $\sum (\text{CFU}_i \cdot x_i) \geq \text{CFU_min}$
 - Time limit: $\sum (\text{total_time}_i \cdot x_i) \leq \text{Time_max}$
 - Integrality: $x_i \in \{0,1\}$

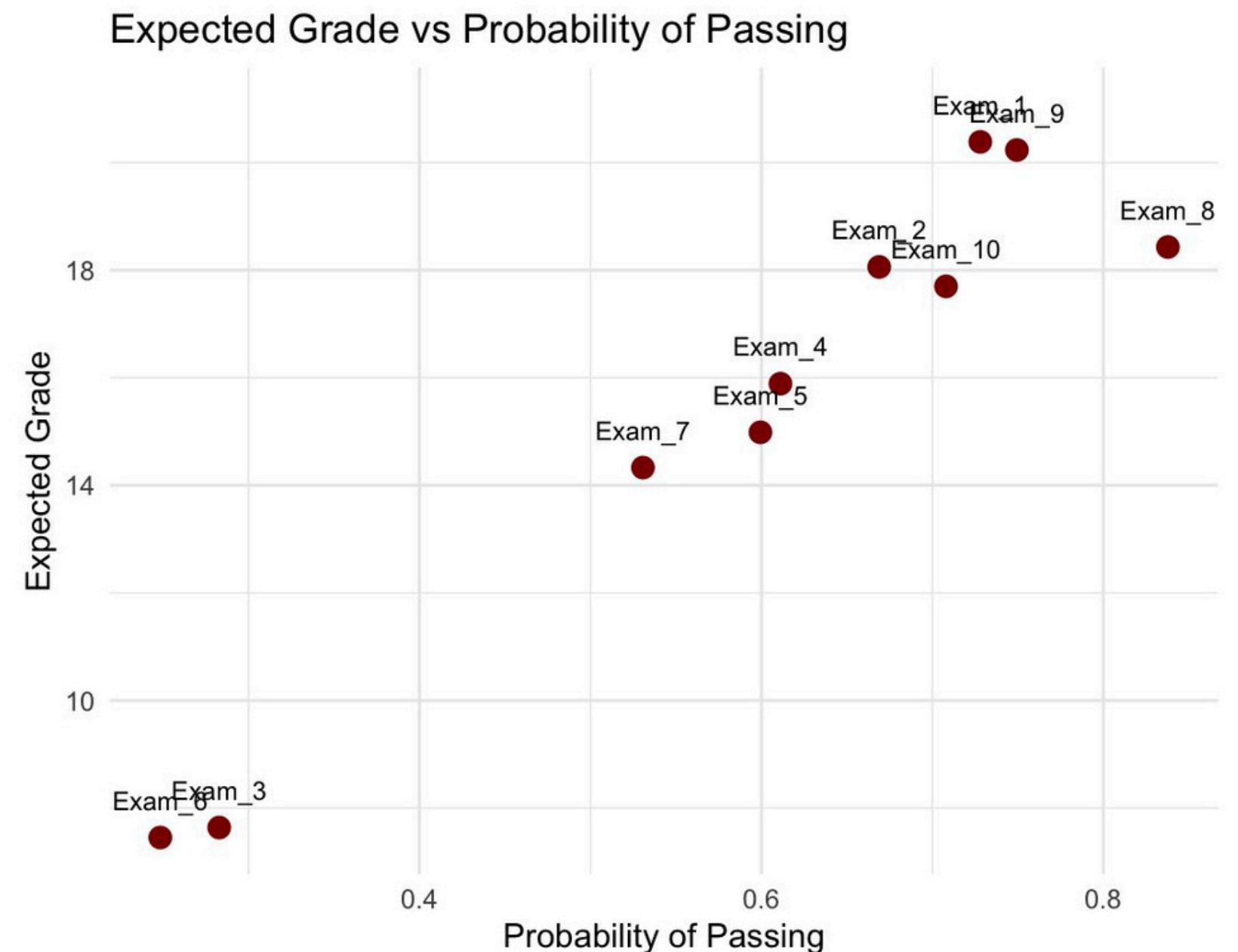
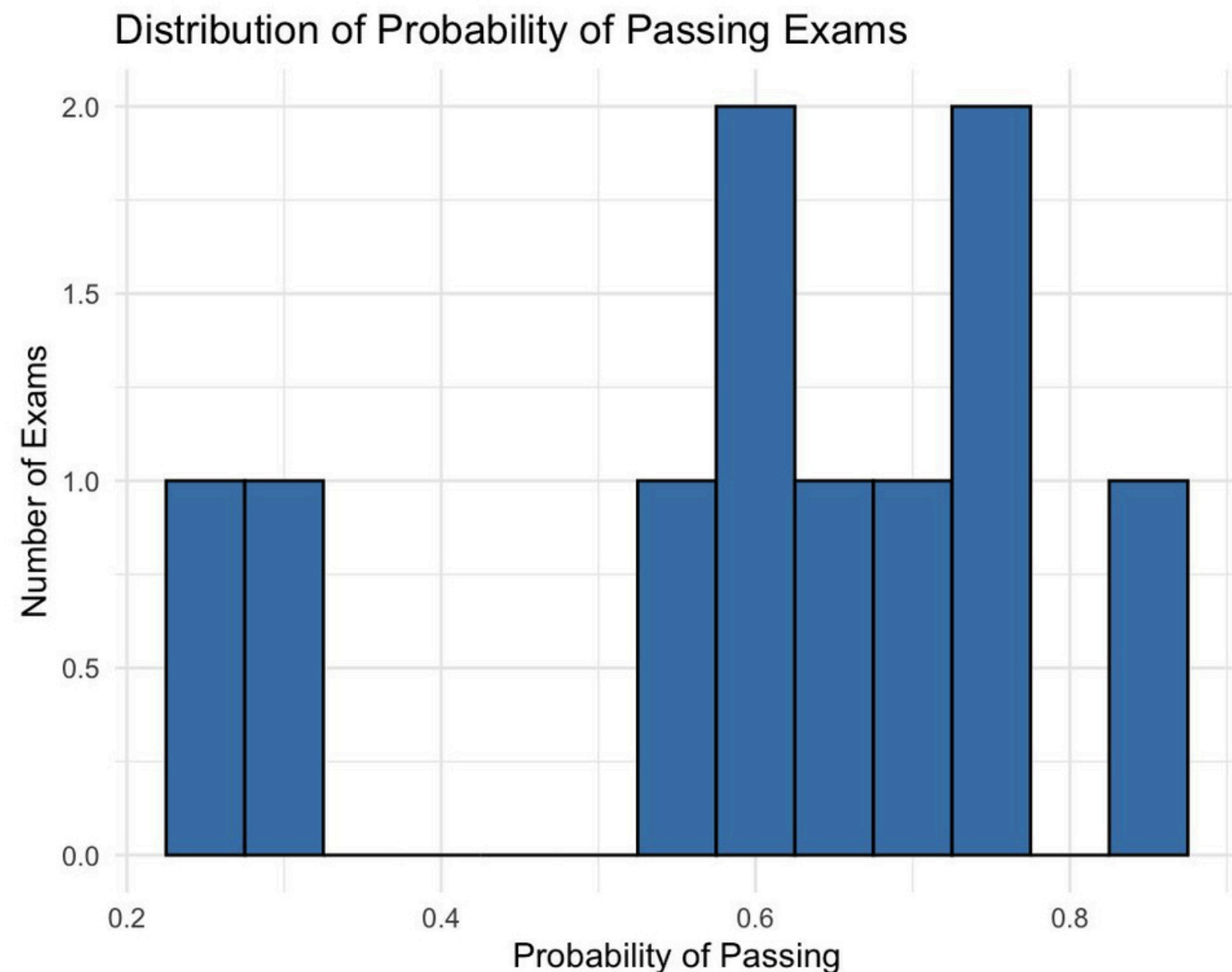
Why Introduce Uncertainty

- Deterministic assumption is unrealistic: exams may not be passed
- Each exam has a probability of passing p_{pass} in $(0,1)$
- Risk: choosing only high-grade exams might reduce the chance of success
- Need a model that accounts for expected outcomes, not just nominal grades

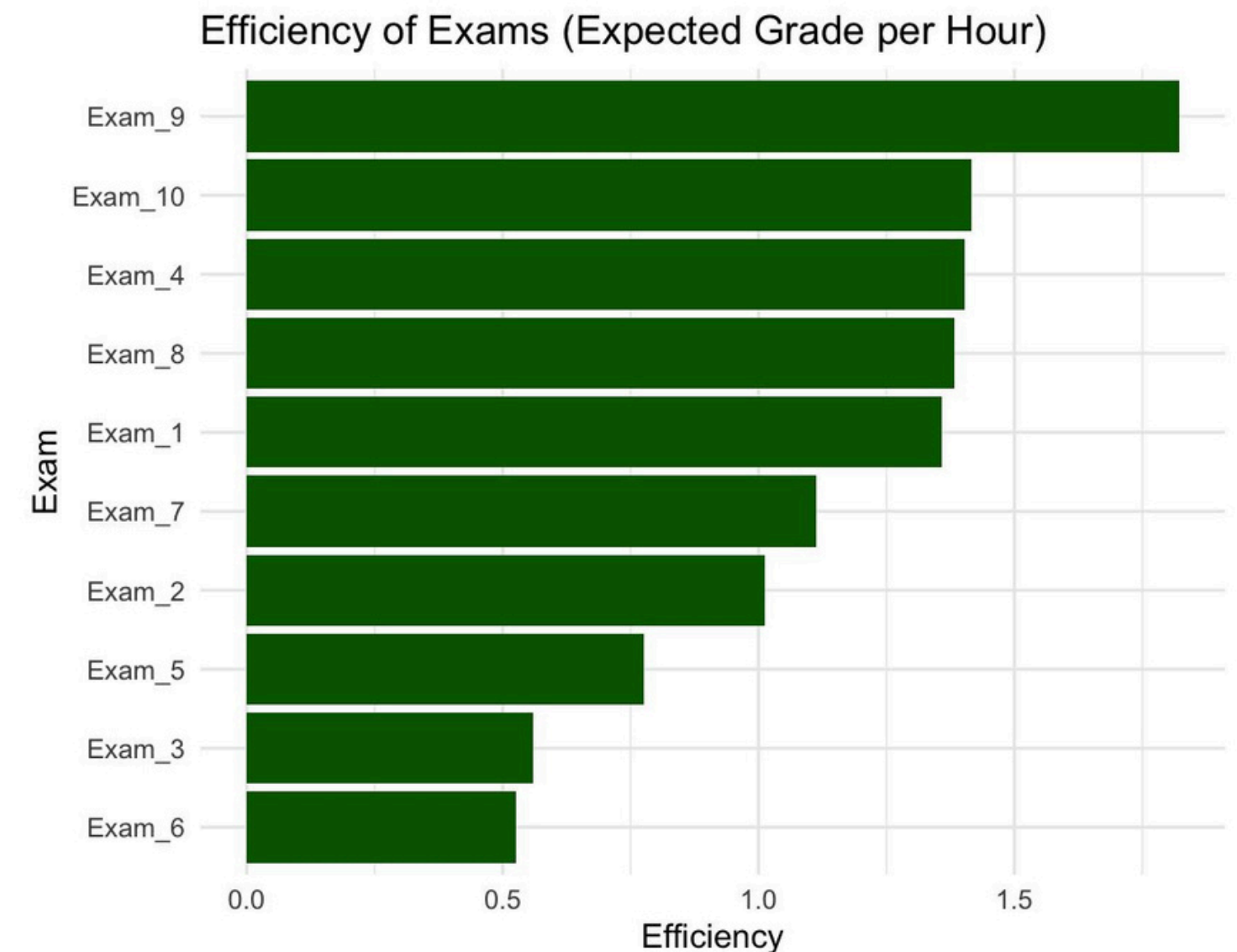
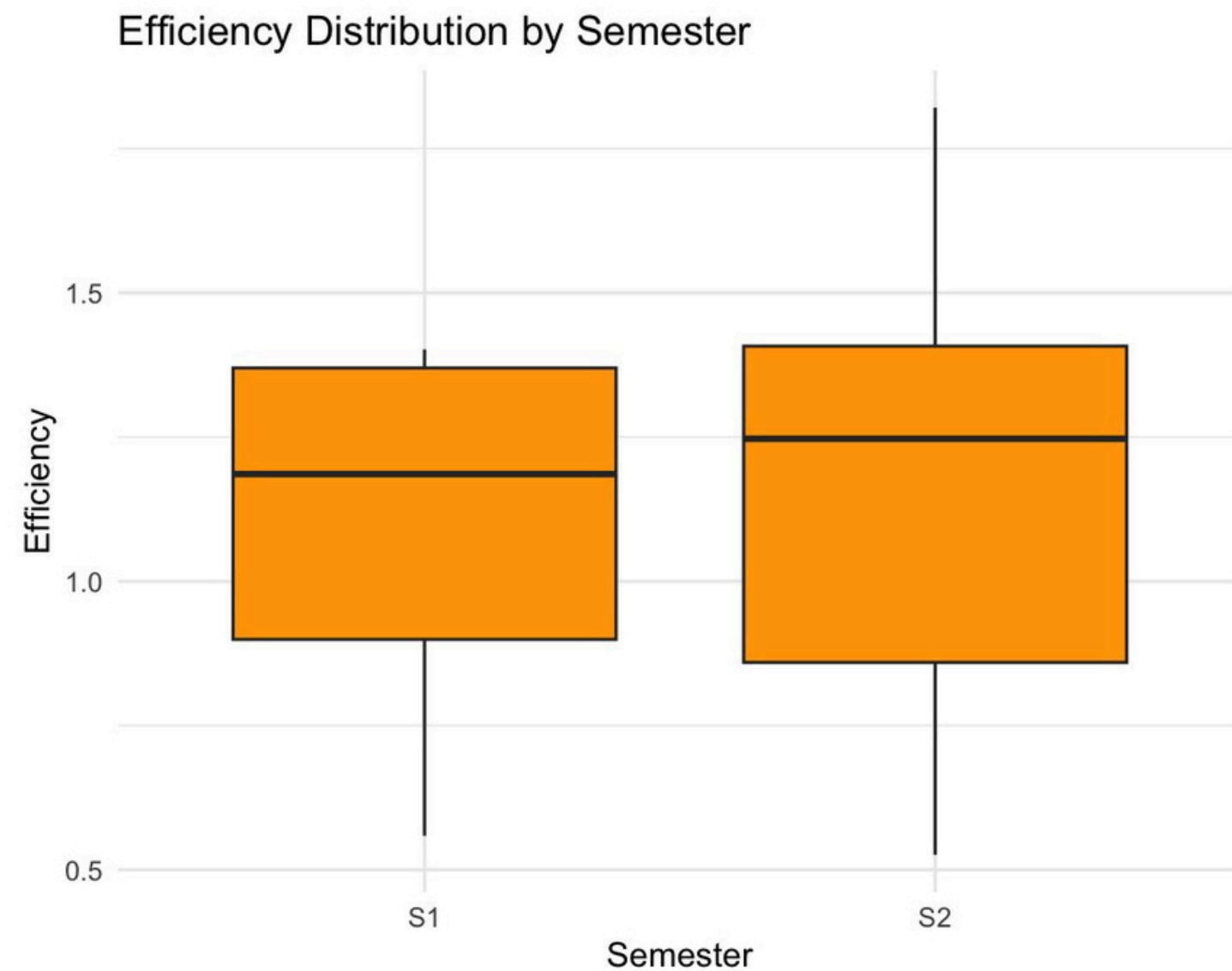
Stochastic Extension (Expected Value Model)

- Exams: $p_i = p_{\text{pass}_i}$; seminars: $p_i = 1$
- Expected contribution for an exam: $E[\text{grade}_i] = p_i \times \text{grade}_i$
- New objective (risk-adjusted performance): $\max \sum (p_i \cdot \text{grade}_i \cdot x_i)$
- Constraints unchanged: CFU_min, Time_max, x_i binary

Exploratory Analysis: Uncertainty Patterns



Exploratory Analysis: Efficiency and Timing



Workflow, Conclusions, and Extensions

- Workflow:
 1. Simulate dataset (exams + seminars)
 2. Compute expected_grade and efficiency
 3. Produce descriptive plots and summaries
 4. Formulate deterministic MILP
 5. Extend to stochastic expected-value objective
- Conclusions: optimization provides a transparent rule for selection under constraints; uncertainty changes priorities vs the deterministic baseline
- Possible extensions: chance constraints (minimum probability of meeting CFU), scenario-based stochastic programming, robust optimization, multi-objective (grades + CFU + risk)