An Analytical Expression for Fermi Level Versus Sheet Carrier Concentration for HEMT Modeling

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Abstract—**A simple second-order analytical expression of Fermi-level variation with two-dimensional electron gas density in a high electron mobility transistor (HEMT) has been developed. This empirical expression was found to give better results near cutoff and in saturation than the linear approximation currently being used in many models. It can be used in the development of a more accurate charge control model and hence in the development of an improved analytical model for the HEMT.**

I. Introduction and Discussion

ONE OF THE main limitations of many existing analytical high electron mobility transistor (HEMT) models is that they assume that the Fermi level in the quantum well is a linear function of the sheet carrier concentration n_s. This approximation is valid for $5 \times 10^{11}\ \text{cm}^{-2} < n_s < 1.5 \times 10^{12}\ \text{cm}^{-2}$, as shown in Fig. 1 [1]. In an actual device n_s can be much lower than $5 \times 10^{11}\ \text{cm}^{-2}$ near cutoff.

The first important analytical model of the HEMT was developed by Delagebeaudeuf and Linh in 1982 [2]. An important limiting feature of Delagebeaudeuf and Linh's model was that Fermi-level variation with electron density in the quantum well was neglected in order to get an analytical model. If it is accounted for, then a few equations related to the model must be solved numerically. This numerical requirement results because the E_f variation with n_s in the quantum well is quite complicated and is given by (1) and (2) [2]. The energies of the two lowest subbands are

$$E_0 = \gamma_0 n_s^{2/3}$$
$$E_1 = \gamma_0 n_s^{2/3} \tag{1}$$

where γ_0 and γ_1 are constants, typically obtained from experiments, and the electron density in the two-dimensional electron gas (2DEG) is

$$n_s = \frac{DkT}{q} \ln\left[(1 + e^{q/kT(E_f - E_0)})(1 + e^{q/kT(E_f - E_1)})\right]. \tag{2}$$

Neglecting the variation in electron sheet density (n_s) with the Fermi level (E_f) imposes a fundamental limitation on the accuracy of the Delagebeaudeuf and Linh model. As a first-order correction to this, Drummond *et al.* [1] proposed a linear approximation for the E_f versus n_s relation. Equation (2) is quadratic with respect to $\exp(qE_f/kT)$. To obtain a linear approximation, this equation was solved and plotted for temperatures of 300, 77, and 4 K as shown in Fig. 1. From this figure a linear relation for the E_f variation with n_s was obtained graphically. For n_s between 5×10^{11} and 1.5×10^{12} cm^{-2}, these dependencies can be approximated as

$$E_f = \Delta E_{f0}(T) + a n_s \tag{3}$$

where

$$a \approx 0.125 \times 10^{-16}\ \text{V} \cdot \text{m}^2$$

$$\Delta E_{f0} \approx 0\ \text{eV at 300 K}$$

$$\Delta E_{f0} \approx 0.025\ \text{eV at 77 K}.$$

This approximation, represented in Fig. 1 as a dotted line, has been used extensively and helped in the improvement of the charge control mode [3] and in the estimation of the equilibrium sheet carrier density n_{s0}.

The charge control equation obtained using the linear $E_f(n_s)$ dependence is given by

$$n_s = \frac{\epsilon}{qd}(V_g - \phi_b + V_{P2} - E_f + \Delta E_c) \tag{4}$$

where d is the thickness of the doped and undoped GaAs and AlGaAs layers, V_g is the applied gate voltage, ϕ_b is the Schottky-barrier height, V_{p2} is the AlGaAs pinchoff potential, and ΔE_c is the conduction-band discontinuity. This equation is quite general and shows that as the gate bias increases, the Fermi level and sheet carrier concentration both decrease. This behavior can be seen in Fig. 1. Referenced to the curve representing the numerical (exact) solution to (1) and (2) at 300 K, it is seen that a decrease in sheet carrier concentration from 10^{12} to 10^{10} cm^{-2} corresponds to a decrease in Fermi potential of approximately 0.25 V. For the same decrease in n_s, the linear approximation of (3) corresponds to an E_f drop of only 0.1 V, which shows its inaccuracy at small n_s values. As seen in the figure, this inaccuracy also exists for large values of n_s. Thus, models into which the linear approximation is incorporated will be inaccurate in the cutoff and deep

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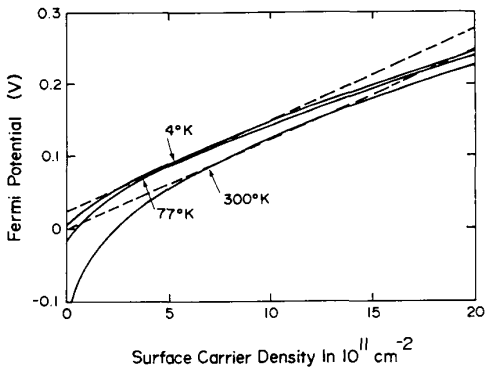

Fig. 1. E_f versus n_s relation and the linear approximation. Solid lines represent (1) and (2). Dashed lines represent linear approximation given by (3).

saturation regions. One of the main objectives of this work was to find a more accurate, yet simple approximation for this problem. Essentially exact solutions [6] require excessive computation and are thus not considered here for design applications such as SPICE.

II. Analytical Expression for $E_f(n_s)$

An empirical formula for $E_f(n_s)$ is required to account for the Fermi-level variation with n_s. This expression should be such that if substituted into the charge control equation given by (4), an analytical solution for n_s is possible. An exponential function may give a better representation of the function, but makes it impossible for an analytical solution for n_s to be obtained. Examining the charge control equation, one obvious solution is to represent the function $E_f = f(n_s)$ by a polynomial. Any polynomial of order higher than two, however, will not only complicate the charge control model, but will make it difficult to obtain an analytical model for current–voltage relations.

Equations (1) and (2) give the E_f versus n_s relation. Equation (2) is modified and rewritten as

$$e^{qn_s/DkT}e^{q/kT(E_0+E_1)} = e^{q/kT(E_0+E_1)} + e^{q/kT(E_f+E_1)} + e^{q/kT(E_f+E_0)} + e^{2q/kTE_f}. \quad (5)$$

If we say

$$R = e^{q/kTE_0} \quad (6a)$$

$$S = e^{q/kTE_1} \quad (6b)$$

$$Y = e^{q/kTE_f} \quad (6c)$$

then (5) can be written as

$$RSe^{qn_s/DkT} = RS + SY + RY + Y^2 \quad (7)$$

which is a quadratic in Y and can be solved as

$$Y = -\left(\frac{R+S}{2}\right) \pm \sqrt{\left(\frac{R+S}{2}\right)^2 - RS[1-e^{qn_s/DkT}]} \quad (8)$$

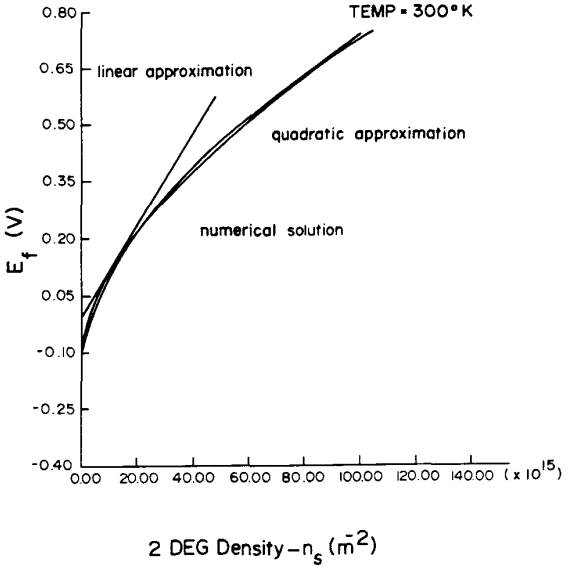

Fig. 2. A comparison of the numerical solution of (1) and (2), the linear model (3), and the second-order model (13). It can be seen that the latter model follows the numerical model more closely than the linear one.

and the Fermi level can be obtained from (7c) as

$$E_f = \frac{kT}{q} \ln (Y). \quad (9)$$

Now, E_0 and E_1 are functions of n_s as defined by (1). From (2), (6), (8), and (9) therefore, E_f can be obtained as a function of n_s. The Fermi level for different values of n_s will be calculated and used for obtaining an empirical formula.

A simple polynomial of the form

$$E_f = an_s^2 + bn_s + c \quad (10)$$

cannot be used for a least squares method to get an empirical formula, since the dependent variable is very much smaller than the independent variable, thus producing significant computational inaccuracies. Instead, a polynomial of the form

$$n_s = aE_f^2 + bE_f + c \quad (11)$$

was used and the coefficients a, b, c obtained by fitting the data obtained from the solution of (2), (6), (8), and (9). Data fitting was done by utilizing a least squares method (LSM) using the singular value decompositon (SVD) technique [4]. The quantity E_f obtained from (11) is

$$E_f = -\frac{b}{2a} + \frac{1}{\sqrt{a}}\left(n_s + \frac{b^2}{4a} - c\right)^{1/2}. \tag{12}$$

This can be written as

$$E_f = K_1 + K_2(n_s + K_3)^{1/2} \tag{13}$$

where

$$K_1 = -\frac{b}{2a} \tag{14a}$$

$$K_2 = \frac{1}{\sqrt{a}} \tag{14b}$$

$$K_3 = \frac{b^2}{4a} - c. \tag{14c}$$

At 300 K, $K_1 = -0.20829$, $K_2 = 0.3029 \times 10^{-8}$, and $K_3 = 0.9666 \times 10^{15}$. Equations (13) and (14) describe the final second-order polynomial expression to represent Fermi-level variation with electron sheet density in the quantum well, which is more accurate than the linear approximation given by (3). In Figs. 2 and 3 the linear model from (1) and (2), and this new model are compared. The improved agreement obtained with (13) over the entire range of n_s values is readily observed. This expression has been used [5] to calculate the I–V characteristics of HEMT's and is seen to fit the experimental data better in the cutoff region than other models.

III. Conclusion

In conclusion, we have presented a second-order expression for the Fermi-level variation with sheet electron concentration in the two-dimensional electron gas of a HEMT. The expression gives a better fit to the numerical solution than the conventional linear approximation. Even though both models are inaccurate in the cutoff region, our analytical expression gives values closer to the numerical model at greatly reduced computational complexity. Because of its reduced computational requirements, such an analytical model should prove useful in device design programs such as SPICE.

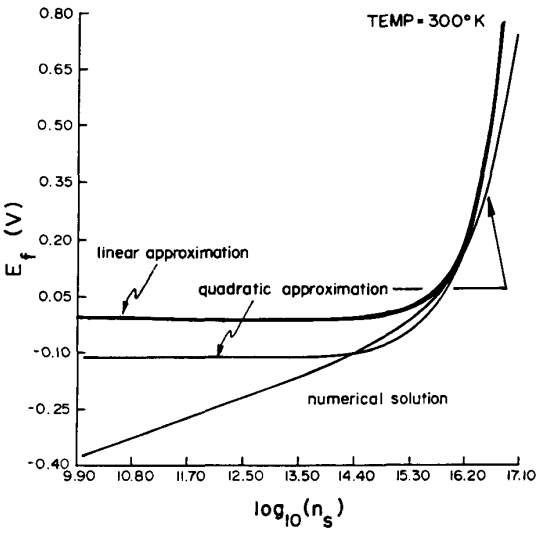

Fig. 3. The same data in Fig. 2 is plotted on a log scale. Here it can be seen that both models fail in deep saturation. The second-order model deviates less than the linear model from the numerical solution.

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