A simplified system for indoor airflow simulation

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Contents

Project context

The 4fastsim-ibat[1] project aims to reduce and control energy consumption in existing buildings while working on identifying potential energy-saving and testing their efficiency before any improvement work is done.
This project is a collaboration between two engineering companies: Cemosis[2] and Synapse-Concept[3].

[1] https://www.cemosis.fr/projects/4fastsim-ibat/

[2] https://www.cemosis.fr/

[3] https://www.synapse-concept.com

Presentation of the internship

The objective of this internship is to deepen the study of the 0-equation model which had been approached during the project of the second semester of the CSMI master course that Mariam Grigoryan, Anita Klein and myself did.

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- Study of the turbulence model from a numerical point of view.
- Study of stabilization.
- Understand the use of the fluid and heat-fluid toolboxes.

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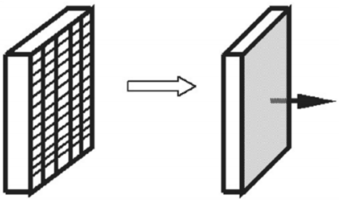
N-point ASOM

«N-point ASOM» combines the positive features of both direct ASOM and momentum method.



The essential of N-point ASOM is to replace the real diffuser by several simple openings so as to reduce thenumber of grids for numerical calculation, while maintaining the inlet momentum and mass flows.

The zero-equation turbulence model

To simulate room airflow quickly, Chen Qingyan and Xu Weiran[5] developed a zero-equation turbulence model by directly numerical simulation data. The model uses a single algebraic equation to express the turbulent viscosity

$$\boxed{\mu_t = 0.03874\rho U l},$$

where l is the distance to the nearest closure and U is the local mean velocity.

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Based on this equation, the Reynolds averaged Navier-Stokes equations are closed. The governing equations of mass, momentum and energy for indoor can be written as follows:

$$\begin{cases} \rho\dfrac{\partial u}{\partial t} + \rho(u\cdot\nabla)u - \nabla\cdot\sigma = \rho\beta(T_0 - T)g & (1.1) \\ \nabla\cdot u = 0 & (1.2) \\ \rho\dfrac{\partial T}{\partial t} + \rho u\cdot\nabla T - \nabla\cdot(\Gamma_{T,eff}\nabla T) = \dfrac{J}{C_p} & (1.3) \\ \mu_{eff} = \mu_l + \mu_t & (1.4) \end{cases}$$

[5] Chen Qingyan, Xu Weiran. *A zero-equation turbulence model for indoor air flow simulation*. Energy and Building 1998;28(2):137–44.

The zero-equation turbulence model

In their work Chen and Xu «Chen-Xu» have estimated the effective diffusive coefficient for temperature in Equation (1.3), $\Gamma_{T,eff}$, by:

$$\boxed{\Gamma_{T,eff} = \frac{\mu_{eff}}{\mathrm{Pr}_{eff}}}$$

where the effective Prandtl number[6], Pr_{eff}, is 0.9.

[6] https://en.wikipedia.org/wiki/Prandtl_number

Strong formulation[7]

We obtain a non-linear system of three coupled equations

$$
\begin{cases}
\rho \dfrac{\partial u}{\partial t} + \rho (u \cdot \nabla) u - \nabla \cdot \sigma = \rho \beta (T_0 - T) g & (2.1) \\
\nabla \cdot u = 0 & (2.2) \\
\dfrac{\partial T}{\partial t} + u \cdot \nabla T - \nabla \cdot (\kappa \nabla T) = \dfrac{J}{\rho C_p} & (2.3)
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This system is supplemented with boundary conditions and initial values for the different fields of interest.

[7] Wahl Jean-Baptiste. *The Reduced Basis Method Applied to Aerothermal Simulations*. Ph.D. thesis, Université de Strasbourg, 2018.

Variationnal formulation

In order to write the variational formulation, we now introduce definite function spaces $\forall t$.
We have the solution function spaces associated respectively with the speed and pressure:

$$V = H^1_{(u_D, \Gamma_D^F)}(\Omega)$$
$$Q = \mathcal{L}^2(\Omega)$$

as well as the space of the fluid velocity test functions:

$$W = H^1_{(0, \Gamma_D^F)}(\Omega)$$

We multiply equation (2.1) (resp. (2.2)) with $v \in W$ (resp. $q \in Q$).

We then integrate each of the equations on the domain Ω, and we perform an integration by part of the term containing the stress tensor σ:

$$\boxed{\begin{array}{c} \rho \int_\Omega \frac{\partial u}{\partial t} \cdot v + \rho \int_\Omega (u \cdot \nabla) u \cdot v + \int_\Omega \sigma : \nabla v - \int_\Gamma \sigma n \cdot v = \int_\Omega \rho \beta (T_0 - T) g \cdot v \\ \int_\Omega q \nabla \cdot u = 0 \end{array}}$$

Variationnal formulation

We now introduce the solution space assiociated with the temperature:

$$X = H^1_{(T_D, \Gamma_D^T)}(\Omega)$$

as well as the space of the temperature test functions:

$$Y = H^1_{(0, \Gamma_D^T)}(\Omega)$$

Then, we multiply equation (2.3) with a test function $r \in Y$. We then integrate each of the equations on the domain Ω, and we perform an integration by part of the term containing the stress tensor σ, which gives us:

$$\boxed{\int_\Omega \frac{\partial T}{\partial t} r + \int_\Omega (u \cdot \nabla T) r + \int_\Omega \kappa \nabla T \cdot \nabla r - \int_\Gamma \kappa (\nabla T \cdot n) r = \int_\Omega \frac{J}{\rho C_p} r}$$

Variationnal formulation

Finally, we must take into account the boundary conditions. Thus, we obtain the incompressible Navier-Stokes variational formulation for the problem we have considered:

Find $(u, p, T) \in V \times Q \times X$ such that $\forall (v, q, r) \in W \times Q \times Y$ we have:

$$\boxed{\begin{cases} \rho \int_\Omega \frac{\partial u}{\partial t} \cdot v + \rho \int_\Omega (u \cdot \nabla) u \cdot v + \int_\Omega \sigma : \nabla v = \int_\Omega \rho \beta (T_0 - T) g \cdot v \\ \int_\Omega q \nabla \cdot u = 0 \\ \int_\Omega \frac{\partial T}{\partial t} r + \int_\Omega (u \cdot \nabla T) r + \int_\Omega \kappa \nabla T \cdot \nabla r = \int_\Omega \frac{J}{\rho C_p} r + \int_{\Gamma_N^T} \phi_\Gamma r \end{cases}}$$

Spatial discretization[8]

We start by describing the approximation spaces that we will use to write the discrete variational formulation. Let:

$$
\begin{aligned}
V_\delta &= H^1_{(u_D,\Gamma^F_D)} \cap [P^M_c(\Omega_\delta)]^d \\
Q_\delta &= P^N_c(\Omega_\delta) \\
X_\delta &= H^1_{(T_D,\Gamma^T_D)} \cap [P^K_c(\Omega_\delta)]^d
\end{aligned}
$$

be the discrete spaces associated respectively with the velocity, the pressure and the temperature.
The polynomial degrees M and N cannot be chosen freely.

[8] Vincent Chabannes. *Vers la simulation des écoulements sanguins. Médecine humaine et pathologie*. Université de Grenoble, 2013. Français. NNT : 2013GRENM061. tel-00923731v2

Spatial discretization (Babuška–Brezzi condition)

The discrete problem is well posed if and only if the spaces V_δ and Q_δ are such that there exists a constant β_δ such that:

$$\inf_{q_\delta \in Q_\delta} \sup_{v_\delta \in V_\delta} \frac{\int_{\Omega_\delta} q_\delta \nabla \cdot v_\delta}{\|q_\delta\|_{\mathcal{L}^2(\Omega_\delta)} \|v_\delta\|_{H^1(\Omega_\delta)}} \geqslant \beta_\delta$$

If this constraint is not checked, the numerical solution can reveal instabilities. This is the case when $M = N$.

We choose to use one of the more common finite element choices, the Taylor-Hood elements[9]. The polynomial orders are then connected by $N = M - 1$.

[9] A. Ern and J.L. Guermond. *Theory and practice of finite elements*, volume 159. Springer Verlag, 2004.

Spatial discretization (Boundary conditions)

Since we have non-homogeneous Dirichlet conditions, we also need to introduce the discrete spaces of test functions associated respectively with the speed and temperature:

$$W_\delta = H^1_{(0,\Gamma^F_{D,\delta})}(\Omega_\delta) \cap [P^N_c(\Omega_\delta)]^d$$
$$Y_\delta = H^1_{(0,\Gamma^T_{D,\delta})}(\Omega_\delta) \cap [P^K_c(\Omega_\delta)]^d$$

The discrete variational formulation results in the following problem:

Find $(u_\delta, p_\delta, T_\delta) \in V_\delta \times Q_\delta \times X_\delta$ such that $\forall (v, q, r) \in W_\delta \times Q_\delta \times Y_\delta$

$$\boxed{\begin{cases} \rho \int_\Omega \frac{\partial u_\delta}{\partial t} \cdot v + \rho \int_\Omega (u_\delta \cdot \nabla) u_\delta \cdot v + \int_\Omega \sigma_\delta : \nabla v = \int_\Omega \rho\beta(T_0 - T_\delta) g \cdot v \\ \int_\Omega q \nabla \cdot u_\delta = 0 \\ \int_\Omega \frac{\partial T_\delta}{\partial t} r + \int_\Omega (u_\delta \cdot \nabla T_\delta) r + \int_\Omega \kappa \nabla T_\delta \cdot \nabla r = \int_\Omega \frac{J}{\rho C_p} r + \int_{\Gamma^T_N} \phi_\Gamma r \end{cases}}$$

Time discretization

We will now approach the derivatives in time using implicit schemes called backward differentiation formulation (BDF). These schemes can be written for an arbitrary order q and they are thus denoted by $\mathrm{BDF_q}$.

We will present $\mathrm{BDF_q}$ diagrams up to order 4 in the following.
Let Δt be the time step assumed to be constant over time, we have $t_0 = t_i$ and $t_n = t_0 + n\Delta t$.

q	β_{-1}	β_0	β_1	β_2	β_3
1	1	1			
2	3/2	2	-1/2		
3	11/6	3	-3/2	1/3	
4	25/12	4	-3	4/3	-1/4

Time discretization

This formula describes the approximation of the q-order time derivative of the speed using the β_j coefficients:

$$\frac{\partial u^{n+1}}{\partial t} \approx \frac{\beta_{-1}(q)}{\Delta t} u_\delta^{n+1} - \sum_{j=0}^{q-1} \frac{\beta_j(q)}{\Delta t} u_\delta^{n-j}$$

This expression is made up of two terms. The first contains the unknown u_δ^{n+1}. The second shows the solutions of the previous time steps.

We define:

$$f^{n+1} = \rho\beta(T_0 - T_\delta^{n+1})g + \rho \sum_{j=0}^{q^F-1} \frac{\beta_j(q^F)}{\Delta t} u_\delta^{n-j}$$

$$g^{n+1} = \frac{J}{\rho C_p} + \sum_{j=0}^{q^T-1} \frac{\beta_j(q^T)}{\Delta t} T_\delta^{n-j}$$

where q^F and q^T are diagram orders for speed and temperature, respectively.

Time discretization

We finish the temporal discretization by adding the initial conditions necessary to be able to correctly construct the temporal derivatives.

Finally, the discrete variational formulation of our standard problem is:

Find $(u_\delta, p_\delta, T_\delta) \in V_\delta \times Q_\delta \times X_\delta$ such that $\forall (v, q, r) \in W_\delta \times Q_\delta \times Y_\delta$

$$\boxed{\begin{cases} \rho \dfrac{\beta_{-1}(q^F)}{\Delta t} \displaystyle\int_\Omega \frac{\partial u_\delta^{n+1}}{\partial t} \cdot v + \rho \int_\Omega (u_\delta^{n+1} \cdot \nabla) u_\delta^{n+1} \cdot v \\ \qquad\qquad + \displaystyle\int_\Omega \sigma_\delta^{n+1} : \nabla v = \int_\Omega f^{n+1} \cdot v \\ \qquad\qquad \displaystyle\int_\Omega q \nabla \cdot u_\delta^{n+1} = 0 \\ \dfrac{\beta_{-1}(q^T)}{\Delta t} \displaystyle\int_\Omega \frac{\partial T_\delta^{n+1}}{\partial t} r + \int_\Omega (u_\delta^{n+1} \cdot \nabla T_\delta^{n+1}) r \\ \qquad\qquad + \displaystyle\int_\Omega \kappa \nabla T_\delta^{n+1} \cdot \nabla r = \int_\Omega g^{n+1} r + \int_{\Gamma_N^T} \phi_\Gamma r \end{cases}}$$

Extrapolation

In order to simplify and speed up the calculations, we will use an extrapolation of the velocity and separate the equations.

Let u^* be the extrapolation of u_δ^{n+1}, the problem becomes:

$$\text{Find } T_\delta \in X_\delta \text{ such that } \forall r \in Y_\delta$$

$$\frac{\beta_{-1}(q^T)}{\Delta t}\int_\Omega \frac{\partial T_\delta^{n+1}}{\partial t} r + \int_\Omega (u^* \cdot \nabla T_\delta^{n+1}) r + \int_\Omega \kappa \nabla T_\delta^{n+1} \cdot \nabla r = \int_\Omega g^{n+1} r + \int_{\Gamma_N^T} \phi_\Gamma r$$

Turbulence viscosity is

$$\mu_t = 0.03874 \rho U^* l,$$

where l is the distance to the nearest closure and U^* is the extrapolated local mean velocity.

Extrapolation

Then

$$\text{Find } (u_\delta, p_\delta) \in V_\delta \times Q_\delta \text{ such that } \forall (v, q) \in W_\delta \times Q_\delta$$

$$\begin{cases} \rho \dfrac{\beta_{-1}(q^F)}{\Delta t} \displaystyle\int_\Omega \dfrac{\partial u_\delta^{n+1}}{\partial t} \cdot v + \rho \int_\Omega (u^* \cdot \nabla) u_\delta^{n+1} \cdot v + \int_\Omega \sigma_\delta^{n+1} : \nabla v = \int_\Omega f^{n+1} \cdot v \\ \displaystyle\int_\Omega q \nabla \cdot u_\delta^{n+1} = 0 \end{cases}$$

Benchmarks

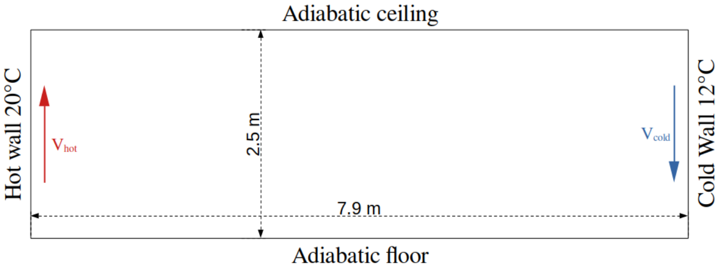
Gmsh

ParaView

FEEL++

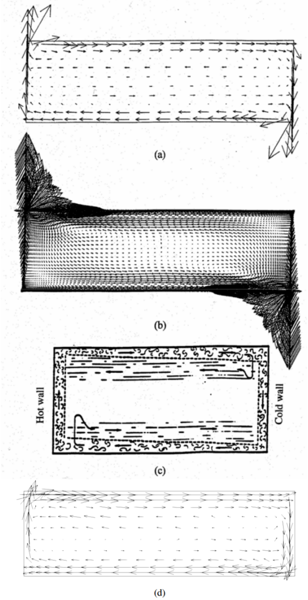
Natural convection

For natural convection, the experimental data of Olson and Glicksman [10] will be used.



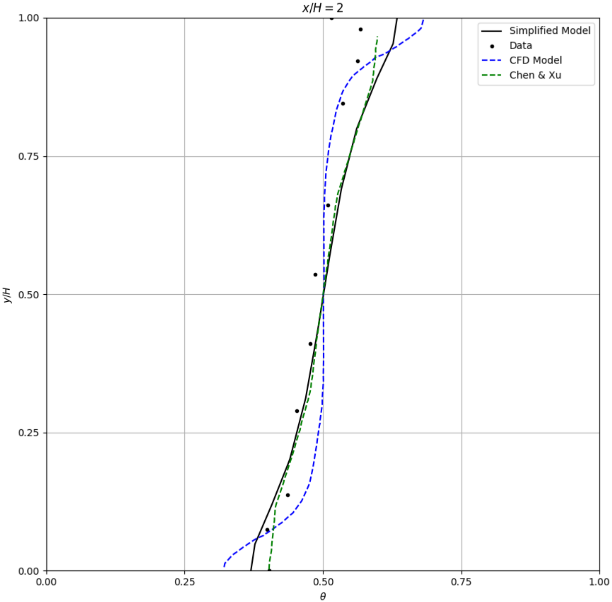
[10] Olson, D.A. and Glicksman, L.R. 1991. *Transient natural convection in enclosures at high Rayleigh number*, ASME J. Heat Transfer , 113, 635-642.

Natural convection



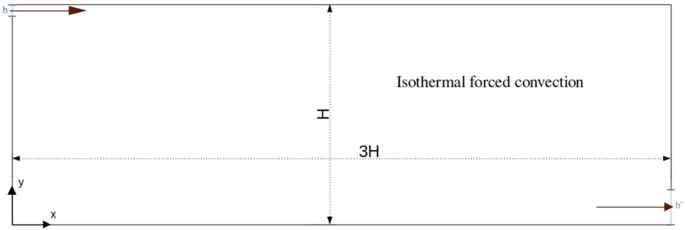
(a) Chen and Xu, (b) CFD, (c) smoke visualization, (d) Feel++.

Natural convection



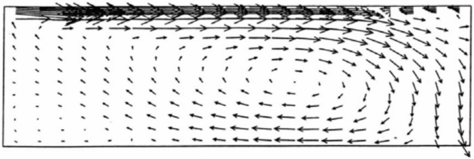
Forced advection

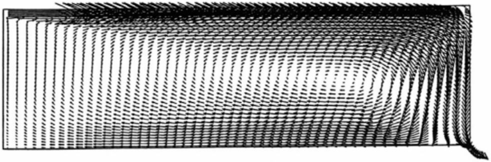
The forced convection case uses the experimental data from Restivo[11].
The Reynolds number is 5000 based on bulk supply velocity and the height of air supply outlet. The air supply outlet $h = 0.056 \times H$, and exhaust inlet $h' = 0.16 \times H$.

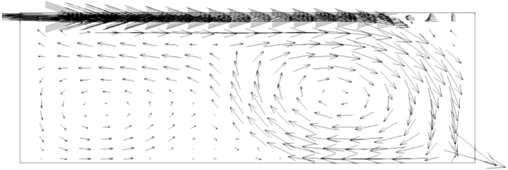


[11] Restivo, A. 1979. *Turbulent Flow in Ventilated Rooms* , Ph.D. Thesis, University of London, U.K.

Forced advection


(a)


(b)


(c)

(a) Chen and Xu, (b) CFD, (c) Feel++.

Forced advection

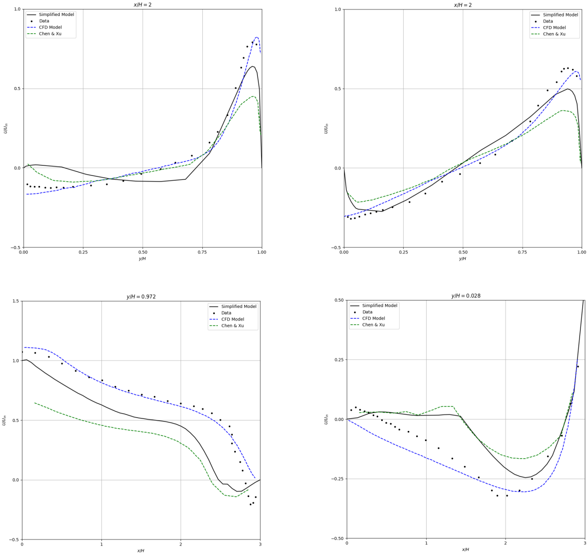
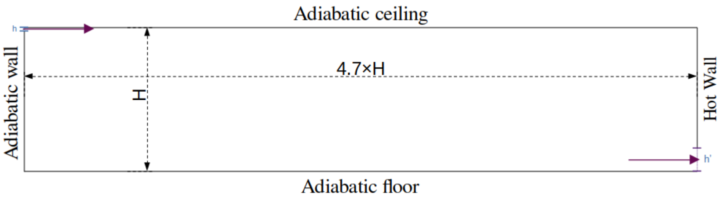


Figure: Comparison of velocity profiles in different sections of the room.

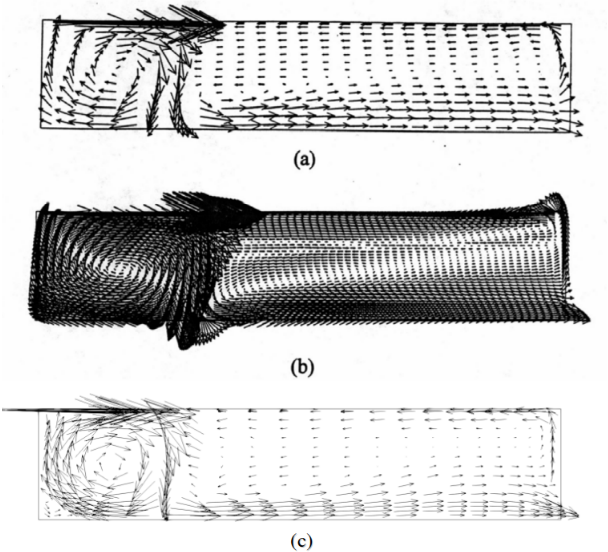
Mixed advection

The mixed convection case uses the experimental data from Schwenke [12]. The case is similar to the forced convection but the room length is $4.7 \times H$ and the height of the air supply outlet $h = 0.025 \times H$. The right wall is heated but the ceiling and floor are adiabatic.



[12] Schwenke, H. 1975. *Über das Verhalten elener horizontaler Zuluftstrahlen im begrenzten Raum*, Luft- und Kaltetechnik , 5, 241-246.

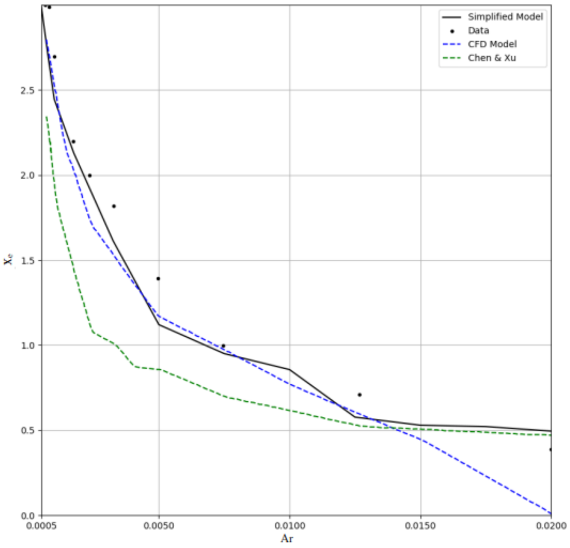
Mixed advection



(a) Chen and Xu, (b) CFD, (c) Feel++.

Mixed advection

Comparison of the penetration length versus Archimedes number for the room with mixed convection.



Conclusion

Chen and Xu demonstrated the capability of the simplified model by applying it to predict the airflow with natural convection, forced convection, mixed convection, and displacement ventilation in rooms. The predicted results are compared with experimental data and the results of CFD simulations. The simplified method can predict reasonably good indoor airflow patterns and the distributions of air temperature and contaminant concentrations.

Our results seem to show that the simplified model coupled with the use of Feel++ obtains better predictions than those of Chen and Xu.

I would like to conclude this report by thanking Cemosis for having me as in intern, and in particular Vincent Chabannes for our weekly meetings and his availability to answer my questions even during his vacation.