Using uWUE to remove dependence of ET on GPP

This document is primarily just to make sure I get my units right, as I'm new to all of this stuff.

Penman montieth is given by:

$$\lambda E = \frac{\Delta A + \frac{\rho_a c_p D_{ref}}{r_a}}{\Delta + \gamma(1 + \frac{r_s}{r_a})} \tag{1}$$

where:

$$r_s = \frac{1}{LAI \cdot g_{st}} \tag{2}$$

Using the Medlyn model (from Frank et. al. 2017) we have:

$$g_{st} = g_w = 1.6 \left(1 + \frac{g_{1M}}{\sqrt{D_s}}\right) \frac{A}{c_s} \tag{3}$$

Here, g_{1M} has units of kPa $^{0.5}$, A is the net CO $_2$ assimilation rate in μ mol m $^{-2}$ s $^{-1}$, c_s is the CO $_2$ concentration in ppm (supposed to be at leaf surface). The problem is the A term that we need to get rid of. Also g_w as above has units of mol (air?) m $^{-2}$ s $^{-1}$.

we can use:

$$\text{uWUE} = \frac{\text{GPP} \cdot \sqrt{\text{VPD}}}{ET} \tag{4}$$

This has units of $\frac{\text{g C hPA}^{0.5}}{\text{kg H}_2\text{O}}$.

So let's do these unit conversions, starting with g_w unit converstion. Note that the mean molecular mass of dry air is 28.9645 g/mol. So then we have:

$$g_{st\,-\,m/s} = g_w \frac{mol}{m^2 s^1} \frac{28.9645 gair}{molair} \frac{1kg}{1000g} \frac{m^3}{\rho\; kg} = \frac{m}{s} \tag{5}$$

so:

$$g_{st\,-\,m/s} = g_w \frac{R_{air}}{\rho_{air}} \tag{6}$$

where:

$$R_{air} = .0289645 \frac{\text{kg}}{\text{mol}} \tag{7}$$

Now we just need to convert uWUE from gC to μ mol C:

$$uWUE_{\mu\text{ mol C}} = uWUE \frac{\text{mol C}}{\text{12.011g C}} \frac{1.e6\;\mu\text{ mol}}{mol} \tag{8}$$

so:

$$uWUE = \frac{uWUE\; 1.e6}{12.011} \tag{9}$$