

h -parameters is studied more extensively. For a value of $h = 2 \cdot 10^{-14} \text{ cm}^4/\text{s}$ there are also two ranges with a linear decay of the junction voltage. In the first linear range the lifetime after Davies is determined to be 0.7τ . The second linear range begins at $t = 4 \tau$. In this region the true carrier lifetime again can be determined with a good approximation from the Davies formula.

The influence of a reduction of the forward current on the time dependence of the junction voltage has also been examined. At a current turn-off from 100 A/cm^2 and a value of $h = 2 \cdot 10^{-14} \text{ cm}^4/\text{s}$ for the recombination parameters in the end regions, the same time dependencies, as before at current turn-off from 3000 A/cm^2 , appear qualitatively. The lifetime after Davies is found now to be 0.8τ in the first linear decay of the junction voltage, while it is identical with the true lifetime in the second linear range.

III. SUMMARY

The results of our numerical investigation can be summarized as follows. The evaluation of the linear decay of the open-circuit voltage after Davies gives the true value of the carrier lifetime in the i-region of p-i-n rectifiers, only if the recombination in the end regions is negligible. Taking into account the recombination in the end regions the Davies formula gives a

smaller value for the lifetime, when the first linear decay range of the open-circuit voltage is evaluated. The error in the determination of the carrier lifetime amounts to 40 percent for an initial current density of 3000 A/cm^2 and h -parameters of $10^{-13} \text{ cm}^4/\text{s}$. Moreover, it increases with increasing current density and with increasing h -parameters. However, the true value of the carrier lifetime can be determined after Davies in the second linear range of the open-circuit voltage decay with a high accuracy.

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Metal-(n) AlGaAs-GaAs Two-Dimensional Electron Gas FET

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Abstract—Theoretical calculations have been developed for a two-dimensional electron gas FET (TEGFET) constituted by a AlGaAs (n)-GaAs (n⁻ or p⁻) heterostructure in which the Schottky gate is deposited on the AlGaAs (n) top layer. The theory takes into account: i) the subband splitting in the two-dimensional electron gas (2-DEG); and ii) the existence of an undoped AlGaAs spacer layer which has been found to enhance the electron mobility.

The sheet carrier concentration of the TEGFET has been calculated, and a simple analytical formula has been established for the charge control in large and small gate FET.

I. INTRODUCTION

THE PRESENCE of an inversion or accumulation electron layer located at the interface of certain heterojunctions was predicted about twenty years ago by Anderson [1]. In

1969, Esaki and Tsu [2] proposed a heterostructure in which ionized impurities and free electrons could be spatially separated giving rise to a reduced Coulomb scattering. In 1978, Dingle *et al.* [3] observed mobility enhancement in modulation-doped superlattices. More recently, various experiments concerning modulation-doped GaAs-AlGaAs single heterojunctions and superlattices have shown the high electron mobility behavior [4]–[8] of the two-dimensional gas [9]–[13]. At the same time, several papers were published showing the use of such structures for FET applications. The two-dimensional electron gas FET (TEGFET) has the Schottky gate deposited on the GaAs layer [14], [15] or on the $\text{Al}_x\text{Ga}_{1-x}\text{As}$ layer [16], [17]. Low noise TEGFET's have been fabricated in our laboratory with the latter structure: 2.3-dB noise figure, with 10.3 dB associated gain at 10 GHz [18]. It was also demonstrated that the latter structure presents higher mobility [19] and higher FET performance [17] than the former structure.

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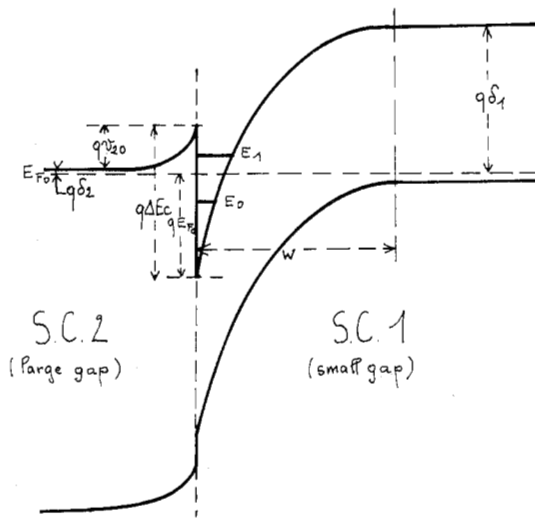


Fig. 1. $\text{Al}_x\text{Ga}_{1-x}\text{As}$ (n)-GaAs (p) heterojunction at equilibrium and isolated.

In this paper we shall consider transistors having the metal-AlGaAs (n)-GaAs (n⁻ or p⁻) structure. The TEGFET with GaAs as the uppermost layer has been treated recently [20]. The theoretical approach we have in this work differs slightly from the previously published paper: we take into account the subband splitting in the two-dimensional electron gas (2-DEG) and the existence of an undoped AlGaAs spacer layer which spatially separates electrons from their parent-donor impurities.

We shall study successively:

- i) the equilibrium of a heterostructure which consists of two different semiconductors: a large gap n-type one and a small gap n⁻ or p⁻-type one, such that the Anderson conditions for the existence of an electron accumulation or inversion layer in the latter are satisfied;
- ii) the control of the 2-DEG layer sheet density by a Schottky gate deposited on the large gap semiconductor;
- iii) the performances of FET structures using such materials.

All the numerical estimations will concern the case of GaAs (p⁻)/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$ (n) heterostructures, the experimental data relative to the subbands splitting when the GaAs is n⁻-type being is insufficient at this date. Nevertheless, the analytical treatment given below is valid for both n⁻- and p⁻-cases.

II. TREATMENT OF THE TWO-DIMENSIONAL ELECTRON LAYER

The objective in this section is the calculation of the Fermi-level position in the 2-DEG, the free electron surface density and the electric field at the interface. Only two subbands will be taken into account and the first question concerns the splitting of these subbands. We shall use a semi-empirical formulation derived from an approximate well model and corrected to take into account the most important published results.

Fig. 1 gives the used notations (a p-type small gap semiconductor is assumed). It starts from the assumption of a quasi-constant electric field \mathcal{E} in the potential well (triangular well approximation).

The solution for the longitudinal quantized energy is then well approximated by the formula

$$E_n(\text{eV}) \# \left(\frac{\hbar^2}{2m_l} \right)^{1/3} \left(\frac{3}{2} \pi q \mathcal{E} \right)^{2/3} \left(n + \frac{3}{4} \right)^{2/3} \quad (1)$$

where m_l^* is the longitudinal effective mass. For GaAs and considering only the lower and the excited subband, one obtains

$$\begin{aligned} E_0(\text{eV}) &\# 1.83 \times 10^{-6} \mathcal{E}^{2/3} \\ E_1(\text{eV}) &\# 3.23 \times 10^{-6} \mathcal{E}^{2/3} \quad \mathcal{E} \text{ in V/m.} \end{aligned} \quad (2)$$

The splitting given by (2) is certainly exaggerated resulting from the sublinear increase of the conduction band edge energy with the variable x ; keeping as a reasonable approximation the $\frac{2}{3}$ power law for E_0 and E_1 versus \mathcal{E} , we shall modify the two numerical coefficients of (2) to be in agreement with the most important published experimental results.

First we shall establish a relation between the interface electric field \mathcal{E}_i , and the electron sheet concentration

i) p⁻-Type Case

In the small gap semiconductor the electric field obeys the Poisson equation

$$\frac{d\mathcal{E}_1}{dx} = -\frac{q}{\epsilon_1} [n(x) + N_{A_1}] \quad (3)$$

$n(x)$ being the bulk free electron concentration and N_{A_1} the ionized acceptors density in the small gap semiconductor. Integration between the limit of the depletion region ($\mathcal{E}_1 = 0$) and the interface ($\mathcal{E}_1 = \mathcal{E}_i$) gives

$$\epsilon_1 \mathcal{E}_i = qn_s + qN_{A_1} w_1 \quad (4)$$

where ϵ_1 is the dielectric permittivity in the small gap semiconductor, n_s the 2-DEG electron density, and w_1 the space charge region width. For MESFET's applications, where a good mobility is required, N_{A_1} is chosen very low (the material is then qualified as "undoped" or "nonintentionally doped") to reduce the impurity scattering in the 2-DEG layer. Then the second term in the second member of (4) is negligible and we have a good approximation

$$\epsilon_1 \mathcal{E}_i \# qn_s. \quad (5)$$

ii) n⁻-Type Case

The electric field \mathcal{E}_1 obeys then the Poisson equation

$$\frac{d\mathcal{E}_1}{dx} = -\frac{q}{\epsilon_1} (n(x) - N_{D_1}) \quad (6)$$

where N_{D_1} is the ionized donors density in the semiconductor. As previously noted, if d_1 is the small gap semiconductor layer width and if $\mathcal{E}(d_1) \neq 0$ (neutrality at d_1) the integration of (6) gives

$$\epsilon_1 \mathcal{E}_i = qn_s - qN_{D_1} d_1. \quad (7)$$

If N_{D_1} and d_1 are chosen sufficiently small we have

$$\epsilon_1 \mathcal{E}_i \# qn_s \quad (8)$$

that is the same result than for the p⁻-type case.

From these assumptions and (2), (5), and (8) the subbands

positions are given by the relations

$$\begin{aligned} E_0 &\# \gamma_0 (n_s)^{2/3} \\ E_1 &\# \gamma_1 (n_s)^{2/3} \end{aligned} \quad (9)$$

where γ_0 and γ_1 are adjustable parameters which otherwise would be given by

$$\begin{aligned} \gamma_0 &= 2.26 \times 10^{-12} \\ \gamma_1 &= 4 \times 10^{-12} \quad (\text{Système International (SI)}) \end{aligned} \quad (10)$$

according to (2), (5), or (8).

The treatment of the 2-DEG layer will be complete when a relation between n_s and the Fermi-level position will have been established. The density of states (associated with a single quantized energy level) for a two-dimensional system being a constant is

$$D = \frac{qm^*}{\pi \hbar^2} \quad (11)$$

(spin degeneracy 2, valley degeneracy 1) it is easy to obtain the relation between E_F , E_0 , E_1 , and n_s . The calculation is performed in Appendix I and gives

$$n_s = D \frac{kT}{q} \log [(1 + e^{q/kT(E_F - E_0)})(1 + e^{q/kT(E_F - E_1)})] \quad (12)$$

which at low temperatures, reduces to

$$n_s = D(E_F - E_0) \quad (13)$$

or

$$n_s = D(E_1 - E_0) + 2D(E_F - E_1) \quad (14)$$

when the second subband is, respectively, unoccupied or occupied. From the published results performed at low temperatures by Shubnikov De Haas or cyclotron resonance experiments [10]–[13], an estimation of γ_0 and γ_1 can be done. In the p⁺-case one finds

$$\begin{aligned} \gamma_0 &= 2.5 \times 10^{-12} \\ \gamma_1 &= 3.2 \times 10^{-12} \quad (\text{SI}) \end{aligned} \quad (15)$$

and from the measured cyclotron mass

$$D = 3.24 \times 10^{17} \text{ m}^{-2} \cdot \text{V}^{-1}. \quad (16)$$

For the n⁺-case few results are published but the band bending being less pronounced the subband splitting is certainly smaller than for the p⁺-case. This can explain why some authors [4] have observed the two subbands occupation at low temperature when the small gap semiconductor is n-type and when the measured value of n_s is moderate.

III. EQUILIBRIUM

Fig. 1 shows the band diagram of the studied heterostructure in the p⁺-type case at equilibrium and isolated from the influence of any external contact. Two of the presumed subbands in the notch are shown but their position is only illustrative. In the calculation we shall take into account the presence near the interface of an undoped AlGaAs spacer layer of thickness:

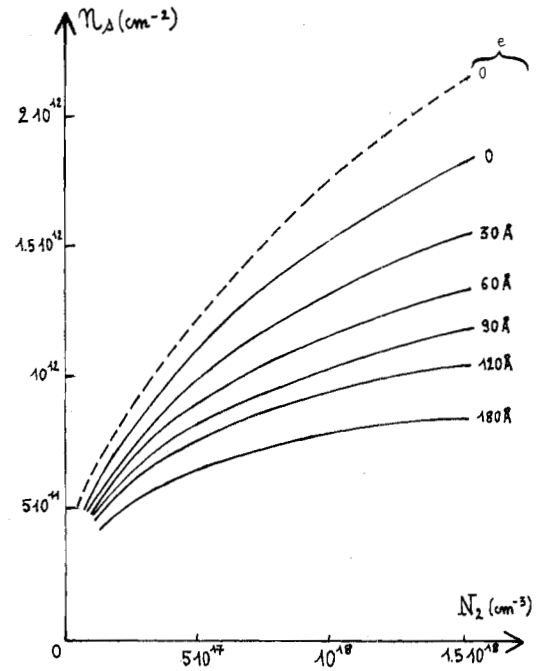


Fig. 2. Sheet concentration of the 2-DEG. GaAs is taken p-type ($N_{A1} = 10^{14} \text{ cm}^{-3}$) and Al concentration in AlGaAs is 0.3. Full curves correspond to various thicknesses e , dash curve to the case $e = 0$ but ignore the two-dimensional character of the electron sheet.

e which reduces the electron scattering in the 2-DEG by the ionized impurities remaining in the large gap semiconductor space charge region. We shall compute the electric displacement vector at the interface in the large gap semiconductor by assuming the total depletion approximation in its space charge layer. The band bending being v_{20} , one easily obtains (see Appendix II-A)

$$\epsilon_2 \mathcal{E}_{i_2} = \sqrt{2q\epsilon_2 N_2 v_{20} + q^2 N_2^2 e^2} - qN_2 e. \quad (17)$$

By examination of the Fig. 1,

$$v_{20} = \Delta E_c - \delta_2 - E_{F0}. \quad (18)$$

Neglecting interface states we have, according to the Gauss law,

$$\epsilon_2 \mathcal{E}_{i_2} \# \epsilon_1 \mathcal{E}_{i_1} \# qn_s. \quad (19)$$

Then, from (5) (or (8)), (12), (17), and (19) we must solve

$$\begin{aligned} \sqrt{2q\epsilon_2 N_2 v_{20} + q^2 N_2^2 e^2} - qN_2 e &= qn_s \\ &= DkT \log [(1 + e^{q/kT(E_{F0} - E_0)})(1 + e^{q/kT(E_{F0} - E_1)})] \end{aligned} \quad (20)$$

where E_0 and E_1 given by (9) are functions of n_s . The solution is obtained numerically according to the following procedure.

We start from an arbitrary low value for E_{F0} (which can be negative); from (17), (18), and (19) n_s is deduced and E_0 , E_1 computed with (9). The deduced value for n_s must also satisfy (12) and, if it is not the case, E_{F0} is increased until (20) is verified.

The result in the p⁺-type case (using the γ_0 and γ_1 values given by (10)) $x = 0.3$ (Al content in AlGaAs) and various values of e is shown as a function of N_2 (AlGaAs doping) at $T = 300 \text{ K}$ in Fig. 2 (an arbitrary low value for the acceptor

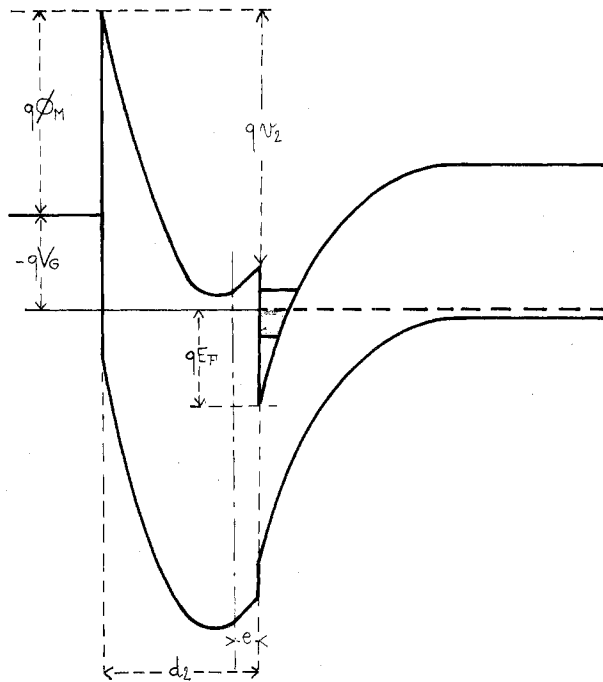


Fig. 3. $\text{Al}_x\text{Ga}_{1-x}\text{As}$ (n)-GaAs (p) heterojunction in the 2-DEG control regime by a Schottky gate.

density in GaAs has been chosen but the result is quite insensitive to this value). In this figure, is also reported the curve which corresponds to the case where the two-dimensional behavior is ignored (20). The temperature dependence of n_s is small and the frequently observed decrease of n_s at low temperatures must be attributed to carriers freeze-out in AlGaAs (reduction of the number of ionized impurities).

IV. CHARGE CONTROL BY A SCHOTTKY GATE IN CONTACT WITH THE LARGE GAP SEMICONDUCTOR

A. Charge Control Law

Fig. 3 shows the band diagram of the heterostructure submitted to the influence of a Schottky gate in contact with the large gap semiconductor. We suppose as effective the interpenetration of the two depletion layers, that is, we have assumed a sufficiently high reverse voltage on the Schottky gate or a sufficiently small width of the large gap semiconductor layer.

With the notations used in the Fig. 3, we establish in the Appendix II-B the relation

$$\epsilon_2 \mathcal{E}_{i_2} = \frac{\epsilon_2}{d_2} (V_{p_2} - v_2) \quad (21)$$

where

$$V_{p_2} = \frac{qN_2}{2\epsilon_2} (d_2 - e)^2. \quad (22)$$

But we have by examination of Fig. 3 the relation

$$v_2 = \phi_M - V_G + E_F - \Delta E_c \quad (23)$$

then

$$\epsilon_2 \mathcal{E}_{i_2} = \frac{\epsilon_2}{d_2} (V_{p_2} - \phi_M - E_F + \Delta E_c + V_G). \quad (24)$$

In the absence of interface states we have then, according to (19)

$$Q_s = qn_s = \frac{\epsilon_2}{d_2} (V_{p_2} - \phi_M - E_F + \Delta E_c + V_G). \quad (25)$$

But E_F , which is a function of V_G , is always very small as compared to the other terms and we obtain the approximate but important result,

$$Q_s \approx \frac{\epsilon_2}{d_2} (V_G - V_{\text{off}}) \quad (26)$$

where

$$V_{\text{off}} = \phi_M - \Delta E_c - V_{p_2} \quad (27)$$

is the "off voltage" which annihilates the 2-DEG. Having neglected E_F to obtain these results it is obvious that (26) and (27) are insensitive to the exact positions of the subbands hence they are valid for the two cases where the small gap semiconductor type is p^- or n^- .

If the interface states charge Q_i cannot be neglected, (26) remains valid but (27) becomes

$$V_{\text{off}} = \phi_M - \Delta E_c - V_{p_2} - \frac{d_2}{\epsilon_2} Q_i. \quad (28)$$

B. Threshold Voltage for Charge Control

For a given large gap semiconductor layer width, there exists a threshold voltage $V_{G\text{th}}$ separating the charge control regime from the equilibrium state. It can be obtained by equalizing the two expressions of $\epsilon_2 \mathcal{E}_{i_2}$ (17) and (24) for the equilibrium value E_{F0} of the Fermi-energy. The result can be put in the form

$$V_{G\text{th}} = \phi_M - \delta_2 - \left(\sqrt{\frac{qN_2 d_2^2}{2\epsilon_2}} - \sqrt{(\Delta E_c - \delta_2 - E_{F0}) + \frac{qN_2 e^2}{2\epsilon_2}} \right)^2 \quad (29)$$

where (18) has also been used.

V. FET APPLICATIONS

$V_c(x)$ being the channel voltage under the gate at abscissa x , and V_G the gate potential, the effective voltage for charge control at x is

$$V_{\text{eff}}(x) = V_G - V_c(x) \quad (30)$$

and (26) must be rewritten as

$$Q_s(x) \approx \frac{\epsilon_2}{d_2} (V_G - V_c(x) - V_{\text{off}}). \quad (31)$$

The channel current expression at x is

$$I = Q_s(x) Z v(x) \quad (32)$$

where Z is the gate width and $v(x)$ the electron velocity at x .

The scattering limitation of the electron velocity is not well known for such a two-dimensional system. In addition to phonon scattering and intervalley electron transfer, other scattering mechanisms can occur such as electron-electron scattering [21], intersubband scattering, and real space electron scatter-

ing [22], etc. The hierarchy of these effects is not clearly established at this date and we will only assume a limitation of the electron velocity occurring at some critical field \mathcal{E}_c .

Using the approximation

$$v = \mu \mathcal{E}, \quad \text{for } \mathcal{E} < \mathcal{E}_c \quad (33)$$

$$v = v_s, \quad \text{for } \mathcal{E} \geq \mathcal{E}_c \quad (34)$$

which is well known in the case of conventional FET [23], one can give an analytical treatment of the problem. This approximation is purely formal and does not involve any assumption on the dominant scattering mechanism.

At fields less than \mathcal{E}_c we have

$$I = \mu Z \frac{\epsilon_2}{d_2} (V_G - V_c(x) - V_{\text{off}}) \frac{dV_c}{dx} \quad (35)$$

where $dV_c/dx = \mathcal{E}$ is the electric field.

The current I being conservative and the channel voltage $V_c(x)$ increasing from source to drain, the electric field is maximum close to the drain and the velocity saturation will occur first at the drain side of the gate region. First we consider the case of very small drain voltage V_D (linear regions of the I - V_D characteristics). From (35) we have

$$I = \mu Z \frac{\epsilon_2}{d_2} (V_G - V_{\text{off}}) \frac{V_c(L) - V_c(0)}{L}. \quad (36)$$

But, R_S and R_D being, respectively, the source and drain access resistances

$$V_c(0) = R_S I \quad (37)$$

$$V_c(L) = V_D - R_D I. \quad (38)$$

Then from (36)-(38)

$$\frac{V_D}{I} = R_S + R_D + \frac{L d_2}{\mu Z \epsilon_2 (V_G - V_{\text{off}})} \quad (39)$$

and the FET acts as a pure controllable resistor. At drain voltages such that the drain side electric field is less than \mathcal{E}_c , the integration of (35) gives

$$V_c(x) = V_G - V_{\text{off}} - \sqrt{(V_G - V_{\text{off}} - V_c(0))^2 - \frac{2 d_2 I x}{\mu Z \epsilon_2}}. \quad (40)$$

The electric field at abscissa x : $\mathcal{E}(x) = dV_c/dx$ is then easily obtained. Expressing the condition

$$\mathcal{E}(L) = \mathcal{E}_c \quad (41)$$

one obtains the saturated current expression

$$I_S = \frac{Z \epsilon_2 v_s}{d_2} (\sqrt{\mathcal{E}_c^2 L^2 + (V_G - V_{\text{off}} - R_S I_S)^2} - \mathcal{E}_c L). \quad (42)$$

For a large gate FET, (42) reduces to

$$I_S = \frac{\mu Z \epsilon_2}{2 d_2 L} (V_G - V_{\text{off}} - R_S I_S)^2 \quad (43)$$

and for a short gate FET, the control is linear

$$I_S = \frac{Z \epsilon_2 v_s}{d_2} (V_G - V_{\text{off}} - R_S I_S - \mathcal{E}_c L) \quad (44)$$

an expression valid if $V_G - V_{\text{off}} - R_S I_S \gg \mathcal{E}_c L$. In that case the linear relationship between I_S and V_G has been experimentally verified (18), (24).

VI. CONCLUSION

Experimental results have shown that the heterojunction 2-DEG MESFET having the Schottky contact deposited on AlGaAs is a promising candidate as low noise FET's. They also have demonstrated that subband splitting occurs in the 2-DEG, and mobility is enhanced if an undoped AlGaAs spacer layer is grown between AlGaAs (n) and GaAs (p⁻).

Theoretical calculations developed in this paper, take these data into account. The sheet carrier concentration in the 2-DEG has been deduced and simple analytical formulas have been established for the charge control in large and small gate FET's.

APPENDIX I

ELECTRON SHEET CONCENTRATION IN THE 2-DEG AS A FUNCTION OF FERMI-LEVEL POSITION AND TEMPERATURE (TWO SUBBANDS ASSUMED)

For a two-dimensional density of states given by D between E_0 and E_1 and equal to $2D$ for energies greater than E_1 , we have, using, Fermi-Dirac, statistics

$$n_s = D \int_{E_0}^{E_1} \frac{dE}{1 + e^{\frac{q(E - E_F)}{kT}}} + 2D \int_{E_1}^{\infty} \frac{dE}{1 + e^{\frac{q(E - E_F)}{kT}}}.$$

Applying the easily verifiable formula

$$\int \frac{dx}{1 + e^x} = -\log(1 + e^{-x})$$

we obtain (12).

APPENDIX II

ELECTRIC DISPLACEMENT VECTOR AT THE INTERFACE IN THE LARGE GAP SEMICONDUCTOR

A. Equilibrium, Isolated Case

Using the depletion approximation, the voltage $V_2(x)$ in the space charge region of the semiconductor 2 obeys the Poisson equation

$$\frac{d^2 V_2}{dx^2} = -\frac{q}{\epsilon_2} N_2(x).$$

Taking the heterojunction interface as origin the conditions

$$\begin{aligned} V_2(0) &= 0 \\ \left(\frac{dV_2}{dx}\right)_{x=-w_2} &= 0 \\ \left(\frac{dV_2}{dx}\right)_{x=0} &= -\mathcal{E}_{i2} \end{aligned}$$

(w_2 : being the space charge layer width) must be fulfilled. One obtains successively

$$\mathcal{E}_{i_2} = -\frac{q}{\epsilon_2} \int_0^{-w_2} N_2(x) dx$$

$$V_2(-w_2) = v_{20} = \mathcal{E}_{i_2} w_2 - \frac{q}{\epsilon_2} \int_0^{-w_2} dx \int_0^x N_2(x') dx'.$$

For our case

$$N_2(x) = 0, \quad \text{for } -e < x < 0$$

$$N_2(x) = N_2, \quad \text{for } x < -e$$

then

$$\epsilon_2 \mathcal{E}_{i_2} = qN_2(w_2 - e)$$

$$v_{20} = \frac{qN_2}{2\epsilon_2} (w_2^2 - e^2)$$

and (17) results.

B. 2-DEG Charge Control Regime

In the charge control regime the region between the Schottky contact and the heterojunction interface is totally depleted and the electrostatic potential obeys the Poisson equation with

$$N_2(x) = 0, \quad \text{for } -e < x < 0$$

$$N_2(x) = N_2, \quad \text{for } -d_2 < x < -e.$$

The origin is also taken at the interface and we arbitrarily choose $V_2(0) = 0$. A double integration gives

$$V_2(-d_2) = -v_2 = \mathcal{E}_{i_2} d_2 - \frac{q}{\epsilon_2} \int_0^{-d_2} dx \int_0^x N_2(x') dx'$$

that is,

$$v_2 = \frac{qN_2}{2\epsilon_2} (d_2 - e)^2 - \mathcal{E}_{i_2} d_2$$

and (21) results.

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