Multiphysics Modeling with Feelpp

Application to the modeling of LNCMI magnets

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Presentation of LNCMI

National Laboratory of Intense Magnetic Field is an user magnet research facility run by the CNRS.

The laboratory provides magnet time for research :

- RMN
- Supraconductivity

Two sites:

- Grenoble : continue magnetic field
- Toulouse : pulsed magnetic field

View of LNCMI

LNCMI ElectroMagnets

LNCMI-Grenoble manufactures three types of magnets:

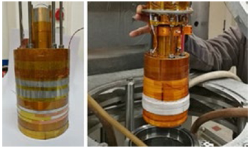
- Resistive magnets:

37 T Magnet

- Hybrid magnet of 42 T in assembly

Superconductor part of Hybrid Magnet

- Superconductor magnets:



HTS mock-up : 32 T in background field of 20 T

Collaboration with CEMOSIS

LNCMI need to simulate accurately the magnet before manufacturing.

LNCMI uses the software Feelpp developed by CEMOSIS (Center of Modelization and Simulation of Strasbourg).

Actual Models :

- HifiMagnet
- 1D

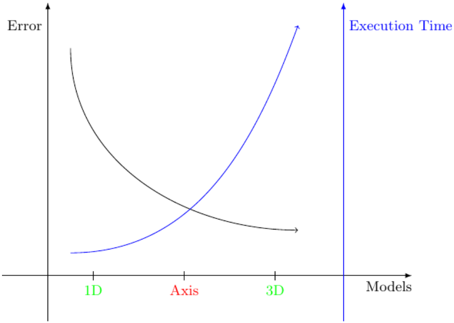
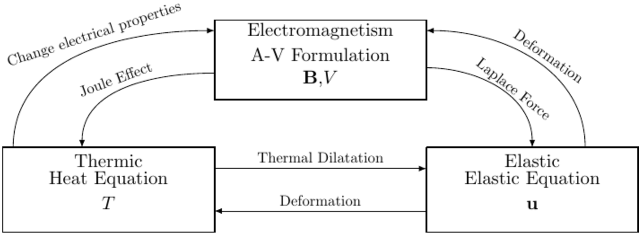


Diagram of models

Goals

The initial goal was to model the high temperature superconductor magnets but before we need implement Multiphysic model on CFPDEs :

- Electromagnetism : static, transient
- Thermic : static, transient
- Elastic : static, transient
- Coupling problem : static, transient



Coupling diagram

Goals

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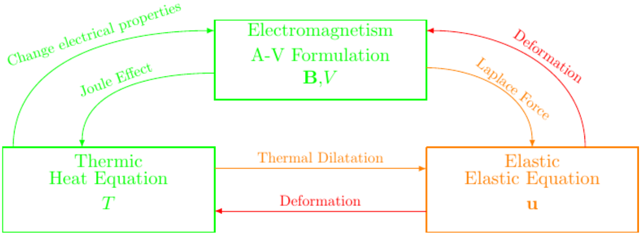

Coupling diagram

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What is CFPDEs ?

CFPDEs solves the equations of the type :

$$d\frac{\partial u}{\partial t} + \nabla \cdot (-c\nabla u - \alpha u + \gamma) + \beta \cdot \nabla u + au = f$$

with :

- $u : \mathbb{R}^m \to \mathbb{R}$ scalar unknown
- $d \in \mathbb{R}$ damping or mass coefficient
- $c \in \mathbb{R}$ or $\mathbb{R}^{m\times m}$ diffusion coefficient
- $\alpha \in \mathbb{R}$ conservative flux convection coefficient
- $\gamma \in \mathbb{R}^m$ conservative flux source term
- $\beta \in \mathbb{R}^m$ convection coefficient
- $a \in \mathbb{R}$ absorption or reaction coefficient
- $f \in \mathbb{R}$ source term

Example of Heat Equation :

$$\begin{cases} \rho C_p \frac{\partial T}{\partial t} - k\Delta T = Q \text{ on } \Omega_c \\ \frac{\partial T}{\partial n} = 0 \text{ on } \Gamma_{Nc} \\ k\frac{\partial T}{\partial n} = h\,(T - T_c) \text{ on } \Gamma_{Rc} \end{cases}$$

with :

- T the temperature
- k thermal conductivity
- ρ the density
- C_p the thermal capacity
- T_c the temperature of cooling
- h convective coefficient
- Q the source term from Joules effect

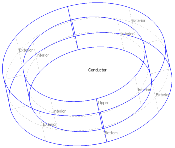
We identify the coefficients :

- $c = k$
- $f = Q$

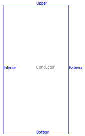
Axisymmetry

An **Axisymmetrical Geometry** :

- Symmetric by the axis Oz
- described on O_{rz} plan
- described by two coordinates : (r, z)



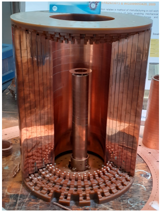
Torus in cartesian representation



Torus in axisymmetric representation

Approximation of solenoidal magnets in axisymmetric :

- save time of execution
- loose precision



Cut of helices insert

Adaptation to Axisymmetrical : Scalar Case I

- Write the Heat Equation in Axisymmetry :

$$\text{(Heat Axis)} \begin{cases} -k\Delta T(r,z) = Q \text{ on } \Omega_c^{axis} \\ \frac{\partial T}{\partial n}(r,z) = 0 \text{ on } \Gamma_{Nc}^{axis} \\ k\,\frac{\partial T}{\partial n}(r,z) = h\,(T(r,z) - T_c) \text{ on } \Gamma_{Rc}^{axis} \end{cases} \text{ with } \Delta T = \frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2}$$

- Jacobian of cylindric coordinates $\phi : (x, y, z) \to (r, \theta, z)$:

$$|Jac(\phi)| = r$$

- For boundary condition :

$$d\Gamma = \sqrt{\left(\frac{\partial z}{\partial \tau}\right)^2 + \left(\frac{\partial r}{\partial \tau}\right)^2}\, r(\tau)\, d\tau$$

with $\tau \to \begin{pmatrix} r(\tau) \\ z(\tau) \end{pmatrix}$ a parametrization
For our geometry :

$$\tau = r,\ \frac{\partial z}{\partial r} = 0 \Longrightarrow d\Gamma = rdr$$

- Write the weak formulation in axisymmetric :

$$\int_{\Omega_c^{axis}} k\,\tilde{\nabla} T \cdot \tilde{\nabla}\phi\, r\, drdz + \int_{\Gamma_R^{axis}} h\,T\,\phi\, r\, d\Gamma = \int_{\Omega_c^{axis}} Q\,\phi\, r\, drdz + \int_{\Gamma_R^{axis}} h\,T_c\,\phi\, r\, d\Gamma \text{ with } : \tilde{\nabla} = \begin{pmatrix} \partial_r \\ \partial_z \end{pmatrix}$$

- Identify the coefficients :
  - $c = r\,k$
  - $f = r\,\sigma\left(\frac{U}{2\pi\, r}\right)^2$

Adaptation to Axisymmetrical : Scalar Case II

Coefficients on JSON File for Heat problem :

```
"Materials":
{
    "Conductor":
    {
        "k":380,        // W/m/K
        "sigma":58e+6,  // S.m-1
        "rho":10000,    // kg/m3
        "Cp":380,       // J/K/kg

        "heat_c":"k:k",
        "heat_f":"sigma*(U/2/pi)*(U/2/pi)/(x*x+y*y):sigma:U:x:y
",
        "heat_d":"rho*Cp:rho:Cp"
    }
},
```

3D Cartesian

$\Longrightarrow$

```
"Materials":
{
    "Conductor":
    {
        "k":380,        // W/m/K
        "sigma":58e+6,  // S.m-1
        "rho":10000,    // kg/m3
        "Cp":380,       // J/K/kg
        "heat_c":"k*x:k:x",
        "heat_f":"sigma*(U/2/pi)/x*(U/2/pi)/x*x:sigma:U:x",
        "heat_d":"rho*Cp*x:rho:Cp:x"
    }
},
```

Axi

Adapatation to Axisymmetrical : Scalar Case III

Boundary Condition on JSON File for Heat problem :

```
"BoundaryConditions":
{
    "heat":
    {
        "Robin":
        {
            "Interior":
            {
                "expr1":"h:h",
                "expr2":"h*T_c:h:T_c"
            },
            "Exterior":
            {
                "expr1":"h:h",
                "expr2":"h*T_c:h:T_c"
            }
        }
    }
},
```

3D Cartesian

$\Longrightarrow$

```
"BoundaryConditions":
{
    "heat":
    {
        "Robin":
        {
            "Interior":
            {
                "expr1":"h*x:h:x",
                "expr2":"h*T_c*x:h:T_c:x"
            },
            "Exterior":
            {
                "expr1":"h*x:h:x",
                "expr2":"h*T_c*x:h:T_c:x"
            }
        }
    }
},
```

Axi

Adaptation to Axisymmetrical : Vectorial Case I

CFPDEs also solves the equations of the vectorial type :

$$d\frac{\partial \mathrm{u}}{\partial t} + \nabla \cdot (-c\nabla \mathrm{u} + \gamma) + \beta \cdot \nabla \mathrm{u} + a\mathrm{u} = \mathrm{f}$$

Example of Elastic Equation :

- on Torus geometry
- Laplace Forces $\mathrm{F}_{\mathrm{laplace}} = \mathrm{J} \times \mathrm{B}$
- Thermal Dilatation

$$\begin{cases} -\nabla \cdot \bar{\bar{\sigma}} = \mathrm{F}_{\mathrm{laplace}} & \text{on } \Omega_c & \text{(Static Elas-1)} \\ \mathrm{u} = 0 & \text{on } \Gamma_{Delas} & \text{(D elas)} \\ \bar{\bar{\sigma}} \cdot \mathrm{n} = 0 & \text{on } \Gamma_{Nelas} & \text{(N elas)} \end{cases}$$

with :

- displacement : $\mathrm{u} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$
- stress tensor : $\bar{\bar{\sigma}} = (\lambda \nabla \cdot \mathrm{u} - \sigma_T)\, Id + \mu \left(\nabla \mathrm{u} + \nabla \mathrm{u}^T \right)$, with λ, μ the Lame's coefficients, $\sigma_T = (3\lambda + 2\mu)\alpha_T (T - T_0)$ stress of thermal dilatation

and notations :

Divergence of tensor :

$$\nabla \cdot \bar{\bar{\sigma}} = \begin{pmatrix} \frac{\partial \bar{\bar{\sigma}}_{xx}}{\partial x} + \frac{\partial \bar{\bar{\sigma}}_{xy}}{\partial y} + \frac{\partial \bar{\bar{\sigma}}_{xz}}{\partial z} \\ \frac{\partial \bar{\bar{\sigma}}_{yx}}{\partial x} + \frac{\partial \bar{\bar{\sigma}}_{yy}}{\partial y} + \frac{\partial \bar{\bar{\sigma}}_{yz}}{\partial z} \\ \frac{\partial \bar{\bar{\sigma}}_{zx}}{\partial x} + \frac{\partial \bar{\bar{\sigma}}_{zy}}{\partial y} + \frac{\partial \bar{\bar{\sigma}}_{zz}}{\partial z} \end{pmatrix}$$

Scalar product of tensor :

$$\bar{\bar{\sigma}} \cdot \mathrm{n} = \begin{pmatrix} \bar{\bar{\sigma}}_{xx}\, n_x + \bar{\bar{\sigma}}_{xy}\, n_y + \bar{\bar{\sigma}}_{xz}\, n_z \\ \bar{\bar{\sigma}}_{yx}\, n_x + \bar{\bar{\sigma}}_{yy}\, n_y + \bar{\bar{\sigma}}_{yz}\, n_z \\ \bar{\bar{\sigma}}_{zx}\, n_x + \bar{\bar{\sigma}}_{zy}\, n_y + \bar{\bar{\sigma}}_{zz}\, n_z \end{pmatrix}$$

Adaptation to Axisymmetrical : Vectorial Case II

- Write Elastic Equation in Axisymmetric :

$$-\begin{pmatrix} \frac{\partial \bar{\bar{\sigma}}_{rr}}{\partial r} + \frac{\partial \bar{\bar{\sigma}}_{rz}}{\partial z} + \frac{\bar{\bar{\sigma}}_{rr} - \bar{\bar{\sigma}}_{\theta\theta}}{r} \\ \frac{\partial \bar{\bar{\sigma}}_{rz}}{\partial r} + \frac{\partial \bar{\bar{\sigma}}_{zz}}{\partial z} + \frac{\bar{\bar{\sigma}}_{rz}}{r} \end{pmatrix} \quad = \quad F_{\text{laplace}}$$

$$\tilde{\nabla} \cdot \left(-\mu \tilde{\nabla} u + \begin{pmatrix} -\lambda \left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} \right) - \mu \frac{\partial u_r}{\partial r} - \sigma_T & -\mu \frac{\partial u_z}{\partial r} \\ -\mu \frac{\partial u_r}{\partial z} & -\lambda \left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} \right) - \mu \frac{\partial u_z}{\partial z} - \sigma_T \end{pmatrix} \right) - \begin{pmatrix} \frac{2\mu}{r} \left(\frac{\partial u_r}{\partial r} - \frac{u_r}{r} \right) \\ \frac{\mu}{r} \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right) \end{pmatrix} = F_{\text{laplace}}, \ \text{with } \tilde{\nabla} = \begin{pmatrix} \partial_r \\ \partial_z \end{pmatrix}$$

- Write the equation in form :

$$\tilde{\nabla} \cdot \left(-\hat{c} \tilde{\nabla} u + \hat{\gamma} \right) = \hat{f}$$

with :

- $\hat{c} = \mu$
- $\hat{\gamma} = \begin{pmatrix} -\lambda \left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} \right) - \mu \frac{\partial u_r}{\partial r} - \sigma_T & -\mu \frac{\partial u_z}{\partial r} \\ -\mu \frac{\partial u_r}{\partial z} & -\lambda \left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} \right) - \mu \frac{\partial u_z}{\partial z} - \sigma_T \end{pmatrix}$
- $\hat{f} = \begin{pmatrix} F_{laplace\ r} + \frac{2\mu}{r} \left(\frac{\partial u_r}{\partial r} - \frac{u_r}{r} \right) \\ F_{laplace\ z} + \frac{\mu}{r} \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right) \end{pmatrix}$

Adaptation to Axisymmetrical : Vectorial Case III

- Do weak formulation :

$$\begin{aligned}
\int_{\Omega_c^{axis}} \left(\tilde{\nabla}\cdot\left(-\hat{c}\tilde{\nabla}u+\hat{\gamma}\right)\right)\cdot\xi\, rdrdz &= \int_{\Omega_c^{axis}} \hat{f}\cdot\xi\, rdrdz \\
\int_{\Omega_c^{axis}} \left(\hat{c}\tilde{\nabla}u-\hat{\gamma}\right)\cdot\tilde{\nabla}(r\xi)\, drdz &= \int_{\Omega_c^{axis}} \hat{f}\cdot\xi\, rdrdz \\
\int_{\Omega_c^{axis}} \left(\left(r\hat{c}\tilde{\nabla}u-r\hat{\gamma}\right)\cdot\tilde{\nabla}\xi+\begin{pmatrix}\hat{c}\frac{\partial u_r}{\partial r}-\hat{\gamma}_{00}\\ \hat{c}\frac{\partial u_z}{\partial r}-\hat{\gamma}_{10}\end{pmatrix}\cdot\xi\right) drdz &= \int_{\Omega_c^{axis}} \hat{f}\cdot\xi\, rdrdz
\end{aligned}$$

- Identify the coefficients :

$$\begin{aligned}
c &= r\hat{c} = r\mu \\
\gamma &= r\hat{\gamma} \\
&= \begin{pmatrix} -r\lambda\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}\right)-r\mu\frac{\partial u_r}{\partial r}-r\sigma_T & -r\mu\frac{u_z}{\partial z} \\ -r\mu\frac{\partial u_r}{\partial z} & -r\lambda\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}\right)-r\mu\frac{\partial u_z}{\partial z}-r\sigma_T \end{pmatrix} \\
f &= \hat{f}+\begin{pmatrix}-\hat{c}\frac{\partial u_r}{\partial r}+\hat{\gamma}_{00}\\ -\hat{c}\frac{\partial u_z}{\partial r}+\hat{\gamma}_{10}\end{pmatrix} \\
&= \begin{pmatrix} rF_{laplace\ r}-\lambda\frac{\partial u_r}{\partial r}-(\lambda+2\mu)\frac{u_r}{r}-\lambda\frac{\partial u_z}{\partial z}-\sigma_T \\ rF_{laplace\ z}\end{pmatrix}
\end{aligned}$$

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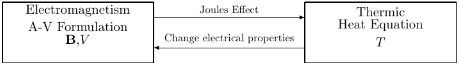
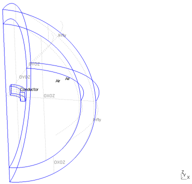
Thermo-magnetism coupled problem

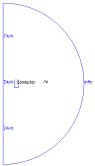


Coupling diagram

	Cartesian	Axisymmetric
Electromagnetism Magnetic Potential Field A Magnetic Potential Scalar V $\Phi = rA_\theta$	$\sigma \frac{d\mathbf{A}}{dt} + \frac{1}{\mu}\Delta \mathbf{A} = -\sigma \nabla V$ on Ω $\nabla \cdot (\sigma \nabla V) = -\nabla \cdot \left(\sigma \frac{\partial \mathrm{A}}{\partial t}\right)$ on Ω_c $\mathrm{A} = 0$ on Γ_D $\frac{\partial \mathrm{A}}{\partial \mathrm{n}} = 0$ on Γ_N $V = V_0$ on Γ_{DV} $\frac{\partial V}{\partial \mathrm{n}} = 0$ on Γ_{NV}	$\sigma \frac{\partial \Phi}{\partial t} - \frac{1}{\mu}\Delta \Phi + \frac{2}{\mu\, r}\frac{\partial \Phi}{\partial r} + \sigma \frac{\partial V}{\partial \theta} = 0$ on Ω $V = \frac{U}{2\pi}\theta$ on Ω_c^{axis} $A_\theta = 0$ on Γ_D^{axis} $\frac{\partial A_\theta}{\partial \mathrm{n}^{axis}} = 0$ on Γ_N^{axis}
	σ electrical conductivity	
Heat equation Temperature T	$\rho C_p \frac{\partial T}{\partial t} - k\Delta T = \sigma \lVert \nabla V + \frac{\partial \mathrm{A}}{\partial t} \rVert^2$ on Ω_c $\frac{\partial T}{\partial \mathrm{n}} = 0$ $-k\frac{\partial T}{\partial \mathrm{n}} = h\,(T - T_c)$ on Γ_{Rc}	$\rho C_p \frac{\partial T}{\partial t} - k\Delta T = \sigma \left(\frac{1}{r}\frac{\partial V}{\partial \theta} + \frac{\partial A_\theta}{\partial t}\right)^2$ on Ω_c^{axis} $\frac{\partial T}{\partial \mathrm{n}^{axis}} = 0$ on Γ_{Nc}^{axis} $-k\frac{\partial T}{\partial \mathrm{n}^{axis}} = h\,(T - T_c)$ on Γ_{Rc}^{axis}
	k thermal conductivity, C_p thermal conductivity, ρ density, T_c temperature of cooling	



Geometry in Cartesian



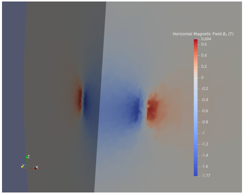
Geometry in Axisymmetric

Thermo-magnetism coupled problem

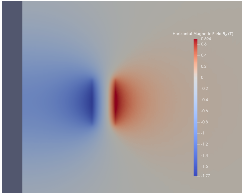
Main results

I export at stationnary state :

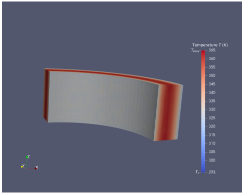
- Magnetic Field B
- Temperature T



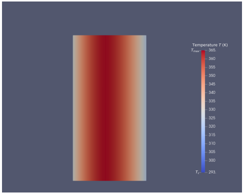
Horizontal magnetic Field $B_z(T)$ in Cartesian



Horizontal magnetic Field $B_z(T)$ in Axisymmetric



Temperature $T(K)$ in Cartesian



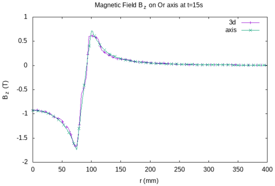
Temperature $T(K)$ in Axisymmetric

Thermo-magnetism coupled problem

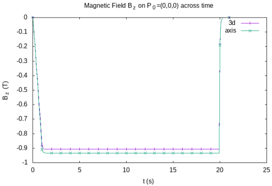
Main results

I export at transient state :

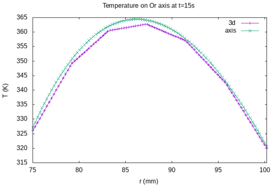
- Magnetic Field B
- Temperature T

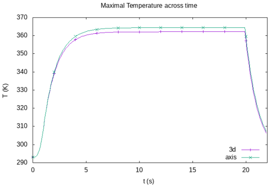


Horizontal magnetic Field $B_z(T)$ on Or axis



Horizontal magnetic Field $B_z(T)$ on $P_0 = (0, 0, 0)$ in Cartesian





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Conclusions

I implement a multiphysic model with CFPDES :

- Adapt CFPDEs in axisymmetrical geometry
- Static case : elastic, thermic and electromagnetism
- Transient case : thermic and electromagnetism physics

In progress :

- Automatic generation of cfg, json file with CFPDEs

To go further

To continue the model :

- Implement the elastic transient equation
- Add elastic physics to transient multiphysic model
- Develop models of real cases
- Implement a model of superconductor magnet

Opinions :

PRO	CONS
Many equation solved Coupled equations Reduce time of developement	Exports not adaptative More equation (order 2 in time) Error Messages Complicate changement of coordinates

Difficulties :

- Manage the version
- organize my work
- french/english

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